CLASS IMBALANCE AND ACTIVE LEARNING

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7.1 INTRODUCTION

The rich history of predictive modeling has culminated in a diverse set of techniques capable of making accurate predictions on many real-world problems. Many of these techniques demand training, whereby a set of instances with ground-truth labels (values of a dependent variable) are observed by a model-building process that attempts to capture, at least in part, the relationship between the features of the instances and their labels. The resultant model can be applied to instances for which the label is not known, to estimate or predict the labels. These predictions depend not only on the functional structure of the model itself, but on the particular data with which the model was trained. The accuracy of the predicted labels depends highly on the model's ability to capture an unbiased and sufficient understanding of the characteristics of different classes; in problems where the prevalence of classes is imbalanced, it is necessary to prevent the resultant

model from being skewed towards the majority class and to ensure that the model is capable of reflecting the true nature of the minority class.

Another consequence of class imbalance is observed in domains where the ground truth labels in the dataset are not available beforehand and need to be gathered on-demand at some cost. The costs associated with collecting labels may be due to human labor or as a result of costly incentives, interventions or experiments. In these settings, simply labeling all available instances may not be practicable, due to budgetary constraints or simply a strong desire to be cost efficient. In a similar fashion to predictive modeling with imbalanced classes, the goal here is to ensure that the budget is not predominantly expended on getting the labels of the majority class instances, and to make sure that the set of instances to be labeled have comparable number of minority class instances as well.

In the context of learning from imbalanced datasets, the role of active learning can be viewed from two different perspectives. The first perspective considers the case where the labels for all the examples in a reasonably large, imbalanced dataset are readily available. The role of active learning in this case is to reduce, and potentially eliminate, any adverse effects that the class imbalance can have on the model's generalization performance. The other perspective addresses the setting where we have prior knowledge that the dataset is imbalanced, and we would like to employ active learning to select informative examples both from the majority and minority classes for labeling, subject to the constraints of a given budget. The first perspective focuses on active learning's ability to address class imbalance, whereas the second perspective is concerned with the impact of class imbalance on the sampling performance of the active learner. The intent of this chapter is to present a comprehensive analysis of this interplay of active learning and class imbalance. In particular, we first present techniques for dealing with the class imbalance problem using active learning and discuss how active learning can alleviate the issues that stem from class imbalance. We show that active learning, even without any adjustments to target class imbalance, is an effective strategy to have a balanced view of the dataset in most cases. It is also possible to further improve the effectiveness of active learning by tuning its sampling strategy in a class-specific way. Additionally, we will focus on dealing with highly skewed datasets and their impact on the selections performed by an active learning strategy. Here, we discuss the impact significant class imbalance has on active learning, and illustrate alternatives to traditional active learning that may be considered when dealing with the most difficult, highly skewed problems.

7.2 ACTIVE LEARNING FOR IMBALANCED PROBLEMS

The intent of this section is to provide the reader with some background on the active learning problem in the context of building cost-effective classification models. We then discuss challenges encountered by active learning heuristics

in settings with significant class imbalance. We then discuss strategies specialized in overcoming the difficulties imposed by this setting.

7.2.1 Background on Active Learning

Active Learning is specialized set of machine learning techniques developed for reducing the annotation costs associated with gathering the training data required for building predictive statistical models. In many applications, unlabeled data comes relatively cheaply when compared to the costs associated with the acquisition of a ground-truth value of the target variable of that data. For instance, the textual content of a particular web page may be crawled readily, or the actions of a user in a social network may be collected trivially by mining the web logs in that network. However, knowing with some degree of certainty the topical categorization of a particular web page, or identifying any malicious activity of a user in a social network is likely to require costly editorial review. These costs restrict the number of examples that may be labeled, typically to a small fraction of the overall population. Because of these practical constraints typically placed on the overall number of ground truth labels available, and the tight dependence of the performance of a predictive model on the examples in its training set, the benefits of careful selection of the examples are apparent. This importance is further evidenced by the vast research literature on the topic.

While an in-depth literature review is beyond the scope of this chapter, for context we provide a brief overview of some of the more broadly cited approaches in active learning. For a more thorough treatment on the the history and details of active learning, we direct the reader to the excellent survey by Settles [50]. Active learning tends to focus on two scenarios-(i) stream-based selection, where unlabeled examples are presented one-at-atime to a predictive model, which feeds predicted target values to a consuming process and subsequently applies an active learning heuristic to decide if some budget should be expended gathering this example's class label for subsequent re-training. (ii) pool-based active learning, on the other hand, is typically an offline, iterative process. Here, a large set of unlabeled examples are presented to an active learning system. During each epoch of this process, the active learning system chooses one or more unlabeled examples for labeling and subsequent model training. This proceeds until the budget is exhausted or some stopping criterion is met. At this time, if the predictive performance is sufficient, the model may be incorporated into an end system that feeds the model with unlabeled examples and consumes the predicted results. A diagram illustrating these two types of active learning scenarios is presented in Figures 7.2 and 7.1 respectively. Due to the greater attention given to pool-based active learning in recent scientific literature and the additional power available to an active learning system capable of processing a large representation of the problem space at once, the remainder of this chapter focuses on the latter of these two scenarios, the pool-based setting.

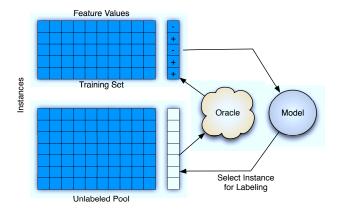


Figure 7.1 Pool-Based Active Learning

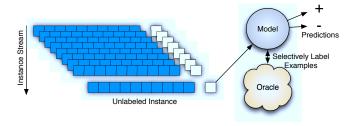


Figure 7.2 Stream-Based Active Learning

The most common techniques in active learning have focused on selecting examples from a so-called "region of uncertainty", the area nearest to the current model's predictive decision boundary. Incorporating uncertainty into active data acquisition dates back to research focused on optimal experimental design [18], and has been amongst the earliest successful examples of active machine learning techniques [8, 29]. The intuition behind uncertainty-based selection is that this region surrounding a model's decision boundary is where that model is most likely to make mistakes. Incorporating labeled examples from this region may improve the model's performance along this boundary, leading to gains in overall accuracy.

Many popular subsequent techniques are specializations of uncertainty selection, including query-by-committee-based approaches [19, 9, 20], where,

¹For instance, the simplest case when performing binary classification would involve choosing $x' = \arg\max_x \min_y P(y|x), y \in 0, 1$.

given an ensemble of (valid) predictive models, examples are selected based on the level of disagreement elicited amongst the ensemble, and the popular "simple margin" technique proposed by Tong and Koller [55], where, given a current parameterization of a support vector machine, w_j , the example x_i is chosen that comes closest to the decision boundary, $x_i = \arg\min_{x'} |w_j\Phi(x)|$, where $\Phi(\cdot)$ is a function mapping an example to an alternate space utilized by the kernel function in the support vector machine: $k(u, v) = \Phi(u)\Phi(v)$.

Expected-utility-based approaches, where examples are chosen based on the estimated expected improvement in a certain objective achieved by incorporating a given example into the training set.² Such techniques often involve costly nested cross-validation where each available example is assigned all possible label states [44, 40, 22].

7.2.2 Dealing with Class Imbalance Problem in Active Learning

Selecting examples from an unlabeled pool with substantial class imbalance may pose several difficulties for traditional active learning. The greater proportion of examples in the majority class may lead to a model that prefers one class over another. If the labels of examples selected by an active learning scheme are thought of as a random variable, the innate class imbalance in the example pool would almost certainly lead to a preference for majority examples in the training set. Unless properly dealt with,³ this over-representation may simply lead to a predictive preference for the majority class when labeling. Typically, when making predictive models in an imbalanced setting, it is the minority class that is of interest. For instance, it is important to discover patients who have a rare but dangerous ailment based on the results of a blood test, or infrequent but costly fraud in a credit card company's transaction history. This difference in class preferences between an end-system's needs and a model's tendencies causes a serious problem for active learning (and predictive systems in general) in imbalanced settings. Even if the problem of highly imbalanced (though correct in terms of base rate) training set problem can be dealt with, the tendency for a selection algorithm to gather majority examples creates other problems. The nuances of the minority set may be poorly represented in the training data, leading to a "predictive misunderstanding" in certain regions of the problem space; while a model may be able to accurately identify large regions of the minorityspace, portions of this space may get mislabeled, or labeled with poor quality, due to underrepresentation in the training set. At the extreme, disjunctive sub-regions may get missed entirely. Both of these problems are particularly acute as the class imbalance increases, and are discussed in greater detail

²Note that this selection objective may not necessarily be the same objective used during the base model's use time. For instance, examples may be selected according to their contribution to the reduction in problem uncertainty.

³For instance, by imbalanced learning techniques described throughout this book.

in Section 7.6. Finally, in the initial stages of active learning, when the base model is somewhat naïve, the minority class may get missed entirely as an active learning heuristic probes the problem space for elusive but critical minority examples.

7.2.3 Addressing the Class Imbalance Problem with Active Learning

As we will demonstrate in Section 7.3, active learning presents itself as an effective strategy for dealing with moderate class imbalance even, without any special considerations for the skewed class distribution. However, the previously discussed difficulties imposed by more substantial class imbalance on the selective abilities of active learning heuristics have led to the development of several techniques that have been specially adapted to imbalanced problem settings. These skew-specialized active learning techniques incorporate an innate preference for the minority class, leading to more balanced training sets and better predictive performance in imbalanced settings. Additionally, there exists a category of density-sensitive active learning techniques, techniques that explicitly incorporate the geometry of the problem space. By incorporating knowledge of independent dimensions of the unlabeled example pool, there exists a potential for better exploration, resulting in improved resolution of rare sub-regions of the minority class. We detail these two broad classes of active learning techniques below.

7.2.3.1 Density-Sensitive Active Learning Utility-based selection strategies for active learning attribute some score, $\mathcal{U}(\cdot)$, to each instance x encapsulating how much improvement can be expected from training on that instance. Typically, the examples offering a maximum $\mathcal{U}(x)$ are selected for labeling. However, the focus on individual examples may expose the selection heuristic to outliers, individual examples that achieve a high utility score, but do not represent a sizable portion of the problem space. Density-sensitive active learning heuristics seek to alleviate this problem by leveraging the entire unlabeled pool of examples available to the active learner. By explicitly incorporating the geometry of the input space when attributing some selection score to a given example, outliers, noisy examples, and sparse areas of the problem space may be avoided. Below are some exemplary active learning heuristics that leverage density sensitivity.

Information Density: this is a general density-sensitive paradigm compatible with arbitrary utility-based active selection strategies and a variety of metrics used to compute similarity [51]. In this case, a meta-utility score is computed for each example based not online on a traditional utility score, $\mathcal{U}(x)$, but on a measurement of that example's similarity to other instances in the problem space. Given a similarity metric between two points, $\sin(x, x')$, information density selects examples according to:

$$\mathcal{U}_m(x) = \mathcal{U}(x) \left(\frac{1}{|X|} \sum_{x' \in X} \sin(x, x') \right)^{\beta}$$

Here, β is a hyper-parameter controlling the tradeoff between the raw instance-specific utility, $\mathcal{U}(x)$ and the similarity component in the overall selection criterion.

Zhu et al. [63] develop a similar technique to the information density technique of Settles and Craven, selecting instances according a uncertainty-based criterion modified by a density factor: $U_n(x) = U(x)KNN(x)$, where KNN(x) is the average cosine similarity of the K nearest neighbors to x. The same authors also propose sampling by clustering, a density-only active learning heuristic where the problem space is clustered, and the points closest to the cluster centeroids are selected for labeling.

Pre-Clustering: here it is assumed that the problem is expressed as a mixture model comprising K distributions, each component model completely encoding information related to the labels of member examples—the label y is conditionally independent of the covariates x given knowledge of it's cluster, k [41]. This assumption yields a joint distribution describing the problem: p(x, y, k) = p(x|k)p(y|k)p(k), yielding a poster probability on y:

$$p_k(y|x) = \sum_{k=1}^{K} p(y|k) \frac{p(x|k)p(k)}{p(x)}$$

In essence, this a density weighted mixture model used for classification. The K clusters are created by a application of typical clustering techniques of the data, with a cluster size used to estimate p(k), and p(y|k) is estimated via a logistic regression considering a cluster's representative example. A probability density is inferred for each cluster, in the example case presented in the above work, an isotropic normal distribution is used, from which p(x|k) can be estimated. Examples are then selected from an uncertainty score computed via the above posterior model weighted by the probability of observing a given x:

$$U_k(x) = (1 - |p_k(y|x)|) p(x)$$

Of course, there exists a variety of other techniques in the research literature designed to explicitly incorporate information related to the problem's density into an active selection criterion. McCallum and Nigam [36] modify a query-by-committee to use an exponentiated KL-divergence-based uncertainty metric and combine this with semi-supervised learning in the form of an expectation maximization procedure. This combined semi-supervised active learning has the benefit of ignoring regions that can be reliably "filled in" by a semi-supervised procedure, while also selecting those examples that may benefit this EM process.

Donmez et al. [12] propose a modification of the density-weighted technique of Nguyen and Smeulders. This modification simply selects examples according to the convex combination of the density weighted technique and traditional uncertainty sampling. This hybrid approach is again incorporated into a so-called dual active learner, where uncertainty sampling is only incorporated once the benefits of pure density sensitive sampling seem to be diminishing.

Alternate Density-Sensitive Heuristics: Donmez and Carbonell [11] incorporate density into active label selection by performing a change of coordinates into a space whose metric expresses not only euclidian similarity but also density. Examples are then chosen based on a density-weighted uncertainty metric designed to select examples in pairs— one member of the pair from each side of the current decision boundary. The motivation is that sampling from both sides of the decision boundary may yield better results than selecting from one side in isolation.

Through selection based on an "unsupervised" heuristic estimating the utility of label acquisition on the pool of unlabeled instances, Roy and McCallum incorporate the geometry of the problem space into active selection implicitly [44]. This approach attempts to quantify the improvement in model performance attributable to each unlabeled example, taken in expectation over all label assignments:

$$\mathcal{U}_E = \sum_{y' \in Y} \hat{p}(y'|x) \sum_{x' \neq X} \mathcal{U}_e(x'; x, y = y')$$

Here, the probability of class membership in the above expectation comes from the base model's current posterior estimates. The utility value on the right side of the above equation, $U_e(x'; x, y = y')$, comes from assuming a label of y' for example x, and incorporating this pseudo-labeled example into the training set temporarily. The improvement in model performance with the inclusion of this new example is then measured. Since a selective label acquisition procedure may result in a small or arbitrarily biased set of examples, accurate evaluation through nested cross validation difficult. To accommodate this, Roy and McCallum propose two uncertainty measures taken over the pool of unlabeled examples, $x' \neq x$. Specifically, they look at the entropy of the posterior probabilities of examples in the pool, and the magnitude of the maximum posterior as utility measures, both estimated after the inclusion of the "new" example. Both metrics favor "sharp" posteriors, an optimization minimizing uncertainty rather than model performance; instances are selected by their reduction in uncertainty taken in expectation over the entire example pool.

7.2.3.2 Skew-Specialized Active Learning Additionally, there exists a body of research literature on active learning specifically to deal with class imbalance problem. Tomanek [54] investigates query-by-committee-based approaches to sampling labeled sentences for the task of named entity recognition. The

goal of their selection strategy is to encourage class-balanced selections by incorporating class-specific costs. Unlabeled instances are ordered by a class-weighted, entropy-based disagreement measure, $-\sum_{j\in\{0,1\}}b_j\frac{V(k_j)}{|C|}\log\frac{V(k_j)}{|C|},$ where $V(k_j)$ is the number of votes from a committee of size |C| that an instance belongs to a class k_j . b_j is a weight corresponding to the importance of including a certain class; a larger value of b_j corresponds to a increased tendency to include examples that are thought to belong to this class. From a window W of examples with highest disagreement, instances are selected greedily based on the model's estimated class membership probabilities so that the batch selected from the window has the highest probability of having a balanced class membership.

SVM-based active learning has been shown [17] to be a highly effective strategy for addressing class imbalance without any skew-specific modifications to the algorithm. Bloodgood and Shanker [5] extend the benefits of SVM-based active learning by proposing an approach that incorporates class specific costs. That is, the typical C factor describing an SVM's misclassification penalty is broken up into C_{+} and C_{-} , describing costs associated with misclassification of positive and negative examples, respectively, a common approach for improving the performance of support vector machines in cost-sensitive settings. Additionally, cost sensitive support vector machines is known to yield predictive advantages in imbalanced settings by offering some preference to an otherwise overlooked class, often using the heuristic for setting class specific costs: $\frac{C_+}{C_-} = \frac{|\{x|x \in -\}|}{|\{x|x \in +\}|}$, a ratio in inverse proportion to the number of examples in each class. However, in the active learning setting, the true class ratio is unknown, and the quantity $\frac{C_+}{C}$ must be estimated by the active learning system. Bloodgood and Shanker show that it is advantageous to use a preliminary stage of random selection in order to establish some estimate of the class ratio, and then proceed with example selection according to the uncertainty-based "simple margin" criterion using the appropriately tuned cost-sensitive SVM.

Active learning has also been studied as a way to improve the generalization performance of resampling strategies that address class imbalance. In these settings, active learning is used to choose a set of instances for labeling, with sampling strategies used to improve the class distribution. Ertekin [16] presented VIRTUAL, a hybrid method of oversampling and active learning that forms an adaptive technique for resampling of the minority class instances. The learner selects the most informative example x_i for oversampling, and the algorithm creates a synthetic instance along the direction of x_i 's one of k neighbors. The algorithm works in an online fashion and builds the classifier incrementally without the need to retrain on the entire labeled dataset after creating a new synthetic example. This approach, which we present in detail in Section 7.4, yields an efficient and scalable learning framework.

Zhu and Hovy [62] describe a bootstrap-based oversampling strategy (BootOS) that, given an example to be resampled, generates a bootstrap example based on all the k neighbors of that example. At each epoch, the examples with the greatest uncertainty are selected for labeling and incorporated into a labeled set, L. From L, the proposed oversampling strategy is applied, yielding a more balanced data set, L', a data set that is used to retrain the base model. The selection of the examples with the highest uncertainty for labeling at each iteration involves resampling the labeled examples and training a new classifier with the resampled dataset, therefore, scalability of this approach may be a concern for large scale datasets.

In the next section, we demonstrate that the principles of active learning are naturally suited to address the class imbalance problem and that active learning can in fact be an effective strategy to have construct a balanced view of an otherwise imbalanced dataset, without the need to resort to resampling techniques. It is worth noting that the goal of the next section is not to cast active learning as a replacement for resampling strategies. Rather, our main goal is to demonstrate how active learning can alleviate the issues that stem from class imbalance, and to present active learning as an alternate technique that should be considered in case a resampling approach is impractical, inefficient or ineffective. In problems where resampling is the preferred solution, we show in Section 7.4 that the benefits of active learning can still be leveraged to address class imbalance. In particular, we present an adaptive oversampling technique that uses active learning to determine which examples to resample in an online setting. These two different approaches show the versatility of active learning and the importance of selective sampling to address the class imbalance problem.

7.3 ACTIVE LEARNING FOR IMBALANCED DATA CLASSIFICATION

As outlined in Section 7.2.1, active learning is primarily considered as a technique to reduce the number of training samples that need to be labeled for a classification task. From a traditional perspective, the active learner has access to a vast pool of unlabeled examples, and it aims to make a clever choice to select the most informative example to obtain its label. However, even in the cases where the labels of training data are already available, active learning can still be leveraged to obtain the informative examples through training sets [49, 6, 24]. For example, in large-margin classifiers such as Support Vector Machines (SVM), the *informativeness* of an example is synonymous with its distance to the hyperplane. The farther an example is to the hyperplane, the more the learner is confident about its true class label; hence there is little, if any, benefit that the learner can gain by asking for the label of that example. On the other hand, the examples close to the hyperplane are the ones that yield the most information to the learner. Therefore, the most commonly used

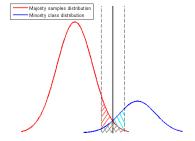


Figure 7.3 Data within the margin is less imbalanced than the entire data.

active learning strategy in SVMs is to check the distance of each unlabeled example to the hyperplane and focus on the examples that lie closest to the hyperplane, as they are considered to be the most informative examples to the learner [55].

The strategy of selecting examples within the margin also strongly addresses the problems that arise from imbalanced classes. Consider the class distributions of an imbalanced dataset presented in Figure 7.3. The shaded region corresponds to the class distribution of the data within the margin. As shown in the figure, the imbalance ratio of the classes within the margin is much smaller than the class imbalance ratio of the entire dataset. Therefore, any selection strategy that focuses on the examples in the margin most likely ends up with a more balanced class distribution than that of the entire dataset.

Throughout this section, the discussion is constrained to standard twoclass classification problems using Support Vector Machines (SVMs). The next subsection presents a brief overview of SVMs, followed by the working principles of an efficient active learning algorithm in Section 7.3.2. We explain the advantage of using online SVMs with the active sample selection in Section 7.3.3.

7.3.1 Support Vector Machines

Support Vector Machines [56] are well known for their strong theoretical foundations, generalization performance and ability to handle high dimensional data. In the binary classification setting, let $((x_1, y_1) \cdots (x_n, y_n))$ be the training dataset where x_i are the feature vectors representing the instances and $y_i \in (-1, +1)$ be the labels of the instances. Using the training set, SVM builds an optimum hyperplane – a linear discriminant in a higher dimensional feature space – that separates the two classes by the largest margin. This

hyperplane is obtained by minimizing the following objective function:

$$\min_{\mathbf{w}, \mathbf{b}, \xi_i} \frac{1}{2} \mathbf{w} \cdot \mathbf{w}^T + C \sum_{i=1}^{N} \xi_i$$
 (7.1)

subject to
$$\begin{cases} \forall i \ y_i(\mathbf{w}^T \Phi(x_i) - b) \ge 1 - \xi_i \\ \forall i \ \xi_i \ge 0 \end{cases}$$
 (7.2)

where **w** is the norm of the hyperplane, b is the offset, y_i are the labels, $\Phi(\cdot)$ is the mapping from input space to feature space, and ξ_i are the slack variables that permit the non-separable case by allowing misclassification of training instances. In practice the convex quadratic programming (QP) problem in (7.1) is solved by optimizing the dual cost function. The dual representation of (7.1) is given as

$$\max W(\alpha) \equiv \sum_{i=1}^{N} \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j K(\mathbf{x_i}, \mathbf{x_j})$$
 (7.3)

subject to
$$\begin{cases} \forall i \ 0 \le \alpha_i \le C \\ \sum_{i=1}^N \alpha_i y_i = 0 \end{cases}$$
 (7.4)

where y_i are the labels, $\Phi(\cdot)$ is the mapping from the input space to the feature space, $K(\mathbf{x_i}, \mathbf{x_j}) = \langle \Phi(\mathbf{x_i}), \Phi(\mathbf{x_j}) \rangle$ is the kernel matrix and the α_i 's are the Lagrange multipliers which are non-zero only for the training instances which fall in the margin. Those training instances are called support vectors and they define the position of the hyperplane. After solving the QP problem, the norm of the hyperplane \mathbf{w} can be represented as

$$\mathbf{w} = \sum_{i=1}^{n} \alpha_i \Phi(x_i) \tag{7.5}$$

7.3.2 Margin-based Active Learning with SVMs

Note that in (7.5), only the support vectors affect the SVM solution. This means that if SVM is retrained with a new set of data which only consist of those support vectors, the learner will end up finding the same hyperplane. This emphasizes the fact that not all examples are equally important in training sets. Then the question becomes how to select the most informative examples for labeling from the set of unlabeled training examples. This section focuses on a form of selection strategy called margin-based active learning. As was highlighted earlier, in SVMs the most informative example is believed to be the closest one to the hyperplane since it divides the version space into two equal parts. The aim is to reduce the version space as fast as possible to reach the solution faster in order to avoid certain costs associated with the problem. For the possibility of a non-symmetric version space, there are more

complex selection methods suggested by [55], but it has been observed that the advantage of those are not significant, considering their high computational costs.

Active Learning with Small Pools: The basic working principle of margin-based active learning with SVMs is: i) train an SVM on the existing training data, ii) select the closest example to the hyperplane, and iii) add the new selected example to the training set and train again. In classical active learning [55], the search for the most informative example is performed over the entire dataset. Note that, each iteration of active learning involves the recomputation of each training example's distance to the new hyperplane. Therefore, for large datasets, searching the entire training set is a very time consuming and computationally expensive task.

One possible remedy for this performance bottleneck is to use the "59 trick" [53], which alleviates a full search through the entire dataset, approximating the most informative examples by examining a small constant number of randomly chosen samples. The method picks L ($L \ll \#$ training examples) random training samples in each iteration and selects the best (closest to the hyperplane) among them. Suppose, instead of picking the closest example among all the training samples $X_N = (x_1, x_2, \dots, x_N)$ at each iteration, we first pick a random subset X_L , $L \ll N$ and select the closest sample x_i from X_L based on the condition that x_i is among the top p% closest instances in X_N with probability $(1-\eta)$. Any numerical modification to these constraints can be met by varying the size of L, and is independent of N. To demonstrate, the probability that at least one of the L instances is among the closest p is $1-(1-p)^L$. Due to the requirement of $(1-\eta)$ probability, we have

$$1 - (1 - p)^{L} = 1 - \eta (7.6)$$

which follows the solution of L in terms of η and p

$$L = \log \eta / \log(1 - p) \tag{7.7}$$

For example, the active learner will pick one example, with 95% probability, that is among the top 5% closest instances to the hyperplane, by randomly sampling only $\lceil \log(.05)/\log(.95) \rceil = 59$ examples regardless of the training set size. This approach scales well since the size of the subset L is independent of the training set size N, requires significantly less training time and does not have an adverse effect on the classification performance of the learner.

7.3.3 Active Learning with Online Learning

Online learning algorithms are usually associated with problems where the complete training set is not available. However, in cases where the complete training set is available, the computational properties of these algorithms can be leveraged for faster classification and incremental learning. Online learning

techniques can process new data presented one at a time, either as the result of active learning or random selection, and can integrate the information of the new data to the system without training on all previously seen data, thereby allowing models to be constructed incrementally. This working principle of online learning algorithms leads to speed improvements and a reduced memory footprint, making the algorithm applicable to very large datasets. More importantly, this incremental learning principle suits the nature of active learning in a much more naturally than the batch algorithms. Empirical evidence indicates that a single presentation of each training example to the algorithm is sufficient to achieve training errors comparable to those achieved by the best minimization of the SVM objective [6].

7.3.4 Performance Metrics

Classification accuracy is not a good metric to evaluate classifiers in applications facing class imbalance problems. SVMs have to achieve a tradeoff between maximizing the margin and minimizing the empirical error. In the non-separable case, if the misclassification penalty C is very small, the SVM learner simply tends to classify every example as negative. This extreme approach maximizes the margin while making no classification errors on the negative instances. The only error is the cumulative error of the positive instances which are already few in numbers. Considering an imbalance ratio of 99 to 1, a classifier that classifies everything as negative, will be 99% accurate. Obviously, such a scheme would not have any practical use as it would be unable to identify positive instances.

For the evaluation of these results, it is useful to consider several other prediction performance metrics such as g-means, AUC and PRBEP which are commonly used in imbalanced data classification. g-means [27] is denoted as $g = \sqrt{sensitivity} \cdot specificity$ where sensitivity is the accuracy on the positive instances given as TruePos./(TruePos.+FalseNeg.) and specificity is the accuracy on the negative instances given as TrueNeg./(TrueNeg. + FalsePos.).

The Receiver Operating Curve (ROC) displays the relationship between sensitivity and specificity at all possible thresholds for a binary classification scoring model, when applied to independent test data. In other words, ROC curve is a plot of the true positive rate against the false positive rate as the decision threshold is changed. The area under the ROC curve (AUC) is a numerical measure of a model's discrimination performance and shows how successfully and correctly the model ranks and thereby separates the positive and negative observations. Since the AUC metric evaluates the classifier across the entire range of decision thresholds, it gives a good overview about the performance when the operating condition for the classifier is unknown or the classifier is expected to be used in situations with significantly different class distributions.

Precision Recall Break-Even Point (PRBEP) is another commonly used performance metric for imbalanced data classification. PRBEP is the accuracy of the positive class at the threshold where precision equals to recall. Precision is defined as TruePos./(TruePos. + FalsePos.) and recall is defined as TruePos./(TruePos. + FalseNeg.)

7.3.5 Experiments and Empirical Evaluation

We study the performance of the algorithm on various benchmark real-world datasets, including MNIST, USPS, several categories of Reuters-21578 collection, five topics from CiteSeer, and three datasets from the UCI repository. The characteristics of the datasets are outlined in [17]. In the experiments, an early stopping heuristic for active learning is employed, since it has been shown that active learning converges to the solution faster than the random sample selection method [17]. A theoretically sound method to stop training is when the examples in the margin are exhausted. To check if there are still unseen training examples in the margin, the distance of the newly selected example is compared against the support vectors of the current model. If the new selected example by active learning (closest to the hyperplane) is not closer than any of the support vectors, it is concluded that the margin

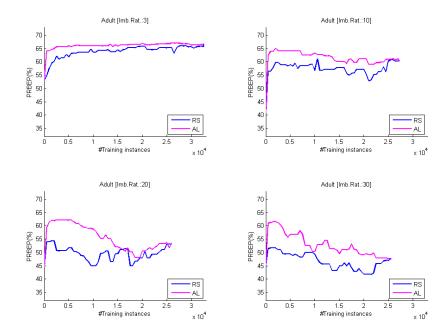


Figure 7.4 Comparison of PRBEP of AL and RS on the adult datasets with different imbalance ratios (Imb.R.=3, 10, 20, 30).

Table 7.1 Comparison of g-means and AUC for AL and RS with entire training data (Batch). SV ratios are given at the saturation point. Data efficiency corresponds to the percentage of training instances which AL processes to reach saturation.

Dataset		g-means (%)		AUC (%)		Imb.	SV-/	Data
		Batch	AL	Batch	AL	Rat.	SV+	Efficiency
x	Corn	85.55	86.59	99.95	99.95	41.9	3.13	11.6%
	Crude	88.34	89.51	99.74	99.74	19.0	2.64	22.6%
	Grain	91.56	91.56	99.91	99.91	16.9	3.08	29.6%
Reuters	Interest	78.45	78.46	99.01	99.04	21.4	2.19	30.9%
cen	Money-fx	81.43	82.79	98.69	98.71	13.4	2.19	18.7%
~	Ship	75.66	74.92	99.79	99.80	38.4	4.28	20.6%
	Trade	82.52	82.52	99.23	99.26	20.1	2.22	15.4%
	Wheat	89.54	89.55	99.64	99.69	35.7	3.38	11.6%
	AI	87.83	88.58	94.82	94.69	4.3	1.85	33.4%
ee.	COMM	93.02	93.65	98.13	98.18	4.2	2.47	21.3%
CiteSeer	CRYPT	98.75	98.87	99.95	99.95	11.0	2.58	15.2%
Ü	DB	92.39	92.39	98.28	98.46	7.1	2.50	18.2%
	OS	91.95	92.03	98.27	98.20	24.2	3.52	36.1%
П	Abalone-7	100.0	100.0	100.0	100.0	9.7	1.38	24.0%
nC	Letter-A	99.28	99.54	99.99	99.99	24.4	1.46	27.8%
	Satimage	82.41	83.30	95.13	95.75	9.7	2.62	41.7%
	USPS	99.22	99.25	99.98	99.98	4.9	1.50	6.8%
MNIST-8		98.47	98.37	99.97	99.97	9.3	1.59	11.7%

is exhausted. A practical implementation of this idea is to count the number of support vectors during the active learning training process. If the number of the support vectors stabilizes, it implies that all possible support vectors have been selected by the active learning method.

As the first experiment, examples are randomly removed from the minority class in *Adult* dataset to achieve different data imbalance ratios, comparing SVM-based active learning (AL) and random sampling (RS).⁴ For brevity, AL with small pools is referred to as AL since the small pools heuristic is utilized for all active learning methods considered later. Comparisons of PRBEP in Figure 7.4 show an interesting behavior. As the class imbalance ratio is increased, AL curves display peaks in the early steps of the learning. This implies that by using an early stopping criteria AL can give higher

 $^{^4}$ Here, the random process is assumed to be uniform; examples are selected with equal probability from the available pool.

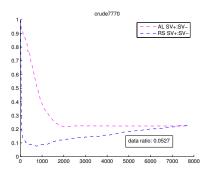


Figure 7.5 Support Vector ratios in AL and RS

prediction performance than RS can possibly achieve even after using all the training data. The learning curves presented in Figure 7.4 demonstrate that the addition of instances to a model's training after finding those most informative instances can be detrimental to the prediction performance of the classifier, since this may cause the model to suffer from overfitting. Figure 7.4 curves show that generalization can peak to a level above that can be achieved by using all available training data. In other words, it is possible to achieve better classification performance from a small informative subset of the training data than what can achieved using all available training data. This finding agrees with [49] and strengthens the idea of applying an early stopping to active learning algorithms.

For further comparison of the performance of the performance of a model built on all available data (Batch) and AL subject to early halting criteria, refer to Table 7.1, comparing the g-means and the AUC values for these two methods. The data efficiency column for AL indicates that by processing only a portion of the examples from the training set, AL can achieve similar or even higher generalization performance than that of Batch which sees all the training examples. Another important observation from Table 7.1 is that support vector imbalance ratios in the final models are much less than the class imbalance ratios of the datasets. This confirms the discussion of Figure 7.3. The class imbalance ratio within the margins are much less than the class imbalance ratio of the entire data and active learning can be used to reach those informative examples which most likely become support vectors without seeing all the training examples.

Figure 7.5 investigates how the number of support vectors changes when presented with examples selected according to AL and RS. Because the base rate of the dataset gathered by RS approaches that of the example pool, the support vector imbalance ratio quickly approaches the data imbalance ratio. As learning continues, the learner should gradually see all the instances within the final margin and the support vector imbalance ratio decreases. At the end of training with RS, the support vector imbalance ratio is the data

imbalance ratio within the margin. The support vector imbalance ratio curve of AL is drastically different than RS. AL intelligently picks the instances closest to the margin in each step. Since the data imbalance ratio within the margin is lower than data imbalance ratio, the support vectors in AL are more balanced than RS during learning. Using AL, the model saturates by seeing only 2000 (among 7770) training instances and reaches the final support vector imbalance ratio. Note that both methods achieve similar support vector imbalance ratios when learning finishes, but AL achieves this in the early steps of the learning.

It is also interesting to consider the performance of active learning as a selection heuristic in light of more conventional sampling strategies. Here, AL is compared to traditional under-sampling of the majority class (US), and an oversampling method (SMOTE), both examples of resampling techniques which require preprocessing. It has been shown that oversampling at random does not help to improve prediction performance [25] therefore a more complex oversampling method is required. Synthetic Minority Oversampling Technique (SMOTE) oversamples the minority class by creating synthetic examples rather than with replacement. The k nearest positive neighbors of all positive instances are identified and synthetic positive examples are created and placed randomly along the line segments joining the k minority class nearest neighbors.

For additional comparison, the method of assigning different costs (DC) to the positive and negative classes as the misclassification penalty parameter is examined. For instance, if the imbalance ratio of the data is 19:1 in favor of the negative class, the cost of misclassifying a positive instance is set to be 19 times greater than that of misclassifying a negative one. We use the online SVM package LASVM⁵ in all experiments. Other than the results of the methods addressing the class imbalance problem, the results of Batch algorithm with the original training set are provided to form a baseline. LASVM is run in random sampling mode for US, SMOTE and DC.

We present the comparisons of the methods for g-means performance metric for several datasets in Figure 7.6. The right border of the shaded pink area is the place where the aforementioned early stopping strategy is applied. The curves in the graphs are averages of 10 runs. For completeness, all AL experiments were allowed to continue to select examples until exhaustion, bypassing any early stopping. Table 7.2 presents the PRBEP of the methods and the total running times of the SMOTE and AL on 18 benchmark and real-world datasets. The results for active learning in Table 7.2 depict the results in the early stopping points. The results for the other methods in Table 7.2 depict the values at the end of the curves— when trained with the entire dataset— since those methods do not employ any early stopping criteria. We did not apply early stopping criteria to the other methods because, as observed

⁵Available at http://leon.bottou.org/projects/lasvm

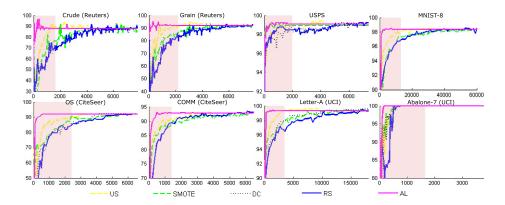


Figure 7.6 Comparisons of g-means. The right border of the shaded area corresponds to the early stopping point.

from Figure 7.6, no early stopping criteria would achieve a comparable training time with of AL's training time without a significant loss in their prediction performance based on convergence time. The other methods converge to similar levels of g-means when nearly all training instances are used, and applying an early stopping criteria would have little, if any, effect on their training times.

Since AL involves discarding some instances from the training set, it can be perceived as a type of under-sampling method. Unlike traditional US, which discards majority samples randomly, AL performs an intelligent search for the most informative ones adaptively in each iteration according to the current hyperplane. Datasets where class imbalance ratio is high such as *corn*, *wheat*, *letter* and *satimage* observe significant decrease in PRBEP of US (see Table 3). Note that US's under-sampling rate for the majority class in each category is set to the same value as the final support vector ratio which AL reaches in the early stopping point and RS reaches when it sees the entire training data. Although the class imbalance ratio provided to the learner in AL and US are the same, AL achieves significantly better PRBEP performance metric than US. The Wilcoxon signed-rank test (2-tailed) reveals that the zero median hypothesis can be rejected at the significance level 1% (p=0.0015), implying that AL performs statistically better than US in these 18 datasets. These results reveal the importance of using the informative instances for learning.

Table 7.2 gives the comparison of the computation times of the AL and SMOTE. Note that SMOTE requires significantly long preprocessing time which dominates the training time in large datasets, e.g., MNIST-8 dataset. The low computation cost, scalability and high prediction performance of AL suggest that AL can efficiently handle the class imbalance problem.

7.4 ADAPTIVE RESAMPLING WITH ACTIVE LEARNING

The analysis in Section 7.3.5 shows the effectiveness of active learning on imbalanced datasets—without employing any resampling techniques. This section extends the discussion on the effectiveness of active learning for imbalanced data classification, and demonstrates that even in cases where resampling is the preferred approach, active learning can still be used to significantly improve the classification performance.

In supervised learning, a common strategy to overcome the rarity problem is to resample the original dataset to decrease the overall level of class imbalance. Resampling is done either by oversampling the minority (positive) class and/or undersampling the majority (negative) class until the classes are approximately equally represented [7, 26, 27, 31]. Oversampling, in its simplest form, achieves a more balanced class distribution either by duplicating minority class instances, or introducing new synthetic instances that belong to the minority class [7]. No information is lost in oversampling since all original instances of the minority and the majority classes are retained in the oversampled dataset. The other strategy to reduce the class imbalance

Table 7.2 Comparison of PRBEP and training time.

Metric		PRBEP					Training time (sec.)	
Dataset		Batch	US	SMOTE	DC	AL	SMOTE	AL
	Corn	91.07	78.57	91.07	89.28	89.29	87	16
	Crude	87.83	85.70	87.83	87.83	87.83	129	41
	Grain	92.62	89.93	91.44	91.94	91.94	205	50
ers	Interest	76.33	74.04	77.86	75.57	75.57	116	42
Reuters	Money-fx	73.74	74.30	75.42	75.42	76.54	331	35
\mathbb{R}	Ship	86.52	86.50	88.76	89.89	89.89	49	32
	Trade	77.77	76.92	77.77	77.78	78.63	215	38
	Wheat	84.51	81.61	84.51	84.51	85.92	54	25
	AI	78.80	80.68	78.99	78.79	79.17	1402	125
CiteSeer	COMM	86.59	86.76	86.59	86.59	86.77	1707	75
eS	CRYPT	97.89	97.47	97.89	97.89	97.89	310	19
Ċīŧ	DB	86.36	86.61	86.98	86.36	86.36	526	41
	OS	84.07	83.19	84.07	84.07	84.07	93	23
П	Abalone-7	100.0	100.0	100.0	100.0	100.0	16	4
UCI	Letter-A	99.48	96.45	99.24	99.35	99.35	86	3
1	Satimage	73.46	68.72	73.46	73.93	73.93	63	21
USPS		98.44	98.44	98.13	98.44	98.75	4328	13
MNIST-8		97.63	97.02	97.74	97.63	97.74	83,339	1,048

is under-sampling, which eliminates some majority class instances mostly by random sampling.

Even though both approaches address the class imbalance problem, they also suffer some drawbacks. The under-sampling strategy can potentially sacrifice the prediction performance of the model, since it is possible to discard informative instances that the learner might benefit. Oversampling strategy, on the other hand, can be computationally overwhelming in cases with large training sets— if a complex oversampling method is used, a large computational effort must be expended during preprocessing of the data. Worse, oversampling causes longer training time during the learning process due to the increased number of training instances. In addition to suffering from increased runtime due to added computational complexity, it also necessitates an increased memory footprint due to the extra storage requirements of artificial instances. Other costs associated with the learning process (i.e., extended kernel matrix in kernel classification algorithms) further increase the burden of oversampling.

7.4.1 VIRTUAL: Virtual Instance Resampling Technique Using Active Learning

In this section, the focus is on the oversampling strategy for imbalanced data classification and investigate how it can benefit from the principles of active learning. Our goal is to remedy the efficiency drawbacks of oversampling in imbalanced data classification and use an active learning strategy to generate minority class instances only if they can be useful to the learner. Virtual (Virtual Instance Resampling Technique Using Active Learning) [16] is a hybrid method of oversampling and active learning that forms an adaptive technique for resampling of the minority class instances. In contrast to traditional oversampling techniques that act as an offline step that generate virtual instances of the minority class prior to the training process, VIRTUAL leverages the power of active learning to intelligently and adaptively oversample the data during training, removing the need for an offline and separate preprocessing stage. Similar to the discussions in the previous section, Virtual also employs an online SVM-based active learning strategy. In this setting, the informativeness of instances are measured by their distance to their hyperplane, and the most informative instances are selected as the support vectors. VIRTUAL targets the set of support vectors during training, and resamples new instances based on this set. Since most support vectors are found during early stages of training, corresponding virtual examples are also created in the early stages. This prevents the algorithm from creating excessive and redundant virtual instances and integrating the resampling process into the training stage improves the efficiency and generalization performance of the learner compared to other competitive oversampling techniques.

7.4.1.1 Active Selection of Instances Let S denote the pool of real and virtual training examples unseen by the learner at each active learning step. Instead of searching for the most informative instance among all the samples in S, VIRTUAL employs the small-pool active learning strategy that discussed in section 7.3.2. From the small pool, VIRTUAL selects an instance that is closest to the hyperplane according to the current model. If the selected instance is a real positive instance (from the original training data) and becomes a support vector, VIRTUAL advances to the oversampling step, explained in the following section. Otherwise, the algorithm proceeds to the next iteration to select another instance.

Algorithm 7.1 VIRTUAL

```
Define: X = \{x_1, x_2, \cdots, x_n\}: training instances X_R^+: positive real training instances S: pool of training instances for SVM v: \# virtual instances to create in each iteration L: size of the small set of randomly picked samples for active sample selection
```

```
1. Initialize S \leftarrow X
2. while S \neq \emptyset
        // Active sample selection step
3.
        d_{min} \leftarrow \infty
4.
       for i \leftarrow 1 to L
5.
           x_i \leftarrow RandomSelect(S)
6.
7.
           If d(x_j, hyperplane) < d_{min}
8.
                d_{min} \leftarrow d(x_j, hyperplane)
9.
                candidate \leftarrow x_i
10.
            end
11.
        end
        x_s \leftarrow candidate
12.
        // Virtual Instance Generation
13.
        If x_s becomes SV and x_s \in X_R^+
14.
15.
            K \leftarrow k nearest neighbors of x_s
            for i \leftarrow 1 to v
16.
                x_m \leftarrow RandomSelect(K)
17.
                // Create a virtual positive instance x_{s,m}^v between x_s and x_m
18.
19.
                \lambda=random number between 0 and 1
                x_{s,m}^v = \lambda \cdot x_s + (1 - \lambda)x_m
20.
21.
                S \leftarrow S \cup x_{s,m}^v
22.
            end
23.
        end
24.
        S \leftarrow S - x_s
25. end
```

7.4.1.2 Virtual Instance Generation VIRTUAL oversamples real minority instances (instances selected from the minority class of the original training data) which become support vectors in the current iteration. It selects the k nearest minority class neighbors $(x_{i\rightarrow 1}\cdots x_{i\rightarrow k})$ of x_i based on their similarities in the kernel transformed higher dimensional feature space. We limit the neighboring instances of x_i to the minority class so that the new virtual instances lie within the minority class distribution. Depending on the amount of over-sampling required, the algorithm creates v virtual instances. Each virtual instance lies on any of the line segments joining x_i and its neighbor $x_{i\rightarrow j}$ (j=1,...,k). In other words, a neighbor $x_{i\rightarrow j}$ is randomly picked and the virtual instance is created as $\bar{x}_v = \lambda \cdot x_i + (1-\lambda)x_{i\rightarrow j}$, where $\lambda \in (0,1)$ determines the placement of \bar{x}_v between x_i and $x_{i\rightarrow j}$. All v virtual instances are added to S and are eligible to be picked by the active learner in the subsequent iterations.

The pseudocode of Virtual given in Algorithm 7.1 depicts the two processes described above. In the beginning, the pool S contains all real instances in the training set. At the end of each iteration, the instance selected is removed from S, and any virtual instances generated are included in the pool S. In this pseudocode, Virtual terminates when there are no instances in S.

7.4.2 Remarks on VIRTUAL

We compare VIRTUAL with a popular oversampling technique SMOTE. Figure 7.7 shows the different behavior of how SMOTE and VIRTUAL create virtual instances for the minority class. SMOTE creates virtual instance(s) for each positive example (see Figure 7.7(a)), whereas VIRTUAL creates the majority of virtual instances around the positive canonical hyperplane (shown with a dashed line in Figure 7.7(b)). Note that a large portion of virtual instances created by SMOTE are far away from the hyperplane and thus are not likely to be selected as support vectors. VIRTUAL, on the other hand, generates virtual instances near the real positive support vectors adaptively in the learning process. Hence the virtual instances are near the hyperplane and thus are more informative.

We further analyze the computation complexity of SMOTE and VIRTUAL. The computation complexity of VIRTUAL is $O(|SV(+)| \cdot v \cdot \mathcal{C})$, where v is the number of virtual instances created for a real positive support vector in each iteration, |SV(+)| is the number of positive support vectors and \mathcal{C} is the cost of finding k nearest neighbors. The computation complexity of SMOTE is $O(|X_R^+| \cdot v \cdot \mathcal{C})$, where $|X_R^+|$ is the number of positive training instances. \mathcal{C} depends on the approach for finding k nearest neighbors. The naive implementation searches all N training instances for the nearest neighbors and thus $\mathcal{C} = kN$. Using advanced data structure such as kd-tree, $\mathcal{C} = k \log N$. Since |SV(+)| is typically much less than $|X_R^+|$, VIRTUAL incurs lower computation overhead than SMOTE. Also, with fewer virtual instances

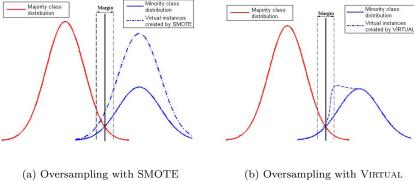
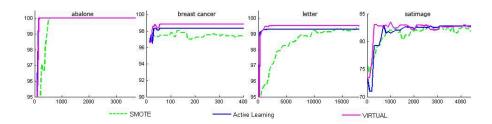


Figure 7.7 Comparison of oversampling the minority class with SMOTE and VIRTUAL.



Comparison of SMOTE, AL and VIRTUAL on UCI datasets. We present the g-means (%) (y-axis) of the current model for the test set vs. the number of training samples (x-axis) seen.

created, the learner is less burdened with VIRTUAL. We demonstrate with empirical results that the virtual instances created with VIRTUAL are more informative and the prediction performance is also improved.

7.4.3 **Experiments**

We conduct a series of experiments on Reuters-21578 and four UCI datasets to demonstrate the efficacy of VIRTUAL. The characteristics of the datasets are detailed in [16]. We compare VIRTUAL with two systems, Active Learning (AL) and SMOTE. AL adopts the traditional active learning strategy without preprocessing or creating any virtual instances during learning. SMOTE, on the other hand, preprocesses the data by creating virtual instances before training and uses random sampling in learning. Experiments elicit the advantages of adaptive virtual sample creation in Virtual.

In Figures 7.8 and 7.9 provide details on the behavior of the three algorithms, SMOTE, AL and VIRTUAL. For the Reuters datasets (Figure 7.9), note that in all the 10 categories VIRTUAL outperforms AL in g-means metric after saturation. The difference in performance is most pronounced in the more imbalanced categories, e.g. corn, interest and ship. In the less imbalanced datasets such as acq and earn, the difference in g-means of both methods is less noticeable. The g-means of SMOTE converges much slower than both AL and VIRTUAL. However, SMOTE converges to higher g-means than AL in some of the categories, indicating that the virtual positive examples provide additional information that can be used to improve the model. VIRTUAL converges to the same or even higher g-means than SMOTE while generating fewer virtual instances. For the UCI datasets (Figure 7.8), VIRTUAL performs as well as AL in abalone in g-means and consistently outperforms AL and SMOTE in the other three datasets.

In Table 7.3, the support vector imbalance ratio of all the three methods are lower than the data imbalance ratio, and VIRTUAL achieves the most balanced ratios of positive and negative support vectors in the Reuters datasets. Despite that the datasets used have different data distributions, the portion of virtual instances which become support vectors in VIRTUAL consistently and significantly higher than that in SMOTE. These results confirm the previous discussion that VIRTUAL is more effective in generating informative virtual instances.

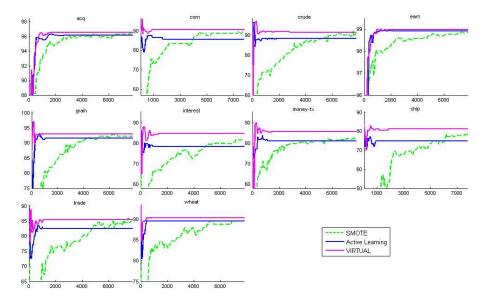


Figure 7.9 Comparison of SMOTE, AL and VIRTUAL on 10 largest categories of *Reuters-21578*. We show the g-means (%) (y-axis) of the current model for the test set versus the number of training samples (x-axis) seen.

Table 7.3 Support vectors with SMOTE (SMT), AL and VIRTUAL. Imb.Rt. is the data imbalance ratio and #SV(-)/#SV(+) represents the support vector imbalance ratio. The rightmost two columns compare the portion of the virtual instances selected as support vectors in SMOTE and VIRTUAL.

Dataset		Imb.	#S	V(-)/#	±SV(+)	$\#SV_V(+)/\#V.I.$			
		Rt.	SMT	AL	Virtual	SMT	Virtual		
	acq	3.7	1.24	1.28	1.18	2.4%	$\boldsymbol{20.3\%}$		
	corn	41.9	2.29	3.08	1.95	17.1%	$\boldsymbol{36.6\%}$		
	crude	19.0	2.30	2.68	2.00	10.8%	50.4 %		
	earn	1.7	1.68	1.89	1.67	6.0%	24.2 %		
er	grain	16.9	2.62	3.06	2.32	7.2%	$\boldsymbol{42.3\%}$		
Reuters	interest	21.4	1.84	2.16	1.66	13.3%	$\boldsymbol{72.2\%}$		
R	money-fx	13.4	1.86	2.17	1.34	8.2%	31.1%		
	$_{ m ship}$	38.4	3.45	4.48	2.80	20.0%	$\boldsymbol{66.5\%}$		
	trade	20.1	1.89	2.26	1.72	15.4%	$\boldsymbol{26.6\%}$		
	wheat	35.7	2.55	3.43	2.22	12.3%	$\boldsymbol{63.9\%}$		
	abalone	9.7	0.99	1.24	0.99	30.4%	69.2 %		
NCI	breast	1.9	1.23	0.60	0.64	2.9%	$\boldsymbol{39.5\%}$		
	letter	24.4	1.21	1.48	0.97	0.98%	$\boldsymbol{74.4\%}$		
	satimage	9.7	1.31	1.93	0.92	37.3%	53.8 %		

Table 7.4 presents g-means and the total learning time for SMOTE, AL and VIRTUAL. Classical batch SVM's g-means values are also provided as a reference point. In Reuters datasets, VIRTUAL yields the highest g-means in all categories. Table 4 shows the effectiveness of adaptive virtual instance generation. In categories corn, interest and ship with high class imbalance ratio, VIRTUAL gains substantial improvement in g-means. Compared to AL, VIRTUAL requires additional time for the creation of virtual instances and selection of those which may become support vectors. Despite this overhead, VIRTUAL's training times are comparable with that of AL. In the cases where minority examples are abundant, SMOTE demands substantially longer time to create virtual instances than VIRTUAL. But as the rightmost columns in Table 3 show, only a small fraction of the virtual instances created by SMOTE become support vectors. Therefore SMOTE spends much time to create virtual instances that will not be used in the model. On the other hand, VIRTUAL has already a short training time and uses this time to create more informative virtual instances. In Table 4, the numbers in parentheses give the ranks of the g-means prediction performance of the four approaches. The values in bold correspond to a win and VIRTUAL wins in nearly all datasets. The Wilcoxon signed-rank test (2-tailed) between Virtual and its nearest competitor SMOTE reveals that the zero median hypothesis can be rejected at the significance level 1% ($p = 4.82 \times 10^{-4}$), implying that

Table 7.4 g-means and total learning time using SMOTE, AL and VIRTUAL. 'Batch' corresponds to the classical SVM learning in batch setting without resampling. The numbers in brackets denote the rank of the corresponding method in the dataset.

Dataset			Total learning time (sec.)								
	Dataset	Batch	SMOTE	AL	Virtual	SMOTE	AL	Virtual			
Reuters	acq	96.19 (3)	96.21 (2)	96.19 (3)	96.54(1)	2271	146	203			
	corn	85.55 (4)	89.62 (2)	86.59 (3)	90.60(1)	74	43	66			
	crude	88.34 (4)	91.21 (2)	88.35 (3)	91.74(1)	238	113	129			
	earn	98.92 (3)	98.97 (1)	98.92 (3)	98.97 (1)	4082	121	163			
	grain	91.56 (4)	92.29 (2)	91.56 (4)	93.00 (1)	296	134	143			
	interest	78.45 (4)	83.96 (2)	78.45 (4)	84.75 (1)	192	153	178			
\mathbb{R}	money-fx	81.43 (3)	83.70 (2)	81.08 (4)	85.61 (1)	363	93	116			
	ship	75.66 (3)	78.55 (2)	74.92 (4)	81.34 (1)	88	75	76			
	trade	82.52 (3)	84.52 (2)	82.52 (3)	85.48 (1)	292	72	131			
	wheat	89.54 (3)	89.50 (4)	89.55 (2)	90.27 (1)	64	29	48			
IOU	abalone	100 (1)	100 (1)	100 (1)	100 (1)	18	4	6			
	breast	98.33 (2)	97.52 (4)	98.33 (2)	98.84 (1)	4	1	1			
	letter	99.28 (3)	99.42 (2)	99.28 (3)	99.54(1)	83	5	6			
	satimage	83.57 (1)	82.61 (4)	82.76 (3)	82.92 (2)	219	18	17			

VIRTUAL performs statistically better than SMOTE in these 14 datasets. These results demonstrate the importance of creating synthetic samples from the informative examples rather than all the examples.

7.5 DIFFICULTIES WITH EXTREME CLASS IMBALANCE

Practical applications rarely provide us with data that have equal numbers of training instances of all the classes. However, in many applications, the imbalance in the distribution of naturally occurring instances is extreme. For example, when labeling web pages to identify specific content of interest, uninteresting pages may outnumber interesting ones by a million to one or worse (consider identifying web pages containing hate speech, in order to keep advertisers off them, cf. [4]).

Prior sections have detailed techniques that have been developed to cope with moderate class imbalance. However, as class imbalances tends towards the extreme, active learning strategies can fail completely— and this failure is not simply due to the challenges of learning models with skewed class distributions, which has received a good bit of study and has been addressed throughout this book. The lack of labeled data compounds the problem, because techniques cannot concentrate on the minority instances, as the techniques are unaware which instances to focus on.

Figure 7.10 compares the AUC of logistic regression text classifiers induced by labeled instances selected with uncertainty sampling and with random sampling. The learning task is to differentiate sports web pages from nonsports pages. Depending on the source of the data (e.g., different impression streams from different on-line advertisers), one could see very different degrees of class skew in the population of relevant web pages. The panels in Figure 7.10, left-to-right, depict increasing amounts of induced class skew. On the far left, we see that for a balanced class distribution, uncertainty sampling is indeed better than random sampling. For a 10:1 distribution, uncertainty sampling has some problems very early on, but soon does better than random sampling—even more so than in the balanced case. However, as the skew begins to get large, not only does random sampling start to fail (it finds fewer and fewer minority instances, and its learning suffers), uncertainty sampling does substantially worse than random for a considerable amount labeling expenditure. In the most extreme case shown, both random sampling and uncertainty sampling simply fail completely. Random sampling effectively does not select any positive examples, and neither does uncertainty sampling.⁷

A practitioner well-versed in the active learning literature may decide she should use a method other than uncertainty sampling in such a highly skewed domain. A variety of techniques have been discussed in Sections 7.2, 7.3, and 7.4 for performing active learning specifically under class imbalance, including [54, 5, 62, 17, 23], as well as for performing density-sensitive active learning, where the geometry of the problem space is specifically included when making selections, including [63, 11, 41, 61, 36]. While initially appealing, as problems become increasingly difficult, these techniques may not provide results better than more traditional active learning techniques—indeed class skews may be sufficiently high as to thwart these techniques completely [4].

As discussed later in Section 7.8.1, Attenberg and Provost [4] proposed an alternative way of using human resources to produce labeled training set, specifically tasking people with finding class-specific instances ("guided learning") as opposed to labeling specific instances. In some domains, finding such instances may even be cheaper than labeling (per instance). Guided learning can be much more effective per instance acquired; in one of Attenberg and Provost's experiments it outperformed active learning as long as searching for class-specific instances was less than eight times more expensive (per instance) than labeling selected instances. The generalization performance of guided learning is shown in Figure 7.12, discussed below in Section 7.8.1 for the same setting as Figure 7.10.

 $^{^6}$ 10,000:1 — still orders of magnitude less skewed than some categories

 $^{^{7}}$ The curious behavior of AUC< 0.5 here is due to overfitting. Regularizing the logistic regression "fixes" the problem, and the curve hovers about 0.5. See another article in this issue for more insight on models exhibiting AUC< 0.5 [42].

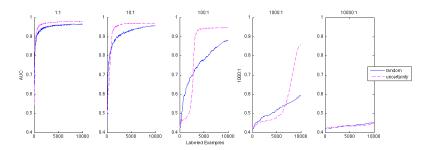


Figure 7.10 Comparison of random sampling and uncertainty sampling on the same data set with induced skews ranging from 1:1 to 10,000:1.

7.6 DEALING WITH DISJUNCTIVE CLASSES

Even more subtly still, certain problem spaces may not have such an extreme class skew, but may still be particularly difficult because they possess important but very small disjunctive sub-concepts, rather than simple continuously dense regions of minority and majority instances. Prior research has shown that such "small disjuncts" can comprise a large portion of a target class in some domains [59]. For active learning, these small subconcepts act like rare classes: if a learner has seen no instances of the subconcept, how can it "know" which instances to label? Note that this is not simply a problem of using the wrong loss function: in an active learning setting, the learner does not even know that the instances of the subconcept are misclassified if no instances of a subconcept have yet been labeled. Nonetheless, in a research setting (where we know all the labels) using an undiscriminative loss function, such as classification accuracy or even the area under the ROC curve (AUC), may result in the researcher not even realizing that an important subconcept has been missed.

To demonstrate how small disjuncts influence (active) model learning, consider the following text classification problem: separating the science articles from the non-science articles within a subset of the 20 Newsgroups benchmark set (with an induced class skew of 80 to 1). Figure 7.11 examines graphically the relative positions of the minority instances through the active learning. The black curve shows the AUC (right vertical axis) of the models learned by a logistic regression classifier using uncertainty sampling, rescaled as follows. At each epoch we sort all instances by their predicted probability of membership in the majority class, $\hat{P}(y=0|x)$. The blue dots in Figure 7.11 represent the minority class instances, with the value on the left vertical axis showing their relative position in this sorted list. The x-axis shows the active learning epoch (here each epoch requests 30 new instances from the pool). The blue trajectories mostly show instances' relative positions changing. Minority instances drop down to the very bottom (certain minority) either because

they get chosen for labeling, or because labeling some other instance caused the model to "realize" that they are minority instances.

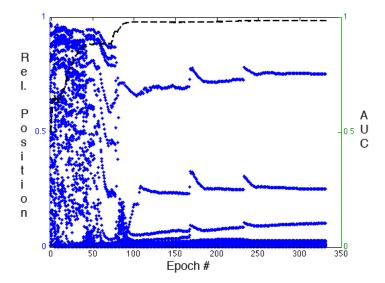


Figure 7.11 A comparison of the learned model's ordering of the instance pool, along with the quality of the cross-validated AUC.

We see that, early on, the minority instances are mixed all throughout the range of estimated probabilities, even as the generalization accuracy increases. Then the model becomes good enough that, abruptly, few minority class instances are misclassified (above $\hat{P}=0.5$). This is the point where the learning curve levels off for the first time. However, notice that there still are some residual misclassified minority instances, and in particular that there is a cluster of them for which the model is relatively certain they are majority instances. It takes many epochs for the active learning to select one of these, at which point the generalization performance increases markedly—apparently, this was a subconcept that was strongly misclassified by the model, and so it was not a high priority for exploration by the active learning.

On the 20 Newsgroups data set we can examine the minority instances for which \hat{P} decreases the most in that late rise in the AUC curve (roughly, they *switch* from being misclassified on the lower plateau to being correctly classified afterward). Recall that the minority (positive) class here is "Science" newsgroups. It turns out that these late-switching instances are members of the cryptography (sci.crpyt) subcategory. These pages were classified as non-Science presumably because before having seen any positive instances of the subcategory, they looked much more like the many computer-oriented subcategories in the (much more prevalent) non-Science category. As soon as

a few were labeled as Science, the model generalized its notion of Science to include this subcategory (apparently pretty well).

Density-sensitive active learning techniques did not improve upon uncertainty sampling for this particular domain. This was surprising, given the support we have just provided for our intuition that the concepts are disjunctive. One would expect a density-oriented technique to be appropriate for this domain. Unfortunately in this domain—and we conjecture that this is typical of many domains with extreme class imbalance—the *majority* class is even more disjunctive than the minority class. For example, in 20 Newsgroups, Science indeed has four very different subclasses. However, non-Science has 16 (with much more variety). Techniques that (for example) try to find as-of-yet unexplored clusters in the instance space are likely to select from the vast and varied majority class. We need more research on dealing with highly disjunctive classes, especially when the less interesting class is more varied than the main class of interest.

7.7 STARTING COLD

The *cold start problem* has long been known to be a key difficulty in building effective classifiers quickly and cheaply via active learning [63, 12]. Since the quality of data selection directly depends on the understanding of the space provided by the "current" model, early stages of acquisitions can result in a vicious cycle of uninformative selections, leading to poor quality models and therefore additional poor selections.

The difficulties posed by the cold start problem can be particularly acute in highly skewed or disjunctive problem spaces; informative instances may be difficult for active learning to find due to their variety or rarity, potentially leading to substantial waste in data selection. Difficulties early in the active learning process can, at least in part, be attributed to the base classifier's poor understanding of the problem space. This cold start problem is particularly acute in otherwise difficult domains. Since the value of subsequent label selections depends on base learner's understanding of the problem space, poor selections in the early phases of active learning propagate their harm across the learning curve.

In many research papers active learning experiments are "primed" with a preselected, often class-balanced training set. As pointed out by [4] if the possibility and procedure exists to procure a class-balanced training set to start the process, maybe the most cost-effective model-development alternative is not to do active learning at all, but to just continue using this procedure. This is exemplified in Figure 7.12 [4], where the red lines show the effect of investing resources to continue to procure a class-balanced, but

⁸How interesting a class is could be measured by its relative misclassification cost, for example.

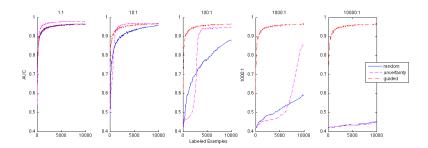


Figure 7.12 Comparison of random sampling and uncertainty sampling and guided learning on the problem seen in Figure 7.10.

otherwise random, training set (as compared with the active acquisition shown in Figure 7.10).

7.8 ALTERNATIVES TO ACTIVE LEARNING FOR IMBALANCED PROBLEMS

In addition to traditional label acquisition for unlabeled examples, there are other sorts of data that may be acquired at a cost for the purpose of building or improving statistical models. The intent of this section is to provide the reader with an brief overview of some alternative techniques for active data acquisition for predictive model construction in a cost-restrictive setting. We begin this setting with a discussion of class-conditional example acquisition, a paradigm related to active learning where examples are drawn from some available unlabeled pool in accordance to some predefined classproportion. We then go on in Section 7.8.2 to touch on active feature labeling and active dual supervision. These two paradigms attempt to to replace or supplement traditional supervised learning with class-specific associations on certain feature values. While this set of techniques require specialized models, significant generalization performance can often be achieved at a reasonable cost by leveraging explicit feature/class relationships. This is often appealing in the active setting, where it is occasionally less challenging to identify classindicative feature values than it is to find quality training data for labeling, particularly in the imbalanced setting.

7.8.1 Class-Conditional Example Acquisition

Imagine as an alternative to the traditional active learning problem setting, where an oracle is queried in order to assign examples to specially selected unlabeled examples, a setting where an oracle is charged with selecting exemplars from the underlaying problem space in accordance to some predefined class ratio. Consider as a motivational example, the problem

of building predictive models based on data collected through an "artificial nose" with the intent of "sniffing out" explosive or hazardous chemical compounds [33, 35, 34]. In this setting, the reactivity of a large number of chemicals is already known, representing label-conditioned pools of available instances. However, producing these chemicals in a laboratory setting and running the resultant compound through the artificial nose may be an expensive, time-consuming process. While this problem may seem quite unique, many data acquisition tasks may be cast into a similar framework.

A much more general issue in selective data acquisition is the amount of control ceded to the "oracle" doing the acquisition. The work discussed so far assumes that an oracle will be queried for some specific value, and the oracle simply returns that value. However, if the oracle is actually a person, he or she may be able to apply considerable intelligence and other resources to "guide" the selection. Such guidance is especially helpful in situations where some aspect of the data is rare—where purely data-driven strategies are particularly challenged.

As discussed throughout this work, in many practical settings, one class is quite rare. As an example motivating the application of class-conditional example acquisition in practice, consider building a predictive model from scratch designed to classify web pages containing a particular topic of interest. While large absolute numbers of such web pages may be present on the web, they may be outnumbered by uninteresting pages by a million to one or worse (take, for instance, the task of detecting and removing hate speech from the web [4]). As discussed in Section 7.5, such extremely imbalanced problem settings present a particularly insidious difficulty for traditional active learning techniques. In a setting with a 10,000 : 1 class ratio, a reasonably large labeling budget could be expended with out observing a single minority example.⁹

Previously, we have discussed a plethora of active learning techniques specifically tuned for the high-skew setting [54, 5, 62, 17] as well as techniques where the geometry and feature-density of the problem space are explicitly included when making instance selections [63, 23, 11, 41, 61, 36], these techniques, as initially appealing as they may seem, may fail just as badly as traditional active learning techniques. Class skew and subconcept rarity discussed in Section 7.6 may be sufficient to thwart them completely [4, 3].

However, in many of these extremely difficult settings, we can task humans to search the problem space for rare cases, using tools (like search engines) and possibly interacting with the base learner. Consider the motivating example of hate speech classification on the web (from above). While an active learner may experience difficulty exploring the details of this rare class, a human oracle armed with a search interface is likely to expose examples of hate speech quite easily. In fact, given the coverage of modern web search engines, a human

 $^{^{9}}$ Note that in practice, such extremely imbalanced problem settings may actually be quite common.

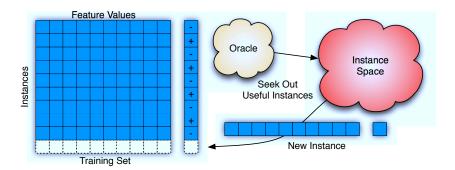


Figure 7.13 Guided Learning: An oracle selecting useful examples from the instance space.

can produce interesting examples from a much larger sample of the problem space, far beyond that which is likely to be contained in a sample pool for active learning. This is critical due to hardware-imposed constraints on the size of the pool an active learner is able to choose from—e.g., a random draw of several hundred thousand examples from the problem space may not even contain any members of the minority class or of rare disjuncts!

Guided Learning is the general process of utilizing oracles to search the problem space, using their domain expertise to seek instances representing the interesting regions of the problem space. Figure 7.13 presents the general guided learning setting. Here, given some interface enabling search over the domain in question, an oracle searches for interesting examples, which are either supplemented with an implicit label by the oracle, or sent for explicit labeling as a second step. These examples are then added to the training set and a model is retrained. Oracles can leverage their background knowledge of the problem being faced. In addition to simply being charged with the acquisition of class-specific examples, by allowing the oracle to interact with the base learner, confusing instances, those that "fool" the model can be sought out from the problem space and used for subsequent training in a form of human-guided uncertainty sampling. This interaction with the base learner can be extended a step further—by allowing humans to challenge the predictive accuracy of the problem space may potentially reveal "problem areas", portions of the example space where the base model performs poorly that might not be revealed through traditional techniques such as crossvalidation studies [1].

Guided learning, along with alternative problem settings such as that faced by the artificial nose discussed above deal with situations where an oracle is able to provide "random" examples in arbitrary class proportions. It now becomes interesting to consider just what this class proportion should be? This problem appears to face the inverse of the difficulties faced by active

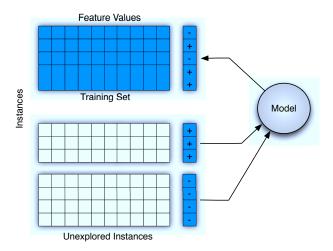


Figure 7.14 Active Class Selection: Gathering instances from random class-conditioned fonts in a proportion believed to offer greatest improvement in generalization performance.

learning—labels essentially come for free, while the independent feature values are *completely* unknown and must be gathered at a cost. In this setting, it becomes important to consider the question: "In what proportion should classes be represented in a training set of a certain size?" [58].

Let's call the problem of proportioning class labels in a selection of nadditional training instances, "Active Class Selection" (ACS) [33, 35, 34, 58]. This process is exemplified in Figure 7.14. In this setting, there is assumed to be large, class-conditioned (virtual) pools of available instances with completely hidden feature values. At each epoch, t, of the ACS process, the task is to leverage the current model when selecting examples from these pools in a proportion believed to have the greatest effectiveness for improving the generalization performance of this model. The feature values for each instance are then collected and the complete instances are added to the training set. The model is reconstructed and the processes is repeated until n examples are obtained (because the budget is exhausted or some other stopping criterion is met, such as a computational limit). Note that this situation can be considered to be a special case of the instance completion setting of Active Feature-Value Acquisition (cf. [38]). It is a degenerate special case because, prior to selection, there is no information at all about the instances other than their classes.

For the specific problem at the heart of ACS, the extreme lack of information to guide selection leads to the development of unique uncertainty and utility estimators, which, in the absence of predictive covariates, require

unique approximations.¹⁰ While alternative approaches to active class selection have emerged, for thematic clarity, uncertainty-based and expected utility-based approaches will be presented first. Note that because effective classification requires that both sides of a prediction boundary be represented, unlike typical active learning techniques, active class selection typically samples classes from their respective score distributions [45, 46].

7.8.1.1 Uncertainty-based approaches This family of techniques for performing active class selection is based on the volatility in the predictions made about certain classes—those classes whose cross-validated predictions are subject to the most change between successive epochs of instance selection are likely to be based upon an uncertain predictor, and amenable to refinement by the incorporation of additional training data [33, 34]. Analogous to the case of more traditional uncertainty-based data acquisition, several heuristics have been devised to capture the notion of variability.

One measure of the uncertainty of a learned model is how volatile its predictive performance is in the face of new training data. Take a typical learning curve, for instance, those presented in Figure 7.6. Notice that the modeling is much more volatile at the left side of the figure, showing large changes in generalization performance for the same amount of new training data. We can think that as the predictor gains knowledge of the problem space, it tends to solidify in the face of data, exhibiting less change and greater certainty. For ACS, we might wonder if the learning curves will be equally steep regardless of the class of the training data [33, 35, 34]. With this in mind, we can select instances at epoch t from the classes in proportion to their improvements in accuracy at t-1 and t-2. For example, we could use cross-validation to estimate the generalization performance of the classifier with respect to each class, $\mathcal{A}(c)$; class c can then be sampled according to:

$$p_{\mathcal{A}}^t(c) \propto \frac{\max\left\{0, \mathcal{A}^{t-1}(c) - \mathcal{A}^{t-2}(c)\right\}}{\sum_{c'} \max\left\{0, \mathcal{A}^{t-1}(c') - \mathcal{A}^{t-2}(c')\right\}},$$

Alternatively, we could consider general volatility in class members' predicted labels, beyond improvement in the model's ability to predict the class. Again, by using cross-validated predictions at successive epochs, it is possible to isolate members of each class, and observe changes in the predicted class for each instance. For example, when the predicted label of a given instance changes between successive epochs, we can deem the instance to have been redistricted [33, 35, 34]. Again considering the level of volatility in a model's predictions to be a measurement of uncertainty, we can sample classes at epoch t according to each classes' proportional measure of redistricting:

¹⁰In realistic settings, for instance, such as potential application for ACS, guided learning, this lack of information assumption may be softened.

$$p_{\mathcal{R}}^{t}(c) \propto \frac{\frac{1}{|c|} \sum_{x \in c} \mathbb{I}(f^{t-1}(x) \neq f^{t-2}(x))}{\sum_{c'} \frac{1}{|c'|} \sum_{x \in c'} \mathbb{I}(f^{t-1}(x) \neq f^{t-2}(x))},$$

where $\mathbb{I}(\cdot)$ is an indicator function taking the value of 1 if its argument is true and 0 otherwise. $f^{t-1}(x)$ and $f^{t-2}(x)$ are the predicted labels for instance x from the models trained at epoch t-1 and t-2, respectively [33, 35, 34].

7.8.1.2 Expected Class Utility The previously described active class selection heuristics are reliant on the assumption that adding examples belonging to a particular class will improve the predictive accuracy with respect to that class. This does not directly estimate the utility of adding members of a particular class to a model's overall performance. Instead, it may be preferable to select classes whose instances' presence in the training set will reduce a model's misclassification cost by the greatest amount in expectation.

Let $cost(c_i|c_j)$ be the cost of predicting c_i on an instance x whose true label is c_j . Then the expected empirical misclassification cost over a sample dataset, \mathbb{D} , is:

$$\hat{R} = \frac{1}{|\mathbb{D}|} \sum_{x \in \mathbb{D}} \sum_{i} \hat{P}(c_i|x) \operatorname{cost}(c_i|y),$$

Where y is the correct class for a given x. Typically in the active class selection setting, this expectation would be taken over the training set (e.g. $\mathbb{D} = T$), preferably using cross-validation. In order to reduce this risk, we would like to select examples from class c leading to the greatest reduction in this expected risk [35].

Consider a predictive model $\hat{P}^{T \cup c}(\cdot|x)$, a model built on the training set, T, supplemented with an arbitrary example belonging to class c. Given the opportunity to choose an additional class-representative example to the training pool, we would like to select the class that reduces expected risk by the greatest amount:

$$\bar{c} = \arg\max_{c} U(c),$$

Where

$$U(c) = \frac{1}{|\mathbb{D}|} \sum_{x \in \mathbb{D}} \sum_{i} \hat{P}^{T}(c_i|x) \operatorname{cost}(c_i|y) - \frac{1}{|\mathbb{D}|} \sum_{x \in \mathbb{D}} \sum_{i} \hat{P}^{T \cup c}(c_i|x) \operatorname{cost}(c_i|y).$$

Of course the benefit of adding additional examples on a test dataset is unknown. Furthermore, the impact of a particular class's examples may vary depending on the feature values of particular instances. In order to cope with these issues, we can estimate via cross-validation on the training set. Using sampling, we can try various class-conditional additions and compute the expected benefit of a class across that class's representatives in T, assessed on the testing folds. The above utility then becomes:

$$\hat{U}(c) = E_{x \in c} \left[\frac{1}{|\mathbb{D}|} \sum_{x \in \mathbb{D}} \sum_{i} \hat{P}^{T}(c_{i}|x) \operatorname{cost}(c_{i}|y) - \frac{1}{|\mathbb{D}|} \sum_{x \in \mathbb{D}} \sum_{i} \hat{P}^{T \cup c}(c_{i}|x) \operatorname{cost}(c_{i}|y) \right].$$

Note that it is often preferred to add examples in batch. In this case, we may wish to sample from the classes in proportion to their respective utilities:

$$p_{\hat{U}}^t(c) \propto \frac{\hat{U}(c)}{\sum_{c'} \hat{U}(c')}.$$

Further, diverse class-conditional acquisition costs can be incorporated, utilizing $\frac{\hat{U}(c)}{\omega_c}$ in place of $\hat{U}(c)$, where ω_c is the (expected) cost of acquiring the feature vector of an example in class c.

7.8.1.3 Alternative Approaches to ACS In addition to uncertainty-based and utility-based techniques, there are several alternative techniques for performing active class selection. Motivated by empirical results showing that barring any domain-specific information, when collecting examples for a training set of size n, a balanced class distribution tends to offer reasonable AUC on test data [57, 58], a reasonable baseline approach to active class selection is simply to select classes in balanced proportion.

Search strategies may alternately be employed in order to reveal the most effective class ratio at each epoch. Utilizing a nested cross-validation on the training set, the space of class ratios can be explored, with the most favorable ratio being utilized at each epoch. Note that it is not possible to explore all possible class ratios in all epochs, without eventually spending too much on one class or another. Thus, as we approach n we can narrow the range of class ratios, assuming that there is a problem-optimal class ratio that will become more apparent as we obtain more data [58].

It should be noted that many techniques employed for building classification models assume an identical or similar training and test distribution. Violating this assumption may lead to biased predictions on test data where classes preferentially represented in the training data are predicted more frequently. In particular increasing the prior probability of a class increases the posterior probability of the class, moving the classification boundary for that class so that more cases are classified into that class" [47, 43]. Thus in settings where instances are selected specifically in proportions different from those seen in the wild, posterior probability estimates should be properly calibrated to be aligned with the test data, if possible [43, 15, 58].

Prior work has repeatedly demonstrated the benefits of performing active class selection beyond simply selecting random examples from an example pool for acquisition or simply using uniformly balanced selection. However, in many cases, simply casting what would typically be an active learning problem into an ACS problem, and selecting examples uniformly amongst the classes can provide results far better than what would be possible with active learning alone. For instance, the learning curves presented in Figure 7.12 compare such uniform guided learning with active learning and simple random selection. Providing the model with an essentially random but class-balanced training set far exceeds the generalization performance possible for an active learning strategy or by random selection once the class skew becomes substantial. More intelligent ACS strategies may make this difference even more pronounced, and should be considered if the development effort associated with incorporating such strategies would be outweighed by the savings coming from reduced data acquisition costs.

7.8.2 Feature-Based Learning and Active Dual Supervision

While traditional supervised learning is by far the most prevalent classification paradigm encountered in the research literature, it is not the only approach for incorporating human knowledge into a predictive system. By leveraging, for instance, class associations with certain feature values, predictive systems can be trained that offer potentially excellent generalization performance without requiring the assignment of class labels to individual instances. Consider, the example domain of predicting the sentiment of movie reviews. In this context it is clear that the presence of words like "amazing" and "thrilling" carry an association with the positive class, while terms like "boring" and "disappointing" evoke negative sentiment [2]. Gathering this kind of annotation leverages an oracle's prior experience with the class polarity of certain feature values—in this case, the emotion that certain terms tend to evoke. The systematic selection of feature-values for labeling by a machine learning system is referred to as Active Feature-Value Labeling, ¹¹ (AFL). The general setting where class associations are actively sought for both featurevalues and particular examples is known as Active Dual Supervisions (ADS). The process of selection for AFL and ADS can be seen in Figures 7.15 and 7.16 respectively.

Of course, incorporating the class polarities associated with certain feature-values typically requires specialized models who's functional form has been designed to leverage feature-based background knowledge. While a survey of models for incorporating such feature-value/class polarities is beyond the scope of this chapter, an interested reader is advised to seek any number of related papers (cf. [37, 48, 60, 32, 10, 28, 13]). However, while sophisticated

 $^{^{11}}$ For brevity, this is often shortened to Active Feature Labeling, a moniker that is best suited for domains with binary features

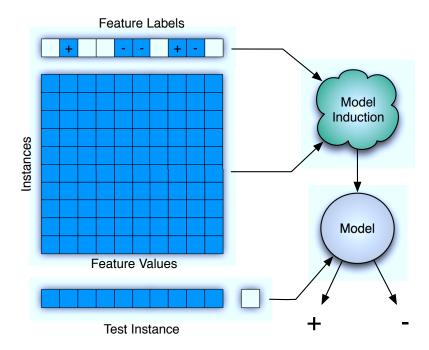


Figure 7.15 Active Feature-Value Labeling: Selecting feature-values for association with a certain class-polarity

models of this type have been devised, practical, yet very simple solutions are easy to imagine. Consider at the most basic extreme class assignment rules, where the presence of certain feature-values (or combinations of feature-values) connect an example with a particular class. A rudimentary "dual" model can also be constructed by simply averaging the class predictions of a traditional supervised machine learning model with a model based on feature-value/class relationships.

Given the option of incorporating prior knowledge associated with certain feature-values into the predictive behavior of a model, the question then becomes: which feature examples should be selected for labeling? Initially, this may seem to be a deflection, replacing the search for useful information of one kind with another. However, there are several reasons why seeking feature-values to "label" may be preferred to labeling examples. The most obvious reason for selecting feature-values for labeling is that traditional is often "slow". It may take many labeled movie reviews to teach the model that "amazing" has a positive association, and not the other uninformative terms that just happen to occur in positive reviews by coincidence. In this way, the

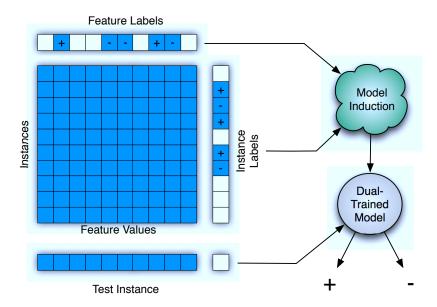


Figure 7.16 Active Dual Supervision: Simultaneous acquisition of label information for both feature-values and instances

cold-start problem faced in active learning¹² may be less acute in AFL and ADS— while complex feature/class relationships may be difficult to achieve using AFL, reasonable generalization performance is often achievable with few requests to an oracle. Second, the labor costs associated with assigning a class polarity to a certain feature value is often quite low— it is easy for a human to associate the term "terrible" with negative movie reviews, while labeling one particular movie review as positive or negative requires reading the entire document. Note, that of course not every term is polar; in the ongoing example of movie reviews, it is easy to imagine that a few terms have a natural association with the positive or negative class, while most terms on their own don't have such a polarity. However, this imbalance between polar and non-polar feature values is often far less acute than the imbalance between classes in many machine learning problems. A problem domain with a class imbalance of 1,000,000: 1 may still have one in ten features exhibit some meaningful class association. Perhaps even more importantly, the ratio between positively and negatively linked feature-values (for instance) may be far more balanced than the ratio between those classes in the wild. In fact, there is not necessarily a relationship between the base rate and the ratio

¹²Recall Section 7.7

of strongly identifiable positively and negatively associated feature values. While selecting useful feature-values in practice is still often a challenging problem, experience has shown that it is often more informative to select random features for labeling than random examples. More intelligent selection heuristics can make this preference for AFL and ADS even stronger.

Again, giving a thorough survey of selection heuristics for performing AFL is beyond the scope of this chapter. However, we will provide the reader with a brief overview of the techniques typically employed for such tasks. As in many selective data acquisition tasks for machine learning, we see two common themes: uncertainty-based selection and expected-utility-based approaches. Below we will briefly present some more popular techniques for AFL delineated accordingly. We will then briefly discuss techniques for ADS, selecting features and examples for labeling simultaneously.

7.8.2.1 Uncertainty-based AFL By far the most prevalent class of active learning heuristics, uncertainty-based approaches do not have a direct analogue to the selection problem faced in ADS; feature-value/class associations factor quite differently into the formation and training of machine learning models than more traditional example labels. Nonetheless, thematically similar approaches to the ADS problem have been developed, where features are selected to labeling according to some notion of stability within the current model. As in traditional uncertainty selection in active learning, the primary difference amongst these uncertainty-based ADS techniques is the manner by which feature uncertainty is estimated. In the simplest case, using a linear model, the coefficients on particular feature values may be interpreted as a measure of uncertainty, with lower coefficient magnitude corresponding to a greater degree of uncertainty [52]. The Naíve Bayes-like model used presented in [37] presents a more appealing option—feature-value label uncertainty can be measured by looking at the magnitude of the log-odds of the feature-value likelihoods: $\left|\log\frac{p(f|+)}{p(f|-)}\right|$ for feature-value f and classes + and -. Again, a smaller value corresponds to increased uncertainty.

A range of other techniques for Uncertainty-based AFL exist. Through the creation of one-term pseudo-documents Godbole et al [21] coerce the notion of feature label uncertainty into a more traditional instance uncertainty framework for text classification tasks. By incorporating label information on each feature-value with unlabeled examples, Druck et al. [13] create a corresponding Generalized Expectation term that rates the model's predicted class distribution conditioned on the presence of the particular feature. This rating penalizes these predicted class distributions according to their KL-divergence from reference distributions constructed using labeled features. Similarly, Liang et al. [30] learn from labeled examples and actively selected constraints in the form of expectations with some associated noise from particular examples. Druck et al. [14] analyze several uncertainty-based selection techniques for gathering feature labels when training conditional random fields. Finding that the total uncertainty (measured as the sum of

the marginal entropies) tends to favor more frequent features. As a remedy, they propose an uncertainty scheme where the mean uncertainty is weighted by the log of the counts of the associated feature values.

Interestingly, even for reasonable uncertainty estimators, feature/class uncertainty may not be a desirable criterion for selection. Consider the above discussion regarding the preponderance of uninformative features. Clearly, in the case of document classification, terms like "of" and "the" will seldom have any class polarity. At the same time, these terms are likely to have a high degree of uncertainty, leaving uncertainty-based approaches to perform poorly in practice. Preferring to select features based on *certainty*, that is, selecting those features with the least uncertainty seems to work much better in practice [52, 39].

7.8.2.2 Expected Utility-based AFL As with traditional uncertainty (and certainty) sampling for active learning, the query corresponding to the greatest level of uncertainty my not necessarily be the query offering the greatest level of information to the model. This is particularly true in noisy or complex environments. Instead of using a heuristic to estimate the information value of a particular feature label, it is possible to estimate this quantity directly. Let q enumerate over all possible feature-values that may be queried for labels. We can estimate the expected utility of such a query by: $EU(q) = \sum_{k=1}^{K} P(q = c_k) \frac{U(q=c_k)}{\omega_q}$, where $P(q = c_k)$ is the probability of the instance or feature queried being associated with class c_k , ω_q is the cost of query q and \mathcal{U} is some measure of the utility of q.¹³ This results in the decision-theoretic optimal policy, which is to ask for feature labels which, once incorporated into the data, will result in the highest increase in classification performance in expectation [2, 39].

7.8.2.3 Active Dual Supervision Active dual supervision is concerned with situations where it is possible to query an oracle for labels associated with both feature-values and examples. Even though such a paradigm is concerned with the simultaneous acquisition of feature and example labels, the simplest approach is treating each acquisition problem separately, and then mixing selections somehow. Active interleaving is performs a separate (un)certinaty-based ordering on features and on examples, and chooses selections from the top of each ordering according to some predefined proportion. The different nature of feature-value and example uncertainty values lead to incompatible quantities existing on different scales, preventing a single, unified ordering. However, expected utility can be used to compute a single unified metric encapsulating the value of both types of data acquisition. As above, we are estimating the utility of a certain feature of example query q as: $EU(q) = \sum_{k=1}^{K} P(q=c_k) \frac{U(q=c_k)}{\omega_q}$. Using a single utility function for both features and

¹³For instance, cross-validated accuracy or log-gain may be used.

examples and incorporating label acquisition costs, costs and benefits of the different types of acquisition can be optimized directly [2].

7.9 CONCLUSION

This chapter presents a broad perspective on the relationship between active learning — the selective acquisition of labeled examples for training statistical models — and imbalanced data classification tasks where at least one of the classes in the training set is represented with much fewer instances than the other classes. Our comprehensive analysis of this relationship leads to the identification of two common associations, namely, (i) the ability of active learning to deal with the data imbalance problem that, when manifested in a training set, typically degrades the generalization performance of an induced model, and (ii) the impact class imbalance may have on the abilities of an otherwise reasonable active learning scheme to select informative examples, a phenomenon that is particularly acute as the imbalance tends towards the extreme.

Mitigating the impact of class imbalance on the generalization performance of an a predictive model, in Sections 7.3 and 7.4 we present active learning as an alternative to more conventional resampling strategies. An active learning strategy may select a data set that is both balanced and extremely informative in terms of model training. Early stopping criteria for halting the selection process of active learning can further improve the generalization of induced models. That is, a model trained on a small but informative subsample often offers performance far exceeding what can be achieved by training on a large dataset drawn from the natural, skewed base rate. The abilities of active learning to provide small, balanced training from large, but imbalanced problems are enhanced further by Virtual, introduced in Section 7.4. Here, artificially generated instances supplement the pool of examples available to the active selection mechanism.

Additionally, it is noted throughout that abilities of an active learning system tend to degrade as the imbalance of the underlying distribution increases. While at more moderate imbalances, the quality of the resulting training set may still be sufficient to provide usable statistical models, but at more substantial class imbalances, it may be difficult for a system based upon active learning to produce accurate models. Throughout Sections 7.2, 7.3, and 7.4, we illustrate a variety of active learning techniques specially adapted for imbalanced settings, techniques that may be considered by practitioners facing difficult problems. In Sections 7.5 and 7.6, we note that as a problem's class imbalance tends towards the extreme, the selective abilities of an active learning heuristic may fail completely. We present several alternative approaches for data acquisition in Section 7.8, mechanisms that may alleviate the difficulties active learning faces in problematic domains. Among these alternatives are guided learning and active class selection in Subsection 7.8.1,

and using associations between specific feature values and certain classes in Subsection 7.8.2.

Class imbalance presents a challenge to statistical models and to machine learning systems in general. Because the abilities of these models are so tightly coupled with the data used for training, it is crucial to consider the selection process that generates this data. This chapter discusses specifically this problem. It is clear that when building models for challenging imbalanced domains, active learning is an aspect of the approach that shouldn't be ignored.

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