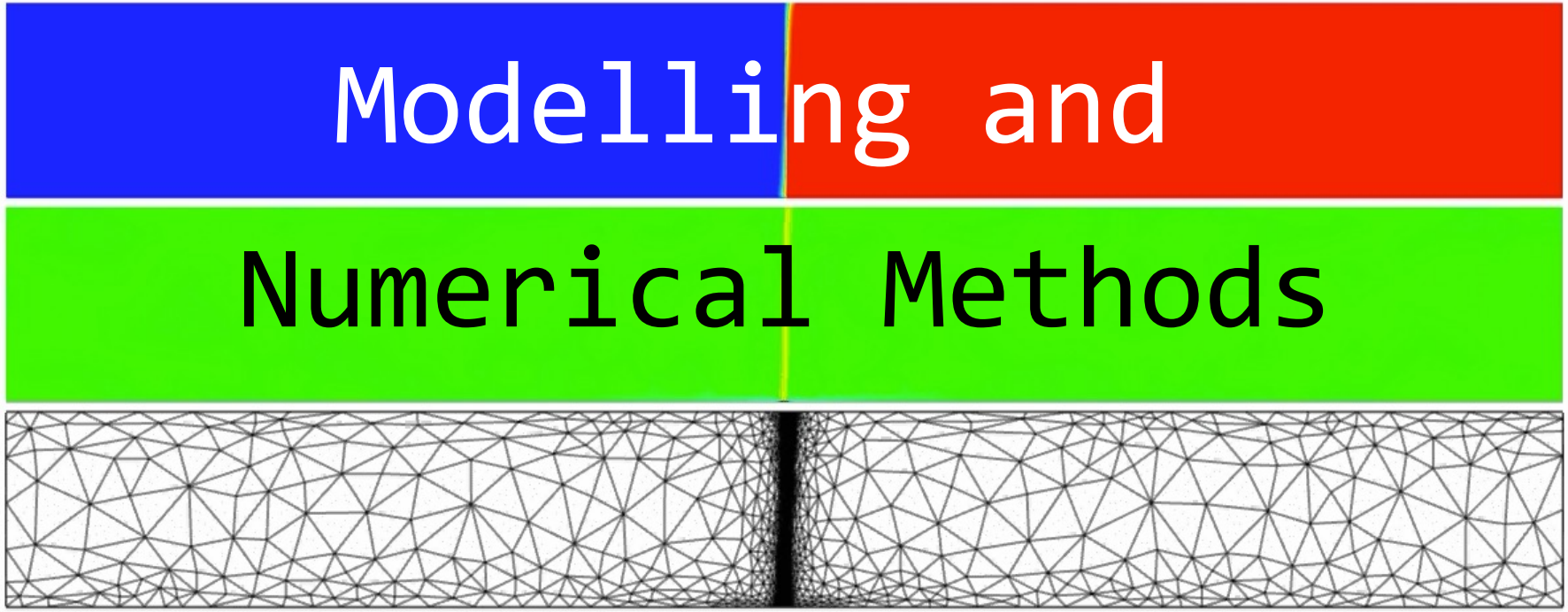
Modelling and Numerical Methods



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Aims

- Part 1: introduce mathematical essentials and physical equations for modelling a range of dynamic processes

- Part 2: provides complementary background on numerical methods that can be used to solve the equations of these physical systems

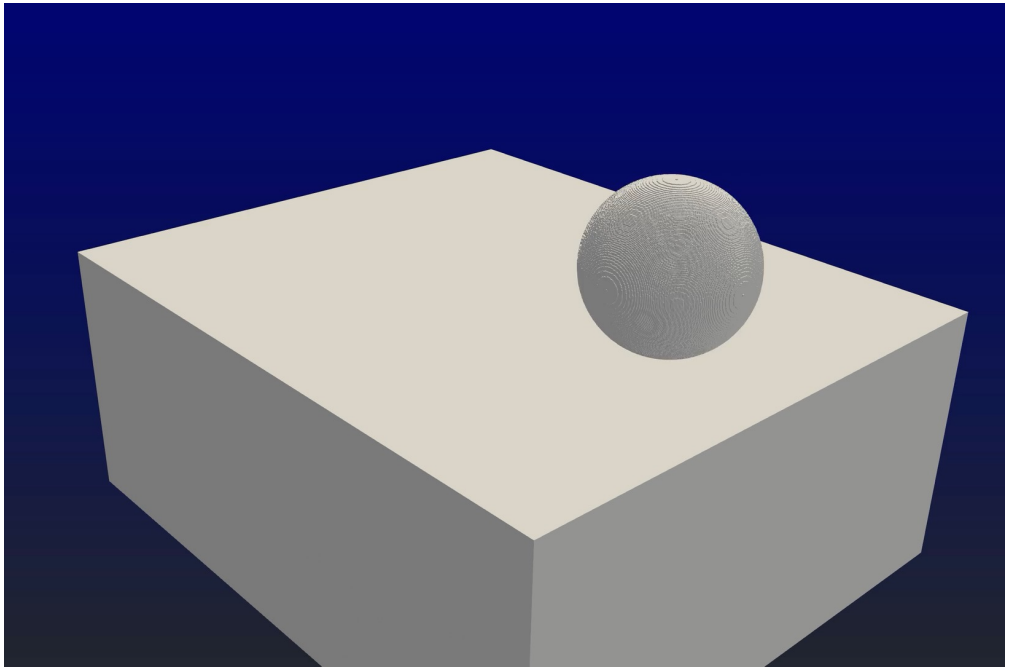
Example: compressible flow of deformable solids

$$\frac{D\rho}{Dt} = -\rho \nabla . \mathbf{u}$$

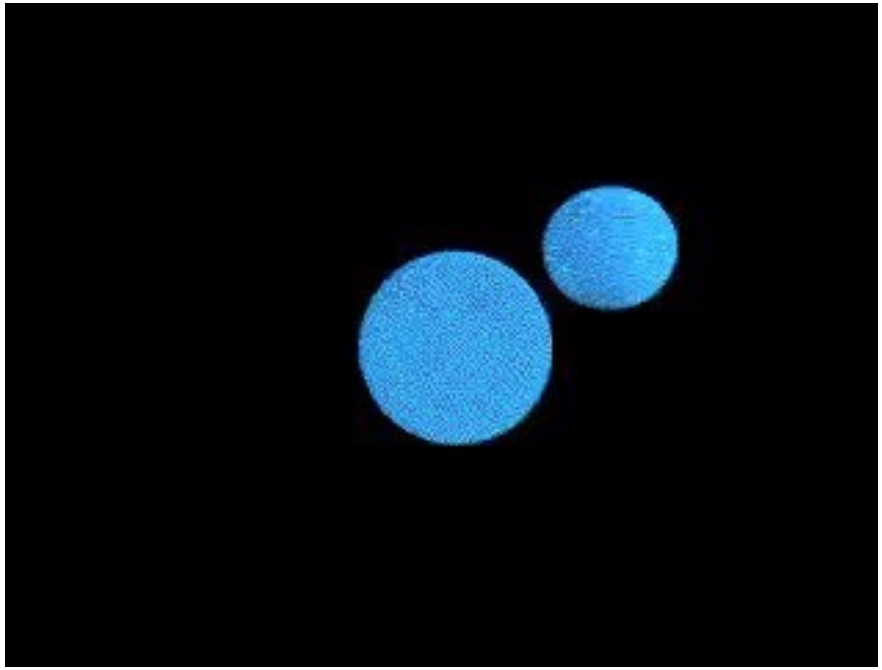
$$\rho \frac{D\mathbf{u}}{Dt} = \mathbf{f} - \nabla p + \nabla \cdot \bar{\bar{s}}$$

$$\rho \frac{DE}{Dt} = -p \nabla \cdot \mathbf{u} + \mathrm{tr}(\bar{\bar{s}} \cdot \nabla \mathbf{u})$$

Example: compressible flow of deformable solids



Halim, S.H., Crawford, I.A., Collins, G.S., Joy, K.H., Davison, T.M., 2020. Assessing the survivability of biomarkers within terrestrial material impacting the lunar surface. Icarus 114026. https://doi.org/10.1016/j.icarus.2020.114026



Canup, R.M., 2004. Simulations of a late lunar-forming impact. Icarus 168, 433–456. https://doi.org/10.1016/j.icarus.2003.09.028

Structure of course

- ***Week 1 Mathematical background:*** *vectors, tensors, conservation principles in continuum mechanics and some analytical solutions*

- ***Week 2-3 Numerical methods and solutions****: Examples and numerical methods to solve fluid mechanical problems including common types of ODEs, PDEs.*

- ***Assessment*** *In week 3: (1) in-class analytical timed assessment, (2) numerical assessment*

Some of you may already be familiar with the basic background that will be covered in part of the lectures, but may have been taught to you in different way, review will be useful, and everyone will have same background for other course(s).

Outline of course

- *Part 1: Analytical background*
  - **1.** Intro vector/tensor calculus (*SG*)
  - **2.** Stress tensor (*SG*)
  - **3.** Kinematics and strain (*SG*)
  - **4.** Conservation equations (*SG*)
  - **5.** Dimensional Analysis (*SN*)
- *Part 2: Numerical techniques (advanced)*
  - **6.** Interpolation and quadrature (*MP*)
  - **7.** Ordinary differential equations (*MP*)
  - **8.** Partial differential equations and finite difference (*MP*)
- *Part 3: Numerical solutions*
  - **9.** Potential flow (*SN*)
  - **10.** Navier-Stokes (*SN*)
  - **11.** Nonlinear rheology and turbulence (*SN*)
  - **12.** Finite Element Method (*MP*)

Schedule & Assessment

- **Lectures**
  - Morning (09:00—12:00)
  - In person and streamed/recorded
  - Using ppt and/or Jupyter notebooks

- **Workshops**
  - Afternoon (14:00—17:00)
  - In person with GTA assistance

- **2 Assessments** – Each worth 50%
  - **Coursework 1 (analytical):** *In-Class Test*, Wednesday 25 January 2023, 10:00—11:30 am.
  - **Coursework 2 (numerical):** *Jupyter Notebook*, released Wednesday 25 January 2023, due 5 pm Friday 27 January 2023.

Course Schedule

Monday	Tuesday	Wednesday	Thursday	Friday
09-Jan	10-Jan	11-Jan	12-Jan	13-Jan
lecture **workshop**	**lecture** **workshop**	**lecture**	**lecture** **workshop**	**lecture** **workshop**
16-Jan	17-Jan	18-Jan	19-Jan	20-Jan
lecture **workshop**	**lecture** **workshop**	**lecture**	**lecture** **workshop**	**lecture** **workshop**
23-Jan	24-Jan	25-Jan	26-Jan	27-Jan
lecture **workshop**	**lecture** **workshop**	**cw1 (in class)** **cw2 released**	**work on** **cw2**	**cw2 due 5** **pm**

lectures will be live and recorded. **workshops** will be GTA assisted. there are two pieces of **coursework**, **cw1**: analytical in-class, **cw2**: numerical/notebook