Mathematical models in biology, 2021Q1
Mini-projects' proposals
B2: Single cell neurophysiology

Ferran Arqué, Dimitris Lagos

Consider model introduced in the `xpp` file `fhn-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).

2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.

3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.

4. Plot the $f - I$ curve in terms of I_{app}.

5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and a.

6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v, w)\, dt + \sigma\, dW_t,$$

where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\, dt + \sigma \sqrt{dt}\, \xi,$$

where ξ is a random number drawn from a $N(0, 1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0, 100]$ ms. Compute the average and the standard deviation for every point of the trajectory and plot the result.

   (b) Plot the $f - I$ curve in terms of I_{app} for $\sigma \in \{0.1\, j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

Hint: Use `xppaut` for the deterministic part and create your own `Matlab` for the stochastic part. I will provide you with some additional hints (videos or files) about (a) performing 2-parametric bifurcation diagrams and (b) generate random numbers from a Gaussian distribution.

Mathematical models in biology, 2021Q1
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B2: Single cell neurophysiology
Abraham Cano, Philip Mitchell

Consider model introduced in the `xpp` file `HHred-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).
2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.
3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.
4. Plot the $f - I$ curve in terms of I_{app}.
5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and $GNABAR$.
6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v, w)\, dt + \sigma\, dW_t,$$

where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\, dt + \sigma \sqrt{dt}\, \xi,$$

where ξ is a random number drawn from a $N(0, 1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0, 100]$ ms. Compute the average and the standard deviation for every point of the trajectory and plot the result.
   (b) Plot the $f - I$ curve in terms of I_{app} for $\sigma \in \{0.1\, j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

Hint: Use `xppaut` for the deterministic part and create your own `Matlab` for the stochastic part. I will provide you with some additional hints (videos or files) about (a) performing 2-parametric bifurcation diagrams and (b) generate random numbers from a Gaussian distribution.

Mathematical models in biology, 2021Q1
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B2: Single cell neurophysiology
Jaume Colom

Consider model introduced in the `xpp` file `ML1-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).

2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.

3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.

4. Plot the $f-I$ curve in terms of I_{app}.

5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and $v3$.

6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v,w)\,dt + \sigma\,dW_t,$$

   where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\,dt + \sigma\sqrt{dt}\,\xi,$$

   where ξ is a random number drawn from a $N(0,1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0,100]$ ms. Compute the average and the standard deviation for every point of the trajectory and plot the result.

   (b) Plot the $f-I$ curve in terms of I_{app} for $\sigma \in \{0.1\,j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

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Team 4

Consider model introduced in the `xpp` file `ML2-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).

2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.

3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.

4. Plot the $f-I$ curve in terms of I_{app}.

5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and $v3$.

6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v,w)\,dt + \sigma\,dW_t,$$

where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\,dt + \sigma\sqrt{dt}\,\xi,$$

where ξ is a random number drawn from a $N(0,1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0,100]\,\text{ms}$. Compute the average and the standard deviation for every point of the trajectory and plot the result.

   (b) Plot the $f-I$ curve in terms of I_{app} for $\sigma \in \{0.1\,j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

Hint: Use `xppaut` for the deterministic part and create your own `Matlab` for the stochastic part. I will provide you with some additional hints (videos or files) about (a) performing 2-parametric bifurcation diagrams and (b) generate random numbers from a Gaussian distribution.

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Team 5

Consider model introduced in the `xpp` file `ML3-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).
2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.
3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.
4. Plot the $f - I$ curve in terms of I_{app}.
5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and $v3$.
6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v, w)\, dt + \sigma\, dW_t,$$

where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\, dt + \sigma \sqrt{dt}\, \xi,$$

where ξ is a random number drawn from a $N(0, 1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0, 100]$ ms. Compute the average and the standard deviation for every point of the trajectory and plot the result.

   (b) Plot the $f - I$ curve in terms of I_{app} for $\sigma \in \{0.1\, j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

Hint: Use `xppaut` for the deterministic part and create your own `Matlab` for the stochastic part. I will provide you with some additional hints (videos or files) about (a) performing 2-parametric bifurcation diagrams and (b) generate random numbers from a Gaussian distribution.

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Team 6

Consider model introduced in the `xpp` file `INapH-2020.ode`.

1. Present the equations in a standard written layout (as in a paper; it is just a mere "translation"of the ODE file for the sake of presentation to the reader).
2. Perform the bifurcation diagram in terms of I_{app} and describe the bifurcations that you observe, emphasizing those involving limit cycles (appearance or vanishing), and give the I_{app} interval where we observe (a) subthreshold activity, (b) bistability and (c) spiking (action potentials) activity.
3. Take a representative I_{app} for each of the 2 or 3 (depending on the model) activity regions (a), (b) and (c). Localize and classify the equilibria (singular points) and classify them topologically.
4. Plot the $f - I$ curve in terms of I_{app}.
5. Perform the 2-parametric bifurcation diagram in terms of I_{app} and gNa.
6. Now, add Gaussian noise to the voltage equation. Mathematically speaking, one should write the corresponding differential equation as

$$dv = f(v, w)\,dt + \sigma\,dW_t,$$

where w is the gating variable, σ is the noise intensity and dW_t is an increment of the so-called *Wiener process*, which is Gaussian with zero mean and variance equal to 1 and proportional to dt. To avoid a crash course in stochastic differential equations (SDEs), the easiest implementation of this SDE is the *Euler-Maruyama* method consisting of applying the Euler's method adding the nature of the Gaussian noise:

$$v_{n+1} = v_n + f(v_n, w_n)\,dt + \sigma\sqrt{dt}\,\xi,$$

where ξ is a random number drawn from a $N(0, 1)$ distribution (this involves using a random number generator).

   (a) Take a value of I_{app} in the subthreshold region and obtain 100 realizations of $v(t)$, for $t \in [0, 100]$ ms. Compute the average and the standard deviation for every point of the trajectory and plot the result.

   (b) Plot the $f - I$ curve in terms of I_{app} for $\sigma \in \{0.1\,j\}_{j=1}^{20}$. What do you observe? Explain the differences with respect to σ.

Hint: Use `xppaut` for the deterministic part and create your own `Matlab` for the stochastic part. I will provide you with some additional hints (videos or files) about (a) performing 2-parametric bifurcation diagrams and (b) generate random numbers from a Gaussian distribution.