HW Problem 13 Chebyshev Approximation

Sunday, October 11, 2020 2:54 PM

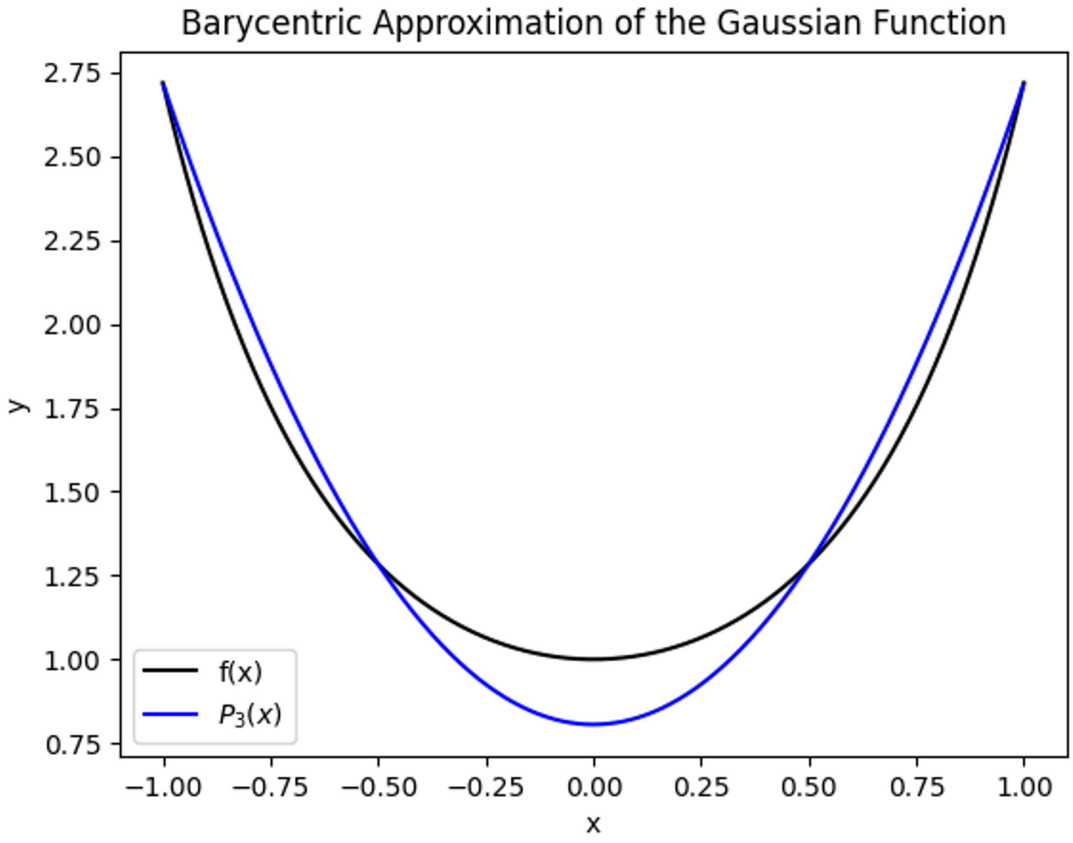
The gaussian function is defined by

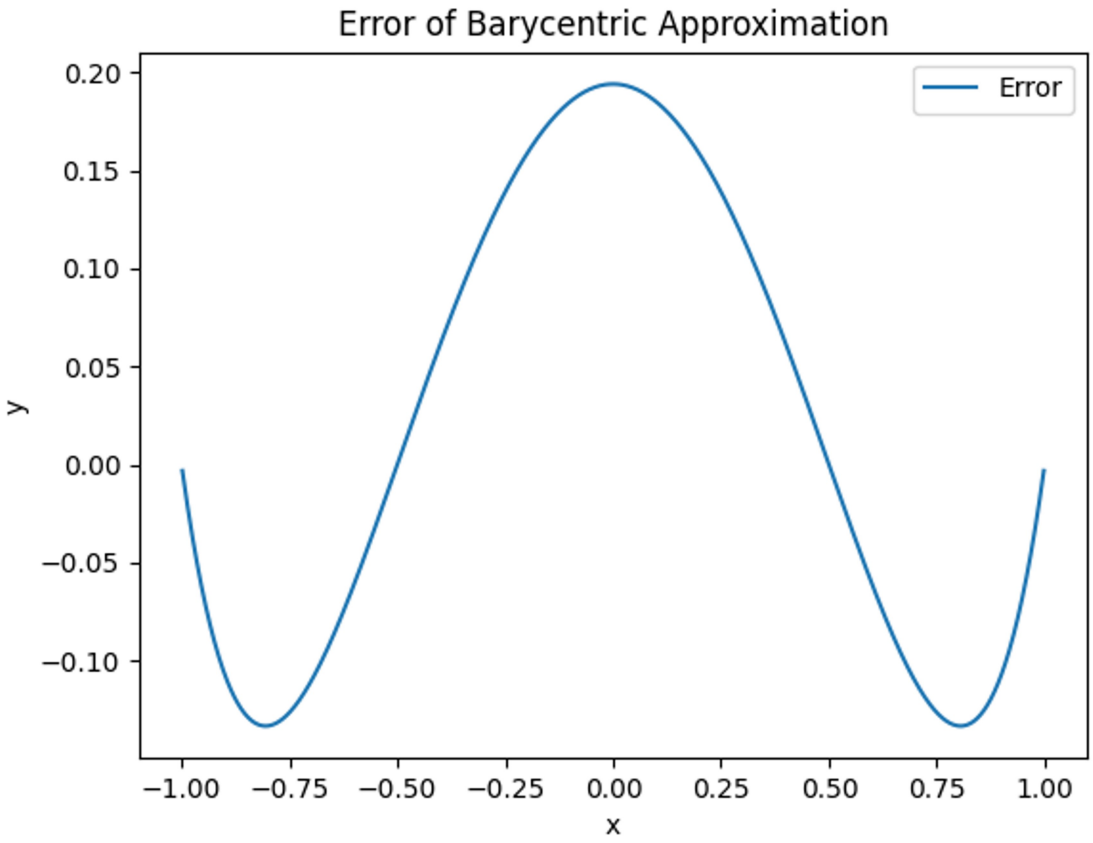
$$f(x) = e^{-x^2}.$$

(A) Write the barycentric form of the interpolating polynomial $p_3(x)$ for the gaussian function on the interval $[-1, 1]$ using the extremal Chebyshev points.

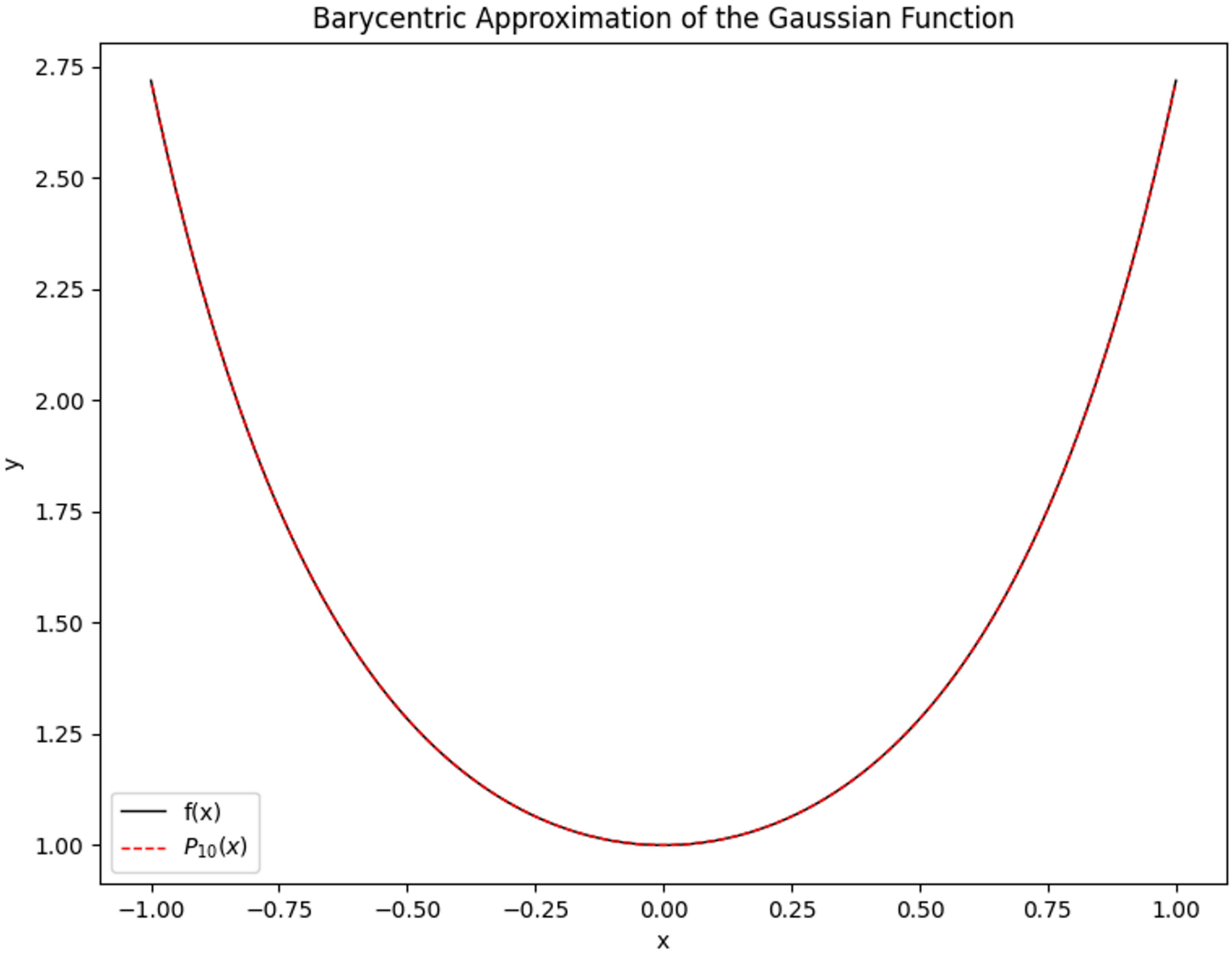
$$P_3(x) = \frac{\frac{1.359}{x-1}}{\frac{0.5}{x-1}} + \frac{\frac{-1.284}{x-0.5}}{\frac{-1}{x-0.5}} + \frac{\frac{1.284}{x+0.5}}{\frac{1}{x+0.5}} + \frac{\frac{-1.359}{x+1}}{\frac{-0.5}{x+1}}$$

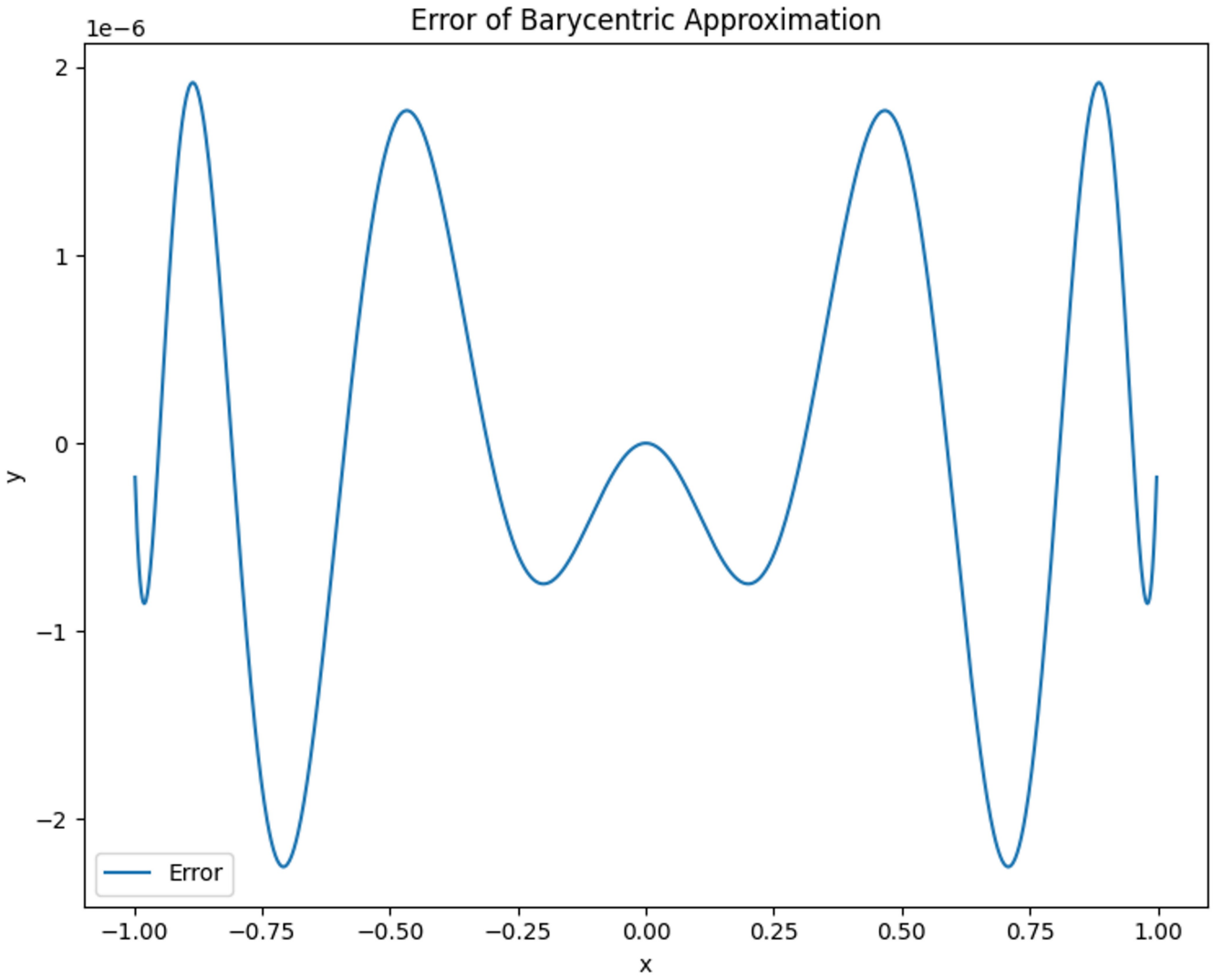
(B) Use MATLAB to plot the gaussian function and the $p_3(x)$ you obtained in (A) on the same axes. Also plot the error on a separate graph.




Error of Barycentric Approximation
Error
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
y
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
x

(C) Use MATLAB to plot a tenth-order interpolating polynomial $p_{10}(x)$ using the barycentric form of the polynomial and the extremal Chebyshev points on the same axis as the gaussian. Also plot the error on a separate graph.





(D) Compare and contrast the two interpolating polynomials.

Compared to the Lagrange barycentric approximation of the Gaussian function in HW11, the third order Chebyshev approximation has slightly more error. However, the higher order P_{10} approximation has less error, especially on the endpoints of the function.