Momento lineal

$\vec{p} = m\vec{v}$ → velocidad

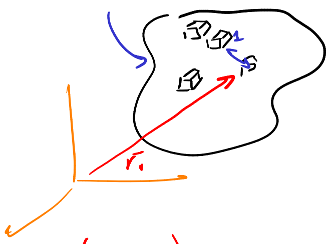
↳ masa

Si se cumple 2ª

1ra ley → $\vec{F} = \frac{d}{dt}\vec{p} = \dot{\vec{p}}$ $\boxed{\dot{f} = \frac{d}{dt} f}$ df

1 partícula → N partículas

↳ 1 cuerpo físico



$\vec{r}_i \to \Delta m_i \dot{\vec{r}}_i = \vec{p}_i$

$\vec{f}_i = \dot{\vec{p}}_i = \Delta m_i \ddot{\vec{r}}_i$

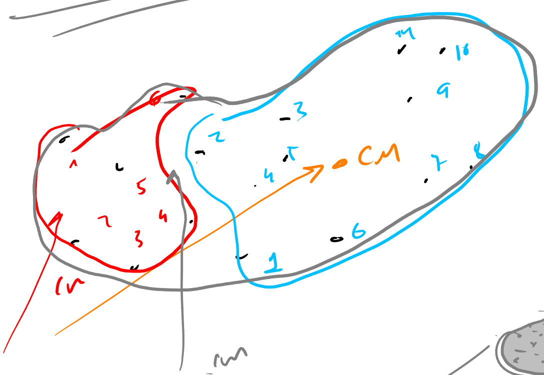
$\vec{f}_i^{\,ext} \quad , \quad \vec{f}_i^{\,int}$

$\sum_{j=1,\, j\neq i}^{N} f_{ij} = f_{i1} + f_{i2} + \dots f_{iN}$

$\vec{F}^{ext} = \sum_i f_i^{ext} + \sum_{i,j,\, i\neq j} f_{ij} = \sum_i \frac{d}{dt}\vec{p}_i = \frac{d}{dt}\left(\sum \vec{p}_i\right)$

$f_{ij} + f_{ji} = 0$ $\qquad \vec{P}$

Sist. N partículas



CM → Partícula puntual

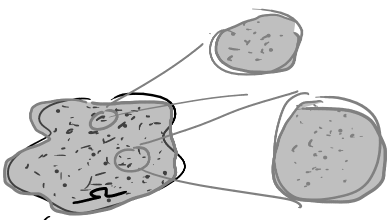
$\vec{R}_{cm} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i}$ → ¿Cuáles son las partículas que suma?

Suma de Riemann

$\sum m_i \to \sum_i \Delta m_i \xrightarrow{\Delta m_i \to 0} \int_\Omega dm = M$

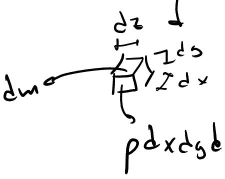
Cuerpo sólido

↳ 10^{23} $\frac{\text{partículas}}{cm^3}$



$\sum \Delta m_i \vec{r}_i \xrightarrow{\Delta m_i \to 0} \boxed{\int_\Omega \vec{r}'\, dm}$

$\iiint_\Omega \rho(\vec{r}')\, \vec{r}'\, dV$ $\quad \boxed{\vec{r}' \in \Omega}$



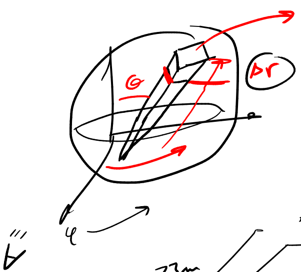
dm, dz, dy, dx

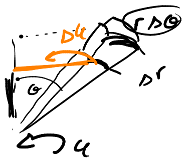
$\rho\, dx\, dy\, dz$

si $\Omega \in \mathbb{R}^3$

$dm = \rho\, dV = \rho\, dx\, dy\, dz$

volumétrica

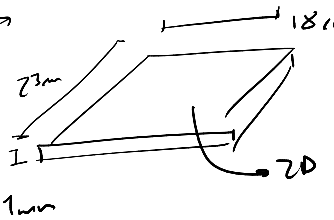




$x = 0,\ \varphi = \pi/2$ $\quad , \ y = 0,\ \varphi = 0$

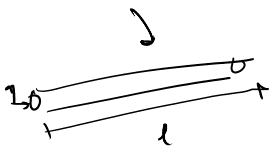
$r \sin\theta\, \Delta\varphi$

$dV = r^2 \sin\theta\, d\theta\, d\varphi\, dr$



$dm = \sigma\, dx\, dy = \sigma\, dA$

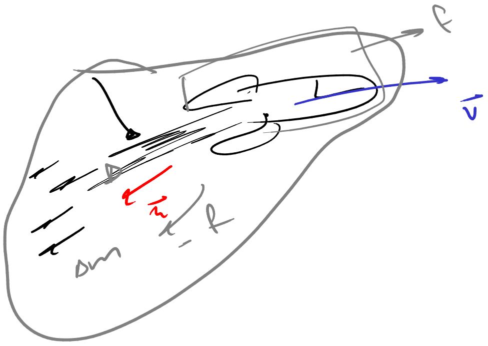
$\vec{R}_{cm} = \frac{1}{M} \iint_\Omega \vec{r}'\, \sigma(\vec{r}')\, dA$

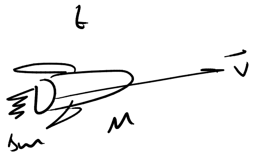


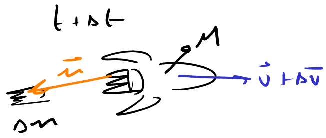
$dm = \lambda\, dx = \lambda\, dl$

$\vec{R}_{cm} = \frac{1}{m} \int_\Omega \vec{r}'\, \lambda(\vec{r}')\, dl$

Masa no constan







$$\vec{P}(t) = (\Delta m + M)\vec{v}$$

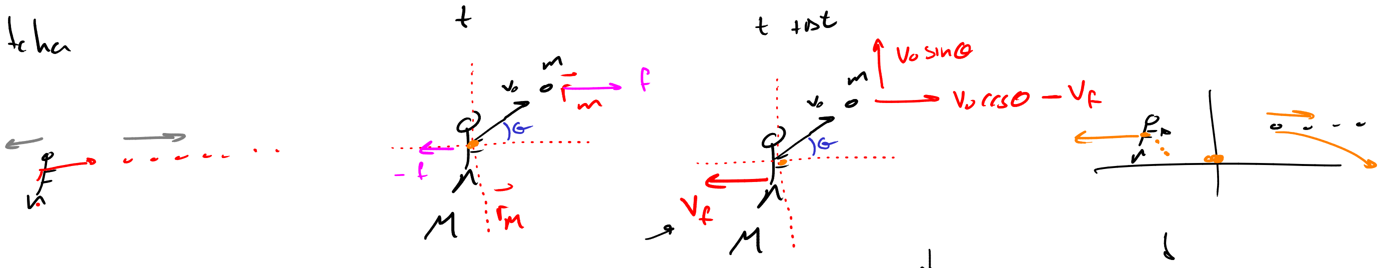
Δm

$\vec{v} + \Delta\vec{v} + \vec{u}$

$$\vec{P}(t+\Delta t) = \Delta m(\vec{v} + \Delta\vec{v} + \vec{u}) + M(\vec{v} + \Delta\vec{v})$$

$$\boxed{\frac{\Delta\vec{P}}{\Delta t} \xrightarrow[\Delta t \to 0]{} M\frac{d\vec{v}}{dt} - \vec{u}\frac{dM}{dt} = \vec{F}^{ext}}$$

Gotcha



$P_x(t) = (M+m)0$

$P_x(t) = 0$

$P_x(t+\Delta t) = -M V_f + m(V_0 \cos\theta - V_f) = 0$

$\vec{P} = \vec{P}_{yo} + \vec{P}_{bola} \quad \dot{\vec{P}} = \vec{F} \rightarrow \dot{P}_x = 0 \quad P_x(t) = P_x(t+\Delta t) = 0$

$\dot{P}_y = -(M+m)g$

$$V_f = \frac{V_0 m \cos\theta}{m+M}$$

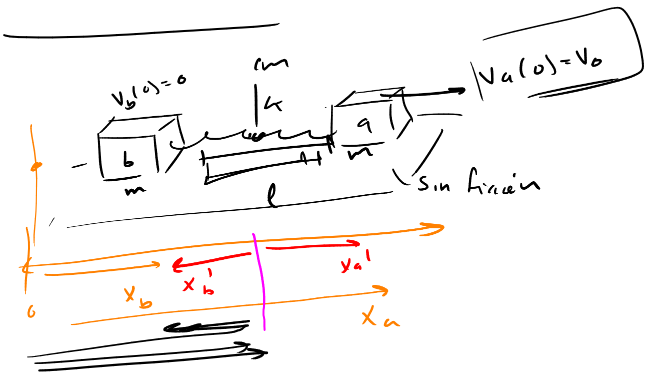
$M \ddot{x}_M = -f_{Mm} = f_{mM} = -m \ddot{x}_m$

↳ tras integrar → $M \dot{x}_M(t) - M \dot{x}_M(0) = -\int_0^t f\,dt = m \dot{x}_m(0) - m \dot{x}_m(t)$

$M \dot{x}_M(t) + m \dot{x}_m(t) = M \dot{x}_M(0) + m \dot{x}_m(0) \rightarrow P_x = 0$

$\dot{P}_x = 0$

$$\vec{X} = \frac{m \vec{r}_m + M \vec{r}_M}{m+M} \rightarrow \dot{X} = \frac{m \dot{x}_m + M \dot{x}_M}{m+M} = 0$$



$m_a = m_b = m$

$x_b = X + x'_b$

$$X = \frac{m x_a + m x_b}{2m} = \frac{1}{2}(x_a + x_b)$$

$$x'_b = x_b - X = x_b - \frac{1}{2}(x_a + x_b)$$

$$= \frac{1}{2}(x_b - x_a) = \frac{m_a}{m_a + m_b}(x_b - x_a)$$

$$x'_a = -\frac{1}{2}(x_b - x_a) = \frac{-m_b}{m_a + m_b}(x_b - x_a)$$

$F = -k\,\big||x_b - x_a| - l\big| = -k\,|x'_b - x'_a - l|$

$m \ddot{x}'_a = -k(x'_a - x'_b - l) \quad (1)$

$m \ddot{x}'_b = +k(x'_a - x'_b - l) \quad (2)$

$(1) - (2)$

$m(\ddot{x}'_a - \ddot{x}'_b) = -2k(x'_a - x'_b - l)$

$$\frac{d^2}{dt^2}\underbrace{(x'_a - x'_b - l)}_{y} = -\frac{2k}{m}\underbrace{(x'_a - x'_b - l)}_{y}$$

$$\boxed{\ddot{y} = -\frac{2k}{m} y}$$

$y(t) = y_0 \cos(\omega t + \varphi_0) \quad \omega = \sqrt{\frac{2k}{m}}$

$$x'_a - x'_b = \ell\left(\cos\left(\sqrt{\tfrac{2k}{m}}\,t\right)+1\right) = x_a - x_b$$

$$\dot x'_a - \dot x'_b = -\ell\sqrt{\tfrac{2k}{m}}\,\sin\left(\sqrt{\tfrac{2k}{m}}\,t\right) = \dot x_a - \dot x_b \qquad \vec P = 0 = m(\dot x'_a + \dot x'_b) = 0$$

$$\left\{\begin{array}{l} \dot x_a = \dot x'_a + \dot X \\ \dot x_b = \dot x'_b + \dot X \end{array}\right\} \qquad \dot X = \frac{\dot x_a + \dot x_b \cancel{-(\dot x'_a + \dot x'_b)}^{\,0}}{2} = \frac12(\dot x_a + \dot x_b)$$

cte

$$\dot X(t) = \dot X(0) = \frac12\left(\dot x_a(0) + \cancel{\dot x_b(0)}^{\,0}\right) = \frac12 v_0 \qquad (\dot x_a(0) = v_0)$$

$$X = \frac12 v_0 t + \frac{x_a(0)+x_b(0)}{2}$$

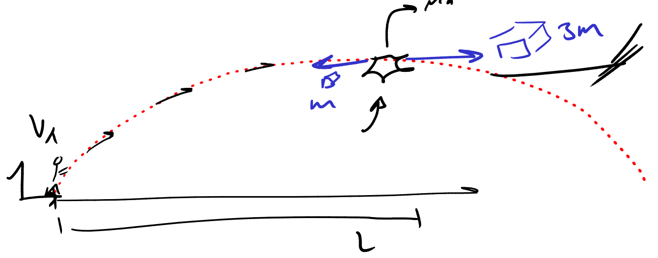
$$x'_a = \frac12 v_0 t + X_0 + \ell(\cos(\sqrt{\ }t)+1) \qquad x'_a = \frac12\Delta x +$$

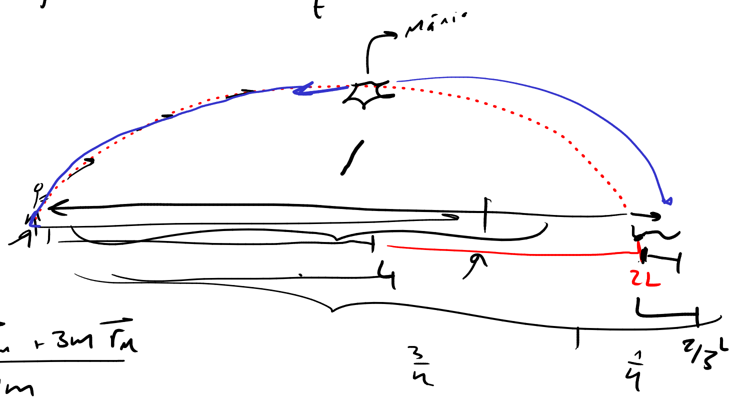
$$x'_b = \frac12 v_0 + X_0 + \ell(\cos\sqrt{\ }t)+1)$$

t

→ ¿Qué tan lejos llegó?

t





$$M = m + 3m = 4m \qquad \vec R = \frac{m\vec r_m + 3m\vec r_M}{4m}$$

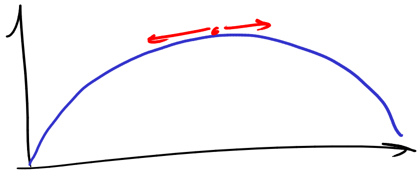
$$X = \frac{m x_m + 3m X_M}{4m} = \frac{x_m + 3X_M}{4} \checkmark =$$

Calculando cuando caen $X = 2L$, $x_m = 0$ $\to$ $2L = \frac34 X_M \to X_M = \left(2L + \frac23 L\right)$

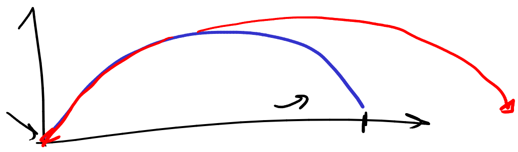
$$\dot X = \frac{m\dot x_m + 3m\dot X_M}{4m} = \frac{\dot x_m + 3\dot X_M}{4} = v_0 \rightarrow \text{No está acabado.}$$

$$\to x'_m = x_m - X = x_m - \frac{x_m}{4} - \frac34 X_M = \frac34(x_m - X_M) = \frac34\left(-\frac83\right)L = -2L$$

$$\to X'_M = X_M - \frac{x_m}{4} - \frac34 X_M = -\frac14(x_m - X_M) = -\frac14\left(-\frac83\right) = \frac23 L$$



Rcm (1 obj)



Rcm (+rnLs)

$$R_{cm} = ZL = \frac{\cancelto{0}{mr} + mR}{m+M} = \frac{3}{4}R$$