# 365 V DataScience

## Customer Analytics in Python

**Price Elasticity Formula Derivation** 

#### Price Elasticity of Purchase Probability

Y: Outcome (purchase probability)

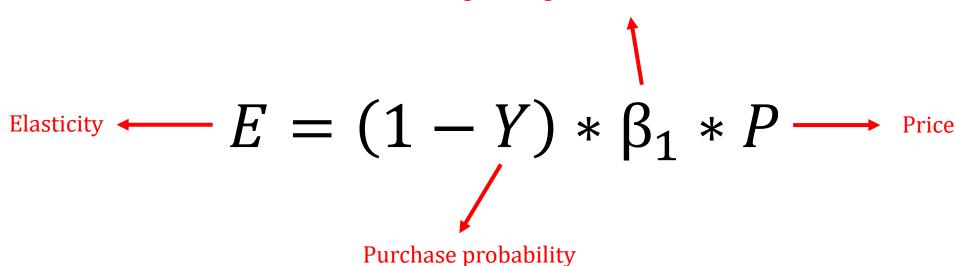
P: Price

E: Elasticity

$$Y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}$$

Y (purchase probability) is determined by a logistic regression

Logistic regression coefficient of Price



### Price Elasticity of Purchase Probability

Y: Outcome (purchase probability)

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$$Y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}$$

Y (purchase probability) is determined by a logistic regression

$$E = \frac{\frac{dY}{Y}}{\frac{dP}{P}} = ->$$
 We start from the general price elasticity formula

$$=\frac{dY}{dP}*\frac{P}{V}=$$
 -> We simply rearrange the terms of the equation

$$=\frac{dY}{d(\beta_0+\beta_1Price+\beta_2Promotion+\cdots)}*\frac{d(\beta_0+\beta_1Price+\beta_2Promotion+\cdots)}{dP}*\frac{P}{Y}=-> \text{ We apply the chain rule to determine the partial derivative of Y w.r.t. P, where the green expression is the interim function}$$

$$= \frac{e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}}{[1 + e^{-(\beta_0 + \beta_1 Price + \beta_2 Promotion + \cdots)}]^2} * \beta_1 * \frac{P}{Y} = \xrightarrow{-> \text{We find the partial derivative of Y, w.r.t. the green expression}} -> \text{We find the partial derivative of the green expression, w.r.t. P } (\beta_1)$$

$$=Y*(1-Y)*\beta_1*\frac{P}{Y}=->$$
 We replace the value of Y from the logistic regression expression

$$= (1 - Y) * \beta_1 * P$$
 -> We reach the final formula for the price elasticity of purchase probability



#### **Brand Choice Cross Price Elasticity**

 $E_{cross}$ : Cross-price elasticity

Y<sub>i</sub>: Purchase probability of our own brand

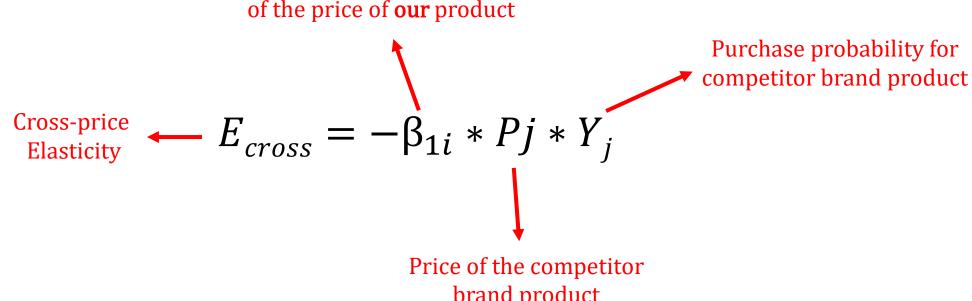
Y<sub>i</sub>: Purchase probability of our competitor brand

P<sub>i</sub>: Price of competitor brand

$$Y_i = \frac{e^{(\beta_{0i} + \beta_{1i}Price_i + \beta_{2i}Promotion_i + \cdots)}}{\sum_{k=1}^{I} e^{(\beta_{0i} + \beta_{1i}Price_k + \beta_{2i}Promotion_k + \cdots)}}$$

Y (purchase probability of a given brand i) is determined by a softmax function (multinomial logistic regression)

Logistic regression coefficient of the price of **our** product



brand product

### **Brand Choice Cross Price Elasticity**

E<sub>cross</sub>: Cross-price elasticity

Y<sub>i</sub>: Purchase probability of our own brand

Y<sub>i</sub>: Purchase probability of competitor brand

P<sub>i</sub>: Price of competitor brand

$$Y_{i} = \frac{e^{(\beta_{0i} + \beta_{1i}Price_{i} + \beta_{2i}Promotion_{i} + \cdots)}}{\sum_{k=1}^{I} e^{(\beta_{0i} + \beta_{1i}Price_{k} + \beta_{2i}Promotion_{k} + \cdots)}}$$

Y (purchase probability of a given brand i) is determined by a softmax function (multinomial logistic regression)

$$E_{cross} = \frac{\frac{dYi}{Yi}}{\sqrt{\frac{dP_j}{P_j}}} = ->$$
 We start from the general price elasticity formula

$$= \frac{dY_i}{dP_i} * \frac{P_j}{Y_i} = -$$
 We simply rearrange the terms of the equation

$$= \frac{dY_i}{d(\beta_0 + \beta_{1i}Price_j + \cdots)} * \frac{d(\beta_{0j} + \beta_{1ij}Price_j + \cdots)}{dP_j} * \frac{P_j}{Y_i} = ->$$
We apply the chain rule to determine the partial derivative of  $Y_i$  w.r.t.  $P_j$ 

= 
$$Y_i * (-Y_j) * \beta_{1i} * \frac{P_j}{Y_i} = ^{->}$$
 We find the partial derivative of  $Y_i$ , w.r.t. the green expression (from the softmax derivative)  $Y_i * Y_i = ^{->}$  We find the partial derivative of the green expression, w.r.t.  $P(\beta_{1i})$ 

$$=-Yj*eta_{1i}*P_{j}$$
 -> We reach the final formula for the price elasticity of purchase probability

#### For a softmax function:

$$\frac{\partial Y_i}{\partial x_j} = Y_i (\delta_{ij} - Y_j)$$
where  $\delta_{ij}$  is 1 if i=j, 0 otherwise\*

