Lab 6

Lab Assignment (QR method)

Problem 1 *Find a QR decomposition for the matrix based on Gram-Schmidt method.*

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A{=}QR \Rightarrow \begin{bmatrix} | & | & | \\ a_1 & a_2 & a_3 \\ | & | & | \end{bmatrix} = \begin{bmatrix} | & | & | \\ q_1 & q_2 & q_3 \\ | & | & | \end{bmatrix} \begin{bmatrix} | & | & | \\ r_1 & r_2 & r_3 \\ | & | & | \end{bmatrix}.$$

Using Gram-Schmidt's method we'll get.

$$v_1 = a_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad ||v_1|| = \sqrt{1}$$

$$\boxed{q_1 = \frac{1}{||v_1||} v_1 = \frac{1}{\sqrt{1}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

$$a_1 = \sqrt{1} q_1 = q_1$$

$$\boxed{r_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}$$

$$v_2 = a_2 - (q_1 \cdot a_2) q_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad ||v_2|| = \sqrt{1}$$

Learning outcomes:
Author(s): Unknown

$$q_2 = \frac{1}{||v_2||} v_2 = \frac{1}{\sqrt{1}} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$a_2 - (1)q_1 = \sqrt{1} q_2 \Rightarrow a_2 = q_1 + q_2$$

$$r_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$v_3 = a_3 - (q_1 \cdot a_3)q_1 - (q_2 \cdot a_3)q_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} - \left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$||v_3|| = \sqrt{3^2} = 3$$

$$q_3 = \frac{1}{||v_3||} v_3 = \frac{1}{3} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$a_3 - 2q_1 - q_2 = 3q_3 \Rightarrow a_3 = 2q_1 + q_2 + 3q_3$$

$$r_3 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} = QR.$$

We can see that orthonormal condition is satisfied because

$$QQ^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I.$$