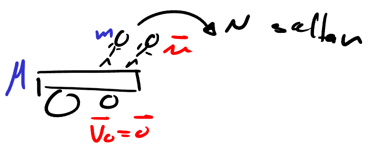
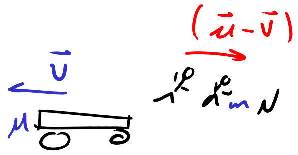
Resolución de problemas

1)





→ Todos saltan al mismo tiempo

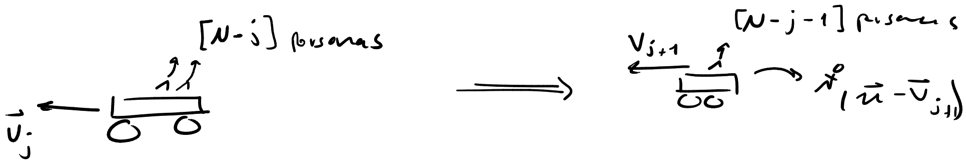
$$P(t_f) = -MV + Nm(u-V)$$
$$= -V(M+Nm) + Nmu$$

$$P(t_i) = 0$$

$$\Rightarrow P(t_f) = 0 \Rightarrow V^{todos} = u\left(\frac{Nm}{M+Nm}\right) \quad \ldots (1)$$

Saltando de uno en uno

2) Supongamos que ya saltaron $j-1$ personas y el carrito va a $-\vec{V}_{j+1}$



$$P(t_j) = -V_j\{M+[N-j]m\}$$

$$P(t_{j+1}) = -V_{j+1}\{M+[N-j-1]m\} + (u-V_{j+1})m$$
$$= -V_{j+1}\{M+m[N-j-\cancel{1}+\cancel{1}]\} + um$$

$$\Rightarrow (V_{j+1}-V_j)[M+m(N-j)] = um$$

$$\Rightarrow V_{j+1} = V_j + \frac{um}{M+m(N-j)}$$

Hagamos $j+1 = N \Rightarrow j = N-1 \Rightarrow N-j = N-(N-1) = 1$

$$\Rightarrow V_N = V_{N-1} + \frac{um}{M+m}$$

$j+1 = N-1 \Rightarrow j = N-2 \Rightarrow N-j = N-(N-2) = 2$

$$\Rightarrow V_N = V_{N-1} + \frac{um}{M+m} = V_{N-2} + u\Big(\underbrace{\frac{m}{M+m}}_{N} + \underbrace{\frac{m}{M+2m}}_{N-1}\Big)$$

entonces

$$V_N^{1\times1} = um\Big(\underbrace{\underbrace{\frac{1}{M+m}}_{\substack{j=0,\ N\\ 1}} + \underbrace{\frac{1}{M+2m}}_{\substack{N-1\\ 2}} + \cdots \underbrace{\frac{1}{M+Nm}}_{\substack{j=N-(N-1)\\ j=1}}}_{N\text{ términos}}\Big) + \cancel{V_0}^{\,0}_{\;j=N-N=0} \quad \ldots (2)$$

Comparemos (1) y (2)

$$(1) \to V^{todos} = Nmu\frac{1}{M+Nm} = mu\Big(\overbrace{\frac{1}{M+Nm} + \frac{1}{M+Nm} + \cdots \frac{1}{M+Nm} + \frac{1}{M+Nm}}^{N\text{ términos}}\Big)$$

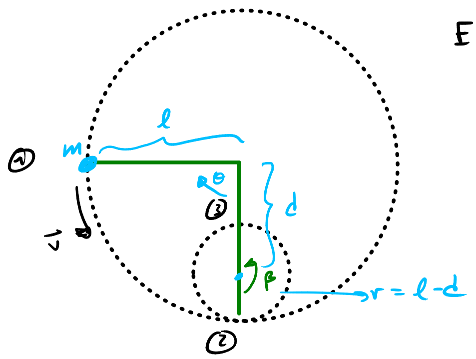
el mismo

$$(2) \to V^{1\times1} = mu\Big(\underbrace{\frac{1}{M+m} + \frac{1}{M+2m} + \cdots + \frac{1}{M+(N-1)m} + \frac{1}{M+Nm}}_{N\text{ términos}}\Big)$$

$$\Rightarrow \quad V^{todos} - V^{1\times1} = m\,\mathcal{U} \sum_{j=1}^{N} \left(\frac{1}{M+Nm} - \frac{1}{M+jm} \right) = m\,\mathcal{U} \sum_{j=1}^{N-1} \left(\frac{1}{M+Nm} - \frac{1}{M+jm} \right)$$

Como $j < N \Rightarrow M + jm < M + Nm \Rightarrow \frac{1}{M+Nm} < \frac{1}{M+jm}$

$$\Rightarrow \quad V^{todos} - V^{1\times1} < 0 \Rightarrow \underline{\underline{V^{todos} < V^{1\times1}}}$$



$E = T + \mathcal{U} = \frac{1}{2} m v^2 + mgh$

De ① — ② $\quad h = \ell(1 - \cos\theta)$

En ① $\quad V = 0,\ \theta = \pi/2 \Rightarrow E = mg\ell$

② $\quad \theta = 0 \Rightarrow E = mg\ell = \frac{1}{2} m v^2 \Rightarrow v^2 = 2g\ell$

De ② — ③ $\quad h = r(1 - \cos\beta)$

② $\quad v = \sqrt{2g\ell}\ ,\ \beta = 0 \Rightarrow E' = mg\ell$

③ $\quad \beta = \pi \Rightarrow E' = mg\ell = \frac{1}{2} m v'^2 + mgr2$

$$\Rightarrow \quad (v')^2 = 2g\ell - 4gr = 4g\left(\frac{\ell}{2} - r\right) = 4g\left(\frac{\ell}{2} - (\ell - d)\right) = 4g\left(d - \frac{\ell}{2}\right) \geq 0 \quad \ldots (3)$$

Si queremos que sea un círculo la trayectoria

En ③ $\quad a_c = \frac{v'^2}{r} = g \Rightarrow (v')^2 = gr \Rightarrow 4\cancel{g}\left(d - \frac{\ell}{2}\right) = \cancel{g}(\ell - d)$

$\Rightarrow 4d - 2\ell = \ell - d$

$\Rightarrow 5d = 3\ell \Rightarrow d = \frac{3}{5}\ell \quad \ldots\ldots (4)$

Vemos que de ③

$d \geq \ell/2$

y $\quad d = \frac{3}{5}\ell$

$\frac{d}{\ell} \geq \frac{1}{2}$ → Esta es la condición necesaria

$\frac{d}{\ell} = \frac{3}{5}$