365 DataScience

Customer Analytics in Python

Price Elasticity Formula Derivation

Price Elasticity of Purchase Probability

Y: Outcome (purchase probability)
P: Price
E: Elasticity

$$Y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}$$

Y (purchase probability) is determined by a logistic regression

Logistic regression coefficient of Price

Elasticity $\leftarrow$ $$E = (1 - Y) * \beta_1 * P$$ $\rightarrow$ Price

Purchase probability

Price Elasticity of Purchase Probability

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$$Y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}$$

Y (purchase probability) is determined by a logistic regression

$$E = \frac{dY}{Y} \Big/ \frac{dP}{P} =$$ -> We start from the general price elasticity formula

$$= \frac{dY}{dP} * \frac{P}{Y} =$$ -> We simply rearrange the terms of the equation

$$= \frac{dY}{d(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)} * \frac{d(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}{dP} * \frac{P}{Y} =$$ -> We apply the chain rule to determine the partial derivative of Y w.r.t. P, where the green expression is the interim function

$$= \frac{e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}{[1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}]^2} * \beta_1 * \frac{P}{Y} =$$ -> We find the partial derivative of Y, w.r.t. the green expression
-> We find the partial derivative of the green expression, w.r.t. P (β_1)

$$= Y * (1 - Y) * \beta_1 * \frac{P}{Y} =$$ -> We replace the value of Y from the logistic regression expression

$$= (1 - Y) * \beta_1 * P$$ -> We reach the final formula for the price elasticity of purchase probability

Brand Choice Cross Price Elasticity

E_{cross} : Cross-price elasticity
Y_i : Purchase probability of our own brand
Y_j : Purchase probability of our competitor brand
P_j : Price of competitor brand

$$Y_i = \frac{e^{(\beta_{0i}+\beta_{1i}Price_i+\beta_{2i}Promotion_i+\cdots)}}{\sum_{k=1}^{I} e^{(\beta_{0i}+\beta_{1i}Price_k+\beta_{2i}Promotion_k+\cdots)}}$$

Y (purchase probability of a given brand i) is determined by a softmax function (multinomial logistic regression)

Logistic regression coefficient of the price of **our** product

Purchase probability for competitor brand product

Cross-price Elasticity

$$E_{cross} = -\beta_{1i} * Pj * Y_j$$

Price of the competitor brand product

Brand Choice Cross Price Elasticity

E_{cross}: Cross-price elasticity
Y_i: Purchase probability of our own brand
Y_j: Purchase probability of competitor brand
P_j: Price of competitor brand

$$Y_i = \frac{e^{(\beta_{0i}+\beta_{1i}Price_i+\beta_{2i}Promotion_i+\cdots)}}{\sum_{k=1}^{I} e^{(\beta_{0i}+\beta_{1i}Price_k+\beta_{2i}Promotion_k+\cdots)}}$$

Y (purchase probability of a given brand i) is determined by a softmax function (multinomial logistic regression)

For a softmax function:

$$\frac{\partial Y_i}{\partial x_j} = Y_i(\delta_{ij} - Y_j)$$

where δ_{ij} is 1 if i=j, 0 otherwise*

$$E_{cross} = \frac{dYi}{Yi} \Big/ \frac{dP_j}{P_j} =$$ -> We start from the general price elasticity formula

$$= \frac{dY_i}{dP_j} * \frac{P_j}{Y_i} =$$ -> We simply rearrange the terms of the equation

$$= \frac{dY_i}{d(\beta_0+\beta_{1i}Price_j+\cdots)} * \frac{d(\beta_{0j}+\beta_{1ij}Price_j+\cdots)}{dP_j} * \frac{P_j}{Y_i} =$$ -> We apply the chain rule to determine the partial derivative of Y_i w.r.t. P_j

$$= Y_i * (-Yj) * \beta_{1i} * \frac{P_j}{Y_i} =$$ -> We find the partial derivative of Y_i, w.r.t. the green expression (from the softmax derivative)
-> We find the partial derivative of the green expression, w.r.t. P (β_{1i})

$$= -Yj * \beta_{1i} * P_j$$ -> We reach the final formula for the price elasticity of purchase probability

*This is the general case. The price elasticity of purchase probability (own brand) was a particular case.
If you are interested, you can practice by using this proof to derive the purchase probability elasticity