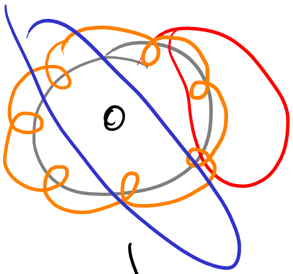
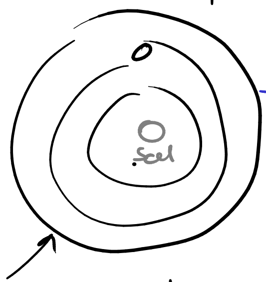
Movimiento planetario

Ptolomeo ↓



→ Mediciones experimentales

Copérnico → Navaja de Ockham
↓ → Más sencillo



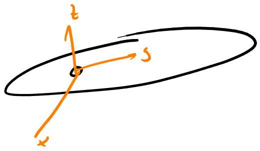
→ Modelo con diseño divino

T

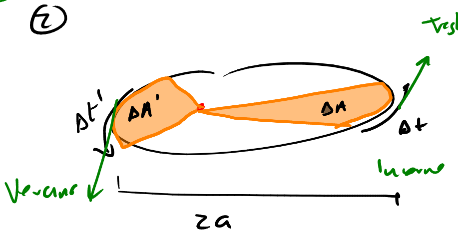
↳ Galileo
↳ Kepler → Miope

↳ Kepler formula sus 3 leyes

① ✓



② ✓



$\Delta A = \Delta A'$ si $\Delta t = \Delta t'$

③ ✓ $T^2 \sim a^3$

① y ② → Campo central

$$\vec{F} = -G\frac{Mm}{r^2}\hat{e}_r$$ → Son aproximadamente círculos

$\frac{1}{r^2}$ → (3)

aprox. $\omega = 2\pi\nu = \frac{2\pi}{T}$ $\quad \nu = \frac{1}{T}$

$$ma_c = mr\omega^2 = mr\frac{4\pi^2}{T^2} \sim \frac{r}{T^2} \sim \frac{r}{r^3} \sim \frac{1}{r^2}$$

$$\int_{r_0}^{r} \frac{d}{dr}U = \int_{r_0 \to \infty}^{r} G\frac{Mm}{r^2} \Rightarrow U = -\frac{K}{r}$$

① $u = 1/r \Rightarrow du = -dr/r^2$

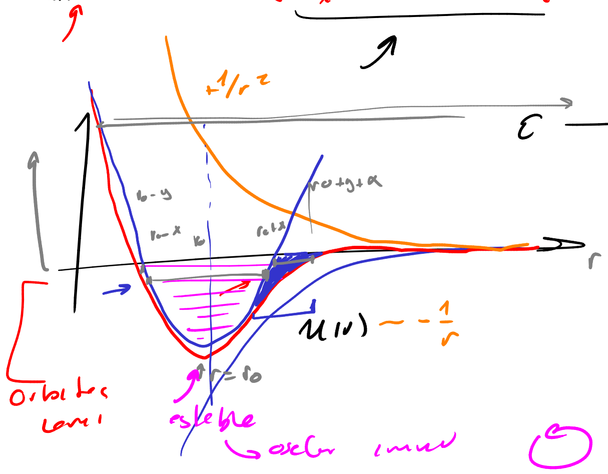
$$\Delta\theta = \theta - \theta_0 = \frac{\ell}{m}\int \frac{dr/r^2}{\sqrt{\frac{2}{m}E - \frac{2}{m}\frac{\ell^2}{mr^2} + \frac{2k}{m}\frac{1}{r}}}$$

$$\sqrt{\frac{2m}{\ell^2}E + \frac{2\ell^2}{m^2}u^2 + \frac{2mk}{\ell^2}u}$$

$U = -\frac{k}{r} + \frac{k}{r_0}$

$r_0 \to \infty$

$$U_{ef} = \frac{\ell^2}{2m}\frac{1}{r^2} - \frac{k}{r}$$



E → $E > 0$ → abiertas
$E < 0$ → cerradas

$E \geq \frac{K}{r_0}$

$ax^2 + bx + c$

$a(x^2 + \frac{2}{2}\frac{b}{a}x) + c$

$a(x^2 + 2(\frac{b}{2a})x + (\frac{b}{2a})^2) + c - a(\frac{b}{2a})^2$ (= d)

$a(x - (\frac{b}{2a}))^2 + d$

$u = \frac{1}{r} \Rightarrow du = -dr/r^2$

$$\Delta\theta = \int \frac{dr/r^2}{\sqrt{\frac{2mE}{\ell^2} + \frac{2mk}{\ell^2}\frac{1}{r} - \frac{1}{r^2}}}$$

$$= -\int \frac{du}{\sqrt{\frac{2mE}{\ell^2} + \frac{2mk}{\ell^2}u - u^2}} = -\int \frac{du}{\left[\frac{2mE}{\ell^2} + \left(\frac{mk}{\ell^2}\right)^2 - \left(u - \frac{mk}{\ell^2}\right)^2\right]^{1/2}}$$

a ; $(u-b)$

$$-u^2 + \frac{2mk}{\ell^2}u - \left(\frac{mk}{\ell^2}\right)^2 + \left(\frac{mk}{\ell^2}\right)^2$$

$$-\left(u - \left(\frac{mk}{\ell^2}\right)\right]$$

$$= \frac{1}{\sqrt{a}} \int \frac{-du}{\left(1 - \frac{(u-b)^2}{a}\right)^{1/2}}$$

$$\beta = \frac{u-b}{\sqrt{a}} \Rightarrow d\beta = \frac{du}{\sqrt{a}}$$

$\beta = \cos\chi \rightarrow d\beta = -\sin\chi\, d\chi$

$d\chi = \frac{-d\beta}{\sin\chi}$

$$= \int \frac{-d\beta}{\sqrt{1-\beta^2}}$$

$\sqrt{1-\beta^2} = \sqrt{1-\cos^2} = \sin\chi$

$$= \int \frac{-d\beta}{\sin\chi} = \int d\chi = \chi = \arccos\left(\frac{\frac{1}{r} - (mk/\ell^2)}{\sqrt{\frac{2Em}{\ell^2} + \left(\frac{mk}{\ell^2}\right)^2}}\right)$$

$$r = \frac{mk/\ell^2}{1 + \epsilon\cos(\alpha - \alpha_0)}$$

$$\epsilon = \sqrt{1 + \frac{2E\ell^2}{m}}$$

semieje

$$\frac{(x-x_0)^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ec. secciones cónicas centradas en un foco

↳ ¿Por qué?