IMPERIAL COLLEGE LONDON

MSc EXAMPLE ASSESSMENT

For internal students of Imperial College London

Taken by students of MSc Applied Computational Science and Engineering

Modelling and Numerical Methods Coursework 1

90 minutes

This assessment comprises one question with six parts. Please answer ALL parts of the question. The marks are out of a total of 50 marks.

Note that the content for part (e) was only covered as optional in 2023, and will not be assessed in the 2023 coursework

The assessments are run as open-book assessments, and as such we have worked hard to create a coursework that assesses synthesis of knowledge rather than factual recall. Be aware that access to the internet, notes or other sources of factual information in the time provided may not be too helpful and may well limit your time to successfully synthesise the answers required.

The use of the work of another student, past or present, constitutes plagiarism. Giving your work to another student to use may also constitute an offence. Collusion is a form of plagiarism and will be treated in a similar manner. This is an individual assessment and thus should be completed solely by you.

The College will investigate all instances where an assessment offence is reported or suspected, using plagiarism software, vivas and other tools, and apply appropriate penalties to students. In all assessments, we will analyse performance against performance on the rest of the course and against data from previous years and use an evidence-based approach to maintain a fair and robust assessment.

As with all assessments, the best strategy is to read the question carefully and answer as fully as possible, taking account of the time and number of marks available.

Please write clearly, annotate any graphs or sketches and explain your answers. Only partial marks will be given for correct answers without an explanation or derivation. Make sure you put a page number on each page of your answer script. GOOD LUCK.

(1) Consider the *steady state* velocity field:

$$\mathbf{v}(x, y) = \begin{pmatrix} x^2 y \\ -xy^2 \end{pmatrix} \text{ m s}^{-1}.$$

(a) Calculate the velocity gradient tensor $\nabla\mathbf{v}$, as well as the divergence $\nabla \cdot \mathbf{v}$ and curl $\nabla \times \mathbf{v}$ of the velocity field. [*10 marks*]

(b) Determine the strain *rate* tensor $\underline{\underline{\mathbf{D}}}$ as a function of x and y. Verify that at the point A ($x = 1, y = 3$):

$$\underline{\underline{\mathbf{D}}} = \begin{bmatrix} 6 & -4 \\ -4 & -6 \end{bmatrix} \text{ s}^{-1}.$$

[*5 marks*]

(c) Using your answers to Parts (a) and (b), describe the nature of the flow near point A and sketch how you would expect a small square centred at point A to deform over a short time interval. [*10 marks*]

(d) Assuming that the velocity field describes the flow of a *Newtonian* fluid with shear viscosity $\eta = 0.25$ MPa-s and that the pressure $p = 1$ MPa at point A, show that the 2D Cauchy stress tensor at point A is given by:

$$\underline{\underline{\boldsymbol{\sigma}}} = \begin{bmatrix} 2 & -2 \\ -2 & -4 \end{bmatrix} \text{ MPa}.$$

[*3 marks*]

(e) By solving the eigenvalue problem for the stress tensor at point A, find the principal stresses and principal stress directions. What is the angle between the maximum principal stress direction and the x-axis?

[*7 marks*]

(f) Imagine that a Hookean solid is subjected to the same stress state as the fluid at point A. Express the principal strains experienced by the solid in terms of the Lamé parameters λ and μ. If $\lambda = 2\mu$, describe the deformation of the solid in terms of μ. *Partial credit will be given for describing the deformation in the reference frame of the principal stress directions. Full credit can be achieved for also describing the deformation in the original coordinate frame.*

[*15 marks*]