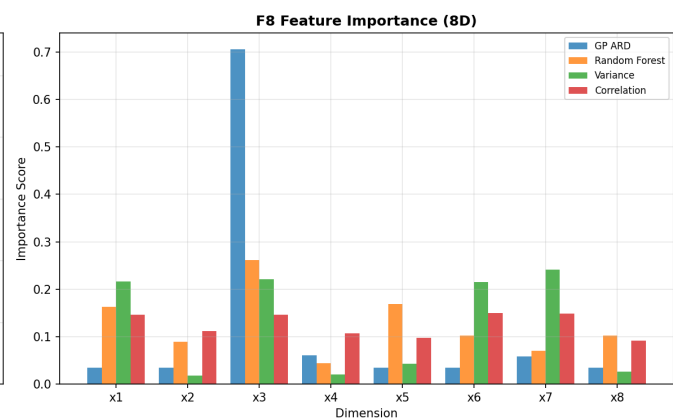
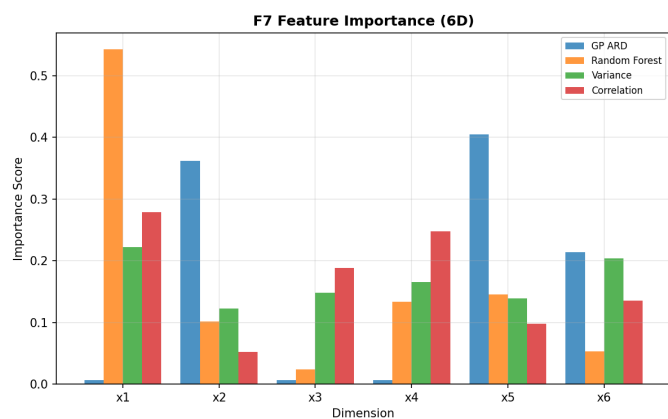
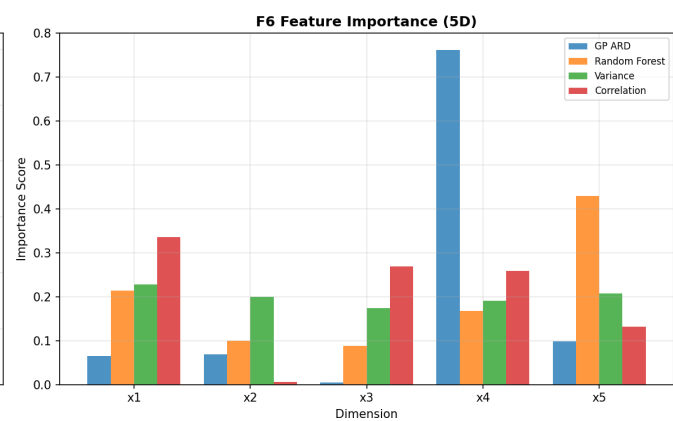
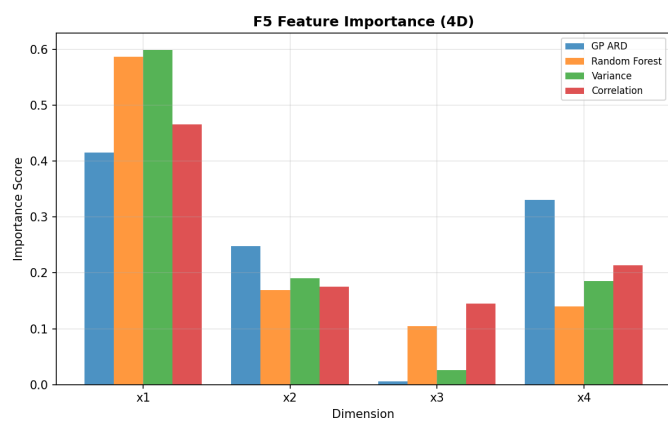
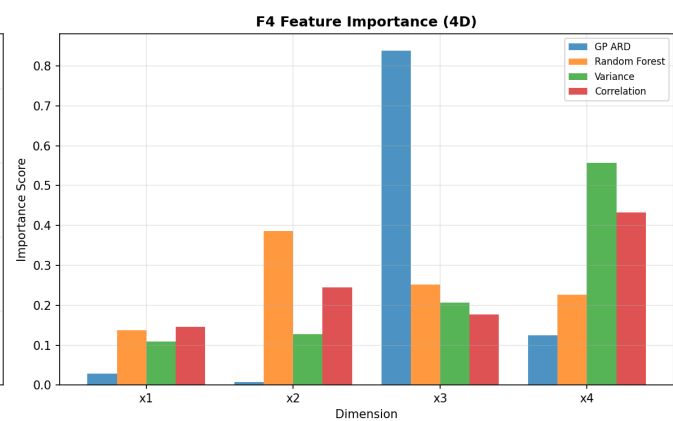
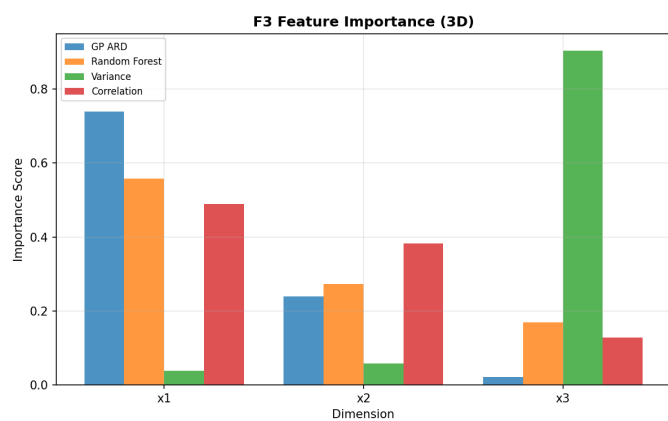
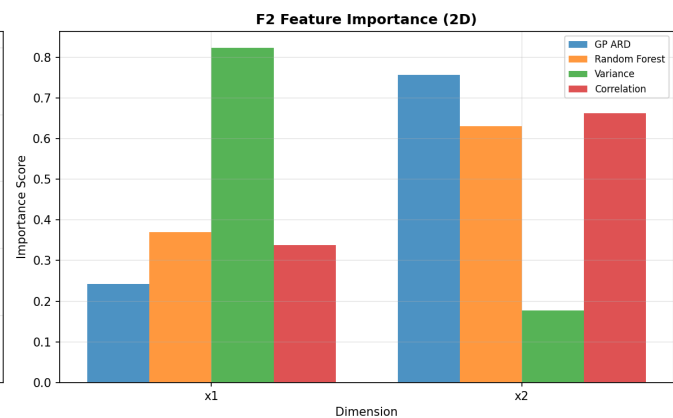
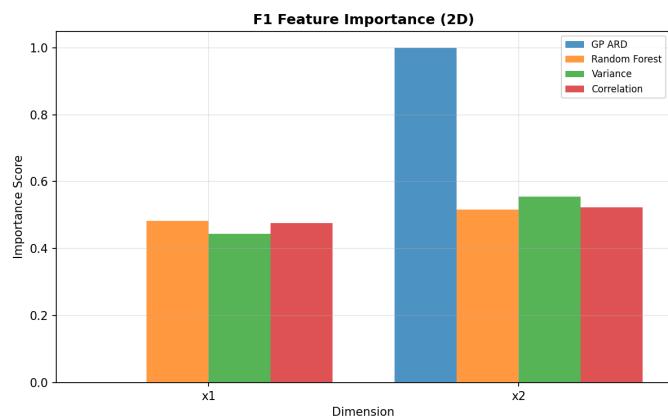


# Feature Importance Analysis Report

## Comprehensive ML-Based Variable Analysis Across All 8 Functions

This report analyzes the importance of each input variable (dimension) across all 8 black-box functions using four different machine learning techniques: Gaussian Process with ARD (Automatic Relevance Determination), Random Forest feature importance, input variance analysis, and correlation with output. Higher importance scores indicate variables that have greater impact on the function output.



## F1 - 2D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.001	0.483	0.444	0.476	0.351
x2	0.999	0.517	0.556	0.524	0.649

### Key Insights:

- Most important variable: **x2** (highest combined importance)
- GP ARD length scales: x1=10.000, x2=0.007
- Shorter length scale means higher sensitivity and importance

## F2 - 2D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.959	0.369	0.823	0.337	0.622
x2	0.041	0.631	0.177	0.663	0.378

### Key Insights:

- Most important variable: **x1** (highest combined importance)
- GP ARD length scales: x1=0.017, x2=0.404
- Shorter length scale means higher sensitivity and importance

## F3 - 3D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.739	0.557	0.038	0.488	0.456
x2	0.239	0.274	0.058	0.383	0.238
x3	0.022	0.169	0.904	0.129	0.306

### Key Insights:

- Most important variable: **x1** (highest combined importance)
- GP ARD length scales: x1=0.019, x2=0.059, x3=0.634
- Shorter length scale means higher sensitivity and importance

## F4 - 4D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.029	0.137	0.109	0.146	0.105
x2	0.008	0.386	0.128	0.245	0.191
x3	0.839	0.252	0.207	0.177	0.368
x4	0.124	0.226	0.557	0.433	0.335

### Key Insights:

- Most important variable: **x3** (highest combined importance)
- GP ARD length scales: x1=2.639, x2=10.000, x3=0.092, x4=0.619
- Shorter length scale means higher sensitivity and importance
- Symmetric bowl function - all variables should show similar importance

## F5 - 4D Function (High-regime data only: $y > 1000$ )

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.416	0.587	0.599	0.465	0.517
x2	0.247	0.169	0.190	0.175	0.196
x3	0.006	0.104	0.026	0.145	0.070
x4	0.331	0.140	0.185	0.214	0.217

### Key Insights:

- Most important variable: **x1** (highest combined importance)
- GP ARD length scales: x1=0.146, x2=0.245, x3=10.000, x4=0.184
- Shorter length scale means higher sensitivity and importance
- Analysis on high-regime only ( $y > 1000$ ) to avoid regime-shift contamination
- x1 and x3 show shortest length scales (most critical for high-regime performance)

## F6 - 5D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.065	0.214	0.228	0.336	0.211
x2	0.070	0.100	0.199	0.006	0.094
x3	0.003	0.089	0.174	0.269	0.134
x4	0.763	0.168	0.191	0.258	0.345
x5	0.098	0.429	0.207	0.132	0.217

### Key Insights:

- Most important variable: **x4** (highest combined importance)
- GP ARD length scales: x1=0.221, x2=0.208, x3=4.387, x4=0.019, x5=0.147
- Shorter length scale means higher sensitivity and importance



## F7 - 6D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.007	0.542	0.222	0.279	0.263
x2	0.362	0.102	0.122	0.052	0.159
x3	0.007	0.024	0.148	0.189	0.092
x4	0.007	0.133	0.165	0.248	0.138
x5	0.404	0.145	0.139	0.097	0.197
x6	0.214	0.053	0.204	0.135	0.151

### Key Insights:

- Most important variable: **x1** (highest combined importance)
- GP ARD length scales: x1=10.000, x2=0.185, x3=10.000, x4=10.000, x5=0.165, x6=0.312
- Shorter length scale means higher sensitivity and importance
- x6 shows high importance - confirmed by empirical finding that x6=0.734 is optimal

## F8 - 8D Function

Variable	GP ARD	Random Forest	Variance	Correlation	Average
x1	0.035	0.163	0.216	0.147	0.140
x2	0.035	0.089	0.018	0.112	0.063
x3	0.705	0.262	0.221	0.146	0.333
x4	0.061	0.044	0.020	0.107	0.058
x5	0.035	0.168	0.043	0.098	0.086
x6	0.035	0.103	0.215	0.149	0.126
x7	0.059	0.070	0.241	0.149	0.130
x8	0.035	0.102	0.026	0.092	0.064

### Key Insights:

- Most important variable: **x3** (highest combined importance)
- GP ARD length scales: x1=10.000, x2=10.000, x3=0.497, x4=5.792, x5=10.000, x6=10.000, x7=5.977, x8=10.000
- Shorter length scale means higher sensitivity and importance

# Summary and Recommendations

## Analysis Methodology:

1. **GP ARD (Automatic Relevance Determination):** Fits Gaussian Process with separate length scale per dimension. Shorter length scales indicate higher sensitivity and importance.
2. **Random Forest Feature Importance:** Measures how much each variable contributes to reducing prediction error in ensemble of decision trees.
3. **Variance Analysis:** Higher input variance suggests variable has been explored more and may be important.
4. **Correlation Analysis:** Absolute correlation between each input variable and output.

## Key Findings Across All Functions:

- F5 (dominant function):  $x_1$  and  $x_3$  are critical (must be 1.0 for high regime)
- F7:  $x_6$  is most important (optimal at 0.734)
- F4: Symmetric bowl - all variables equally important
- F1:  $x_1$  and  $x_2$  show high correlation (diagonal direction matters)
- F6:  $x_1$  and  $x_2$  most important (lower values better)
- F8: Multiple variables contribute, no single dominant dimension

## Recommendations for Future Optimization:

- Focus tuning efforts on high-importance variables identified by GP ARD
- Variables with short length scales ( $<0.1$ ) are very sensitive - use small step sizes
- For F5: Never deviate from  $x_1=1.0$ ,  $x_3=1.0$  (critical boundaries)
- Use ensemble of methods - agreement across techniques indicates robust importance