Contents


```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% ASEN 3111 - CA4
%
% Created By: Johnathan Tucker
%
% Collaborators:
%
% The purpose of the script is to act as a driver that will execute the
% functions necesary to solve questions two and three of CA4. Note that the
% solution to question 1 is the PLLT function itself
%
% Created Date: 4/7/2020
%
% Change Log:
%           - 4/7/2020 Code the PLLT function
%           - 4/8/2020 Code up questions two and three
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Housekeeping

```
clc;
clear all;
close all;
tic
%Global formatting commands to imporve graphing looks:
set(groot,'defaulttextinterpreter','latex');
set(groot,'defaultAxesTickLabelInterpreter','latex');
set(groot,'defaultLegendInterpreter','latex');
```

Question 1 Solution

Please see the attached PLLT equation

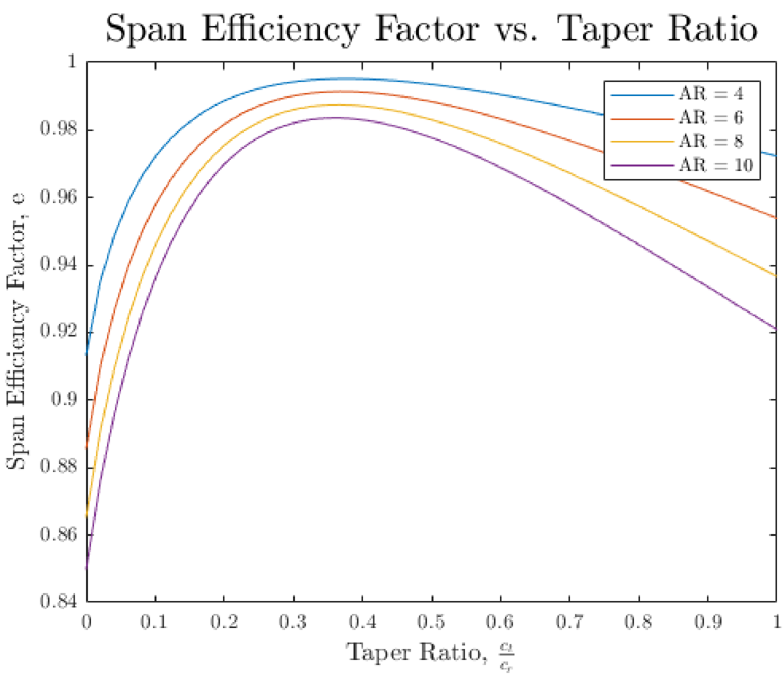
Question 2 Solution

```
Question_2();
```

```
For 1000 odd terms at sea level and a velocity of 150 mph:
Lift = 103887.052554 [N]
Induced Drag = 1918.144772 [N]

The number of odd terms required for a 5% relative error in
the lift and induced drag solutions is: 4
The number of odd terms required for a 5% relative error in
the lift and induced drag solutions is: 7

The number of odd terms required for a 1% relative error in
the lift solution is: 9
The number of odd terms required for a 1% relative error in
the induced drag solution is: 16

The number of odd terms required for a 0.1% relative error in
the lift solutions is: 27
The number of odd terms required for a 0.1% relative error in
the induced drag solutions is: 48
```

Question 3 Solution

```
Question_3();
```



Functions Called

The following functions were built and called as a part of this assignment

```
function [e,c_L,c_Di] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N)
%PLLT Performs Prandtl lifting line theory calculations that take
% advantage of the Fourier sine series to obtain circulation. This
% ultimately leads to the calculation and output of span efficiency factor,
% coefficient of lift, and coefficient of induced drag.
%
% Author: Johnathan Tucker
%
% Collaborators: N/A

% Inputs:
%           b: Span [ft]
%           a0_t: cross-sectional lift slope at the wingtips [-]
%           a0_r: cross-sectional lift slope at the wing roots [-]
%           c_t: chord at the tips [ft]
%           c_r: chord at the wing root [ft]
%           aero_t: zero-lift angle of attack at tips [deg]
%           aero_r: zero-lift angle of attack at wing root [de]
%           geo_t: geometric angle of attack at tips [deg]
%           geo_r: geometric angle of attack at wing root [deg]
%           N: number of odd terms in series
%
% Outputs:
%           e: span efficiency factor [-]
%           c_L: coefficient of lift [-]
%           c_Di: coefficient of induced drag [-]
% Last Revised: 3/26/2020
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Convert all variables to compatible units
aero_t = aero_t*(pi/180);
aero_r = aero_r*(pi/180);
geo_t = geo_t*(pi/180);
geo_r = geo_r*(pi/180);
%% First create the theta vector
i = 1:1:N;
theta_vec = i.*(pi./(2*N));
%% Now create anonymous functions or vectors for wing properties
% Wing Taper
c = @(theta) c_r - (c_r - c_t).*cos(theta);
% Variable cross-sectional lift slope
a_0 = @(theta) a0_r - (a0_r - a0_t).*cos(theta);
```

```
% Aerodynamic Twist
alpha_lzero = aero_r - (aero_r - aero_t).*cos(theta_vec);
% Geometric Twist
alpha = geo_r - (geo_r - geo_t).*cos(theta_vec);

%% Now Create the matrix to solve for the Fourier Coefficients
% Preallocate to save time
A = zeros(N,N);
% Prior to this I'll create a vector that is N in length of odd values
odds = 1:2:2*N;
% First we need to iterate through every theta value
for j = 1:N
    % Then we'll need to iterate through every coefficient multiplier
    % corresponding to this theta value
    for k = 1:N
        A(j,k) = (4*b*sin(odds(k)*theta_vec(j)))/...
            (a_0(theta_vec(j))*c(theta_vec(j))) + odds(k)*sin(odds(k)*...
            theta_vec(j))/sin(theta_vec(j));
    end
end
%% With the A matrix we can now solve for the for the Fourier Coefficients
% First get the "b" vector using the geometric and aerodynamic twist
b_vec = alpha - alpha_lzero;
coeffs = A\b_vec';

%% Now get the required outputs
% Get the aspect ratio
AR = (b^2)/((c_r + c_t)*(b/2));

% Get the c_L
c_L = coeffs(1)*pi*AR;

% Get the delta vector
delta = odds(2:end).*(coeffs(2:end)'./coeffs(1)').^2;
delta = sum(delta);
% Use delta to solve for span efficiency
e = 1/(1+delta);

% Finally calculate c_Di
c_Di = (c_L^2)/(pi*e*AR);

end



function [Cl, CP] = Vortex_Panel(x,y,V_inf,alpha,plotcp,increment)
%Vortex_Panel Performs the calculations detailed in the Kuethe and Chow
%document
%
% Author: Johnathan Tucker
%
% Collaborators: N/A
%
% Corrected using the reference solution posetd on canvas.
%
% This function takes in the x and y values from the NACA airfoil function,
% the free stream velocity, the angle of attack, plotcp flag, and plot
% increments. It outputs the coefficient of pressure, coefficient of lift,
% and displays a plot of coefficient of pressure vs x/c
%
% Last Revised: 3/26/2020
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Get the chord length for calculating Cl
c = max(x) - min(x);

% Begin the translation from 4tran to MATLAB
% Create necessary variables
M = length(x) - 1;
MP1 = M + 1;
alpha = alpha *(pi/180);


for I = 1:M
    IP1 = I + 1;
    X(I) = 0.5*(x(I) + x(IP1));
    Y(I) = 0.5*(y(I) + y(IP1));
```

```
        S(I) = sqrt( (x(IP1) - x(I))^2 + (y(IP1) - y(I))^2 );
        theta(I) = atan2( (y(IP1) - y(I)) , (x(IP1) - x(I)));
        sine(I) = sin(theta(I));
        cosine(I) = cos(theta(I));
    end
    RHS = sin(theta - alpha);

    for I = 1:M
        for J = 1:M
            if I == J
                CN1(I,J) = -1;
                CN2(I,J) = 1;
                CT1(I,J) = 0.5*pi;
                CT2(I,J) = 0.5*pi;
            else
                A = -(X(I) - x(J))*cosine(J) - (Y(I) - y(J))*sine(J);
                B = (X(I) - x(J))^2 + (Y(I) - y(J))^2;
                C = sin(theta(I) - theta(J));
                D = cos(theta(I) - theta(J));
                E = (X(I) - x(J))*sine(J) - (Y(I) - y(J))*cosine(J);
                F = log(1 + S(J)*(S(J) + 2.*A)/B);
                G = atan2(E*S(J), B + A*S(J));
                P = (X(I) - x(J))*sin(theta(I) - 2.*theta(J)) +...
                    (Y(I) - y(J))*cos(theta(I) - 2.*theta(J));
                Q = (X(I) - x(J))*cos(theta(I) - 2.*theta(J)) -...
                    (Y(I) - y(J))*sin(theta(I) - 2.*theta(J));
                CN2(I,J) = D + .5*Q*F/S(J) - (A*C + D*E)*G/S(J);
                CN1(I,J) = .5*D*F + C*G - CN2(I,J);
                CT2(I,J) = C + .5*P*F/S(J) + (A*D - C*E)*G/S(J);
                CT1(I,J) = .5*C*F - D*G - CT2(I,J);
            end
        end
    end

    for  I = 1:M
         AN(I,1) = CN1(I,1);
         AN(I,MP1) = CN2(I,M);
         AT(I,1) = CT1(I,1);
         AT(I,MP1) = CT2(I,M);
         for J = 2:M
             AN(I,J) = CN1(I,J) + CN2(I,J-1);
             AT(I,J) = CT1(I,J) + CT2(I,J-1);
         end
    end

    AN(MP1,1) = 1;
    AN(MP1,MP1) = 1;
    for J = 2:M
        AN(MP1,J) = 0;
    end

    RHS(MP1) = 0;

    % Solve the system of equations using A\b instead of Cramers
    GAMA = AN\RHS';

    for I = 1:M
        V(I) = cos(theta(I) - alpha);
        for J = 1:MP1
            V(I) = V(I) + AT(I,J)*GAMA(J);
            CP(I) = 1 - V(I)^2;
        end
    end
    % Change from gamma prime to gamma via Kuethe and Chow
    Gamma = 0;
    for i = 1:M
        Gamma = Gamma + 2*pi*0.5*(GAMA(i) + GAMA(i+1))*S(i);
    end
    % Solve for CL by calculating capital GAMMA inline using CA3 Notes formulas
    Cl = 2*Gamma;
    %% Create the pressure plot
    % This flag is if only one cp plot is wanted
    if plotcp == 1
        figure
        cp_lower = CP(1:(length(x)+1)/2);
        cp_upper = CP((length(x)+1)/2:end);
```

```
        scatter(x((length(x)+1)/2:end-1)./c,cp_upper,'r')
        hold on
        scatter(x(1:(length(x)+1)/2)./c,cp_lower,'b')
        title('$Coefficeint\:of\:Pressure\:vs\:\\frac{x}{c}$','Interpreter','latex')
        xlabel('$x-distance\:[\% Chord]$','Interpreter','latex')
        ylabel('$Coefficient\:of\:Presure$','Interpreter','latex')
        legend('$Upper\:Surface$','$Lower\:Surface$','Interpreter','latex')

    % This flag is for the cp subplots required for question two
    elseif plotcp == 2
        hold on
        subplot(2,2,increment)
        cp_lower = -CP(1:(length(x)+1)/2);
        cp_upper = -CP((length(x)+1)/2:end);
        scatter(x(1:(length(x)+1)/2)./c,cp_lower,'b')
        hold on
        scatter(x((length(x)+1)/2:end-1)./c,cp_upper,'r')
        title(strcat('CP vs $\frac{x}{c}$ with: $\alpha$ = ',num2str(alpha*180/pi),'$^\circ$'),'Interpreter','latex');
        sgtitle('CP vs $\frac{x}{c}$ at Different $\alpha$ Values','Interpreter','latex');
        xlabel('$\frac{x}{c}$','Interpreter','latex')
        ylabel('$-CP$','Interpreter','latex')
        legend('$Lower\:Surface$','$Upper\:Surface$','Interpreter','latex')
    end

    end


    function Question_2()
    %Question_2 Performs all calculations and outputs for question 2 in CA4
    %
    % Author: Johnathan Tucker
    %
    % Collaborators: N/A
    %
    % This function has no inputs or direct outputs. However, it does display
    % the number of odd numbers required to achieve 5, 1, and 0.1 percent relative
    % error between an "exact" Lift and induced drag and the calculatedd Lift
    % and induced drag.
    %
    % Last Revised: 4/8/2020
    %% Create constants
    b = 100; % [ft]
    c_r = 15; % [ft]
    c_t = 5; % [ft]
    geo_r = 5; % [deg]
    geo_t = 0; % [deg]
    V_inf = 150*5280/3600; % [ft/s]
    rho_SL = 0.0023769; % [slugs/ft^3]
    S = (c_r + c_t)*(b/2); % [ft^2]
    %% Get the root and tip aerodynamic twist
    % First get the x and y values for each airfoil
    [naca_0012_x,naca_0012_y] = NACA_Airfoil(0/100,0/10,12/100,1,150);
    [naca_2412_x,naca_2412_y] = NACA_Airfoil(2/100,4/10,12/100,1,150);

    % Then get the coefficient of lift for the airfoils for each angle of
    % attack
    aoa_vec = linspace(-5,10);
    for i = 1:length(aoa_vec)
        [Cl_0012(i),~] = Vortex_Panel(naca_0012_x,naca_0012_y,V_inf,aoa_vec(i),0,0);
        [Cl_2412(i),~] = Vortex_Panel(naca_2412_x,naca_2412_y,V_inf,aoa_vec(i),0,0);
    end

    % Now get aero_r and a0_r
    fit_2412 = polyfit(aoa_vec, Cl_2412,1);
    aero_r = fit_2412(2)/fit_2412(1);
    a0_r = fit_2412(1)*180/pi;

    % Now get aero_r and a0_r
    fit_0012 = polyfit(aoa_vec, Cl_0012,1);
    aero_t = -fit_0012(2)/fit_0012(1);
    a0_t = fit_0012(1)*180/pi;

    %% Begin error calculations
    % Now get an "exact" Lift and induced drag using a high number of panels
    N = 1000;
    [~,exact_cl,exact_cdi] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N);
```

```
exact_L = 0.5*rho_SL*(V_inf^2)*S*exact_cl;
exact_Di = 0.5*rho_SL*(V_inf^2)*S*exact_cdi;

exact_L_SI = 0.5*rho_SL*(V_inf^2)*S*exact_cl*4.44822;
exact_Di_SI = 0.5*rho_SL*(V_inf^2)*S*exact_cdi*4.44822;
% Print out the calculated lift and drag in Newtons
fprintf("For 1000 odd terms at sea level and a velocity of 150 mph:\n");
fprintf("Lift = %f [N]\n",exact_L_SI)
fprintf("Induced Drag = %f [N]\n\n",exact_Di_SI)
% Using the exact values calculate the number of panels it takes to achieve
% 5, 1, and 0.1 percent relative error
N = 1;
error_L = 100;
error_Di = 100;

% Create a loop to get the 0.1 percent error
while true
    [~,cl,cdi] = PLLT(b,a0_t,a0_r,c_t,c_r,aero_t,aero_r,geo_t,geo_r,N);
    L = 0.5*rho_SL*(V_inf^2)*S*cl;
    Di = 0.5*rho_SL*(V_inf^2)*S*cdi;

    error_L(N) = (abs(L - exact_L)/exact_L) * 100;
    error_Di(N) = (abs(Di - exact_Di)/exact_Di) * 100;

    if error_L(N) <=0.1 && error_Di(N) <= 0.1
        break
    end
    N = N + 1;
end

% Now find the number of panels where the error is first below 5 percent
% for both Lift and Drag
num_five_perc_L = find(lt(error_L,5),1,'first');
num_five_perc_Di = find(lt(error_Di,5),1,'first');
% Print the number of panels need for five percent error
fprintf("The number of odd terms required for a 5%% relative error in \nthe lift and induced drag solutions is: %d\n",num_five_perc_L);
fprintf("The number of odd terms required for a 5%% relative error in \nthe lift and induced drag solutions is: %d\n\n",num_five_perc_Di);

% Now find the number of panels where the error is first below 1 percent
% for both Lift and Drag
num_one_perc_L = find(lt(error_L,1),1,'first');
num_one_perc_Di = find(lt(error_Di,1),1,'first');
% Print the number of panels need for five percent error
fprintf("The number of odd terms required for a 1%% relative error in \nthe lift solution is: %d\n",num_one_perc_L);
fprintf("The number of odd terms required for a 1%% relative error in \nthe induced drag solution is: %d\n\n",num_one_perc_Di);


% Finally display the number of panels needed to achieve a relative error
% less than 0.1 percent in both lift and drag
num_point_1_perc_L = find(lt(error_L,.1),1,'first');
fprintf("The number of odd terms required for a 0.1%% relative error in \nthe lift solutions is: %d\n",num_point_1_perc_L);
fprintf("The number of odd terms required for a 0.1%% relative error in \nthe induced drag solutions is: %d\n\n",N);
end


function Question_3()
%Question_3 Performs all calculations and outputs for question 3 in CA4
%
% Author: Johnathan Tucker
%
% Collaborators: N/A
%
% This function has no inputs or direct outputs. However, it does create a
% plot of span efficiency factor versus taper ratio for different aspect
% ratios.
%
% Last Revised: 4/8/2020
%% First Create any necessary
% Use some odd term number greater than 20
N = 50;

% Create a vector of taper ratios
taper_ratio_vec = linspace(0,1,N);

% Using thin airfoil theory with non varying lift slope
a0_r = 2*pi; % [rad]
```

```
a0_t = 2*pi; % [rad]

% Use a constant geometric aoa
geo_r = 5; % [deg]
geo_t = 5; % [deg]

% Vector of Aspect Ratios
AR_vec = [4,6,8,10];

% Assume a zero lift aoa of 0 deg
aero_r = 0; % [deg]
aero_t = 0; % [deg]

% The span should be constant reuse the span from Q2
b = 100; % [ft]

%% Solve for the c_t and c_r values at the constant span for each AR
% First iterate through each aspect ratio
for i = 1:length(AR_vec)
    % Solve for the c_r values
    c_r(i,:) = (2*b)./(AR_vec(i).*(1+taper_ratio_vec)');
end
% Solve for the c_t values
c_t = c_r.*taper_ratio_vec;

%% Solve for the span efficiency factor values
% Iterate through each aspect ratio value
for i = 1:length(AR_vec)
    % Iterate through the taper ratio vector
    for j = 1:length(taper_ratio_vec)
        % Calculate the span efficiency factor
        [e,~,~] = PLLT(b,a0_t,a0_r,c_t(i,j),c_r(i,j),aero_t,aero_r,geo_t,geo_r,N);
        e_vec(i,j) = e;
    end
end

%% Plot the results
figure
plot(taper_ratio_vec,e_vec(1,:))
hold on
plot(taper_ratio_vec,e_vec(2,:))
hold on
plot(taper_ratio_vec,e_vec(3,:))
hold on
plot(taper_ratio_vec,e_vec(4,:))
title("Span Efficiency Factor vs. Taper Ratio",'FontSize',18)
xlabel("Taper Ratio, $\frac{c_t}{c_r}$",'FontSize',12)
ylabel("Span Efficiency Factor, e",'FontSize',12)
legend("AR = 4", "AR = 6", "AR = 8", "AR = 10")

function [x,y] = NACA_Airfoil(m,p,t,c,N)
%NACA_Airfoil Performs the calculations necessary to get the x and y
%vectors that describe the specified NACA airfoil
%
% Author: Johnathan Tucker
%
% Collaborators: N/A

% This function takes in the max chord value "m", the location of max chord
% "p", the thickness "t", the chord length "c", and the number of panels to
% use "N". This function outputs the x and y vectors that describe the
% specified NACA airfoil.
%
% Last Revised: 3/26/2020
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Create vector for percentage of chord length
x = linspace(0,c,N/2);
% Create a vector of half thicknesses
y_t = (t/0.2)*c*(0.2969.*sqrt(x./c) - 0.1260.*(x./c) - 0.3516.*(x./c).^2 +...
    0.2843.*(x./c).^3 - 0.1036.*(x./c).^4);

% I need to find the index of x that is closest to the p*c value
[~,index] = min(abs(x-p*c));
% Now loop through the x values using a conditional statement that
% replicates the peicewise function
for i = 1:length(x)
```

```
        if i <= index
            y_c(i) = m.*(x(i)./p^2).*(2*p - x(i)./c);
        else
            y_c(i) = m.*((c-x(i))./(1-p)^2).*(1 + x(i)./c - 2*p);
        end
    end
    %Adding a check for NaN values
    y_c(isnan(y_c)) = 0;

    % Create zeta to solve for the upper and lower x/y values
    zeta = atan2(diff(y_c),diff(x));
    zeta = [zeta,0];
    % Solve for x_u and x_l
    x_u = x - y_t.*sin(zeta);
    x_l = x + y_t.*sin(zeta);

    % Solve for y_u and y_l
    y_u = y_c + y_t.*cos(zeta);
    y_l = y_c - y_t.*cos(zeta);

    % Combine upper and lower vectors to get final x and y
    x = [flip(x_u),x_l(2:end)];
    y = -[flip(y_u),y_l(2:end)];
    end
```

```
toc
```

```
Elapsed time is 6.291406 seconds.
```

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