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A Green Vehicle Routing Problem

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ABSTRACT

A Green Vehicle Routing Problem (G-VRP) is formulated and solution techniques are developed to aid organizations with alternative fuel-powered vehicle fleets in overcoming difficulties that exist as a result of limited vehicle driving range in conjunction with limited refueling infrastructure. The G-VRP is formulated as a mixed integer linear program. Two construction heuristics, the Modified Clarke and Wright Savings heuristic and the Density-Based Clustering Algorithm, and a customized improvement technique, are developed. Results of numerical experiments show that the heuristics perform well. Moreover, problem feasibility depends on customer and station location configurations. Implications of technology adoption on operations are discussed.

1. Introduction

In the United States (US), the transportation sector contributes 28% (US EPA, 2009) of national greenhouse gas (GHG) emissions. This is in large part because 97% of US transportation energy comes from petroleum-based fuels (US DOT, 2010). Efforts have been made over many decades to attract drivers away from personal automobiles and onto public transit and freight from trucks to rail. Such efforts are aimed at reducing vehicle miles traveled by road and, thus, fossil-fuel usage. Other efforts have focused on introducing cleaner fuels, e.g. ultra low sulfur diesel, and efficient engine technologies, leading to reduced emissions for the same miles traveled and greater mileage per gallon of fuel used. While each such effort has its benefits, only a multi-faceted approach can engender the needed reduction in fossil-fuel usage.

As part of such a multi-faceted approach, renewed attention is being given to efforts to exploit alternative, greener fuel sources, namely, biodiesel, electricity, ethanol, hydrogen, methanol, natural gas (liquid-LNG or compressed-CNG), and propane (US DOE, 2010). Municipalities, government agencies, nonprofit organizations and private companies are converting their fleets of trucks to include Alternative Fuel Vehicles (AFVs), either to reduce their environmental impact voluntarily or to meet new environmental regulations. This focus on truck conversion is desirable because, while medium- and heavy-duty trucks comprise only 4% of the vehicles on the roadways (US FHWA, 2008), they contribute nearly 19.2% of US transportation-based GHG emissions (US DOT, 2010). Moreover, truck traffic has had the greatest growth rate of all vehicle traffic, increasing 77% for heavy-duty trucks and 65.6% for light-duty trucks compared with only 3.3% for passenger cars between 1990 and 2006 (US DOT, 2010).

The US currently has energy policies in place with the aim of reducing fossil-fuel use so as to reduce GHG emissions, break dependency on foreign oil, increase homeland security and support renewable energy use (e.g. the Energy Policy Act (EPAct), 1992, 2005; Executive Order (EO) 13423 and the Energy Independence and Security Act (EISA), 2007). These policies have led to the creation of regulations, mandates, tax incentives, etc. that motivate or require companies and agencies to use AFVs. In fact, federal agencies with a fleet of 20 motor vehicles or more are required to reduce petroleum consumption by a minimum of 2% per year through the end of fiscal year 2015 from the 2005 baseline usage. These agencies are required by executive

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order to increase their alternative fuel use by 10% per year relative to the previous year (EO 13423, 2007). This executive order replaced an earlier order (EO 13149, 2000) requiring a 20% reduction in petroleum use by 2005 in comparison to base year 1999. The replacement was needed, because no EPAct-covered agency could meet the reduction goal due to insufficient alternative fueling infrastructure. Federal fleets are also required to maximize use of diesel with biodiesel blends (B20) by replacing medium- and heavy-duty gasoline vehicles with diesel vehicles that can use such biodiesel blends. This requirement applies to agencies at locations where there is sufficient B20 infrastructure (current or planned). In addition, the US DOE sponsors a program called Clean Cities (US DOE, 2011a) with over 100 local coalitions to support reduction in petroleum use in the transportation sector.

Agencies consider numerous factors in the selection of a particular vehicle type, including fuel availability and geographic distribution of fueling stations in the service area, vehicle driving range, vehicle and fuel cost, fuel efficiency, and fleet maintenance costs. The lack of a national infrastructure for refueling AFVs presents a significant obstacle to alternative fuel technology adoption by companies and agencies seeking to transition from traditional gasoline-powered vehicle fleets to AFV fleets (Melaina and Bremson, 2008). In fact, approximately 98% of the fuel used in the federal government's 138,000 AFV fleet (of which, 92.8% in 2008 are flex-fuel vehicles that can run on gasoline or ethanol-based E85 fuel) continues to be conventional gasoline as a result of a lack of opportunity for refueling using the alternative fuel for which the vehicles were designed (US DOE, 2010). Moreover, existing alternative fueling stations (AFSs) are distributed unevenly across the country and within specific regions. Additional operational challenges exist as a result of the reduced driving range of most AFVs.

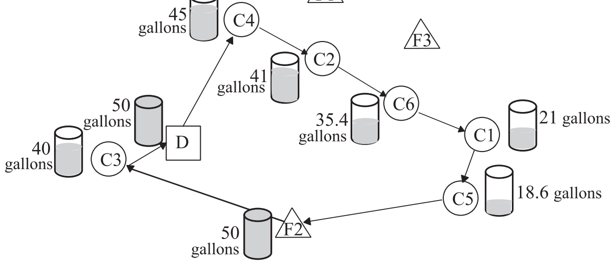
Similar challenges exist for privately owned AFV fleets as noted in various reports (e.g. Chandler et al., 2000, 2002; US DOE, 1997, 2001, 2006; ATA, 2010). FedEx, in its overseas operations, employs AFVs that run on biodiesel, liquid natural gas (LNG) or compressed natural gas (CNG). In US operations, hybrid vehicles have dominated, while LPG, biodiesel and CNG use is limited to regions with access to appropriate AFSs (Bohn, 2008).

This paper is concerned with those companies or agencies that employ a fleet of vehicles to serve customers or other entities located over a wide geographical region. Such companies rely on tools to aid in forming low cost tours, so as to save money and time resulting from travel to customer locations. These routes typically begin at a depot, visit multiple customers and then return to the depot. The problem of assigning customers to vehicles and ordering the customer visits in forming these tours is known as the Vehicle Routing Problem (VRP). A variant of the VRP, the Green Vehicle Routing Problem (G-VRP), is introduced herein that accounts for the additional challenges associated with operating a fleet of AFVs.

In this paper, techniques are developed to aid an organization with an AFV fleet in overcoming difficulties that exist as a result of limited refueling infrastructure. These techniques plan for refueling and incorporate stops at AFSs so as to eliminate the risk of running out of fuel while maintaining low cost routes. The G-VRP is formulated as a mixed-integer linear program (MILP). Given a complete graph consisting of vertices representing customer locations, AFSs, and a depot, the G-VRP seeks a set of vehicle tours with minimum distance each of which starts at the depot, visits a set of customers within a pre-specified time limit, and returns to the depot without exceeding the vehicle's driving range that depends on fuel tank capacity. Each tour may include a stop at one or more AFSs to allow the vehicle to refuel en route.

The G-VRP is illustrated on a simple example problem in Fig. 1. This example involves only one truck with a fuel tank capacity of $Q = 50$ gallons and fuel consumption rate of $r = 0.2$ gallons per mile (or 5 miles per gallon fuel efficiency (Fraer et al., 2005). Three AFSs are available in the region. The vehicle begins its tour at depot D and must visit customers C1–C6 before returning to the depot. To visit these customers, a minimum distance of 339 miles must be traversed. Travel of such a distance would consume 67.8 gallons, 17.8 more gallons of fuel than the vehicle's tank can hold. Thus, the vehicle needs to visit at least one AFS in order to serve all customers and return to depot D. The G-VRP takes into account the vehicle's fuel tank capacity limitation and chooses the optimal placement of AFS visits within the tour. Accounting for fuel limitations, the optimal solution to the G-VRP involves a stop at one AFS and requires the traversal of 354 miles. Thus, the tour length is 15 miles longer than the minimum tour length, where fuel tank capacity is assumed to be unlimited.

As the VRP is known to be an NP-hard problem (indicating that the computational effort required for its solution grows exponentially with increasing problem size), and the VRP is a special case of the G-VRP, the G-VRP is NP-hard. Thus, exact solution of large, real-world problem instances will be difficult to obtain. Two heuristics, the Modified Clarke and Wright



	D/F_0	C1	C2	C3	C4	C5	C6	F1	F2	F3
D/F_0	0	148	55	32	70	140	73	60	45	75
C1	-	0	93	180	99	12	72	102	80	40
C2	-	-	0	85	20	83	28	45	75	90
C3	-	-	-	0	100	174	99	90	50	93
C4	-	-	-	-	0	85	49	25	60	55
C5	-	-	-	-	-	0	73	89	55	80
C6	-	-	-	-	-	-	0	50	85	64
F1	-	-	-	-	-	-	-	0	100	82
F2	-	-	-	-	-	-	-	-	0	58
F3	-	-	-	-	-	-	-	-	-	0

Fig. 1. Illustrative example of a solution to the G-VRP.

Savings (MCWS) heuristic and the Density-Based Clustering Algorithm (DBCA), along with a customized improvement technique, are proposed for solution of such larger problem instances. These techniques are intended to provide decision support for a company or agency operating a fleet of AFVs for which limited fueling stations exist. These heuristics provide fast solution capability. Their steps show how the additional problem constraints can be tackled within construction and improvement heuristics. Moreover, they provide intuition for the development of more sophisticated implementations. A natural extension, for example, would be to incorporate the proposed concepts within a tabu search procedure. Numerical experiments were designed and conducted to assess heuristic performance as a function of customer location configuration, and station density and distribution. The techniques are also applied on a hypothetical problem instance meant to replicate a medical textile supplier company's daily operations in the Washington, DC metropolitan area.

2. Background

A number of works in the literature present optimization-based approaches designed specifically for siting AFSs. The majority of these works were motivated by the Hydrogen Program that was created during the G.W. Bush administration and supported by a diverse group of governmental and private sponsors (Nicholas et al., 2004; Kuby and Lim, 2005, 2007; Upchurch et al., 2007; Lin et al., 2008a,b; Bapna et al., 2002). Other works focus on military applications and consider issues pertaining to the limited capacity of fuel tanks (e.g. Mehrez et al., 1983; Mehrez and Stern, 1985; Melkman et al., 1986; Yamani et al., 1990; Yuan and Mehrez, 1995). Numerous works address the classical VRP with capacity and distance constraints (e.g. Laporte et al., 1985); however, such works do not consider the opportunity to extend a vehicle's distance limitation as a consequence of actions taken while en route. Of greater relevance is the multi-depot VRP in which vehicles can stop at satellite facilities (also referred to as replenishment or inter-depot facilities) to replenish or unload (e.g. Bard et al., 1998; Chan and Baker, 2005; Crevier et al., 2007; Kek et al., 2008; Tarantilis et al., 2008). Such opportunity for reloading aims to overcome capacity limitations of the vehicles, thus, permitting longer routes and reduced return travel to the central depot. In another related work, Ichimori and Hiroaki (1981) addressed a shortest path problem for a single vehicle en route to a single destination in which stops to refuel are explicitly considered.

It appears that no work in the literature directly addresses the G-VRP or a direct variant thereof. While solution techniques developed to address related problems cannot be applied directly in solution of the G-VRP in which fuel tank limits guide distances that can be traveled, the MILP formulation of the G-VRP developed in the next section builds on concepts conceived in (Bard et al., 1998). Bard et al. formulated a VRP with Satellite Facilities (VRPSF) problem as an MILP with capacity and tour duration limitation constraints. Vehicles with capacity limitations have the option to stop at satellite facilities to reload in order to serve customer demand at the vertices. Subtour elimination constraints that employ time relationships, as well as concepts used for tracking capacity utilization, employed by Bard et al., are exploited herein.

3. Problem definition and formulation

The G-VRP is defined on an undirected, complete graph $G = (V,E)$, where vertex set V is a combination of the customer set $I = \{v_1, v_2, \ldots, v_n\}$, the depot v_0, and a set of $s \geqslant 0$ AFSs, $F = \{v_{n+1}, v_{n+2}, \ldots, v_{n+s}\}$. The vertex set is $V = \{v_0\} \cup I \cup F = \{v_0, v_1, v_2, \ldots, v_{n+s}\}$, $|V| = n + s + 1$. It is assumed that in addition to the AFSs, the depot can be used as a refueling station and all refueling stations have unlimited capacities. The set $E = \{(v_i,v_j): v_i,v_j \in V, i < j\}$ corresponds to the edges connecting vertices of V. Each edge (v_i,v_j) is associated with a non-negative travel time t_{ij}, cost c_{ij} and distance d_{ij}. Travel speeds are assumed to be constant over a link. In addition, no limit is set on the number of stops that can be made for refueling. When refueling is undertaken, it is assumed that the tank is filled to capacity.

The G-VRP seeks to find at most m tours, one for each vehicle, that starts and ends at the depot, visiting a subset of vertices including AFSs when needed such that the total distance traveled is minimized. Vehicle driving range constraints that are dictated by fuel tank capacity limitations and tour duration constraints meant to restrict tour durations to a pre-specified limit $T_{\max}$, apply. It is assumed that all customers can be served by a vehicle that begins its tour at the depot and returns to the depot after visiting the customer directly within $T_{\max}$. Without loss of generality, to reflect real-world service area designs, it is assumed that all customers can be visited directly by a vehicle beginning and returning to the depot with at most one visit to an AFS. This does not preclude the possibility of choosing a tour that serves multiple customers and contains more than one visit to an AFS.

The formulation distinguishes between visits to AFSs and the depot from customer visits. This is because each AFS may be visited more than once or not at all. In addition, the depot must be visited at the start and end of each tour and can serve, when desired, as an AFS. Customers, on the other hand, must be visited exactly once. To permit multiple (and possibly zero) visits to a subset of the vertices, while requiring exactly one visit to other vertices, graph G is augmented (to create $G' = (V',E')$ with a set of s' dummy vertices, $\Phi = \{v_{n+s+1}, v_{n+s+2}, \ldots, v_{n+s+s'}\}$, one for each potential visit to an AFS or depot serving as an AFS. $V' = V \cup \Phi$. Associated with each refueling station $v_f \in F$ is n_f dummy vertices for $f = 0, \ldots, n + s$. The number of dummy vertices associated with each AFS, n_f, is set to the number of times the associated v_f can be visited. n_f should be set as small as possible so as to reduce the network size, but large enough to not restrict multiple beneficial visits. This technique involving dummy vertices was introduced by Bard et al. (1998) for their application involving stops at intermediate depots for reloading vehicles with goods for delivery.

Additional notation used in formulating the G-VRP is defined next.

I_0 Set of customer vertices and depot, $I_0 = \{v_0\} \cup I$
F_0 Set of AFS vertices and depot, $F_0 = \{v_0\} \cup F'$, where $F' = R \cup \Phi$
p_i Service time at vertex i (if $i \in I$, then p_i is the service time at the customer vertex, if $i \in F$, p_i is the refueling time at the AFS vertex, which is assumed to be constant)
r Vehicle fuel consumption rate (gallons per mile)
Q Vehicle fuel tank capacity
Decision variables
x_{ij} Binary variable equal to 1 if a vehicle travels from vertex i to j and 0 otherwise
y_j Fuel level variable specifying the remaining tank fuel level upon arrival to vertex j. It is reset to Q at each refueling station vertex i and the depot
τ_j Time variable specifying the time of arrival of a vehicle at vertex j, initialized to zero upon departure from the depot

The mathematical formulation of the G-VRP is as follows:

$$\min \sum_{\substack{i,j \in V' \\ i \neq j}} d_{ij} x_{ij} \tag{1}$$

s.t.

$$\sum_{\substack{j \in V' \\ j \neq i}} x_{ij} = 1, \quad \forall i \in I \tag{2}$$

$$\sum_{\substack{j \in V' \\ j \neq i}} x_{ij} \leqslant 1, \quad \forall i \in F_0 \tag{3}$$

$$\sum_{\substack{i \in V' \\ j \neq i}} x_{ji} - \sum_{\substack{i \in V' \\ j \neq i}} x_{ij} = 0, \quad \forall j \in V' \tag{4}$$

$$\sum_{j \in V' \setminus \{0\}} x_{0j} \leqslant m \tag{5}$$

$$\sum_{j \in V' \setminus \{0\}} x_{j0} \leqslant m \tag{6}$$

$$\tau_j \geqslant \tau_i + (t_{ij} - p_j)x_{ij} - T_{\max}(1 - x_{ij}), \quad i \in V', \forall j \in V' \setminus \{0\} \text{ and } i \neq j \tag{7}$$

$$0 \leqslant \tau_0 \leqslant T_{\max} \tag{8}$$

$$t_{0j} \leqslant \tau_j \leqslant T_{\max} - (t_{j0} + p_j), \quad \forall j \in V' \setminus \{0\} \tag{9}$$

$$y_j \leqslant y_i - r \cdot d_{ij} x_{ij} + Q(1 - x_{ij}), \quad \forall j \in I \text{ and } i \in V', i \neq j \tag{10}$$

$$y_j = Q, \quad \forall j \in F_0 \tag{11}$$

$$y_j \geqslant \min\{r \cdot d_{j0}, r \cdot (d_{jl} + d_{l0})\}, \quad \forall j \in I, \forall l \in F' \tag{12}$$

$$x_{i,j} \in \{0, 1\}, \quad \forall i, j \tag{13}$$

The objective (1) seeks to minimize total distance travelled by the AFV fleet in a given day. Constraints (2) ensure that each customer vertex has exactly one successor: a customer, AFS or depot vertex. Constraints (3) ensure that each AFS (and associated dummy vertices) will have at most one successor vertex: a customer, AFS or depot vertex. Flow conservation is ensured through constraints (4) by which the number of arrivals at a vertex must equal the number of departures for all vertices. Constraints (5) ensure that at most m vehicles are routed out of the depot and constraints (6) ensure that at most m vehicles return to the depot in a given day. A copy of the depot is made to distinguish departure and arrival times at the depot, which is necessary for tracking the time at each vertex visited and preventing the formation of subtours. The time of arrival at each vertex by each vehicle is tracked through constraints (7). Constraints (7) along with constraints (8) and (9) make certain that each vehicle returns to the depot no later than $T_{\max}$. Constraints (8) specify a departure time from the depot of zero ($\tau_0 = 0$) and an upper bound on arrival times upon return to the depot. Lower and upper bounds on arrival times at customer and AFS vertices given in constraints (9) ensure that each route is completed by $T_{\max}$. Constraints (10) track a vehicle's fuel level based on vertex sequence and type. If vertex j is visited right after vertex i ($x_{ij} = 1$) and vertex i is a customer vertex, the first term in constraints (10) reduces the fuel level upon arrival at vertex j based on the distance traveled from vertex i and the vehicle's fuel consumption rate. Time and fuel level tracking constraints, constraints (7) and (10), respectively, serve to eliminate the possibility of subtour formation. Constraints (11) reset the fuel level to Q upon arrival at the depot or an AFS vertex. Constraints (12) guarantee that there will be enough remaining fuel to return to the depot directly or by way of an AFS from any customer location en route. This constraint seeks to ensure that the vehicles will not be stranded. One could extend this constraint to permit return paths that visit more than one AFS. These constraints are implemented through the Java CPLEX interface using if–then logic. Finally, binary integrality is guaranteed through constraints (13).

The main difficulty in solving any VRP is ensuring that subtours will not be created. In traditional VRP formulations, a set of constraints known as subtour elimination constraints are included. In the G-VRP formulation presented herein, subtours are prevented through the combination of constraints (7)–(9) acting together.

The formulation of the G-VRP presented in this section builds on the VRPSF formulation by Bard et al. (1998) designed for a delivery routing problem with satellites at which goods can be loaded en route to customers. Similar notation was employed where possible. The G-VRP differs from the VRPSF in several substantial ways. First, the VRPSF does not consider distance restrictions based on fuel tank capacity. As such, the possibility of running out of fuel en route to a customer is not considered. Second, fuel is consumed along the network edges, while goods are consumed at the network vertices. Thus, capacity limitations associated with the VRPSF cannot serve in modeling fuel usage limitations. Third, determination of upper and lower bounds on arrival times at the vertices are complicated by refueling needs. This is because there are many more combinations of possible vertex sequences than in the VRPSF and the number of AFSs in an instance of the G-VRP will likely exceed the number of satellite facilities in a typical VRPSF. The additional combinations are due to the fact that in the G-VRP, it is possible that refueling will be required even before arriving at a single customer and travel to a refueling station must be considered from every customer en route. This differs from the VRPSF, where reloading at a satellite facility need only be considered when supplies (i.e. goods) must be replenished. Finally, satellite facilities are strategically located, while locations of the AFSs are typically beyond the company's control, possibly affecting the difficulty associated with determining good routes.

4. Solution of the G-VRP

The vehicle driving range (or fuel tank capacity) limitations and existence of a subset of vertices (the AFSs) that can, but need be, visited, as well as the possibility of extending a vehicle's driving range as a result of a visit to a site along the tour, introduce complications that are not present in classical VRPs or most variants thereof. Thus, heuristics designed for the classical VRP or related variants cannot be applied directly in solving the G-VRP. Not only might such heuristics result in solutions that perform poorly, but these solutions may not even be feasible. Two heuristics customized for the G-VRP are proposed herein for solution of large problem instances: the MCWS heuristic and DBCA. The Clarke and Wright Savings algorithm (Clarke and Wright, 1964) designed for the classical VRP, and customized for its variants, is modified to create the MCWS heuristic so as to tackle the challenges introduced by the G-VRP. The DBCA builds on concepts from the Density Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm proposed in Ester et al. (1996), for the purpose of discovering clusters of arbitrary shapes in large spatial databases, such as satellite images and X-rays. In addition, two tour improvement techniques involving within-tour edge interchanges and across-tour vertex exchanges designed for the G-VRP that can be applied in series once a tour is constructed are presented herein.

4.1. The MCWS Heuristic

MCWS heuristic

Step 1: Create *n back-and-forth* vehicle tours (v_0–v_i–v_0), each starting at the depot v_0, visiting a customer vertex $v_i \in I$ and ending at the depot. Add each created tour to the *tours list*.

Step 2: Calculate the tour duration and distance for all tours in the *tours list*. Check for feasibility of all initial *back-and-forth* tours with regard to driving range and tour duration limitation constraints and categorize them as feasible or infeasible. Place all feasible tours in the *feasible tours list* and the remainder in the *infeasible tour list*.

Step 3: For each tour in the *infeasible tour list*, calculate the cost of an AFS insertion between customer vertices v_i and the depot v_0, $c(v_i, v_0) = d(v_i, v_f) + d(v_f, v_0) - d(v_i, v_0)$ for every AFS ($v_f \in F'$). For every such tour, insert an AFS with the least insertion cost. If both driving range and tour duration limitation constraints are met after the insertion of an AFS, add the resulting tour to the *feasible tours list*. If the driving range constraint is not met with the addition of any AFS, discard the tour. No starting tour containing more than one AFS is considered.

Step 4: Compute the savings associated with merging each pair of tours in the *feasible tours list*. To do so, first identify all vertices that are adjacent to the depot in a tour. Create a *savings pair list* (*SPL*) that includes all possible pairs of these vertices (v_i, v_j) with the condition that each pair is formed by vertices that belong to different tours. Compute the savings associated with each pair of vertices in the *SPL*, $s(v_i, v_j) = d(v_0, v_i) + d(v_0, v_j) - d(v_i, v_j)$, where $((v_i, v_j) \in I \cup F')$. Rank the pairs in the *SPL* in descending order of savings $s(v_i, v_j)$.

Step 5:

While SPL is not empty

Select and remove the topmost pair of vertices (v_i, v_j) in the *SPL* and merge their associated tours.

For the selected (v_i, v_j), check driving range and tour duration limitation constraints.

If both constraints are met, add the resulting tour to the *feasible tours list*.

If the resulting tour duration is less than $T_{\max}$, but violates the driving range constraint, compute the insertion cost $c(v_i, v_j) = d(v_i, v_f) + d(v_f, v_j) - d(v_i, v_0) - d(v_j, v_0)$ for savings pair $((v_i, v_j) \in I \cup F')$ for every AFS ($v_f \in F'$). Insert the

AFS between v_i and v_j with the least insertion cost for which the resulting tour is feasible. Check for redundancy: If the tour contains more than one AFS, consider whether it is possible to remove one or more of the AFSs from the tour. Remove any redundant AFSs. Add the resulting tour to the *feasible tours list*.

If any tour has been added to the *feasible tours list*, return to Step 4. Otherwise, *stop*.

The MCWS heuristic terminates with a set of tours that together form a feasible solution to the G-VRP in which constraints (5) and (6) are relaxed. The heuristic continues until no tours in the *feasible tour list* can be further merged. The number of tours in the final *feasible tours list* is the smallest that can be attained through the merge process of Step 5. This procedure is consistent with including a secondary objective of minimizing fleet size. If the final number of tours is less than m, then the entire set of customers can be served with fewer than m vehicles. If it is greater than m, then the heuristic was unable to obtain a solution with m or fewer vehicles. The best solution obtained, i.e. with the smallest number of required vehicles, is provided. This relaxation of constraints (5) and (6) in this way, as opposed to declaring infeasibility, permits the decision-maker to consider the impact of conversion to alternative fuels with limited refueling stations on needed fleet size.

An intrinsic quality of solutions of nearly all VRPs and their variants is acyclicity. Moreover, in most variants, every vertex must be visited once and only once. In the G-VRP, cycle formation is allowed and AFS vertices can be visited more than once, by more than one vehicle, or not visited at all. This is illustrated through a series of small examples depicted in Fig. 2. In Fig. 2a, a single vehicle visits F1 twice, forming a cycle. In Fig. 2b, there are two vehicles visiting F1 once each. These sequences allow an AFS vertex to be visited by more than one vehicle. In Fig. 2c, F1 is not visited at all.

Fig. 3 illustrates additional characteristics of this problem class that affect the merging process. As depicted in Fig. 3a, two tours that visit the same AFS can be merged with only a deletion in the links incident on the depot. No additional links are

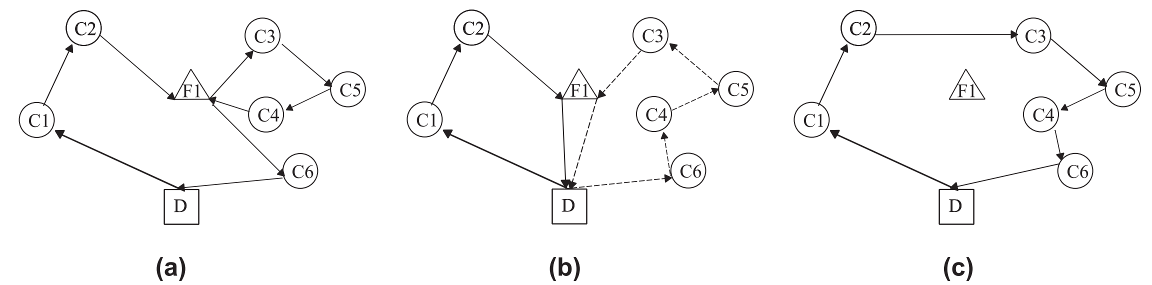


Fig. 2. Possible feasible G-VRP solutions.

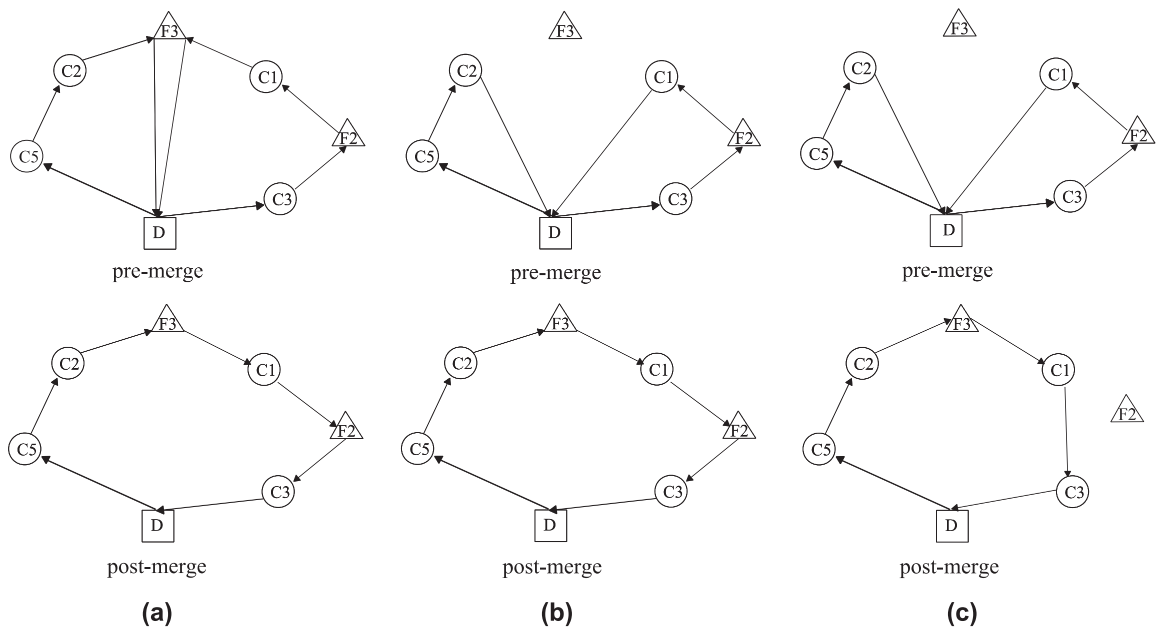


Fig. 3. Characteristics of merging in the G-VRP.

required. Moreover, tours that cannot be merged directly may be merged if an AFS is included as depicted in Fig. 3b. When a tour containing an AFS is included in a merge that involves an additional AFS visit, as in 3b, it may be that inclusion of an AFS from an original tour is redundant. This AFS can be dropped from the final post-merge tour, resulting in, for example, the tour depicted in Fig. 3c.

4.2. The Density-Based Clustering Algorithm

A second heuristic, the DBCA, is introduced that exploits the spatial properties of the G-VRP. The relative location of customers and AFSs, as well as their distributions over space, significantly affect feasibility and number of required AFS visits. Like many clustering approaches, the DBCA decomposes the VRP into two clustering and routing subproblems.

The key idea of the DBCA is that for each vertex of a cluster, the neighborhood of a given radius (ε) must contain at least a minimum number of vertices (*minPts*). That is, a density threshold is employed with *minPts*. Fig. 4 illustrates the DBCA on a 20 customer and three AFS example, where clusters are formed for $minPts \geqslant 4$ and $\varepsilon = 30$ miles.

The ε-neighborhood of a vertex v_j, denoted by $N_\varepsilon(v_j)$, is defined by the set of vertices that are within a radius of ε from $v_j, N_\varepsilon(v_j) = \{v_i \in V' | d_{ij} \leqslant \varepsilon\}$ (*Definition 1*, Ester et al., 1996). By using ε-neighborhood notation, a cluster can be formed by ensuring that each constituent vertex has at least *minPts* vertices in its ε-neighborhood (e.g. the ε-neighborhood of vertex 5, for $\varepsilon = 30$ miles, includes four vertices as depicted in Fig. 4). A vertex v_i is said to be *directly density-reachable* from a vertex v_j with respect to ε and *minPts* if the following conditions are satisfied (*Definition 2*, Ester et al., 1996):

i. $v_i \in N_\varepsilon(v_j)$ and
ii. $|N_\varepsilon(v_j)| \geqslant minPts$

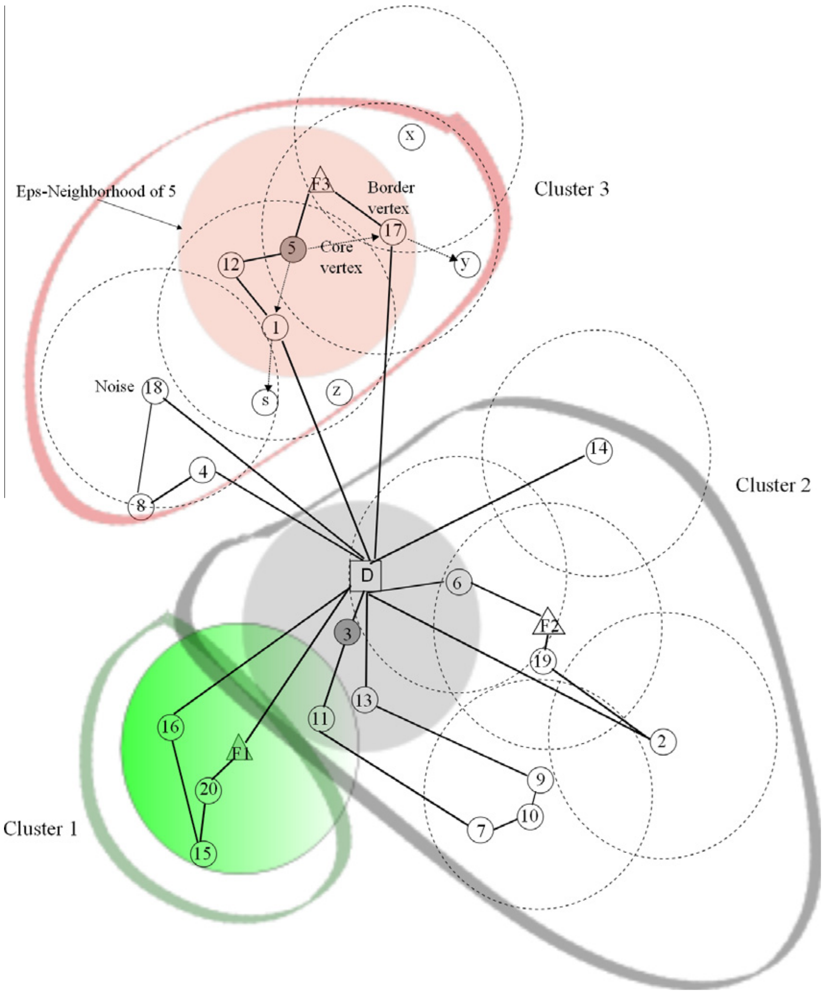


Fig. 4. Forming clusters by DBSCAN Algorithm.

According to this definition v_i is direct-density reachable from v_j, but the opposite may not always be true if $|N_\varepsilon(v_i)| < minPts$ (i.e. condition (*ii*) is not met). Condition (*ii*) is called the *core vertex condition*. Vertices that do not satisfy this condition are called *noise* vertices. For example, in Fig. 4, vertices 17, F3, 12 and 1 are border vertices, and are directly density reachable from vertex 5. However, vertex 5 is not direct-density reachable from any of these vertices. Thus, vertex 5 is a core vertex and is used as a seed to form cluster 3.

A vertex v_i is *density-reachable* from a vertex v_m with respect to ε and *minPts* if there is a chain of vertices that satisfy direct density-reachability for each consecutive vertex pair (*Definition 3*, Ester et al., 1996). In Fig. 4, vertices *y* and *x* are density-reachable from vertex 5 via vertex 17. Density-reachability is a transitive, but not symmetric relation. A vertex v_i is *density-connected* to a vertex v_p with respect to ε and *minPts* if there is a vertex v_m such that both v_i and v_p are density reachable from v_m (*Definition 4*, Ester et al., 1996). For example, vertices *y* and *s* are density-connected through vertex 5 in Fig. 4. Using these concepts, clusters are formed by identifying sets of *density-connected* vertices based on a core vertex. Elements of each set are assigned a common cluster ID. In Fig. 4, three core vertices are identified (5, 3 and F1) and three clusters are formed.

Notation used in the DBCA are given next, followed by details of the DBCA.

m	number of required routes corresponding to number of clusters
ε	radius parameter used in determining a vertex' ε-neighborhood
minPts	minimum number of vertices in an ε-neighborhood of a vertex
$[\varepsilon_{\min}, \varepsilon_{\max}]$	search interval for ε
$[minPts_{\min}, minPts_{\max}]$	the interval for density threshold for which DBCA searches for different clustering schemes

DBCA ($[\varepsilon_{\min}, \varepsilon_{\max}]$ and $[minPts_{\min}, minPts_{\max}]$)

Step 1: Clustering

For each combination of ε and *MinPts*:
- For all v_i in V'
  - Determine the ε-neighborhood of vertex v_i with respect to ε and *minPts*
  - If v_i satisfies core vertex condition (*ii*), v_i is a core vertex. Assign a cluster ID to vertex v_i and all vertices in its ε-neighborhood.
- For each vertex v_i with a cluster ID
  - For each v_j with no cluster ID that is density connected to vertex v_i
    - Assign the cluster ID of v_i to v_j.
- For each vertex v_i with no cluster ID
  - Assign the cluster ID of the vertex v_j with cluster ID closest to v_i.

This step ends with a set of clusters for each combination of ε and *MinPts* pair. The depot is added to any cluster in which it is not already included.

Step 2: Routing

For each set of clusters corresponding to each pairing of ε and *MinPts*
- Run MCWS to construct vehicle tours.

Step 3: Identify Set of Routes

Calculate the total distance traveled by all vehicles for the resulting set of tours corresponding to each (ε, *MinPts*) pair from Step 2 and identify the parameter combination (ε, *MinPts*) that results in the least distance traveled and output the corresponding set of tours.

Like the MCWS heuristic, the DBCA terminates with a set of tours that form a feasible solution to the G-VRP for which constraints (5) and (6) have been relaxed.

In typical cluster-first, route-second heuristics for the VRP, customers in a single cluster are served with a single vehicle and clusters are formed such that vehicle capacity limitations are not exceeded. However, in the DBCA, clusters are formed without regard for imposed limitations, because there is no simple check to ensure that customers in a cluster can be served by a single vehicle without violating tour duration and vehicle driving range constraints. Thus, more than one tour may be required to serve customers in a given cluster. For example, two tours are formed in cluster 3 and three in cluster 2 as shown in Fig. 4.

4.3. Improvement heuristic

The MCWS heuristic and DBCA construct a set of feasible tours. An improvement technique can be applied on the resulting set of feasible tours in an effort to reduce the total distance that must be traveled. Concepts involving inter-tour vertex exchange and within-tour two-vertex interchange and reordering are customized for the G-VRP. Beginning with a set of tours, inter-tour vertex exchange is applied by considering an exchange of one vertex between every pair of tours. For each pair of tours, two vertices are selected for a position exchange. If the total distance of both tours together is reduced as a result of the exchange and steps can be taken to maintain feasibility, the exchange is executed. Within-tour two-vertex interchange and reordering is applied next in which every pair of vertices is considered for an exchange. The position within the tour of the two chosen vertices is exchanged, creating a new tour ordering. If the new tour ordering is infeasible, the exchange is not performed. Otherwise, if one or both of the chosen vertices for the exchange are AFSs, AFS redundancy is checked and AFS relocation or exchange with an alternate unscheduled AFS is considered so as to minimize the tour length. The improvement heuristic terminates with a set of tours for the G-VRP for which constraints (5) and (6) have been relaxed. The total distance required to carry out the tours will be no worse than that required of the initial tours to which the procedure is applied.

5. Numerical experiments

Numerical experiments were conducted to assess the quality of solutions obtained through the proposed heuristics on randomly generated small problem instances through comparison with exact solutions obtained through direct solution of the G-VRP formulation. The experiments were devised to allow consideration of the impact of customer and AFS location configuration and AFS density on the solution. A larger, more realistic G-VRP was devised using a medical textile supply company's depot location in Virginia. A customer pool for this company was created based on hospital locations in Virginia (VA), Maryland (MD) and the District of Colombia (DC) using Google Earth. Conversion to biodiesel (B20 or higher) was considered, because of the modest density of biodiesel fueling stations in the region. Such conversion will lead to significant reductions in carbon monoxide, particulate matter, sulfates, and hydrocarbon as compared with diesel fuel, as well as lifecycle GHG emissions (US EPA, 2002). Actual biodiesel stations located in the region in the summer of 2009 were obtained from a US DOE website (US DOE, 2009). Experiments were designed to analyze the impact of fleet conversion for this company to biodiesel using the developed heuristics.

In both sets of experiments, unless otherwise stated, a fuel tank capacity of 60 gallons and fuel consumption rate of 0.2 gallons per mile were set based on average values for biodiesel-powered AFVs (Fraer et al., 2005). The average vehicle speed is assumed to be 40 miles per hour (mph) and the total tour duration limitation was assumed to be 11 h. Service times were assumed to be 30 min at customer locations and 15 min at AFS locations.

The construction and improvement heuristics were implemented in Java and compiled using Eclipse. Exact solutions were obtained by implementing the model using ILOG's CPLEX Concert Technology (version 11.2, 2009) in Java, which allowed Java objects to be used in building the optimization model. The experiments were run on a desktop with Pentium (4) CPU, 32-bit platform with 3.20 GHz processor and 2.00 GB of RAM, while ILOG CPLEX runs were made on a Xeon (R) CPU 5160 3.00 GHz processor, 64-bit platform with 16.00 GB of RAM.

5.1. Experiments on small instances

Random problem instances were generated so as to maintain the properties of one of four general scenario categories as defined in Table 1.

Each instance was randomly generated assuming a grid of 330 by 300 miles based on an area similar in size to MD, VA and the DC. The depot location was fixed and assumed to be located near the center of the grid in all scenarios. Three AFSs were fixed and assumed to be located between the depot and the grid boundaries in westerly, northerly and southeasterly directions for S1 and S2. Specific instances for each scenario are identified by an alternating pattern of numbers and letters indicating, e.g. in 20c3sU1, the number of customers (20), AFSs (3), how the AFSs are distributed over space (U or C indicating that they are uniformly distributed or clustered, respectively), and instance number (1–10 for each instance) for S1 and S2. For S3 and S4, the pattern indicates the S1 or S2 instance (Scenario 1, instance 2 in S1_2i6s) and number of AFSs (6 AFSs in S1_2i6s).

The computation time limit in CPLEX was set to 100,000 s with an optimal solution tolerance of 10^{-3}. The results are presented in Tables 2–5. In these tables, the values of the objective function obtained for each instance is given in two lines, the first which provides the heuristic objective function value prior to implementation of the improvement techniques and the second which gives the objective function value (in italics) after the improvement techniques have been applied. The computational times required to run the heuristics were on the order of seconds and are not reported here. The DBCA was run multiple times, each for a different (ε, *minPts*)-pair. The best achieved results are provided.

Table 1
Small instance test scenario categories.

Scenario	Description	Details
S1	Impact of spatial customer configuration (uniform)	10 randomly generated instances of 20 uniformly distributed customer locations with 3 fixed AFS locations
S2	Impact of spatial customer configuration (clustered)	10 instances of 20 clustered customer locations with 3 fixed AFS locations
S3	Impact of spatial AFS configuration	10 instances, half selected from S1 and half from S2, each instance with 6 AFSs generated randomly
S4	Impact of station density	10 instances, half created from 1 instance of S1 and half from 1 instance of S2, by increasing the number of AFSs gradually from 2 to 10 in increments of 2

Table 2
S1, Impact of spatial customer configuration (uniform) results.

Sample	CPLEX			MCWS		DBCA $15 \leqslant \varepsilon \leqslant 150$, $1 \leqslant minPts \leqslant 10$	
	Exact solution (miles)	Number of tours	Customers served	Total cost (miles)	Difference (%)	Total cost (miles)	Difference (%)
20c3sU1	1797.51	6	20	1843.52	2.56	1843.52	2.56
				1818.35	1.16	*1797.51*	0.00
20c3sU2	1574.82	6	20	1614.15	2.50	1614.14	2.50
				1614.15	2.50	*1613.53*	2.46
20c3sU3	1765.9	7	20	1969.64	11.54	1969.64	11.25
				1969.64	11.54	*1964.57*	11.25
20c3sU4	1482.00	5	20	1513.45	2.12	1508.41	1.78
				1508.41	1.78	*1487.15*	0.35
20c3sU5	1689.35	6	20	1802.93	6.72	1802.93	6.72
				1752.73	3.75	*1752.73*	3.75[a]
20c3sU6	1643.05	6	20	1713.39	4.28	1713.39	4.28
				1668.16	1.53	*1668.16*	1.53[a]
20c3sU7	1715.13	6	20	1730.45	0.89	1730.45	0.89
				1730.45	0.89	*1730.45*	0.89
20c3sU8	1709.43	6	20	1766.36	3.33	1766.36	3.33
				1718.67	0.54	*1718.67*	0.54
20c3sU9	1708.84	6	20	1718.43	0.56	1718.43	0.56
				1714.43	0.33	*1714.43*	0.33
20c3sU10	1261.15	5	20	1309.52	3.84	1309.52	3.84
				1309.52	3.84	*1309.52*	3.84
Average					*2.79*		*2.49*

[a] Indicates when a single cluster is formed at the end of the clustering step of DBCA.

Table 3
S2, Impact of spatial customer configuration (clustered) results.

Sample	CPLEX			MCWS		DBCA $15 \leqslant \varepsilon \leqslant 150$, $1 \leqslant minPts \leqslant 10$	
	Exact solution (miles)	Number of tours	Customers served	Total cost (miles)	Difference (%)	Total cost (miles)	Difference (%)
20c3sC1	1235.21	5	20	1340.36	8.51	1340.36	8.51
				1300.62	*5.30*	*1300.62*	*5.30*
20c3sC2	1539.94	5	19	1553.53	0.88	1553.53	0.88
				1553.53	*0.88*	*1553.53*	*0.88*[a]
20c3sC3	985.41	4	12	1083.12	9.92	1083.12	9.92
				1083.12	*9.92*	*1083.12*	*9.92* [a]
20c3sC4	1080.16	5	18	1135.90$^{(5)}$	5.16	1135.90$^{(5)}$	5.16
				1135.90$^{(5)}$	*5.16*	*1091.78*$^{(4)}$	*1.08*
20c3sC5	2190.68	7	19	2190.68	0.00	2190.68	0.00
				2190.68	*0.00*	*2190.68*	*0.00*[a]
20c3sC6	2785.86	9	17	2887.55	3.65	2887.55	3.65
				2883.71	*3.51*	*2883.71*	*3.51*[a]
20c3sC7	1393.98	5	6	1703.40	22.20	1703.40	22.20
				1701.40	*22.05*	*1701.40*	*22.05*[a]
20c3sC8	3319.71	10	18	3319.74	0.00	3319.74	0.00
				3319.74	*0.00*	*3319.74*	*0.00*[a]
20c3sC9	1799.95	6	19	1811.05	0.62	1811.05	0.62
				1811.05	*0.62*	*1811.05*	*0.62*[a]
20c3sC10	2583.42	8	15	2667.23	3.24	2667.23	3.24
				2648.84	*2.53*	*2644.11*	*2.35*
Average					*5.00*		*4.57*

[a] Indicates when a single cluster is formed at the end of the clustering step of DBCA.

To ensure that the results are comparable, the heuristics were run and the number of tours required for the best found solution was used in constraints (5) and (6) of the formulation in obtaining the corresponding optimal solution. When the two heuristics obtained solutions with a different number of tours, as was the case in a few instances, the smaller number of tours was employed in the exact solution. In a number of instances (e.g. S2_4i2s in Table 5), no feasible solution could be obtained. That is, it was not possible to directly visit all customers with one AFS visit, or within the maximum tour duration, requirements of the heuristics. Thus, those customers that could not be served directly with a visit to one AFS or within the maximum tour duration were eliminated from the problem instance. The number of required tours as identified from heuristic solutions and final number of customers considered in each instance are provided in the tables.

Table 4
S3, Impact of spatial AFS configuration results.

Sample	CPLEX			MCWS		DBCA $15 \leqslant \varepsilon \leqslant 150$, $1 \leqslant minPts \leqslant 10$	
	Exact solution (miles)	Number of tours	Customers served	Total cost (miles)	Difference (%)	Total cost (miles)	Difference (%)
S1_2i6s	1578.15	6	20	1614.15	2.28	1614.15	2.28
				1614.15	*2.28*	*1614.15*	*2.28*
S1_4i6s	1438.89	5	20	1599.56$^{(6)}$	*11.17*	1599.56$^{(6)}$	*11.17*
				1561.30$^{(6)}$	*8.51*	*1541.46*$^{(5)}$	*7.13*
S1_6i6s	1571.28	6	20	1626.94	3.54	1626.94	3.54
				1616.20	*2.86*	*1616.20*	*2.86*
S1_8i6s	1692.34	6	20	1937.87$^{(6)}$	14.51	1937.87$^{(7)}$	*14.51*
				1902.51$^{(6)}$	12.42	*1882.54*$^{(6)}$	*11.24*
S1_10i6s	1253.32	5	20	1309.52	4.48	1309.52	4.48
				1309.52	*4.48*	*1309.52*	*4.48*[a]
S2_2i6s	1645.8	6	20	1648.24	0.15	1648.24	0.15
				1645.80	*0.00*	*1645.80*	*0.00*
S2_4i6s	1505.06	6	19	1505.06	0.00	1505.06	0.00
				1505.06	0.00	*1505.06*	0.00[a]
S2_6i6s	2842.08	10	20	3127.43	10.04	3127.43	10.04
				3115.10	*9.61*	*3115.10*	*9.61*[a]
S2_8i6s	2549.98	9	16	2724.12	6.83	2724.12	6.83
				2722.55	*6.77*	*2722.55*	*6.77*
S2_10i6s	1606.65[b]	6	16	2068.93	28.77	2068.93	28.77
				1995.62	24.21	*1995.62*	24.21[a]
Average					*5.21*		*4.93*

[a] Indicates when a single cluster is formed at the end of the clustering step of DBCA.
[b] Best feasible solution found with <11.30% guarantee difference from optimal.

Table 5
S4, Impact of station density results.

Sample	CPLEX			MCWS		DBCA $15 \leqslant \varepsilon \leqslant 150$, $1 \leqslant minPts \leqslant 10$	
	Exact solution (miles)	Number of tours	Customers served	Total cost (miles)	Difference (%)	Total cost (miles)	Difference (%)
S1_4i2s	1582.22	6	20	1589.6	0.47	1589.6	0.47
				1582.2	0.00	*1582.2*	*0.00*[a]
S1_4i4s	1504.1	6	20	1599.6	6.35	1599.6	6.35
				1580.52	5.08	*1580.52*	*5.08*[a]
S1_4i6s	1397.28	5	20	1599.60$^{(6)}$	14.48	1599.6$^{(6)}$	14.48
				1561.29$^{(6)}$	11.74	*1541.46*$^{(5)}$	*10.32*
S1_4i8s	1376.98	6	20	1599.60	16.17	1599.6	16.17
				1561.29	13.39	*1561.29*	*13.39*[a]
S1_4i10s	1397.28	5	20	1568.60	12.26	1568.00	12.22
				1536.04	9.93	*1529.73*	*9.48*
S2_4i2s	1080.16	5	18	1135.8	5.16	1135.89	5.16
				1135.89	5.16	*1117.32*	*3.44*
S2_4i4s	1466.9	6	19	1522.72	3.81	1522.72	3.81
				1522.72	3.81	*1522.72*	3.81[a]
S2_4i6s	1454.96	6	20	1788.22	22.91	1788.22	22.91
				1786.21	22.77	*1730.47*	*18.94*
S2_4i8s	1454.96	6	20	1788.22	22.91	1788.22	22.91
				1786.21	22.77	*1786.21*	*22.77*
S2_4i10s	1454.93[b]	6	20	1787.22	22.84	1787.22	22.84
			20	*1783.63*	22.59	*1729.51*	*18.87*
Average					*10.51*		*9.69*

[a] Indicates when a single cluster is formed at the end of the clustering step of DBCA.
[b] Best feasible solution found with <3.5% guarantee difference from optimal.

It is often in the cases for which the original problem is infeasible that the heuristics perform the worst. The heuristics perform well, however, on average with a gap of 2.64%, 4.78%, 5.07%, and 10.1% from optimal for S1, S2, S3 and S4 instances, respectively, as indicated in Tables 2–5. The performance of the heuristics was better for S1 and S2 instances of Tables 2 and 3 in which there are limited AFSs and their locations are strategically located than for S3 and S4 instances of Tables 4 and 5 in which there are double the numbers of AFSs, but their locations were randomly chosen. In many instances, the heuristics find the optimal solution, but in the worst-case, the solution is nearly 23% from optimal. The improvement heuristics contributed modestly to improving the solutions obtained (an average of 0.9% reduction in objective function value for MCWS and 1.5%

for DBCA). In closer investigation of the solutions with higher optimality gaps, it was noted that it was often the case that these solutions differed from the optimal solution in terms of the number of AFS stations included in the solution tours. Thus, including steps in the improvement procedures that permit changes in the number of AFS stations may produce more significant improvements.

In general, the results of the two heuristics were very similar; although, whenever there is a difference in solutions obtained, the DBCA finds the better solution. This similarity in the obtained solutions may be a consequence of the small size of the problem instances. That is, there are few feasible solutions and these techniques often narrow in on the same solutions. Moreover, the heuristics are expected to obtain identical solutions when the DBCA produces a single cluster from the first stage. Those instances in which this arises are noted in Tables 2–5. Out of the 13 instances in which the DBCA produces a better solution than the MCWS, the DBCA's solution uses fewer routes to serve the customers in three instances. While there were differences in the number of AFS visits included in the final tours of all three techniques, no consistent pattern was noted. In approximately half the instances, the heuristics employed one fewer or one additional AFS within the final set of tours as compared with the number employed in the optimal set of tours.

The impact of AFS density is examined in S4 (Table 5). Results of these instances indicate that more customers could be served as the number of AFSs increased. Thus, the number of infeasible instances was reduced. Note that it was not possible to visit all customers in three of the clustered customer instances (S2_4i6s, S2_8i6s, S2_10i6s) despite the increased number of AFS options and different location configurations (Table 4). As the number of AFSs increases, the total cost of the optimal solution decreases for the same number of served customers (Table 5). With a larger number of AFS options, the distance required to incorporate needed AFS visits can only decrease. Of course, whether or not an additional AFS will be beneficial depends on its location.

5.2. Real-world case study

There are 21 publicly available biodiesel stations in VA, MD and DC considered together (US DOE, 2009). Four customer-based scenarios were considered as described in Table 6 in which all 21 AFS locations are considered as options unless otherwise specified.

The MCWS heuristic and DBCA were employed in solving the problem instances. Results from Scenarios 1 and 2 runs are provided in Table 7. Additional runs were made to show the heuristic solution when no driving range limitation (i.e. an infinite fuel tank capacity) is assumed. These results are also provided in Table 7. Results from Scenario 3 and 4 are provided in Figs. 5 and 6, respectively.

The original instance (111c) results in Table 7 are compared with and without driving range limitations. Given the AFS infrastructure, the results indicate that 20 AFVs are required to serve the same number of customers served by 17 vehicles for which no driving range limitations would apply. Additionally, an increase by 19% in driving distance is required to serve the same set of customers when driving range limitations are imposed (i.e. through vehicle fleet conversion to biodiesel AFVs). As the number of customers increased from 200 to 500, the difference between those customers that could not be served when no driving range limitations were enforced as compared to when such limitations were required increased from 2 to 21.

The graph in Fig. 5 indicates that as the number of AFSs increases from 21 to 28 (a roughly 33% increase), the total distance traveled decreases by 295 miles (a roughly 5% decrease). Increased AFS availability can reduce AFV fleet operational costs; however, cost savings depends heavily on the specific locations of the added stations. This is illustrated in the numerical experiments. An increase by three AFSs from 21 to 24 led to a reduction in travel distance by 213 miles as indicated in Fig. 5, but an increase from 24 to 26 AFSs resulted in only a four mile reduction. Thus, it may be beneficial for the company to seek partnerships with agencies or companies that own private fueling stations in well-positioned locations or maintain one or more of its own refueling facilities located strategically within an operational area.

As indicated in Fig. 6, as the driving range is increased from 200 to 400 miles, the required travel distance decreased by 2337 miles. Any increase in driving range beyond 400 miles did not result in an improved solution, indicating that all customers could be served given the 21 AFSs located in the region. For example, a fleet of 25 vehicles each with a 250 mile driving range can serve 107 customers traveling 6835 miles. A fleet of only 17 vehicles would be required to serve all 109 customers if the driving range of the vehicles is increased to 400 miles. Moreover, the total distance required to serve the customers would decrease to 4731 miles based on the heuristic solutions.

Table 6
Real world case study scenarios.

Scenario	Description	Details
1	Transitioning to AFV	111 customers
2	Impact of increasing number of customers	Number of customers increased in increments of 50 from 200 to 500, adding customers at random locations within the study area to customer pool from Scenario 1, keeping AFS locations fixed
3	Impact of increased AFS availability	Identical to Scenario 1, but with additional AFSs located strategically, increased in increments of 2 from 22 to 28
4	Impact of driving range limits	Identical to Scenario 1, but driving range increased from 200 miles to 500 miles in 50 mile increments

Table 7
Heuristic solution results.

Instance	Without driving range limit (MCWS)			Modified Clarke and Wright Algorithm (MCWS)			Density Based Clustering Algorithm (DBCA) $15 \leqslant \varepsilon \leqslant 150$, $1 \leqslant minPts \leqslant 30$		
	Total cost (miles)	Number of tours	Customers served	Total cost (miles)	Number of tours	Customers served	Total cost (miles)	Number of tours	Customers served
111c	4745.90 *4731.22*	17	109	5750.62 *5626.64*	20	109	5750.62 *5626.64*	20	109
200c	9358.63 *9355.56*	32	196	10617.02 *10428.59*	35	190	10617.83 *10413.59*	36	191[a]
250c	11691.43 *11668.388*	40	244	11965.10 *11886.61*	41	235	11965.10 *11886.61*	41	236[a]
300c	14782.08 *14762.41*	50	293	14331.30 *14242.56*	49	281	14331.30 *14229.92*	49	282[a]
350c	17677.70 *17661.00*	59	343	16610.25 *16471.79*	57	329	16610.25 *16460.30*	57	329
400c	19968.97 *19936.75*	67	393	19568.56 *19472.10*	67	378	19196.71 *19099.04*	66	373
450c	23168.02 *21336.91*	77	443	21952.48 *21854.17*	75	424	21952.48 *21854.19*	75	424
500c	25032.38 *25024.94*	83	492	24652.15 *24527.46*	84	471	24652.15 *24517.08*	84	471

[a] Indicates when a single cluster is formed at the end of the clustering step of DBCA.

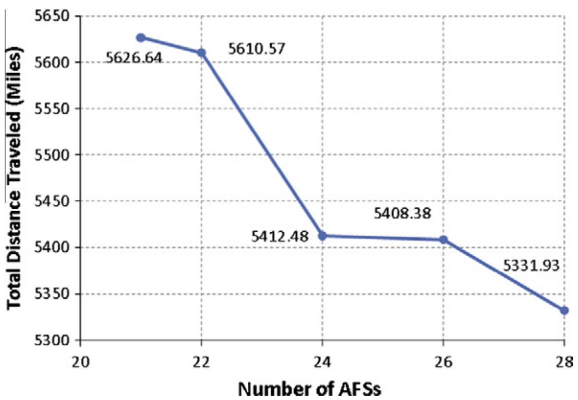
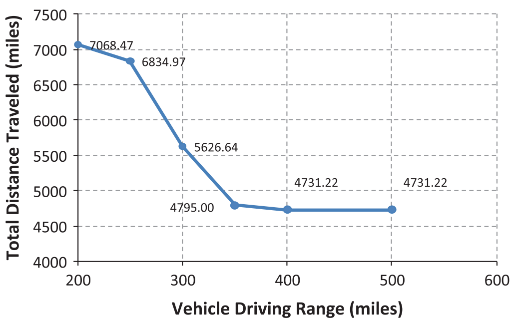


Fig. 5. Effect of increasing AFSs for instance 111c.



Driving range (miles)	Total Cost (miles)	Number of tours	Customers Served
200	7068.47	28	98
250	6834.97	25	107
300	5626.64	20	109
350	4795.00	17	109
400	4731.22	17	109
500	4731.22	17	109

Fig. 6. Effect of vehicle driving range on total distance traveled.

6. Concluding remarks

In this paper, the G-VRP is formulated and techniques were proposed for its solution. These techniques seek a set of vehicle tours that minimize total distance traveled to serve a set of customers while incorporating stops at AFSs in route plans so as to eliminate the risk of running out of fuel. Numerical experiments showed that these techniques perform well compared to exact solution methods and that they can be used to solve large problem instances. The ability to formulate the G-VRP, along with the solution techniques, will aid organizations with AFV fleets in overcoming difficulties that exist as a result of limited refueling infrastructure and will allow companies considering conversion to a fleet of AFVs to understand the potential impact of their decision on daily operations and costs. These techniques can help companies in evaluating possible reductions in the number of customers that can be served or increase in fleet size needed to serve an existing customer base, as well as any increase in required distance traveled as a result of driving range limitations and added fueling stops.

The problem posed herein is likely to exist for many years to come. Alternative fuel and vehicle fuel economy legislation dates back to the Clean Air Act (CAA) of 1970. After four decades, limited infrastructure remains a significant barrier to alternative fuel use adoption in practice. Even today, there are only 7483 AFSs, supporting seven different fuel types in the US (US DOE, 2011b), while there are 161,768 gasoline stations nationwide as of 2007 (US DOE, 2010).

The contributions of this work include: (1) conceptualization of a class of vehicle routing problems involving vehicles with limited fueling capacity and limited fuel station availability; (2) development and testing of efficient heuristics for solution of large, real-world problem instances, including the specific steps for tracking fuel levels as fuel is consumed and replenished, and extending a vehicle's distance limitation by incorporating optional visits to non-unique fueling stations while en route; (3) insights into the impact of geographic distribution of stations and customers on operational viability; and (4) a tool to support institutions in reducing their carbon footprint given currently available vehicle technologies and limited deployment of supporting infrastructure. Thus, concepts proposed herein can be directly applied today and will have applicability in future technology adoption as new technologies are introduced nationwide.

The formulation and solution techniques are applicable for any fuel choice. The techniques account for service times at the stations and, thus, the proposed approach is directly relevant in modeling conversion to electric vehicles in which significant time may be spent at stations for the purpose of recharging the battery and for possible programs that would permit the trading of a depleted battery for a fully charged one while en route. Moreover, this approach can be used in seeking optimal tours for gasoline or diesel powered fleets that involve special refueling arrangements.

The developed formulation and solution techniques presume that fuel usage is directly related to distance traveled. The model could be extended to consider more complex fuel-usage models, consideration of fuel prices and heterogeneous fleets in which vehicles may have different driving range limitations or be powered by different sources of fuel.

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References

ATA (American Trucking Association), 2010. Is Natural Gas a Viable Alternative to Diesel for the Trucking Industry? White Paper 0610, June 2010. <http://www.truckline.com/AdvIssues/Energy/Natural%20Gas/Natural%20Gas%20Alternative%20-%20White%20Paper%200610.pdf> (accessed 23.04.11).

Bapna, R., Thakur, L.S., Nair, S.K., 2002. Infrastructure development for conversion to environmentally friendly fuel. European Journal of Operational Research 1423, 480–496.

Bard, J., Huang, L., Dror, M., Jaillet, P., 1998. A branch and cut algorithm for the VRP with satellite facilities. IIE Transactions 30, 821–834.

Bohn, J., 2008. FedEx Implements Green Fleet Initiative. <http://www.worktruckonline.com/Channel/Fuel-Management/Article/Story/2008/09/FedEx-Implements-Green-Fleet-Initiative.aspx> (accessed 15.09.10).

Chan, Y., Baker, S.F., 2005. The multiple depot, multiple traveling salesmen facility-location problem: vehicle range, service frequency and heuristic implementations. Mathematical and Computer Modeling 41, 1035–1053.

Chandler, K., Norton, P., Clark, N., 2000. Raley's LNG Truck Fleet: Final Results. Alternative Fuel Truck Evaluation Project, Prepared by DOE/NREL.

Chandler, K., Walkowicz, K., Clark, N., 2002. UPS CNG Truck Fleet Results: Final Results. Alternative Fuel Truck Evaluation Project, Prepared by DOE/NREL.

Clarke, G., Wright, J.W., 1964. Scheduling of vehicle from central depot to a number of delivery points. Operations Research 12, 568–581.

Crevier, B., Cordeau, J-F., Laporte, G., 2007. The multi-depot vehicle routing problem with inter-depot routes. European Journal of Operational Research 176, 756–773.

EISA (the Energy Independence and Security Act), 2007. Public Law 110-140, December 19, 2007.

EPAct (The Energy Policy Act), 1992. Public Law 102-486, October 24, 1992.

Executive Order (EO) 13149, 2000. Greening the Government through Federal Fleet and Transportation Efficiency. Federal Register, vol. 65, No. 81, Wednesday, April 26, 2000.

Executive Order (EO) 13423, 2007. Strengthening Federal Environmental, Energy, and Transportation Management. Federal Register, vol. 72, No. 17, Friday, January 26, 2007.

EPAct (The Energy Policy Act), 2005. Public Law 109-58, August 8, 2005.

Ester, M., Kriegel, H.P., Sander, J., Xu, X., 1996. A density-based algorithm for discovering clusters in large spatial databases. In: Proceedings of the International Conference on Knowledge Discovery and Data Mining (KDD.96), Portland, Oregon, pp. 226–231.

Fraer, R., Dinh, H., Kenneth, P., Robert, L., McCormick, C.K., 2005. Operating Experience and Teardown Analysis for Engines Operated on Bio-diesel Blends (B20). SAE Technical Paper No. 2005-01-3641.

Ichimori, T., Hiroaki, I., 1981. Routing a vehicle with the limitation of fuel. Journal of the Operations Research Society of Japan 24 (3), 277–281.

Kek, A.G.H., Cheu, R.L., Meng, Q., 2008. Distance-constrained capacitated vehicle routing problems with flexible assignment of start and end depots. Mathematical and Computer Modelling 47, 140–152.

Kuby, M., Lim, S., 2005. The flow-refueling location problem for alternative-fuel vehicles. Socio-Economic Planning Science 39, 125–145.

Kuby, M., Lim, S., 2007. Location of alternative-fuel stations using the flow-refueling location model and dispersion of candidate sites on arcs. Networks and Spatial Economics 7, 129–152.

Laporte, G., Nobert, Y., Desrochers, M., 1985. Optimal routing under capacity and distance restrictions. Operations Research 33 (5), 1050–1073.

Lin, Z., Ogden, J., Fan, Y., Chena, C.-W., 2008a. The fuel-travel-back approach to hydrogen station siting. International Journal of Hydrogen Energy 33 (12), 3096–3101.

Lin, Z., Chena, C.-W., Ogden, J., Fan, Y., 2008b. The least cost hydrogen for southern California. International Journal of Hydrogen Energy 33 (12), 3009–3014.
Mehrez, A., Stern, H.I., 1985. Optimal refueling strategies for a mixed-vehicle fleet. Naval Research Logistics Quarterly 32, 315–328.
Mehrez, A., Stern, H.J., Ronen, D., 1983. Vehicle fleet refueling strategies to maximize operational range. Naval Research Logistics Quarterly 30, 319–342.
Melaina, M.W., Bremson, J., 2008. Refueling availability for alternative fuel vehicle markets: sufficient urban station coverage. Energy Policy 36 (8), 3223–3231.
Melkman, A., Stern, H.I., Mehrez, A., 1986. Optimal refueling sequence for a mixed fleet with limited refuelings. Naval Research Logistics Quarterly 33, 759–762.
Nicholas, M., Handy, S., Sperling, D., 2004. Using geographic information systems to evaluate siting and networks of hydrogen stations. Transportation Research Record 1880, 126–134.
Tarantilis, C.D., Zachariadis, E.E., Kiranoudis, C.T., 2008. A hybrid guided local search for the vehicle-routing problem with intermediate replenishment facilities. INFORMS Journal on Computing 20 (1), 154–168.
Upchurch, C., Kuby, M., Lim., S., 2007. A capacitated model for location of alternative-fuel stations. Geographical Analysis 41 (1), 85–106.
US DOE, Department of Energy, 1997. Running Line-Haul Trucks on Ethanol: The Archer Daniels Midland Experience. Prepared and Originally Published by the Center for Transportation Technologies at the National Renewable Energy Laboratory.
US DOE, Department of Energy, 2001. UPS CNG Truck Fleet Start-Up Experience. Alternative Fuel Truck Evaluation Project, Prepared by DOE/NREL.
US DOE, Department of Energy, 2006. Federal Fleet Compliance with EPACT and EO 13149, Fiscal Year 2006. <http://www.afdc.energy.gov/afdc/pdfs/ff22_federal_fleet_report.pdf> (accessed 20.11.10).
US DOE, Department of Energy, 2009.The Alternative Fuels and Advanced Vehicles Data Center (AFDC). <http://www.afdc.energy.gov/afdc/locator/stations/state> (accessed 24.06.09).
US DOE, Department of Energy, 2010. Transportation Data Book, Edition 29 by Davis, S.C., Diegel, S.W., Boundy, R.G. Oak Ridge National Laboratory, ORNL-6985.
US DOE, Department of Energy, 2011a. What is Clean Cities? Brochure prepared by the National Renewable Energy Laboratory (NREL), the US DOE, Office of Energy Efficiency and Renewable Energy, DOE/GO-102011-3309, May 2011.
US DOE, Department of Energy, 2011b. The Alternative Fuels and Advanced Vehicles Data Center (AFDC). <http://www.afdc.energy.gov/afdc/fuels/stations_counts.html> (accessed 28.04.11).
US DOT, Department of Transportation, 2010. The Transportation's Role in Reducing US Greenhouse Gas Emissions. <http://ntl.bts.gov/lib/32000/32700/32779/DOT_Climate_Change_Report_-_April_2010_-_Volume_1_and_2.pdf> (accessed 10.11.10).
US EPA, Environmental Protection Agency, 2002. A Comprehensive Analysis of Biodiesel Impacts on Exhaust Emissions. Draft Technical Report, EPA420-P-02-001.
US EPA, Environmental Protection Agency, 2009. Inventory of US Greenhouse Gas Emissions and Sinks: 1990–2007, EPA 430-R-09-004.
US FHWA, Federal Highway Administration, 2008. Highway Statistics 2008. <http://www.fhwa.dot.gov/policyinformation/statistics/2008/vm1.cfm> (accessed 21.10.10).
Yamani, A., Hodgson, T.J., Martin-Vega, L.A., 1990. Single aircraft mid-air refueling using spherical distances. Operations Research 38 (5), 792–800.
Yuan, Y., Mehrez, A., 1995. Refueling strategies to maximize the operational range of a nonidentical vehicle fleet. European Journal of Operational Research 83 (1), 167–181.