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4.1 Równanie transportu ciepła

$$-k(x)\frac{d^2u(x)}{dx^2}=0$$

$$u(2)=0$$

$$\frac{du(0)}{dx}+u(0)=20$$

$$k(x)=\begin{cases}1 & \text{dla } x\in[0,1]\\ 2 & \text{dla } x\in(1,2]\end{cases}$$

$$-k(x)u''=0 \qquad |:(-k(x))$$

$$u''=0 \qquad |\cdot v(x)\ ,\ v\in V=\{f\in H^1: f(2)=0\}$$

$$v\cdot u''=0$$

$$\int_0^2 v\cdot u''\,dx=0$$

$$\int v\cdot u''\,dx=\left|\begin{matrix}f=v & f'=v'\\ g'=u'' & g=u'\end{matrix}\right|=v\cdot u'-\int v'\cdot u'\,dx$$

$$\left[v\cdot u'-\int v'u'\,dx\right]_0^2=\underbrace{\overset{0}{v(2)}u'(2)}_{0}-v(0)u'(0)-\int_0^2 v'u'\,dx$$

$$u'(0)+u(0)=20$$

$$u'(0)=20-u(0)$$

$$-v(0)\cdot(20-u(0))-\int_0^2 v'u'\,dx=0$$

$$v(0)\cdot u(0)-\int_0^2 v'u'\,dx=20v(0)$$

$$B(u, v) = v(0) \cdot u(0) - \int_0^2 v'u'\,dx$$

$$L(v) = 20 \cdot v(0)$$