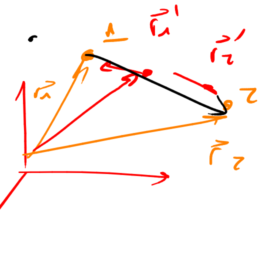
Campo central (Fuerza central)

① → Caso general $\qquad \vec{F} = f(r)\,\hat{e}_r$



② → Problema de Kepler

↳ Mecánica de los planetas

$$\vec{r}_1' = -\frac{m_2}{m_1+m_2}\,\vec{r}$$

$$\vec{r}_2' = \frac{m_1}{m_1+m_2}\,\vec{r}$$

$$\vec{r} = r\hat{e}_r = (\vec{r}_2 - \vec{r}_1) = (\vec{r}_2' - \vec{r}_1')$$

→ Duda: Potencial efectivo U_{eff}

2 cuerpos × 3 posibles dirección → 6 ecuaciones $\begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \\ z_1 \\ z_2 \end{pmatrix} \to \begin{pmatrix} r_1 \\ r_2 \\ \theta_1 \\ \theta_2 \end{pmatrix}$, $\quad z_1, z_2 = 0$

↳ $4 \begin{pmatrix} r_1 \\ r_2 \\ \theta_1 \\ \theta_2 \end{pmatrix} \to 2 \begin{pmatrix} r \\ \theta \end{pmatrix}$ → 2do grad.

→ $l = mrv = mr^2\dot{\theta}$ (cte)

→ $\dot{\theta} = \frac{l}{mr^2}$ $\qquad E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + U(r)$

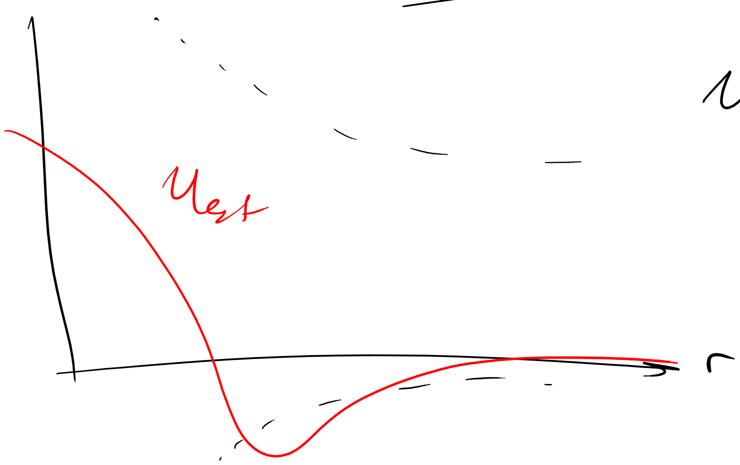
1 partícula → 2 dim.

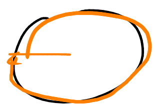
$$E = \frac{1}{2}m\dot{r}^2 + \underbrace{\left(\frac{l^2}{2m}\frac{1}{r^2} + U(r)\right)}_{U_{eff}(r)}$$

1 partícula → 1 dim. $\qquad \boxed{F_{ef} = -\frac{d}{dr}U_{eff}}$

Replanteamos nuestro problema como uno nuevo, con una fuerza distinta pero que nos de los mismos resultados.

$U: \mathbb{R}^+ \to \mathbb{R}$





Dudas:

$$\dot{\vec r} = (\dot r \hat e_r + r\dot\theta \hat e_\theta)\mu$$ ¿Sabemos de dónde sale?

$$\mu \ddot{\vec r} = \underbrace{\mu(\ddot r - r\dot\theta^2)}_{f(r)} \hat e_r + \mu\underbrace{(r\ddot\theta + 2\dot r\dot\theta)}_{=0} \hat e_\theta = f(r)\,\hat e_r$$

$$\frac{d\vec L}{dt} = \vec 0$$

$$E = \frac{1}{2} m \dot r^2 + U_{ef}(r)$$ cte

$$U_{ef}(r) = \overbrace{\frac{l^2}{2m}\frac{1}{r^2}} + \overbrace{U(r)}$$

$$\dot r = \frac{dr}{dt} = \sqrt{\frac{2}{m}}\left(E - U_{ef}(r)\right)^{1/2} \longrightarrow \int_{r_0}^{r} \frac{dr}{\sqrt{\frac{2}{m}}\left(E - U_{ef}(r)\right)^{1/2}} = \Delta t = t - t_0 = g(r)$$

$$r = r(t)$$

$$\dot\theta = \frac{d\theta}{dt} = \frac{l}{mr^2} \rightarrow \Delta\theta = l\int_{t_0}^{t} \frac{dt}{mr^2(t)} = \theta(t) - \theta_0$$

$\boxed{r = r(\theta)}$

$$\frac{dr}{d\theta}, \quad \frac{d\theta}{dr} \rightarrow \frac{d\theta}{dt}, \quad \frac{dr}{dt} = \frac{1}{\left(\frac{dt}{dr}\right)}$$

$$\frac{d\theta}{dt}\frac{dt}{dr} = \frac{d\theta}{dt}\frac{1}{(dr/dt)} = \frac{l/mr^2}{\sqrt{\frac{2}{m}}\left(E - U_{ef}(r)\right)^{1/2}} = \frac{d\theta}{dr}$$

$$\int_{\theta_0}^{\theta} d\theta = \Delta\theta = \int_{r_0}^{r} \frac{l}{m}\frac{dr/r^2}{\left(\frac{2}{m}E - \frac{2}{m}U_{ef}\right)^{1/2}} = h(r) = \theta - \theta_0$$

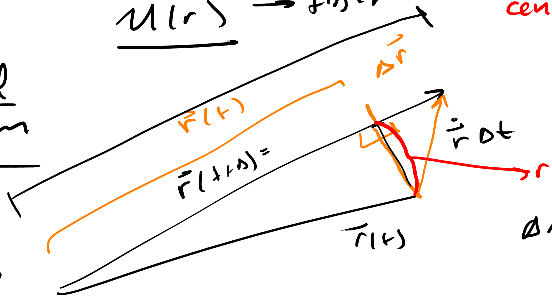
$$r = r(\theta)$$

Campo central $\longrightarrow$ Fuerza central

$U(r) \rightarrow f(r)\hat e_r$

Velocidad de áreas

$$\rightarrow \frac{dA}{dt} = \text{cte} = \frac{l}{2m}$$



$$\Delta A = \frac{1}{2} b h \simeq \frac{1}{2}(r + \Delta r)(r\Delta\theta)$$

$$\frac{\Delta A}{\Delta t} \xrightarrow{\Delta t \to 0} \frac{dA}{dt} = \frac{1}{2} r^2\dot\theta + \frac{1}{2}\cancel{\Delta r\, r\dot\theta}$$

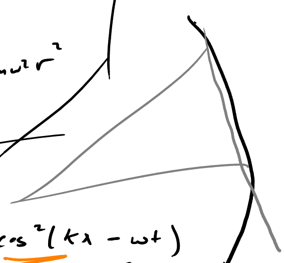
$\Delta r \to 0$

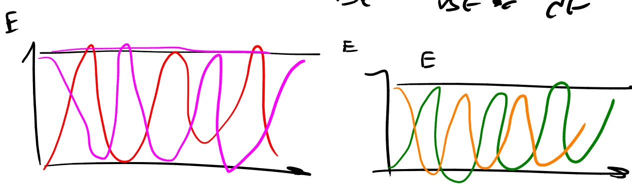
$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

diatómica

m_1 m_2 $U = \frac{1}{2} m\omega^2 r^2$

↳ campo central





$$\varepsilon_i = m\omega^2 \cos^2(kx - \omega t)$$

Primeros vecinos

$$\vec r_{ij} = \vec r_i - \vec r_j$$

$$U(r) = U(r_{ij})$$

$$E = \sum_i \varepsilon_i = \sum_i \frac{1}{2} m v_i^2 + \frac{1}{2}\sum_{ij} U(r_{ij}) \longrightarrow 1 \text{ vecino}, \text{ vecinos } N$$

Caso límite

$$\vec{r}_1' = -\frac{m_2 m_1}{m_1+m_2}\frac{\vec{r}}{m_1} = -\frac{\mu}{m_1}\vec{r} \approx -\frac{\mu}{m_1}\vec{r} \approx \vec{r}$$

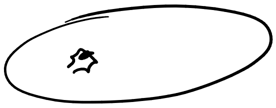
$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} = \frac{m_1+m_2}{m_1 m_2} \quad \text{Si} \quad m_2 \gg m_1 \quad \frac{1}{m_1} \gg \frac{1}{m_2}$$

$$\approx \frac{1}{m_1}$$

$$\vec{r}_2' = \frac{m_1 m_2}{m_1+m_2}\frac{\vec{r}}{m_2} = \frac{\mu}{m_2}\vec{r} \xrightarrow{\text{cuv}} \vec{r}_2' \approx \vec{0} \qquad \mu \approx m_1 \qquad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\frac{1}{m_2} \ll \frac{1}{m} \approx \frac{1}{\mu} \qquad \text{CM} \Rightarrow \vec{r}_1$$

1ra $\longrightarrow$ Planeta



$$r(\theta) = \frac{r_0}{1+\varepsilon\cos(\theta-\theta_0)}$$ $\longrightarrow$ con un foco en el origen

$$0 < \varepsilon < 1$$

$$M_{sol} \gg M_{Jup} \gg M_{\text{los demás}}$$