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Journal of Materials Processing Technology 129 (2002) 663–666

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

Fuzzy–grey prediction of cutting force uncertainty in turning

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Abstract

To predict the extent of turning force uncertainty quantitatively, this paper proposes a fuzzy–grey prediction procedure based on the symmetric fuzzy number, linear planning theory and grey set theory. To verify the developed procedure, a measuring system of turning force is schematised to acquire the evaluating data. The comparison between the prediction results and the measured data demonstrates that the prediction is an extent of variable force rather than a certain point for the given turning conditions, and that the measured force drops into the extent with a smaller relative error. In addition, the procedure needs less experimental data in modelling. This work is new and original, and helpful for engineering application.

Keywords: Uncertainty; Fuzzy–grey; Prediction; Turning force

1. Introduction

The existing uncertainties in machining processes have been becoming outstanding problems. For cutting force, the uncertainty phenomena are derived from many process factors, including some quantified, unquantified, known and unknown factors. If ignoring the unquantified and unknown factors, it may be considered that the uncertainty budget comes from two aspects [3]: (1) the contributions of the actual force measuring system (including the calibration procedure, the drift compensation procedure, the averaging and rounding procedure in the software, and the temperature of the dynamometer); (2) contributions of variations of process parameters (such as cutting speed, feed rate, and depth of cut).

The estimation methods of uncertainty for any system may include probability distribution, fuzzy evaluation, the least-squares regression, and so on. So far, a few papers concerning the uncertainty evaluation of machining process have been published [1,3]. For evaluating turning force uncertainty, taking into account the calibration error in a measuring system and cutting parameter errors, a model [3] is proposed from least-squares regression and error compensation, in which the uncertainty prediction result is a certain value rather than an extent. Considering the variations of grinding parameters and omitting the calibration errors of a measuring system, a fuzzy regression model [1] is presented from the symmetric fuzzy number and linear planning, in which the uncertainty estimation result of grinding force is an extent rather than a certain value.

It is more adequate that the expression of uncertainty prediction is an extent rather than a certain point. From engineering practice, it is significant how to predict the extent of cutting force uncertainty. The model [1] uses the extent form for evaluating grinding force uncertainty, but the relative error of uncertainty estimation needs to be improved. To investigate this problem, the authors in this paper have paid attention to grey theory, which is good at estimating uncertainty for any system, and plan to develop a fuzzy–grey prediction procedure for turning force uncertainty. In addition, a measuring system of turning force is schematised to verify the proposed procedure.

2. Fuzzy–grey prediction of turning force uncertainty

The empirical model of the main cutting force in a turning operation can be expressed as follows:

$$F_z = C_z(a_p)^x(f)^y \tag{1}$$

where C_z is the coefficient, a_p the depth of cut (mm), f the feed rate (mm/rev), and x and y are exponents. After taking logarithms, Eq. (1) is transformed as

$$\log(F_z) = \log(C_z) + x\log(a_p) + y\log(f) \tag{2}$$

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PII: S0924-0136(02)00677-5

Letting $Y = \log(F_z)$, $b_0 = \log(C_z)$, $b_1 = x$, $X_1 = \log(a_p)$, $b_2 = y$, $X_2 = \log(f)$, Eq. (2) becomes the crisp form of a linear model as follows:

$$Y = b_0 + b_1X_1 + b_2X_2 \tag{3}$$

Now, let the fuzzy form corresponding to Eq. (3) be defined as

$$Y(X_p) = A_0 + A_1X_{1p} + A_2X_{2p} \tag{4}$$

where the symmetric triangle form of the membership function in the fuzzy set is selected. The centre and width of the bottom side of the triangle are, respectively, expressed as C and W. Symmetric fuzzy number $Y(X_p) = [C(X_p), W(X_p)] = [\log F_z]_p$; $A_0 = [C_0, W_0]$; $A_1 = [C_1, W_1]$; and $A_2 = [C_2, W_2]$. The crisp variable $X_{1p} = \log(a_p)_p$, and $X_{2p} = \log(f)_p$. The index p indicates the code of experiments (i.e. $p = 1, 2, \ldots, n$).

To have the measured force drop into the estimated extent from Eq. (4), and to minimise the sum of width W_i of the symmetric fuzzy number, linear planning equations must be met as follows:

$$\text{Objective}: \quad \sum W(X_p) \to \min. \tag{5}$$

$$\text{Constraints}: \quad \log(F_z)_p \le C(X_p) + (1-h)W(X_p) \tag{6}$$

$$\log(F_z)_p \ge C(X_p) - (1-h)W(X_p) \tag{7}$$

where $W(X_p) = W_0 + W_1X_{1p} + W_2X_{2p}$; $C(X_p) = C_0 + C_1X_{1p} + C_2X_{2p}$; width $W_i \ge 0$ and centre $C_i \ge 0$; index $i = 0, 1, 2$. The fuzzy membership h meets $0 < h < 1$; and $\log(F_z)_p$ is the logarithm of the measured force.

Considering feed rate $0 < f < 1$, and to facilitate programming calculation, $X_{1p} = \log(10^m f)_p$ is usually used, where $m \ge 2$. In addition, Eqs. (4)–(7) can logically be extended to the modelling of other cutting state variables, such as cutting temperature.

After finding out the fuzzy number $Y(X_p) = [C(X_p), W(X_p)] = [\log F_z]_p$ in Eq. (4), taking its anti-logarithm as $F_z = [\exp(C), \exp(W)]$, then the estimated extent of turning force uncertainty, will be obtained.

From a defined range of turning parameters, a group of measurement experiments is schematised, and the experimental series is encoded as $p = 1, 2, \ldots, n$. For the pth experiment, measuring the actual cutting force, and substituting its cutting parameters into Eqs. (4)–(7) the fuzzy estimation extent of force uncertainty is obtained, and one is able to observe if the measured force value drops into the estimated extent.

The research of Wang et al. [1,2] has shown that the fuzzy regression model described above has obvious features, i.e., more data samples in modelling for better estimation, satisfactory output of the model within the range of modelling data, and poor estimation without the range of modelling data. Grey set theory [4] is good at predicting the uncertainty, and can overcome the drawbacks arising in the fuzzy regression model.

Based on Eq. (4), assume the output series of the fuzzy model as $EY = \{Y(X_p)\} = \{[C_p, W_p]\} = \{[Y_u(p), Y_s(p)]\}$, where $Y_u(p) = (C_p - W_p)$, $Y_s(p) = (C_p + W_p)$, and $p = 1, 2, \ldots, n$. Pick out q data $(q < n)$ as the original data series of grey prediction of cutting force uncertainty, which may be expressed as

$$Y_{u0} = \{Y_{u0}(1), Y_{u0}(2), \ldots, Y_{u0}(j), \ldots, Y_{u0}(q)\},$$
$$Y_{s0} = \{Y_{s0}(1), Y_{s0}(2), \ldots, Y_{s0}(j), \ldots, Y_{s0}(q)\}$$

where $j = 1, 2, \ldots, q < n$. For series Y_{u0} and Y_{s0}, their one-accumulation-generation series are, respectively, defined as follows:

$$Y_{u1} = \{Y_{u1}(1), \ldots, Y_{u1}(j), \ldots, Y_{u1}(q)\},$$
$$Y_{s1} = \{Y_{s1}(1), \ldots, Y_{s1}(j), \ldots, Y_{s1}(q)\}$$

where the elements $Y_{u1}(j)$ and $Y_{s1}(j)$ may be expressed as

$$Y_{k1}(j) = \sum_{i=1}^{j} Y_{k0}(i) \tag{8}$$

where $j = 1, 2, \ldots, q < n$ and $k = u$ or s. For series Y_{u1} and Y_{s1}, their neighbour-mean-generation are, respectively, defined as

$$Z_{u1} = \{Z_{u1}(2), \ldots, Z_{u1}(i), \ldots, Z_{u1}(q)\},$$
$$Z_{s1} = \{Z_{s1}(2), \ldots, Z_{s1}(i), \ldots, Z_{s1}(q)\}$$

where $i = 2, \ldots, q < n$. The elements $Z_{u1}(i)$ and $Z_{s1}(i)$ are expressed as

$$Z_{k1}(i) = 0.5Y_{k1}(i) + 0.5Y_{k1}(i-1)$$

where $k = u$ or s. Assume that the column vector $Y = [Y_{k0}(2), \ldots, Y_{k0}(q)]^{\mathrm{T}}$, and that the matrix B is defined as

$$B = \begin{bmatrix} -Z_{k1}(2), 1 & -Z_{k1}(3), 1 & \cdots & -Z_{k1}(q), 1 \end{bmatrix}$$

where $k = u$ or s. Then, the estimation values of parameters a and b in the grey differential equation $\mathrm{d}Y_{k1}/\mathrm{d}t + aY_{k1}(t) = b$ can be determined as follows:

$$[a, b]^{\mathrm{T}} = (B^{\mathrm{T}}B)^{-1}B^{\mathrm{T}}Y \tag{9}$$

From Eq. (8), $Y_{k1}(0) = Y_{k0}(1)$. The solution of the grey differential equation may be written as

$$Y_{sk1}(1) = Y_{k0}(1) \tag{10}$$

$$Y_{sk1}(i) = \left(Y_{k0}(1) - \left(\frac{b}{a}\right)\right)\exp(-a(i-1)) + \left(\frac{b}{a}\right) \tag{11}$$

where $i = 2, 3, \ldots, q$.

Eqs. (10) and (11) can be used to determine the simulation series Y_{sk1} of Y_{k1}. Therefore, the simulation series Y_{sk0} of Y_{k0} is determined as follows:

$$Y_{sk0}(1) = Y_{k0}(1) \tag{12}$$

$$Y_{sk0}(i) = Y_{sk1}(i) - Y_{sk1}(i-1) \tag{13}$$

where $i = 2, 3, \ldots, q$, $k = u$ or s. Notes: $i = q$ for the simulation of the original series Y_{u0} and Y_{s0}, and $i > q$ for the prediction of the cutting force uncertainty.

Table 1
Experimental details

Number	Component	Characteristics
1	Measuring sensor	Resistance–strain cell
2	Analogue signal amplifier	YD-4A resistance–strain voltage amplifier
3	Digital measuring instrument	12 bit A/D converter
4	Laboratory software	Original program developed in C++ and MASM
5	Conditions for data acquisition	Sampling frequency 50 HZ; evaluating window; sample 500–1000
6	Computer	586 microcomputer

3. Experimental details

To verify the fuzzy–grey prediction procedure, a measuring system of turning force needs to be schematised so as to acquire the modelling and evaluating data. The characteristics of the measuring system are shown in Table 1. Other machining conditions include #45 medium-carbon steel workpieces with normalising, hardness HB187, diameter 81 mm; a CA6140 lathe with spindle 380 rpm, cutting speed 96 m/min; a YT15 carbide cutter.

4. Prediction and experimental evaluation

Based upon the experimental conditions shown in Table 1, the experimental records of turning forces are listed in Table 2, where all data are used in fuzzy modelling with Eqs. (4)–(7) of cutting force uncertainty, because of the poor fuzzy estimation without the range of modelling data [1,2]. To facilitate programming calculation, let $m = 2$ in $X_{1p} = \log(10^m f)_p$. The fuzzy estimations of turning force uncertainty are shown in Table 3. The corresponding fuzzy regression model is determined as

$$Y(X_p) = [\log F_z]_p = A_0 + A_1[\log(a_p)]_p + A_2[\log(100f)]_p$$

where $A_0 = [1.272603, 0.1092808]$; $A_1 = [0.8845206, 0.0]$; and $A_2 = [0.856115, 0.0]$. Fuzzy membership $h = 0.5$. Relative error Δ_u or Δ_s is defined as

$$\Delta = \frac{\text{estimation} - \text{measurement}}{\text{measurement}}$$

Table 2
The experimental records of turning forces

Number	Depth of cut, a_p (mm)	Feed rate, f (mm/rev)	Main cutting force, F_z (N)
1	2	0.1	439
2	2	0.2	878
3	2	0.3	1129
4	2	0.4	1443
5	2	0.5	1756

Table 3
Fuzzy estimation of turning force uncertainty

Number	Measured force	Fuzzy extent	Lower Δ_u (%)	Upper Δ_s (%)
1	439	[415, 516]	−5.5	+17.5
2	878	[749, 932]	−14.7	+6.20
3	1129	[1063, 1324]	−5.8	+17.3
4	1443	[1363, 1696]	−5.5	+17.5
5	1756	[1646, 2047]	−6.35	+16.6

If selecting data nos. 1–3 in Table 3 for modelling the fuzzy–grey prediction, the fuzzy–grey prediction results of turning force uncertainty are shown in Table 4, where data nos. 1–3 correspond to the simulations of the fuzzy–grey modelling data which comes from fuzzy estimation data nos. 1–3, and data nos. 4 and 5 represent the predictions without the range of fuzzy–grey modelling data. Moreover, the parameters in the differential equation are determined as $[a_s, b_s] = [-0.3475, 590.7376]$, $[a_u, b_u] = [-0.3466, 475.3764]$; the relative error is defined as

$$\Delta = \frac{\text{prediction} - \text{measurement}}{\text{measurement}}$$

If selecting data nos. 1–4 in Table 3 for modelling the fuzzy–grey prediction, the fuzzy–grey prediction results of turning force uncertainty are shown in Table 5, where data nos. 1–4 correspond to the simulations of the fuzzy–grey modelling data which comes from fuzzy estimation data nos. 1–4, and data no. 5 represents the prediction without the range of fuzzy–grey modelling data. Moreover, parameters in the differential equation are determined as $[a_s, b_s] = [-0.2872, 674.6824]$, $[a_u, b_u] = [-0.2874, 541.5835]$.

Table 4
Fuzzy–grey prediction of turning force uncertainty (partial information model with three modelling data)

Number	Measured force	Fuzzy–grey extent	Lower Δ_u (%)	Upper Δ_s (%)
1	439	[415.00, 516.00]	−5.5	+17.5
2	878	[740.10, 920.80]	−15.7	+4.9
3	1129	[1046.6, 1303.4]	−7.3	+15.4
4	1443	[1480.1, 1845.1]	+2.6	+27.9
5	1756	[2093.2, 2611.8]	+19.2	+48.7

Table 5
Fuzzy–grey prediction of turning force uncertainty (new information model with four modelling data)

Number	Measured force	Fuzzy–grey extent	Lower Δ_u (%)	Upper Δ_s (%)
1	439	[415.00, 516.00]	−5.5	+17.5
2	878	[765.60, 953.30]	−12.8	+8.5
3	1129	[1020.6, 1270.5]	−9.6	+12.5
4	1443	[1360.4, 1693.2]	−5.7	+17.3
5	1756	[1813.5, 2256.6]	+3.3	+28.5

Table 6
Fuzzy–grey prediction of turning force uncertainty (new–old alternation model with three modelling data)

Number	Measured force	Fuzzy–grey extent	Lower Δ_u (%)	Upper Δ_s (%)
2	878	[749.00, 932.00]	−14.7	+6.2
3	1129	[1056.9, 1316.4]	−6.4	+16.6
4	1443	[1353.4, 1684.2]	−6.2	+16.7
5	1756	[1733.2, 2154.6]	−1.3	+22.7

If selecting data nos. 2–4 in Table 3 for modelling the fuzzy–grey prediction, the fuzzy–grey prediction results of turning force uncertainty are shown in Table 6, where data nos. 2–4 correspond to the simulations of the fuzzy–grey modelling data which comes from fuzzy estimation data nos. 2–4, and data no. 5 represents the prediction without the range of fuzzy–grey modelling data. Moreover, parameters in the differential equation are determined as $[a_s, b_s] = [-0.2464, 931.306]$, $[a_u, b_u] = [-0.2473, 746.3059]$.

5. Discussion

After observing the data in Table 4, it is found that data nos. 1–3 are better simulations of the original data which comes from nos. 1–3 in Table 3, but the extents somewhat drift to the left; data no. 4, although it is without the range of modelling data, gives a better prediction with less relative error of +2.6%, i.e., the measured force locates at the left side of the lower limit of the prediction extent; data no. 5 gives a worse prediction, i.e., the measured force locates at the left of the lower limit of prediction extent but with a greater relative error of +19.2%.

Observing the data in Table 5, it is also found that data nos. 1–4 are better simulations of the original data which comes from nos. 1–4 in Table 3; data no. 5, although it is without the range of modelling data, gives a better prediction with less relative error of +3.3%, i.e. the measured force locates at the left side of the lower limit of the prediction extent.

Continuously observing the data in Table 6, it is found that data nos. 2–4 are better simulations of original data which come from nos. 2–4 in Table 3; data no. 5, although it is without the range of modelling data, is better prediction with less relative error −1.3%, that is, the measured force locates at the right side of the lower limit of the prediction extent.

Consequently, the fuzzy–grey prediction procedure may simulate the modelling data well, and also validly predict the extent of cutting force uncertainty without the range of modelling data samples, but a valid prediction can only be at the first point (such as no. 4 in Table 4 and no. 5 in Tables 5 and 6) after the final modelling data (such as no. 3 in Table 4, or no. 4 in Tables 5 and 6). Moreover, it is noticed that the prediction of the new–old alternation model in Table 6 is better than the partial information model in Table 4, which is prior to the new information model in Table 5. Differing from the least-squares regression, the neural network and fuzzy regression, the grey method needs less experimental data in modelling, and can validly predict the uncertainty extents for the future.

6. Conclusions

A new and original fuzzy–grey prediction procedure of cutting force uncertainty is proposed, and also evaluated in a measuring system mounted on a lathe. A comparison between the prediction of the fuzzy–grey model and the measured data shows that the developed procedure has better simulation for the modelling data series, and also better prediction for the evaluating data series. In addition, the procedure needs less experimental data in modelling, and can validly predict the uncertainty extents for the future.

Acknowledgements

The authors are grateful to Professor T.J. Chen for guidance. This work was supported by the National Natural Science Foundation of China (funding no. 59905008).

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