# Question 1

Force balance on annulus (same as for Newtonian as done in class):

$$2\pi r \Delta r \left( P(z) - P(z+\Delta z) \right)$$

$$+ \tau_{rz}(r+\Delta z) \, 2\pi (r+\Delta r) \Delta z$$

$$- \tau_{rz}(r) \, 2\pi r \Delta z = 0$$

$\div$ by $2\pi \Delta z \Delta r$

$$-r\,\frac{P(z+\Delta z)-P(z)}{\Delta z}$$

$$+\,\frac{\tau_{rz}(r+\Delta r)(r+\Delta r)-\tau_{rz}(r)r}{\Delta r}$$

$$=0$$

$\lim \Delta r \to 0$
$\lim \Delta z \to 0$

$$-r\frac{\partial P}{\partial z}+\frac{\partial}{\partial r}\left(\tau_{rz}r\right)=0$$

$$\frac{\partial P}{\partial z}=-\frac{\Delta P}{L}$$

$$\frac{d}{dr}\left(\tau_{rz}r\right)=-r\frac{\Delta P}{L}$$

$$\tau_{rz}=-\frac{\Delta P}{2L}r+\frac{A}{r}$$

symmetry: At $r=0$

$\tau_{rz}=0$

$\therefore \quad A=0$

$$\tau_{rz} = -\frac{\Delta P}{2L} r$$

$\tau_{rz}$ is negative

$$\therefore \quad \left|\frac{du_z}{dr}\right| = -\frac{du_z}{dr}$$

$$\tau_{rz} = -k\left(-\frac{du_z}{dr}\right)^n$$

$$\left(-\frac{du_z}{dr}\right)^n = \frac{\Delta P}{2Lk} r$$

$$\frac{du_z}{dr} = -\left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}} r^{\frac{1}{n}}$$

$$u_z = \frac{-n}{n+1} \left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}} r^{\frac{n+1}{n}}$$

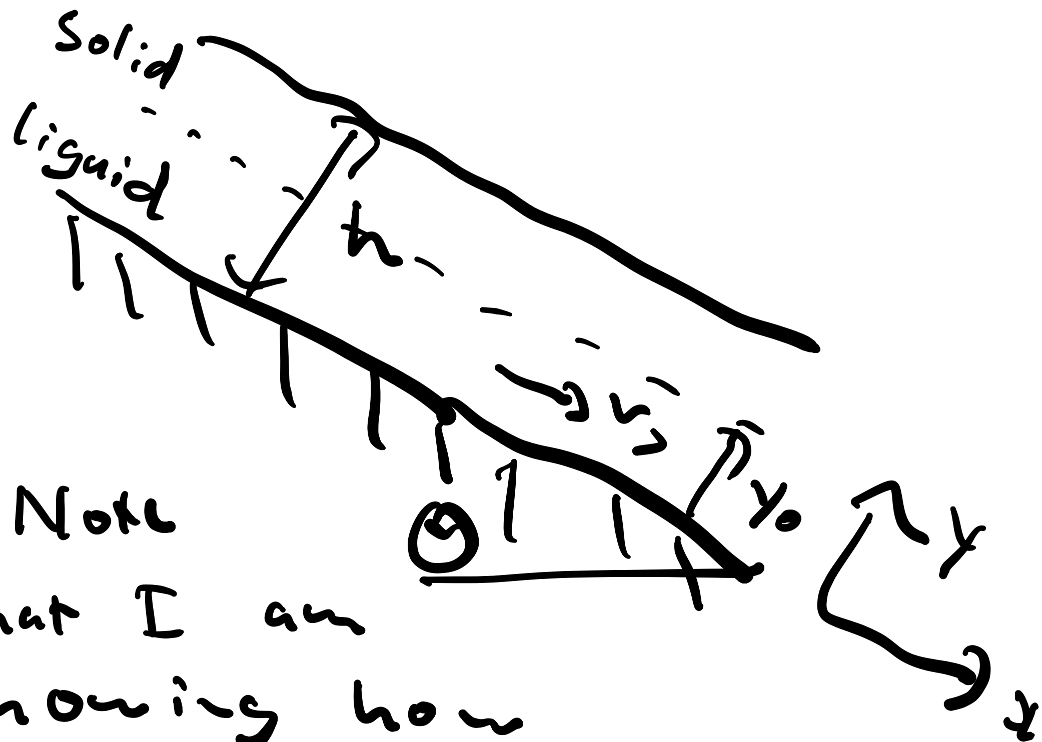
$$+ B$$

at $r = R$ $u_z = 0$

$$\therefore B = \frac{n}{n+1} \left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}} R^{\frac{n+1}{n}}$$

$$\therefore u_z = \frac{n}{n+1} \left(\frac{\Delta P}{2Lk}\right)^{\frac{1}{n}} \left(R^{\frac{n+1}{n}} - r^{\frac{n+1}{n}}\right)$$

# Question 2




Note that I am showing how to do it with diffrent axes than I used in class (and the angle is from the horizontal rather than the vertical):

Force balance in y direction (only gravity and viscous forces):

$$\left(\tau_{yx}(y+\Delta y) - \tau_{yx}(y)\right)\Delta x + \rho g_x \Delta x \Delta y = 0$$

$\div \Delta x \Delta y$

$$\frac{\tau_{yx}(y+\Delta y) - \tau_{yx}(y)}{\Delta y} + \rho g_x = 0$$

$$\lim_{\Delta y \to 0} \quad \frac{d\tau_{yx}}{dy} = -\rho g_x$$

$$g_x = \sin\theta\, g$$

$$\tau_{yx} = -\rho g \sin(\theta)\, y + A$$

No shear at the free surface!

$$\tau_{yx} = 0 \quad \text{at } y = h$$

$$\tau_{yx} = \rho g \sin(\theta)(h - y)$$

Solid/liquid like transition

$$\tau_{yx} = \tau_0$$

$$\tau_0 = \rho g \sin(\theta)(h - y_0)$$

$$y_0 = h - \frac{\tau_0}{\rho g \sin\theta}$$

Liquid region

$y < y_0$

$\tau_{yx}$ is +ve everywhere (note that this is the opposite of when the axes run from the surface)

$$\therefore \left|\frac{dv_x}{dy}\right| = \frac{dv_x}{dy}$$

$$\tau_{yx} = \left(\mu_\infty \frac{dv_x}{dy} + \tau_0\right)$$

$$\mu_\infty \frac{dv_x}{dy} + \tau_0 = \rho g \sin\theta \, (h-y)$$

$$\frac{dv_x}{dy} = \frac{\rho g \sin\theta}{\mu_\infty}(h-y) - \frac{\tau_0}{\mu_\infty}$$

$$v_x = \frac{\rho g \sin\theta}{\mu_\infty}\left(hy - \frac{y^2}{2}\right) - \frac{\tau_0}{\mu_p} y + B$$

at $y = 0$ $v_x = 0$

$\therefore B = 0$

if $y < y_0$

$$v_x = \frac{\rho g \sin\theta}{\mu_p}\left(hy - \frac{y^2}{2}\right) - \frac{\tau_0}{\mu_p} y$$

if $y > y_0$

$$v_x = \frac{\rho g \sin\theta}{\mu_p}\left(hy_0 - \frac{y_0^2}{2}\right) - \frac{\tau_0}{\mu_p} y_0$$

where $y_0 = h - \frac{\tau_0}{\rho g \sin\theta}$

Initially just at rest:

$y_0 = 0$

$$h = \frac{\tau_{0\,original}}{\rho g \sin\theta}$$

$\tau_{0\,original} = 4905\ Pa$

After rain:

$\tau_0 = 3924$

$y_0 = 0.2\ m$

$$v_{max} = \frac{\rho g \sin\theta}{m_a}\left(h y_0 - \frac{y_0^2}{2}\right) - \frac{\tau_0}{m_a} y_0$$

$V_{max} = 2.981 \ m/s$