HW Problem 11 Lagrange Barycentric Approximation

Friday, October 9, 2020 1:13 PM

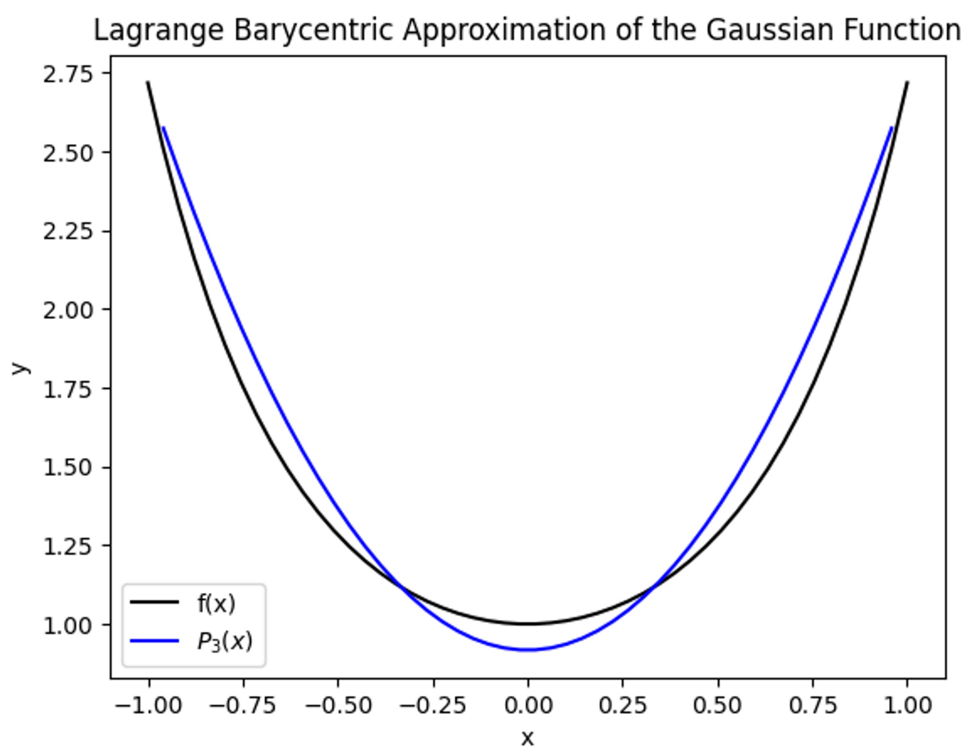
The gaussian function is defined by

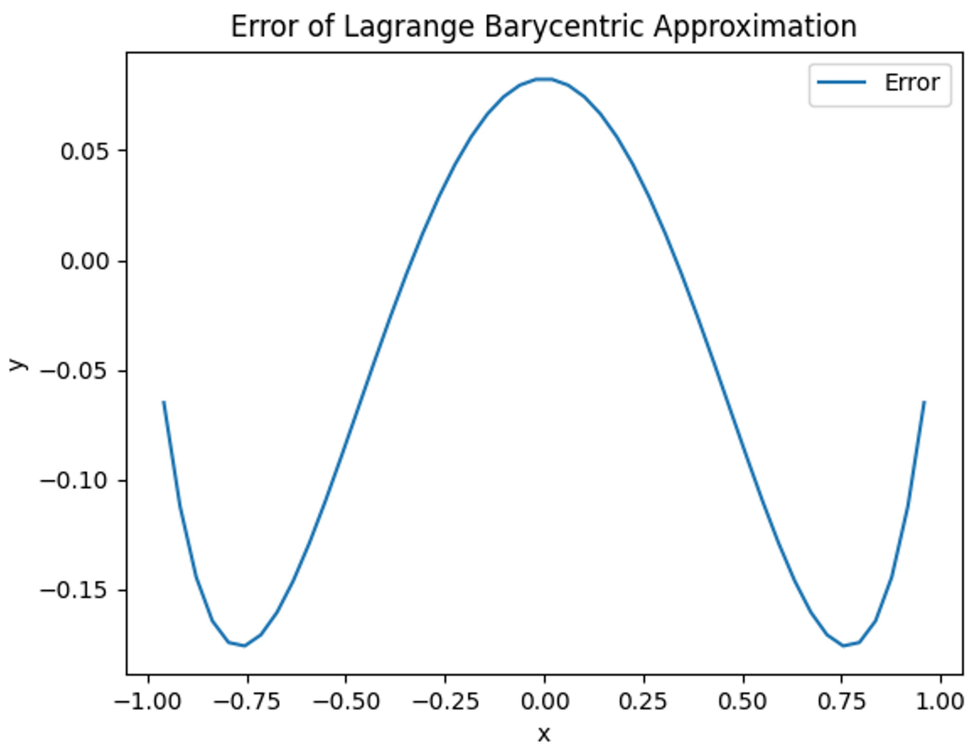
$$f(x) = e^{-x^2}.$$

(A) Write the barycentric form of the interpolating polynomial $p_3(x)$ for the gaussian function on the interval $[-1, 1]$ using four evenly spaced fitting points.

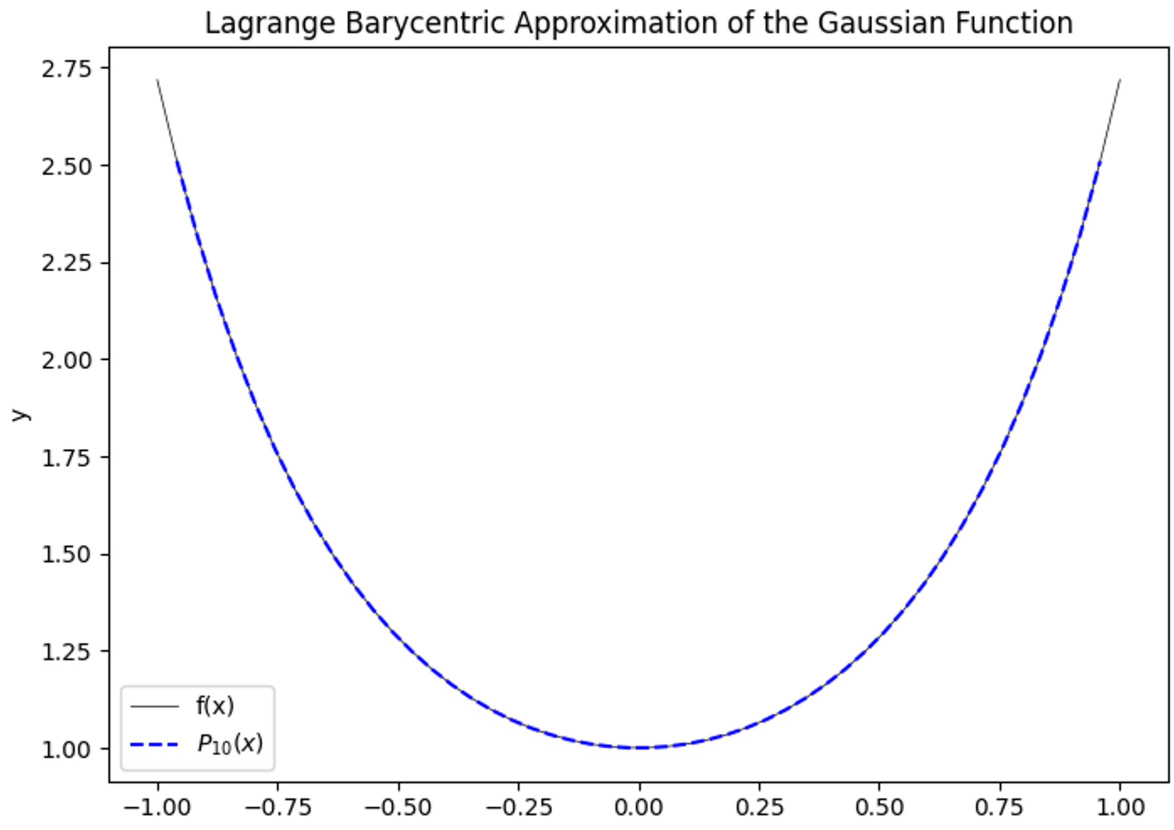
$$p_3(x) = \frac{\frac{2.718}{x+1} + \frac{-3.353}{x+0.\overline{3}} + \frac{3.353}{x-0.\overline{3}} + \frac{-2.718}{x-1}}{\frac{1}{x+1} + \frac{-3}{x+0.\overline{3}} + \frac{3}{x-0.\overline{3}} + \frac{-1}{x-1}}$$

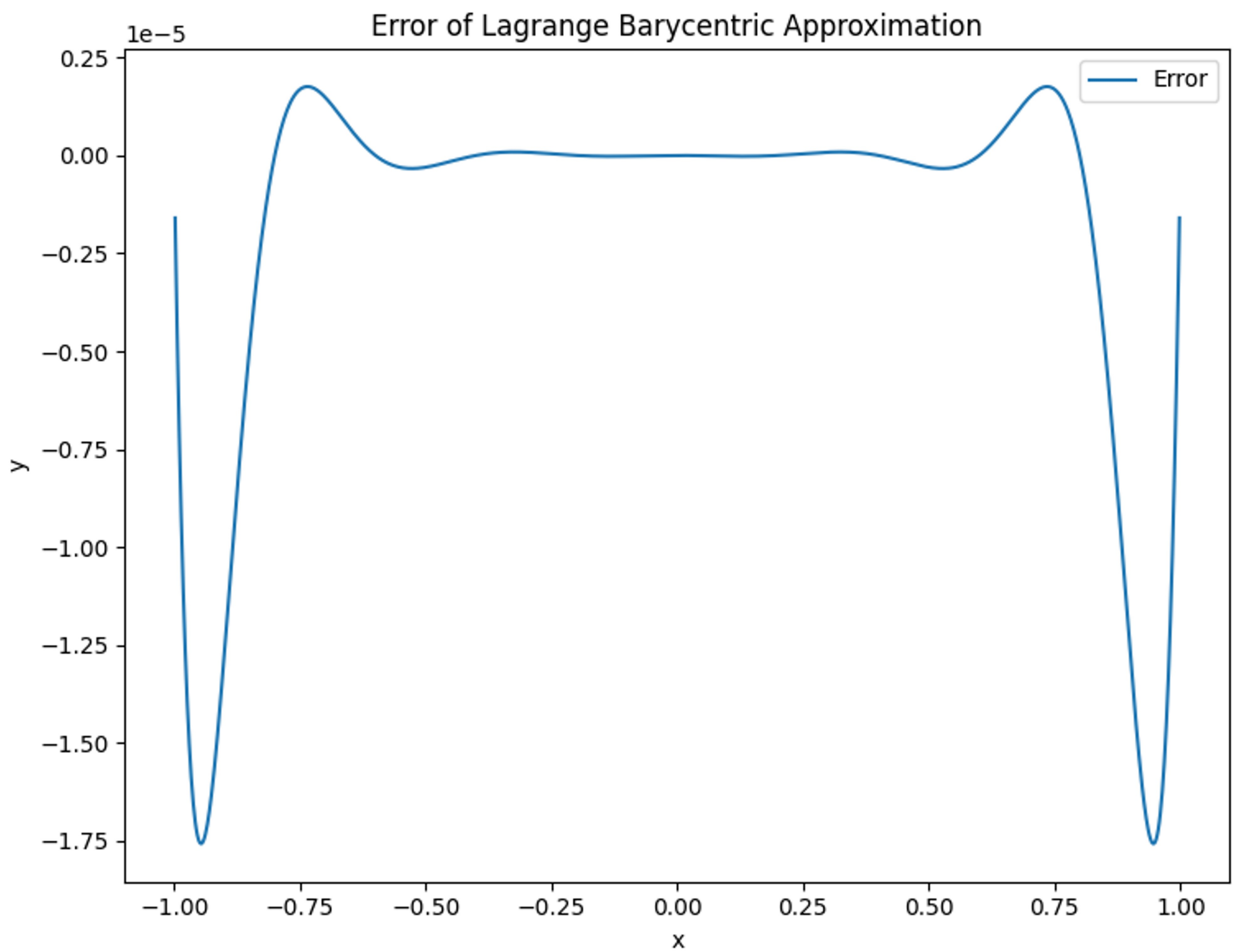
(B) Use MATLAB to plot the gaussian function and the $p_3(x)$ you obtained in (A) on the same axes. Also plot the error on a separate graph.





(C) Use MATLAB to plot a tenth-order interpolating polynomial $p_{10}(x)$ using the barycentric form of the polynomial and eleven evenly spaced fitting points on the same axis as the gaussian. Also plot the error on a separate graph.


Lagrange Barycentric Approximation of the Gaussian Function
y
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
f(x)
$P_{10}(x)$
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00


Error of Lagrange Barycentric Approximation
1e−5
Error
y
0.25
0.00
−0.25
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

(D) Compare and contrast the two interpolating polynomials.

Clearly the tenth-order interpolating polynomial has much less error than the third-order polynomial. The error for the first polynomial is on the magnitude of E^{-1} while the tenth-order error is on a scale of E^{-5}. Plotting both the gaussian function and tenth-order barycentric approximation together makes them virtually indistinguishable.