The parameter :-

$t=7, \; b=7, \; r=3, \; K=3$

$\lambda =$ no of blocks in which a pair of treatment occurs together.

$$\lambda = \frac{r(K-1)}{t-1} \Rightarrow \frac{3(3-1)}{7-1} = \frac{6}{6} = 1$$

Hypothesis :-

H_0^1 :

H_1^1 :

H_0^2 :

H_1^2 :

$$CF = \frac{G^2}{n} = \frac{Y_{..}^2}{bK} \Rightarrow \frac{(2715)^2}{7 \times 3} = \frac{7371225}{21}$$

$$CF = 351010.7143$$

Total sum of squares TSS

$$TSS = \sum_i \sum_j Y_{ij}^2 - CF$$

$$= 353611 - 351010.7143$$

$$TSS = 2600.2857$$

Blocks S.S (unadjusted)

$$BSS = \frac{\sum Y_{.j}^2}{K} - CF$$

$= \frac{1056375}{3} - CF$

$BSS = \cancel{35027}\ 1114.28577 //.$

Adjusted treatment totals

$$Q_i = Y_{i.} - \frac{1}{K}\left(\sum_{j=1}^{b} I_{ij}\, Y_{.j}\right)$$

where $I_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ treatment appears in } j^{th} \text{ block} \\ 0, & \text{otherwise} \end{cases}$

$$Q_1 = 351 - \frac{1}{3}(1 \times 381 + 381 + 378)$$

$$Q = -29$$

$$Q_2 = 365 - \frac{1}{3}(1 \times 381 + 402 + 372)$$

$$Q_2 = -20$$

$$Q_3 = 388 - \frac{1}{3}(1 \times 402 + 366 + 378)$$

$$= -6$$

$$Q_4 = 419 - \frac{1}{3}(1 \times 381 + 366 + 435)$$

$$Q_4 = 25$$

$$Q_5 = 438 - \frac{1}{3}(1 \times 402 + 435 + 381)$$

$$Q_5 = 32$$

$$Q_6 = 361 - \frac{1}{3}(1 \times 366 + 381 + 372)$$

$Q_6 = -12$

$Q_7 = -2$

$\therefore$ adjusted treatments

$$Tr.ss\ adj = \frac{K \sum Q_i^2}{\lambda t}$$

$$= \frac{3 \times 2963074}{1 \times 7}$$

$$= \frac{888}{7} = 1317.43$$

unadjusted $T.SS. = \frac{\sum Y_{i.}^2}{r} - CF$

$$= \frac{1059145}{3} - 35100.7143$$

unadjusted $TSS = 2037.6190$

$ESS = TSS - BSS(\text{unadjusted}) - TrSS(\text{adjusted})$

$= 2600.2857 - 1114.2857 - 1317.43$

$ESS = 168.58$

ANOVA.

	d.f	S.S	MSS	F-ratio
[illegible]	$t-1=6$	1317.43	219.571	10.4197
[illegible]	$6-1=6$	1114.2857	185.7142	8.0

[illegible] $(bk-1)-(t-1)-(b-1) = 8$ 168.58   21.0725

[illegible] $bk-1 = 20$ 2600.2857   130.0142

$F_{cal} > F_{(0.05)}(6,8)$

$F_{cal} > \cancel{10.4197}\ 3.58$

$10.4197 \geq 3.58$

$\therefore$ we reject H_0, hence all concentrations are not same.

$BSS(adj) = Trss(adj) + BSS(unadjusted) - Trss(unadj) = 394.0967$

ANOVA.

S.V	d.f	S.S	MSS	F-ratio
[illegible] (adj)	6	2037.6190	339.6031	
[illegible]	6	394.08	65.68	3.11
[illegible]	8	168.58	21.0725	
[illegible]	20	2600.2857	130.014	

$F_{cal} > F_{0.05}(6,8)$

$F_{cal} > 3.58$

$3.11 < 3.58$

$\therefore$ we donot reject H_0.

$\therefore$ all days are same.

H_0^1: all concentration (treatments) are same

H_1^1: all concentration are not same.

H_0^2: all days (blocks) are same

H_1^2: all days (blocks) are not same

(IV) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\dfrac{2k\hat{\sigma}^2}{\lambda t}} = \sqrt{\dfrac{2K\,MSSE}{\lambda t}}$

i – bloc.
k = res.

$$\sqrt{\frac{2\times3\times21.0725}{1\times7}} = \sqrt{18.0621}$$

$$S.E(Q_i - Q_i) = 4.2$$

(3). <u>Model</u> :-

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \delta_k$$

when, $\alpha_i =$, $\beta_j =$

$i = 1, ---, t$
$j = 1, ---, b$
$k = 1, ---, r$

<u>Assumptions</u>:-

i) α_i's, β_j's & δ_k's are fixed effect

ii) $\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

<u>Hypothesis</u> :-

H_0^1 : The four manorial treatment are same
H_1 : The four manorial treatment are not same.

H_0^2
H_1^2 :

$$\lambda = \frac{r(k-1)}{t-1} = \frac{3(3-1)}{4-1} = \frac{6}{3} = 2$$

H_0^3
H_1^3 :

$t = 4$, $b = 4$, $r = 3$, $K = 3$, $n = b\times K = 12$

Treatm.

	A	B	C	D
	432	442	540	
	468	490	562	340
	568	584	556	384
				526.
$y_{i..}$	1468	1516	1658	1250

$$CF = \frac{G^2}{n} = \frac{34715664}{12} = 2892972$$

$$TSS = \Sigma\Sigma y_{ijk}^2 - CF$$

$$= 2961104 - CF$$

$$TSS = 68132.$$

~~BSS(~~ $RSS = \frac{\Sigma Y_{..k}^2}{b} - CF$

$$RSS = 30150$$

$$BSS(unadj) = \frac{\Sigma y_{.j.}^2}{k} - CF$$

$$BSS(un) = 11150.666$$

1445.333

adjusted treatment totals

$$Q_i = y_{i..} - \frac{\Sigma n_{ij} Y_{.j.}}{K} \quad , i = 1, ---, t$$

where $n_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ treatment appears in block } j \\ 0, & O.w \end{cases}$

$$Q_1 = 1468 - \left(\frac{1372 + 1572 + 1392}{3}\right)$$

$Q_1 = 22.67$

$Q_2 = 9.33$

$Q_3 = 1658 - \frac{(1372+1562+1556)}{3}$

$Q_3 = 158$

$Q_4 = 1250 - \frac{(1372+1556+1392)}{3}$

$Q_4 = -190$

Adjusted Tr. ss is,

$$Trss(adj) = \frac{k}{\lambda t} \sum_{i=1}^{t} Q_i^2$$

$$= \frac{3}{2 \times 4} \times 61664.9778$$

$$Trss(adj) = 23124.3666$$

$$ESS = TSS - \underset{unadj}{(BSS)} - \underset{(adj)}{Tr.ss} - RSS.$$

$$= 68132 - 11150.666 - 23124.366 - 30150$$

$$ESS = 3706.974$$

ANOVA.

SV	d.f	S.S.	MSS.	F ratio
Treat (adj)	4-1=3	23124.366	7708.122	6.23
row	3-1=2	30150	15075	
Block (und	4-1=3	11150.66	3716.88	
Error	(3+2+3-11) = 3	3706.974	1235.658	
Total	n-1=12-1 11	68132		

$F_{cal} \geq F_{\alpha(3,3)}$
$6.23 \geq 9.28$

$\therefore$ we donot reject H_0.

The ~~no~~ four manorial treatments are same.

(iii) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\frac{2K\,MSSE}{\lambda t}}$

$= \sqrt{\frac{2 \times 3 \times \cancel{7708.122} \; 1235.658}{2 \times 4}}$

$= \cancel{\sqrt{5781.0915}} \; \sqrt{926.7435}$

$S.E(\hat{\alpha}_i - \hat{\alpha}_j) = 30.44$ //.

(2) ANOVA TABLE

S.V.	d.f.	S.S	M.SS	F value	p-value
Treat	4	7.6133	1.9033	3.1263	0.04445
Block	9	3.5124	0.39027	0.6410	0.74734
Residual	16	9.7409	0.60881		

Hypothesis

H_0: Treatment effects are same V/S
H_1: Treatment effects are not same.

Conclusion :- $F_1 = 3.1263$
$F_1 \geq F_{cal}$

$$3.1263 \geq F_\alpha(4,16)$$
$$3.1263 \geq 3.01$$

Hence we reject H_0, Hence all the treatment effect are not same.

$$F_\alpha \geq F_{cal}$$
$$0.6410 \geq F(9,16)$$
$$0.6410 \geq 2.54$$

we do not reject H_0, Hence all the treatment effect are same.

P - 4

(1) (i) the model is

$$y_{ij} = \mu + \alpha_i + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij} \quad , i = 1, \dots, t$$
$$j = 1, \dots, r$$

where

y_{ij} :- value of observation for the jth response variable for the ith treatment

μ :- overall mean

α_i :- ith Treatment effect

β :- linear regression coefficient of Y on X

x_{ij} :- jth observation on the covariate for the ith treatment.

$\bar{x}_{..}$:- sample mean of x observation

ε_{ij} :- random errors.

Assumptions

$\varepsilon_{ij} \sim N(0, \sigma^2)$

$\Sigma \alpha_i = 0$

Calculation :

$r = 5,\ t = 3,\ n = r \times t = 5 \times 3 = 15$

$X_{..} = 185 \quad Y_{..} = 714.9$

$C.F_{XX} = \frac{X_{..}^2}{n} = \frac{185^2}{15} = 2281.666$

$CF_{YY} = \frac{Y_{..}^2}{n} = \frac{714.9^2}{15} = 34072.134$

$CF_{XY} = \frac{X_{..} Y_{..}}{n} = \frac{185 \times 714.9}{15} = 8817.1$

$TSS_{XX} = \Sigma\Sigma X_{ij}^2 - CF_{XX} = 2317 - 2281.666$

$TSS_{XX} = 35.334$

$TSS_{YY} = \Sigma\Sigma Y_{ij}^2 - CF_{YY} = \cancel{2}$

$TSS_{YY} = 49.296$

$TSS_{XY} = \Sigma\Sigma X_{ij} Y_{ij} - CF_{XY}$

$= 8783.4 - CF_{XY} = -33.7$

$Tr.SS_{YY} = \frac{\Sigma Y_{ij}^2}{t} - CF_{YY} =$

$= \frac{102263.52}{3} - 34072.134 = 15.706$

$$Tr.SS_{xx} = \frac{\sum x_{i.}^2}{r} - CF_{xx}$$

$$= \frac{11427}{5} - 2281.666$$

$$= \cancel{1527.334}\ 3.734$$

$$Tr.SS_{xy} = \frac{\sum x_{i.} y_{i.}}{r} - CF_{xy}$$

$$= \frac{44056.3}{5} - CF_{xy}$$

$$= -5.84.$$

$$E_{yy} = TSS_{yy} - Tr.SS_{yy}$$

$$= 49.296 - 15.706$$

$$= 33.59$$

$$E_{xx} = TSS_{xx} - Tr.SS_{xx}$$

$$= 35.334 - 3.734$$

$$= 31.6$$

$$E_{xy} = TSS_{xy} - Tr.S_{xy}$$

$$= -33.7 - (-5.84)$$

$$= -27.86$$

(ii). Hypothesis

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

H_0: there is no effect of covariate.

H_1: the effect of covariate is there

$$SSE = E_{yy} - \frac{E_{xy}^2}{E_{xx}} \Rightarrow 33.159 - \frac{(-27.86)^2}{31.6}$$

$$= 33.15 - 24.5626$$

$$SSE = 8.5874$$

$$MSE = \frac{SSE}{n-t-1} = \frac{8.5874}{15-3-1} = 0.78$$

$$F_{cal} = \frac{E_{xy}^2 / E_{xx}}{MSE} \sim F_{(\beta, n-t-1)}$$

$$= \frac{24.5626}{0.78} \sim F^{(0.05)}_{(1,11)}$$

$$= 31.49$$

$$\therefore F^{(0.05)}_{(1,11)} = 4.84$$

$$\therefore F_{cal} > F_{tab}$$

Conclusion $31.49 > 4.84$

∴ we reject H_0

∴ the effect of covariate is there.

(ii) $H_0: \alpha_i = 0 \quad \forall i$

~~H_0 the glue formulations are~~ not significant.

H_0: there is no significant difference in the glue formation.

$H_1: \alpha_i \neq 0$

There is a significant difference in glue formation.

$T_{SSS} = SSE - SSE$

$$SSE' = TSS_{YY} - \frac{TSS_{XY}^2}{TSS_{XX}}$$

$$= 49.296 - \frac{(-33.7)^2}{35.334}$$

$$= 49.296 - 32.1415$$

$$SSE' = 17.1545$$

$$F_0 \frac{(SSE' - SSE)/t-1}{MSSE} \sim F(t-1, n-t-1)$$

$$F_0 = \frac{(17.1545 - 8.5874)/3-1}{0.78}$$

$$F_0 = \frac{4.28355}{0.78} = 5.491$$

$F(t-1, n-t-1) \Rightarrow F(3-1, 15-3-1) \Rightarrow F(2,11) = 3.98$

~~ANOVA Table~~

$\therefore F_{cal} > F_{tab}$

$\therefore 5.491 > 3.98$

$\therefore$ we reject H_0

$\therefore$ there is a significant difference in glue formation.

ANCOVA Table.

S.V.	Sum of Squares & Cross products: x	xy	y	d.f	adjusted treatment SS for Regression S.S	MSS	F
Treatment	$T_{xx}=3.734$	-5.84	15.706	.	$SSE=$		
Error	$E_{xx}=31.6$	-27.86	33.15		8.5874		
Total	$TSS=35.334$	-33.7	49.296	$n-t-1=11$	$SSE'=17.1545$	0.78	$F_0=5.49$
adj treatment				$n-2=13$	$Tr.SS=SSE'-SSE$		
				$t-1=2$	4.28355		

(iv) $SE \text{ of adj } (\bar{y}_{i.} - \bar{y}_{j.}) = \sqrt{MSE\left(\frac{2}{r} + \frac{\bar{x}_{i.} - \bar{x}_{j.}}{E_{xx}}\right)}$

$\bar{x}_{i.} \to$ 1 treatment mean.

$\bar{x}_{j.} \to$ 2 treatment mean. 0.67

$$= \sqrt{MSE\left(\frac{2}{5} + \frac{b}{31.6}\right)}$$

$$= 0.68$$

(d). The linear model is,

$$y_{ij} = \mu + \alpha_i + \xi_j + \beta(x_{ij} - \bar{x}_{..}) + \epsilon_{ij}$$

$$i = 1, \dots, t$$

$$j = 1, \dots, b$$

where, $y_{ij} =$, $\mu =$, $\alpha_i =$, $\xi_j =$, $\beta =$, $\epsilon_{ij} =$

Hypothesis.

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

1

~~Soln~~ Here $t=3$, $b=4$, $n=tb=12$.

$CF_{xx} = \frac{X_{..}^2}{n} = \frac{290^2}{12} = 7008.33$

$CF_{yy} = \frac{Y_{..}^2}{n} = \frac{483^2}{12} = \frac{233289}{12} = 19440.75$

$CF_{xy} = \frac{X_{..}Y_{..}}{n} = \frac{\cancel{11949}\ 290\times483}{12} = \cancel{995.75}\ 11672.5$

$TSS_{xx} = \sum\sum X_{ij}^2 - CF_{xx}$

$= 7262 - CF_{xx} = 253.67$

$TSS_{yy} = \sum\sum Y_{ij}^2 - CF_{yy}$

$TSS_{yy} = 19781 - 19440.75 = 340.25$

$TSS_{xy} = 1\cancel{1949}\ \sum\sum X_{ij}Y_{ij} - CF_{xy}$

$= 11949 - 11672.5$

$TSS_{xy} = 276.5$

$Tr_{yy} = \frac{\sum Y_{i.}^2}{b} - CF_{yy}$

$= \frac{78597}{4} - 19440.75$

$= 19649.25 - 19440.75$

$Tr_{yy} = 208.5$

$Tr_{xx} = \frac{\sum X_{i.}^2}{b} - CF_{xx} \Rightarrow$

$Tr_{xx} = \frac{28468}{4} - CF_{xx} = 7117 - CF_{xx} = 108.67$

$$Trxy = \frac{\sum x_{i\cdot} y_{i\cdot}}{b} - CF_{xy} = \frac{47292}{4} - CF_{xy}$$

$$Trxy = 11823 - CF_{xy} \Rightarrow 150.5$$

$$Byy = \frac{\sum Y_{\cdot j}^2}{t} - CF_{yy} = \cancel{5846}\ \frac{58463}{3} - CF_{yy}$$

$$= 19487.666 - CF_{yy}$$

$$Byy = 46.9166$$

$$Bxx = \frac{\sum X_{\cdot j}^2}{t} - CF_{xx} = \frac{21146}{3} - CF_{xx} = 40.33$$

$$Bxy = \frac{\sum X_{\cdot j} Y_{\cdot j}}{t} - CF_{xy}$$

$$= \frac{35140}{3} - 11672.5 = 11713.33 - CF_{xy}$$

$$Bxy = 40.833$$

$$Eyy = TSS_{yy} - TrSS_{yy} - Byy$$

$$= 340.25 - 208.5 - 46.9166$$

$$= 84.8334$$

$$Exx = TSS_{xx} - TrSS_{xx} - B_{xx}$$

$$= 253.67 - 108.67 - 40.33$$

$$= 104.67$$

$$Exy = TSS_{xy} - TrSS_{xy} - Bxy$$

$$= 276.5 - 150.5 - 40.833$$

$$= 85.167$$

$$\hat{\beta} = \frac{Exy}{Exx} = \frac{85.167}{104.67} = 0.8136$$