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Prediction analysis of solid waste generation based on grey fuzzy dynamic modeling

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Abstract

Successful planning of a solid waste management system depends critically on the prediction accuracy of solid waste generation. But the prediction condition of generation trend in many developing countries is quite different from those in developed countries. The lack of sampling and analysis in many developing countries due to insufficient budget and unavailable management task force has resulted in a situation where the historical record of solid waste generation and composition can never be completed in the long term. To effectively handle these problems with only limited samples and fulfil the prediction analysis of solid waste generation with reasonable accuracy, a special analytical technique must be developed and applied before the subsequent system planning for urban solid waste management is carried out. This study presents a new theory — grey fuzzy dynamic modeling — for the prediction of solid waste generation in the urban area based on a set of limited samples. The practical implementation has been accessed by a case study in the city of Tainan in Taiwan. It shows that such a new forecasting technique may achieve better prediction accuracy than those of the conventional grey dynamic model, least-squares regression method, and the fuzzy goal regression technique. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Forecasting; Grey systems theory; Fuzzy sets theory; Grey fuzzy dynamic modeling; Solid waste generation

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1. Introduction

Both planning and design of solid waste management systems require accurate prediction of solid waste generation [1]. Conventional forecasting methods for solid waste generation frequently use the demographic and socioeconomic factors in a per-capita basis. The per-capita coefficients may be taken as fixed over time or they may be projected to change with time. Grossman et al. [2] extended such considerations by including the effects of population, income level, and the dwelling unit size in a linear regression model. Niessen and Alsobrook [3] conducted similar estimates by providing some other extensive variables characterizing waste generation. However, without considering the dynamic impacts, the static formation of those models inherently limits the applicability to the situation in which underlying relationships have not significantly changed over time. To improve such a situation, dynamic properties in the process of solid waste generation must be fully characterized in the model formulation. Econometric forecasting, one of the alternatives to static models, is an indirect technique in which the future forecasts are derived from current forecasts of the independent variables themselves. To explore the effect of lagged influence of independent variables, Chang et al. [4] further applied a geometric lag econometric analysis, incorporating related temporal socioeconomic or demographic factors simultaneously, for the forecasting of solid waste generation in Taiwan. In addition, Chang and Lin [1] took the recycling impact into account as an intervention event in a time series statistical model that is designed to account for the varying trend of solid waste generation in the city of Taipei in Taiwan. Such an analysis creates profound impacts in dealing with the possible structural change of generation trend of solid waste in metropolitan regions where the concept of resource conservation and recycling starts to be emphasized by the people. These traditional forecasting methods, however, need a completed socioeconomic and environmental database to build the essential mathematical models. But many municipalities in the developing countries may not have sufficient budget and management task force available to maintain a long-term and large-scale sampling and analysis program. A revised forecasting method, that is suitable for the situation where only very limited samples are available for forecasting practice, is needed to handle the prediction analysis for solid waste generation with reasonable accuracy.

Most of the traditional statistical forecasting models, such as the geometry average method, saturation curve method, least-squares regression method, and the curve extension method, are designed based on the configuration of semi-empirical mathematical models. The structure of these models is simply an expression of cause–effect or an illustration of trend extension in order to verify the inherent systematic features that are recognized as related to the observed database. In light of the evolution of structured or semi-structured forecasting techniques that have been developed in the scientific community, the fuzzy forecasting and grey dynamic modeling are viewed as two promising approaches for handling forecasting issues under uncertain environments. Chang [5] proposed the fuzzy seasonality forecasting technique for handling non-homogenous database. Chang and coworkers [6,7] presented a revised fuzzy forecasting technique — the fuzzy goal regression — to explore the underlying cost structure corresponding to various types of municipal incinerators and wastewater treatment plants in Taiwan. The fuzzy goal regression model, which is derived from the fuzzy goal programming (FGP) technique, abandons the conventional regression goals of minimization for sum of squared error but pursues the highest prediction accuracy using flexible fuzzy membership functions to fit in the observed database. On the other hand, the grey dynamic model (i.e. the so-called GM model in this study) was developed by Deng [8] in China. The GM model is particularly designed for handling the situation in which only limited data are available for forecasting practice while system environment is not well-defined and fully understood (i.e. the so-called grey environment in this study). Morita et al. [9] proposed an interval prediction model based on the grey system theory to balance the supply and demand in a power system planning and operation program. Xia [10] presented a new grey system prediction model to forecast the slipping time of a landslide and to successfully predict the common monitoring information change of a landslide. Additional effort has been made for developing some hybrid soft forecasting techniques in the literature. For example, Huang and Huang [11] applied fuzzy and gradient descent methods to form a new grey dynamic control mechanism. Huang and Huang [12,13] further discussed the integration and application of both fuzzy and grey dynamic models. They successfully integrated the genetic algorithms with the conventional GM model. Tien and Chen [14] combined the ARIMA models and GM model together to formulate a new forecasting model that is denoted as GDM(2, 2, 1) in their study.

This analysis serves as a companion study of Chang and Lin [1]. The previous development of a GM model was based on the similarity characteristics between the real world physical systems and the energy systems in which any type of data set in series can be described by a simulated exponential function after the essential grey pre-treatment process. The simulated exponential function can be viewed as a general solution of a pseudo-differential equation that is designed to illustrate the grey system. The conventional least-squares regression method, used in the GM model was simply utilized as a means for the determination of those parameters in the simulated exponential function. However, it is recognized that the least-squares regression method is not suitable for handling the forecasting issues with only limited samples in the real world systems. This analysis replaces the least-squares regression method used in the GM model by the fuzzy goal regression method in order to present a better mathematical function and, as a result, to improve the overall prediction accuracy in a grey environment. A case study of solid waste generation in the city of Tainan in Taiwan demonstrates the significance and applicability of such an analytical approach.

2. Methodology

2.1. The basic concepts of grey system and grey dynamic model

Grey systems theory, which was first derived by Deng [8], describes random variables as a changeable interval number that varies with time factors and uses 'color' to represent the degree of uncertainty in a dynamic system. It implies that a grey environment is the system that consists of partially known and partially unknown information. It is believed that the uncertainties existing in the whitening process mainly come from the insufficiency of understandable information. Therefore, by way of increasing the system information, the degree of uncertainty could be changed or diminished over time. The output from a grey environment must have a certain degree of implication for the behaviors in this system. Therefore, instead of analyzing the characteristics of the grey systems directly, the grey systems theory exploits the accumulated generating operation (AGO) technique to outline the system behaviors because the external and latent behaviors will become more apparent due to the decrease of the random intensity after the AGO practice. In other words, the AGO practice may reduce the white noise embedded in the input data from a statistical sense. GM model always describes the real world physical system as an energy system in a continuous domain. Hence, all the systematic responses in the real world physical system should be consistent with some sort of exponential pattern. The measured database with discrete features therefore can be formulated as a typical type of solution based on a so-called 'pseudo-differential equation'. With the solution from the proposed pseudo-differential equation (i.e. the simulated exponential function), the next output from a grey environment can be predicted based on a few observed samples or data. Such an approach does not need to pay more effort to directly formulating the underlying complicated physiochemical property or socioeconomic behavior in an unknown system.

Since the GM model can characterize such an unknown system and be able to forecast effectively based on a few data, it has showed its practicality in utilizing insufficient database. Therefore, the GM model is proven as an effective method especially for many environmental or socioeconomic issues in China [15,16] where the situation of lacking long-term monitoring database is vast. The original idea to model such a grey system is to use a simulated exponential function, which could be a general solution of a differential equation. However, the conventional differential equation(s) can only be used to represent a real world physical system with respect to continuous type of observations. In order to replace the continuous type of observations by a set of discrete type of observations, a pseudo-differential equation must be defined and applied for representing the grey system. Since the general solution of the pseudo-differential equation (i.e. the grey differential equation in the grey systems theory) can be defined by the exponential function or its mixture, it is thus regarded as representative of the real world grey physical system. Such an exponential function may be thought of as an illustration of being consistent with the energy dissipation or accumulation phenomenon inherent in a specific grey system, in which the gross impacts are expressed by those lumped-parameters defined in the grey differential equation (i.e. the GM model).

The mathematical expression for various levels of GM model (i.e. the grey differential equation) is GM(n, h), as shown as Eq. (1), in which n corresponds to the dimension and h corresponds to the number of variables

$$\sum_{i=0}^{n} a_i \frac{\mathrm{d}^{n-i} X_1^{(1)}}{\mathrm{d}t^{n-i}} = \sum_{i=1}^{h-1} b_i X_{i+1}^{(1)} \tag{1}$$

where the variables $X_i^{(k)}(k)$ are defined for those discrete types of observations after some sort of grey pre-treatment. The method of grey pre-treatment will be described later. The boundary conditions of Eq. (1) include:

$$X_1^{(0)}(0) = X_1^{(1)}(1) \tag{2}$$

$$X_i^{(1)}(k) = \sum_{j=1}^{k} X_i^{(0)}(j) \tag{3}$$

The most commonly used GM model is the GM(1, 1) model which represents the simplest form of the grey differential equation, as defined by Eq. (4).

$$\frac{\mathrm{d}X^{(1)}}{\mathrm{d}t} + a\xi^{(1)} = b \tag{4}$$

Fig. 1 illustrates the analytical procedure for building a generic GM(n, h) model. The steps required for building a GM(1,1) model, which is just a specific case for building a generic GM(n, h) model, are summarized in the following contexts.

2.1.1. Step 1

The first step is to perform the grey relation analysis for screening the effective data base required for modeling analysis and then determine the essential variables in the framework of the GM(1,1) model. Thus, the data base screened for GM analysis can be expressed based on the originally-observed data set (ODS) $X^{(0)} = [x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n-1), x^{(0)}(n)]$, where $x^{(0)}(i)$ corresponds to the system output at time period i.

2.1.2. Step 2

In order to prepare the grey generated space $X^{(1)}$ from the initial data base, a data processing technique — accumulated generating operation (AGO) — has to be applied, that is designed particularly for removing the white noise existing in the ODS. The output of this AGO is expressed as a first-order accumulated generating sequence, $X^{(1)} = [x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n-1), x^{(1)}(n)]$, where $x^{(1)}(i)$ corresponds to the system output at time period i. The generation of higher order accumulated generating sequences may be obtained after an iterative procedure by following the same logic, that may contribute to the provision of a data set with lower white noise.

2.1.3. Step 3

The GM(1,1) model, using the corresponding grey differential equation as shown in Eq. (4), can then be built based on the first-order accumulated generating sequence $X(1)$, in which $\xi^{(1)}$ corresponds to the background of the differential equation, a is the state parameter, and b is the control or disturbance parameter. In the GM(1,1) model, as shown in Eq. (2), $\xi^{(1)}$ is defined by $\xi^{(1)} = [0.5X^{(1)}(k) + 0.5X^{(1)}(k-1)]$. The building of a generic GM(n, h) model may be carried out following the same logic. However, except for the grey differential equation as we discussed here, some other types of mathematical equations, such as the difference equation, exponential equation and the multinomial equation, are also feasible for the expression of a typical grey system. When the prediction accuracy is poor, we may try to utilize the rest of the formulations to improve the prediction accuracy.

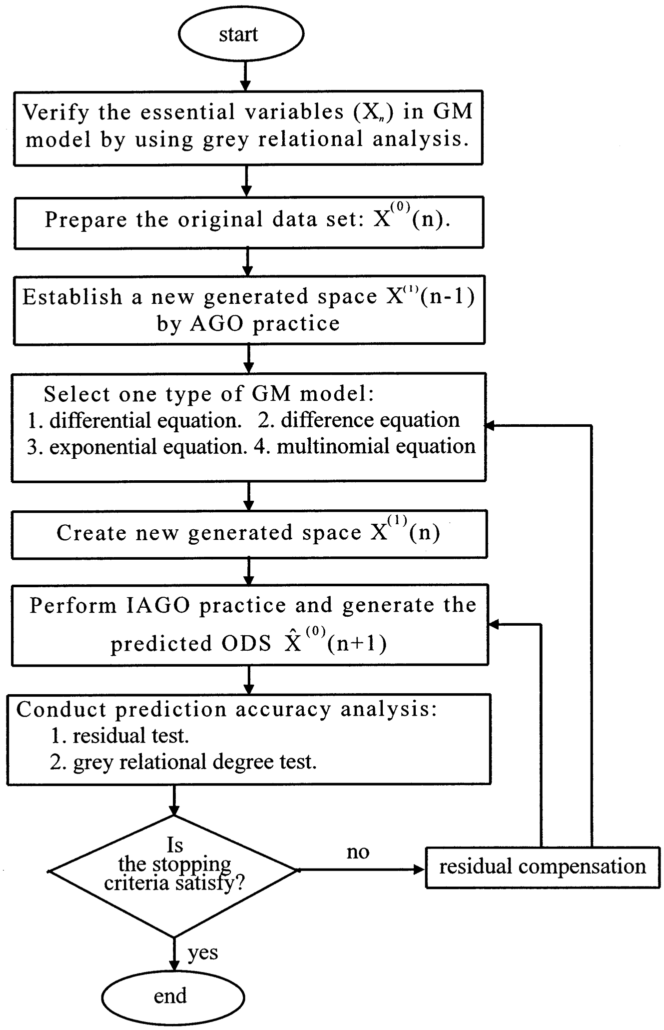

Fig. 1. The flowchart for the construction of a GM model.

2.1.4. Step 4

After building the required GM model, the analytical solution (i.e. the general solution) of the corresponding grey differential equation can be found through the identification of the parameters a and b. Both parameters as defined in Eq. (4) can be obtained by using the least-squares method when the identification of the entire GM(1,1) model is viewed as a conventional regression practice. Let β be an estimated parameter vector for all a and b, then the least-squares estimate for $\hat{\beta}$ is expressed as in Eq. (5)

$$\hat{\beta} = [\hat{a}, \hat{b}] = [B^T B]^{-1} B^T Y_n \tag{5}$$

where

$$B = \begin{bmatrix} -\frac{1}{2}(x_1^{(1)} + x_2^{(1)}), & 1 \\ -\frac{1}{2}(x_2^{(1)} + x_3^{(1)}), & 1 \\ \cdots & \cdots \\ \cdots & \cdots \\ -\frac{1}{2}(x_{n-1}^{(1)} + x_n^{(1)}), & 1 \end{bmatrix} \tag{6}$$

$$Y_n = [x_2^{(0)}, x_3^{(0)}, \ldots, x_n^{(0)}] \tag{7}$$

2.1.5. Step 5

While the estimated value $\hat{\beta}$ is integrated into the GM(1,1) model, the solutions can be converted back to a discrete sequence as defined in Eqs. (8) and (9). The solutions include two parts. The fitting output of $\hat{X}^{(1)}(k+1)$ at time $k+1$ is first obtained by Eq. (5) and then Eq. (8) is used as a manipulation basis to perform inverse accumulated generating operation (i.e. denoted as $[\text{AGO}]^{-1}$ or IAGO) by which the final result, $\hat{X}^{(0)}(k+1)$ yields in Eq. (9)

$$\hat{X}^{(1)}(k+1) = \left(X^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right) \exp(-\hat{a}k) + \frac{\hat{b}}{\hat{a}}; \quad k = 0, 1, 2, \ldots n-1 \tag{8}$$

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k); \quad k = 0, 1, 2, \ldots n-1 \tag{9}$$

2.1.6. Step 6

The last step is to examine the accuracy of prediction by the GM model. Two methods, including the residual test and the grey relational degree test [10,14,15], are frequently used for examining the prediction accuracy. The residual test is simply a method for a direct comparison between the original ODS and the predicted ODS. The grey relational degree test is required to examine the problem with a much more detailed procedure. It includes three steps for a complete analysis. The first step is to choose the reference sequence, $X_0 = (x_0(1), x_0(2), \ldots, x_0(n))$, and select one of the predicted sequences, $X_i = (x_i(1), x_i(2), \ldots, x_i(n))$, $i = 1, 2, \ldots, m$, for a comparative study. The second step is to use Eq. (10), for the generation of the grey relational coefficient $\gamma(x_0(k), x_i(k))$ at point k. Finally, the grey relational grade is derived from Eq. (11) for the justification of the prediction accuracy

$$\gamma(x_0(k),\ x_i(k)) = \frac{\min_j \min_k |x_0(k) - x_j(k)|}{|x_0(k) - x_i(k)| + \zeta\ \max_j \min_k |x_0(k) - x_j(k)|} + \frac{\max_j \max_k |x_0(k) - x_j(k)|}{|x_0(k) - x_i(k)| + \zeta \max_j \min_k |x_0(k) - x_j(k)|} \tag{10}$$

$$\gamma(x_0,\ x_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_0(k),\ x_i(k)) \tag{11}$$

where $\zeta \in (0,\ 1)$ is called the distinguishing coefficient, $j = 1,\ 2, \ldots,\ n$, and $k = 1,\ 2, \ldots,\ n$.

2.2. The basic concept of fuzzy sets theory and fuzzy goal regression

A different type of uncertainty embedded especially in human's feeling or decision-making is the fuzziness. The focus of fuzzy sets theory is thus placed upon its non-statistical characteristics in nature. While the random variable is used for the description of uncertain statistical implication, the fuzzy membership function, that refers to the similarity of an element that belongs to a subjectively described set, is defined for illustrating the imprecision existing in real world system. The more an element or object can be said to belong to a fuzzy set A, the closer to 1 is its grade of membership.

Fuzzy sets theory is frequently applied in recent research for various types of systems analysis, covering forecasting, optimization, reasoning, and control issues [17–19]. In particular, Tanaka et al. [20] first mentioned that we must deal with a fuzzy structure of the systems in the regression model where human estimation is influential. Since then, fuzzy regression analysis, representing an alternative to the statistical regression technique, has matured rapidly in the last few years. Its application potential is verified by many practices in engineering planning, design, operation, management, and control. For example, the fuzzy linear regression is thus recognized as a mapping process based on a set of observations. Fuzzy parameters are used for such a linkage between the independent variables and the dependent variable. To find the solution of fuzzy parameters, an equivalent linear programming model has to be solved. However, it is found that the prediction accuracy of a fuzzy linear regression model is not always better than that of conventional least-squares regression model, although fuzzy regression allows the inclusion of expert knowledge or fuzzy information in the model in advance. Chang et al. [6] further proposed a revised approach — the fuzzy goal regression model — to improve the performance of previous fuzzy regression methodology. This paper further applied the fuzzy goal regression technique to estimate the parameters of the GM model. To ease the application, the following sections review the principles of fuzzy goal regression model.

The central idea of the fuzzy goal regression model is derived from an integration of both fuzzy goal programming and fuzzy linear regression techniques. Since the solution approach of fuzzy mathematical programming is generally designed to satisfy the fuzzy objective(s) and constraint(s) simultaneously, a decision space in a fuzzy environment is thus defined as the intersection of those membership functions corresponding to those fuzzy objective(s) and constraint(s) [21]. Therefore, if $\{\mu_{G_1}, \mu_{G_2},\ldots, \mu_{G_{2m}}\}$ and $\{\mu_{C_1}, \mu_{C_2},\ldots, \mu_{C_p}\}$ are denoted as the fuzzy membership functions for those fuzzy objectives $\{G_1, G_2,\ldots, G_m\}$ and fuzzy constraints $\{C_1, C_2,\ldots, C_p\}$ respectively in a decision space X, all the fuzzy membership functions of G_m and C_p can be combined together to form a new decision space D, which stands for a resultant fuzzy set generated from the intersection of all related G_m and C_n as shown below.

$$D = G_1 \cap G_2 \cap \ldots \cap G_m \cap C_1 \cap C_2 \cap \ldots \cap C_p \tag{12}$$

Since the decision D is defined as a fuzzy set, the optimal decision is any alternative $s \in S$ that can maximize the minimum attainable aspiration levels in decision making. The aspiration level of each fuzzy membership function is actually a common membership value achieved in the decision set, $\mu_D(s)$. Thus, the max min convolution requires maximizing the minimum membership values of those elements, as below [21]:

$$\max_s \mu_D = \max_s \min\{\mu_{G_1}, \mu_{G_2},\ldots, \mu_{G_{2m}}, \mu_{C_1}, \mu_{C_2},\ldots, \mu_{C_p}\} \tag{13}$$

Such an operation is actually an analogy of the non-fuzzy environment as the selection of activities simultaneously satisfies all the objective(s) and constraint(s). Hence, a goal programming problem with multiple fuzzy goals can be simplified as

$$CX_i \lesssim f1 \tag{14}$$

$$CX_i \gtrsim f2 \tag{15}$$

$$A_k X \leq B_k \tag{16}$$

$$A_l X \geq B_l \tag{17}$$

$$X \geq 0 \tag{18}$$

where '$\gtrsim$' and '$\lesssim$' denote the notion of fuzzified version of '$\geq$' and '$\leq$', which have the linguistic interpretation 'approximately greater than or equal to' and 'approximately less than or equal to', respectively. Eqs. (14) and (15) represent the fuzzy goal constraint. Eqs. (16) and (17) express the functional or definitional constraints with '$\geq$' and '$\leq$' relationships in the goal programming model. In a deterministic goal programming model, these goal constraints are generally expressed by an integration of the objective function, deviational variables, and the target values. However, the membership functions introduced from the FGP model are used to replace the function of those deviational variables.

In general, the non-increasing and non-decreasing linear membership functions are frequently used for the inequality constraints with 'approximately less than or equal to' and 'approximately greater than or equal to' relationships, respectively, as shown in Fig. 2. It can be assumed that the membership values are linearly decreasing in Eq. (14) over the 'tolerance interval' δ_i and linearly increasing in Eq. (15) over the 'tolerance interval' δ_j. Hence the most sensitive part of aspiration

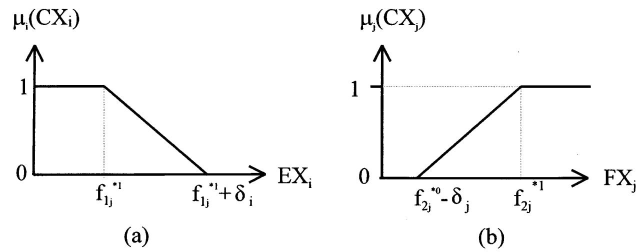

Fig. 2. The membership functions for fuzzy goal programming.

levels for decision makers is the range of tolerance intervals, δ_i and δ_j corresponding to these fuzzy goals. If a fuzzy goal programming model is designed to maximize several fuzzy goals with the fuzzy implication of 'the higher, the better', subject to a set of deterministic constraints, the problem can be further solved by introducing an intermediate control variable α [21]. Such an intermediate control variable α is a decision variable corresponding to the minimum membership values of the available decision maker's aspiration levels achieved in the maximization process.

By introducing the intermediate control variable, α, the fuzzy goal programming model, defined in Eqs. (14)–(17), becomes:

$$\text{Max } \alpha \tag{19}$$

subject to

$$\mu(CX_i) \geq \alpha \tag{20}$$

$$A_k X \leq B_k \qquad \forall k \tag{21}$$

$$A_l X \geq B_l \qquad \forall l \tag{22}$$

$$X \geq 0 \tag{23}$$

Hence, the fuzzy structure represented in Eqs. (19) and (20) may be applied in the following fuzzy goal regression analysis. In the fuzzy goal regression analysis, the proposed linear regression model ($\Sigma a_i x_i$) can be viewed as a linear function that is designed to approach a target value of dependent variable Y. The regression effort for each set of observations, corresponding to dependent variables and independent variables, is to achieve the goal where the target value is approximated by a set of fuzzy parameters, a_i, multiplied by the observed input data, x_i. For n sets of observations, we have n fuzzy goals to be handled in the fuzzy programming model. The problem can therefore be stated as a maximization process of the minimum degree of fitting for these goal constraints. However, the achievement of each goal constraint, expressed by a triangular membership function, can be regarded as a superimposition of two membership functions, as shown in Fig. 3. One is a non-increasing membership function and the other is a non-decreasing membership function. The highest degree of fitting may be attained when the observed value of dependent variable Y is equal to the predicted value of $\Sigma a_i x_i$ (Fig. 4). Thus, each goal constraint has to be separately expressed by two inequality constraints in the following method

$$\text{Max} \sum_{i=1}^{n} (h_{i1} + h_{i2}) \tag{24}$$

subject to:

$$\sum_{i=1}^{n} a_i x_i \leq Y_i + (1 - h_{i1})(k_2)(y_i) \quad \forall i = 1,\ldots, n \tag{25}$$

$$\sum_{i=1}^{n} a_i x_i \geq Y_i - (1 - h_{i1})(k_1)(y_i) \quad \forall i = 1,\ldots, n \tag{26}$$

$$0 \leqq h_{i1}, \quad h_{i2} \leqq 1 \tag{27}$$

$$X \geqq 0 \tag{28}$$

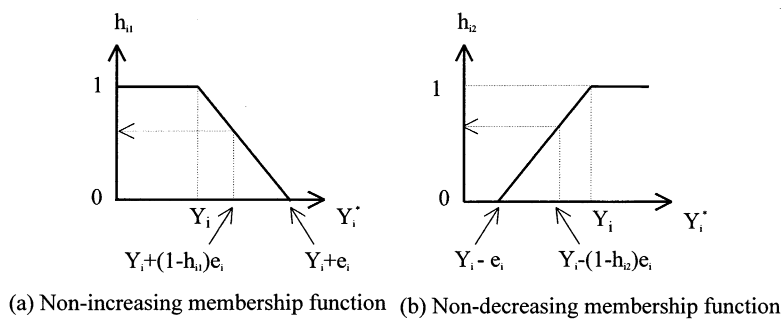

Fig. 3. The membership functions used in fuzzy goal regression model.

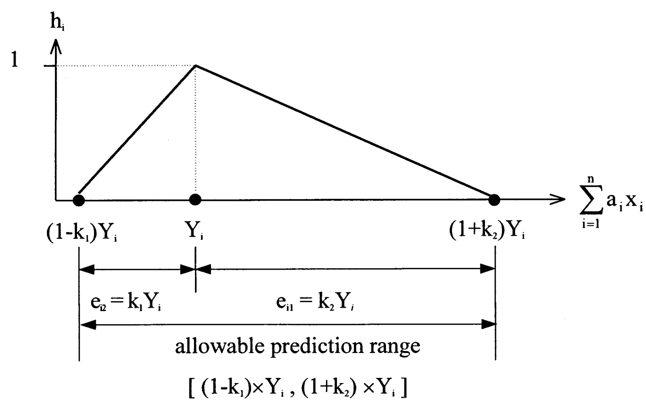

Fig. 4. The permitted range of Y^* based on given fuzzy data Y and its degree of fitting.

If considering the relative importance between various objectives and the effect resulting from the slopes of membership functions, Eq. (24) can be revised in a form of Eq. (29). After carrying out a series of analytical procedures in the above, a multiple linear regression analysis can be integrated into a linear programming practice. It is believed that such a new forecasting method with the aid of the principles of fuzzy goal programming and fuzzy linear regression techniques may avoid the errors from the incomplete database and present better prediction accuracy.

$$\text{Max} \sum_{i=1}^{n} \left(\frac{w_i}{k_1} h_{i1} + \frac{w_i}{k_2} h_{i2} \right) \tag{29}$$

3. Development of the grey fuzzy dynamic model

It is noted that the values of a and b in the GM model determine the quality of prediction accuracy. Due to the difficulty for least-squares regression method to obtain suitable a and b values with reasonable prediction accuracy in the circumstance of insufficient samples, the chance to use the fuzzy goal regression model for finding the appropriate a and b values is always interesting. The grey dynamic model (GFM) can then be developed by the integration of these two forecasting techniques. The parameter identification technique in the GM model is therefore replaced by the fuzzy goal regression technique such that better prediction of a and b values are anticipated as the construction and operation of a GM model remains.

4. Case study

4.1. System environment of Tainan City

Tainan City, located in the southern part of Taiwan, is divided into seven administrative districts. Fig. 5 illustrates the geographical location and solid waste management facilities of Tainan City. The task of waste collection and shipping are handled by the Environmental Protection Bureau in which seven independent collection teams are organized for cleaning up the waste streams in those administrative districts respectively. Only one incinerator a the capacity of 900 tonnes/day is being built for handling future solid waste in the metropolitan region. It will start up formal operation in the year 2000. One of the remained questions is to determine if the incineration capacity is large enough to handling the increasing trend of solid waste generation in the city of Tainan. Previously studies focus on the determination of optimal solid waste management strategy that illustrates possible interactions between recycling and incineration options [22] and its uncertainty analysis in decision-making [23]. This analysis serves as a companion study of Chang and Wang [22,23] and tries to provide a systematic estimation if the recycling effect is not regarded as an influential factor of solid waste generation.

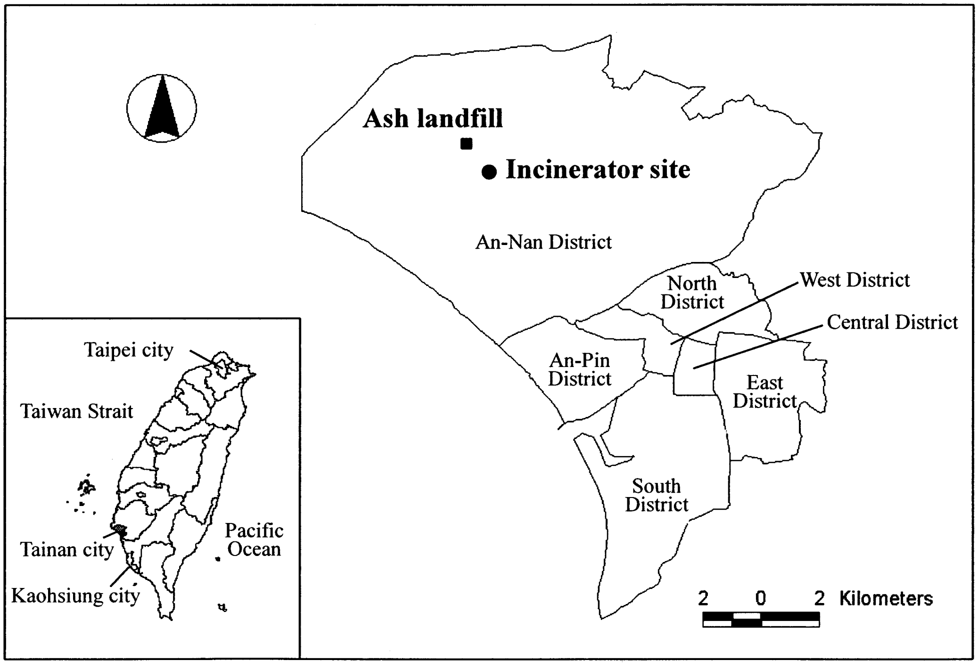

Fig. 5. The geographic location of Tainan City.

The major difficulty existing for the prediction analysis based on the conventional regression techniques actually rests upon the inefficiency when encountering the incomplete database of solid waste generation. Table 1 lists the database available for this case study. With respect to developing countries, the minimum size of database is three for performing a basic GM or GFM modeling analysis. Two out of three are prepared for calibration while the remaining one is used for verification of the proposed model. Using the GM(1,1) model to predict the future trend of solid waste generation with such a 10-year record seems reasonable. Extra measures for system planning of solid waste management is not required. This paper, however, first presents the application of the GFM(1,1) model for an advanced analysis. A strong improvement may be anticipated by performing a sensitivity analysis for the GFM method. Thus, the parameter sensitivity of the GFM model for determining the robustness of the GFM method will be fully discussed in Section 4.3.

4.2. Results and discussions

The implementation of the GFM model has to be accessed via three steps. The first step is to divide the ODS into two sub-sequences. One is used to calibrate the forecasting models and the other is reserved for the verification of the prediction accuracy. The second step is to establish the calibration and verification task and

Table 1
The amount of solid waste generation in Tainan City

	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998
Solid waste (tonnes/day)	262	278	322	367	405	410	427	465	525	576	629	672	733	766

Table 2
The comparative study of prediction analysis using GM(1,1) and GFM(1,1) models as tools[a]

Data type	Year	Observation	GM (1,1)	Predicted error (%)	GFM (1,1)	Predicted error (%)
Original data set	1985	262	262	0	262	0
Sequence for calibration	1986	278	311	11.8	289	3.9
	1987	322	337	4.7	313	−2.7
	1988	367	364	−0.8	340	−7.5
	1989	405	394	−2.7	368	−9.2
	1990	410	425	3.7	399	−2.8
	1991	427	460	7.7	432	1.2
	1992	465	537	15.5	468	0.7
	1993	525	580	19.4	508	−3.3
Sequence for verification	1994	576	627	17.7	550	−4.5
	1995	629	678	16.4	596	−5.3
	1996	672	732	17.7	646	−3.9
	1997	733	791	7.9	700	−4.5
	1998	766	855	11.6	759	−0.9
Predicted sequence	1999		924		823	
	2000		999		891	
	2001		1105		941	
	2002		1204		997	
	2003		1299		1087	

[a] Unit: tonnes per day.

the final step is to fulfill the prediction analysis for practical application. In this practice, the GM model serves as a base model for the purpose of comparison only. Table 2 depicts a comparative study of prediction analysis using both GM and GFM models as tools. Table 3 presents the final evaluation of prediction analysis for these two methods. Both grey relational analysis and residual test indicate that the GFM(1,1) method may exhibit a better prediction accuracy than the GM(1,1) method. Although the GM method can find out the general tendency of an unknown system, it still cannot fully avoid the imprecision resulting from the insufficient database.

Table 3
The prediction accuracy of GM and GFM modeling analysis[a]

	Grey relational test (γ)	Residual test $(S_2/S_1)^2$
GM(1,1)	0.68	0.32
GFM(1,1)	0.81	0.19

[a] The symbols S_1 and S_2 are defined as the S.D. as commonly used in statistical analysis.

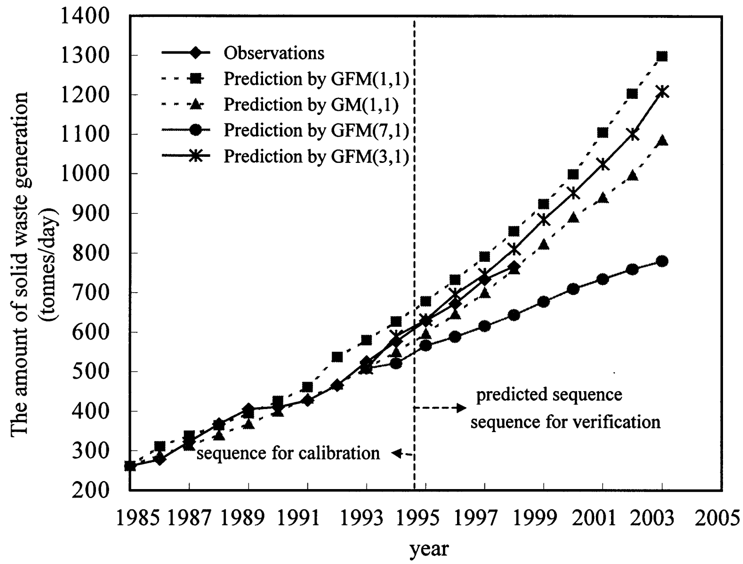

Fig. 6. The sensitivity analysis using various GM and GFM models.

The prediction analysis of solid waste generation characterized by the GFM method suggests that the planned incinerator with 900 tonnes per day capacity cannot handle the growing demand of solid waste generation in the near future. Once the proposed recycling program is unsuccessful, additional treatment and disposal alternatives should be taken into account in time.

4.3. Sensitivity analysis

Fig. 6 describes the sensitivity analysis through a comparative approach. Varying the dimension of the GFM model exhibits a significant impact on the prediction accuracy. This implies that the dimension chosen in the grey differential equation should be limited to some extent so as to improve the overall prediction accuracy. The observation in Fig. 6 also indicates that the best application potential of the GFM technique is existing especially in the case of GFM(3,1). Once the number of dimensions used in the grey differential equation becomes larger than three, the performance of the GFM model would become worse than that of the base model (i.e. GM(1,1)). The selection of several different dimensions in the GFM model, however, may generate a set of predicted curves in which an allowable range for reflecting the practical variability in the prediction of solid waste generation can be offered. Such an allowable range may be defined by the predicted area between the curves formed by the GFM(1,1) and GFM(7,1) models in this practice.

5. Conclusion

Experience indicates the estimation of solid waste generation is crucial for the subsequent system planning of solid waste management in the metropolitan region from both short- and long-term perspectives [1,4,22–24]. However, a complete record of solid waste generation and composition is not always present. This analysis develops an effective tool for tackling those forecasting problems that are lacking a significant amount of data for determining regression models and that have vague relationships between the dependent variables and those socio-economic factors. The central idea is to utilize the fuzzy goal regression technique to improve the conventional grey dynamic model so as to minimize the discrepancy between the predicted values and the observed values. With respect to developing countries, the minimum size of database is three in practising a basic GM or GFM modeling analysis. Two are prepared for calibration, while the remaining one is used for verification. A case study of solid waste generation in the city of Tainan in Taiwan demonstrates the application potential of such an approach.

In fact, not only the selection of dimension and decision variable but also the choice of data sequence for the calibration and verification in the GFM modeling analysis may influence the prediction accuracy. This paper selected different dimensions in the GFM model to proceed a sensitivity analysis that may generate a set of predicted curves as an allowable range in the prediction of solid waste generation. The sensitivity analysis, as shown in Fig. 6, also presents a comparative study between the conventional GM method and the newly derived GFM method. The general finding is that the number of dimension in the grey differential equation is an influential factor especially in models with higher dimension.

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