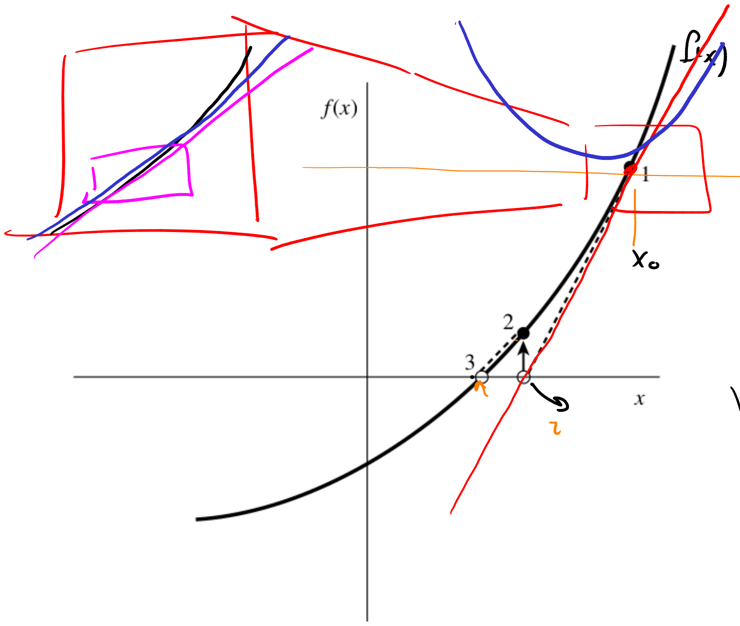
Métodos para resolver raíces: Newton - Raphson



$f(x)\ ,\ f'(x)$

x_0

Serie de Taylor alrededor de x_0

$b \qquad m\ (x-x_0)$

$$f(x) \approx f(x_0) + f'(x_0)\ (x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2$$

$$\boxed{f(x) = m(x-x_0)+b} \rightarrow \text{Rectas}$$

$$f(x) = \sum \frac{d^n f(x)}{dx^n}\Bigg|_{x=x_0} \frac{(x-x_0)^n}{n!}$$

· Serie de Taylor

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \dots$$

· Raíz

$$f(x) = 0$$

$$f(x_0) + f'(x_0)\ (x-x_0) = y(x) = 0$$

$$(x_i - x_{i-1}) = -\frac{f(x_{i-1})}{f'(x_{i-1})} \Rightarrow x_i = -\frac{f(x_{i-1})}{f'(x_{i-1})} + x_i$$

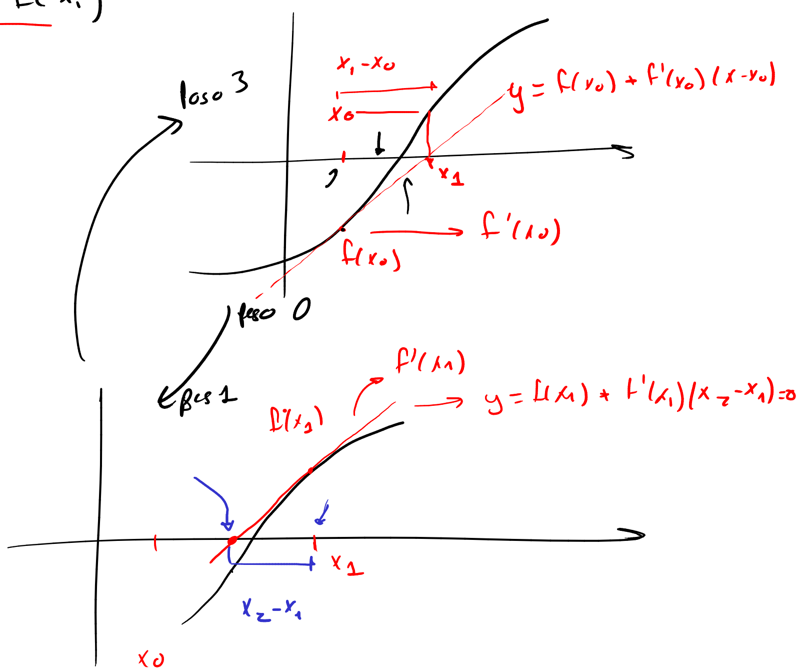
raiz = x_i

while $|\Delta x| >$ tol and $i < n\_max$:

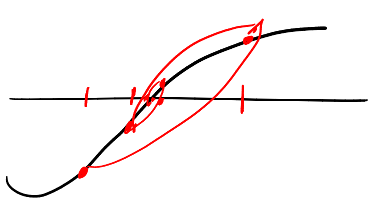
  $\Delta x = -f(x_{i-1}) \,/\, f'(x_i)$

  raiz = raiz + Δx

  $i = i + 1$



Paso



$$f(x) = x^3 + 7x + \sin(x)\, x^2$$

$$\longrightarrow f'(x) = 3x^2 + 7 + x^2\cos(x) + 2x\sin(x)$$

Recuerdo → convergencia bisección

$\varepsilon_n = \frac{1}{2^n}\varepsilon_0$ → por fuerza original

$\boxed{\varepsilon_n = \frac{\varepsilon_{n-1}}{2}}$ → Convergencia lineal

$\Delta\varepsilon = -\frac{f(x)}{f'(x)}$ ← $f(x) = \cancel{f(x_0)} + \underbrace{f'(x_0)\varepsilon_i + f''(x_0)\varepsilon_i^2}$ 0

$\varepsilon_{i+1} - \cancel{\varepsilon_i} = -\frac{f'(x_0)\cancel{\varepsilon_i} + f''(x_0)\varepsilon_i^2}{f'(x_0)} = -\cancel{\frac{f'(x_0)\varepsilon_i}{f'(x_0)}} - \frac{f''(x_0)\varepsilon_i^2}{f'(x_0)} = -\cancel{\varepsilon_i} - \frac{f''(x_0)}{f'(x_0)}\varepsilon_i^2$

↓

$\boxed{\varepsilon_{i+1} = -\frac{f''(x_0)}{f'(x_0)}\varepsilon_i^{\cancel{2}}}$ $\qquad \varepsilon_i = -\frac{f(x_0)}{f'(x_0)}$

$f'(x) = 2\cos(2x) - [-\sin(x)\sin(3x) + \cos(x)3\cos(3x)]$

↓

$2\cos(2x) + \sin(x)\sin 3x - 3\cos(x)\cos(3x)$

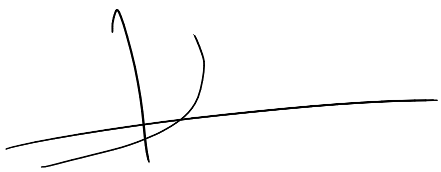
$f(x) = \sin(2x) - \cos(x)\sin(3x)$

$f'(x) = 2\cos(2x) - [-\sin(x)\sin(3x) + 3\cos(x)\cos(3x)]$

$= 2\cos(2x) + \sin(x)\sin(3x) - 3\cos(x)\cos(3x)$

$-\cos(4x) - 2\cos(x)\cos(3x)$

$\sin\alpha\sin\beta - \cos\alpha\cos\beta = -\cos(\alpha+\beta)$



$e^x - 1 = 0 \Rightarrow e^x = 1$

$\ln(e^x) = x = \ln(1) = 0$