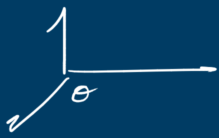
Esquema de Newton

↳ 1ra Ley ⟶ "Marco de referencia inercial"



↳ 2da Ley ⟶ $\vec{F}_{net} = \frac{d\vec{P}}{dt}$, $\vec{p} = m\vec{v}$

intrínsecas

Externas

derivada

↳ 3ra Ley

↳ $\vec{F}_{ij} = -\vec{F}_{ji}$ ⟶ cuerpos distintos

$dW = -\vec{F}\cdot d\vec{\ell}$

\+ 2da Ley de Newton

$\Delta\left(\frac{1}{2} m \dot{r}\right) = -\Delta W$

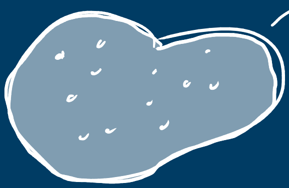
$= -\Delta U$

$-\nabla U = \vec{F}_{aplicada}$

$E_T = E_c + U \equiv cte$

$\frac{dE_T}{dt} = 0$

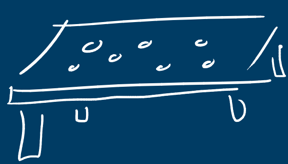
1 partícula ⟶ Sistemas de partículas (N)



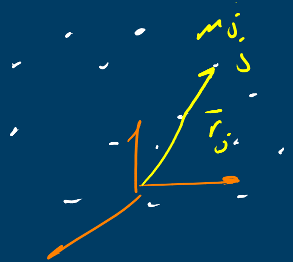
→ Planetas

→ átomos

→ Bolas de billar



N partículas



$j = \{1, \ldots, N\}$

$\frac{d^2}{dt}(m_j \dot{\vec{r}}_j) = m_j \ddot{\vec{r}}_j = \dot{\vec{P}}_j = \vec{F}_j = \vec{F}_j^{int} + \vec{F}_j^{ext}$

$\vec{F}_j^{int} = \vec{f}_{j1} + \vec{f}_{j2} + \ldots + \vec{f}_{jN}$

$= \sum_{\substack{j=1 \\ j\neq i}}^{N} \vec{f}_{ji}$

Si sumamos para todas las j's

$\underbrace{\cancel{\vec{f}_{a,b} + \vec{f}_{b,a}}}_{} \rightarrow 0$ 3ra Ley

$\sum_{j=1}^{N} \dot{\vec{P}}_j = \sum_{j=1}^{N} \vec{F}_j = \sum_{j=1}^{N} \vec{F}_j^{ext} + \cancel{\sum_{\substack{i,j=1 \\ i\neq j}}^{N} \vec{f}_{ij}} \rightarrow 0$

$$\sum_{i=1}^{N} \dot{\vec{p}}_i = \underbrace{\sum_{i=1}^{N} \vec{F}_i^{ext}} = \underline{\vec{F}^{ext}} = \sum_{i=1}^{N} \frac{d}{dt}\vec{p}_j = \frac{d}{dt}\left(\sum_{i=1}^{N}\vec{p}_i\right) = \underline{\frac{d}{dt}\vec{P}}$$

$$\vec{F}^{ext} = \frac{d}{dt}\vec{P} \longrightarrow N \text{ partículas} \longrightarrow \vec{F}^{ext} = M\ddot{\vec{R}}$$

centro de masa

masa total

$$\vec{F}_i^{ext} = \frac{d}{dt}\vec{p}_i = m_i\dot{\vec{p}}_i$$

Promedio ponderado

$$\frac{d}{dt}\left(\sum m_i\dot{\vec{r}}_i\right) = \underline{M\ddot{\vec{R}} = \sum m_i\ddot{\vec{r}}_i} \longrightarrow \boxed{\vec{R} = \frac{\sum m_i\vec{r}_i}{\sum m_i}}$$

Sistemas continuos

$$\sum_i \Delta m_i\vec{r}_i = \vec{R}\sum \Delta m_i \xrightarrow[\Delta m \to 0]{} \int_V \vec{r}\,dm = \vec{R}M$$

$$\Rightarrow \vec{R} = \frac{1}{M}\int_V \vec{r}\,dm = \frac{1}{M}\int_V \vec{r}\rho\,dV$$

¿Por qué el centro de masa?

$$\frac{d\vec{P}}{dt} = \vec{F}^{ext} \longrightarrow \int \frac{d\vec{P}}{dt}\,dt = \Delta\vec{P} = \underbrace{\int_{t}^{t+\Delta t} \vec{F}^{ext}\,dt}_{\text{Impulso}} \longrightarrow \Delta t \ll 1$$

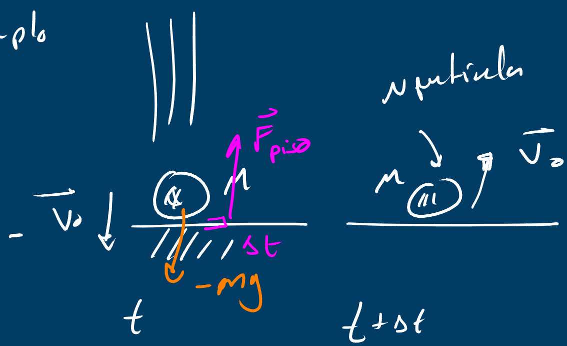
Cantidad integral

$$\Delta\vec{P} = \text{fijo} = \int_0^{\overset{\text{Muy chico}}{\Delta t}} \underbrace{\vec{F}^{ext}}_{\text{Muy grande}}\,dt = \int_0^{\overset{\text{Muy grandes}}{\Delta t}} \underbrace{\vec{F}^{ext}}_{\text{Muy pequeñas}}\,dt$$

Complicado de conocer

¿Qué nos dice el impulso de un sistema?

Ejemplo



$\vec{P} = -M\vec{V}_0$ $\qquad$ $\vec{P}(t+\Delta t) = M\vec{V}_0$

$$\Delta\vec{P} = 2M\vec{V}_0 = \int_0^{\Delta t} \vec{F}^{ext}\,dt$$

si $\Delta t \approx 10^{-3}\,s$ $\quad \vec{F}^{ext} \approx \underbrace{\langle \vec{F}^{ext} \rangle}_{cte}$

$$2M\vec{V}_0 = \Delta t \langle \vec{F}^{ext} \rangle$$

$$\Delta\vec{P} = 2M\vec{V}_0 = \hat{e}_z \int_0^{\Delta t} (-mg)\,dt + \langle \vec{F}_{piso} \rangle \int_0^{\Delta t} dt$$

$$\Rightarrow 2M\vec{V}_0 = \hat{e}_z(-mg)\overbrace{\Delta t}^{\ll 1} + \langle \vec{F}_{piso} \rangle \Delta t$$

$$\approx \langle \vec{F}_{piso} \rangle \Delta t \longrightarrow$$ si $\Delta t \ll 1$

Fuerzas de contacto

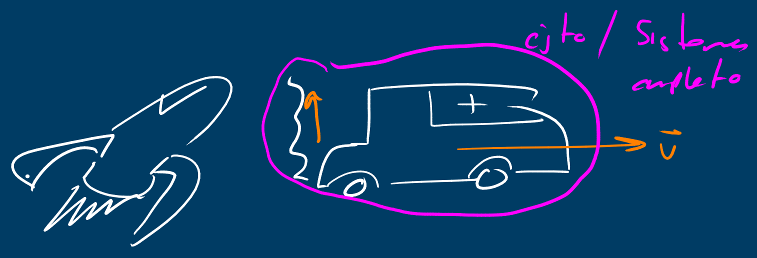
↓

Colisiones

Sistemas de masa variable

Colección N partículas vs. N partículas individuales

↳ Masa variable



M = vehículo

Δm = combustible

$$-\frac{dM}{dt} = \frac{dm}{dt}$$

$$\vec{P}(t) = (M + \Delta m)\vec{v} \qquad \vec{P}(t+\Delta t) = M(\vec{v} + \Delta\vec{v}) + \Delta m(\vec{v} + \Delta\vec{v} + \vec{u})$$

$$\frac{\Delta\vec{P}}{\Delta t} = \left(\vec{P}(t+\Delta t) - \vec{P}(t)\right)/\Delta t$$

$$= \left(M\Delta\vec{v} + \Delta m(\Delta\vec{v} + \vec{u})\right)/\Delta t$$

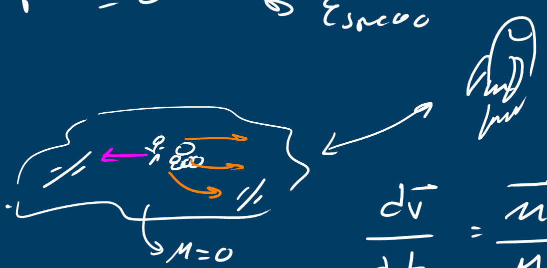
$$\lim_{\Delta t \to 0} \frac{\Delta\vec{P}}{\Delta t} = \frac{d\vec{P}}{dt} = \vec{F}^{ext} = \underbrace{M}_{M}\frac{d\vec{v}}{dt} + \vec{u}\frac{dm}{dt}$$

masa expulsada

$$\frac{dM}{dt} = -\frac{dm}{dt}$$

$$\vec{F}^{ext} = M\frac{d\vec{v}}{dt} - \vec{u}\frac{dM}{dt}$$

$\vec{F}^{ext} = \vec{0}$ → Espacio



$$\frac{d\vec{v}}{dt} = \frac{\vec{u}}{M}\frac{dM}{dt} \rightarrow \int \frac{d\vec{v}}{dt}dt = \Delta\vec{v} = \int \frac{\vec{u}\,dM}{M} = \vec{u}\int\frac{dM}{M}$$

sup cte

$\ln M$

$\vec{v}(t_0) = \vec{0}$

$M(t_0) = M_0$

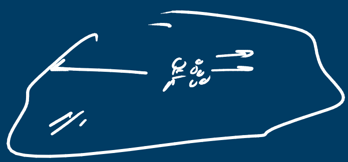
$$\vec{v}(t) = \vec{u}\ln\left(\frac{M(t)}{M_0}\right)$$

$$\vec{F}^{ext} = \vec{0} = \dot{\vec{P}}$$

$$\vec{v}(t) = \vec{u} \ln\left(\frac{M(t)}{M_0}\right)$$

Ecuación del cohete de Tsiolkovski

No hay una dependencia explícita en el tiempo



$$\vec{F}^{ext} = -Mg\hat{e}_z = \dot{\vec{P}}$$

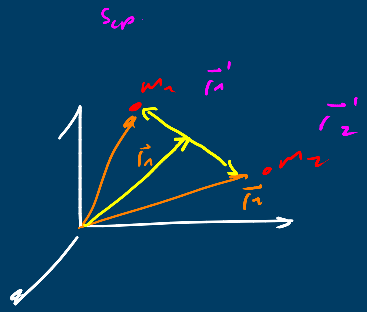
$$\int \frac{d\vec{v}}{dt}\,dt = \Delta\vec{v} = \vec{u}\ln\left(\frac{M(t)}{M}\right) - g\Delta t$$

Despegan los cohetes

→ Más combustible se quema al principio

Sistemas $N=2$ partículas

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$



$$\vec{r}_i = \vec{R} + \vec{r}_i' \Rightarrow \underline{\vec{r}_i' = \vec{r}_i - \vec{R}}$$ | transformación de coordenadas

$$\underline{\underline{\ddot{\vec{R}} = 0}} = \frac{\vec{F}^{ext}}{M}$$

$$\underline{\vec{r}_1'} = \vec{r}_1 - \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} = \frac{(\cancel{m_1}+m_2)\vec{r}_1 - \cancel{m_1\vec{r}_1} - m_2\vec{r}_2}{m_1+m_2} = \frac{m_2\overline{(\vec{r}_1 - \vec{r}_2)}}{m_1+m_2}$$

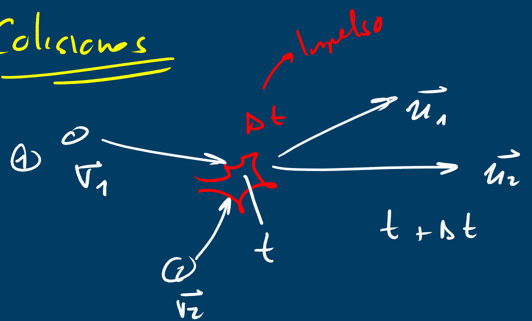
$$\underline{\vec{r}_2'} = -\frac{m_1\overline{(\vec{r}_1 - \vec{r}_2)}}{m_1+m_2}$$

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2} \longrightarrow \underbrace{(m_1+m_2)}_{M_{total}}\dot{\vec{R}} = m_1\dot{\vec{r}}_1 + m_2\dot{\vec{r}}_2 = \vec{p}_1 + \vec{p}_2$$ ← S.R. inercial

$$m_1\dot{\vec{r}}_1' + m_2\dot{\vec{r}}_2' = m_1\left(\frac{m_2}{m_1+m_2}\right)(\dot{\vec{r}}_1 - \dot{\vec{r}}_2) + m_2\left(\frac{-m_1}{m_1+m_2}\right)(\dot{\vec{r}}_1 - \dot{\vec{r}}_2) = 0$$

$$\underline{\vec{P}_{cm} = 0 \Rightarrow \dot{\vec{P}}_{cm} = 0}$$ → cantidad conservada

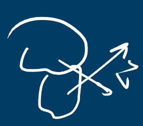
Colisiones



$t' < t \quad \vec{v}_1, \vec{v}_2$

$t' > t+\Delta t \quad \vec{u}_1, \vec{u}_2$

¿Δt? →



↳ Deformación
Calentamiento } Hay pérdidas de energía →

$$\Delta\left(\tfrac{1}{2}mv^2\right) = Q$$

↳ Inelásticas

↓

Colisiones elásticas

$$\Delta\left(\tfrac{1}{2}mv^2\right) = 0$$

$t \qquad t+\Delta t$

↳ $$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 u_1^2 + \frac{1}{2}m_2 u_2^2$$

Colisión elástica, observa desde el CM.

$$\Delta E_c = 0 \longrightarrow \frac{1}{2} m_1 (\overset{cm}{v_1'})^2 + \frac{1}{2} m_2 (v_2')^2 = \frac{1}{2} m_1 (u_1')^2 + \frac{1}{2} m_2 (u_2')^2$$

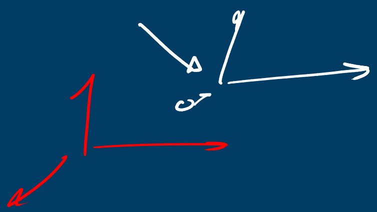
$$\vec{P}_{cm} = 0 \longrightarrow m_1 \vec{v}_1' + m_2 \vec{v}_2' = 0$$

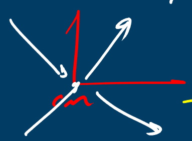
$$m_1 \vec{u}_1' + m_2 \vec{u}_2' = 0$$

Despejamos $\longrightarrow \frac{1}{2}\left(m_1 + \frac{m_2^2}{m_1}\right)(v'_{1/2})^2 = \frac{1}{2}\left(m_1 + \frac{m_2^2}{m_1}\right)(u'_{1/2})^2$

$$\Rightarrow \quad v_1' = \pm u_1' \quad \text{y} \quad v_2' = \pm u_2'$$

Visto desde fuera



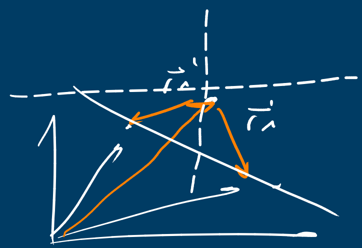


$\longrightarrow \vec{P}_{cm} = 0$

$\longrightarrow$ Velocidades simétricas!

Sistema de $N=2$ partículas

$$R_{cm} = \frac{m_1\vec r_1 + m_2\vec r_2}{m_1+m_2}$$



$$\boxed{\vec R_{cm} + \vec r_i{}' = \vec r_i}$$ sep.

$$\ddot R_{cm} = 0 = \frac{\vec F^{ext}}{M} = 0$$

$$\boxed{\vec r_i{}' = \vec r_i - \vec R_{cm}}$$ Transformación de coordenadas

$$\vec r_i{}' = \vec r_1 - \frac{m_1 r_1 + m_2 r_2}{m_1+m_2} = \frac{r_i m_1 + r_i m_2 - m_1 r_1 + m_2 r_2}{m_1+m_2}$$

$$\Rightarrow \vec r_1{}' = \frac{m_1(\vec r_i - \vec r_1) + m_2(\vec r_i - r_2)}{m_1+m_2}$$

$\vec r_i = \vec r_1$

$$\dot{\vec R} = \frac{m_1\dot{\vec r}_1 + m_2\dot{\vec r}_2}{m_1+m_2}, \quad \dot{\vec r}_1{}' = \frac{m_2}{m_1+m_2}(\dot{\vec r}_1 - \dot{\vec r}_2), \quad \dot{\vec r}_2{}' = -\frac{m_1}{m_1+m_2}(\dot{\vec r}_1 - \dot{\vec r}_2)$$

Vemos ahora que $(m_1+m_2)\dot{\vec R} = \vec P = m_1\dot{\vec r}_1 + m_2\dot{\vec r}_2 \longrightarrow$ lab.

$$m_1\vec r_1{}' + m_2\vec r_2{}' = 0 \longrightarrow CM \longrightarrow \vec P = 0$$

$$\boxed{\frac{d}{dt}\vec P = 0}$$

sin ext.

¿Colisiones

$\Delta \vec P|_t$ $\Delta t/\varepsilon 0$ $\Delta P(t+\Delta t)$

instantánea

Definir / colisión

$\Delta E_c = Q$ | colisión inelástica

$\Delta E_c = 0$ | colisión elástica

Vistas desd CM.

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$m_1 v_1 + m_2 v_2 = 0 \longrightarrow v_2$$

$$m_1 u_1 + m_2 u_2 = 0 \longrightarrow u_2$$

$$\frac{1}{2}\left(m_1 + \frac{m_1^2}{m_2}\right) v_1^2 = \frac{1}{2}\left(m_1 + \frac{m_1^2}{m_2}\right) u_1'^2$$

$$v_1 = \pm u_1$$

$$v_2 = \pm u_2$$