Introduction to Grey System Theory

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Outline

Grey System theory was initiated in 1982 [7]. As far as information is concerned, the systems which lack information, such as structure message, operation mechanism and behaviour document, are referred to as Grey Systems. For example, the human body, agriculture, economy, etc., are Grey Systems.

Usually, on the grounds of existing grey relations, grey elements, grey numbers (denoted by $\otimes$) one can identify which Grey System is, where "grey" means poor, incomplete, uncertain, etc. The goal of Grey System and its applications is to bridge the gap existing between social science and natural science. Thus, one can say that the Grey System theory is interdisciplinary, cutting across a variety of specialized fields, and it is evident that Grey System theory stands the test of time since 1982.

As the case stands, the development of the Grey System—as well as theoretical topic—is coupled with clear applications of the theory in assorted fields.

The concecpt of the Grey System, in its theory and successful application, is now well known in China. The application fields of the Grey System involve agriculture [23, 77—81, 91], ecology [59], economy [61, 102, 103, 104], meteorology [58, 74, 91], medicine [55, 89], history [63, 64], geography [1], industry [6], earthquake [73, 87, 88], geology [76, 119], hydrology [98, 112], irrigation strategy [26], military affairs, sports [116], traffic [67], management [30, 97, 105], material science [82, 83], environment [108], biological protection [69, 70], judicial system [100], etc.

Projects which have been successfully completed with the Grey System theory and its applications are as follows:

1. Regional economic planning for several provinces in China;
2. To forecast yields of grain for some provinces in China;

3. To analyse agricultural economy in China;
4. To make satisfactory planning of irrigation (watershed of the People's Victory Channel in Henan Province);
5. To build models available for biological protection;
6. To control the water level for boilors by grey prediction control;
7. To estimate the economic effect;
8. To build a diagnosis model available for medicine;
9. To forecast weather.

As far as properties of implement fields are concerned, it can be seen that the greatest part of them can be referred to as abstract systems, which are short of physical prototypes, according to the terminology used throughout the Grey System theory. They can be called latent Grey Systems.

The aims of Grey System theory are to provide theory, techniques, notions and ideas for resolving (analysing) latent and intricate systems, for example:

1. To establish a non-function model instead of regressive analysing;
2. To define and constitute Grey Process replacing the stochatic process and to find the real time techniques instead of statistical model to deal with the grey process, in order to obtain an approach to modelling with few data, avoiding searching for data in quantities;
3. To turn the disorderly raw data into a more regular series by grey generating techniques for the benefit of modelling instead of modelling with original data;
4. To build a differential model—so called grey model (GM)—by using the least 4 data to replace difference modelling in vast quantities of data;
5. To develop a novel family of grey forecasting methods instead of time series and regressive methods—all of these may be referred to as an approach to deal with grey processes;
6. To develop innovational techniques and concepts for decision making—so called grey decision making;
7. To develop novel control techniques. e. g., the grey forecasting control replacing classical control which is referred to as afterward control, also relational control, generating control and programming control;
8. To study the mechanism theory, including grey sequence theory and grey structure theory;
9. To study feeling and emotion functions and fields with whitening functions;
10. Based on grey relations, grey elements and grey numbers to be used to study grey mathematics instead of classical mathematics study.

The essential contents and topics of Grey System theory encompass the following areas:

grey relational space
grey generating space
grey forecasting

grey decision making
grey control
grey mathematics
grey theory

Why have so many researchers in the fields of software science utilized Grey System theory, and why has there been dialogue between them and the Grey System theory? At least two reasons immediately spring to mind. First, the software researchers tend to be methodologically accepting, when they are grey mathematically efficient and sure of being grey methodologically innovative; second, some of the element techniques and concepts of Grey System theory have been implemented and investigated by a variety of areas and even incorporated in some theoretical schema, such as grey market and economic programming.

Grey Relational Space

The grey relational space (GRS) [11, 18] is one that describes the posture relationships between one main factor and all the other factors in a given system. A GRS is a binary set denoted by $(\mathbf{X}, \mathbf{\Gamma})$ where $\mathbf{X}$ is a collection composed of sequences x_i to be compared and reference sequence x_0, $\mathbf{\Gamma}$ is a map set called grey relational map set, $\gamma \in \Gamma$ is an appointed relational map in GRS. Assume that

$$\gamma(x_0(k), x_i(k))$$

is an image at point k from the series to real number with map γ and $\gamma(x_0, x_i)$ is an image at all points with $k = 1, 2, 3, \cdots, n$, where

$$x_0 = (x_0(1), \cdots, x_0(n))$$
$$x_i = (x_i(1), \cdots, x_i(n))$$

Let $\gamma(x_0, x_i)$ satisfy that

$$\gamma(x_0, x_i) = \frac{1}{n} \sum_{k=1}^{n} \gamma(x_0(k), x_i(k))$$

then $\gamma(x_0(k), x_i(k))$ is said to be a grey relational coefficient at the point k and $\gamma(x_0, x_i)$ to be a grey relational grade, iff Γ satisfies the following axioms [18]:

(A1) (Norm Interval)

$$\gamma(x_0(k),\ x_i(k)) \in (0,\ 1],\ \forall k$$
$$\gamma(x_0(k),\ x_i(k)) = 1,\ \text{iff}\ x_0(k) = x_i(k),\ \forall k$$
$$\gamma(x_0(k),\ x_i(k)) = 0,\ \text{iif}\ x_0 \in \varphi,\ x_i \in \varphi$$

where φ is an empty set.

(A2) (Duality Symmetric)

$$\gamma(x_0(k),\ x_i(k)) = \gamma(x_i(k),\ x_0(k)),\ \text{iff}$$
$$\mathbf{X} = \{x_0,\ x_i\}$$

(A3) (Wholeness)

$$\gamma(x_0(k),\ x_i(k)) \neq \gamma(x_i(k),\ x_0(k)),$$

almost always, iff

$$\mathbf{X} = \{x_j | j = 0,\ 1,\ \cdots,\ n,\ n > 2\}$$

(A4) (Aproachability)

$\gamma(x_0(k),\ x_i(k))$ decreases along with $\Delta(k)$ increasing, where

$$\Delta(k) = [(x_0(k) - x_i(k))^2]^{\frac{1}{2}}$$

Now, we have a following expression for $\gamma(x_0(k),\ x_i(k))$ which satisfies all of the mentioned axioms, that which

$$\gamma(x_0(k),\ x_i(k)) = \frac{\min\limits_{i}\min\limits_{k}|x_0(k) - x_i(k)| + \zeta \max\limits_{i}\max\limits_{k}|x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \zeta \max\limits_{i}\max\limits_{k}|x_0(k) - x_i(k)|}$$

where $\zeta \in (0,\ 1)$ is of the distinguished coefficient [53].

Remark. The norm interval $(0,1]$ which means $\forall \gamma(x_0(k),\ x_i(k)) \in (0,1]\ \forall k$ was first developed by Deng (1985)[18], while the norm interval $[a,\ b]$, $\gamma(x_0(k),\ x_i(k)) \in [a,\ b]$ where a, b are arbitrary real numbers was developed by Guo (1985)[53]. The concept of grey relational grade of interval was proposed by Wang (1985)[94].

A variety of successful applications for GRS have been exploited by many scientists in different areas, such as material science [83], biological analysis [84], seismological reseach [87], judical system [100], economic analysis [117], and geology [76].

Grey Generating Space

In the viewpoint of Grey System theory, the concept and generating techniques are important ideas by which the disorderly raw data can be turned to a regular series for the benefit of grey modelling, transferred to a dimensionless series in order to obtain an appropriate fundamental for grey analysing and changed into a unidirectional series for decision making.

Let x, y, ψ be the series and $x \in \mathbf{X}, y \in \mathbf{Y}, \psi \in \mathbf{\Phi}$, where $\mathbf{X}, \mathbf{Y}, \mathbf{\Phi}$ are collections of dimension n. If

$$x = \sum_{k=1}^{n} y(k)\psi_k$$
$$y(k) \in y \in \mathbf{Y},\ \psi_k \in \mathbf{\Phi},\ x \in \mathbf{X}$$

then $\mathbf{\Phi}$ is a basis space, ψ_k a basis of $\mathbf{\Phi}$, x an image of y and $\mathbf{X}$ an image space spanned by the basis ψ_k and the coordinates $\mathbf{Y}$.

Hence

$$(\ \mathbf{X};\ \mathbf{\Phi}\ /\ \mathbf{Y}\ )$$

is a generating space [40].

Suppose that

$$x^{(0)} = (x^{(0)}(1),\ \cdots,\ x^{(0)}(n))$$
$$x^{(1)} = (x^{(1)}(1),\ \cdots,\ x^{(1)}(n))$$
$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m)$$

and that

$$x^{(0)} \in \mathbf{Y},\ x^{(1)} \in \mathbf{X},\ \psi_k^{(1)} \in \mathbf{\Phi}$$
$$\psi_k^{(0)} = (0,\ \cdots,\ 0,\ \underset{\substack{k\text{th}\\ \text{column}}}{1},\ 0 \cdots 0)$$
$$\psi_k^{(1)} = (0,\ \cdots,\ 0,\ \underset{\substack{k\text{th}\\ \text{column}}}{1},\ \cdots\ 1)$$

$$\psi_k^{(1)}(m) \in \psi_k^{(1)}$$

$$\psi_k^{(1)}(m) = \sum_{\sigma=1}^{m} \psi_k^{(0)}(\sigma)$$

Then

$$x^{(1)} = \sum_{k=1}^{n} x^{(0)}(k)\psi_k^{(1)}$$

By AGO the above generating operation is denoted, i.e.

$$\text{AGO } x^{(0)} = x^{(1)}$$

$$x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m)$$

thus AGO $x^{(0)}$ is said to be an Accumulated Generating Operation which is vital to grey modelling. Similarly, the jth order AGO is as follows:

$$x^{(i)}(k) = \sum_{m=1}^{k} x^{(i-1)}(m), \quad i = 1, 2, \cdots$$

and that

$$x^{(i)} = \sum_{k=1}^{n} x^{(i-1)}(k)\psi_k^{(1)}$$

Moreover, some theorems concerning the jth order AGO of $x^{(0)}$ have been established [40].

Assuming that

$$\alpha^{(0)}(x^{(i)}(k)) = x^{(i)}(k)$$

$$\alpha^{(1)}(x^{(r)}(k)) = \alpha^{(0)}(x^{(r)}(k)) - \alpha^{(0)}(x^{(r)}(k-1))$$

$$\vdots$$

$$\alpha^{(i)}(x^{(r)}(k)) = \alpha^{(i-1)}(x^{(r)}(k)) - \alpha^{(i-1)}(x^{(r)}(k-1))$$

$$r = 1, 2, \cdots$$

then we have

$$\alpha^{(1)}(x^{(1)}) = \sum_{k=1}^{n} x^{(1)}(k)\psi_k^{(1)}$$

$$\psi_k^{(1)} = (0, \cdots 0, \underset{\substack{k\text{th} \\ \text{colum}}}{1}, -1, 0 \cdots 0)$$

$$\psi_k^{(1)} \in \Phi, \quad x^{(1)}(k) \in x^{(1)} \in \mathbf{Y}, \quad \alpha^{(1)}(x^{(1)}) \in \mathbf{X}$$

By IAGO the above-mentioned generating operation is denoted, such that

$$\text{IAGO } x^{(1)} = x^{(0)} = \alpha^{(1)}(x^{(1)})$$

Thus IAGO $x^{(1)}$ is said to be an Inverse Accumulated Generating Operation.

Summing up the results of grey generating operations, we have

1. (AGO): $x^{(1)}(k) = \sum_{m=1}^{k} x^{(0)}(m)$

2. (IAGO): $\alpha^{(i)}(x^{(j)}(k)) = \alpha^{(i-1)}(x^{(j)}(k)) - \alpha^{(i-1)}(x^{(j)}(k-1))$

3. (MEAN): $z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1)$

4. (P): $\bar{x} = (\bar{x}(1), \cdots, \bar{x}(n)), \quad x = (x(1), \cdots, x(n)),$

   $x(k) = \bar{x}(k) / x(p), \quad x(p) = \frac{1}{n}\sum_{m=1}^{n} \bar{x}(m)$

5. (INIT): $\bar{x} = (x(1), \cdots, \bar{x}(n)), \quad x = (x(1), \cdots, x(n)),$

   $x(k) = \bar{x}(k) / \bar{x}(1)$

By AGO, the following important results can be obtained; the non-negative, smooth, discrete function can be transferred into a series, extended according to an approximate exponential law [36,41] which is called the grey exponential law and by which a reform to establish a suitable foundation in building a differential model is said to be completed.

Grey Modeling

On the subject of differential modeling, T.C. Hxia follows that, for biological phenomena, economy and biological medicine, one had better build a differential model, but this has been impossible in the past. In Grey System theory a dynamic model with a group of differential equations is developed [15, 20, 42, 44], which is called grey differential model (GM). To do this Deng (1985)[20] inferred:

1. A stochastic process whose amplitudes vary with time is referred to as a grey process;
2. The grey modelling is based on the generating series rather than on the raw one;
3. The grey derivative, parallel shooting and grey differential equation are

defined and proposed in order to build a GM:

4. To build a GM model, only a few data (as few as 4) are needed to distinguish it.

Let $x^{(0)}$ be a raw series, $x^{(1)} = \text{AGO} x^{(0)}$, then the following equation

$$x^{(0)}(k) + az^{(1)}(k) = b, \quad k = 1, 2, \cdots, n, \cdots$$

is a grey differential model, called GM(1,1) as it includes only one variable $x^{(0)}$, where

$$z^{(1)}(k) = 0.5 x^{(1)}(k) + 0.5 x^{(1)}(k-1), \quad k = 1, 2, \cdots, n, \cdots$$

$a, b,$ are the coefficients; in Grey System theory terms, a is said to be a developing coefficient and b the grey input, $x^{(0)}(k)$ is a grey derivative which maximizes the information density for a given series to be modelled.

According to the least square method, we have

$$\hat{\mathbf{a}} = \begin{bmatrix} a \\ b \end{bmatrix} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{y}_N$$

there

$$\mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ \cdots & \cdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \qquad \mathbf{y}_N = \begin{bmatrix} x^{(0)}(2) \\ \cdots \\ x^{(0)}(n) \end{bmatrix}$$

Here $\mathbf{B}$ is called a data matrix.

By regarding the following equation

$$\frac{dx^{(1)}}{dt} + a \otimes (x^{(0)}) = b$$

as a shadow for $x^{(0)}(k) + az^{(1)}(k) = b$
We have

$$\mathcal{X}^{(1)} = \otimes(x^{(1)}) = \{ x^{(1)} | \forall x^{(1)} \in \mathcal{X}^{(1)} \}$$
$$= \left\{ \tilde{\otimes}(x^{(1)}) | \tilde{\otimes}(x^{(1)}) \longrightarrow \begin{array}{c} x^{(1)}(t + \Delta t) / \Delta t \\ x^{(1)}(t - \Delta t) / \Delta t \end{array} \right\}$$

where $\otimes(x^{(1)})$ is said to be a background grey number for

$$dx^{(1)} / dt, \quad \tilde{\otimes}(x^{(1)})$$

is the whitening value of grey number $\otimes(x^{(1)})$ and

$$\tilde{\otimes}(x^{(1)}) \xrightarrow{\qquad} \begin{array}{c} x^{(1)}(t + \Delta t) / \Delta t \\ x^{(1)}(t - \Delta t) / \Delta t \end{array}$$

means the parallel morphism or shooting from $\tilde{\otimes}(x^{(1)})$ to the components of $dx^{(1)} / dt$ [20, 35], then the response equation for GM(1,1) are as follows

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak} + \frac{b}{a}$$
$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k).$$

where $\hat{x}^{(1)}(k)$, $\hat{x}^{(0)}(k)$ means calculating values of $x^{(1)}$ and $x^{(0)}$ at point k.

A grey differential equation having N variables is called GM(1,N), whose expression can be written as follows:

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{i=2}^{N} b_i x_i^{(1)}(k), \quad k = 1, 2, \cdots$$

where b_i is said to be an ith influence coefficient, which means that x_i exercises influence on x_1 (the behaviour variable). Then we have

$$\hat{\mathbf{a}} = [a, b_2, \cdots, b_N]^T$$

By means of least square method, we have

$$\hat{\mathbf{a}} = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{y}_N$$

$$\mathbf{B} = \begin{bmatrix} -z_1^{(1)}(2) & x_2^{(1)}(2) & \cdots & x_N^{(1)}(2) \\ \cdots & \cdots & & \cdots \\ -z_1^{(1)}(n) & x_2^{(1)}(n) & \cdots & x_N^{(1)}(n) \end{bmatrix}, \quad \mathbf{y}_N = \begin{bmatrix} x_1^{(0)}(2) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix}$$

Model GM(1,1) plays an important role in grey forecasting, grey programming and grey control. Model GM(1,N) has laid an important foundation for regional economic programming and grey multivariable control [42, 102].

Grey Forecasting

The subjects of grey forecasting [21, 43] include:

     series forecasting
     calamities forecasting
     season calamities forecasting
     topological forecasting
     systematic forecasting

As all of these are based on GM(1,1), we call them grey forecasting, whose objective is to unify the field and to bridge the gap between grey process theory and practice. Our intention is to make forecasting useful for decision and policy makers who need future predictions.

Grey Series Forecasting

Grey series prediction is a common technique available for forecasting—which is referred to essential prediction—and direct use of GM(1,1) will enable us to know where and how the events to be forecasted would appear.

Let $x^{(0)}$ be a raw series, AGO $x^{(0)} = x^{(1)}$, $\hat{x}^{(0)}(\xi)$ the prediction value at point ξ, then the series prediction process can be written as

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: x^{(0)} \rightarrow \hat{x}^{(0)}(\xi)$$
$$x^{(0)} = (x^{(0)}(1), \cdots, x^{(0)}(n)), \quad \xi > n,$$

where IAGO and AGO are maps which transfer $\hat{x}^{(1)}$ to $\hat{x}^{(0)}$ and $x^{(0)}$ to $x^{(1)}$ respectively, and GM implies the grey modelling for GM(1,1) by $x^{(1)}$. For a series prediction, a variety of successful instances have been completed in different areas, such as grain yields prediction [62, 79, 114], economic prediction [108, 92, 102], material science [82], epidemiology [89], electrical power [75], traffic [67], technological progress [85], seismology [88], judicial systems [100], engineering [120] and agriculture [117, 118].

Calamities Prediction

While the amplitudes of events to be forecasted refer to an appointed interval called "grey number of pests", then the predictions to know when the event will occur belong to calamities prediction [21, 43].

Let $x^{(0)}$ be a raw series

$$x^{(0)} = (x^{(0)}(1), \cdots, x^{(0)}(n))$$

$\otimes_L, \otimes_u$ represent grey numbers of pests such that

$$\otimes_L = \{x^{(0)}(k) | x^{(0)}(k) \in x^{(0)}, x^{(0)}(k) \leqslant \zeta\}$$
$$\otimes_u = \{x^{(0)}(k) | x^{(0)}(k) \in x^{(0)}, x^{(0)}(k) \geqslant \zeta\}$$

where ζ is said to be a boundary value of pests and $\otimes_L$ is a grey number of low pests which means that the pest value is less than ζ; $\otimes_u$ is a grey number of upper pests in which the pest value is greater than ζ. If we have a subseries $x_\zeta^{(0)}$ of $x^{(0)}$ such that:

$$x_\zeta^{(0)} = (x^{(0)}(1_\zeta), \cdots, x^{(0)}(n_\zeta))$$
$$x^{(0)}(m_\zeta) \in \otimes_L \cap x^{(0)}, \quad m = 1, 2, \cdots n$$

or

$$x^{(0)}(m_\zeta) \in \otimes_u \cap x^{(0)}, \quad m = 1, 2, \cdots n$$

then

$$\otimes_L = \{x^{(0)}(m_\zeta) | x^{(0)}(m_\zeta) \in x^{(0)}, x^{(0)}(m_\zeta) \leqslant \zeta\}$$
$$\otimes_u = \{x^{(0)}(m_\zeta) | x^{(0)}(m_\zeta) \in x^{(0)}, x^{(0)}(m_\zeta) \geqslant \zeta\}$$

Suppose that p_ζ, p_h and p_g are maps such that

$$\mathbf{p}_\zeta: x^{(0)} \rightarrow x_\zeta^{(0)}$$
$$\mathbf{p}_h: x^{(0)}(k) \rightarrow k$$
$$\mathbf{p}_h: x^{(0)}(m_\zeta) \rightarrow m_\zeta$$
$$\mathbf{p}_g: k_\zeta \rightarrow k$$

then the calamity prediction can be written as follows

$$\mathbf{p}_g \cdot \mathbf{p}_h \cdot \mathbf{p}_\zeta: x^{(0)} \rightarrow (p_g(1_\zeta), p_g(2_\zeta), \cdots, p_g(n_\zeta))$$
$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: (p_g(1_\zeta), \cdots, p_g(n_\zeta)) \rightarrow (t_\xi)$$

where t_ξ is a time to be forecasted when pests appear and (t_ξ) is a series of t_ξ.

A variety of instances of pest predictions become effective in different areas, such as meteorology [115, 58] and agriculture [78].

Seasonal Calamity Prediction

The pests to be forecasted which happen in a certain span of time of a year are referred to as a seasonal calamity. It is obvious since an assignment to forecast the seasonal pests is similar to the above-stated prediction—namely, the calamity prediction—but their procedures are different [21, 43].

Let $h(k)$ be a pest time (date) located in a special span of a year; its scalar quantity refers to a relative value regarding the minimum value among the samples of pest date, and h is the series of $h(k)$

$$h = (h(1), \cdots, h(n))$$

h_k is the straight line given by

$$h_k = m_k t + b$$
$$h_k \in [\min(h(k-1), h(k)), \max(h(k-1), h(k))], \forall k$$
$$t \in [k-1, k]$$
$$m_k = h(k) - h(k-1)$$
$$b_k = kh(k-1) - (k-1)h(k)$$

where h_k is a straight line to link $h(k-1)$ with $h(k)$, then the collection

$$Y = \{h_k | k = 1, 2, \cdots\}$$

is a line graph called the seasonal pest graph.

Suppose d_i = const which means a fixed date of pest and assume that d_i cuts across Y at points 1, 2, $\cdots$ n and it is said to be a contour line in the pest graph. Let $p_i(k)$ be the kth span from point 1 to k, then

$$p_i = (p_i(1), p_i(2), \cdots, p_i(n_i), \quad d_i \Rightarrow p_i$$

is a series for modelling, iff p_i is non-empty, the potency of p_i is more than 4 and

$$\rho(\xi) = \frac{p_i(\xi)}{\sum\limits_{m=1}^{i-1} p_i(m)} \leqslant \varepsilon < 1, \quad \varepsilon \in [0,1]$$

The process of seasonal pest prediction thus can be written as

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO} \quad \mathbf{p}_i \Rightarrow \hat{\mathbf{p}}_i^{(0)}(\xi)$$
$$\xi > n_i$$

where $\hat{p}_i^{(0)}(\xi)$ is a year to be forecasted in which the seasonal pest would occur at date d_i.

Instances would be to forecast the date that the first frost would occur in Shanxi Province [91] and the date that the seasonal wind would appear in Hunan and Guangxi Provinces.

Topological Prediction

According to the above-mentioned procedures and techniques for seasonal pest prediction, forecasting the future form of a given curve is called topological prediction [21, 43].

Let us suppose that Y is a graph concerning a given curve such that

$$Y = \{h_k | k = 1, 2, \cdots\}$$

$\{d_i\}$ represents a subset of points belonging to a set of points which may be located at a straight line to the given or another curve. Thus:

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO} \quad \{p_i\} \Rightarrow \{\hat{p}_i^{(0)}(\xi)\}, \quad \{d_i\} \Rightarrow \{p_i\}$$

is a topological prediction model referred to $\{d_i\}$ and $\{\hat{p}_i^{(0)}(\xi)\}$ is a future curve to be forecasted. In order to analyse and to deal with a grey process whose curves are not smooth, the topological prediction techniques may be useful.

Systematic Prediction

To forecast a variety of different variables as a whole, referred to a given system, it is said to be a systematic prediction. If the assumption is made that the variables to be forecasted together are subordinated to, and modelled with, GM(1,1) and the results for every variable can be obtained recursively one by one, then such a forecasting is said to be a grey systematic prediction.

Let x_i ($i = 1, 2, \cdots n$) be the behaviour series referred to a given system, then the whole x_i can be forecasted by means of a grey systematic prediction, if the following relationships between variables x_i, $i = 1, 2, \cdots n$ hold,

$$\frac{dx_1^{(1)}}{dt} + a_{11} x_1^{(1)} = a_{12} x_2^{(1)} + \cdots + a_{1n} x_n^{(1)}$$

$$\cdots$$

$$\frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1} x_{n-1}^{(1)} = a_{n-1,n} x_n^{(1)}$$

$$\frac{dx_n^{(1)}}{dt} + a_{n,n} x_n^{(1)} = b$$

where $x_i^{(1)}$ $(i = 1, 2, \cdots n)$ are the background grey numbers concerned with derivatives

$$\frac{dx_i^{(1)}}{dt}, \quad i = 1, 2, \cdots n$$

Let us denote the procedure to model GM(1,1)

$$\frac{dx_n^{(1)}}{dt} + a_{nn} x_n^{(1)} = b$$

by GM_n and by gm_n the result or the model of it. As a function of gm_n, consider the $n-1$th grey differential equation, as follows:

$$\frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1} x_{n-1}^{(1)} = a_{n-1,n} x_n^{(1)}$$

$$\Rightarrow \frac{dx_{n-1}^{(1)}}{dt} + a_{n-1,n-1} x_{n-1}^{(1)} = a_{n-1,n} f(gm_n)$$

Thus we can briefly denote it with

$$GM_{n-1} = (gm_{n-1}, gm_n) = f(gm_{n-1}, gm_n)$$

where GM_{n-1} implies the precedure to model $n-1$th relationship and (gm_{n-1}, gm_n) is the result from GM_{n-1} which involves models gm_n and gm_{n-1}. Hence, the grey systematic prediction can be presented briefly with the following representations:

$$GM_n$$
$$GM_{n-1} = (gm_{n-1}, gm_n)$$
$$\cdots$$

$$GM_1 = (gm_1, gm_2, \cdots gm_n)$$

It is evident that to model a variety of variables together by the grey systematic prediction which is obtained directly from a variety of models, GM(1,1), the complex procedures which are necessary to solve differential equation set by common means, are to be avoided.

The effect and convenience of the grey systematic prediction has been illustrated by many instances; for example, to forecast grain yield, applying fertilizers, yield per mu and irrigated area together in Shanxi Province [79].

Grey Decision-Making

Grey decision-making is primarily concerned with the grey strategy of situation, grey group decision making and grey programming.

Grey Strategy of Situation

Grey strategy of situation deals with the strategy-making based on multi-objects which are contradictory in the ordinary way. The situation named by Grey System theory implies that the events to be attended to are coped over by games; then we have

$$s = (a, b) = (\text{event, game})$$

where s is of an appointed situation, a the event, b the game to be used to cope over the event.

The following are the essentials to make a grey strategy of the situation [22, 46]:

1. Exploiting point-set topology: the construction of situation set can be determined when an event is uncertain and games are unique. Let **A**, **B** be collections for events and games respectively, then we have

$$\mathbf{X} = \mathbf{A} \times \mathbf{B}, \ a_i \in \mathbf{A}, \ b_j \in \mathbf{B}$$

$$\varphi = \{s | s = \overset{2}{\underset{i=1}{\times}} Y_i \ \text{when} \ i \neq j, \ \text{then} \ \mathbf{Y}_i = \mathbf{A}, \ \mathbf{Y}_j = u_j \in \mathbf{O}_j,$$
$$j = 1, 2\}$$

where **O** is a system of open sets, φ is the sub-basis of x, x is the topology of situation.

2. To discover the effects of a given game dealing with a raised event, let us consider the map σ.

$$\sigma: \{s_{ij}\} \xrightarrow{\rho} \{u_{ij}^{(p)}\}, \ s_{ij} \in \mathbf{X}$$

where σ is an effect map from situation s_{ij} to its sample $u_{ij}^{(p)}$ describing the effect value concerning the pth object. By arranging the effect samples of all the objects, $p = 1, 2, \cdots m$, the following effect series is obtained

$$\mathbf{u}_{ij} = \{u_{ij}^{(1)}, \ u_{ij}^{(2)}, \ \cdots, \ u_{ij}^{(m)}\}$$

3. Consider a grey relational space $(\mathbf{X}, \mathbf{\Gamma})$ where $\mathbf{X}$ is a collection of u_{ij}, $\mathbf{\Gamma}$ is a map set in relational space.
   Assuming the grey target is

$$\mathbf{s}_e(\gamma) = \{\gamma(u_0, \ u_{ij}) | 1 \geqslant \gamma(u_0, \ u_{ij}) \geqslant \varepsilon\}$$

where u_0 is a reference sample series, which implies the kenel of target, then s_{ij} is a satisfactory situation and b_j the satisfctory game dealing with event a_i, iff

$$u_{ij} \in s_e(\gamma)$$
$$u_{ij} \Rightarrow s_{ij} \Rightarrow (a_i, \ b_j) \Rightarrow b_j$$

It is also important to make a satisfactory strategy by means of effect measure maps, which transfer the disconformity samples resulting from different objects into identical scalars.

A grey variety of problems have been solved using the grey strategy of situation, such as the irrigation strategy in Henan Province [26], a forestation strategy in Jinagsu Province and the agricultural economic division in Shanxi Province [93, 118].

Decision-Making of Group

A comprehensive treatment of the strategy having three levels, by means of grey relational space, grey statistics, grey clustering and grey prediction, is referred to as a decision-making of group [22, 28, 46].

Citing a case as a group strategy, let us consider an investment strategy whose items to be decided may be an exploited products—a category of investment, namely, the invested number and range which are to be marked by "much", "middle" and "less". The decision levels, namely, the collections of the decision and policy makers, are mass groups whose strategy is usually

determined solely by grey statistics; the expert group whose original strategy is obtained by grey prediction as usual and whose attention is paid only to the exploited products; the final group of the three levels, namely, the managers group, whose master strategy comes from official documentation and the quantitative strategy concerned is obtained by grey clustering in an ordinary way, taking only the category of investment into consideration.

A comprehensive strategy which gives consideration to the three levels can be obtained by unifying the decision series referred to the three groups, respectively, based on grey relational space.

A case in the above-mentioned point is one to harness a channel in Henan Province [26].

Grey Programming

To confirm traditional programming as a foundation, to incorporate grey prediction model in order to make a dynamic programming and to regard the coefficients as grey numbers with provision for adapting to the environment, are the essential ideas and attempts of grey programming [28, 46].

Let us denote the model of linear programming by

$$f(x) = \mathbf{c}^T x = \otimes$$
$$\mathbf{A}x \leqslant \hat{b}$$

where $\otimes$ is a grey target. While the vector $\mathbf{c}$ is referred to as a grey one whose elements are grey numbers denoted with $\otimes(c_i)$, $\mathbf{A}$ is a grey matrix denoted by

$$\mathbf{A} = (\otimes_{ij})$$
$$i = 1, 2, \cdots n$$
$$j = 1, 2, \cdots m$$

$\hat{b}(\xi)$ represents a boundary vector at time ξ which is obtained with GM(1,1) modelling can be written as follows:

$$\text{IAGO} \cdot \text{GM} \cdot \text{AGO}: \ b_k \to \hat{b}_k(\xi)$$

where b_k is a time series associated with the kth coefficient belonging to boundary vector b, and $\hat{b}_k(\xi)$ means the value of b_k to be forecasted at future time ξ, which constitutes $\hat{b}(\xi)$.

Then $f(x^*)$ means a satisfactory cost and x^* is a satisfactory resolution of it, iff $f(x^*)$ falls in a grey target which is given.

Grey linear programming has been brought to practical use, for example,

to adjust the structure of village economy [81] and to make a development plan coordinating the economy, society and science in Hubei Province.

Grey Prediction Control

The control principle of conventional control theory, whether classical or modern, is to control the system behaviour according to the state sample which has already occurred. It is then referred to as an afterward manner for control. The demerits of the afterward manner are as follows:

1. Impossible to avert accidents in advance;
2. Impossible to control timely;
3. Weakly adaptable.

The essential idea of grey prediction control [13, 29, 49, 50] is to control the system behaviour in advance with the control strategy obtained from the prediction controller [6], based on GM(1,1). Figure 1 illustrates the outline for a grey prediction control system. It maintains a desired state within reasonably accurate tolerances even though the output y is varied.
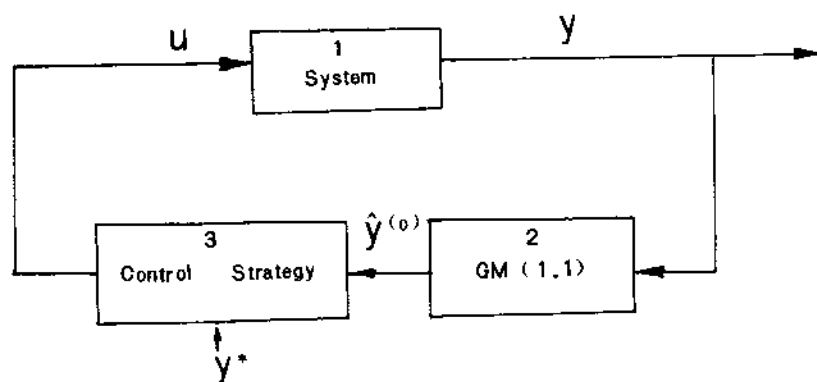

Fig. 1. Grey prediction control system.

In Fig. 1 the output of loop 1 (control object) is denoted as y, the prediction value of the systems behaviour is denoted as $\hat{y}^{(0)}$ which is obtained from controller 2, the assigned quantity is denoted as y^* and u is a control strategy coming from loop 3.

The control principle is as follows: a transducer feeds the sampled data to prediction controller 2, which functions as a computer. The prediction value $\hat{y}^{(0)}$ as a calculated result of controller 2 is delivered to loop 3, which constitutes a control strategy as compared with y^*. The control strategy u stems

from loop 3 as an input, activating loop 3, and then stored therein. Should the behaviour $y^{(0)}$ occur in the future, the stored strategy u will deal with it and lead the systems behaviour to a desired state.

A practical grey prediction controller has been initiated by [6].

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Properties of Multivariable Grey Model GM(1,N) *

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Abstract—This paper develops an exponential solution of the GM(1,N) and studies of its some properties.

Keywords: grey modelling, GM, multivariable grey model.

Introduction

A multivariable grey model denoted by the GM(1,N), which, developed by contributions [1, 2, 3, 4], includes N variables has been applied to a variety of fields for the purpose of modelling and analysing multifactor affairs and problems. But how to obtain conveniently a solution and how to improve the modelling accuracy to yield an appropriate result are still important objectives. Now, let us find an exponential solution for the GM(1,N).

Exponential Model

Definition 1. Let $x(t)$ be a continuous function, Δt the time gap between $x(t)$ and $x(t+\Delta t)$, and $d(x, \Delta t)$ a derivative of $x(t)$ which satisfies the following condition

$$d(x,\Delta t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

where $d(x,\Delta t) = \dfrac{dx}{dt}$