15.093 - OPTIMIZATION METHODS
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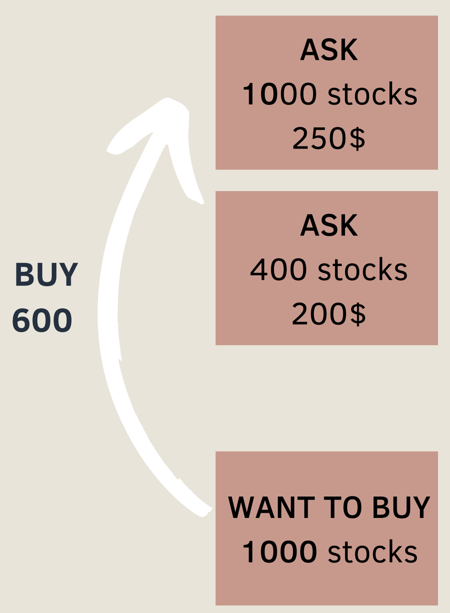
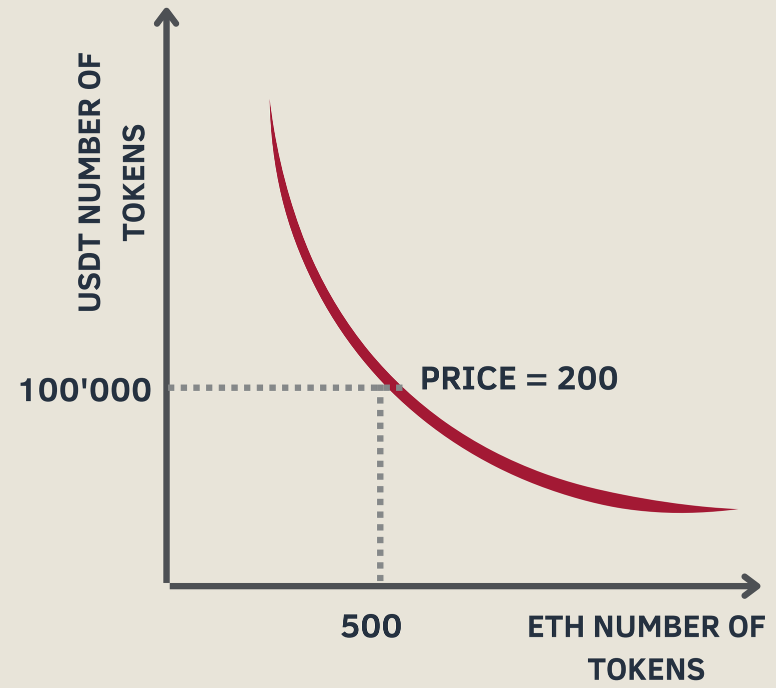
OPTIMAL TRADE EXECUTION IN AUTOMATED MARKET MAKERS

BACKGROUND

ORDER-BOOK MARKETS



AUTOMATED MARKET MAKERS



WHEN WE TRADE, PRICE MOVES IN BOTH MARKET STRUCTURES

WHY DO WE CARE

USD 1.24 BILLION ARE TRADED IN AMMS EVERY 24H

PRACTICAL RELEVANCE

- Crypto exchanges are generally illiquid. Large orders result in significant shifts in prices and high costs, especially in pools with low liquidity
- With growing interest in DeFI, fundraising involves the purchase and resale of project native tokens on AMMs. Optimization can make the financing of new projects possible

ACADEMIC RELEVANCE

- AMMs by construction operate with a Constant Product pricing formula xy =k. The structure of AMMs allows for interesting mathematical modeling in terms of optimization problems and price dynamics

PROBLEM FORMULATION

OPTIMIZING MARKET IMPACT

$$\min \sum_{j=1}^{n} p_j^{post} - p_j^{pre} = \min \sum_{j=1}^{n} \frac{y_j \Delta x_j (2x_j - \Delta x_j)}{x_j (x_j - \Delta x_j)^2}$$

$$\text{s.t} \sum_{j=1}^{n} \Delta x_j = \Delta x$$

$$\Delta x_j \geq 0$$

OPTIMIZING TOTAL COSTS

$$\min \sum_{j=1}^{n} \frac{y_j}{x_j - \Delta x_j} \frac{\Delta x_j}{\Delta x}$$

$$\text{s.t} \sum_{j=1}^{n} \Delta x_j = \Delta x$$

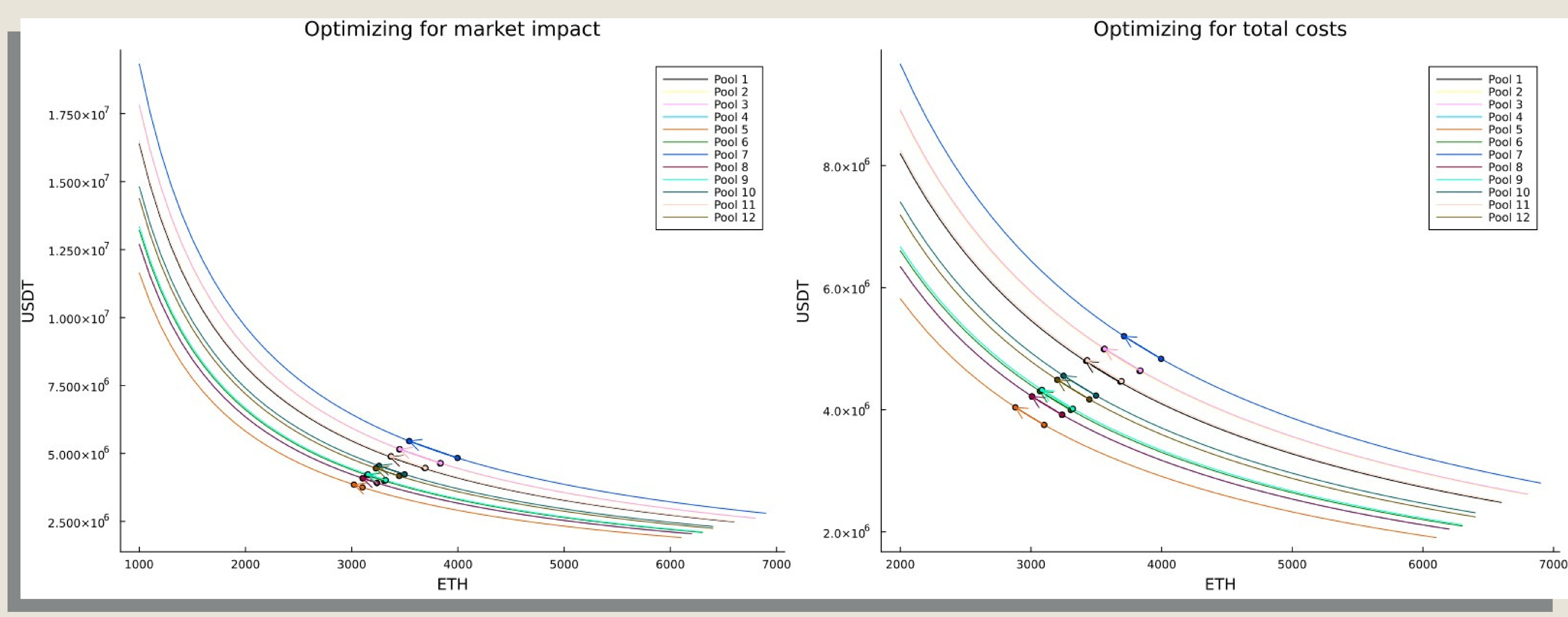
$$\Delta x_j \geq 0$$

DATA & EXPERIMENTS

- Experiments with one of the most widely traded cryptocurrencies, Ethereum (ETH) against the USD token (USDT)
- Market price, initial liquidity and trading fees extracted from Uniswap ETH: USDT pools. Liquidity curves are generated to preserve the magnitude of the constant product formula
- Optimizing for market impact and total costs across 12 AMMs, with and without trading fees. Different scenarios were generated in terms of the liquidity ratio of AMMs and initial market price. Optimization approach is compared to uniform splitting across AMMs or not splitting

FINDINGS

- Optimization approach has a significant edge over not splitting or uniformly splitting an order across AMMs
- The presence of transaction costs increases the necessity of optimized strategies
- Optimization approach can exploit arbitrage opportunities and stabilize the AMMs post-trade



IMPACT

- Improvement of 7% of total costs over baseline with even splits
- Improvement of 17% of total costs over baseline with no splits
- Reduction of market impact by a factor 1.7 over baseline with even splits
- Reduction of market impact by a factor 3.5 over baseline with no splits
- The inner structure of the AMMs allowed us to formulate two realistic optimization problems that are tractable computationally and can be extended to more complex formulations

STOCHASTIC MULTI-PERIOD OPTIMIZATION

Random Walk Approach

$$p_t = f(p_{t-1}, s_t) + \epsilon_t$$

- Solved the Bellman equation. Closed-form strategy derived: Under the simplifications, trade everything at time 0

Binomial Tree Approach

$$p_t^{pre} = r p_{t-1}^{post}$$
$$p_t^{post} = f(p_t^{pre}, s_t)$$

- Solved the two-period binomial model. The strategy depends on the probability of the price going up or down and

EXTENSIONS & FURTHER WORK

- Combine previous random walk stochastic approach over time with trading fees. The invariant k in xy=k becomes time dependent. Measure influence of this addition
- Solve the binomial tree stochastic approach over multiple time periods
- A robust optimization approach. Mix findings with an ML predictive model of prices. Identify an uncertainty set. Optimize the price impact function