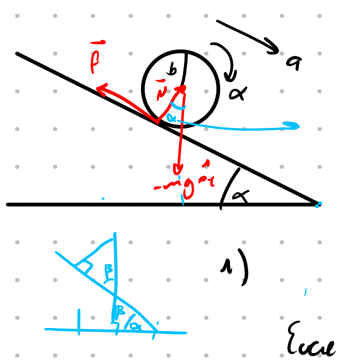
Ejemplos

Rueda sin resbalar

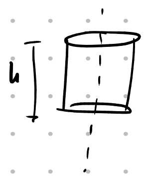


1)

Ecuaciones de movimiento

x: $-f_r + mg\sin\theta = ma \quad \cdots(1)$

y: $N - mg\cos\theta = 0 \quad \cdots(2)$



Medido desde el eje de simetría

$$I = \int s^2 \rho\, dv = \rho \int s^2\, s\, d\varphi\, dz\, ds \qquad \rho = \frac{M}{\pi b^2 h}$$

$$= \rho\, 2\pi h \int s^3 ds = \rho\, 2\pi h \frac{1}{4} b^4 = \frac{M}{\cancel{\pi} b^{\cancel{2}} \cancel{h}} \frac{\cancel{2\pi h}}{\cancel{4}\, 2} b^{\cancel{4}\, 2}$$

$$I = \frac{1}{2} M b^2 \longrightarrow \text{No depende de } h$$

Torca medida desde el centro del objeto

Rueda sin resbalar

$\boxed{a = \alpha b}$

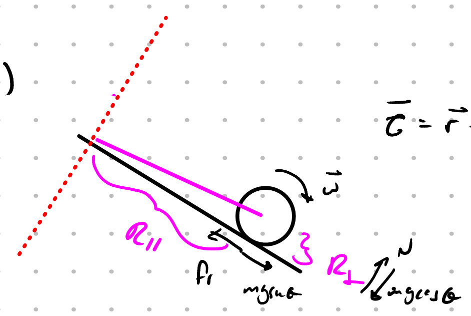
$$\vec{s} \times \vec{f_r} = -\hat{e}_z\, b f_r = -\hat{e}_z I\alpha = -I\frac{a}{b}\hat{e}_z \quad \cdots(3)$$

Sustituyendo (3) en (1)

$$\underbrace{-\left(I\frac{a}{b^2}\right)}_{f_r} + mg\sin\theta = -\left(\frac{1}{2}ma\right) + mg\sin\theta = ma \Rightarrow a = \frac{2}{3} g\sin\theta$$

$I = \frac{1}{2}Mb^2$

2)



$$\vec{\tau} = \vec{r}\times\vec{F} \Rightarrow \vec{\tau}\cdot\hat{e}_z = -R_\perp f_r + \overset{b}{R_\perp}(f - mg\sin\theta) + R_\parallel(\cancel{N - mg\cos\theta})$$

$$= -b\, mg\sin\theta$$

$$\vec{L}\cdot\hat{e}_z = -I_0\omega + (\vec{R}\times M\overset{v = b\omega}{\vec{v}})_z$$

$$= -\frac{1}{2}Mb^2\omega - Mb^2\omega = -\frac{3}{2}Mb^2\omega$$

$$\tau_z = \frac{d}{dt}L_z \Rightarrow -bmg\sin\theta = -\frac{3}{2}Mb^2\alpha =$$

$$\Rightarrow \alpha = \frac{2}{3}g\sin\theta\frac{1}{b} \Rightarrow a = \frac{2}{3}g\sin\theta$$

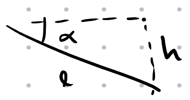
¿Cuál es su velocidad al caer?

Si $v(t=0) = 0$

$$\Delta T_{cm} = \frac{1}{2}M(\cancel{v_i^2} - v_f^2) \longrightarrow T \quad E - T \qquad \Delta T = -\Delta U$$

$$\Rightarrow (mg\sin\theta - f)l = \frac{1}{2}m v_f^2$$

Sup. $f_r = \text{cte} \Rightarrow \int \vec{F}\cdot d\vec{l} = \int (mg\sin\theta - f)\, dl = (mg\sin\theta - f)\, l$



$l = \frac{h}{\sin\theta}$

$$\tau_0 = I_0\alpha = I_0\frac{d\omega}{dt}$$

$\Delta\theta = \omega\Delta t$

$d\theta = \omega\, dt$

$$\Rightarrow \tau_0\, d\theta = I_0\frac{d\omega}{dt}\omega\, dt = d\left(\frac{1}{2}I_0\omega^2\right)$$

$$\int \tau\, d\theta = -b f_r \Delta\theta = \frac{1}{2}I_0(\cancel{\omega_0^2} - \omega^2)$$

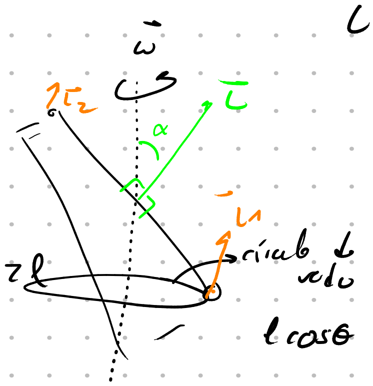
$$= \underbrace{-b\Delta\theta}_{l} \Rightarrow fl = \frac{1}{2}I_0\omega^2 = \frac{1}{2}I_0\left(\frac{v}{b}\right)^2$$

Se s.l

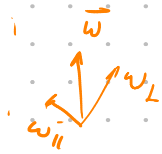
$$l\left(mg\sin\theta - \frac{1}{2}mv^2\right) = \frac{1}{2}I_0\frac{v^2}{b^2}$$

$$\Rightarrow mgh = \frac{3}{4}Mv^2$$

$$\vec{L} = \sum (\vec{r}_i \times \vec{p})$$

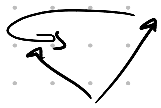


$$|\vec{p}| = m|\vec{v}| = m\omega\, \ell\cos\omega \Rightarrow \vec{L} = 2\ell|\vec{p}| = 2m\omega\, \ell^2\cos\theta$$



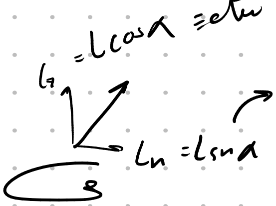
$\omega_\perp = \omega\cos\theta$

$\omega_\parallel = \omega\sin\theta$

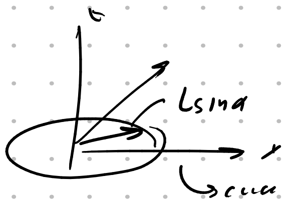


$|\vec{L}| = cte$

gira en el besteño



esto sí gira y depende del tiempo



cuando $t = t_0$

$L_x = L_h\cos(\omega t)$

$= L\sin\alpha\cos\omega t$

$L_y = L\sin\alpha\sin\omega t$

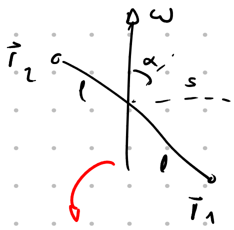
$$\vec{L} = L\cos\alpha\,\hat{e}_z + L\sin\alpha\,(\cos\omega t, \sin\omega t)$$

$$\vec{\tau} = \frac{d\vec{L}}{dt} = L\sin\alpha\,\omega\,(-\cos\omega t, \sin\omega t) = (\tau_x, \tau_y)$$

$\rightarrow 2m\ell^2\omega\cos\alpha$

$$\Rightarrow \|\vec{\tau}\| = \omega L\sin\alpha$$

Tma $\quad \vec{L} = I\vec{\omega} \longrightarrow \vec{L} = \overleftrightarrow{I}\cdot\vec{\omega}$



Sistema de referencia es con $\hat{e}_z$ el eje de giro

$\vec{r}_1$: $x_1 = s\cos\omega t$, $y_1 = s\sin\omega t$, $z_1 = -h$

$\vec{r}_2$: $x_2 = -s\cos\omega t$, $y_2 = -s\sin\omega t$, $z_2 = h$

$s = \ell\cos\alpha$

$h = \ell\sin\alpha$

$$I_{xx} = m_1(y_1^2 + z_1^2) + m_2(y_2^2 + z_2^2) = 2m(s^2\sin^2\omega t + h^2)$$

$$I_{zy} = I_{yz} = -m_1 y_1 z_1 - m_2 y_2 z_2 = 2m\,sh\sin\omega t$$

$$\overleftrightarrow{I} = 2m\begin{pmatrix} s^2\sin^2\omega t + h^2 & -s^2\sin\omega t\cos\omega t & sh\cos\omega t \\ & s^2\cos^2\omega t + h^2 & sh\sin\omega t \\ & & s^2 \end{pmatrix}$$

$$\vec{\omega} = (0, 0, \omega)$$

¿Si sacamos el eje de giro?