26/11/21

Energía $\rightarrow$ $\vec{F} = \nabla U$, $d\vec{F} = -dU$ $\rightarrow$ $E = T + U$ $\rightarrow$ $T = \frac{1}{2}mv^2$

$$\frac{dE}{dt} = 0$$

Momento lineal $\rightarrow$ $\vec{p} = m\vec{v}$ $\rightarrow$ $\frac{d\vec{p}}{dt} = \vec{F}_{ext} = \vec{0}$ $\rightarrow$ $\frac{d\vec{p}}{dt} = 0$

1 sola partícula

N partículas

$$\vec{R}_{cm} = \frac{1}{M}\sum m_i \vec{r}_i$$

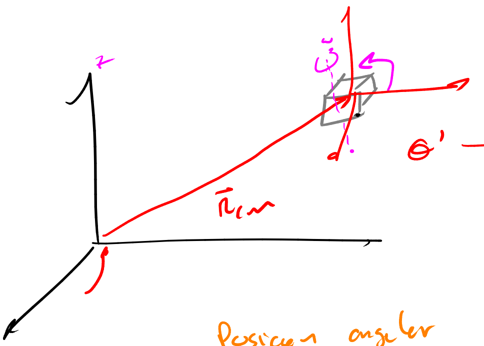
$N = 2$

MCU $\rightarrow$ Movimiento circular

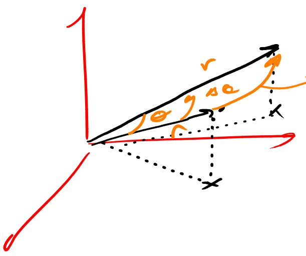
$$\vec{r} = r\,\hat{e}_r$$

$$\dot{\vec{r}} = \cancel{\dot{r}\hat{e}_r} + r\dot{\theta}\hat{e}_\theta$$

$$\hat{e}_r \cdot \hat{e}_\theta = 0$$



$\theta' \rightarrow \vec{R}_{cm}$



longitud de arco

$r\theta = s$

$\theta = 2\pi$

$s = $ longitud $= 2\pi r$

Posición angular

$\theta = \frac{s}{r}$ [=] [adimensional] $\rightarrow$ radianes

$\hookrightarrow$ rad $\rightarrow$ 2π rad $= 1$ vuelta $= 1$ ciclo

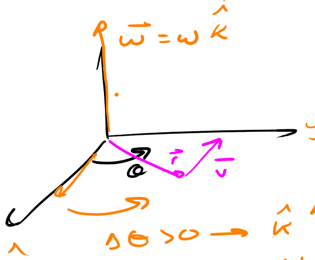
$$\omega = 2\pi f$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\theta(t+\Delta t) - \theta(t)}{\Delta t} \xrightarrow[\Delta t \to 0]{} \frac{d\theta}{dt} = \dot{\theta} = \omega \rightarrow \text{velocidad angular}$$

$$\frac{\Delta\omega}{\Delta t} \xrightarrow[\Delta t \to 0]{} \frac{d\omega}{dt} = \dot{\omega} = \alpha \rightarrow \text{aceleración angular}$$

$\vec{\omega} = \omega\hat{k}$ $\rightarrow$ perpendicular al plano del movimiento

$\{\theta, \omega, \alpha\} \rightarrow$ No son vectores



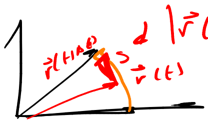
$\Delta\theta > 0 \rightarrow \hat{k}\uparrow \rightarrow$ antihorario

$\Delta\theta < 0 \rightarrow \hat{k}\downarrow \rightarrow$ horario

$$s = r\theta \rightarrow \frac{ds}{dt} = \boxed{r\omega = v_a} \text{ vel. tangencial}$$

(cte)

$$\boxed{r\alpha = a}$$



$$\left|\frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}\right| = \left|\frac{d\vec{r}}{dt}\right| = |\vec{v}|$$

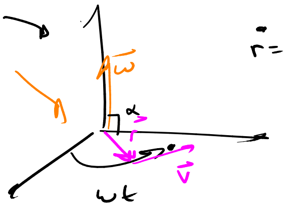
	Lineales	Angulares
variables dinámicas	Δx $\rightarrow$	$r\Delta\theta = \Delta x$
	v $\rightarrow$	$\omega r = v$
	a $\rightarrow$	$\alpha r = a$

¿Existen análogos a otras cantidades?

m (cantidad de la inercia de un cuerpo) $\rightarrow$ $I = mr^2$

$\vec{P} \rightarrow \vec{L} = \vec{r} \times \vec{P}$

$\vec{F} \rightarrow \vec{\tau} = \vec{N} = \vec{r} \times \vec{F}$



$\dot{r}=0$

$$T=\frac{1}{2} m v^2 = \frac{1}{2}(m r^2)\omega^2$$

$$V \longrightarrow \omega$$

$I \longrightarrow$ Momento de inercia de 1 partícula puntual girando alrededor de 1 eje

$v=\omega r$

$$m \longrightarrow I = m r^2$$

$$\frac{1}{2} m v^2 \longrightarrow \frac{1}{2} I \omega^2$$

$$v=\omega r \sin\alpha \longrightarrow \vec v = \vec\omega \times \vec r$$

$$\vec p = m\vec v \qquad I\vec\omega = \vec L$$

Tarea

$$\vec A \times (\vec B \times \vec C) = \vec B(\vec A\cdot\vec C) - \vec C(\vec A\cdot\vec B)$$

Tablas

$$\vec r \times \vec p = \vec r \times [m(\vec\omega\times\vec r)] = m\, \underset{A}{\vec r} \times (\underset{B}{\vec\omega} \times \underset{C}{\vec r})$$

$$= m[\vec\omega\,(\underbrace{\vec r\cdot\vec r}_{r^2}) - \vec r\,(\cancel{\vec r\cdot\vec\omega})]$$

$$= m r^2 \vec\omega = I\vec\omega$$

$$\vec L = I\vec\omega = \vec r \wedge \vec p$$

$$\dot{\vec p} = \vec F$$

$$\vec F = ma \longrightarrow \vec\tau = N - I\alpha$$

$$\frac{d}{dt}(fg) = \dot f g + f \dot g$$

$$\vec\tau = \frac{d\vec L}{dt} = I\dot{\vec\omega} = I\alpha\hat\omega$$

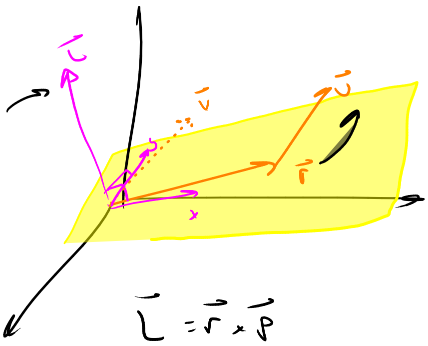
$$\vec\tau = \frac{d\vec L}{dt} = \frac{d}{dt}(\vec r \times \vec p) = \cancel{\frac{d\vec r}{dt}\times\vec p} + \vec r \times \underbrace{\frac{d\vec p}{dt}}_{\vec F}$$

$$\vec v \times m\vec v$$

$$\vec\tau = \vec r \wedge \vec F$$

$$\frac{d\vec r}{dt} \times \vec p = \vec v \times (m\vec v) = m(\cancel{\vec v\times\vec v}) = \vec 0$$

$$|\vec v \times \vec v| = v^2 \sin\alpha = 0 \iff \vec v\times\vec v = \vec 0 \longrightarrow \vec v \times \vec v = \begin{vmatrix} \hat e_x & \hat e_y & \hat e_z \\ v_x & v_y & v_z \\ v_x & v_y & v_z \end{vmatrix} = 0$$



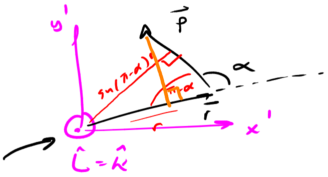
$\vec{L} = \ell \hat{L}$

$\ell > 0$ → gira positiu → antihorari
$\ell < 0$ → gira negatiu → horari

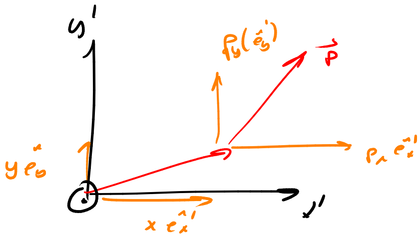
→ mano dreta

$\vec{L} = \vec{r} \times \vec{p}$

$\ell = r\, p \sin\alpha$ → $\sin(\pi - \alpha) = \sin\alpha$
$= r_\perp p$
$= r\, p_\perp$ → Braç palanca



$\vec{p}_\perp$



$y\hat{e}_y' \times p_x \hat{e}_x' = y p_x (\hat{e}_y' \times \hat{e}_x') = -y p_x \hat{e}_z$

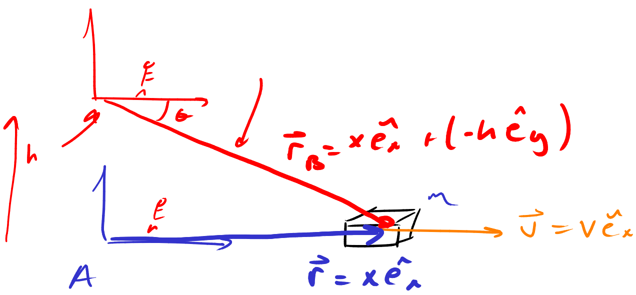
$x\hat{e}_x' \times p_y \hat{e}_y' = x p_y \hat{e}_z$

$\vec{L} = \hat{e}_z (x p_y - y p_x)$

$= \hat{e}_z \begin{vmatrix} x & y \\ p_x & p_y \end{vmatrix}$

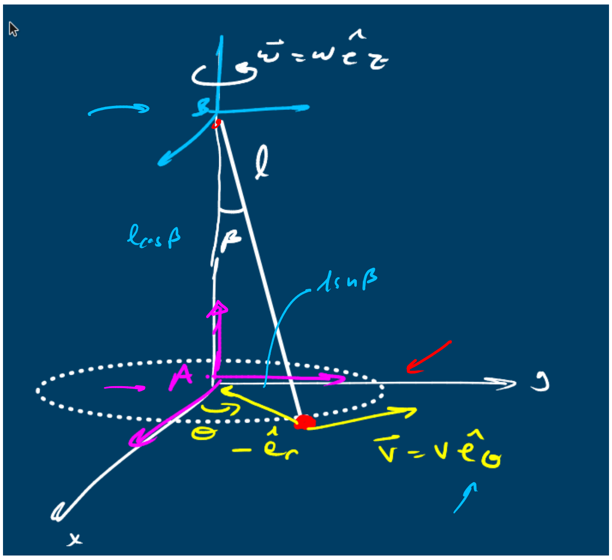
$= \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ x & y & 0 \\ p_x & p_y & 0 \end{vmatrix}$

$\vec{L} = \vec{r} \times \vec{p} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$



$\vec{L}_B = \vec{r}_B \times \vec{p} = mvh(\cancel{\sin\theta\, \hat{e}_x} - \cos\theta\, \hat{e}_y) \times \hat{e}_x$
$= mvh\cos\theta\, \hat{e}_z$

$\vec{L}_A = \vec{r}_A \times \vec{p} = mxv\, \hat{e}_x \times \hat{e}_x = \vec{0}$



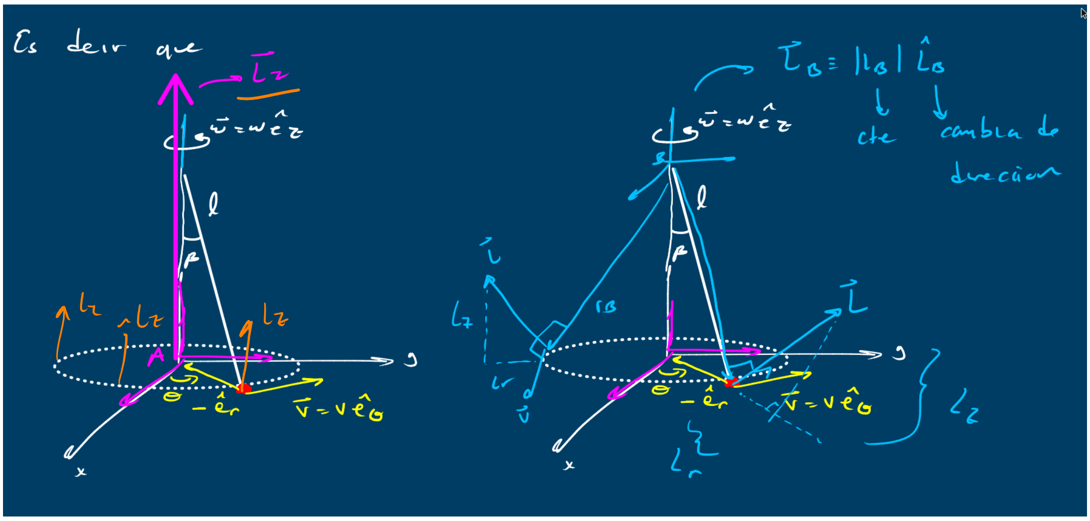
$(\hat e_r, \hat e_\theta, \hat e_z) \longrightarrow$ vectores ortonormales

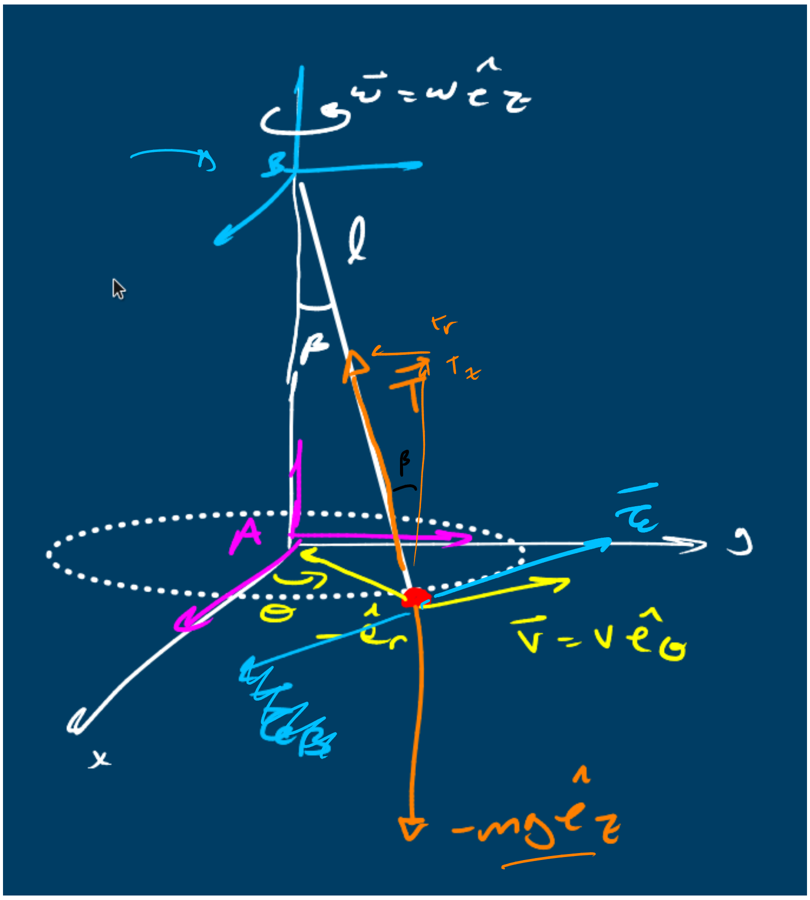
$$\vec L_A = \vec r_A \times m\vec v = m\, l\sin\beta\, v\,(\hat e_r \times \hat e_\theta)$$

$$\vec L_A = m l \sin\beta\, v\, \hat e_z$$

$$\vec L_B = \vec r_B \times \vec v = (\vec r_A - l\cos\beta\, \hat e_z)\times m v\, \hat e_\theta$$

$$= \underbrace{m l v \sin\beta\, \hat e_z}_{\vec L_z = L_z \hat e_z} + \underbrace{m l v\cos\beta\, \hat e_r}_{\vec L_r = L_r \hat e_r}$$





z: $m a_z = 0 = T\cos\beta - mg$

r: $m a_c = -T\sin\beta = m\frac{v}{r^2} = -F_r$

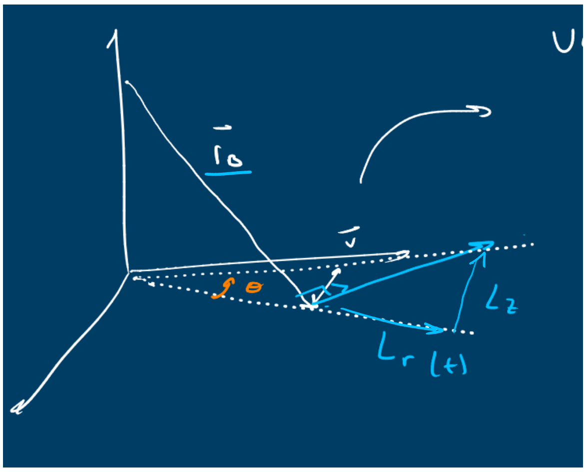
$$\vec\tau = \vec r \times \vec F = \vec r \times (-F\hat e_r) = \vec 0$$

$$\cancel{\frac{d\vec L_A}{dt} = \vec 0 = \vec\tau}$$

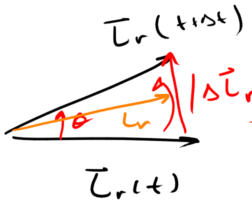
$$\vec\tau = \frac{d\vec L_B}{dt} = \frac{d(\vec L_z + \vec L_r)}{dt}$$

$$\frac{d\hat e_r}{dt} = \omega\, \hat e_\theta$$

$$\frac{d\vec L_r}{dt} = m l v\cos\beta\, \omega\, \hat e_\theta$$



Vista dsd arriba



$|\Delta \vec{L}_r| = L_r \omega \Delta t$

$$\frac{|\Delta \vec{L}_r|}{\Delta t} \approx L_r \omega = \frac{dL_r}{dt}$$

$$\frac{dL_r}{dt} = L_r\omega = m l v \cos\beta\, \omega = m l \omega\left(-\frac{mg}{T}\right) = m l \omega(-mg) = m\, l\omega(-mg) F_r \sin\beta$$

$$= r\,(m F_r mg)$$

$$= r \times (-mg)$$

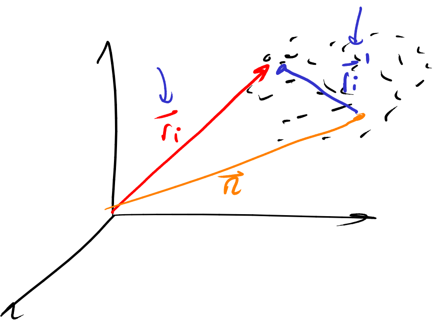
z: $m a_z = 0 = T\cos\beta - mg$

r: $m a_c = -T\sin\beta = m\frac{v}{r^2} = -F_r$

$$\frac{1}{T} = F_r \sin\beta$$

Check

$\frac{\omega}{r} = v \qquad a_c = \frac{v}{r^2}$



$$\vec{L}'_{Tot} = \sum_i \vec{r}_i' \times \vec{p}_i' = \sum_i m_i \vec{r}_i' \times \dot{\vec{r}}_i'$$

$$\frac{d}{dt}\vec{L}'_{TOT} = \sum m_i \left(\cancel{\dot{\vec{r}}_i' \times \dot{\vec{r}}_i'}^{\,0} + \vec{r}_i' \times \ddot{\vec{r}}_i'\right)$$

$$= \sum \vec{r}_i' \times \vec{F}_i$$

$$= \sum \vec{r}_i' \times \vec{F}_i^{(ext)} + \underbrace{\sum \vec{r}_i \times \Big(\sum_{\substack{j=1\\ j\neq i}}^{N} \overbrace{\vec{f}_{ij}}^{\text{internas}}\Big)}$$

$j \leftrightarrow i$

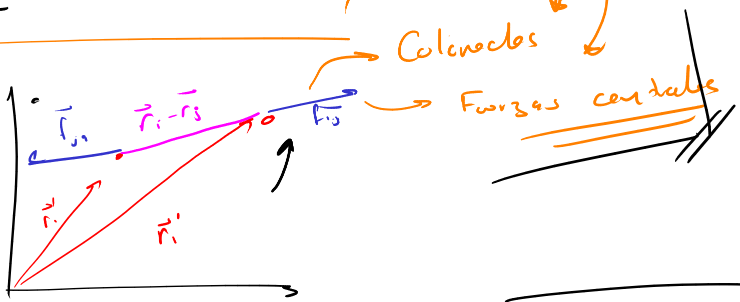
$$\sum_{\substack{i,j\\ i\neq j}} \vec{r}_i' \times \vec{f}_{ij} = \frac{1}{2}\sum_{\substack{i,j\\ i\neq j}} \left(\vec{r}_i' \times \vec{f}_{ij} + \vec{r}_j' \times \vec{f}_{ji}\right) = \frac{1}{2}\sum_{\substack{i,j\\ i\neq j}} \underbrace{(\vec{r}_i' - \vec{r}_j')} \times \vec{f}_{ij} \stackrel{?}{=} \vec{0}$$

$$f_{ij} = -f_{ji}$$

$$\boxed{\frac{d}{dt}\vec{L}'_{Tot} = \sum \vec{r}_i' \times \vec{F}_i = \sum \tau_i = \tau'_{Tot}}$$

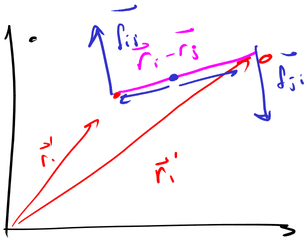
Colineales

Fuerzas centrales



$$\vec{F}(\vec{r},t) = f(\vec{r},t)$$

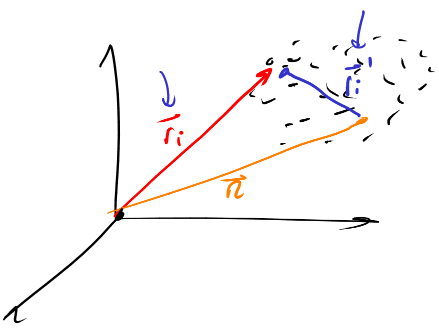
$$\vec{F}(\vec{r},t) = f\left(\vec{r} - \frac{c}{t}\right)$$ Tiempo de retraso



$\hat{e}_z$ → Campos físicos

↳ EM $\quad \vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

→ Fenómenos de retraso



$$\vec{L}_{Tot} = \sum_i m_i (\vec{r}_i \times \dot{\vec{r}}_i) \qquad \vec{r}_i = \vec{R} + \vec{r}_i'$$

$$= \sum_i m_i (\vec{R} + \vec{r}_i') \times (\dot{\vec{R}} + \dot{\vec{r}}_i')$$

$$= \sum_i m_i \left[\vec{R} \times \dot{\vec{R}} + \cancel{\vec{R} \times \dot{\vec{r}}_i' + \vec{r}_i' \times \dot{\vec{R}}}^{\,0} + \vec{r}_i' \times \dot{\vec{r}}_i'\right]$$

$$\sum \vec{R} \times (m\dot{\vec{R}})$$

$$\vec{L}_{Tot} = \vec{L}'_{Tot} + \underbrace{\vec{R} \times \vec{P}}_{\vec{L}_{cm}}$$

$$\left(\sum m_i \dot{\vec{r}}_i'\right) \times \vec{R}$$

$$\sum m_i (\vec{R} - \vec{r}_i)$$

$$M\vec{R} - \left(\sum m_i \vec{R}_i\right)\frac{M}{M} = M\dot{\vec{R}} - \dot{\vec{R}} M = 0$$