Feature & Dimension Analysis

ML Optimization Competition

Complete Analysis Report

Week 7 Comprehensive Study

Analysis Methods:

• Automatic Relevance Determination (ARD)

• Pearson Correlation Analysis

• Gradient Sensitivity Analysis

• Best/Worst Point Comparison

• Multi-Method Consensus

Executive Summary

This report presents a comprehensive feature and dimension importance analysis for 8 optimization functions studied over 6 weeks. Using multiple statistical and machine learning methods, we identified key dimensions driving performance for each function. These insights enabled targeted optimization strategies that achieved breakthrough results, including a 39x improvement on Function 5.

Key Discoveries

Finding	Impact
Function 5: Dimension 3 is 96.6% important	Achieved 5549 points (39x improvement)
Function 7: Dimension 6 is 94% important	Used peer pattern, +72% improvement
Function 3: Dimension 1 is 99.6% important	Simplified optimization strategy
Function 6: Dimension 5 is 89.2% important	Focused trend optimization

Function 1 (2D) - Analysis

Dimension Importance

Status: Limited data (5 weeks of zeros, Week 6 breakthrough)
Challenge: Cannot compute reliable correlations with mostly zero outputs
Week 6 Discovery: First non-zero output (0.0128) at [0.45, 0.45]

Method	Dim 1	Dim 2	Insight
Correlation	N/A	N/A	Insufficient variance
ARD Kernel	Unknown	Unknown	Need more data
Strategy	Equal	Equal	Local exploration

Recommendations

• Continue local exploration around [0.45, 0.45]
• Test gradient ascent in small increments
• Week 7 Strategy: [0.48, 0.48] - symmetric increase

Function 2 (2D) - Analysis

Dimension Importance

Best Performance: 0.1300 (Week 6)
Key Pattern: Both dimensions negatively correlated
Strategy: Push both dimensions LOW

Method	Dim 1	Dim 2	Insight
Correlation	-0.530	-0.649	Both negative
ARD Kernel	48.1%	51.9%	Nearly equal importance
Gradient	Negative	Negative	Decrease both
Recommendation	↓ Lower	↓ Lower	Push toward 0

Week 6 Success Story

ML model correctly predicted improvement by decreasing both dimensions:
• Week 5: [0.08, 0.08] → 0.0463
• Week 6: [0.111, 0.100] → 0.1300 (+181%!)
Lesson: Trust ML when it suggests conservative moves

Week 7 Strategy

• Input: [0.110, 0.099]
• Expected: 0.128
• Method: GP exploitation (maximize mean prediction)

Function 3 (3D) - Analysis

Dimension Importance

Best Performance: -0.0786 (Week 4)
Critical Discovery: Dimension 1 dominates (99.6% importance!)
Challenge: Still producing negative outputs

Method	Dim 1	Dim 2	Dim 3	Insight
Correlation	-0.14	-0.23	-0.35	All weak negative
ARD Kernel	99.6%	0.2%	0.2%	Dim 1 DOMINATES!
Sensitivity	Very High	Very Low	Very Low	Focus on Dim 1
Best Point	0.949	0.966	0.808	Week 4 best
Trend	↑ Higher	↑ Higher	↑ Higher	Push toward 1.0

Week 6 Lesson Learned

Mistake: ML suggested Dim 3 → 0.004 (extreme low)
Result: Performance dropped to -0.1161
Root Cause: Ignored ARD analysis showing Dim 3 barely matters
Lesson: Don't trust ML when it suggests extremes that contradict importance analysis

Week 7 Strategy: Boundary Push

• Input: [0.99, 0.99, 0.99] - Push ALL dimensions to boundary
• Expected: -0.06 (improvement toward 0)
• Goal: Break into positive territory!
• Risk: LOW (clear trend supports this)

Function 4 (4D) - Analysis

Dimension Importance

Best Performance: -3.986 (Week 1) at CENTER [0.5, 0.5, 0.5, 0.5]
Key Pattern: Center is optimal, corners fail catastrophically
Challenge: Any deviation from center performs worse

Method	Dim 1	Dim 2	Dim 3	Dim 4
Correlation	-0.47	-0.37	-0.36	-0.77
ARD Kernel	6.6%	1.1%	54.6%	37.7%
Importance Rank	3rd	4th	1st	2nd
Strategy	Center	Center	Explore	Explore

Exploration History

Week	Input	Output	Result
1	[0.5, 0.5, 0.5, 0.5]	-3.986	BEST (center)
2	[0.3, 0.3, 0.3, 0.3]	-4.306	Worse (too low)
3	[0.44, 0.29, 0.35, 1.25]	-30.129	DISASTER (out of bounds)
6	[0.2, 0.2, 0.95, 0.4]	-19.009	FAIL (corner)

Week 7 Strategy: Unexplored Quadrant

• Input: [0.65, 0.65, 0.65, 0.65] - Uniform moderate-high
• Expected: -4 to -9 (uncertain)
• Goal: Test if there's better region between center and corners
• Risk: MEDIUM (might be worse, but informative)

Function 5 (4D) - BREAKTHROUGH FUNCTION ■

Dimension Importance

Performance: 5549 (Week 5) - 39x improvement!
Critical Discovery: Dimension 3 is 96.6% important
Key Insight: All dimensions strongly positively correlated

Method	Dim 1	Dim 2	Dim 3	Dim 4
Correlation	+0.986	+0.997	+0.994	+0.992
ARD Kernel	1.1%	1.1%	96.6%	1.1%
Importance	Very Low	Very Low	CRITICAL!	Very Low
Strategy	→ 1.0	Variable	→ 1.0	→ High

Performance Progression

Week	Input	Output	Change
1-4	Various	~137	Baseline
5	[0.99, 0.9, 0.98, 0.93]	5549	+3900%!
6	[0.985, 0.905, 0.975, 0.925]	5399	-2.7%
7 (Pred)	[1.0, 0.853, 1.0, 0.977]	6001	+11%!

Week 7 Strategy: Local Search

Method: Gradient descent on GP surface
• Dim 1: 0.99 → 1.00 (+0.01) - Push to boundary
• Dim 2: 0.90 → 0.85 (-0.05) - Slight decrease (trade-off)
• Dim 3: 0.98 → 1.00 (+0.02) - CRITICAL! Push to boundary
• Dim 4: 0.93 → 0.98 (+0.05) - Increase

Predicted: 6001 ± 147 (+452 points!)
Risk: Sharp peak requires careful navigation
Confidence: HIGH - GP learned from 6 weeks of data

Function 6 (5D) - Analysis

Dimension Importance

Best Performance: -1.092 (Week 5)
Critical Discovery: Dimension 5 is 89.2% important
Pattern: Dims 1,2 negative correlation, Dims 3-5 positive

Method	Dim 1	Dim 2	Dim 3	Dim 4	Dim 5
Correlation	-0.77	-0.54	+0.38	+0.42	+0.35
ARD Kernel	2.7%	2.7%	2.7%	2.7%	89.2%
Importance	Low	Low	Low	Low	CRITICAL
Strategy	↓ Low	↓ Low	→ Mid	→ Mid	↑ High

Trend Analysis

Clear improving trend discovered:
• Week 1: -1.521
• Week 2: -1.139 ✓ Better
• Week 5: -1.092 ✓ BEST
• Week 6: -1.231 (overshot - changed all dims too much)

Week 7 Strategy: Focused Trend

• Input: [0.15, 0.15, 0.50, 0.50, 0.70]
• Changes from Week 5 [0.26, 0.18, 0.50, 0.48, 0.41]:
- Dims 1,2: 0.26/0.18 → 0.15 (lower, negative correlation)
- Dims 3,4: 0.50/0.48 → 0.50 (keep stable)
- Dim 5: 0.41 → 0.70 (HUGE increase - 89% important!)

Expected: -0.9 to -1.1
Goal: Push closer to 0, potentially break positive

Function 7 (6D) - Peer Pattern Success

Dimension Importance

Best Performance: 1.435 (Week 6) - +72% improvement!
Critical Discovery: Dimension 6 is 94% important
Success Factor: Learned from peer's winning pattern

Method	Dim 1	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
Correlation	-0.88	-0.50	-0.76	-0.77	-0.76	-0.15
ARD Kernel	1.2%	1.2%	1.2%	1.2%	1.2%	94%
Peer Gradient	0.12	0.45	0.08	0.52	0.23	0.15
Strategy	Low	Mid	Low	Low	Mid	High

Week 6 Success Story

Peer's Pattern: [0.058, 0.492, 0.247, 0.218, 0.420, 0.731] → 1.365
Our Adaptation: [0.060, 0.480, 0.250, 0.200, 0.400, 0.750] → 1.435
Result: Beat peer by 5% while using their pattern!

Key Insight: Peer information is valuable when validated by our analysis

Week 7 Strategy: Exploit Success

- Input: [0.038, 0.462, 0.239, 0.171, 0.378, 0.734]
- Method: GP exploitation (maximize mean prediction)
- Expected: 1.45
- Strategy: Small refinements around Week 6 success

Function 8 (8D) - Steady Optimization

Dimension Importance

Best Performance: 9.676 (Week 6)
Critical Discovery: Dimension 3 is 71.4% important
Pattern: Mixed correlations, avoid extremes

Method	D1	D2	D3	D4	D5	D6	D7	D8
Correlation	-0.98	+0.73	-0.98	+0.70	+0.72	-0.97	-0.98	+0.52
ARD Kernel	4.1%	4.1%	71.4%	4.1%	4.1%	4.1%	4.1%	4.1%
Strategy	↓	↑	Focus	↑	↑	↓	↓	↑

Week 4 Lesson: Avoid Extremes

Mistake: Tested binary pattern [1.0, 0.001, 1.0, 0.001, 0.001, 1.0, 1.0, 0.001]
Result: 4.180 (disaster - lost 5 points)
Lesson: F8 does not respond well to extreme values

Week 7 Strategy: Balanced EI

- Input: [0.177, 0.194, 0.170, 0.194, 0.294, 0.143, 0.109, 0.208]
- Method: Expected Improvement (balanced exploration/exploitation)
- Expected: 9.67
- Focus: Prioritize Dimension 3, avoid extremes

Methodology

1. Automatic Relevance Determination (ARD)

ARD kernels in Gaussian Processes learn individual length scales for each dimension. Shorter length scales indicate higher importance. This is our most reliable method as it's learned directly from the data through maximum likelihood optimization.

Advantages: Data-driven, handles nonlinear relationships, no assumptions
Used for: Primary importance ranking

2. Pearson Correlation Analysis

Measures linear relationship between each input dimension and output. Positive correlation means increasing the dimension improves output; negative means decreasing improves output.

Advantages: Simple, interpretable, shows direction
Limitations: Only captures linear relationships
Used for: Strategy direction (increase vs decrease)

3. Gradient Sensitivity Analysis

Finite difference method to compute numerical gradients: how much does output change when each dimension changes slightly? Larger gradients indicate higher sensitivity.

Advantages: Direct measure of sensitivity
Used for: Validation of importance rankings

4. Best/Worst Point Comparison

Compares input values at best-performing points vs worst-performing points. Large differences indicate that dimension matters; small differences suggest it doesn't.

Advantages: Practical, focuses on what actually worked
Used for: Identifying key patterns

5. Peer Gradient Synthesis

When available, incorporates dimension importance learned by other competitors. Cross-validates our findings and provides additional perspectives.

Example: F7 success came from adapting peer's pattern
Used for: Validation and strategy refinement

Complete Summary Table

Function	Dimensions	Most Important	% Importance	Best Output	Strategy
F1	2D	Unknown	N/A	0.0128	Local exploration
F2	2D	Both equal	48% / 52%	0.1300	Decrease both
F3	3D	Dim 1	99.6%	-0.0786	Push to boundary
F4	4D	Dim 3, 4	55%, 38%	-3.986	Stay near center
F5	4D	Dim 3	96.6%	5549	Maximize Dim 3
F6	5D	Dim 5	89.2%	-1.092	Focus on Dim 5
F7	6D	Dim 6	94.0%	1.435	Peer pattern
F8	8D	Dim 3	71.4%	9.676	Avoid extremes

Key Insights & Lessons

What Worked ✓

- **Multi-method consensus:** Using 5 different methods provided robust insights
- **ARD kernels:** Most reliable for identifying dominant dimensions
- **Conservative ML predictions:** Small perturbations worked well (F2, F7)
- **Peer learning:** F7 improved 72% using adapted peer pattern
- **Correlation-guided exploration:** Provided clear directional guidance
- **Sharp peak handling:** F5 required careful local search, not big jumps

What Didn't Work ✗

- **ML extreme predictions:** F3 Dim3→0.004 failed (ignored importance)
- **Binary exploration:** F8 crashed with [1.0, 0.001, ...] pattern
- **Ignoring dimension importance:** Week 6 F3 mistake
- **Large perturbations on sharp peaks:** F5 lost 151 points from 0.005 change
- **Corner exploration without gradient info:** F4 Week 6 disaster (-19)

Critical Discoveries

- **Dominant dimensions exist:** F3, F5, F6, F7 all have one dimension >85% important
- **Sharp vs smooth landscapes:** F5 requires precision, F2 is more forgiving
- **Center can be optimal:** F4 performs best at [0.5, 0.5, 0.5, 0.5]
- **Exploration has value:** Week 6 losses informed better Week 7 strategy
- **6 weeks enables strong ML:** Enough data for reliable GP predictions

Week 7 Predictions

Function	Week 7 Input	Predicted	Change	Confidence
F1	[0.48, 0.48]	0.013	+0.000	Medium
F2	[0.11, 0.10]	0.128	-0.002	High
F3	[0.99, 0.99, 0.99]	-0.06	+0.02	High
F4	[0.65, 0.65, 0.65, 0.65]	-4.5	-0.5	Medium
F5	[1.0, 0.85, 1.0, 0.98]	6001	+452	Very High
F6	[0.15, 0.15, 0.50, 0.50, 0.70]	-1.10	-0.01	High
F7	[0.04, 0.46, 0.24, 0.17, 0.38, 0.73]	1.45	+0.01	High
F8	[0.18, 0.19, 0.17, 0.19, 0.29, 0.14, 0.11, 0.21]	9.67	-0.01	High
TOTAL		**6007**	**+617**	

Expected Impact

Week 6 Total: 5389 points
Week 7 Predicted: 6007 points
Expected Gain: +617 points (+11%)

Primary Driver: F5 breakthrough (+452 points from Dim 3 optimization)
Risk Assessment: Exploration costs on F3, F4 acceptable with 6 weeks remaining

Conclusion

Through systematic application of multiple dimension importance analysis methods, we identified critical patterns that enabled breakthrough performance. The discovery that several functions have single dominant dimensions (>85% importance) simplified optimization strategies and led to our 39x improvement on Function 5.

Key success factors include: (1) multi-method validation for robust insights, (2) careful balance of ML predictions with domain understanding, (3) learning from both successes and failures, and (4) strategic exploration when time permits.

Week 7 predictions suggest continued strong performance with a total of 6007 points, driven primarily by the Function 5 breakthrough prediction of 6001 points. This represents the culmination of 6 weeks of iterative learning and systematic analysis.