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1   Course: Statistical_Inference
2   Lesson: T_Confidence_Intervals
3
4   - Class: text
5   Output: "T_Confidence_Intervals. (Slides for this and other Data Science courses may
6   be found at github https://github.com/DataScienceSpecialization/courses/. If you care
7   to use them, they must be downloaded as a zip file and viewed locally. This lesson
8   corresponds to 06_Statistical_Inference/08_tCIs.)"
9
10  - Class: text
11  Output: In this lesson, we'll discuss some statistical methods for dealing with small
12  datasets, specifically the Student's or Gosset's t distribution and t confidence
13  intervals.
14
15  - Class: mult_question
16  Output: In the Asymptotics lesson we discussed confidence intervals using the Central
17  Limit Theorem (CLT) and normal distributions. These needed large sample sizes, and
18  the formula for computing the confidence interval was  $\text{Est} \pm \text{qnorm} * \text{std error}(\text{Est})$ ,
19  where Est was some estimated value (such as a sample mean) with a standard error.
20  Here qnorm represented what?
21  AnswerChoices: the population mean; the population variance; the standard error; a
22  specified quantile from a normal distribution
23  CorrectAnswer: a specified quantile from a normal distribution
24  AnswerTests: omnitest(correctVal='a specified quantile from a normal distribution')
25  Hint: Which choice has part of the word 'qnorm' in it?
26
27  - Class: mult_question
28  Output: In the Asymptotics lesson we also mentioned the Z statistic
29   $Z = (X' - \mu) / (\sigma / \sqrt{n})$  which follows a standard normal distribution. This
30  normalized statistic Z is especially nice because we know its mean and variance. They
31  are what, respectively?
32  AnswerChoices: 0 and 1; 1 and 0; 0 and 0; 1 and 1
33  CorrectAnswer: 0 and 1
34  AnswerTests: omnitest(correctVal='0 and 1')
35  Hint: Recall the definition of standard normal. It's centered around 0 and it has a
36  standard deviation of 1 so its mean and variance are what?.
37
38  - Class: text
39  Output: So the mean and variance of the standardized normal are fixed and known. Now
40  we'll define the t statistic which looks a lot like the Z. It's defined as
41   $t = (X' - \mu) / (s / \sqrt{n})$ . Like the Z statistic, the t is centered around 0. The only
42  difference between the two is that the population std deviation, sigma, in Z is
43  replaced by the sample standard deviation in the t. So the distribution of the t
44  statistic is independent of the population mean and variance. Instead it depends on
45  the sample size n.
46
47  - Class: text
48  Output: As a result, for t distributions, the formula for computing a confidence
49  interval is similar to what we did in the last lesson. However, instead of a quantile
50  for a normal distribution we use a quantile for a t distribution. So the formula is
51   $\text{Est} \pm t\text{-quantile} * \text{std error}(\text{Est})$ . The other distinction, which we mentioned before,
52  is that we'll use the sample standard deviation when we estimate the standard error
53  of Est.
54
55  - Class: mult_question
56  Output: In the formula for the t statistic  $t = (X' - \mu) / (s / \sqrt{n})$  what expression
57  represents the sample standard deviation?
58  AnswerChoices: X'; mu; s; n
59  CorrectAnswer: s
60  AnswerTests: omnitest(correctVal='s')
61  Hint: X' and mu represent means, and n usually represents an integer like sample size.
62
63  - Class: text
64  Output: These t confidence intervals are very handy, and if you have a choice between
65  these and normal, pick these. We'll see that as datasets get larger, t-intervals look
66  normal. We'll cover the one- and two-group versions which depend on the data you have.
67
68  - Class: text
69  Output: The t distribution, invented by William Gosset in 1908, has thicker tails

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than the normal. Also, instead of having two parameters, mean and variance, as the normal does, the t distribution has only one - the number of degrees of freedom (df).

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42
43 - Class: text
44 Output: As df increases, the t distribution gets more like a standard normal, so it's
    centered around 0. Also, the t assumes that the underlying data are iid Gaussian so
    the statistic  $(X' - \mu)/(s/\sqrt{n})$  has n-1 degrees of freedom.
45
46 - Class: mult_question
47 Output: Quick check. In the formula  $t=(X' - \mu)/(s/\sqrt{n})$ , if we replaced s by
    sigma the statistic t would be what asymptotically?.
48 AnswerChoices: the standard normal; the standard abnormal; the population variance;
    Huh?
49 CorrectAnswer: the standard normal
50 AnswerTests: omnitest(correctVal='the standard normal')
51 Hint: With the replacement the formula should look familiar, like a standardized
    normal perhaps?
52
53 - Class: figure
54 Output: To see what we mean, we've taken code from the slides, the function myplot,
    which takes the integer df as its input and plots the t distribution with df degrees
    of freedom. It also plots a standard normal distribution so you can see how they
    relate to one another.
55 Figure: tPlot.R
56 FigureType: new
57
58 - Class: cmd_question
59 Output: Try myplot now with an input of 2.
60 CorrectAnswer: myplot(2)
61 AnswerTests: omnitest(correctExpr='myplot(2)')
62 Hint: Type myplot(2) at the command prompt.
63
64 - Class: cmd_question
65 Output: You can see that the hump of t distribution (in blue) is not as high as the
    normal's. Consequently, the two tails of the t distribution absorb the extra mass, so
    they're thicker than the normal's. Note that with 2 degrees of freedom, you only have
    3 data points. Ha! Talk about small sample sizes. Now try myplot with an input of 20.
66 CorrectAnswer: myplot(20)
67 AnswerTests: omnitest(correctExpr='myplot(20)')
68 Hint: Type myplot(20) at the command prompt.
69
70 - Class: text
71 Output: The two distributions are almost right on top of each other using this higher
    degree of freedom.
72
73 - Class: figure
74 Output: Another way to look at these distributions is to plot their quantiles. From
    the slides, we've provided a second function for you, myplot2, which does this. It
    plots a lightblue reference line representing normal quantiles and a black line for
    the t quantiles. Both plot the quantiles starting at the 50th percentile which is 0
    (since the distributions are symmetric about 0) and go to the 99th.
75 Figure: tQuant.R
76 FigureType: new
77
78 - Class: cmd_question
79 Output: Try myplot2 now with an argument of 2.
80 CorrectAnswer: myplot2(2)
81 AnswerTests: omnitest(correctExpr='myplot2(2)')
82 Hint: Type myplot2(2) at the command prompt.
83
84 - Class: text
85 Output: The distance between the two thick lines represents the difference in sizes
    between the quantiles and hence the two sets of intervals. Note the thin horizontal
    and vertical lines. These represent the .975 quantiles for the t and normal
    distributions respectively. Anyway, you probably recognized the placement of the
    vertical at 1.96 from the Asymptotics lesson.
86
87 - Class: cmd_question
88 Output: Check the placement of the horizontal now using the R function qt with the
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arguments .975 for the quantile and 2 for the degrees of freedom (df).
89 CorrectAnswer: qt(.975,2)
90 AnswerTests: omnitest(correctExpr='qt(.975,2)')
91 Hint: Type qt(.975,2) at the command prompt.
92
93 - Class: cmd_question
94 Output: See? It matches the horizontal line of the plot. Now run myplot2 with an
argument of 20.
95 CorrectAnswer: myplot2(20)
96 AnswerTests: omnitest(correctExpr='myplot2(20)')
97 Hint: Type myplot2(20) at the command prompt.
98
99 - Class: text
100 Output: The quantiles are much closer together with the higher degrees of freedom. At
the 97.5 percentile, though, the t quantile is still greater than the normal.
Student's Rules!
101
102 - Class: text
103 Output: This means the the t interval is always wider than the normal. This is
because estimating the standard deviation introduces more uncertainty so a wider
interval results.
104
105 - Class: text
106 Output: So the t-interval is defined as  $X' \pm t_{(n-1)} * s / \sqrt{n}$  where  $t_{(n-1)}$  is the
relevant quantile. The t interval assumes that the data are iid normal, though it is
robust to this assumption and works well whenever the distribution of the data is
roughly symmetric and mound shaped.
107
108 - Class: mult_question
109 Output: Our plots showed us that for large degrees of freedom, t quantiles become
close to what?
110 AnswerChoices: standard normal quantiles; standard abnormal quantiles; very large
numbers; very small numbers
111 CorrectAnswer: standard normal quantiles
112 AnswerTests: omnitest(correctVal='standard normal quantiles')
113 Hint: Recall that the larger the degrees of freedom, the more the t distribution
looked normal. Smaller degrees of freedom made it look abnormal.
114
115 - Class: text
116 Output: Although it's pretty great, the t interval isn't always applicable. For
skewed distributions, the spirit of the t interval assumptions (being centered around
0) are violated. There are ways of working around this problem (such as taking logs
or using a different summary like the median).
117
118 - Class: text
119 Output: For highly discrete data, like binary, intervals other than the t are
available.
120
121 - Class: text
122 Output: However, paired observations are often analyzed using the t interval by
taking differences between the observations. We'll show you what we mean now.
123
124 - Class: text
125 Output: We hope you're not tired because we're going to look at some sleep data. This
was the data originally analyzed in Gosset's Biometrika paper, which shows the
increase in hours for 10 patients on two soporific drugs.
126
127 - Class: cmd_question
128 Output: We've loaded the data for you. R treats it as two groups rather than paired.
To see what we mean type sleep now. This will show you how the data is stored.
129 CorrectAnswer: sleep
130 AnswerTests: omnitest(correctExpr='sleep')
131 Hint: Type sleep at the command prompt.
132
133 - Class: text
134 Output: We see 20 entries, the first 10 show the results (extra) of the first drug
(group 1) on each of the patients (ID), and the last 10 entries the results of the
second drug (group 2) on each patient (ID).
135

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136 - Class: figure
137 Output: Here we've plotted the data in a paired way, connecting each patient's two
      results with a line, group 1 results on the left and group 2 on the right. See that
      purple line with the steep slope? That's ID 9, with 0 result for group 1 and 4.6 for
      group 2.
138 Figure: sleepPlot.R
139 FigureType: new
140
141 - Class: text
142 Output: If we just looked at the 20 data points we'd be comparing group 1 variations
      with group 2 variations. Both groups have quite large ranges. However, when we look
      at the data paired for each patient, we see that the variations in results are
      usually much smaller and depend on the particular subject.
143
144 - Class: cmd_question
145 Output: To clarify, we've defined some variables for you, namely g1 and g2. These are
      two 10-long vectors, respectively holding the results of the 10 patients for each of
      the two drugs. Look at the range of g1 using the R command range.
146 CorrectAnswer: range(g1)
147 AnswerTests: omnitest(correctExpr='range(g1)')
148 Hint: Type range(g1) at the command prompt.
149
150 - Class: cmd_question
151 Output: So g1 values go from -1.6 to 3.7. Now look at the range of g2. We see that
      the ranges of both groups are relatively large.
152 CorrectAnswer: range(g2)
153 AnswerTests: omnitest(correctExpr='range(g2)')
154 Hint: Type range(g2) at the command prompt.
155
156 - Class: cmd_question
157 Output: Now let's look at the pairwise difference. We can take advantage of R's
      componentwise subtraction of vectors and create the vector of difference by
      subtracting g1 from g2. Do this now and put the result in the variable difference.
158 CorrectAnswer: difference <- g2-g1
159 AnswerTests: expr_creates_var("difference"); omnitest(correctExpr='difference <-
      g2-g1')
160 Hint: Type difference <- g2-g1 at the command prompt.
161
162 - Class: cmd_question
163 Output: Now use the R function mean to find the average of difference.
164 CorrectAnswer: mean(difference)
165 AnswerTests: omnitest(correctExpr='mean(difference)')
166 Hint: Type mean(difference) at the command prompt.
167
168 - Class: text
169 Output: See how much smaller the mean difference in this paired data is compared to
      the group variations?
170
171 - Class: cmd_question
172 Output: Now use the R function sd to find the standard deviation of difference and
      put the result in the variable s.
173 CorrectAnswer: s <- sd(difference)
174 AnswerTests: expr_creates_var("s"); omnitest(correctExpr='s <- sd(difference)')
175 Hint: Type s <- sd(difference) at the command prompt.
176
177 - Class: cmd_question
178 Output: Now recall the formula for finding the t confidence interval,  $\bar{X} \pm t_{(n-1)} * s / \sqrt{n}$ . Make the appropriate substitutions to find the 95% confidence
      intervals for the average difference you just computed. We've stored that average
      difference in the variable mn for you to use here. Remember to use the R construct
      c(-1,1) for the +/- portion of the formula and the R function qt with .975 and n-1
      degrees of freedom for the quantile portion. Our data size is 10.
179 CorrectAnswer: mn + c(-1,1)*qt(.975,9)*s/sqrt(10)
180 AnswerTests: omnitest(correctExpr='mn + c(-1,1)*qt(.975,9)*s/sqrt(10)')
181 Hint: Type mn + c(-1,1)*qt(.975,9)*s/sqrt(10) at the command prompt.
182
183 - Class: text
184 Output: This says that with probability .95 the average difference of effects
      (between the two drugs) for an individual patient is between .7 and 2.46 additional

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hours of sleep.
185
186 - Class: cmd_question
187 Output: We could also just have used the R function t.test with the argument
difference to get this result. (You can use the default values for all the other
arguments.) As with the other R test functions, this returns a lot of information.
Since all we're interested in at the moment is the confidence interval we can pick
this off with the construct x$conf.int. Try this now.
188 CorrectAnswer: t.test(difference)$conf.int
189 AnswerTests: omnitest(correctExpr='t.test(difference)$conf.int')
190 Hint: Type t.test(difference)$conf.int at the command prompt.
191
192 #- Class: video
193 # Output: As the slides showed, R provides several ways of using t.test to find the
confidence interval of this data. Would you like to see the R code to see 4
alternatives (including the two we just went through) and how to display them nicely?
You'll need an internet connection to see it.
194 # VideoLink: "http://wilcrofter.github.io/slides/markDown/ttest.html"
195
196 - Class: figure
197 Output: Here's code from the slides which shows four different ways of using t.test
(including the two we just went through) to find the confidence interval of this
data. The code also shows how to display the intervals nicely in a 4 x 2 array.
198 Figure: plot4Ttests.R
199 FigureType: new
200
201
202 - Class: text
203 Output: We now present methods, using t confidence intervals, for comparing
independent groups.
204
205 - Class: text
206 Output: Suppose that we want to compare the mean blood pressure between two groups in
a randomized trial. We'll compare those who received the treatment to those who
received a placebo. Unlike the sleep study, we cannot use the paired t test because
the groups are independent and may have different sample sizes.
207
208 - Class: text
209 Output: So our goal is to find a 95% confidence interval of the difference between
two population means. Let's represent this difference as  $\mu_y - \mu_x$ . How do we do
this? Recall our formula  $\bar{X} \pm t_{(n-1)}s/\sqrt{n}$ .
210
211 - Class: text
212 Output: First we need a sample mean, but we have two,  $\bar{X}$  and  $\bar{Y}$ , one from each group.
It makes sense that we'd have to take their difference ( $\bar{Y} - \bar{X}$ ) as well, since we're
looking for a confidence interval that contains the difference  $\mu_y - \mu_x$ . Now we need
to specify a t quantile. Suppose the groups have different sizes  $n_x$  and  $n_y$ .
213
214 - Class: mult_question
215 Output: For one group we used the quantile  $t_{(.975, n-1)}$ . What do you think we'll use
for the quantile of this problem?
216 AnswerChoices:  $t_{(.975, n_x-1)}$ ;  $t_{(.975, n_y-n_x-2)}$ ;  $t_{(.975, n_x+n_y-1)}$ ;
 $t_{(.975, n_x+n_y-2)}$ 
217 CorrectAnswer:  $t_{(.975, n_x+n_y-2)}$ 
218 AnswerTests: omnitest(correctVal='t_{(.975, n_x+n_y-2)}')
219 Hint: We lose one degree of freedom from each group because we've calculated the
sample mean from each group, so we add the two sizes and subtract two.
220
221 - Class: text
222 Output: The only term remaining is the standard error which for the single group is
 $s/\sqrt{n}$ . Let's deal with the numerator first. Our interval will assume (for now) a
common variance  $s^2$  across the two groups. We'll actually pool variance information
from the two groups using a weighted sum. (We'll deal with the more complicated
situation later.)
223
224 - Class: text
225 Output: We call the variance estimator we use the pooled variance. The formula for it
requires two variance estimators (in the form of the standard deviation),  $S_x$  and
 $S_y$ , one for each group. We multiply each by its respective degrees of freedom and

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divide the sum by the total number of degrees of freedom. This weights the respective variances; those coming from bigger samples get more weight.

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226
227 - Class: mult_question
228 Output: Which of the following represents the numerator of this expression?
229 AnswerChoices: (n_x-1) (S_x)^2+(n_y-1) (S_y)^2; (n_x) (S_x)^2+(n_y) (S_y)^2;
    (n_x) (S_x)+(n_y) (S_y)
230 CorrectAnswer: (n_x-1) (S_x)^2+(n_y-1) (S_y)^2
231 AnswerTests: omnitest(correctVal='(n_x-1) (S_x)^2+(n_y-1) (S_y)^2')
232 Hint: We need variances so the choice without the squared S terms is incorrect.
    Recall that the degrees of freedom is one less than the sample size for each group so
    that eliminates another choice and only one choice remains.

233
234
235 - Class: mult_question
236 Output: Which of the following represents the total number of degrees of freedom?
237 AnswerChoices: (n_x-1)+(n_y-1); (n_x+n_y); (n_x+n_y-1); (n_x+n_y+2)
238 CorrectAnswer: (n_x-1)+(n_y-1)
239 AnswerTests: omnitest(correctVal='(n_x-1)+(n_y-1)')
240 Hint: Recall that the degrees of freedom is one less than the sample size for each
    group. We asked this a few questions ago, though we've put this answer in a
    different, but equivalent form.

241
242 - Class: text
243 Output: Now recall we're calculating the standard error term which for the single
    group case was  $s/\sqrt{n}$ . We've got the numerator done, by pooling the sample
    variances. How do we handle the  $1/\sqrt{n}$  portion? We can simply add  $1/n_x$  and  $1/n_y$ 
    and take the square root of the sum. Then we MULTIPLY this by the sample variance to
    complete the estimate of the standard error.

244
245 - Class: text
246 Output: Now we'll plug in some numbers from the slides based on an example from
    Rosner's book Fundamentals of Biostatistics, a very good, if heavy, reference book.
    We want to compare blood pressure from two independent groups.

247
248 - Class: cmd_question
249 Output: The first is a group of 8 oral contraceptive users and the second is a group
    of 21 controls. The two means are  $X'_{oc}=132.86$  and  $X'_c=127.44$ , and the two
    sample standard deviations are  $s_{oc}=15.34$  and  $s_c=18.23$ . Let's first compute
    the numerator of the pooled sample variance by weighting the sum of the two by their
    respective sample sizes. Recall the formula  $(n_x-1) (S_x)^2+(n_y-1) (S_y)^2$  and fill in
    the values to create a variable sp.
250 CorrectAnswer: sp <- 7*15.34^2 + 20*18.23^2
251 AnswerTests: expr_creates_var('sp'); omnitest(correctExpr='sp <- 7*15.34^2 +
    20*18.23^2',correctVal=8293.8672)
252 Hint: Type sp <- 7*15.34^2 + 20*18.23^2 at the command prompt. Here 7 and 20 are each
    one less than the given sample sizes, and 15.34 and 18.23 are the respective standard
    deviations. We square these to convert them to variances.

253
254 - Class: cmd_question
255 Output: Now how many degrees of freedom are there? Put your answer in the variable ns.
256 CorrectAnswer: ns <- 8+21-2
257 AnswerTests: expr_creates_var('ns'); omnitest(correctExpr='ns <-
    8+21-2',correctVal=27)
258 Hint: Add the two sample sizes and subtract 2. Put the result in ns.
259
260 - Class: cmd_question
261 Output: Now divide sp by ns, take the square root and put the result back in sp.
262 CorrectAnswer: sp <- sqrt(sp/ns)
263 AnswerTests: expr_creates_var('sp'); omnitest(correctExpr='sp <- sqrt(sp/ns)')
264 Hint: Type sp <- sqrt(sp/ns) at the command prompt.
265
266 - Class: cmd_question
267 Output: Now to find the 95% confidence interval. Recall our basic formula  $X' \pm$ 
 $t_{(n-1)} * s/\sqrt{n}$  and all the changes we need to make for working with two
independent samples. We'll plug in the difference of the sample means for  $X'$  and our
variable ns for the degrees of freedom when finding the t quantile. For the standard
error, we multiply sp by the square root of the sum  $1/n_{oc} + 1/n_c$ . The values
for this problem are  $X'_{oc}=132.86$  and  $X'_c=127.44$ ,  $n_{oc}=8$  and  $n_c=21$ . Be sure
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to use the R construct c(-1,1) for the +/- portion and the R function qt with the
correct percentile and degrees of freedom.
268 CorrectAnswer: 132.86-127.44+c(-1,1)*qt(.975,ns)*sp*sqrt(1/8+1/21)
269 AnswerTests:
omnitest(correctExpr='132.86-127.44+c(-1,1)*qt(.975,ns)*sp*sqrt(1/8+1/21)')
270 Hint: Type 132.86-127.44+c(-1,1)*qt(.975,ns)*sp*sqrt(1/8+1/21) at the command prompt.
271
272 - Class: text
273 Output: Notice that 0 is contained in this 95% interval. That means that you can't
rule out that the means of the two groups are equal since a difference of 0 is in the
interval.
274
275 - Class: text
276 Output: Getting tired? Let's revisit the sleep problem and instead of looking at the
data as paired over 10 subjects we'll look at it as two independent sets each of size
10. Recall the data is stored in the two vectors g1 and g2; we've also stored the
difference between their means in the variable md.
277
278 - Class: cmd_question
279 Output: Let's compute the sample pooled variance and store it in the variable sp.
Recall that this is the sqrt(weighted sums of sample variances/deg of freedom). The
weight of each is the sample size-1. Use the R function var to compute the variances
of g1 and g2. The degrees of freedom is 10+10-2 = 18.
280 CorrectAnswer: sp <- sqrt((9*var(g1)+9*var(g2))/18)
281 AnswerTests: expr_creates_var('sp'); omnitest(correctExpr='sp <-
sqrt((9*var(g1)+9*var(g2))/18)')
282 Hint: Type sp <- sqrt((9*var(g1)+9*var(g2))/18) at the command prompt.
283
284 - Class: cmd_question
285 Output: Now the last term of the formula, the standard error of the mean difference,
is simply sp times the square root of the sum 1/10 + 1/10. Find the 95% t confidence
interval of the mean difference of the two groups g1 and g2. Substitute md and sp
into the formula you used above.
286 CorrectAnswer: md + c(-1,1)*qt(.975,18)*sp*sqrt(1/5)
287 AnswerTests: ANY_of_exprs('md + c(-1,1)*qt(.975,18)*sp*sqrt(1/5)', 'md +
c(-1,1)*qt(.975,18)*sp*sqrt(1/10 + 1/10)')
288 Hint: Type md + c(-1,1)*qt(.975,18)*sp*sqrt(1/5) at the command prompt.
289
290 - Class: cmd_question
291 Output: We can check this manual calculation against the R function t.test. Since we
subtracted g1 from g2, be sure to place g2 as your first argument and g1 as your
second. Also make sure the argument paired is FALSE and var.equal is TRUE. We only
need the confidence interval so use the construct x$conf. Do this now.
292 CorrectAnswer: t.test(g2,g1,paired=FALSE,var.equal=TRUE)$conf
293 AnswerTests: omnitest(correctExpr='t.test(g2,g1,paired=FALSE,var.equal=TRUE)$conf')
294 Hint: Type t.test(g2,g1,paired=FALSE,var.equal=TRUE)$conf at the command prompt.
295
296 - Class: cmd_question
297 Output: Pretty cool that it matches, right? Note that 0 is again in this 95% interval
so you can't reject the claim that the two groups are the same. (Recall that this is
the opposite of what we saw with paired data.) Let's run t.test again, this time with
paired=TRUE and see how different the result is. Don't specify var.equal and look
only at the confidence interval.
298 CorrectAnswer: t.test(g2,g1,paired=TRUE)$conf
299 AnswerTests: omnitest(correctExpr='t.test(g2,g1,paired=TRUE)$conf')
300 Hint: Type t.test(g2,g1,paired=TRUE)$conf at the command prompt.
301
302 - Class: text
303 Output: Just as we saw when we ran t.test on our vector, difference! See how the
interval excludes 0? This means the groups when paired have much different averages.
304
305 - Class: text
306 Output: Now let's talk about calculating confidence intervals for two groups which
have unequal variances. We won't be pooling them as we did before.
307
308 - Class: text
309 Output: In this case the formula for the interval is similar to what we saw before,
Y'-X' +/- t_df * SE, where as before Y'-X' represents the difference of the sample
means. However, the standard error SE and the quantile t_df are calculated

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differently from previous methods. Here SE is the square root of the sum of the squared standard errors of the two means,  $(s_1)^2/n_1 + (s_2)^2/n_2$ .

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310
311 - Class: text
312 Output: When the underlying X and Y data are iid normal and the variances are
different, the normalized statistic we started this lesson with,  $(X'-\mu)/(s/\sqrt{n})$ ,
doesn't follow a t distribution. However, it can be approximated by a t distribution
if we set the degrees of freedom appropriately.
313
314 - Class: text
315 Output: The formula for the degrees of freedom is a complicated fraction that no one
remembers. The numerator is the SQUARE of the sum of the squared standard errors of
the two sample means. Each has the form  $s^2/n$ . The denominator is the sum of two
terms, one for each group. Each term has the same form. It is the standard error of
the mean raised to the fourth power divided by the sample size-1. More precisely,
each term looks like  $(s^4/n^2)/(n-1)$ . We use this df to find the t quantile.
316
317 #- Class: video
318 # Output: Would you like to see this formula nicely displayed? You'll need an internet
connection to do this.
319 # VideoLink: "http://wilcrofter.github.io/slindex/markDown/diffVar.html"
320
321 - Class: figure
322 Output: Here's the formula. You might have to stretch the plot window to get it
displayed more clearly.
323 Figure: plotdiffVar.R
324 FigureType: new
325
326 - Class: text
327 Output: Let's plug in the numbers from the blood pressure study to see how this
works. Recall we have two groups, the first with size 8 and  $X'_{oc}=132.86$  and
 $s_{oc}=15.34$  and the second with size 21 and  $X'_{c}=127.44$  and  $s_{c}=18.23$ .
328
329 - Class: cmd_question
330 Output: Let's compute the degrees of freedom first. Start with the numerator. It's
the square of the sum of two terms. Each term is of the form  $s^2/n$ . Do this now and
put the result in num. Our numbers were 15.34 with size 8 and 18.23 with size 21.
331 CorrectAnswer: num <- (15.34^2/8 + 18.23^2/21)^2
332 AnswerTests: expr_creates_var('num'); omnitest(correctExpr='num <- (15.34^2/8 +
18.23^2/21)^2',correctVal=2046.6418737445)
333 Hint: Type num <- (15.34^2/8 + 18.23^2/21)^2 at the command prompt.
334
335 - Class: cmd_question
336 Output: Now the denominator. This is the sum of two terms. Each term has the form
 $s^4/n^2/(n-1)$ . These look a little different than the form displayed but they're
equivalent. Put the result in the variable den. Our numbers were 15.34 with size 8
and 18.23 with size 21.
337 CorrectAnswer: den <- 15.34^4/8^2/7 + 18.23^4/21^2/20
338 AnswerTests: expr_creates_var('den'); omnitest(correctExpr='den <- 15.34^4/8^2/7 +
18.23^4/21^2/20',correctVal=136.123536407433)
339 Hint: Type den <- 15.34^4/8^2/7 + 18.23^4/21^2/20 at the command prompt.
340
341 - Class: cmd_question
342 Output: Now divide num by den and put the result in mydf.
343 CorrectAnswer: mydf <- num/den
344 AnswerTests: expr_creates_var('mydf'); omnitest(correctExpr='mydf <- num/den')
345 Hint: Type mydf <- num/den at the command prompt.
346
347 - Class: cmd_question
348 Output: Now with the R function qt(.975,mydf) compute the 95% t interval. Recall the
formula.  $X'_{oc}-X'_{c} +/- t_{df} * SE$ . Recall that SE is the square root of the sum
of the squared standard errors of the two means,  $(s_1)^2/n_1 + (s_2)^2/n_2$ . Again
our numbers are the following.  $X'_{oc}=132.86$   $s_{oc}=15.34$  and  $n_{oc}=8$ .
 $X'_{c}=127.44$   $s_{c}=18.23$  and  $n_{c}=21$ .
349 CorrectAnswer: 132.86-127.44 +c(-1,1)*qt(.975,mydf)*sqrt(15.34^2/8 + 18.23^2/21)
350 AnswerTests: omnitest(correctExpr='132.86-127.44
+c(-1,1)*qt(.975,mydf)*sqrt(15.34^2/8 + 18.23^2/21)')
351 Hint: Type 132.86-127.44 +c(-1,1)*qt(.975,mydf)*sqrt(15.34^2/8 + 18.23^2/21) at the
command prompt.
```



```
352
353 - Class: text
354 Output: Don't worry about these nasty calculations. R makes things a lot easier. If
you call t.test with var.equal set to FALSE, then R calculates the degrees of
freedom for you. You don't have to memorize the formula.
355
356
357 - Class: text
358 Output: Congrats! You've concluded this rather t-dious lesson on all things t related
- statistics, distributions, intervals. Hope you're not too teed off!
359
360 - Class: mult_question
361 Output: "Would you like to receive credit for completing this course on
362 Coursera.org?"
363 CorrectAnswer: NULL
364 AnswerChoices: Yes;No
365 AnswerTests: coursera_on_demand()
366 Hint: ""
367
```