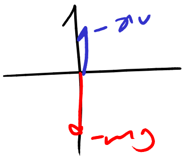
— Revisión de la tarea —

Fluidos → agua/aceite

1) $\vec{F}_f = -m\gamma v\,\hat{v}$ ⟶ $\frac{\vec{v}}{v} \Rightarrow \|\hat{v}\| = 1$

sup. $v(t=0)=0$



$\downarrow -m g\,\hat{j}$

$$\vec{F} = m\vec{a} \longrightarrow -\dot{v} = -g + \gamma v$$

$$-\frac{dv}{dt} = -g + \gamma v$$

$$\underline{\frac{-\gamma}{-\gamma v + g}\frac{dv}{dt}} = -\gamma = \frac{d}{dt}\left(\ln(g-\gamma v)\right)$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$

$$\frac{d}{dt}\left(\ln(g-\gamma v)\right) = \frac{-\gamma\, dv/dt}{g-\gamma v}$$

integramos

$$-\int_0^t \gamma\, dt = -\gamma t = \int_{t=0}^{t=t} \frac{d}{dt}\ln(g-\gamma v)\, dt$$

$$= \ln(g-\gamma v)\Big|_{t=0}^{t=t}$$

$$= \ln(g-\gamma v) - \ln(g)$$

$$\underline{-\gamma t = \ln\left(1-\frac{\gamma v}{g}\right)}$$

Despejar v:

$$\frac{g}{\gamma}e^{-\gamma t} = \left(1-\frac{\gamma v}{g}\right)\frac{g}{\gamma}$$

$$\Rightarrow \quad v = \frac{g}{\gamma}\left(1-e^{-\gamma t}\right)$$

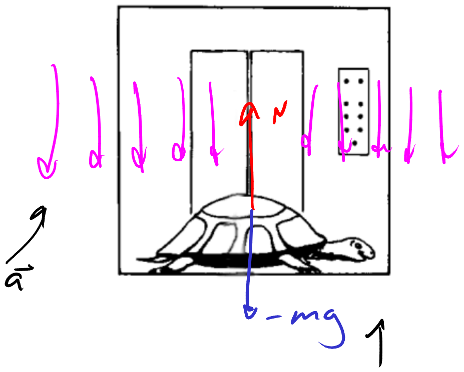
$$\lim_{t\to\infty} v(t) = v_\infty = \frac{g}{\gamma}$$

$$v = \frac{dy}{dt} \longrightarrow \int v\,dt = y-h = \frac{g}{\gamma}\int\left(1-e^{-\gamma t}\right)dt$$

$$= \frac{g}{\gamma}\left[t\Big|_0^t - \frac{1}{-\gamma}e^{-\gamma t}\Big|_0^t\right]$$

$$\underline{y-h = \frac{g}{\gamma}\left(t+\frac{1}{\gamma}e^{-\gamma t}-1\right)}$$

Tortuga



$$N - mg = \pm ma \longrightarrow N = m(g \pm a)$$

mide la báscula

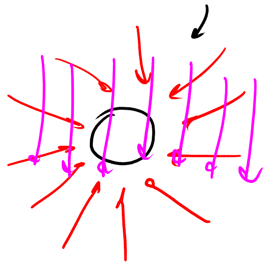
aceleración del elevador

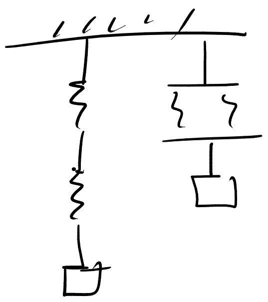
Principio de equivalencia de Einstein $\longleftarrow \begin{cases} g = 0 \\ a = 9.81 \end{cases}$

$$N = m(g \pm a) = \underline{m(9.81)}$$

vs $\longrightarrow$ Análisis global

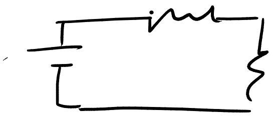
↳ pero localmente es igual



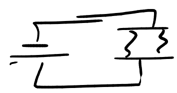


Serie

$$\frac{1}{K_{ef}^{serie}} = \sum\left(\frac{1}{k_i}\right), \quad k_{ef}^{paralelo} = \sum k_i$$



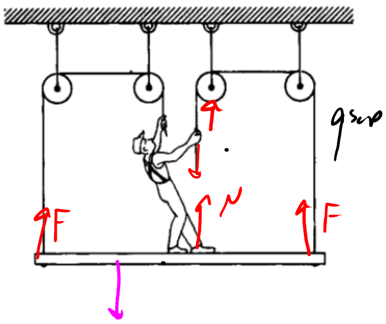
$$R_{eq} = \sum R_i$$

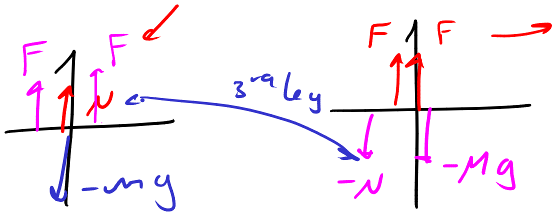


$$\frac{1}{R_{eq}^{para}} = \sum\left(\frac{1}{R_i}\right)$$

3ra ley con la cuerda

Transporte de la fuerza por la cuerda



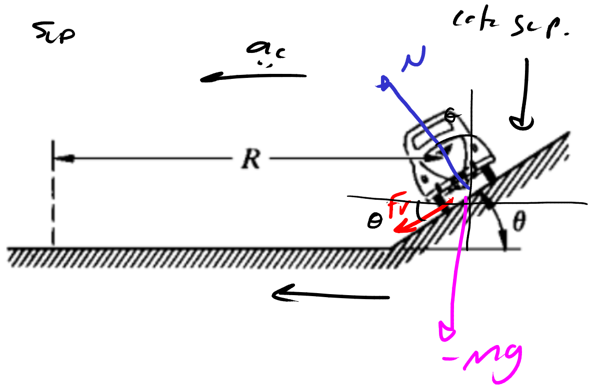


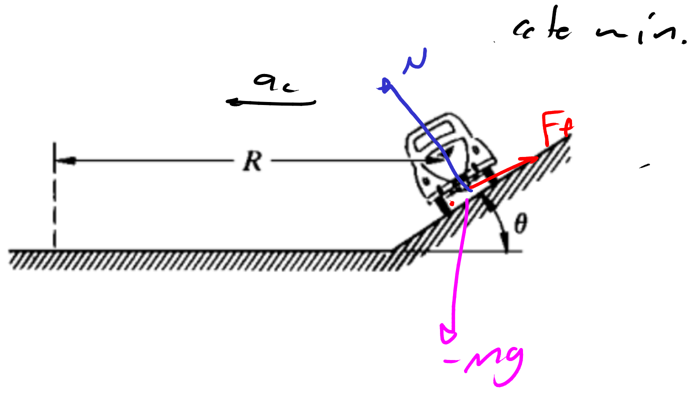
$$N - mg + 2F = ma \qquad 2F - N - Mg = Ma$$

$$N = ma + mg - 2F \qquad N = 2F - Mg - Ma$$

$$-2F + ma + mg = 2F - Mg - Ma$$

$$\boxed{a = \frac{4F - Mg - mg}{M + m}}$$





$$-ma_c = -F_f\cos\theta - N\sin\theta$$

$$0 = -mg + N\cos\theta - F_f\sin\theta$$

$$-ma_c = +F_f\cos\theta - N\sin\theta$$

$$0 = -mg + N\cos\theta + F_f\sin\theta$$

sup. $F_f = \mu N$

$$mg = N\cos\theta - \mu N\sin\theta$$

$$mg = N(\cos\theta - \mu\sin\theta)$$

$$mg = N\cos\theta + N\mu\sin\theta$$

$$mg = N(\cos\theta + \mu\sin\theta)$$

$$-ma_c = -\mu N\cos\theta - N\sin\theta$$

$$= -N(\mu\cos\theta + \sin\theta)$$

$$v^2 = Rg\frac{(\mu\cos\theta + \sin\theta)}{\cos\theta - \mu\sin\theta} \to 0$$

$$-ma_c = \mu N\cos\theta - N\sin\theta$$

$$= N(\mu\cos\theta - \sin\theta)$$

$$v^2 = -Rg\frac{(\mu\cos\theta - \sin\theta)}{\cos\theta + \mu\sin\theta} = 0$$

$\boxed{\text{si } \mu = 1,\ \theta = 45}$ $\to$ $\boxed{V_{max} \to \infty > V > V_{min} \to 0}$

Energía y trabajo

$\vec{F} = m\dot{\vec{v}}$

$$\boxed{dW = \vec{F}\cdot d\vec{r} = \vec{F}\cdot\vec{v}\,dt} \rightarrow \boxed{W_b - W_a = \int_a^b \vec{F}\cdot d\vec{r}} = -\Delta(E_c)$$

$\vec{F}\cdot\vec{v}$: Potencia — Energía/tiempo

$$\boxed{E_c = \frac{1}{2}mv^2}$$

$$\frac{1}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v}) = v\cdot\frac{dv}{dt}$$

$\vec{F} = -\nabla U$

$$\boxed{F^{1D} = -\frac{d}{dx}U(x)}$$

$F = -kx \rightarrow \frac{1}{2}kx^2 = U$

$F = -mg \rightarrow mgx = U$

$F = -\frac{k}{r^2}$ — ley grav. / ley Coulomb.

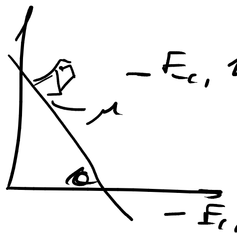
$\rightarrow -\frac{k}{r} = U$

$F = -\nabla U$

$$\Delta U = -\Delta E_c \rightarrow \boxed{E_T = U_1 + E_1 = U_2 + E_2 = cte}$$

$$\vec{F} = -\nabla U + \vec{F}_{dis.} \qquad Q = \int_a^b \vec{F}_{dis}\cdot d\vec{r}$$

$$\boxed{U_a + E_{ca} = U_b + E_b + Q} \rightarrow \boxed{\Delta E_T = Q}$$



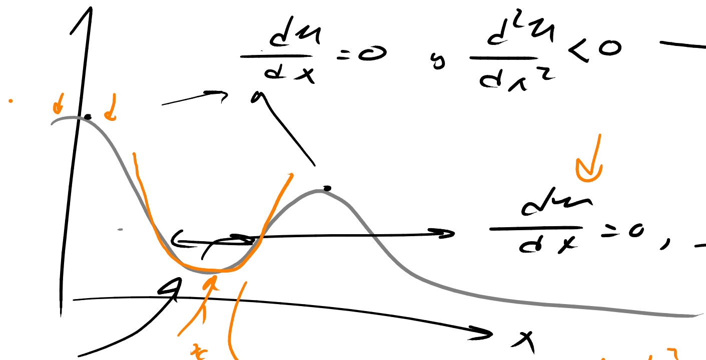
$\mu = cte$

$v = \sqrt{2gh}$ si $\mu = 0$

$v = \sqrt{2gh(1-\mu\cot\theta)}$ si $\mu \neq 0$

$Q = \int f_{dis}\cdot dr \rightarrow$ diagrama de cuerpo libre

$U(x)$



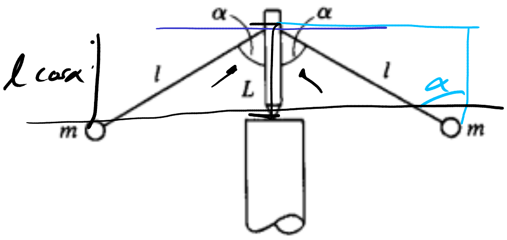
$\frac{dU}{dx} = 0$ y $\frac{d^2U}{dx^2} < 0 \rightarrow$ punto de equilibrio inestable

$\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} > 0 \rightarrow$ punto estable

$U(x) = U(x_0) + \frac{1}{2}\frac{d^2U}{dx^2}(x_0)(x-x_0)^2$

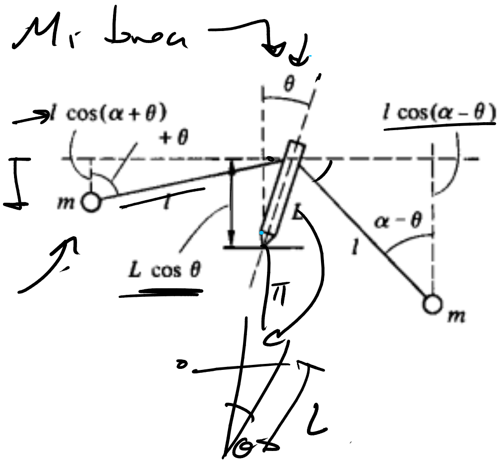
$\rightarrow \vec{F} = -\nabla U = -\frac{d^2U}{dx^2}(x-x_0)\,\hat{e}_x$

$x - x_0 = A\cos(\omega t + \varphi)$ $\quad \omega = \sqrt{k/m}$



$$U = mg\,\underline{z_1} - g\,z_2$$

Mi brazo



$$z_1 = L\cos\theta - l\cos(\alpha+\theta)$$

$$z_2 = L\cos\theta - l\cos(\alpha-\theta)$$

$$U = mg\left[L\cos\theta - l\cos(\alpha-\theta) + L\cos\theta - l\cos(\alpha+\theta)\right]$$

$$\cos(\alpha\pm\theta) = \cos\alpha\cos\theta \mp \sin\theta\sin\alpha$$

$$U = mg(2L\cos\theta - 2l\cos\alpha\cos\theta)$$

$$U(\theta) = mg\cos\theta\,(2L - 2l\cos\alpha) = 2mg(L - l\cos\alpha)\cos\theta$$

$$\theta \ll 1 \rightarrow \cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\frac{dU}{d\theta} = 0 = (2L - 2l\cos\alpha)\,mg(-\sin\theta)$$

$$\theta = 0, \cancel{\pi}$$

$$U(\theta) = -\boxed{\underbrace{2mg(l\cos\alpha - L)}_{U_0}}\left(1 - \frac{\theta^2}{2}\right) = -U_0 + \frac{U_0}{2}\theta^2$$

$$\frac{d^2U}{d\theta^2} = -mg\cos\theta\,(2L - 2l\cos\alpha)$$

$$\frac{m\ddot\theta}{l^2} = F = -\nabla U = -\frac{1}{l}\frac{d}{d\theta}\left(\frac{U_0}{2}\theta^2\right) = -\frac{U_0\theta}{l}$$

$$\frac{d^2U}{d\theta^2} = \underbrace{-mg}_{<0}\,(\underbrace{2L - 2l\cos\alpha}_{<0}) > 0$$

$$\ddot\theta = -\frac{U_0 l}{m}\theta \rightarrow \boxed{\theta = A\cos(\omega t + \varepsilon) \qquad \omega = \sqrt{\frac{l\,U_0}{m}}}$$

$$\boxed{L < l\cos\alpha} \qquad L - l\cos\theta < 0$$