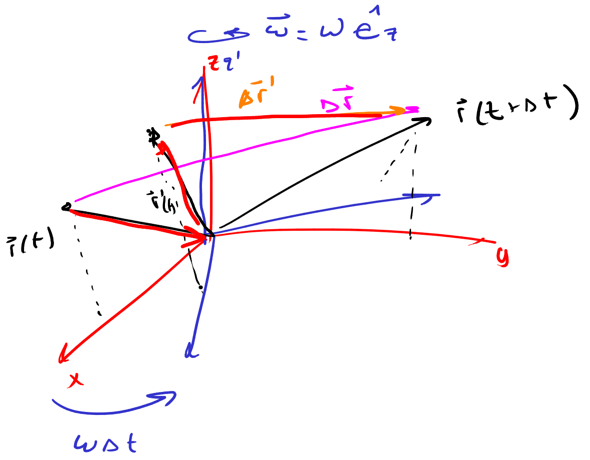
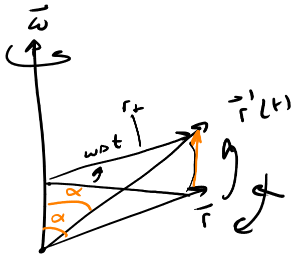
Fuerzas no inerciales (Sistemas rotantes)



$$\underbrace{-\vec r(t) + \vec r'(t)}\ + \overset{\text{obs. rot.}}{\Delta\vec r'} = \overset{\text{obs. in}}{\Delta\vec r} \rightarrow$$



$\omega\Delta t \ll 1$

$\|\omega\times\vec r\|$

$$\omega\Delta t\, r_\perp = \|\vec r' - \vec r\| = (\omega r_\perp)\Delta t = \omega r\sin\alpha\,\Delta t = \|\vec\omega\times\vec r\|\Delta t$$

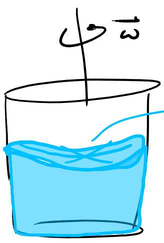
$$\frac{\Delta\vec r}{\Delta t} = \frac{\Delta\vec r'}{\Delta t} + \omega\times\vec r \rightarrow \left(\frac{d}{dt}\right)_{in}\vec B = \left[\left(\frac{d}{dt}\right)_{rot} + \vec\omega\times\right]\vec B$$

usualmente $\dot{\vec\omega} = 0$ / $\vec\omega$ = cte

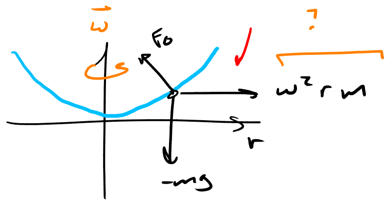
$$\left(\frac{d^2}{dt^2}\right)_{in}\vec r \rightarrow \boxed{\ddot{\vec r}_{in} = \ddot{\vec r}_{rot} + \dot{\vec\omega}\times\vec r \underbrace{- 2\vec\omega\times\dot{\vec r}_{rot}}_{\text{Coriolis}} \underbrace{- \vec\omega\times(\vec\omega\times\vec r)}_{\text{Centrífuga}}}$$

$$\omega = 2\pi f = 2\pi\frac{1}{24\,hs}$$

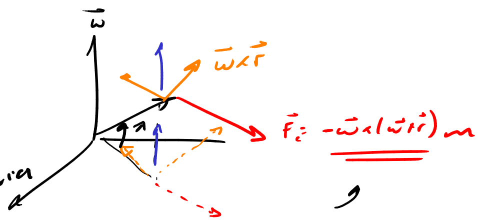
Centrífuga



parabola de revolución



$z \sim r^2$

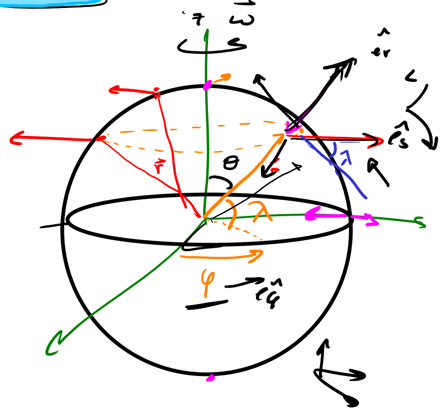


$\vec F_c = -\vec\omega\times(\vec\omega\times\vec r)m$

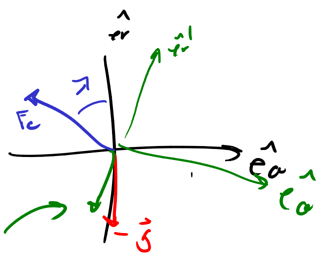
Base polar   Base cilíndrica

$$\{\hat e_r, \hat e_\theta, \hat e_\varphi\} \leftrightarrow \{\hat e_s, \hat e_\varphi, \hat e_z\},\quad \vec r = r\hat e_r = s\hat e_s + z\hat e_z$$

$$r^2 = s^2 + z^2 \qquad s^2 = x^2 + y^2$$



$$\omega^2 r_\perp = \omega^2 r\sin\theta = \omega^2 r\sin(\pi/2 - \lambda) = \omega^2 r\cos\lambda$$



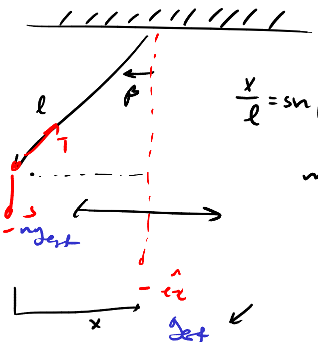
En el Ecuador

Fuerza menor

$$\hat e_r:\ g^{(r)}_{eff} = g - F_c\cos\lambda$$

$$\hat e_\theta:\ g^{(\theta)}_{eff} = -F_c\sin\lambda$$

Qué pasa con los péndulos $\rightarrow \boxed{\vec g_{eff} = g^{(r)}_{eff}\hat e_r + g^{(\theta)}_{eff}\hat e_\theta}$



$$\frac{x}{\ell} = \sin\beta \ll 1 \Rightarrow \sin\beta \approx \beta$$

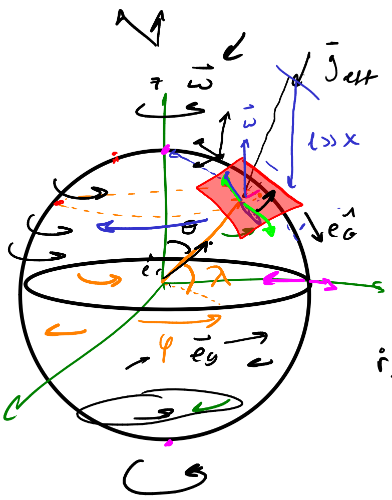
$$mg\sin\beta = m\underbrace{\ell\ddot\beta}_{\ddot x} \rightarrow x(t) = \ell\sin\left(\sqrt{\frac{g_{eff}}{\ell}}\beta\right)$$

La física es la misma pero ahora mis ejes cambiados con g_{eff} en lugar de con $\vec g$.

$$\vec g \longrightarrow \vec g_{eff}$$

F. Coriolis

P. Foucault



$\dot{r}, \dot{\varphi}$

Base esferica de vectores $\to \{\hat{e}_r, \hat{e}_\theta, \hat{e}_\varphi\}$

$$\vec{F}_{coriolis} = (-2\,\vec{\omega} \times \dot{\vec{r}}_{rot})\, m = -2m\,\omega\, v \sin\lambda\, (-\hat{e}_\varphi)$$

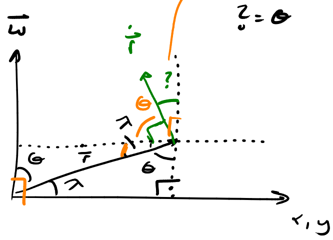
$\hookrightarrow \hat{e}_z \times (-\hat{e}_\theta)$

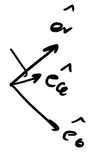
$$\|\vec{\omega} \times \dot{\vec{r}}\| = \omega \|\dot{\vec{r}}\| \sin\lambda = \omega V \sin\lambda$$

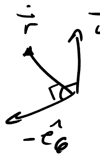
$? = \frac{\pi}{2} - \theta = \lambda$

$? = \theta$

$\frac{\pi}{2} = \theta + \lambda$







$\to$ regla de la mano derecha

$$\text{dir}(\vec{\omega} \times \dot{\vec{r}}_{rot}) = -\hat{e}_\varphi$$

$$\vec{\omega} \times \dot{\vec{r}}_{rot} = \omega v \sin\lambda\, (-\hat{e}_\varphi)$$

$$\vec{F}_{coris} = -2m(\vec{\omega} \times \dot{\vec{r}}_{rot}) = 2m\dot{r}\,\omega \sin\lambda\, \hat{e}_\varphi$$

$$m\ddot{\vec{r}} = m(\cancel{\hat{e}_s \ddot{s}}^{\,0} + \cancel{\hat{e}_z \ddot{z}}^{\,0} + \hat{e}_\varphi (2\dot{s}\dot{\varphi} + s\ddot{\varphi}))$$

$$= 2m\omega \dot{r} \sin\lambda\, \hat{e}_\varphi$$

$$\cancel{m}(r\ddot{\varphi} + 2\dot{r}\dot{\varphi}) = 2\cancel{m}\omega \dot{r} \sin\lambda$$

$$\boxed{r\ddot{\varphi} + 2\dot{r}\dot{\varphi} = 2\omega \dot{r} \sin\lambda}$$

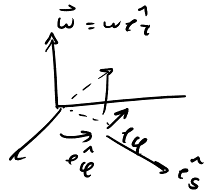
Sup. $\dot{\varphi} = cte \Rightarrow \ddot{\varphi} = 0$

$$\cancel{2}\cancel{\dot{r}}\dot{\varphi} = \cancel{2}\omega \cancel{\dot{r}} \sin\lambda$$

$$\dot{\varphi} = \boxed{\omega \sin\lambda} = 2\pi f = \boxed{\frac{2\pi}{T}} \Rightarrow T = \frac{\omega \sin\lambda}{\dot{\varphi}} = \frac{24\,hrs}{\sin\lambda}$$

$$\dot{\varphi} = \frac{2\pi}{24\,hrs}$$

Coordenadas cilíndricas



$\vec{\omega} = \omega \hat{e}_z$

$\hat{e}_s \times \hat{e}_\varphi = \hat{e}_z$

$$\vec{r} = s\hat{e}_s + z\hat{e}_z = r\hat{e}_r, \quad r^2 = s^2 + z^2 \quad (s^2 = x^2 + y^2)$$

$$\hat{e}_s = (\cos\varphi\, \hat{e}_x + \sin\varphi\, \hat{e}_y)$$

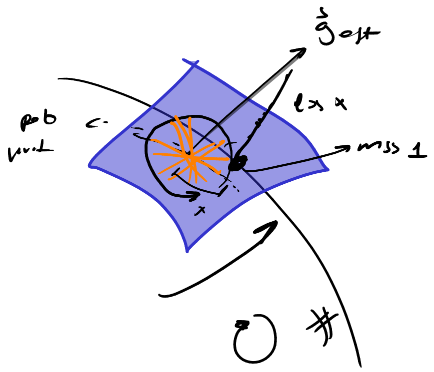
$$\hat{e}_\varphi = -\sin\varphi\, \hat{e}_x + \cos\varphi\, \hat{e}_y$$

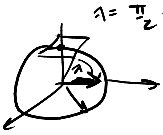
$$\dot{\vec{r}} = \dot{s}\hat{e}_s + s\dot{\hat{e}}_s + \dot{z}\hat{e}_z$$

$$\ddot{\vec{r}} = \ddot{s}\hat{e}_s + 2\dot{s}\dot{\hat{e}}_s + s\ddot{\hat{e}}_s + \ddot{z}\hat{e}_z$$

$$= \ddot{s}\hat{e}_s + 2\dot{s}\dot{\varphi}\hat{e}_\varphi + s\ddot{\varphi}\hat{e}_\varphi + \ddot{z}\hat{e}_z$$

$$\boxed{\dot{s} = \dot{r}\,?}$$



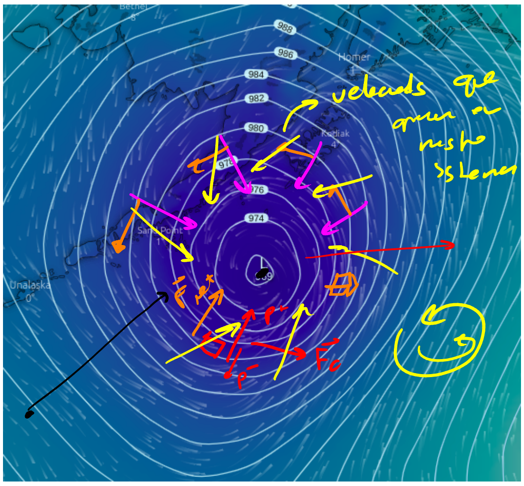


$\lambda = \frac{\pi}{2} \Rightarrow \sin\lambda = 1 \qquad T = 24\,hs$

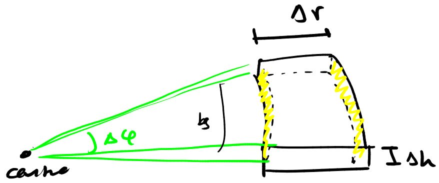
$\lambda = 0 \Rightarrow T \to \infty, \quad F_c \sim \sin\lambda = 0$

Huracanes

H. Norte



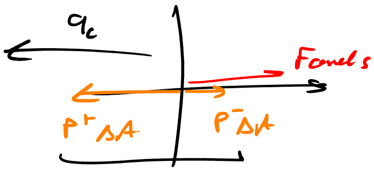
Densidad del volumen de aire $\rho = cte = \frac{dm}{dV} = \frac{\Delta m}{\Delta V}$



$P = \frac{\Delta F}{\Delta A}$ $\qquad \Delta A = r\Delta\varphi\Delta h$

$\Delta V = \Delta A\Delta r$

Diagrama de cuerpo libre



$\Delta P = P^+ - P^-$

$$F_{cor} - \Delta A\Delta P \overset{\sum F}{=} -m a_c \Rightarrow 2mV\omega\sin\lambda - \Delta P\Delta A = -\frac{mV^2}{r}$$

$$\Rightarrow \frac{V^2}{r} = \frac{\Delta P\Delta A}{m} - 2V\omega\sin\lambda$$

gradiente de presión $\quad \nabla P = \frac{dP}{dr}\hat{e}_r$

$$\rho = \frac{\Delta m}{\Delta V} \Rightarrow \frac{1}{m} = \frac{1}{\rho\Delta V} = \frac{1}{\rho\Delta A\Delta r} \Rightarrow \frac{V^2}{r} = -2V\omega\sin\lambda + \frac{1}{\rho}\frac{\Delta P}{\Delta r}$$

$$\Delta r \to 0: \quad \frac{V^2}{r} = -2V\omega\sin\lambda + \frac{1}{\rho}\frac{dP}{dr}$$

Ec. movimiento H. Huracán H.N.

¿Qué pasa si $r \gg 1$? Sup. $V \ll 1$, $V^2 \ll V$ $\Rightarrow 2V\omega\sin\lambda \simeq \frac{1}{\rho}\frac{dP}{dr}$

$$\Rightarrow V = \frac{1}{2\omega\sin\lambda}\frac{1}{\rho}\frac{dP}{dr}$$

$\rho = 1.3\ kg\,m^{-3}$

$P_{atm} = 10^5\ Pa$

$P = 10^4\ Pa$

$\lambda = 45°$

$\omega = 3600\ rad/s$

$\Delta P \simeq 9000\ Pa$

$\Delta r = 300\ km$

$\Rightarrow \frac{\Delta P}{\Delta r} \simeq 3\times10^{-3}\ N \qquad V \simeq 25\ km/hr \sim 30\ km$

Caso general

$$V^2 + (2\omega r\sin\lambda)V - \frac{r}{\rho}\frac{dP}{dr} = 0 \Rightarrow V = -\omega r\sin\lambda + \sqrt{(\omega r\sin\lambda)^2 - \frac{r}{\rho}\left|\frac{dP}{dr}\right|} > 0$$