

$$f(1) = 0$$

$$\underline{m \frac{dr}{dt} = \frac{G M_m}{r^2} e_r}$$

$$\longleftrightarrow \vec{r} = (x, y) \xleftarrow{\downarrow} r(t) \rightarrow$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \arctan\left(\frac{y}{x}\right)$$

A diagram showing a particle moving along a circular path. Two points on the path are connected by a vertical line segment, with a red arrow pointing to it labeled "9 metros". At the lower point, two red arrows represent forces: one pointing vertically downwards and one pointing horizontally to the left. Below the diagram, the equation $\underline{wt} - \underline{\psi} - \underline{e} \sin \psi = 0$ is enclosed in a hand-drawn orange box.

$$\psi(t)$$

$$\hookrightarrow r = a(1 + e \cos \psi)$$

$$x = a \cos \psi$$

$$y = b \sin \varphi$$




Diagram of an ellipse with semi-major axis a and semi-minor axis b . The formula for eccentricity is given as $e = \sqrt{1 - (b/a)^2}$.

$$f(\omega t) \rightarrow [0, 2\pi]$$

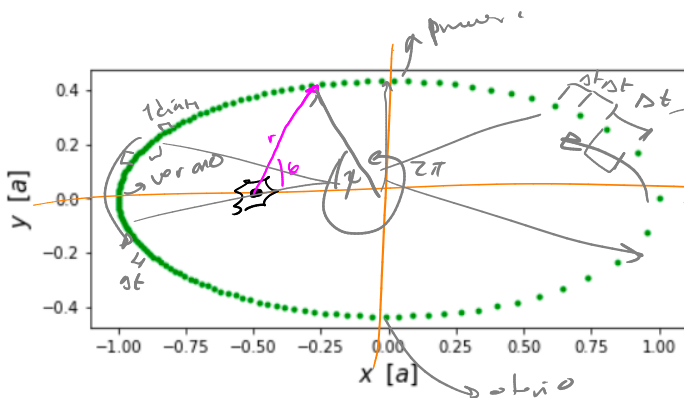
Sei i in $\text{rang}(0, 2\pi)$

$$f(\varphi) = \varphi - c \sin \varphi - \boxed{i}$$

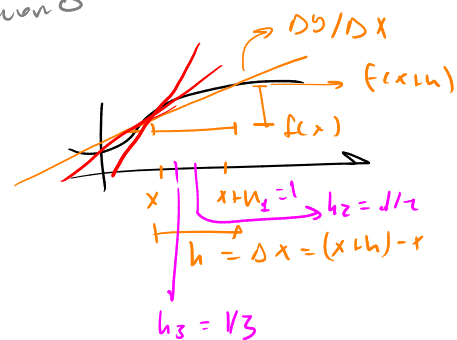
$$f(\gamma; i) = 0$$

$$i \rightarrow t\omega \rightarrow \frac{\psi(t\omega)}{(0, 2\pi)}$$

$[i, \psi]$



¿a qué vel. va? $\left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \vec{v}$



$$f(x) \rightarrow f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\approx \frac{f(x+h) - f(x)}{h}; \quad 0 < h \ll 1 \quad h = 0.001$$

$$f(x+h) = f(x) + f'(x)h + \frac{1}{2} f''(x)h^2 + \frac{1}{3!} f'''(x)h^3 + O(h^4)$$

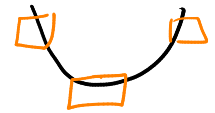
$$\Rightarrow \frac{f(x+h) - f(x)}{h} = f'(x) + \frac{1}{2} h f''(x) + \frac{1}{3!} f'''(x) h^2 + \underline{\underline{O(h^3)}}$$

$$e_h \sim \frac{|f(x)|}{|h|} \xrightarrow{\text{truncation}} \varepsilon_f$$

truncation

$$|f'(x)|h = e_t$$

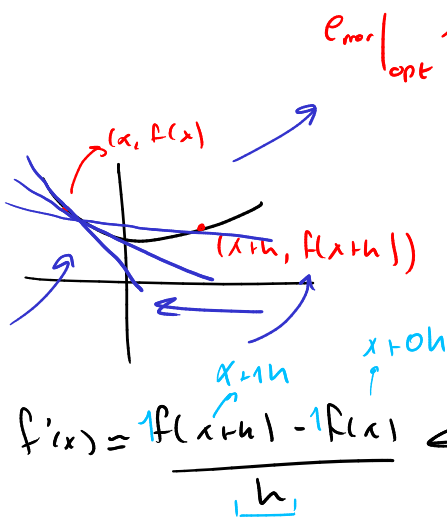
$$e_{\text{tot}} = e_h + e_t = \frac{|f(x)|}{h} \varepsilon_f + |f'(x)|h > 0$$



$$\frac{d[e_{\text{tot}}]}{dh} = \frac{|f(x)|\varepsilon_f}{-h^2} + |f'(x)| = 0 \Rightarrow h_{\text{opt}} = \sqrt{\frac{|f(x)|}{|f''(x)|}} \sqrt{\varepsilon_f}^{1/2}$$

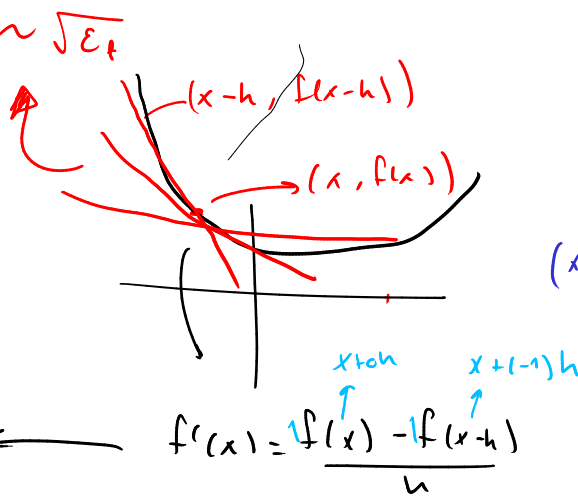
$$e_{\text{tot}}|_{h_{\text{opt}}} = \left(\frac{|f(x)|}{\sqrt{|f''(x)|}} \sqrt{\varepsilon_f} + |f'(x)| \sqrt{\frac{|f(x)|}{|f''(x)|}} \sqrt{\varepsilon_f} \right) = 2 \sqrt{\frac{|f(x)|}{|f''(x)|}} \sqrt{\varepsilon_f}$$

factor de curvatura



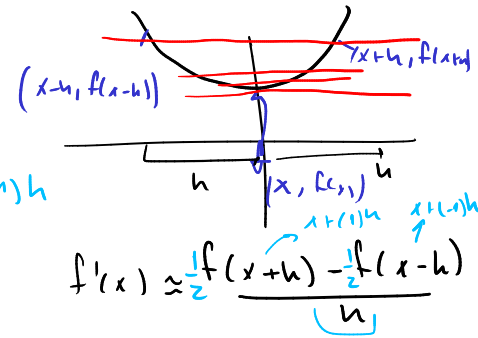
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

Forward Finite Difference



$$f'(x) \approx \frac{f(x) - f(x-h)}{h}$$

Backward Finite Difference



$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Central F-D

Error CFD

$$\rightarrow f(x+h) = f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \frac{1}{3!}f'''(x)h^3 + O(h^4) \quad \dots (a)$$

$$\rightarrow f(x-h) = f(x) - f'(x)h + \frac{1}{2}f''(x)h^2 - \frac{1}{3!}f'''(x)h^3 + O(h^4) \quad \dots (b)$$

Restar (a) - (b)

$$f(x+h) - f(x-h) = (f(x) - f(x)) + 2h f'(x) + \frac{1}{3}f'''(x)h^3 + O(h^4)$$

$$\Rightarrow \frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{1}{2 \cdot 3} f'''(x)h^2 + O(h^4)$$

$$e_h = \frac{|f(x)|}{h} \varepsilon_f$$

$$e_t \sim |f'''(x)|h^2$$

$$e_{\text{err}} = \frac{|f(x)|}{h} \varepsilon_f + |f'''(x)| h^2 \rightarrow \frac{d(e_{\text{err}})}{dh} = 0 = \frac{|f(x)| \varepsilon_f}{-h^2} + |f'''(x)| 2h$$

$$\frac{|f(x)| \varepsilon_f}{2|f'''(x)|} = h^3 \Rightarrow h = \left(\frac{|f(x)|}{|f'''(x)|} \frac{1}{2} \right)^{1/3} \varepsilon_f^{1/3}$$

$$e_{\text{err}}|_{\text{opt}} = \frac{|f(x)|}{2|f'''(x)|} \left(\frac{2|f'''(x)|}{|f(x)|} \right)^{1/3} \underbrace{\frac{\varepsilon_f^{3/3}}{\varepsilon_f^{2/3}}}_{\varepsilon_f^{1/3}} + |f'''(x)| \left(\frac{|f(x)|}{|f'''(x)|} \frac{1}{2} \right)^{2/3} \varepsilon_f^{2/3}$$

$$= |f(x)|^{2/3} |f'''(x)|^{1/3} \frac{3}{2^{2/3}} \varepsilon_f^{1/3} \Rightarrow e_{\text{err}} \sim (\sqrt{\varepsilon_f})^3 < \sqrt{\varepsilon_f}$$

En général, dérivées $\alpha f(x + b_n h)$, y a plus calculer les dérivées numériques comme sigue

CFO $f^{(n)}(x) = \frac{1}{h^n} \sum f(x + b_n h) \rightarrow$ En général

Derivative	Accuracy	-5	-4	-3	-2	-1	0	1	2	3	4	5
1	2					-1/2	0	1/2				
	4				1/12	-2/3	0	2/3	-1/12			
	6			-1/60	3/20	-3/4	0	3/4	-3/20	1/60		
	8		1/280	-4/105	1/5	-4/5	0	4/5	-1/5	4/105	-1/280	
2	2					1	-2	1				
	4				-1/12	4/3	-5/2	4/3	-1/12			
	6			1/90	-3/20	3/2	-49/18	3/2	-3/20	1/90		
	8		-1/560	8/315	-1/5	8/5	-205/72	8/5	-1/5	8/315	-1/560	

$h \rightarrow \text{cte}$

$$f' = \frac{f(x+h) - f(x)}{h}$$

$$f''(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h}$$

Forward finite difference [\[edit source\]](#)

This table contains the coefficients of the **forward** differences, for several orders of accuracy and with uniform grid spacing.^[1]

Derivative	Accuracy	0	1	2	3	4	5	6	7	8
1	1	-1	1							
	2	-3/2	2	-1/2						
	3	-11/6	3	-3/2	1/3					
	4	-25/12	4	-3	4/3	-1/4				
	5	-137/60	5	-5	10/3	-5/4	1/5			
	6	-49/20	6	-15/2	20/3	-15/4	6/5	-1/6		