HW Problem 14 Barycentric Chebyshev Approximation

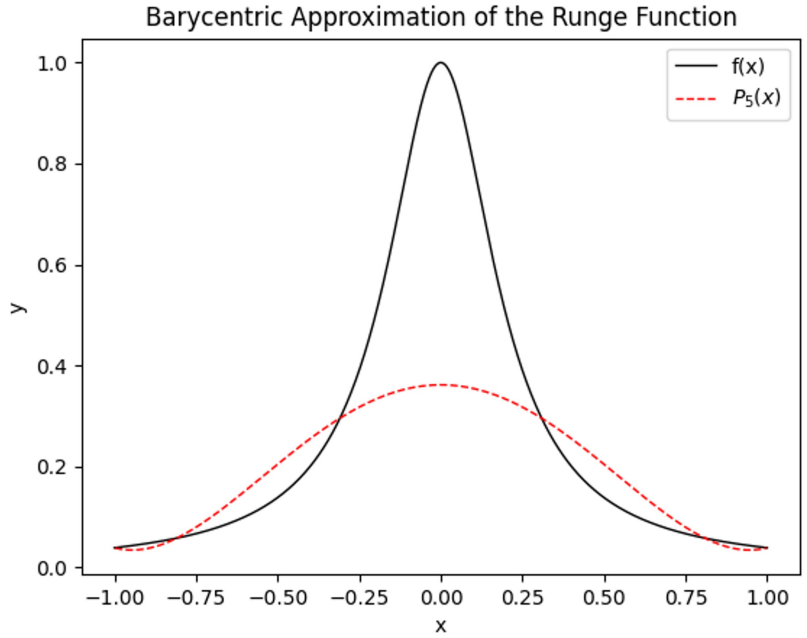
Sunday, October 11, 2020 3:14 PM

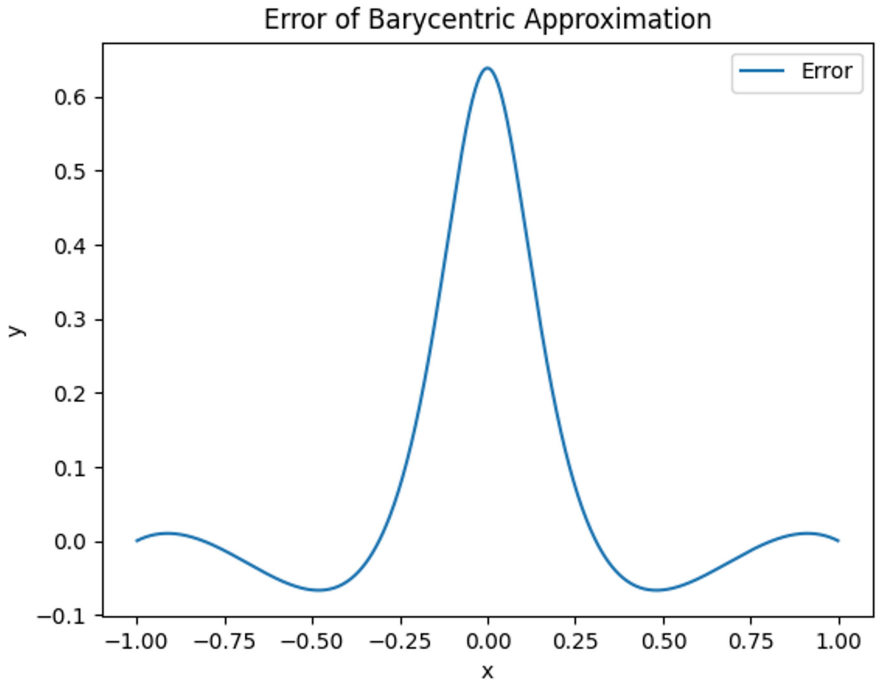
Evenly spaced fitting points are often not well-suited for polynomial interpolation. A famous example is given by Runge to interpolate

$$f(x) = \frac{1}{1 + 25x^2}$$

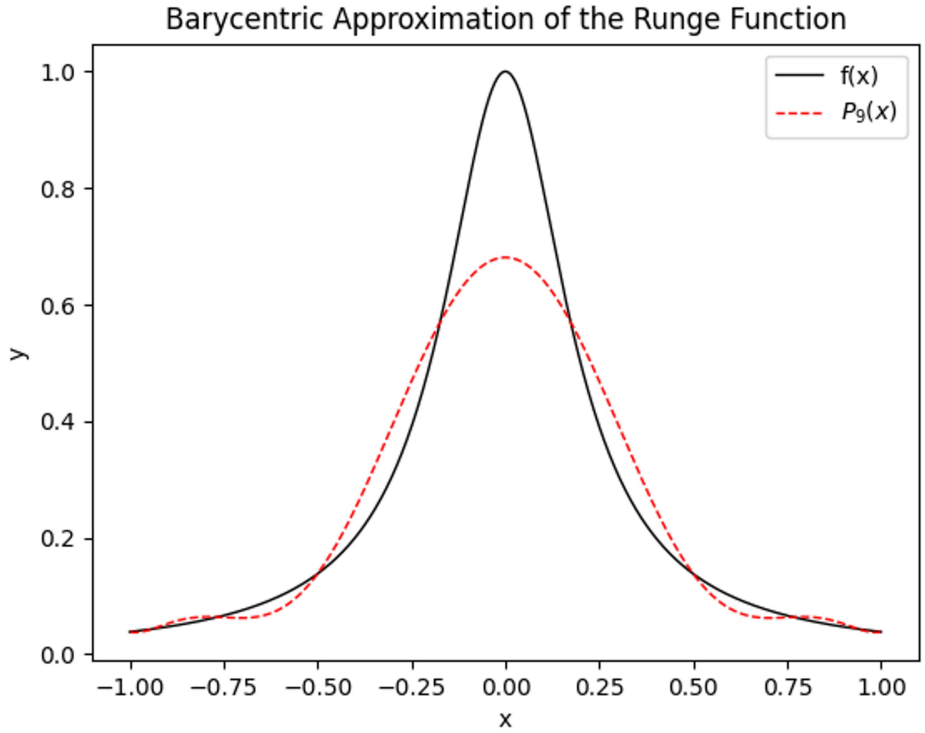
on the interval $[-1, 1]$.

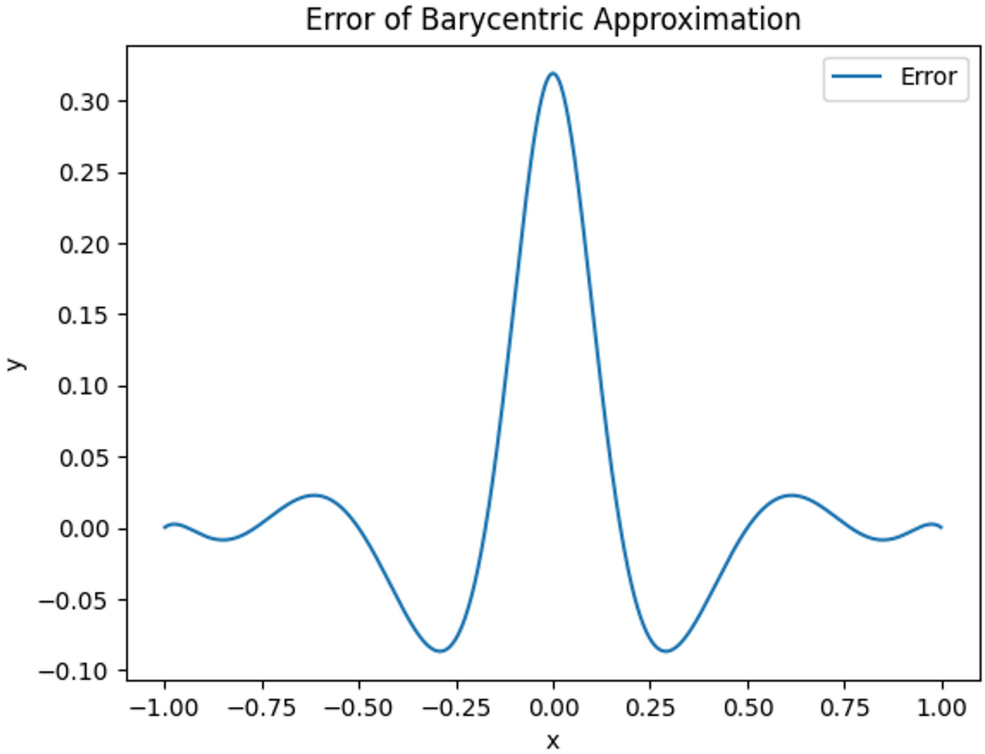
(A) Find a fifth-order approximation to $f(x)$ using six extremal Cheyshev points. Be sure to plot $f(x)$ and $p_5(x)$ on the same axes, and also plot the error $f(x) - p_5(x)$.



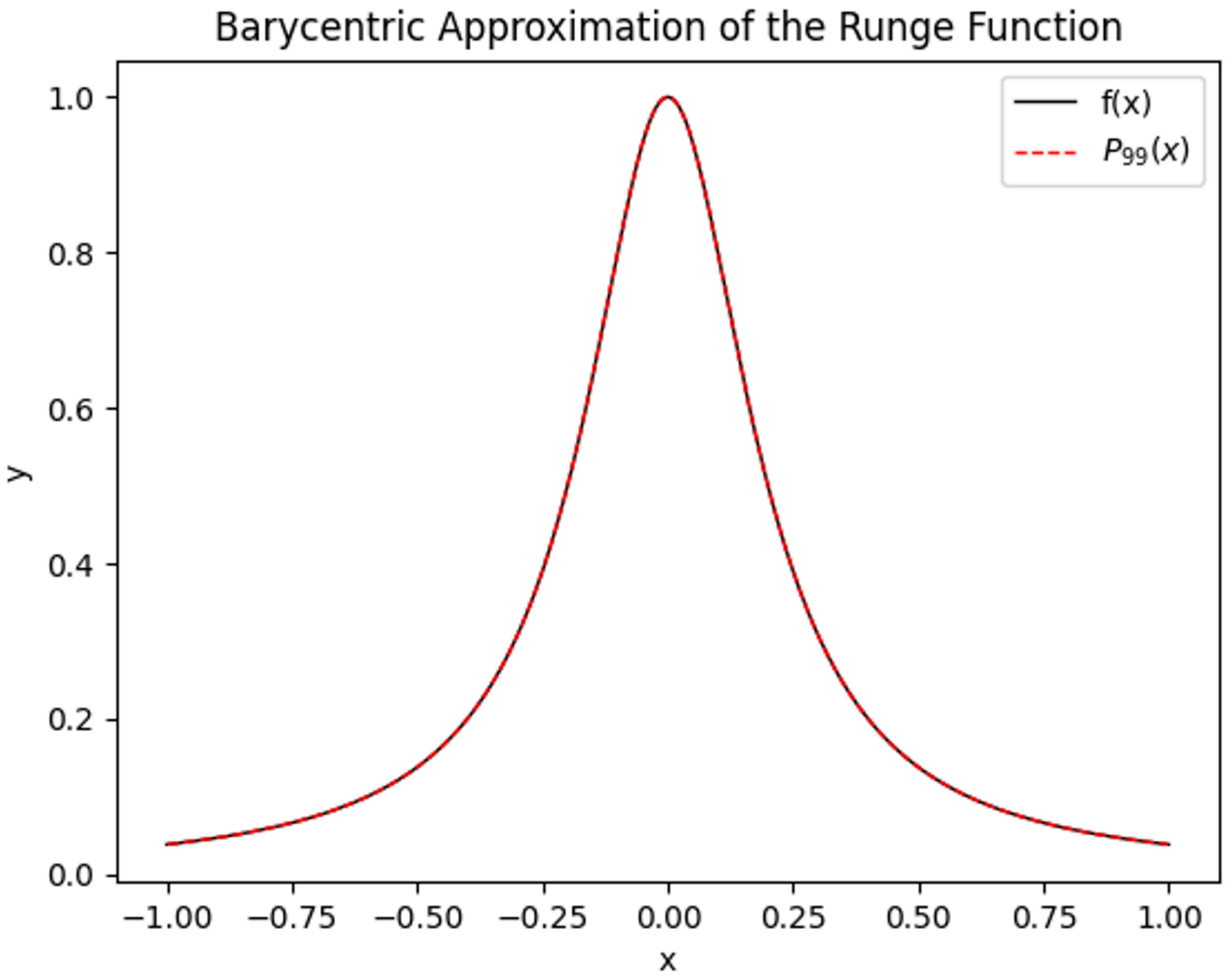


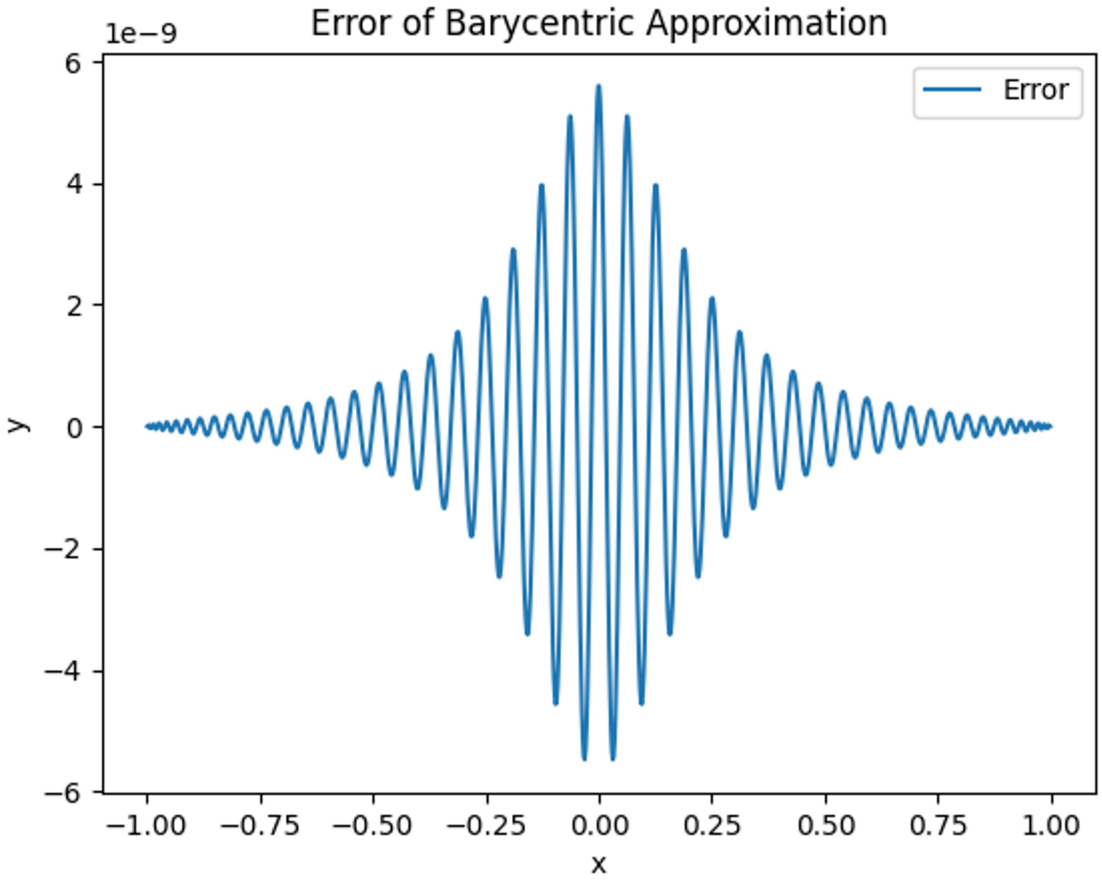
(B) Find a ninth-order approximation to $f(x)$ using ten extremal Cheyshev fitting points. Be sure to plot $f(x)$ and $p_9(x)$ on the same axes, and also plot the error $f(x) - p_9(x)$.





(C) Find a 99^{th}-order approximation to $f(x)$ using one hundred extremal Cheyshev points. Be sure to plot $f(x)$ and $p_{99}(x)$ on the same axes, and also plot the error $f(x) - p_{99}(x)$.





(D) Compare the results for these three approximations with the three approximations from Problem 13.

For the Runge function, lower order Chebyshev approximations seem to have more error compared to the Lagrange approximation of HW12. However, higher order Chebyshev approximations have much less error than the higher order Lagrange approximations of HW12.