HW Problem 12 Lagrange Barycentric Approximation

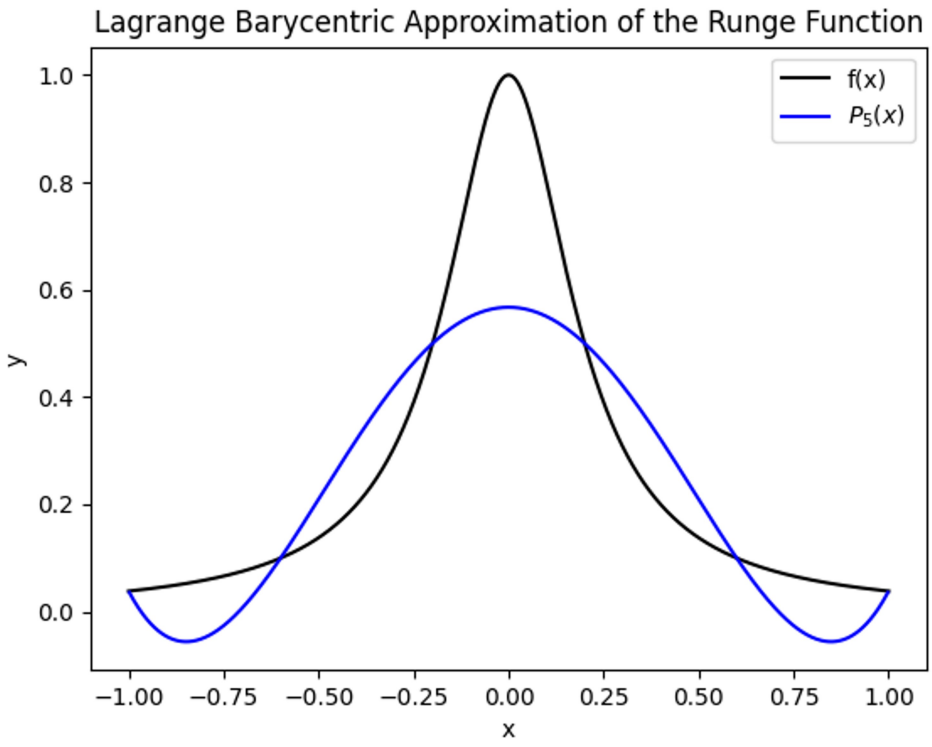
Friday, October 9, 2020 5:15 PM

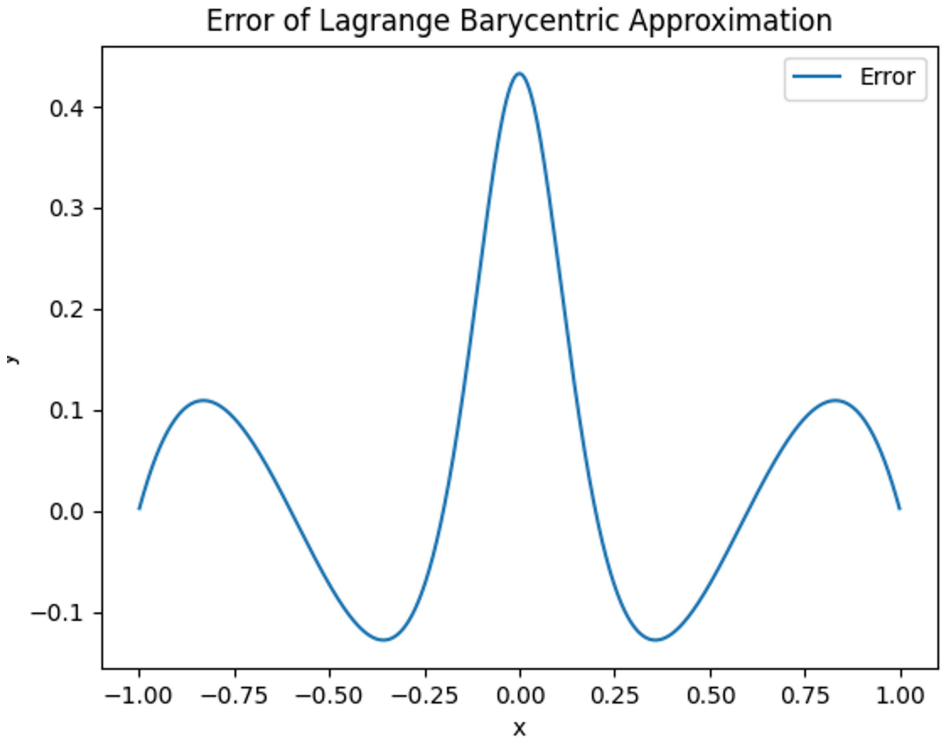
Evenly spaced fitting points are often not well-suited for polynomial interpolation. A famous example is given by Runge to interpolate

$$f(x) = \frac{1}{1 + 25x^2}$$

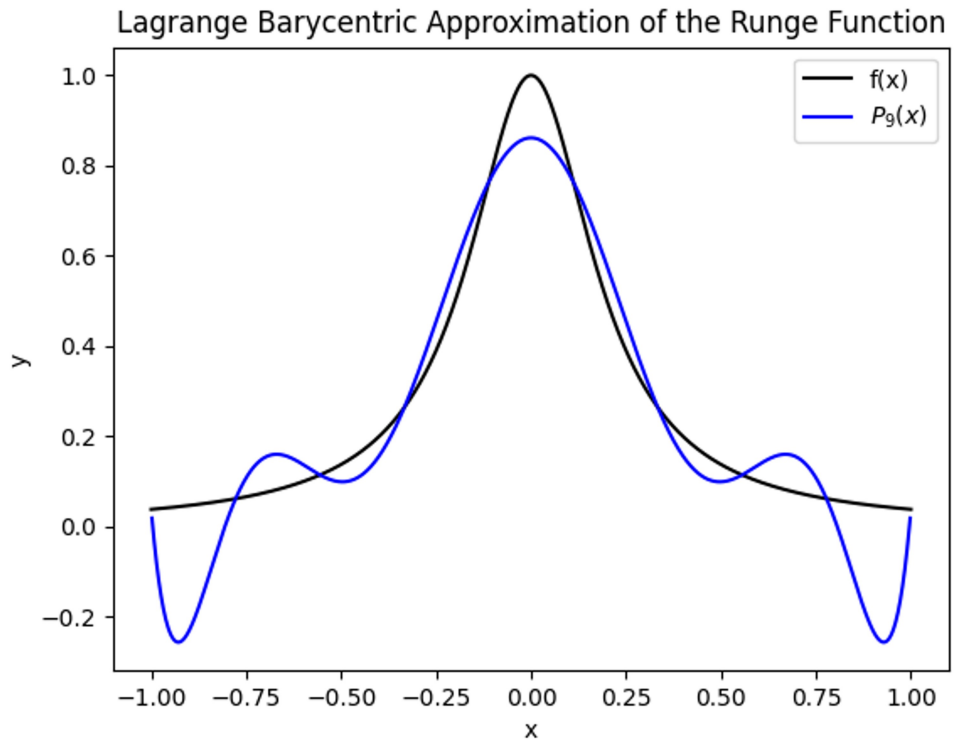
on the interval $[-1, 1]$.

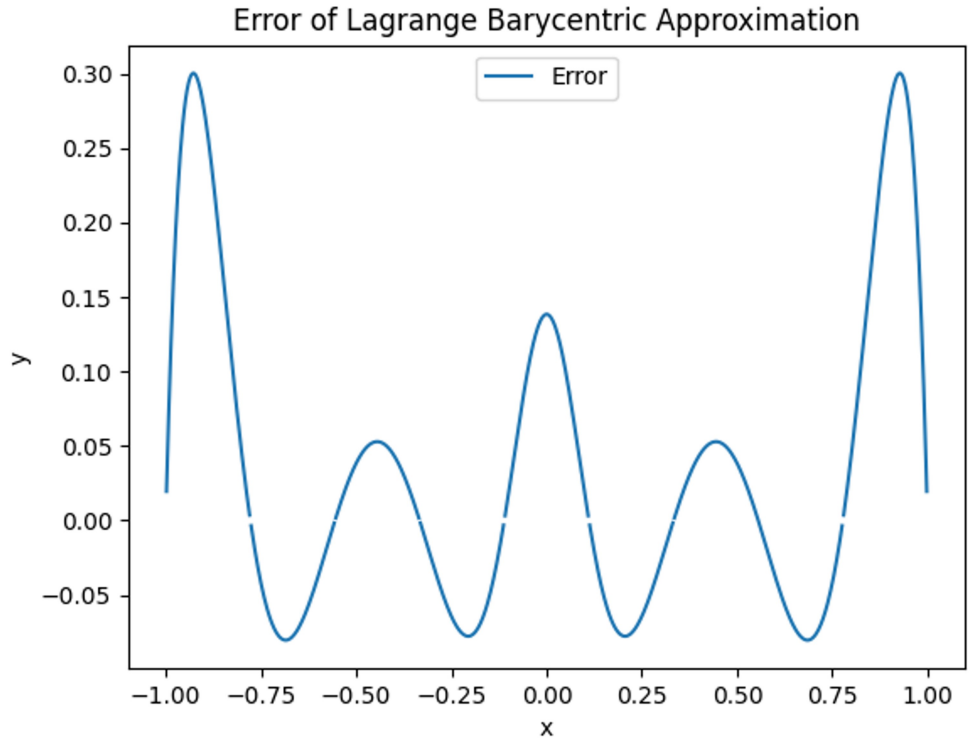
(A) Find a fifth-order approximation to $f(x)$ using six evenly spaced fitting points. Be sure to plot $f(x)$ and $p_5(x)$ on the same axes, and also plot the error $f(x) - p_5(x)$.



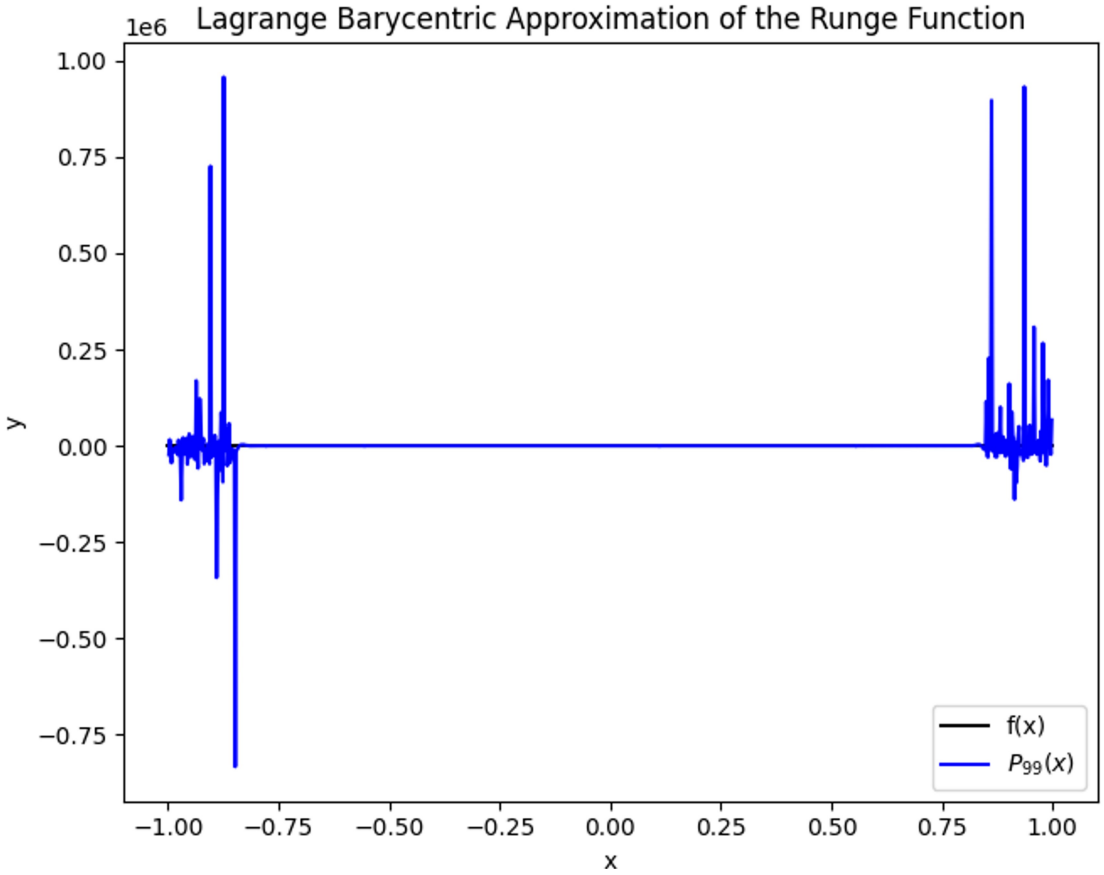


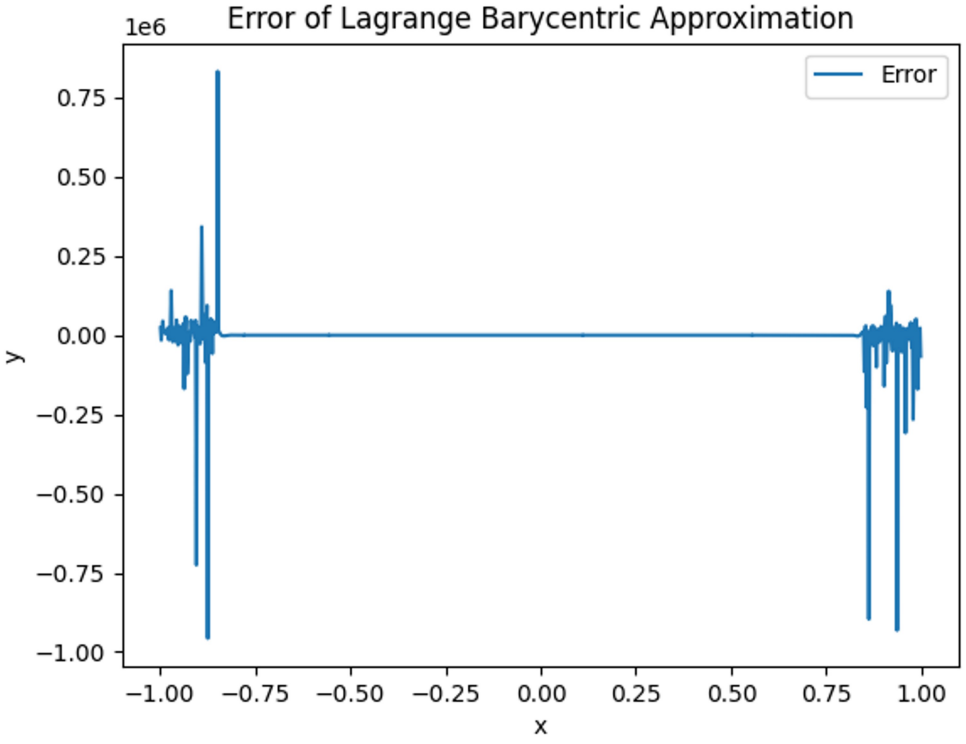
(B) Find a ninth-order approximation to $f(x)$ using ten evenly space fitting points. Be sure to plot $f(x)$ and $p_9(x)$ on the same axes, and also plot the error $f(x) - p_9(x)$.


Lagrange Barycentric Approximation of the Runge Function
f(x)
$P_9(x)$
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x


Error of Lagrange Barycentric Approximation
Error
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

(C) Find a 99^{th}-order approximation to $f(x)$ using one hundred evenly space fitting points. Be sure to plot $f(x)$ and $p_{99}(x)$ on the same axes, and also plot the error $f(x) - p_{99}(x)$.





(D) Compare the results for these three approximations.

In this particular function, higher order approximations don't necessarily always decrease the error of the approximation. For the Runge function, the error particularly towards the end points gets worse with higher-order polynomials.