

```

1  Course: Statistical_Inference
2  Lesson: Asymptotics
3
4  - Class: text
5  Output: "Asymptotics. (Slides for this and other Data Science courses may be found at
github https://github.com/DataScienceSpecialization/courses/. If you care to use
them, they must be downloaded as a zip file and viewed locally. This lesson
corresponds to 07_Statistical_Inference/07_Asymptopia.)"
6
7  - Class: text
8  Output: In this lesson, we'll discuss asymptotics, a topic which describes how
statistics behave as sample sizes get very large and approach infinity. Pretending
sample sizes and populations are infinite is useful for making statistical inferences
and approximations since it often leads to a nice understanding of procedures.
9
10 - Class: text
11 Output: Asymptotics generally give no assurances about finite sample performance, but
they form the basis for frequency interpretation of probabilities (the long run
proportion of times an event occurs).
12
13 - Class: mult_question
14 Output: Recall our simulations and discussions of sample means in previous lessons.
We can now talk about the distribution of sample means of a collection of iid
observations. The mean of the sample mean estimates what?
15 AnswerChoices: population mean; population variance; standard error; sigma^2/n
16 CorrectAnswer: population mean
17 AnswerTests: omnitest(correctVal='population mean')
18 Hint: Which choice has the word 'mean' in it?
19
20 - Class: text
21 Output: The Law of Large Numbers (LLN) says that the average (mean) approaches what
it's estimating. We saw in our simulations that the larger the sample size the better
the estimation. As we flip a fair coin over and over, it eventually converges to the
true probability of a head (.5).
22
23 - Class: text
24 Output: The LLN forms the basis of frequency style thinking.
25
26 - Class: cmd_question
27 Output: To see this in action, we've copied some code from the slides and created the
function coinPlot. It takes an integer n which is the number of coin tosses that will
be simulated. As coinPlot does these coin flips it computes the cumulative sum
(assuming heads are 1 and tails 0), but after each toss it divides the cumulative sum
by the number of flips performed so far. It then plots this value for each of the
k=1...n tosses. Try it now for n=10.
28 CorrectAnswer: coinPlot(10)
29 AnswerTests: omnitest(correctExpr='coinPlot(10)')
30 Hint: Type coinPlot(10) at the command prompt.
31
32 - Class: cmd_question
33 Output: Your output depends on R's random number generator, but your plot probably
jumps around a bit and, by the 10th flip, your cumulative sum/10 is probably
different from mine. If you did this several times, your plots would vary quite a
bit. Now call coinPlot again, this time with 10000 as the argument.
34 CorrectAnswer: coinPlot(10000)
35 AnswerTests: omnitest(correctExpr='coinPlot(10000)')
36 Hint: Type coinPlot(10000) at the command prompt.
37
38 - Class: text
39 Output: See the difference? Asymptotics in Action! The line approaches its asymptote
of .5. This is the probability you expect since what we're plotting, the cumulative
sum/number of flips, represents the probability of the coin landing on heads. As we
know, this is .5 .
40
41 - Class: text
42 Output: We say that an estimator is CONSISTENT if it converges to what it's trying to
estimate. The LLN says that the sample mean of iid samples is consistent for the
population mean. This is good, right?
43

```

```

44 - Class: mult_question
45 Output: Based on our previous lesson do you think the sample variance (and hence
sample deviation) are consistent as well?
46 AnswerChoices: Yes; No
47 CorrectAnswer: Yes
48 AnswerTests: omnitest(correctVal='Yes')
49 Hint: Recall our simulations of sample variances and how, as we increased the sample
size, they converged to the population variance. Sounds like consistency, right?
50
51 - Class: text
52 Output: Now for something really important - the CENTRAL LIMIT THEOREM (CLT) - one of
the most important theorems in all of statistics. It states that the distribution of
averages of iid variables (properly normalized) becomes that of a standard normal as
the sample size increases.
53
54 - Class: text
55 Output: Let's dissect this to see what it means. First, 'properly normalized' means
that you transformed the sample mean  $\bar{X}$ . You subtracted the population mean  $\mu$  from
it and divided the difference by  $\sigma/\sqrt{n}$ . Here  $\sigma$  is the standard deviation
of the population and  $n$  is the sample size.
56
57 - Class: text
58 Output: Second, the CLT says that for large  $n$ , this normalized variable,
 $(\bar{X} - \mu)/(\sigma/\sqrt{n})$  is almost normally distributed with mean 0 and variance 1.
Remember that  $n$  must be large for the CLT to apply.
59
60 - Class: mult_question
61 Output: Do you recognize  $\sigma/\sqrt{n}$  from our lesson on variance? Since the
population std deviation  $\sigma$  is unknown,  $\sigma/\sqrt{n}$  is often approximated by
what?
62 AnswerChoices: the standard error of the sample mean; the variance of the population;
the mean of the population; I give up
63 CorrectAnswer: the standard error of the sample mean
64 AnswerTests: omnitest(correctVal='the standard error of the sample mean')
65 Hint: Recall our many simulation experiments in the variance lesson where we
calculated standard deviations of means using R's sd function, then we calculated an
approximation using a formula involving the population variance and the square root
of the sample size.
66
67 - Class: text
68 Output: Let's rephrase the CLT. Suppose  $X_1, X_2, \dots, X_n$  are independent,
identically distributed random variables from an infinite population with mean  $\mu$  and
variance  $\sigma^2$ . Then if  $n$  is large, the mean of the  $X$ 's, call it  $\bar{X}$ , is
approximately normal with mean  $\mu$  and variance  $\sigma^2/n$ . We denote this as
 $\bar{X} \sim N(\mu, \sigma^2/n)$ .
69
70 - Class: figure
71 Output: To show the CLT in action consider this figure from the slides. It presents 3
histograms of 1000 averages of dice rolls with sample sizes of 10, 20 and 30
respectively. Each average of  $n$  dice rolls ( $n=10, 20, 30$ ) has been normalized by
subtracting off the mean (3.5) then dividing by the standard error,  $\sqrt{2.92/n}$ . The
normalization has made each histogram look like a standard normal, i.e., mean 0 and
standard deviation 1.
72 Figure: cltDice.R
73 FigureType: new
74
75 - Class: text
76 Output: Notice that the CLT said nothing about the original population being normally
distributed. That's precisely where its usefulness lies! We can assume normality of a
sample mean no matter what kind of population we have, as long as our sample size is
large enough and our samples are independent. Let's look at how it works with a
binomial experiment like flipping a coin.
77
78 - Class: text
79 Output: Recall that if the probability of a head (call it 1) is  $p$ , then the
probability of a tail (0) is  $1-p$ . The expected value then is  $p$  and the variance is
 $p-p^2$  or  $p(1-p)$ . Suppose we do  $n$  coin flips and let  $p'$  represent the average of these
 $n$  flips. We normalize  $p'$  by subtracting the mean  $p$  and dividing by the std deviation
 $\sqrt{p(1-p)/n}$ . Let's do this for 1000 trials and plot the resulting histogram.

```

```

80
81 - Class: figure
82 Output: Here's a figure from the slides showing the results of 3 such trials where
each trial is for a different value of n (10, 20, and 30) and the coin is fair, so
 $E(X) = .5$  and the standard error is  $1/(2\sqrt{n})$ . Notice how with larger n (30) the
histogram tightens up around the mean 0.
83 Figure: cltFairCoin.R
84 FigureType: new
85
86 - Class: figure
87 Output: Here's another plot from the slides of the same experiment, this time using a
biased coin. We set the probability of a head to .9, so  $E(X) = .9$  and the standard
error is  $\sqrt{.09/n}$ . Again, the larger the sample size the more closely the
distribution looks normal, although with this biased coin the normal approximation
isn't as good as it was with the fair coin.
88 Figure: cltUnfairCoin.R
89 FigureType: new
90
91 - Class: text
92 Output: Now let's talk about confidence intervals.
93
94 - Class: figure
95 Output: We know from the CLT that for large n, the sample mean is normal with mean
mu and standard deviation  $\sigma/\sqrt{n}$ . We also know that 95% of the area under a
normal curve is within two standard deviations of the mean. This figure, a standard
normal with  $\mu=0$  and  $\sigma=1$ , illustrates this point; the entire shaded portion
depicts the area within 2 standard deviations of the mean and the darker portion
shows the 68% of the area within 1 standard deviation.
96 Figure: stddev1.R
97 FigureType: new
98
99 - Class: text
100 Output: It follows that 5% of the area under the curve is not shaded. By symmetry of
the curve, only 2.5% of the data is greater than the mean + 2 standard deviations
 $(\mu + 2\sigma/\sqrt{n})$  and only 2.5% is less than the mean - 2 standard deviations
 $(\mu - 2\sigma/\sqrt{n})$ .
101
102 - Class: text
103 Output: So the probability that the sample mean  $\bar{X}$  is bigger than  $\mu + 2\sigma/\sqrt{n}$ 
OR smaller than  $\mu - 2\sigma/\sqrt{n}$  is 5%. Equivalently, the probability of being
between these limits is 95%. Of course we could have different sizes of intervals. If
we wanted something other than 95, then we would use a quantile other than 2.
104
105 - Class: text
106 Output: The quantity  $\bar{X} \pm 2\sigma/\sqrt{n}$  is called a 95% interval for
mu. The 95% says that if one were to repeatedly get samples of size n, about 95% of
the intervals obtained would contain mu, the quantity we're trying to estimate.
107
108 - Class: mult_question
109 Output: Note that for a 95% confidence interval we divide  $(100-95)$  by 2 (since we
have both left and right tails) and add the result to 95 to compute the quantile we
need. The 97.5 quantile is actually 1.96, but for simplicity it's often just rounded
up to 2. If you wanted to find a 90% confidence interval what quantile would you want?
110 AnswerChoices: 90; 95; 85; 100
111 CorrectAnswer: 95
112 AnswerTests: omnitest(correctVal='95')
113 Hint: Divide  $(100-90)$  by 2 and add this result to 90.
114
115 - Class: cmd_question
116 Output: Use the R function qnorm to find the 95th quantile for a standard normal
distribution. Remember that qnorm takes a probability as an input. You can use
default values for all the other arguments.
117 CorrectAnswer: qnorm(.95)
118 AnswerTests: omnitest(correctExpr='qnorm(.95)')
119 Hint: Type qnorm(.95) at the command prompt.
120
121 - Class: mult_question
122 Output: As we've seen before, in a binomial distribution in which p represents the
probability or proportion of success, the variance  $\sigma^2$  is  $p(1-p)$ , so the standard

```

error of the sample mean p' is $\sqrt{p(1-p)/n}$ where n is the sample size. The 95% confidence interval of p is then $p' \pm 2\sqrt{p(1-p)/n}$. The 2 in this formula represents what?

AnswerChoices: "the mean of p' ; the variance of p' ; the standard error of p' ; the approximate 97.5% normal quantile"

CorrectAnswer: "the approximate 97.5% normal quantile"

AnswerTests: omnitest(correctVal='the approximate 97.5% normal quantile')

Hint: "Recall the formula for the interval $p' \pm qnorm \cdot \sigma / \sqrt{n}$ "

- **Class:** text

Output: A critical point here is that we don't know the true value of p ; that's what we're trying to estimate. How can we compute a confidence interval if we don't know $p(1-p)$? We could be conservative and try to maximize it so we get the largest possible confidence interval. Calculus tells us that $p(1-p)$ is maximized when $p=1/2$, so we get the biggest 95% confidence interval when we set $p=1/2$ in the formula $p' \pm 2\sqrt{p(1-p)/n}$.

- **Class:** mult_question

Output: Using $1/2$ for the value of p in the formula above yields what expression for the 95% confidence interval for p ?

AnswerChoices: $p' \pm 1/\sqrt{n}$; $p' \pm 1/(2\sqrt{n})$; $p' \pm 2\sqrt{n}$

CorrectAnswer: $p' \pm 1/\sqrt{n}$

AnswerTests: omnitest(correctVal='p\'' $\pm 1/\sqrt{n}$)

Hint: $p(1-p)=1/4$ when $p=1/2$ and the $\sqrt{1/4n}=1/(2\sqrt{n})$. What happens when you multiply this by 2?

- **Class:** mult_question

Output: Here's another example of applying this formula from the slides. Suppose you were running for office and your pollster polled 100 people. Of these 60 claimed they were going to vote for you. You'd like to estimate the true proportion of people who will vote for you and you want to be 95% confident of your estimate. We need to find the limits that will contain the true proportion of your supporters with 95% confidence, so we'll use the formula $p' \pm 1/\sqrt{n}$ to answer this question. First, what value would you use for p' , the sampled estimate?

AnswerChoices: .60; .56; 1.00; .10

CorrectAnswer: .60

AnswerTests: omnitest(correctVal='.60')

Hint: The only sampled number here is the number of people who said they would vote for you. Make it a proportion by dividing it by the sample size.

- **Class:** mult_question

Output: What would you use for $1/\sqrt{n}$?

AnswerChoices: $1/\sqrt{60}$; $1/\sqrt{56}$; $1/100$; $1/10$

CorrectAnswer: $1/10$

AnswerTests: omnitest(correctVal='1/10')

Hint: The sample size is n , and in this case $n=100$. What is $1/\sqrt{100}$?

- **Class:** mult_question

Output: The bounds of the interval then are what?

AnswerChoices: .5 and .7; .46 and .66; .55 and .65; I haven't a clue

CorrectAnswer: .5 and .7

AnswerTests: omnitest(correctVal='.5 and .7')

Hint: We know $p' - 1/\sqrt{n}$ is the lower bound and $p' + 1/\sqrt{n}$ is the upper bound, so use the answers from the two previous answers to fill in values for these variables.

- **Class:** mult_question

Output: How do you feel about the election?

AnswerChoices: confident; unsure; I'll pull out; Perseverance, that's the answer;

CorrectAnswer: confident

AnswerTests: omnitest(correctVal='confident')

Hint: With 95% confidence, between .5 and .7 of the voters support you. You look like a winner to me!

- **Class:** text

Output: Another technique for calculating confidence intervals for binomial distributions is to replace p with p' . This is called the Wald confidence interval. We can also use the R function `qnorm` to get a more precise quantile value (closer to 1.96) instead of our ballpark estimate of 2.

```

169 - Class: cmd_question
170 Output: With the formula  $p' \pm qnorm(.975) * \sqrt{p'(1-p')/100}$ , use the  $p'$  and  $n$ 
values from above and the R construct  $p' + c(-1,1) \dots$  to handle the plus/minus portion
of the formula. You should see bounds similar to the ones you just estimated.
171 CorrectAnswer: .6 + c(-1,1)*qnorm(.975)*sqrt(.6*.4/100)
172 AnswerTests: any_of_exprs('.6 + c(-1,1)*qnorm(.975)*sqrt(.6*.4/100)', '.6 +
c(-1,1)*qnorm(.975)*sqrt(.6*(1-0.6)/100)')
173 Hint: Type  $.6 + c(-1,1) * qnorm(.975) * \sqrt{.6 * .4 / 100}$  at the command prompt.
174
175 - Class: cmd_question
176 Output: As an alternative to this Wald interval, we can also use the R function
binom.test with the parameters 60 and 100 and let all the others default. This
function "performs an exact test of a simple null hypothesis about the probability of
success in a Bernoulli experiment." (This means it guarantees the coverages, uses a
lot of computation and doesn't rely on the CLT.) This function returns a lot of
information but all we want now are the values of the confidence interval that it
returns. Use the R construct  $x\$conf.int$  to find these now.
177 CorrectAnswer: binom.test(60,100)$conf.int
178 AnswerTests: omnitest(correctExpr='binom.test(60,100)$conf.int')
179 Hint: Type  $binom.test(60,100)\$conf.int$  at the command prompt.
180
181 - Class: text
182 Output: Close to what we've seen before, right? Now we're going to see that the Wald
interval isn't very accurate when  $n$  is small. We'll use the example from the slides.
183
184 - Class: figure
185 Output: Suppose we flip a coin a small number of times, say 20. Also suppose we have
a function mywald which takes a probability  $p$ , and generates 30 sets of 20 coin flips
using that probability  $p$ . It uses the sampled proportion of success,  $p'$ , for those 20
coin flips to compute the upper and lower bounds of the 95% Wald interval, that is,
it computes the two numbers  $p' \pm qnorm(.975) * \sqrt{p' * (1-p') / n}$  for each of the
30 trials. For the given true probability  $p$ , we count the number of times out of
those 30 trials that the true probability  $p$  was in the Wald confidence interval.
We'll call this the coverage.
186 Figure: WaldDemo.R
187 FigureType: new
188
189 - Class: cmd_question
190 Output: To make sure you understand what's going on, try running mywald now with the
probability .2. It will print out 30  $p'$  values (which you don't really need to see),
followed by 30 lower bounds, 30 upper bounds and lastly the percentage of times that
the input .2 was between the two bounds. See if you agree with the percentage you
get. Usually it suffices to just count the number of times (out of the 30) that .2 is
less than the upper bound.
191 CorrectAnswer: mywald(.2)
192 AnswerTests: omnitest(correctExpr='mywald(.2)')
193 Hint: Type  $mywald(.2)$  at the command prompt.
194
195 - Class: text
196 Output: Now that you understand the underlying principle, suppose instead of 30
trials, we used 1000 trials. Also suppose we did this experiment for a series of
probabilities, say from .1 to .9 taking steps of size .05. More specifically, we'll
call our function using 17 different probabilities, namely .1, .15, .2, .25, ... .9 .
We can then plot the percentages of coverage for each of the probabilities.
197
198 - Class: figure
199 Output: Here's the plot of our results. Each of the 17 vertices show the percentage
of coverage for a particular true probability  $p$  passed to the function. Results will
vary, but usually the only probability that hits close to or above the 95% line is
the  $p=.5$  . So this shows that when  $n$ , the number of flips, is small (20) the CLT
doesn't hold for many values of  $p$ , so the Wald interval doesn't work very well.
200 Figure: WaldFail.R
201 FigureType: new
202
203 - Class: figure
204 Output: Let's try the same experiment and increase  $n$ , the number of coin flips in
each of our 1000 trials, from 20 to 100 to see if the plot improves. Again, results
may vary, but all the probabilities are much closer to the 95% line, so the CLT works
better with a bigger value of  $n$ .

```

```

205 Figure: WaldPass.R
206 FigureType: new
207
208 - Class: text
209 Output: A quick fix to the problem of having a small  $n$  is to use the Agresti/Coull
interval. This simply means we add 2 successes and 2 failures to the counts when
calculating the proportion  $p'$ . Specifically, if  $X$  is the number of successes out of
the 20 coin flips, then instead of setting  $p'=X/20$ , let  $p'=(X+2)/24$ . We use 24 as
the number of trials since we've added 2 successes and 2 failures to the counts. Note
that we still use 20 in the calculation of the upper and lower bounds.

210
211 - Class: figure
212 Output: Here's a plot using this Agresti/Coull interval, with 1000 trials of 20 coin
flips each. It looks much better than both the original Wald with 20 coin flips and
the improved Wald with 100 coin flips. However, this technique might make the
confidence interval too wide.

213 Figure: ACDemo.R
214 FigureType: new
215
216 - Class: text
217 Output: Why does this work? Adding 2 successes and 2 failures pulls  $p'$  closer to .5
which, as we saw, is the value which maximizes the confidence interval.

218
219 - Class: figure
220 Output: To show this simply, we wrote a function ACCompar, which takes an integer
input  $n$ . For each  $k$  from 1 to  $n$  it computes two fractions,  $k/n$  and  $(k+2)/(n+4)$ . It
then prints out the boolean vector of whether the new  $(k+2)/(n+4)$  fraction is less
than the old  $k/n$ . It also prints out the total number of  $k$ 's for which the condition
is TRUE.

221 Figure: ACComp.R
222 FigureType: new
223
224 - Class: text
225 Output: For all  $k$  less than  $n/2$ , you see FALSE indicating that the new fraction is
greater than or equal to  $k/n$ . For all  $k$  greater than  $n/2$  you see TRUE indicating that
the new fraction is less than the old. If  $k=n/2$  the old and new fractions are equal.

226
227 - Class: cmd_question
228 Output: Try running ACCompar now with an input of 20.
229 CorrectAnswer: ACCompar(20)
230 AnswerTests: omnitest(correctExpr=' ACCompar(20)')
231 Hint: Type ACCompar(20) at the command prompt.
232
233 - Class: text
234 Output: Let's move on to Poisson distributions and confidence intervals. Recall that
Poisson distributions apply to counts or rates. For the latter, we write
 $X \sim \text{Poisson}(\lambda t)$  where  $\lambda$  is the expected count per unit of time and  $t$  is the
total monitoring time.

235
236 - Class: text
237 Output: Here's another example from the slides. Suppose a nuclear pump failed 5
times out of 94.32 days and we want a 95% confidence interval for the failure rate
per day. The number of failures  $X$  is Poisson distributed with parameter  $(\lambda t)$ .
We don't observe the failure rate, but we estimate it as  $x/t$ . Call our estimate
 $\lambda_{\text{hat}}$ , so  $\lambda_{\text{hat}}=x/t$ . According to theory, the variance of our estimated
failure rate is  $\lambda/t$ . Again, we don't observe  $\lambda$ , so we use our estimate of
it instead. We thus approximate the variance of  $\lambda_{\text{hat}}$  as  $\lambda_{\text{hat}}/t$ .

238
239 - Class: mult_question
240 Output: In this example what would you use as the estimated mean  $x/t$ ?
241 AnswerChoices: 5/94.32; 94.32/5; I haven't a clue
242 CorrectAnswer: 5/94.32
243 AnswerTests: omnitest(correctVal='5/94.32')
244 Hint: You need a number of failures divided by some measure of time.
245
246 - Class: cmd_question
247 Output: Set a variable lamb now with this value.
248 CorrectAnswer: lamb <- 5/94.32
249 AnswerTests: expr_creates_var('lamb'); omnitest(correctExpr='lamb <- 5/94.32')

```



```

250     Hint: Type lamb <- 5/94.32 at the R prompt.
251
252 - Class: cmd_question
253 Output: So lamb is our estimated mean and lamb/t is our estimated variance. The
    formula we've used to calculate a 95% confidence interval is est mean +
    c(-1,1)*qnorm(.975)*sqrt(est var). Use this formula now making the appropriate
    substitutions.
254 CorrectAnswer: lamb + c(-1,1)*qnorm(.975)*sqrt(lamb/94.32)
255 AnswerTests: omnittest(correctExpr='lamb + c(-1,1)*qnorm(.975)*sqrt(lamb/94.32)')
256 Hint: Type lamb + c(-1,1)*qnorm(.975)*sqrt(lamb/94.32) at the R prompt.
257
258 - Class: cmd_question
259 Output: As a check we can use R's function poisson.test with the arguments 5 and
    94.32 to check this result. This is an exact test so it guarantees coverage. As with
    the binomial exact test, we only need to look at the conf portion of the result using
    the x$conf construct. Do this now.
260 CorrectAnswer: poisson.test(5,94.32)$conf
261 AnswerTests: omnittest(correctExpr='poisson.test(5,94.32)$conf')
262 Hint: Type 'poisson.test(5,94.32)$conf' at the command prompt.
263
264 - Class: text
265 Output: Pretty close, right? Now to check the coverage of our estimate we'll run the
    same simulation experiment we ran before with binomial distributions. We'll vary our
    lambda values from .005 to .1 with steps of .01 (so we have 10 of them), and for each
    one we'll generate 1000 Poisson samples with mean lambda*t. We'll calculate sample
    means and use them to compute 95% confidence intervals. We'll then count how often
    out of the 1000 simulations the true mean (our lambda) was contained in the computed
    interval.
266
267 - Class: figure
268 Output: Here's a plot of the results. We see that the coverage improves as lambda
    gets larger, and it's quite off for small lambda values.
269 Figure: PoisDemo.R
270 FigureType: new
271
272 - Class: figure
273 Output: Now it's interesting to see how the coverage improves when we increase the
    unit of time. In the previous plot we used t=100 (rounding the 94.32 up). Here's a
    plot of the same experiment setting t=1000. We see that the coverage is much better
    for almost all the values of lambda, except for the smallest ones.
274 Figure: PoisDemoImpr.R
275 FigureType: new
276
277 - Class: text
278 Output: Now for a quick review!
279
280 - Class: mult_question
281 Output: What tells us that averages of iid samples converge to the population means
    that they are estimating?
282 AnswerChoices: the law of small numbers; the law of large numbers; the CLT; the BLT
283 CorrectAnswer: the law of large numbers
284 AnswerTests: omnittest(correctVal='the law of large numbers')
285 Hint: Think Big!
286
287 - Class: mult_question
288 Output: What tells us that averages are approximately normal for large enough sample
    sizes
289 AnswerChoices: the law of small numbers; the law of large numbers; the CLT; the BLT
290 CorrectAnswer: the CLT
291 AnswerTests: omnittest(correctVal='the CLT')
292 Hint: Keep yourself centered!
293
294 - Class: mult_question
295 Output: The Central Limit Theorem (CLT) tells us that averages have what kind of
    distributions?
296 AnswerChoices: normal; abnormal; binomial; Poisson
297 CorrectAnswer: normal
298 AnswerTests: omnittest(correctVal='normal')
299 Hint: Remember the previous question?

```

```

300
301 - Class: mult_question
302 Output: The Central Limit Theorem (CLT) tells us that averages have normal
distributions centered at what?
303 AnswerChoices: the population mean; the population variance; the standard error
304 CorrectAnswer: the population mean
305 AnswerTests: omnitest(correctVal='the population mean')
306 Hint: Remember the old  $E(X') = \mu$ , where  $X'$  is the sample mean and  $\mu$  is the population
mean. Know what I mean?
307
308 - Class: mult_question
309 Output: The Central Limit Theorem (CLT) tells us that averages have normal
distributions with standard deviations equal to what?
310 AnswerChoices: the population mean; the population variance; the standard error
311 CorrectAnswer: the standard error
312 AnswerTests: omnitest(correctVal='the standard error')
313 Hint: Which choice has the word standard in it?
314
315 - Class: mult_question
316 Output: True or False - The Central Limit Theorem (CLT) tells us that averages always
have normal distributions no matter how big the sample size
317 AnswerChoices: True; False
318 CorrectAnswer: False
319 AnswerTests: omnitest(correctVal='False')
320 Hint: Never trust statements with the words ALWAYS or NEVER in them. There are
ALWAYS exceptions to rules.
321
322 - Class: mult_question
323 Output: To calculate a confidence interval for a mean you take the sample mean and
add and subtract the relevant normal quantile times the what?
324 AnswerChoices: standard error; variance; variance/n; mean
325 CorrectAnswer: standard error
326 AnswerTests: omnitest(correctVal='standard error')
327 Hint: You want something like a standard deviation, right? Which choice has the word
standard in it?
328
329 - Class: mult_question
330 Output: For a 95% confidence interval approximately how many standard errors would
you add and subtract from the sample mean?
331 AnswerChoices: 2; 4; 6; 8
332 CorrectAnswer: 2
333 AnswerTests: omnitest(correctVal='2')
334 Hint: Anything above 3 is pretty far from the mean. Also, purists would prefer 1.96
for this.
335
336 - Class: mult_question
337 Output: If you wanted increased coverage what would you do to your confidence interval?
338 AnswerChoices: increase it; decrease it; keep it the same
339 CorrectAnswer: increase it
340 AnswerTests: omnitest(correctVal='increase it')
341 Hint: The key word here is increase. Bigger interval means bigger coverage.
342
343 - Class: mult_question
344 Output: If you had less variability in your data would your confidence interval get
bigger or smaller?
345 AnswerChoices: bigger; smaller
346 CorrectAnswer: smaller
347 AnswerTests: omnitest(correctVal='smaller')
348 Hint: Recall the size of the confidence interval positively depends on standard error
which is  $\sqrt{\text{var}/n}$ . If variance is smaller then so is variability and the interval.
349
350 - Class: mult_question
351 Output: If you had larger sample size would your confidence interval get bigger or
smaller?
352 AnswerChoices: bigger; smaller
353 CorrectAnswer: smaller
354 AnswerTests: omnitest(correctVal='smaller')
355 Hint: Recall the size of the confidence interval positively depends on standard error
which is  $\sqrt{\text{var}/n}$ . If the sample size,  $n$ , gets bigger the standard error gets

```


smaller and so does the interval.

```
356
357 - Class: mult_question
358 Output: A quick fix for small sample size binomial calculations is what?
359 AnswerChoices: add 2 successes and 2 failures; add 2 successes and 4 failures; add 2
    successes and subtract 2 failures; changing data seem dishonest
360 CorrectAnswer: add 2 successes and 2 failures
361 AnswerTests: omnitest(correctVal='add 2 successes and 2 failures')
362 Hint: Adding 2 successes and 2 failures brings the proportion of successes closer to
    1/2 which maximizes the confidence interval.
363
364 - Class: text
365 Output: Congrats! You've concluded this lesson on asymptotics. We hope you feel
    confident and are asymptomatic after going through it.
366
367 - Class: mult_question
368 Output: "Would you like to receive credit for completing this course on
369 Coursera.org?"
370 CorrectAnswer: NULL
371 AnswerChoices: Yes;No
372 AnswerTests: coursera_on_demand()
373 Hint: ""
374
```