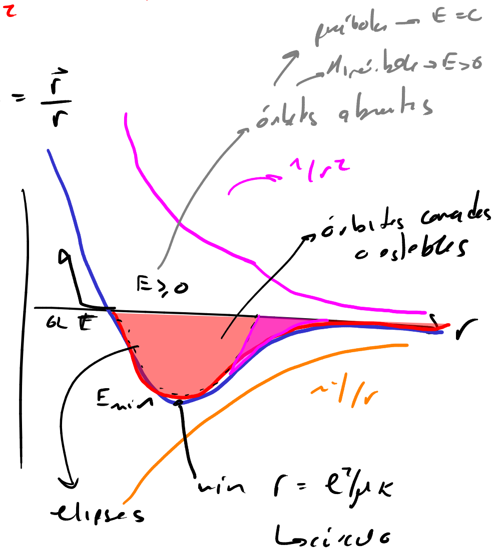
Campo central $\longrightarrow$ $U(r) = -\frac{K}{r}$ $\longrightarrow$ $-\frac{dU}{dr} = f = -\boxed{g\frac{Mm}{r^2}}$ $\longrightarrow$ $\vec{F} = f(r)\hat{e}_r$

Problema de 2 cuerpos $\longrightarrow$ $\vec{r} = \underbrace{\vec{r}_2 - \vec{r}_1}_{\text{s. ref. in}} = \underbrace{\vec{r}_2\,' - \vec{r}_1\,'}_{\text{cm}}$ $\longrightarrow$ $\hat{e}_r = \frac{\vec{r}}{r}$

$$U_{eff}(r) = +\frac{\ell^2}{2\mu}\frac{1}{r^2} + U(r) = \frac{\ell^2}{2\mu}\frac{1}{r^2} - g\frac{Mm}{r}$$

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

$$U_{eff} \xrightarrow[r\to\infty]{} 0$$



$$\Delta\theta = \int \frac{dr/r^2}{\sqrt{\frac{2}{\mu}}\left(E - U_{eff}\right)^{1/2}} \longrightarrow r(\theta) = \frac{\overbrace{\ell^2/\mu K}^{= r_0}}{1 + \epsilon\cos(\theta - \theta_0)}$$

$\theta - \theta_0 = \theta$

$$\epsilon = \sqrt{\frac{2mE}{\ell^2 K} + 1} \implies$$

$\epsilon = 0 \rightarrow$ círculo

$0 < \epsilon < 1 \rightarrow$ elipses

$\epsilon = 1 \rightarrow \exists\theta \quad r \rightarrow \infty$

$\epsilon < 1$

$r^2 = x^2 + y^2$

$x = r\cos\theta$

$y = r\sin\theta$

$$r = \frac{r_0}{1 + \epsilon\cos\theta} \implies r + r\cos\theta\,\epsilon = r_0$$

$$r = r_0 - \underbrace{r\cos\theta}_{x}\,\epsilon$$

$$\implies r^2 = x^2 + y^2 = r_0^2 - 2x\epsilon r_0 + \epsilon^2 x^2$$

$$(1-\epsilon^2)x^2 + 2\epsilon r_0 x + y^2 = r_0^2$$

$$(1-\epsilon^2)\left[x^2 + 2\left(\frac{\epsilon r_0}{(1-\epsilon^2)}\right)x + \left(\frac{\epsilon^2 r_0^2}{(1-\epsilon^2)^2}\right)\right] + y^2 = r_0^2 + \left(\frac{\epsilon^2 r_0^2}{(1-\epsilon^2)^2}\right)$$

$$(1-\epsilon^2)\left(x + \underbrace{\frac{\epsilon r_0}{(1-\epsilon^2)}}_{\text{focal}}\right)^2 + y^2 = r_0^2 + \left(\frac{\epsilon^2 r_0^2}{(1-\epsilon^2)^2}\right)$$

Tarea $\rightarrow$ demostrar que sí estamos en el foco

$\epsilon = 0 \longrightarrow x^2 + y^2 = r_0^2$

$0 < \epsilon < 1$

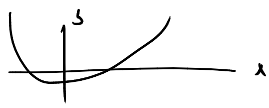
$\epsilon^2 < 1^2$

$1 - \epsilon^2 > 0$

$$\longrightarrow \frac{\left[x + \left(\frac{\epsilon r_0}{1-\epsilon^2}\right)\right]^2}{\underbrace{a^2 = \left(\frac{r_0 + \left(\frac{\epsilon - r_0}{1-\epsilon^2}\right)^2}{1-\epsilon^2}\right)}_{>0}} + \frac{y^2}{\underbrace{r_0 + \left(\frac{\epsilon - r_0}{1-\epsilon^2}\right)^2}_{>0} \rightarrow b^2} = 1 \quad ✓$$

$\epsilon = 1$

$$2\epsilon r_0 x + y^2 = r_0^2$$



$\epsilon > 1$

$$\frac{r_0 + \left(\frac{\epsilon - r_0}{1-\epsilon^2}\right)^2}{1-\epsilon^2} < 0 \quad \text{Hipérbola}$$