ML_HW5

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July 11, 2024

Question 1

A

We denote mean of class one with m_1 and mean of class two with m_2. First we need to find m_1 and m_2:

$$m_1 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}, m_2 = \begin{pmatrix} 8.4 \\ 7.6 \end{pmatrix}$$

Now we subtract means from their corresponding samples:

$$X_1 - m_1 = \{\begin{pmatrix} 1 \\ -2.6 \end{pmatrix}, \begin{pmatrix} -1 \\ 0.4 \end{pmatrix}, \begin{pmatrix} -1 \\ -0.6 \end{pmatrix}, \begin{pmatrix} 0 \\ 2.4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0.4 \end{pmatrix}\}$$

$$X_2 - m_2 = \{\begin{pmatrix} 0.6 \\ 2.4 \end{pmatrix}, \begin{pmatrix} -2.4 \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0.6 \\ -2.6 \end{pmatrix}, \begin{pmatrix} -0.4 \\ -0.6 \end{pmatrix}, \begin{pmatrix} 1.6 \\ 0.4 \end{pmatrix}\}$$

Now we can calculate S_1 and S_2:

$$S_1^2 = \begin{pmatrix} 1 & -2.6 \\ -2.6 & 6.76 \end{pmatrix} + \begin{pmatrix} 1 & -0.4 \\ -0.4 & 0.16 \end{pmatrix} + \begin{pmatrix} 1 & 0.6 \\ 0.6 & 0.36 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 5.76 \end{pmatrix} + \begin{pmatrix} 1 & 0.4 \\ 0.4 & 0.16 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -2 \\ -2 & 13.2 \end{pmatrix}$$

$$S_2^2 = \begin{pmatrix} 0.36 & 1.44 \\ 1.44 & 5.76 \end{pmatrix} + \begin{pmatrix} 5.76 & -0.96 \\ -0.96 & 0.16 \end{pmatrix} + \begin{pmatrix} 0.36 & -1.56 \\ -1.56 & 6.76 \end{pmatrix} + \begin{pmatrix} 0.16 & 0.24 \\ 0.24 & 0.36 \end{pmatrix} + \begin{pmatrix} 2.56 & 0.64 \\ 0.64 & 0.16 \end{pmatrix}$$

$$= \begin{pmatrix} 9.2 & -0.2 \\ -0.2 & 13.2 \end{pmatrix}$$

And now S_w can be calculated:

$$S_w = \begin{pmatrix} 4 & -2 \\ -2 & 13.2 \end{pmatrix} + \begin{pmatrix} 9.2 & -0.2 \\ -0.2 & 13.2 \end{pmatrix} = \begin{pmatrix} 13.2 & -2.2 \\ -2.2 & 26.4 \end{pmatrix}$$

B

$$m_1 - m_2 = \begin{pmatrix} 3 & 3.6 \end{pmatrix} - \begin{pmatrix} 8.4 & 7.6 \end{pmatrix} = \begin{pmatrix} -5.4 & -4 \end{pmatrix}$$

$$S_B = \begin{pmatrix} -5 & -4 \end{pmatrix} \begin{pmatrix} -5.4 \\ -4 \end{pmatrix} = \begin{pmatrix} 29.16 & 21.6 \\ 21.6 & 16 \end{pmatrix}$$

C

$$A = S_w^{-1} S_B$$

$$S_w^{-1} = \frac{1}{343.64} \begin{pmatrix} 26.4 & 2.2 \\ 2.2 & 13.2 \end{pmatrix} = \begin{pmatrix} 0.076 & 0.006 \\ 0.006 & 0.038 \end{pmatrix}$$

$$\Rightarrow A = \begin{pmatrix} 2.34 & 1.73 \\ 0.99 & 0.73 \end{pmatrix}$$

$$\Rightarrow A - \lambda I \begin{pmatrix} 2.34 - \lambda & 1.73 \\ 0.93 & 0.73 - \lambda \end{pmatrix}$$

$$|A - \lambda I| = (\lambda - 2.34) \times (\lambda - 0.73) = 0$$

$$\Rightarrow \lambda^2 - 3.07\lambda - 0.004 = 0$$

$$\lambda_1 = -0.001, \lambda_2 = 3.07$$

Thus the greatest eigen value is **3.07**.

Question 2

TODO