Modelling and Numerical Methods

Lecture 2

Stress and Tensors

Kinematics

Outline

Part 1: Stress and tensors *chapter2.ipynb*

- Cauchy stress tensor
- (Stress) tensor symmetry
- Coordinate transformation (stress) tensors
- Shear and normal stresses
- Tensor invariants

Part 2: Kinematics *chapter3.ipynb*

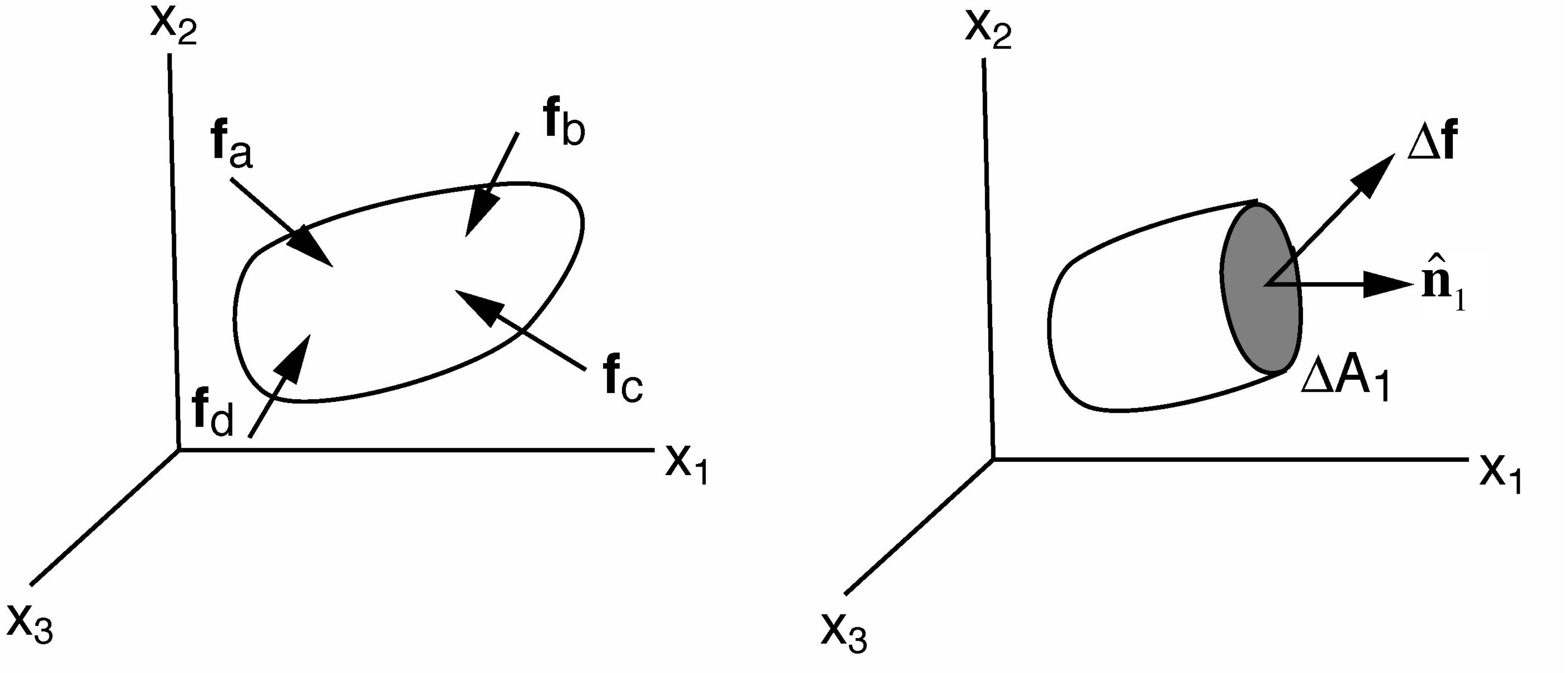
- Material and spatial description of variables

Learning Objectives
Stress and tensors

- Understand meaning of different components of 3D Cauchy stress tensor
- Know how to determine state of stress on given plane
- Be able to decompose a rank 2 tensor into symmetric and anti-symmetric components
- Be able to transform rank 2 tensor to a new basis.
- Be able to determine shear and normal stresses on a plane

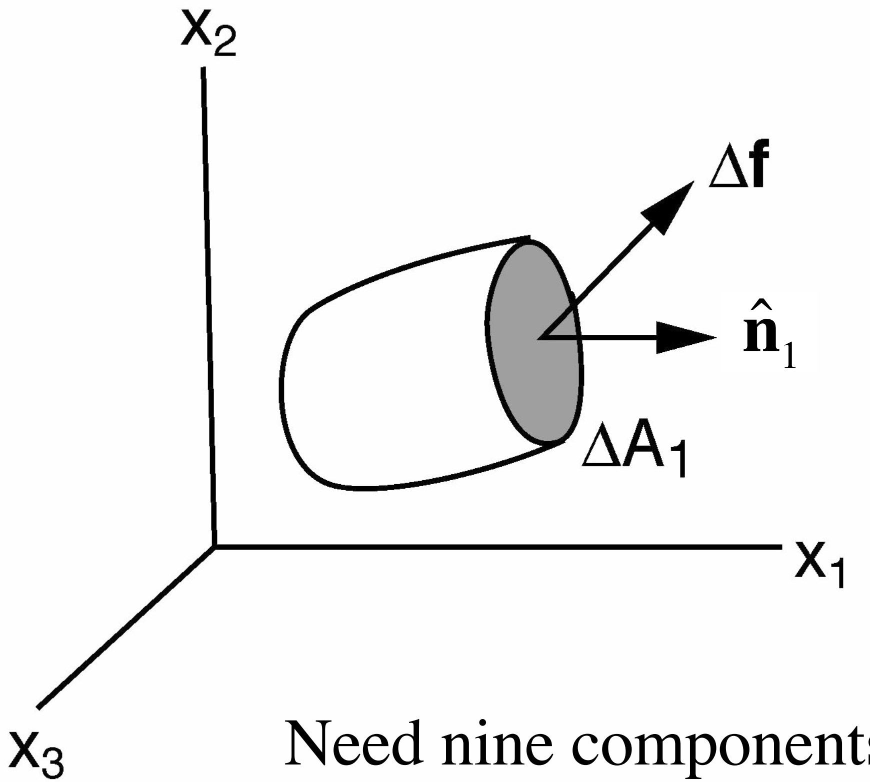
Cauchy Stress

Stress in a point, measured in medium as deformed by the stress experienced.



forces introduce a state of stress in a body

(Other stress measures, e.g., Piola-Kirchhoff tensor, used in Lagrangian formulations)



traction, stress vector

$$\mathbf{t_1} = \mathbf{t}(\hat{\mathbf{n}}_1) = \lim_{\Delta A \to 0} \Delta\mathbf{f}/\Delta A_1$$

$$\mathbf{t_1} = (\sigma_{11}, \sigma_{12}, \sigma_{13})$$

Need nine components to fully describe the stress

$\sigma_{11}, \sigma_{12}, \sigma_{13}$ for ΔA_1
$\sigma_{22}, \sigma_{21}, \sigma_{23}$ for ΔA_2
$\sigma_{33}, \sigma_{31}, \sigma_{32}$ for ΔA_3

first index = orientation of plane
second index = orientation of force

Are nine components sufficient?

Plane area as a vector

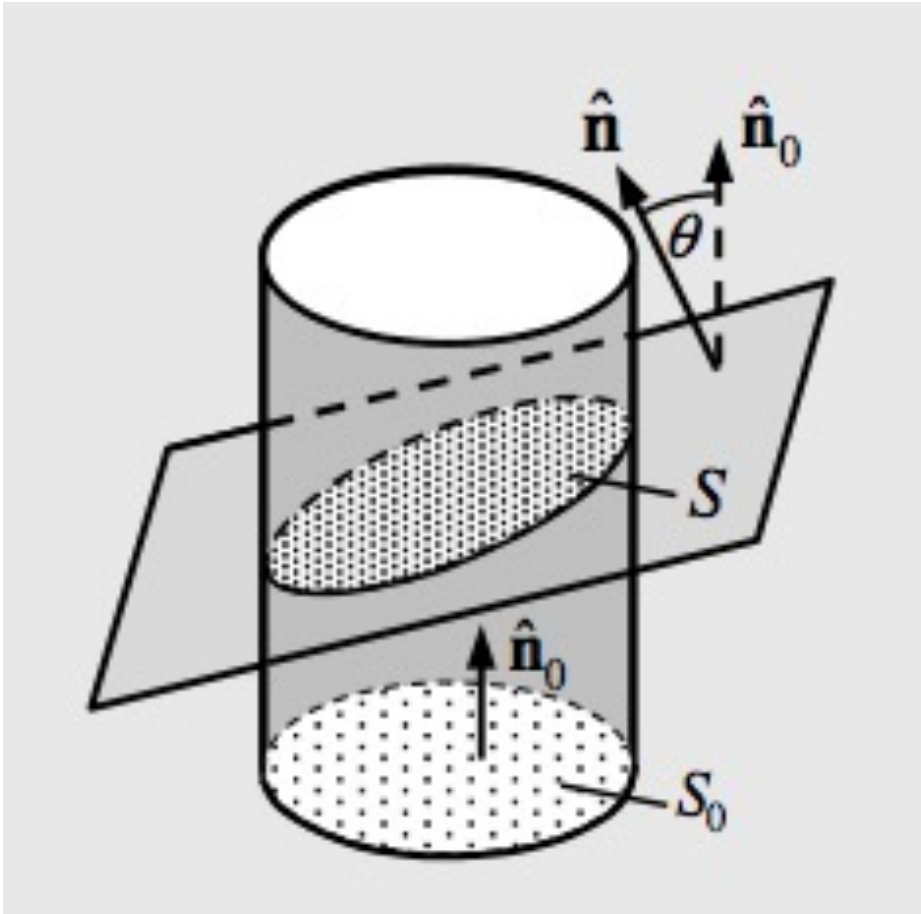
The area of plane S can be defined in terms of vectors assuming S_0 and θ are known.

$$\mathbf{S} = S\hat{\mathbf{n}}$$

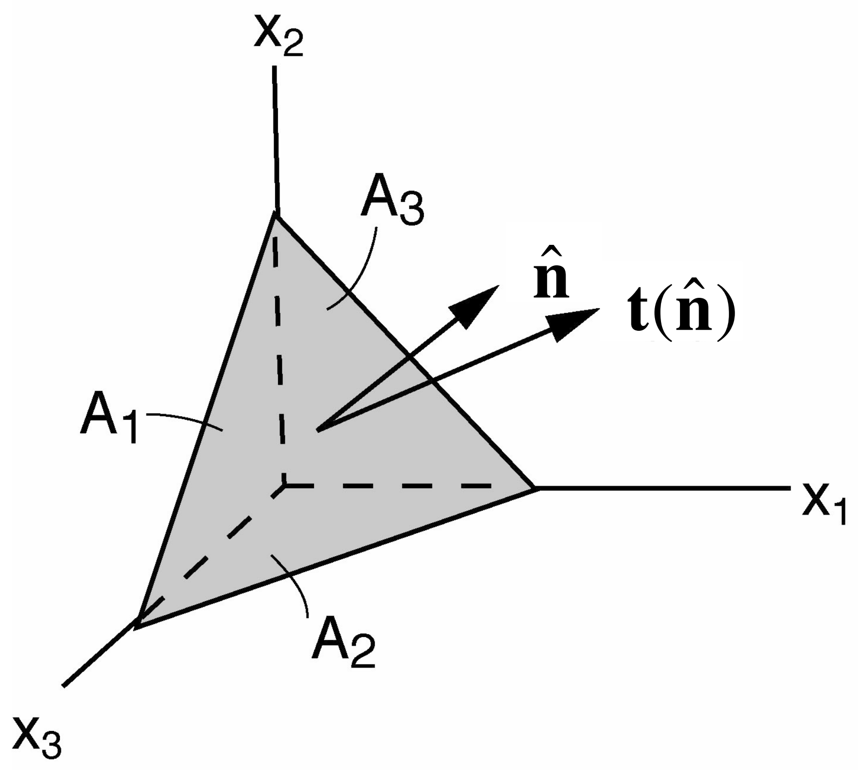
$$S_0 = \mathbf{S} \cdot \hat{\mathbf{n}}_0 =$$

$$S\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_0 = S\cos\theta$$

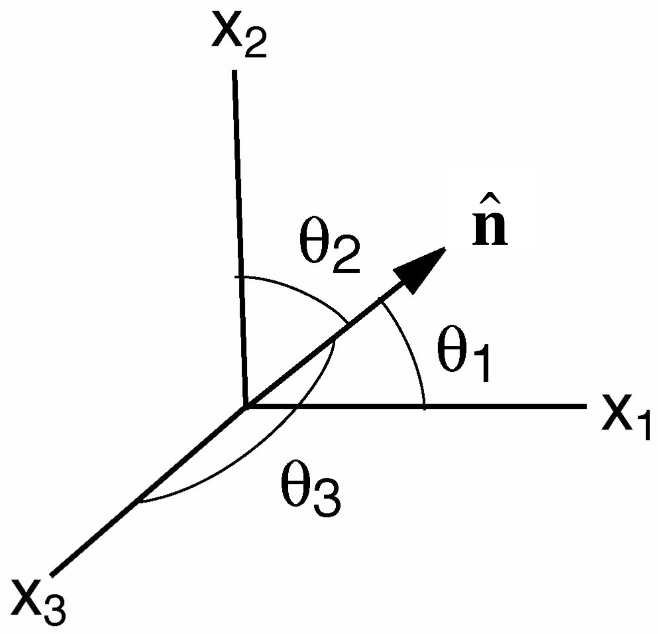
$$\Rightarrow S = S_0 / \cos\theta$$



Are nine components sufficient?
Demonstrate with equilibrium for a tetrahedron



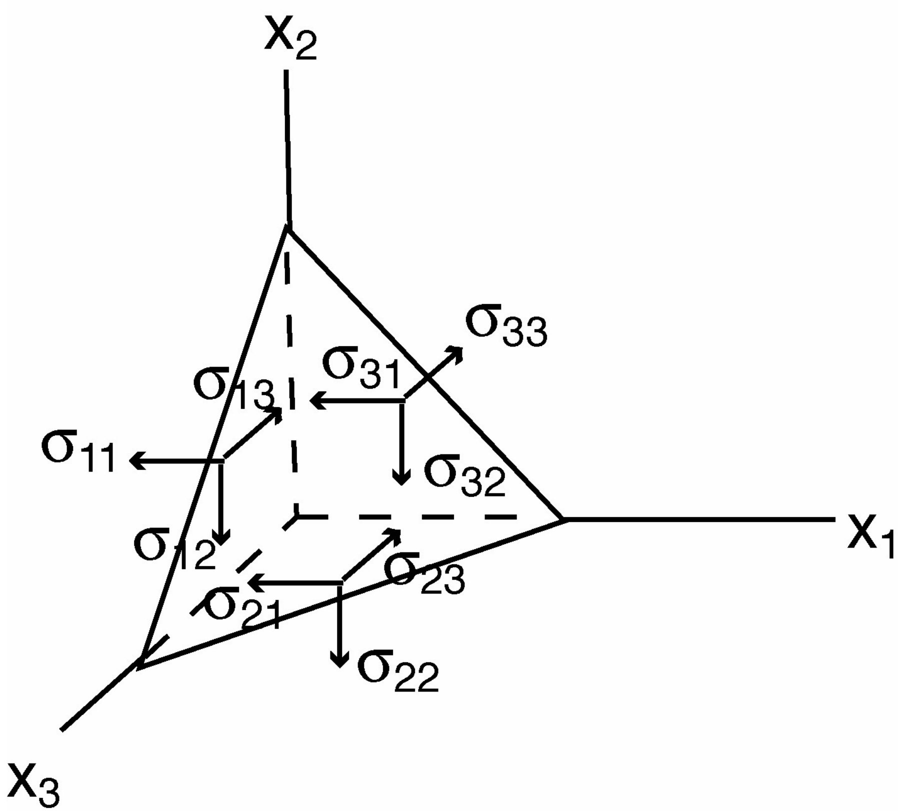
Given: stress on A_1, A_2, A_3
Find: $\mathbf{t}(\hat{\mathbf{n}})$



1: $\hat{\mathbf{n}} = -\hat{\mathbf{x}}_1$, $\Delta A_1 = \Delta A\cos\theta_1$
2: $\hat{\mathbf{n}} = -\hat{\mathbf{x}}_2$, $\Delta A_2 = \Delta A\cos\theta_2$
3: $\hat{\mathbf{n}} = -\hat{\mathbf{x}}_3$, $\Delta A_3 = \Delta A\cos\theta_3$
4: $\hat{\mathbf{n}} = (n_1, n_2, n_3)$, $n_i = \cos\theta_i$, $\Delta A_4 = \Delta A$

$$\Sigma f_1 = t_1\Delta A - \sigma_{11}\Delta A\cos\theta_1 - \sigma_{21}\Delta A\cos\theta_2 - \sigma_{31}\Delta A\cos\theta_3 = 0$$

$$\Sigma f_1 = t_1\Delta A - \sigma_{11}\Delta A\cos\theta_1 - \sigma_{21}\Delta A\cos\theta_2 - \sigma_{31}\Delta A\cos\theta_3 = 0$$



this gives:

$$t_1 = \sigma_{11}n_1 + \sigma_{21}n_2 + \sigma_{31}n_3$$

similarly:

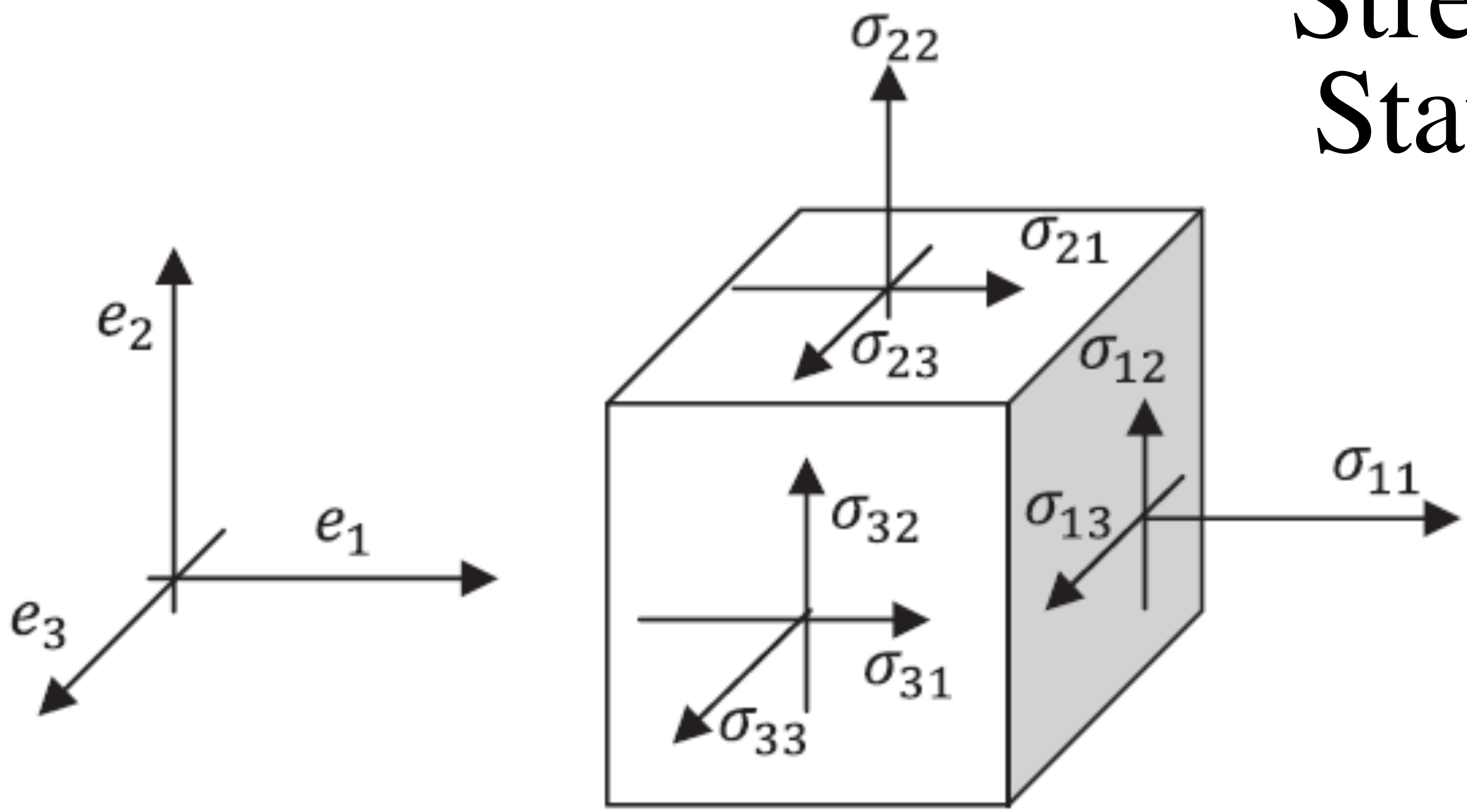
$$t_2 = \sigma_{12}n_1 + \sigma_{22}n_2 + \sigma_{32}n_3$$

$$t_3 = \sigma_{13}n_1 + \sigma_{23}n_2 + \sigma_{33}n_3$$

$$\Rightarrow \boxed{t_i = \sigma_{ji}n_j}$$ (Einstein convention)

How many stress components required in 2D?

3-D Stress State



first index = orientation of plane
second index = orientation of force

Positive if force in direction of normal (as shown)

$t_i = \sigma_{ji} n_j$ $\mathbf{t} = \boldsymbol{\sigma}^T \cdot \hat{\mathbf{n}}$ Transpose: $\sigma_{ji} = \sigma^T_{ij}$

Note: unusual index order

in matrix notation: $\mathbf{t} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \cdot \hat{\mathbf{n}}$

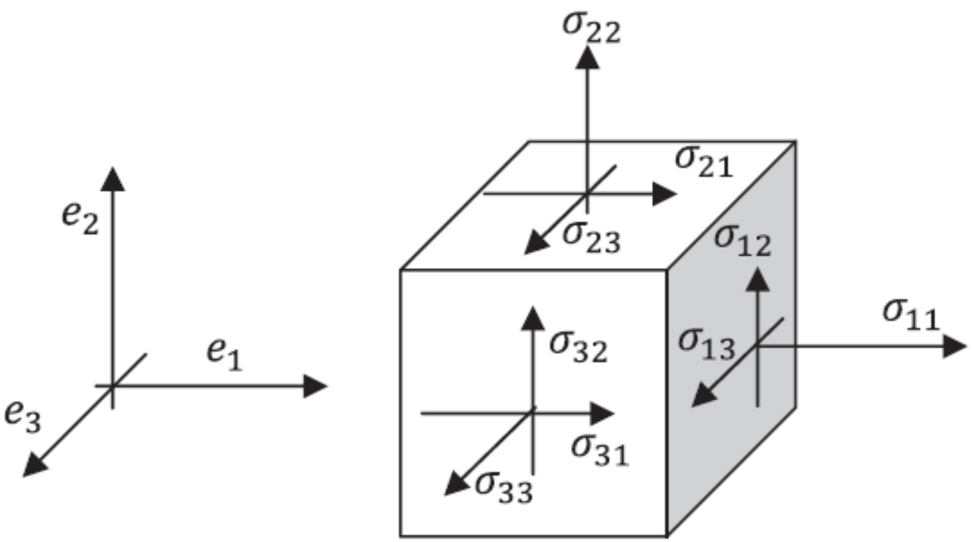
$\mathbf{t}$ and $\hat{\mathbf{n}}$ - tensors of rank 1 (vectors) in 3-D

$\underline{\underline{\boldsymbol{\sigma}}}$ - tensor of rank 2 in 3-D

compression - negative
tension - positive

σ_{ji} where i=j - normal stresses
σ_{ji} where i≠j - shear stresses

***rank 2 tensors** can be written as **square matrices** and have algebraic properties similar to some of those of matrices.*



Stress components

traction on a plane $$\mathbf{t} = \begin{bmatrix} \sigma_{11} & \sigma_{21} & \sigma_{31} \\ \sigma_{12} & \sigma_{22} & \sigma_{32} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix} \cdot \hat{\mathbf{n}}$$

what is (1) $\hat{\mathbf{e}}_1 \cdot \mathbf{t} = \hat{\mathbf{e}}_1 \cdot \boldsymbol{\sigma}^T \cdot \hat{\mathbf{n}}$?

what is (2) $\hat{\mathbf{e}}_1 \cdot \boldsymbol{\sigma}^T \cdot \hat{\mathbf{e}}_1$? what is (3) $\hat{\mathbf{e}}_1 \cdot \boldsymbol{\sigma}^T \cdot \hat{\mathbf{e}}_2$?

Tensor symmetry

A tensor can be symmetric in 1 or more indices

For rank 2:

$S_{ij}= S_{ji}$ => $\mathbf{S}=\mathbf{S}^T$ symmetric

$S_{ij}= -S_{ji}$ => $\mathbf{S}=-\mathbf{S}^T$ antisymmetric

Higher rank:

e.g., $S_{ijk}= S_{jik}$ for all i,j,k => symmetric in i,j

antisymmetric **T** of rank 2

Write out general antisymmetric **T** *rank 2, n=3 => how many independent components?*

symmetric **T** of rank 2

has n(n+1)/2 independent components

Any **T** of rank 2 can be decomposed in symm. and antisymm. part:

$$T_{ij}= (T_{ij}+T_{ji})/2 + (T_{ij}-T_{ji})/2$$

Tensor symmetry

A tensor can be symmetric in one or more indices

For rank 2:

$S_{ij} = S_{ji} \Rightarrow \mathbf{S} = \mathbf{S}^T$ symmetric

$S_{ij} = -S_{ji} \Rightarrow \mathbf{S} = -\mathbf{S}^T$ antisymmetric

Higher rank:

e.g., $S_{ijk} = S_{jik}$ for all i,j,k => symmetric in i,j

antisymmetric $\mathbf{T}$ of rank 2

=> $T_{ij} = 0$ for $i = j$, $\mathrm{trace}(\mathbf{T}) = 0$

has $n(n-1)/2$ independent components

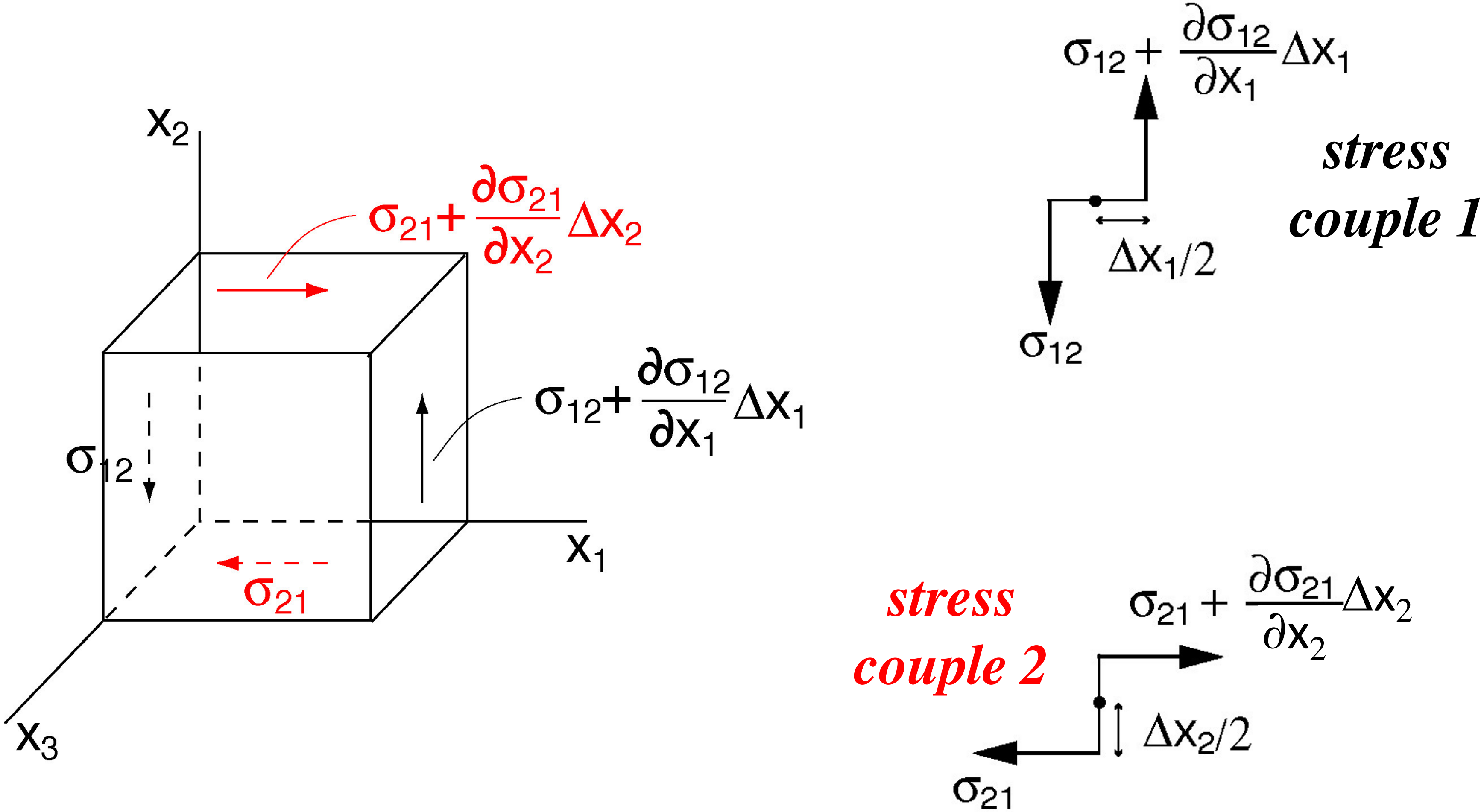
symmetric $\mathbf{T}$ of rank 2

has $n(n+1)/2$ independent components

Any $\mathbf{T}$ of rank 2 can be decomposed in symm. and antisymm. part:

$$T_{ij} = (T_{ij} + T_{ji})/2 + (T_{ij} - T_{ji})/2$$

Symmetry of the stress tensor



assuming static equilibrium

balance of moments in x_3 direction

A balance of moments in x_3 direction:

$$m_3 = \underbrace{[\sigma_{12} + (\sigma_{12} + \Delta x_1 \frac{\partial \sigma_{12}}{\partial x_1})]}_{\text{stress couple 1}} \underbrace{\Delta x_2 \Delta x_3}_{\text{area 1}} \cdot \underbrace{\Delta x_1 / 2}_{\text{arm 1}}$$

$$-\underbrace{[\sigma_{21} + (\sigma_{21} + \Delta x_2 \frac{\partial \sigma_{21}}{\partial x_2})]}_{\text{stress couple 2}} \underbrace{\Delta x_1 \Delta x_3}_{\text{area 2}} \cdot \underbrace{\Delta x_2 / 2}_{\text{arm 2}} = 0$$

$$\Rightarrow [2\sigma_{12} + \Delta x_1 \frac{\partial \sigma_{12}}{\partial x_1})] - [2\sigma_{21} + \Delta x_2 \frac{\partial \sigma_{21}}{\partial x_2})] = 0$$

$$\lim_{\Delta x_1, \Delta x_2} \rightarrow 0 \Rightarrow \boxed{\sigma_{12} = \sigma_{21}}$$

Balancing m_1 and m_2: $\boxed{\sigma_{23} = \sigma_{32}}$ *and* $\boxed{\sigma_{13} = \sigma_{31}}$

*thus, the stress tensor is **symmetric***

$$\mathbf{t} = \boldsymbol{\sigma}^{\mathrm{T}} \cdot \hat{\mathbf{n}} \Rightarrow \mathbf{t} = \boldsymbol{\sigma} \cdot \hat{\mathbf{n}}$$

Take a break

Then try Exercises 1 & 2 in the notebook (*chapter2.ipynb*)

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Stress and tensors

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- Know how to determine state of stress on given plane
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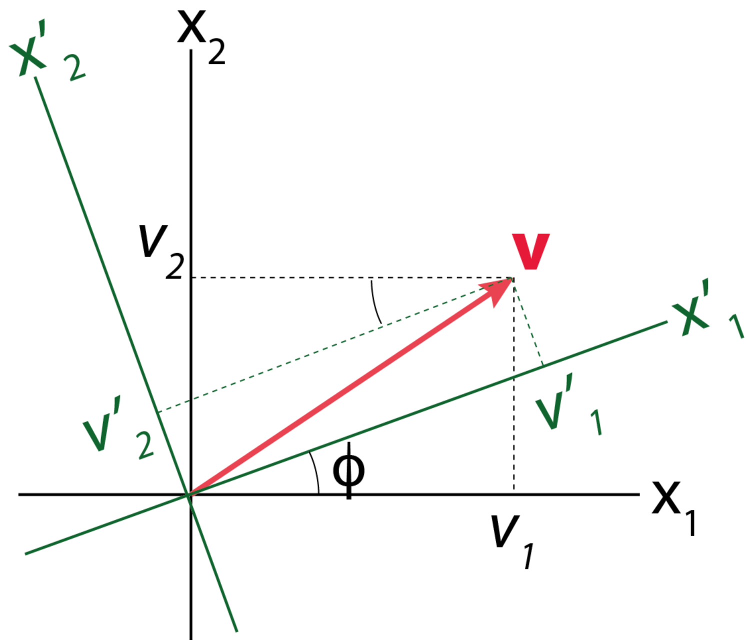
physical parameters should not depend on coordinate frame

$\Rightarrow$ **tensors follow linear transformation laws**

for vectors on orthonormal basis:



$$\mathbf{v}' = \mathbf{A}\mathbf{v}$$

$$\Rightarrow \quad \mathbf{v}' = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \mathbf{v}$$

coefficients α_{ij} depend on angle ϕ between x_1 and x'_1 (or x_2 and x'_2)

$$\mathbf{v}' = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \mathbf{v} = \begin{bmatrix} \cos\phi & \cos(90-\phi) \\ \cos(90+\phi) & \cos\phi \end{bmatrix} \mathbf{v}$$

$$\boxed{\alpha_{ij} = \hat{\mathbf{e}}'_i \cdot \hat{\mathbf{e}}_j}$$

Inverse transform: $v_j = \alpha_{ji}\, v'_i$ $\qquad \alpha_{ji} = \hat{e}_j \cdot \hat{e}'_i$

$$\mathbf{t} = \boldsymbol{\sigma}^T \mathbf{n}$$
$$\mathbf{t}' = \boldsymbol{\sigma}'^T \mathbf{n}'$$

Relation $\boldsymbol{\sigma}'$ to $\boldsymbol{\sigma}$?

In a new coordinate system:

Traction $\mathbf{t}' = \mathbf{A}\mathbf{t} \Rightarrow \mathbf{t} = \mathbf{A}^T\mathbf{t}'$

normal $\mathbf{n}' = \mathbf{A}\mathbf{n} \Rightarrow \mathbf{n} = \mathbf{A}^T\mathbf{n}'$

$\Rightarrow$ *transformation for stress tensor*

$$\mathbf{t}' = \mathbf{A}\boldsymbol{\sigma}^T \mathbf{n}$$
$$\mathbf{t}' = \mathbf{A}\boldsymbol{\sigma}^T \mathbf{A}^T \mathbf{n}'$$
$$\mathbf{t}' = \boldsymbol{\sigma}'^T \mathbf{n}'$$
$$\Rightarrow \boldsymbol{\sigma}'^T = \mathbf{A}\boldsymbol{\sigma}^T \mathbf{A}^T$$

- transformation matrices are orthogonal
  $\alpha_{ij}{}^{-1} = \alpha_{ji}$ $(\mathbf{A}^{-1} = \mathbf{A}^{\mathrm{T}})$
- *remember* $\alpha_{ij} = \hat{\mathbf{e}}'_i \cdot \hat{\mathbf{e}}_j$
  $\alpha_{ij}^{-1} = \hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}'_j = \alpha_{ji} = \alpha_{ij}^T$

$\Rightarrow$ $\sigma'^{T}_{ij} = \alpha_{ik}\sigma^{T}_{kl}\alpha_{jl} = \alpha_{ik}\alpha_{jl}\sigma^{T}_{kl}$ index notation

$\Rightarrow$ each dependence on direction transforms as a vector, requiring two transformations

An *n-dimensional* tensor of *rank r* consists of n^r components

This tensor $T_{i1,i2,\ldots,in}$ is defined relative to a basis of the real, linear n-dimensional space S_n

and under a coordinate transformation **T** transforms as:

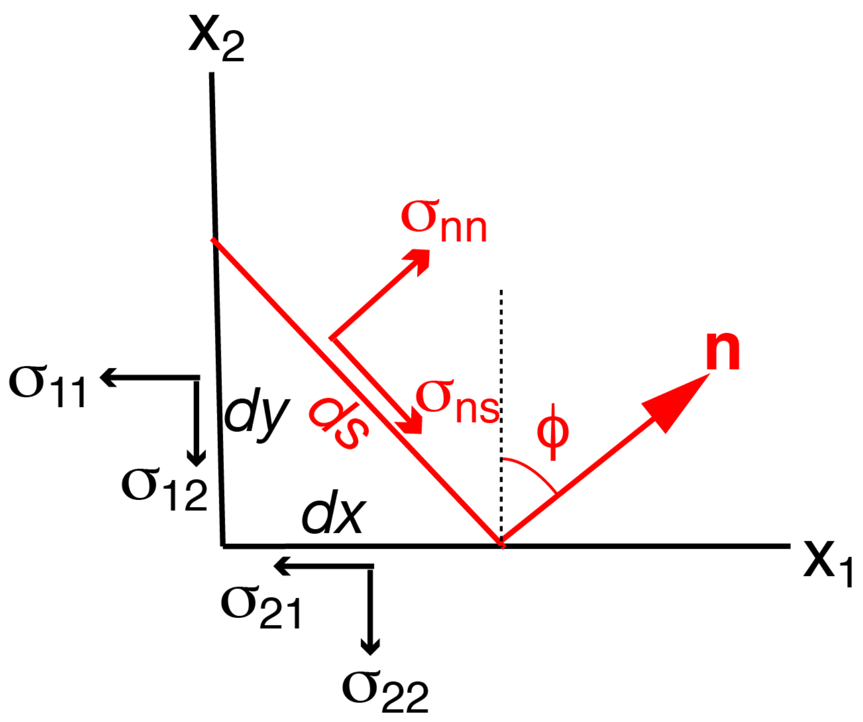
$T'_{ij\ldots n} = \alpha_{ip}\alpha_{jq}\ldots\alpha_{nt}\, T_{pq\ldots t}$ (i.e., one transformation per rank)

For *orthonormal* bases the matrices α_{ik} are *orthogonal* transformations, i.e. $\alpha_{ik}^{-1} = \alpha_{ki}$. (i.e., $\mathbf{A} = \mathbf{A}^T$; columns and rows are orthogonal and have length = 1, i.e., perpendicular unit vectors are transformed to perpendicular unit vectors)

If the basis is *Cartesian*, α_{ik} are *real*.

Transforming the 2-D stress tensor

(determining normal and shear stress on a plane)



Normal to the plane:

$$\hat{\mathbf{n}} = (\sin\phi, \cos\phi)$$

Normal stress on plane:

$$\sigma_{nn} = \hat{\mathbf{n}} \cdot \underline{\underline{\sigma}}^T \cdot \hat{\mathbf{n}}$$

This gives:

$$\sigma_{nn} = (\sin\phi, \cos\phi)\begin{bmatrix}\sigma_{11} & \sigma_{21}\\ \sigma_{12} & \sigma_{22}\end{bmatrix}\begin{pmatrix}\sin\phi\\ \cos\phi\end{pmatrix}$$

Similarly,

$$\sigma_{ns} = \hat{\mathbf{s}} \cdot \underline{\underline{\sigma}}^T \cdot \hat{\mathbf{n}} \qquad \hat{\mathbf{s}} = (\cos\phi, -\sin\phi)$$

verify yourself

Multiplying out:

$$\sigma_{nn} = \sigma_{11}\sin^2\phi + \sigma_{21}\cos\phi\sin\phi + \sigma_{12}\cos\phi\sin\phi + \sigma_{22}\cos^2\phi$$

$$\sigma_{ns} = \sigma_{11}\cos\phi\sin\phi + \sigma_{21}\cos^2\phi - \sigma_{12}\sin^2\phi - \sigma_{22}\cos\phi\sin\phi$$

This is equivalent to the **tensor transformation**

$$\boldsymbol{\sigma}'^T = \mathbf{A}\boldsymbol{\sigma}^T\mathbf{A}^T$$

$$\sigma'_{qp} = \alpha_{pi}\alpha_{qj}\sigma_{ji}$$

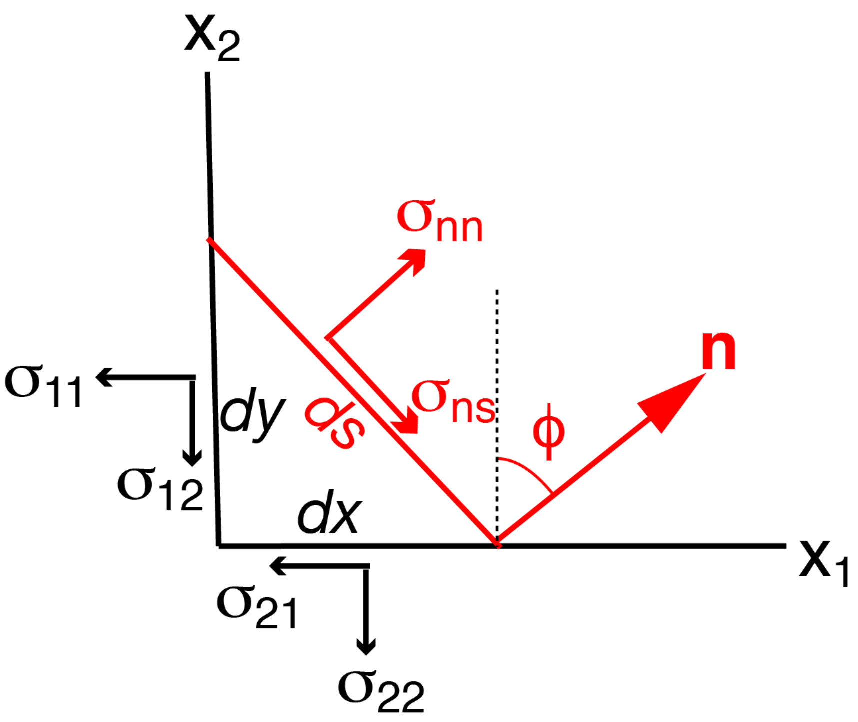
$\sigma'_{nn}=\alpha_{ni}\alpha_{nj}\sigma_{ji}$

$\sigma'_{ns}=\alpha_{si}\alpha_{nj}\sigma_{ji}$

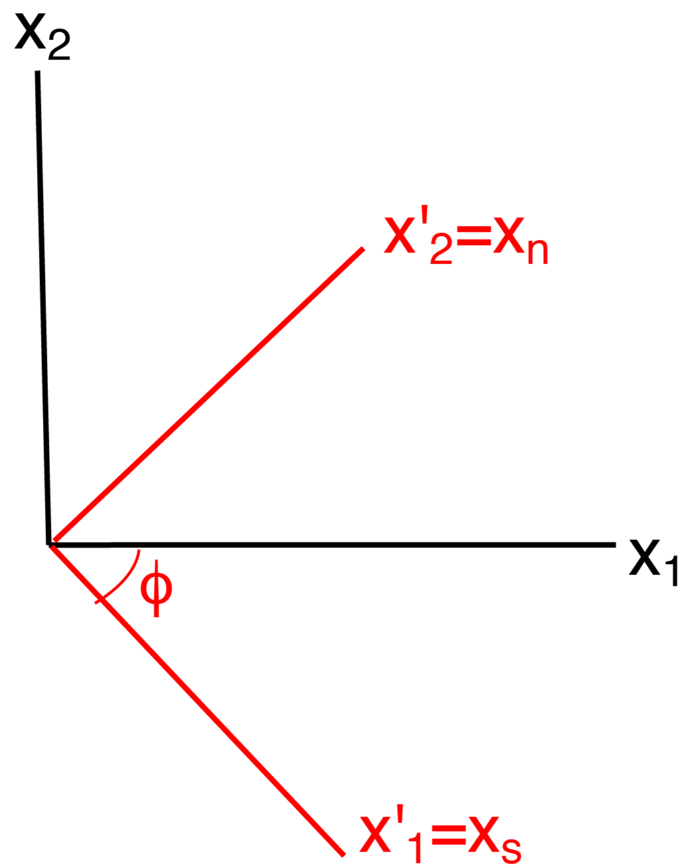
With $\alpha_{n1} = \sin\phi$, $\alpha_{n2} = \cos\phi$, $\alpha_{s1} = \cos\phi$, $\alpha_{s2} = -\sin\phi$

Transforming the 2-D stress tensor

(determining normal and shear stress on a plane)

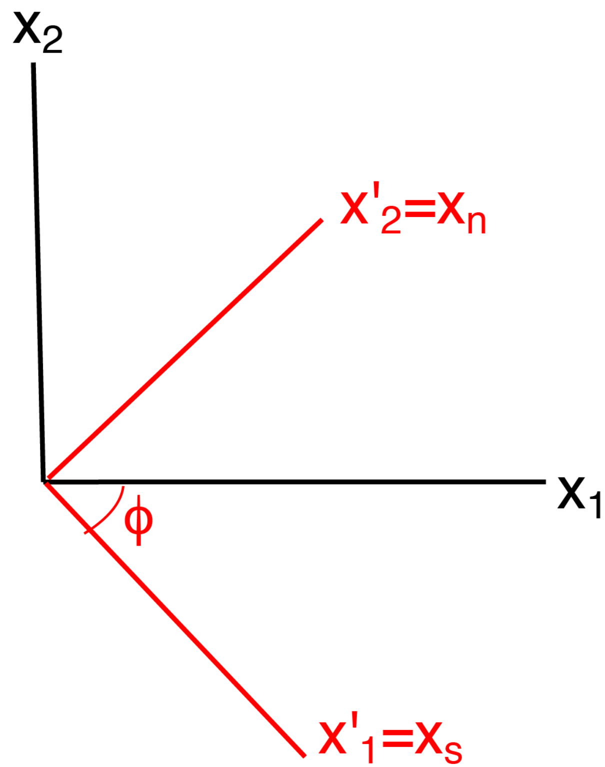


$$dx = \cos\phi \, ds$$
$$dy = \sin\phi \, ds$$



Write out transformation

$\sigma'_{qp}=\alpha_{pi}\alpha_{qj}\sigma_{ji}$



$$\alpha_{ij}=\hat{\mathbf{e}}'_i\cdot\hat{\mathbf{e}}_j$$

$$\alpha_{s1}=\hat{\mathbf{e}}_s\cdot\hat{\mathbf{e}}_1=\cos\phi$$

$$\alpha_{s2}=\hat{\mathbf{e}}_s\cdot\hat{\mathbf{e}}_2=-\sin\phi$$

$$\alpha_{n2}=\hat{\mathbf{e}}_n\cdot\hat{\mathbf{e}}_2=\cos\phi$$

$$\alpha_{n1}=\hat{\mathbf{e}}_n\cdot\hat{\mathbf{e}}_1=\sin\phi$$

In tensor notation:

$\boldsymbol{\sigma}'^{T}=\mathbf{A}\cdot\boldsymbol{\sigma}^{T}\cdot\mathbf{A}^{T}$

In matrix notation:

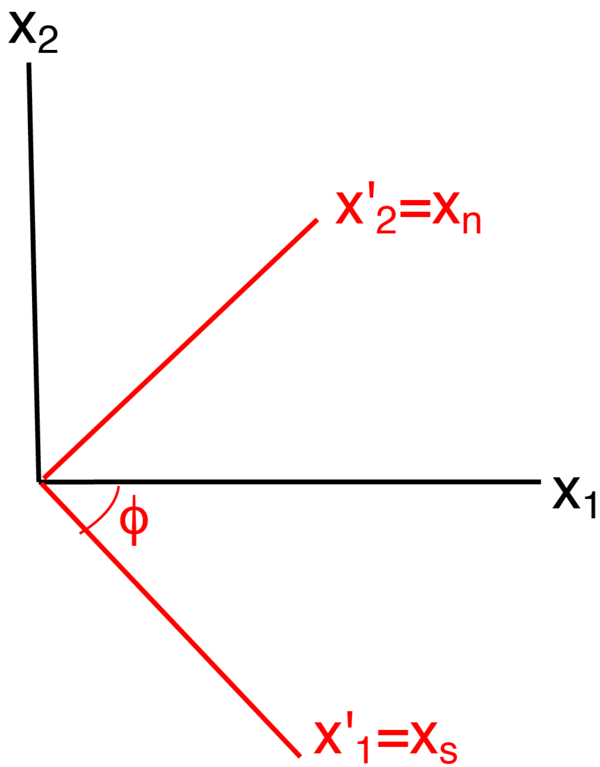
$$\begin{bmatrix}\sigma_{ss} & \sigma_{ns}\\ \sigma_{sn} & \sigma_{nn}\end{bmatrix}=\begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix}\sigma_{11} & \sigma_{21}\\ \sigma_{12} & \sigma_{22}\end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}$$

Write out matrices A and A^T

Check that the expressions for σ_{nn}, σ_{ns} of previous slide obtained

Write out transformation

$\sigma'_{qp} = \alpha_{pi}\alpha_{qj}\sigma_{ji}$



$$\alpha_{ij} = \hat{\mathbf{e}}'_i \cdot \hat{\mathbf{e}}_j$$

$$\alpha_{s1} = \hat{\mathbf{e}}_s \cdot \hat{\mathbf{e}}_1 = \cos\phi$$

$$\alpha_{s2} = \hat{\mathbf{e}}_s \cdot \hat{\mathbf{e}}_2 = -\sin\phi$$

$$\alpha_{n2} = \hat{\mathbf{e}}_n \cdot \hat{\mathbf{e}}_2 = \cos\phi$$

$$\alpha_{n1} = \hat{\mathbf{e}}_n \cdot \hat{\mathbf{e}}_1 = \sin\phi$$

In tensor notation:

$\boldsymbol{\sigma}'^{T} = \mathbf{A}\cdot\boldsymbol{\sigma}^{T}\cdot\mathbf{A}^{T}$

In matrix notation:

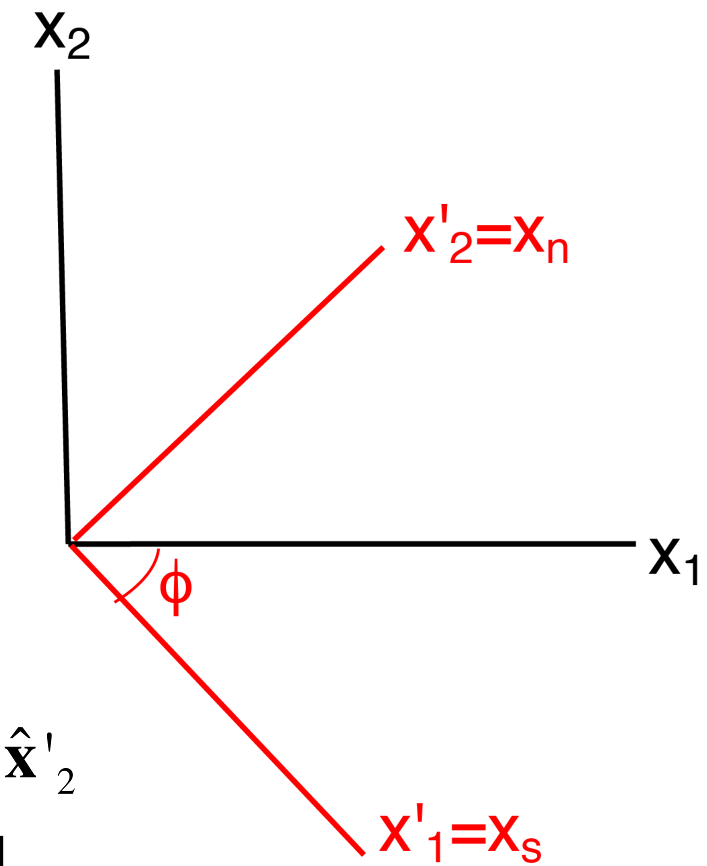
$$\begin{bmatrix} \sigma_{ss} & \sigma_{ns} \\ \sigma_{sn} & \sigma_{nn} \end{bmatrix} = \begin{bmatrix} \alpha_{s1} & \alpha_{s2} \\ \alpha_{n1} & \alpha_{n2} \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \begin{bmatrix} \alpha_{s1} & \alpha_{n1} \\ \alpha_{s2} & \alpha_{n2} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_{ss} & \sigma_{ns} \\ \sigma_{sn} & \sigma_{nn} \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

For $\hat{\mathbf{x}}_1=(1,0)$, $\hat{\mathbf{x}}_2=(0,1)$, $\hat{\mathbf{x}}'_1=(\cos\phi,-\sin\phi)$ $\hat{\mathbf{x}}'_2=(\sin\phi,\cos\phi)$

first **row** of **A** consists of $\hat{\mathbf{x}}'_1$, second of $\hat{\mathbf{x}}'_2$

$$\mathbf{A}=\begin{bmatrix} \mathbf{x}'_1\cdot\mathbf{x}_1 & \mathbf{x}'_1\cdot\mathbf{x}_2 \\ \mathbf{x}'_2\cdot\mathbf{x}_1 & \mathbf{x}'_2\cdot\mathbf{x}_2 \end{bmatrix}=\begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$



*You may recognise **A** as a matrix that describes an **anticlockwise** rigid-body rotation over an angle ϕ*

*A^T describes **clockwise** rotation over angle ϕ*

First **column** of $\mathbf{A}^\mathrm{T}$ consists of $\hat{\mathbf{x}}'_1$, second of $\hat{\mathbf{x}}'_2$

$$\mathbf{A}^\mathrm{T}=\begin{bmatrix} \mathbf{x}_1\cdot\mathbf{x}'_1 & \mathbf{x}_1\cdot\mathbf{x}'_2 \\ \mathbf{x}_2\cdot\mathbf{x}'_1 & \mathbf{x}_2\cdot\mathbf{x}'_2 \end{bmatrix}=\begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}$$

Diagonalizing/Principal Stresses

Real-valued, symmetric rank 2 tensors (square, symmetric matrices) can be diagonalized, i.e. a coordinate frame can be found, such that only the diagonal elements (normal stresses) remain.

For stress tensor, these elements, σ_1, σ_2, σ_3 are called the principal stresses

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

Such a transformation can be cast as:

$\mathbf{T}\cdot\mathbf{x} = \lambda\mathbf{x}$

where $\mathbf{x}_i$ are eigenvectors or characteristic vectors

and λ_i are the eigenvalues, characteristic or principal values

$\Rightarrow(\mathbf{T}-\lambda\mathbf{I})\cdot\mathbf{x} = \mathbf{0}$

Non-trivial solution only if $\det(\mathbf{T}-\lambda\mathbf{I}) = 0$

Stress/Tensor Invariants

Invariants are properties of a tensor that do not change if the coordinate system is changed.
A rank-2 tensor has three invariants:

$$I_1 = \mathrm{tr}(\mathbf{T}) = T_{11} + T_{22} + T_{33}$$

$$I_2 = \mathrm{minor}(\mathbf{T}) = \begin{vmatrix} T_{11} & T_{21} \\ T_{21} & T_{22} \end{vmatrix} + \begin{vmatrix} T_{11} & T_{31} \\ T_{31} & T_{33} \end{vmatrix} + \begin{vmatrix} T_{22} & T_{32} \\ T_{32} & T_{33} \end{vmatrix}$$

$$= T_{11}T_{22} + T_{22}T_{33} + T_{11}T_{33} - T_{21}^2 - T_{32}^2 - T_{31}^2$$

$$I_3 = \det(\mathbf{T}) = \begin{vmatrix} T_{11} & T_{21} & T_{31} \\ T_{21} & T_{22} & T_{32} \\ T_{31} & T_{32} & T_{33} \end{vmatrix} = T_{11}T_{22}T_{33} + 2T_{21}T_{32}T_{31} - T_{11}T_{32}^2 - T_{22}T_{31}^2 - T_{33}T_{21}^2$$

These invariants are important in e.g. diagonalising

Hydrostatic and Deviatoric stress

$\sigma_{ij} = -p\delta_{ij} + \sigma'_{ij}$

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

$\mathrm{tr}(\boldsymbol{\sigma})$ = trace of stress tensor

= sum of normal stresses = $\sigma_{11}+\sigma_{22}+\sigma_{33}$

is an invariant of the stress tensor,

i.e. has same value in any coordinate system,

$\mathrm{tr}(\boldsymbol{\sigma})/3$ = - pressure p = average normal stress = ***hydrostatic stress***

$\Rightarrow$ ***leads to volume change***

$\boldsymbol{\sigma}'_{ij}$ is ***deviatoric stress*** = $\sigma_{ij}+p\delta_{ij}$

$\Rightarrow$ ***leads to shape change***

$\mathrm{tr}(\boldsymbol{\sigma}')$ = ?

Note that in geologic applications $\boldsymbol{\sigma}'_{ij} \neq \sigma_{ij}+\rho g z\delta_{ij}$

Learning Objectives
Stress and tensors

- Understand meaning of different components of 3D Cauchy stress tensor
- Know how to determine state of stress on given plane
- Be able to decompose a rank 2 tensor into symmetric and anti-symmetric components
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Summary
Stress Tensors

- Cauchy stress tensor
- (Stress) tensor symmetry
- Coordinate transformation (stress) tensors
- Shear and normal stresses
- Tensor invariants

Further reading on the topics in the lecture can be done in for example: Lai, Rubin, Kremple (2010): Ch. 2.18 through 2.25, 4.4 through 4.7

Take a break

Then try Exercise 3 and 5 in the *chapter2.ipynb* notebook

Try to finish in the afternoon workshop in
chapter2.ipynb:

Exercise 1, 2, 3, 4, 6

Advanced practise: **Exercise 5**