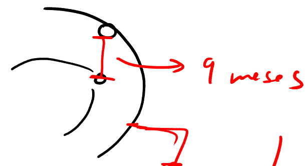
→ Secante → Bisección

→ Newton - Raphson

$f(x) = 0$



$$m\ddot{\vec{r}} = \frac{G M m}{r^2}\hat{e}_r \longleftrightarrow \vec{r} = (x, y) \longleftarrow r(t) \longrightarrow \boxed{\omega t - \psi - e\sin\psi = 0}$$

$r = \sqrt{x^2 + y^2}$

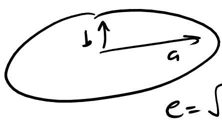
$\varphi = \arctan\left(\frac{y}{x}\right)$

$\psi(t)$

$\hookrightarrow r = a(1 + e\cos\psi)$

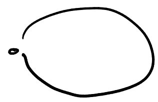
$x = a\cos\psi$

$y = b\sin\psi$



$e = \sqrt{1 - (b/a)^2}$

$\boxed{\omega t} \rightarrow [0, 2\pi]$



for i in rang(0, 2π)

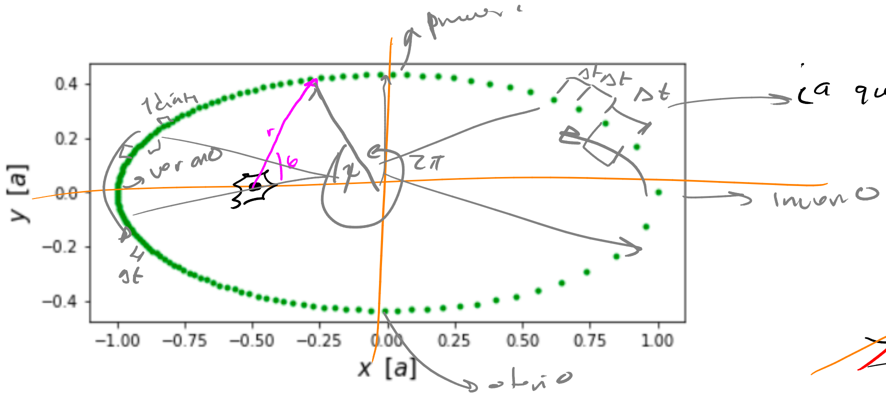
$$\boxed{f(\psi) = \psi - e\sin\psi - \boxed{i}}$$

$f(\psi; i) = 0$

$[i, \psi]$

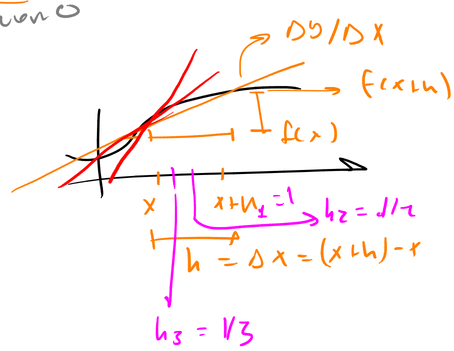
$i \rightarrow t\omega \rightarrow \psi(t\omega)$

$(0, 2\pi)$



¿a qué vel. va?

$$\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \vec{v}$$



$$f(x) \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\approx \frac{f(x+h) - f(x)}{h} \quad ; \quad 0 < h \ll 1 \qquad h = 0.001$$

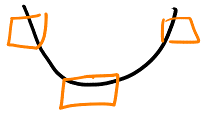
$$f(x+h) = f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \frac{1}{3\cdot 2}f'''(x)h^3 + O(h^4)$$

$$\Rightarrow \boxed{\frac{f(x+h) - f(x)}{h}} = f'(x) + \frac{1}{2}h f''(x) + \frac{1}{3\cdot 2}f'''(x)h^2 + O(h^3)$$

$e_h \sim \frac{|f(x)|}{|h|}\varepsilon_f$ Fraccional

truncamento $|f''(x)|\,h = e_t$

$$e_{ner} = e_h + e_t = \frac{|f(x)|}{h}\varepsilon_f + |f''(x)|\,h > 0$$

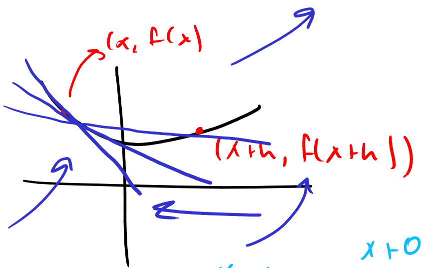


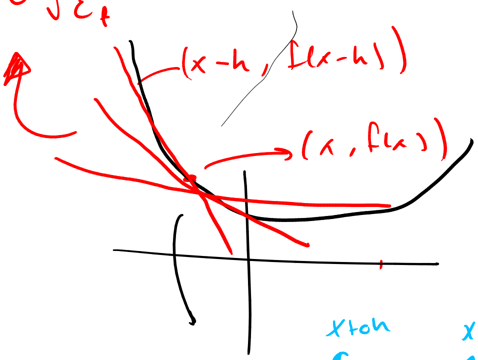
$$\frac{d[e_{ner}]}{dh} = \frac{|f(x)|\varepsilon_f}{-h^2} + |f''(x)| = 0 \Rightarrow h_{opt} = \sqrt{\frac{|f(x)|}{|f''(x)|}}\,\varepsilon_f^{1/2}$$

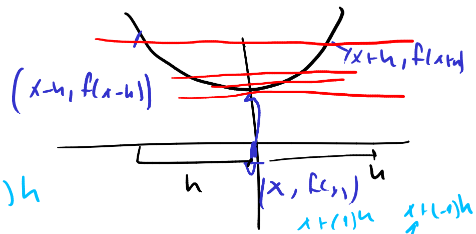
$$e_{ner}\Big|_{h_{opt}} = \left(\frac{|f(x)|\sqrt{|f''(x)|}}{\sqrt{|f(x)|}}\sqrt{\varepsilon_f} + |f''(x)|\sqrt{\frac{|f(x)|}{|f''(x)|}}\sqrt{\varepsilon_f}\right) = 2\sqrt{\frac{|f(x)|}{|f''(x)|}}\sqrt{\varepsilon_f}$$

factor de curvatura

$$e_{ner}\Big|_{opt} \sim \sqrt{\varepsilon_f}$$







$$f'(x) \simeq \frac{\overset{x+1h}{1}f(x+h) - \overset{x+0h}{1}f(x)}{h}$$

Forward Finite Diference

$$f'(x) \simeq \frac{\overset{x+0h}{1}f(x) - \overset{x+(-1)h}{1}f(x-h)}{h}$$

Backward Finite Diference

$$f'(x) \simeq \frac{\overset{x+(1)h}{\tfrac{1}{2}}f(x+h) - \overset{x+(-1)h}{\tfrac{1}{2}}f(x-h)}{h}$$

Central F-D

Error CFD

$$f(x+h) = f(x) + f'(x)\,h + \frac{1}{2}f''(x)\,h^2 + \frac{1}{3\cdot 2}f'''(x)\,h^3 + O(h^4) \quad \ldots (a)$$

$$f(x-h) = f(x) - f'(x)\,h + \frac{1}{2}f''(x)\,h^2 - \frac{1}{3\cdot 2}f'''(x)\,h^3 + O(h^4) \quad \ldots (b)$$

Mostrar $(a)-(b)$

$$f(x+h) - f(x-h) = \cancel{(f(x) - f(x))}^{0} + 2h\,f'(x) + \frac{1}{3}f'''(x)\,h^3 + O(h^4)$$

$$\to \frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{1}{2\cdot 3}f'''(x)\,h^2 + O(h^4)$$

$e_h = \frac{|f(x)|}{h}\varepsilon_f$ $\quad e_t \sim |f'''(x)|\,h^2$

$$e_{ver} = \frac{|f(x)|}{h}\varepsilon_f + |f'''(x)|\,h^2 \longrightarrow \frac{d(e_{ver})}{dt} = 0 = \frac{|f(x)|\,\varepsilon_f}{-h^2} + |f'''(x)|\,2h$$

$$\frac{|f(x)|\,\varepsilon_f}{2|f'''(x)|} = h^3 \Rightarrow h = \left(\frac{|f(x)|}{|f'''(x)|}\frac{1}{2}\right)^{1/3}\varepsilon_f^{1/3}$$

$$3/3 = 2/3 \qquad |f'''(x)|^{1/3}$$

$$e_{ror}\Big|_{opt} = \frac{|f(x)|}{}\left(2\frac{|f'''(x)|}{f(x)}\right)^{1/3}\underbrace{\frac{\varepsilon_f^{3/3}}{\varepsilon_f^{1/3}}}_{\varepsilon_f^{2/3}} + |f'''(x)|^{3/3}\left(\frac{|f(x)|}{|f'''(x)|}\frac{1}{2}\right)^{2/3}\varepsilon_f^{2/3}$$

$$= |f(x)|^{2/3}\,|f'''(x)|^{1/3}\,\frac{3}{2^{2/3}}\,\varepsilon_f^{2/3} \Rightarrow e_{ror} \sim \left(\sqrt{\varepsilon_f}\right)^3 \ll \sqrt{\varepsilon_f}$$

En general definimos $a_n f(x+b_n h)$, y podemos calcular las derivadas numéricas como sigue

CFD $\qquad f^{(n)}(x) = \frac{1}{h^n}\sum f(x+b_n h) \longrightarrow$ En general

$h \to cte$

$\leftarrow b_n$

Derivative	Accuracy	−5	−4	−3	−2	−1	0	1	2	3	4	5
1	2					−1/2	0	1/2				
	4				1/12	−2/3	0	2/3	−1/12			
	6			−1/60	3/20	−3/4	0	3/4	−3/20	1/60		
	8		1/280	−4/105	1/5	−4/5	0	4/5	−1/5	4/105	−1/280	
2	2					1	−2	1				
	4				−1/12	4/3	−5/2	4/3	−1/12			
	6			1/90	−3/20	3/2	−49/18	3/2	−3/20	1/90		
	8		−1/560	8/315	−1/5	8/5	−205/72	8/5	−1/5	8/315	−1/560	

n $\qquad \}a_n \qquad \}a_n$

$$f' = \frac{f(x+h)-f(x)}{h}$$

$$f''(x) = \lim \frac{f'(x-h) - f'(x)}{h}$$

Forward finite difference [edit source]

This table contains the coefficients of the **forward** differences, for several orders of accuracy and with uniform grid spacing:[1]

$\rightarrow b_n$

Derivative	Accuracy	0	1	2	3	4	5	6	7	8
1	1	−1	1							
	2	−3/2	2	−1/2						
	3	−11/6	3	−3/2	1/3					
	4	−25/12	4	−3	4/3	−1/4				
	5	−137/60	5	−5	10/3	−5/4	1/5			
	6	−49/20	6	−15/2	20/3	−15/4	6/5	−1/6		

$n \rightarrow \qquad \}a_n$