<u>Dimensional Analysis Worksheet Solutions</u>

NOTE: With dimensional analysis there is no single correct answer to these questions. These solutions are thus A set of answers, not THE set of answers. The best way to check answers is to see that they are actually dimensionless.

<u>Question 1</u>

a) There are 5 variables (d, γ, g, ρ_h and ρ_l) and 3 dimensions (M, L, T) and so **TWO** dimensionless groups are required.
b) Doing it the proper Buckingham Pi way:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \gamma & g & \rho_h & \rho_l & d \end{array} \\
\begin{array}{c} M \\ L \\ T \end{array} & \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -3 & -3 & 1 \\ -2 & -2 & 0 & 0 & 0 \end{pmatrix}
\end{array}
$$

Setting $x_\gamma = 1$ and $x_{\rho_l} = 0$ results in the following dimensionless group.

The resulting dimensionless group is:

$$\boldsymbol{N = \frac{\gamma}{\rho_h g d^2}}$$

(This is actually the inverse of the Bond number).

c) This dimensionless group represents the ratio of the capillary to gravity force.

<u>Question 2</u>

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} A & \rho & C_P & h & Q & \Delta T & V \end{array} \\
\begin{array}{c} M \\ L \\ T \\ \Theta \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & -3 & 2 & 0 & 2 & 0 & 3 \\ 0 & 0 & -2 & -3 & -3 & 0 & -1 \\ 0 & 0 & -1 & -1 & 0 & 1 & 0 \end{pmatrix}
\end{array}
$$

a) 7 variables and 4 base units mean that there are 3 dimensionless groups
b) With some rearranging we can get:

$$Q^* = \frac{Q}{hA\Delta T}$$

$$V^* = \frac{V\rho C_P \Delta T}{Q}$$

$$\Delta T^* = \frac{\rho^2 {C_P}^3 \Delta T}{h^2}$$

The last one is not very nice and you can probably find a nicer one.

<u>Question 3</u>

a)

$$\frac{\partial \phi}{\partial t} = \frac{2k_1}{\lambda}\phi\frac{\partial \phi}{\partial z} + \frac{k_2}{2\sqrt{\lambda}\sqrt{\phi}}\left(\frac{\partial \phi}{\partial z}\right)^2 + \frac{k_2\sqrt{\phi}}{\sqrt{\lambda}}\frac{\partial^2 \phi}{\partial z^2}$$

where $k_1 = \frac{\rho g}{3\mu C_{PB}}$ and $k_2 = \frac{\gamma\sqrt{3-\frac{\pi}{2}}}{6\mu C_{PB}}$

b) $z^* = z\sqrt{\frac{\rho g}{\gamma}}$ and

$$t^* = t\frac{\sqrt{\gamma \rho g}}{\mu}$$

(other ways of non-dimensionalising the problem are possible).

c)

$$\frac{\partial \phi}{\partial t^*} = Bo\frac{2}{3k_\lambda C_{PB}}\phi\frac{\partial \phi}{\partial z^*} + \frac{\sqrt{3-\frac{\pi}{2}}}{6C_{PB}\sqrt{k_\lambda}}\sqrt{Bo}\left(\frac{1}{2\sqrt{\phi}}\left(\frac{\partial \phi}{\partial z^*}\right)^2 + \sqrt{\phi}\frac{\partial^2 \phi}{\partial {z^*}^2}\right)$$

Where $Bo = \frac{\rho g {d_b}^2}{\gamma}$

Question 4

a+b) Simply replace the derivatives with their approximations and also note that ϕ should be replaced with $\phi_{i,j}$:

$$\phi_{i,j+1} = \phi_{i,j} + \Delta t^* \left(Bo \frac{2}{3k_\lambda C_{PB}} \phi_{i,j} \frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta z^*} \right.$$

$$\left. + \frac{\sqrt{3 - \frac{\pi}{2}}}{6C_{PB}\sqrt{k_\lambda}} \sqrt{Bo} \left(\frac{1}{2\sqrt{\phi_{i,j}}} \left(\frac{\phi_{i+1,j} - \phi_{i-1,j}}{2\Delta z^*} \right)^2 + \sqrt{\phi_{i,j}} \frac{\phi_{i+1,j} + \phi_{i-1,j} - 2\phi_{i,j}}{\Delta z^{*2}} \right) \right)$$

c) For finding the time step:

$$D_{max} = \frac{\sqrt{3 - \frac{\pi}{2}}}{6C_{PB}\sqrt{k_\lambda}} \sqrt{Bo} \sqrt{\phi_{max}}$$

$$v_{max} = \max \left(Bo \frac{2}{3k_\lambda C_{PB}} \phi_{max}, \frac{\sqrt{3 - \frac{\pi}{2}}}{6C_{PB}\sqrt{k_\lambda}} \sqrt{Bo} \frac{1}{2\sqrt{\phi_{min}}} \frac{\partial \phi}{\partial z^*}_{max} \right)$$

The maximum gradient occurs on the first timestep, which given the resolution, is $\frac{\partial \phi}{\partial z^*}_{max} = \frac{\phi_{max} - \phi_{min}}{\Delta z^*}$. Ignoring this gradient condition won’t actually cause proper divergence, but can add wiggles in the formation of the capillary boundary layer.

$\phi_{max} = 0.3$ and $\phi_{min} = 0.01$ as the liquid contents cannot go outside these limits if the code is working correctly.