

Factorio is PSPACE-Complete

Seminararbeit

im Rahmen der Prüfung zum
Bachelor of Science (B.Sc.)

des Studienganges Informatik

an der Dualen Hochschule Baden-Württemberg Karlsruhe

von

Johannes Quast

Abgabedatum:	01. Februar 2018
Bearbeitungszeitraum:	01.10.2017 - 31.01.2018
Matrikelnummer, Kurs:	0000000, TINF15B1
Ausbildungsfirma:	SAP SE Dietmar-Hopp-Allee 16 69190 Walldorf, Deutschland
Betreuer der Ausbildungsfirma:	B-Vorname B-Nachname
Gutachter der Dualen Hochschule:	DH-Vorname DH-Nachname

Eidesstattliche Erklärung

Ich versichere hiermit, dass ich meine Seminararbeit mit dem Thema:

Factorio is PSPACE-Complete

gemäß § 5 der "Studien- und Prüfungsordnung DHBW Technik" vom 29. September 2017 selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe. Die Arbeit wurde bisher keiner anderen Prüfungsbehörde vorgelegt und auch nicht veröffentlicht.

Ich versichere zudem, dass die eingereichte elektronische Fassung mit der gedruckten Fassung übereinstimmt.

Karlsruhe, den May 22, 2023

Gez. Johannes Quast

Nachname, Vorname

Sperrvermerk

Die nachfolgende Arbeit enthält vertrauliche Daten der:

SAP SE
Dietmar-Hopp-Allee 16
69190 Walldorf, Deutschland

Der Inhalt dieser Arbeit darf weder als Ganzes noch in Auszügen Personen außerhalb des Prüfungsprozesses und des Evaluationsverfahrens zugänglich gemacht werden, sofern keine anderslautende Genehmigung vom Dualen Partner vorliegt.

Abstract

- English -

This is the starting point of the Abstract. For the final bachelor thesis, there must be an abstract included in your document. So, start now writing it in German and English. The abstract is a short summary with around 200 to 250 words.

Try to include in this abstract the main question of your work, the methods you used or the main results of your work.

Abstract

- Deutsch -

Dies ist der Beginn des Abstracts. Für die finale Bachelorarbeit musst du ein Abstract in deinem Dokument mit einbauen. So, schreibe es am besten jetzt in Deutsch und Englisch. Das Abstract ist eine kurze Zusammenfassung mit ca. 200 bis 250 Wörtern.

Versuche in das Abstract folgende Punkte aufzunehmen: Fragestellung der Arbeit, methodische Vorgehensweise oder die Hauptergebnisse deiner Arbeit.

Contents

Abkürzungsverzeichnis	VI
List of Figures	VII
1 Introduction	1
1.1 Motivation	1
1.2 Structure	1
1.3 Related work	1
2 Preliminaries	2
2.1 Complexity landscape	2
2.2 Quantified Boolean formula problem (QBF)	3
2.3 Gadget Constructions	4
2.4 0/1/2-Player Games	4
3 Gadget Construction	5
4 PSPACE-Completeness in Factorio	6
4.1 Trains	6
4.2 Transport Belts	6
5 Summary and Outlook	7
Literaturverzeichnis	VIII

Abkürzungsverzeichnis

SAT Boolean satisfiability problem

QBF Quantified Boolean formula problem

List of Figures

2.1	An overview about the complexity landscape relevant for this report. The class PSPACE is a superset of the known classes like P and NP. Just like the famous $P \stackrel{?}{=} NP$ problem, it is still unknown whether $P \stackrel{?}{=} PSPACE$. One interesting aspect, which is already indicated in the figure, is the fact that $PSPACE = NPSPACE$, which follows as a corollary from <i>Savitch's theorem</i> [1, 2].	2
2.2	An arbitrary <i>Turing Machine</i> in state q_1 with a single tape in order to keep the drawings simple. Turing machines are often depicted with multiple tapes [2], but these TMs can be simulated by a TM with just one tape using only quadratically more computation time.	3

1 Introduction

1.1 Motivation

1.2 Structure

1.3 Related work

2 Preliminaries

In order to get a better intuition about the relevant complexity landscape, PSPACE-Completeness and the gadget construction in particular, which will become very important in section 3, a few selected topics are briefly presented on the following pages. Readers which already have a background in theoretical computer science or related fields may skip these sections, as they only contain well known results.

2.1 Complexity landscape

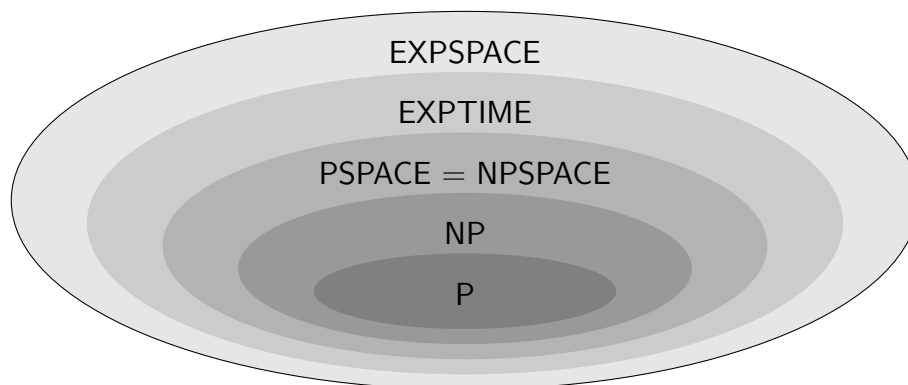


Figure 2.1: An overview about the complexity landscape relevant for this report. The class PSPACE is a superset of the known classes like P and NP. Just like the famous $P \stackrel{?}{=} NP$ problem, it is still unknown whether $P \stackrel{?}{=} PSPACE$. One interesting aspect, which is already indicated in the figure, is the fact that $PSPACE = NPSPACE$, which follows as a corollary from *Savitch's theorem* [1, 2].

In the following sections the most important complexity classes relevant for this report are characterized and described in an intuitive way. As shown in the well known complexity class hierarchy in figure 2.1, classes like P and NP are contained in PSPACE.

Klingt
scheiße

2.1.1 P, NP, PSPACE

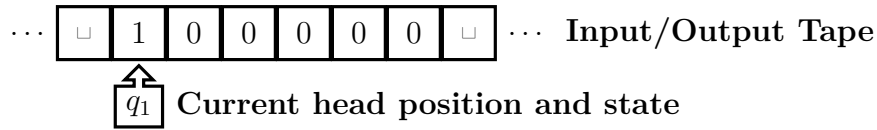


Figure 2.2: An arbitrary *Turing Machine* in state q_1 with a single tape in order to keep the drawings simple. Turing machines are often depicted with multiple tapes [2], but these TMs can be simulated by a TM with just one tape using only quadratically more computation time.

PTIME: PTIME, often abbreviated with just P , is a complexity class which contains all *decision problems* that can be solved by a deterministic turing machine in polynomial time.

With respect to figure 2.1, this means that for any decision problem in **PTIME** there exists a turing machine with can be solved after polynomial many steps.

NPTIME: NPTIME, often abbreviated with just NP , is a complexity class which contains all *decision problems* that can be solved by a **non**-deterministic turing machine in polynomial time.

Another definition which is quite common is the notion of a deterministic turing machine called a *verifier* which can verify a given *certificate* of polynomial length in polynomial time.

2.1.2 The dynamics of PSPACE and NPSPACE

2.1.3 The essence of PSPACE

2.2 QBF

QBF can also be used as a framework to encode and solve a variety of seemingly unrelated problems, e.g. Model Checking [3], Games [4] and many more applications that are beyond the scope of this report. For a more sophisticated overview about possible applications, the reader is referred to [5]. In fact, even with very strong restrictions on the amount

QBF
definieren

and type of quantifiers, these special variants of QBF can still be used to encode many problems [6].

2.3 Gadget Constructions

2.4 0/1/2-Player Games

3 Gadget Construction

4 PSPACE-Completeness in Factorio

Over the past couple of chapters important preliminaries and the essence of gadget constructions were introduced and important results discussed. Of course, these results on it's own are not very useful until we use them to prove PSPACE-Completeness of actual problems. A multitude of such problems is contained in the game “Factorio”, which will we the subject for the rest of the report. In the following we will restrict the analysis to a fraction of the games elements, namely its two most important types of *transport system*: Trains and transport belts. Trains the belts are used to move wares, which are being produced at potentially different locations in the game world, from one place to another.

4.1 Trains

4.2 Transport Belts

5 Summary and Outlook

Literaturverzeichnis

- [1] Savitch, W. J. “Relationships between nondeterministic and deterministic tape complexities”. In: *Journal of Computer and System Sciences* 4.2 (1970), pp. 177–192.
- [2] Arora, S./ Barak, B. *Computational complexity: A modern approach*. Reprint. Cambridge: Cambridge Univ. Press, 2010.
- [3] Baier, C./ Katoen, J.-P. *Principles of model checking*. Cambridge, Mass. and London: MIT, 2008?
- [4] Ani, J. et al. “Trains, Games, and Complexity: 0/1/2-Player Motion Planning Through Input/Output Gadgets”. In: *WALCOM: Algorithms and Computation*. Ed. by Mutzel, P./ Rahman, M. S./ Slamin. Vol. 13174. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2022, pp. 187–198.
- [5] Shukla, A. et al. “A Survey on Applications of Quantified Boolean Formulas”. In: *2019 IEEE 31st International Conference on Tools with Artificial Intelligence (ICTAI)*. IEEE, 2019, pp. 78–84.
- [6] Balabanov, V. et al. “2QBF: Challenges and Solutions”. In: *Theory and Applications of Satisfiability Testing – SAT 2016*. Ed. by Creignou, N./ Le Berre, D. Vol. 9710. Lecture Notes in Computer Science. Cham: Springer International Publishing, 2016, pp. 453–469.