The parameter :-

$t = 7, \; b = 7, \; r = 3, \; K = 3$

$\lambda =$ no of blocks in which a pair of treatment occurs together.

$$\lambda = \frac{r(K-1)}{t-1} \Rightarrow \frac{3(3-1)}{7-1} = \frac{6}{6} = 1$$

Hypothesis :-

H_0^1 :

H_1^1 :

H_0^2 :

H_1^2 :

$$CF = \frac{G^2}{n} = \frac{Y_{..}^2}{bK} \Rightarrow \frac{(2715)^2}{7 \times 3} = \frac{7371225}{21}$$

$$CF = 351010.7143$$

Total sum of squares TSS

$$TSS = \sum_i \sum_j Y_{ij}^2 - CF$$

$$= 353611 - 351010.7143$$

$$TSS = 2600.2857$$

Blocks S.S (unadjusted)

$$BSS = \frac{\sum Y_{.j}^2}{K} - CF$$

$$= \frac{1056375}{3} - CF$$

$BSS = \cancel{35027}\ 1114.2857$ //.

Adjusted treatment totals

$$Q_i = Y_{i.} - \frac{1}{K}\left(\sum_{j=1}^{b} I_{ij} Y_{.j}\right)$$

where $I_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ treatment appears in } j^{th} \text{ block} \\ 0, & \text{otherwise} \end{cases}$

$$Q_1 = 351 - \frac{1}{3}(1 \times 381 + 381 + 378)$$

$$Q = -29$$

$$Q_2 = 365 - \frac{1}{3}(1 \times 381 + 402 + 372)$$

$$Q_2 = -20$$

$$Q_3 = 388 - \frac{1}{3}(1 \times 402 + 366 + 378)$$

$$= 6$$

$$Q_4 = 419 - \frac{1}{3}(1 \times 381 + 366 + 435)$$

$$Q_4 = 25$$

$$Q_5 = 438 - \frac{1}{3}(1 \times 402 + 435 + 381)$$

$$Q_5 = 32$$

$Q_6 = 361 - \frac{1}{3}(1\times 366 + 381 + 372)$

$Q_6 = -12$

$Q_7 = -2$

$\therefore$ adjusted treatments

$$Tr.ss\ adj = \frac{K \sum Q_i^2}{\lambda t}$$

$$= \frac{3 \times 2963074}{1\times 7}$$

$$= \frac{888}{7} = 1317.43$$

unadjusted T.SS. $= \frac{\sum Y_{i.}^2}{r} - CF$

$$= \frac{1059145}{3} - 35100.7143$$

unadjusted TSS $= 2037.6190$

ESS = TSS − BSS(unadjusted) − Trss (adjusted)

$= 2600.2857 - 1114.2857 - 1317.43$

ESS = 168.58

ANOVA.

	d.f	S.S	MSS	F-ratio
	t−1=6	1317.43	219.571	10.4197
	6−1=6	1114.2857	185.7142	8.0

Error $(bk-1)-(t-1)-(b-1) = 8$ 168.58 21.0725

Total $bk-1 = 20$ 2600.2857 130.0142

$F_{cal} > F_{(0.05)}(6,8)$

$F_{cal} > \cancel{10.4197}$ 3.58

$10.4197 \geq 3.58$

$\therefore$ we reject H_0, hence all concentration are not same.

$BSS(adj) = Trs(adj) + BSS(unadjusted) - Trs(unadj) = 394.0967$

ANOVA.

S.V	d.f	S.S	MSS	F-ratio
(unadj)	6	2037.6190	339.6031	
(adj)	6	394.08	65.68	3.11
Error	8	168.58	21.0725	
Total	20	2600.2857	130.014	

$F_{cal} > F_{0.05}(6,8)$

$F_{cal} > 3.58$

$3.11 < 3.58$

$\therefore$ we donot reject H_0.

$\therefore$ all days are same.

H_0^1: all concentration (treatments) are same

H_1^1: all concentration are not same.

H_0^2: all days (blocks) are same

H_1^2: all days (blocks) are not same

(iv) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\frac{2k\hat{\sigma}^2}{\lambda t}} = \sqrt{\frac{2K\,MSSE}{\lambda t}}$

i – block
k = rows.

$$\sqrt{\frac{2\times 3\times 21.0725}{1\times 7}} = \sqrt{18.0621}$$

$$S.E(\hat{\alpha}_i - \hat{\alpha}_{i'}) = 4.2$$

③. <u>model</u> :-

$$y_{ijk} = \mu + \alpha_i + \beta_j + \delta_k$$

where, $\alpha_i =$, $\beta_j =$)

$i = 1, ---, t$
$j = 1, ---, b$
$k = 1, ---, r$

<u>Assumptions</u>:-

i) α_i's, β_j's & δ_k's are fixed effect

ii) $\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

<u>Hypothesis</u> :-

H_0^1 ; The four manorial treatment are same

H_1 ; The four manorial treatment are not same.

H_0^2

H_1^2 ;

$$\lambda = \frac{r(k-1)}{t-1} = \frac{3(3-1)}{4-1} = \frac{6}{3} = 2$$

H_0^3

H_1^3 ;

$t = 4$, $b = 4$, $r = 3$, $K = 3$, $n = b\times k = 12$

Treatm.

	A	B	C	D
	432	442	540	340
	468	490	562	384
	568	584	556	526.
$y_{i..}$	1468	1516	1658	1250

$$CF = \frac{G^2}{n} = \frac{34715664}{12} = 2892972$$

$$TSS = \Sigma\Sigma y_{ijk}^2 - CF$$

$$= 2961104 - CF$$

$$TSS = 68132.$$

~~BSS~~ $RSS = \frac{\Sigma Y_{..k}^2}{b} - CF$

$$RSS = 30150$$

$$BSS(unadj) = \frac{\Sigma y_{.j.}^2}{K} - CF$$

$$BSS(un) = 11150.666$$

1445.333

adjusted treatment totals

$$Q_i = y_{i..} - \frac{\Sigma n_{ij} Y_{.j.}}{K}, \quad i = 1, \ldots, t$$

where $n_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ treatment appear in block } j \\ 0, & O.W \end{cases}$

$$Q_1 = 1468 - \left(\frac{1372 + 1572 + 1392}{3}\right)$$

$Q_1 = 22.67$

$Q_2 = 9.33$

$$Q_3 = 1658 - \frac{(1372+1562+1556)}{3}$$

$Q_3 = 158$

$$Q_4 = 1250 - \frac{(1372+1556+1392)}{3}$$

$Q_4 = -190$

adjusted Tr. ss is,

$$Trss(adj) = \frac{k}{\lambda t} \sum_{i=1}^{t} Q_i^2$$

$$= \frac{3}{2 \times 4} \times 61664.9778$$

$Trss(adj) = 23124.3666$

$$ESS = TSS - \underset{unadj}{(BSS)} - \underset{(adj)}{Tr.ss} - RSS.$$

$= 68132 - 11150.666 - 23124.366 - 30150$

$ESS = 3706.974$

ANOVA.

~~Treat~~ SV	d.f	S.S.	MSS.	F ratio
Treat (adj)	4-1=3	23124.366	7708.122	6.23
row	3-1=2	30150	15075	
Block (und	4-1=3	11150.66	3716.88	
Error	(3+2+3-11) = 3	3706.974	1235.658	
Total	n-1=12-1 11	68132		

$F_{cal} \geq F_{\alpha (3,3)}$

$6.23 \geq 9.28$

∴ we donot reject H_0.

The ~~two~~ four manorial treatments are same.

(iii) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\frac{2K\, MSSE}{\lambda t}}$

$= \sqrt{\frac{2 \times 3 \times \cancel{7708.122}\ 1235.658}{2 \times 4}}$

$= \cancel{\sqrt{5781.0915}}\ \sqrt{926.7435}$

$S.E(\hat{\alpha}_i - \hat{\alpha}_j) = 30.44$ //

(2) ANOVA TABLE

S.V.	d.f.	S.S	M.SS	F value	p-value
Treat	4	7.6133	1.9033	3.1263	0.04445
Block	9	3.5124	0.39027	0.6410	0.74734
Residual	16	9.7409	0.60881		

Hypothesis

H_0 : Treatment effects are same V/S

H_1 : Treatment effersets are not same.

Conclusion :- $F_1 = 3.1263$

$F_1 \geq F_{cal}$

$3.1263 \geq F_\alpha(4,16)$

$3.1263 \geq 3.01$

Hence we reject H_0, Hence all the treatment effect are not same.

$F_\alpha \geq F_{cal}$

$0.6410 \geq F(9,16)$

$0.6410 \geq 2.54$

we do not reject H_0, Hence all the treatment effect are same.

P-4

(1) (i) the model is

$$y_{ij} = \mu + \alpha_i + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij} \quad , i=1,\dots,t \quad j=1,\dots,r$$

where

y_{ij} :- value of observation for the jth response variable for the ith treatment

μ :- overall mean.

α_i :- ith Treatment effect

β :- linear regression coefficient of Y on X

x_{ij} :- jth observation on the covariate for the ith treatment.

$\bar{x}_{..}$ - sample mean of x observation

ε_{ij} :- random errors.

Assumptions

$\varepsilon_{ij} \sim N(0, \sigma^2)$

$\Sigma \alpha_i = 0$

Calculation :-

$r = 5, \; t = 3, \; n = r \times t = 5 \times 3 = 15$

$X_{..} = 185 \qquad Y_{..} = 714.9$

$C.F_{xx} = \frac{X_{..}^2}{n} = \frac{185^2}{15} = 2281.666$

$CF_{yy} = \frac{Y_{..}^2}{n} = \frac{714.9^2}{15} = 34072.134$

$CF_{xy} = \frac{X_{..} Y_{..}}{n} = \frac{185 \times 714.9}{15} = 8817.1$

$TSS_{xx} = \Sigma\Sigma X_{ij}^2 - CF_{xx} = 2317 - 2281.666$

$TSS_{xx} = 35.334$

$TSS_{yy} = \Sigma\Sigma Y_{ij}^2 - CF_{yy} =$

$TSS_{yy} = 49.296$

$TSS_{xy} = \Sigma\Sigma X_{ij} Y_{ij} - CF_{xy}$

$= 8783.4 - CF_{xy} = -33.7$

$Tr.SS_{yy} = \frac{\Sigma Y_{ij}^2}{t} - CF_{yy} =$

$= \frac{102263.52}{3} - 34072.134 = 15.706$

$$Tr.SS_{xx} = \frac{\sum x_{i.}^2}{r} - CF_{xx}$$

$$= \frac{11427}{5} - 2281.666$$

$$= \cancel{1527.334}\ 3.734$$

$$Tr.SS_{xy} = \frac{\sum x_{i.} y_{i.}}{r} - CF_{xy}$$

$$= \frac{44056.3}{5} - CF_{xy}$$

$$= -5.84.$$

$$E_{yy} = TSS_{yy} - Tr.SS_{yy}$$

$$= 49.296 - 15.706$$

$$= 33.159$$

$$E_{xx} = TSS_{xx} - Tr.SS_{xx}$$

$$= 35.334 - 3.734$$

$$= 31.6$$

$$E_{xy} = TSS_{xy} - Tr.S_{xy}$$

$$= -33.7 - (-5.84)$$

$$= -27.86$$

(ii). Hypothesis

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

H_0: there is no effect of covariate.

H_1: the effect of covariate is there

$$SSE = E_{yy} - \frac{E_{xy}^2}{E_{xx}} \Rightarrow 33.159 - \frac{(-27.86)^2}{31.6}$$

$$= 33.15 - 24.5626$$

$$SSE = 8.5874$$

$$MSE = \frac{SSE}{n-t-1} = \frac{8.5874}{15-3-1} = 0.78$$

$$F_{cal} = \frac{E_{xy}^2 / E_{xx}}{MSE} \sim F_{(\beta, n-t-1)}$$

$$= \frac{24.5626}{0.78} \sim F^{(0.05)}_{(1,11)}$$

$$= 31.49$$

$$\therefore F^{(0.05)}_{(1,11)} = 4.84$$

$$\therefore F_{cal} > F_{tab}$$

Conclusion $31.49 > 4.84$

$\therefore$ we reject H_0

$\therefore$ the effect of covariate is there.

(ii) $H_0 : \alpha_1 = 0 \quad \forall i$

~~H_0 the glue formulations are~~ not significant.

H_0: there is no significant difference in the glue formation.

$H_1 : \alpha_1 \neq 0$

There is a significant difference in glue formation.

$T_{SSS} = SSE - SS$

$$SSE' = TSS_{YY} - \frac{TSS_{XY}^2}{TSS_{XX}}$$

$$= 49.296 - \frac{(-33.7)^2}{35.334}$$

$$= 49.296 - 32.1415$$

$$SSE' = 17.1545$$

$$F_0 \frac{(SSE' - SSE)/t-1}{MSSE} \sim F(t-1, n-t-1)$$

$$F_0 = \frac{(17.1545 - 8.5874)/3-1}{0.78}$$

$$F_0 = \frac{4.28355}{0.78} = 5.491$$

$$F(t-1, n-t-1) \Rightarrow F(3-1, 15-3-1) \Rightarrow F(2, 11) = 3.98$$

~~ANOVA Table~~

$\therefore F_{cal} > F_{tab}$

$\therefore 5.491 > 3.98$

$\therefore$ we reject H_0

$\therefore$ there is a significant difference in glue formation

ANCOVA Table.

S.V.	Sum of Squares & Cross products. x	xy	y	d.f	adjusted treatment SS for Regression S.S	MSS	F_{tab}
Treatment	$T_{xx} = 3.734$	-5.84	15.706	.			
Error	$E_{xx} = 31.6$	-27.86	33.15	$n-t-1=11$	$SSE = 8.5874$	0.78	
Total	$TSS = 35.334$	-33.7	49.296	$n-2=13$	$SSE' = 17.1545$		$F_0 = 5.49$
Treatment				$t-1=2$	$Tr.SS = SSE' - SSE$ 4.28355		

(iv) SE of adj $(\bar{y}_{i.} - \bar{y}_{j.}) = \sqrt{MSE\left(\frac{2}{r} + \frac{\bar{x}_{i.} - \bar{x}_{j.}}{E_{xx}}\right)}$

$\bar{x}_{1.} \rightarrow$ 1 treatment mean.

$\bar{x}_{j.} \rightarrow$ 2 treatment mean. 0.67

$$= \sqrt{MSE\left(\frac{2}{5} + \frac{6}{31.6}\right)}$$

$$= 0.68$$

d). The linear model is,

$$y_{ij} = \mu + \alpha_i + \xi_j + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij}$$

$i = 1, \cdots, t$

$j = 1, \cdots, b$

where, $y_{ij} =$, $\mu =$, $\alpha_i =$, $\xi_j =$, $\beta =$, $\varepsilon_{ij} =$

Hypothesis.

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

1

Here $t=3$, $b=4$, $n=tb=12$.

$CF_{xx} = \frac{X_{..}^2}{n} = \frac{290^2}{12} = 7008.33$

$CF_{yy} = \frac{Y_{..}^2}{n} = \frac{483^2}{12} = \frac{233289}{12} = 19440.75$

$CF_{xy} = \frac{X_{..}Y_{..}}{n} = \frac{\overset{290\times 483}{\cancel{11949}}}{12} = \cancel{995.75}\ 11672.5$

$TSS_{xx} = \sum\sum X_{ij}^2 - CF_{xx}$

$= 7262 - CF_{xx} = 253.67$

$TSS_{yy} = \sum\sum Y_{ij}^2 - CF_{yy}$

$TSS_{yy} = 19781 - 19440.75 = 340.25$

$TSS_{xy} = 1\cancel{1949}\ \sum\sum X_{ij} Y_{ij} - CF_{xy}$

$= 11949 - 11672.5$

$TSS_{xy} = 276.5$

$Tr_{yy} = \frac{\sum Y_{i.}^2}{b} - CF_{yy}$

$= \frac{78597}{4} - 19440.75$

$= 19649.25 - 19440.75$

$Tr_{yy} = 208.5$

$Tr_{xx} = \frac{\sum X_{i.}^2}{b} - CF_{xx} \Rightarrow$

$Tr_{xx} = \frac{28468}{4} - CF_{xx} = 7117 - CF_{xx} = 108.67$

$$Tr_{xy} = \frac{\sum x_{i\cdot} y_{i\cdot}}{b} - CF_{xy} = \frac{47292}{4} - CF_{xy}$$

$$Tr_{xy} = 11823 - CF_{xy} \Rightarrow 150.5$$

$$B_{yy} = \frac{\sum Y_{\cdot j}^2}{t} - CF_{yy} = \cancel{5846}\frac{58463}{3} - CF_{yy}$$

$$= 19487.666 - CF_{yy}$$

$$B_{yy} = 46.9166$$

$$B_{xx} = \frac{\sum X_{\cdot j}^2}{t} - CF_{xx} = \frac{21146}{3} - CF_{xx} = 40.33$$

$$B_{xy} = \frac{\sum X_{\cdot j} Y_{\cdot j}}{t} - CF_{xy}$$

$$= \frac{35140}{3} - 11672.5 = 11713.33 - CF_{xy}$$

$$B_{xy} = 40.833$$

$$E_{yy} = TSS_{yy} - TrSS_{yy} - B_{yy}$$

$$= 340.25 - 208.5 - 46.9166$$

$$= 84.8334$$

$$E_{xx} = TSS_{xx} - TrSS_{xx} - B_{xx}$$

$$= 253.67 - 108.67 - 40.33$$

$$= 104.67$$

$$E_{xy} = TSS_{xy} - TrSS_{xy} - B_{xy}$$

$$= 276.5 - 150.5 - 40.833$$

$$= 85.167$$

$$\hat{\beta} = \frac{E_{xy}}{E_{xx}} = \frac{85.167}{104.67} = 0.8136$$

P 2

The model :

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i = 1, --- t$$
$$j = 1, --- b$$

$\alpha_i =$, $\beta_j =$, ε_{ij}

Assumptions

— u —

Hypothesis :-

$H_0^1 = \alpha_1 = \alpha_2 = --- = \alpha_t = 0$ (overall)

$H_1^1 =$

$H_0^2 = \beta_1 = \beta_2 = --- = \beta_b = 0$

$H_1^2 =$

estimate the missing observation

$$\hat{y}_{ij} = \frac{t\, y_{i.}' + b\, y_{.j}' - y_{..}'}{(b-1)(t-1)}$$

where,

$y_{i.}'$ = sum of known observations for i^{th}

$y_{.j}'$ = sum of known observation for j^{th}

$y_{..}$ = sum of all known observations

Given, $b = 4$, $t = 3$

$$\hat{y}_{22} = \frac{3(109.4) + 4(58.7) - 377}{3 \times 2}$$

$$\hat{y}'_{22} = \frac{186}{6}$$

$$\hat{y}'_{22} = 31$$

	A	B	C	$Y_{.j}$
1	32.4	35.6	38.7	106.7
2	28.8	31	29.9	89.7
3	36.5	37.6	29.1	103.2
4	34.4	36.2	37.8	108.4
$Y_{i.}$	132.1	140.4	135.5	

$$C.F = \frac{Y_{..}^2}{b \times t} = \frac{(408)^2}{4\times 3} = \frac{166464}{12} = 13872$$

$$SST_r = \frac{\Sigma Y_{i.}^2}{b} - CF$$

$$= \frac{132.1^2 + 140.4^2 + 135.5^2}{4} - CF$$

$$= 13880.705 - 13872$$

$$SST_r = 8.705$$

$$SSB = \frac{\Sigma Y_{.j}^2}{t} - CF$$

$$= \frac{(106.7^2 + 89.7^2 + \dots 108.4^2)}{3} - CF$$

TSS =

$= 13943.9266 - 13872$

$SSB = 71.926$

$SST = \sum_i \sum_j y_{ij}^2 - CF$

$= 14014.72 - 13872$

$SST = 142.72$

$SSE = TSS - (SST_r + SSB)$

$= 142.72 - 8.705 - 71.926$

$SSE = 62.089$

ANOVA TABLE

SV	d.f	S.S.	MSS	F-ratio
Treatment	$t-1 = 2$	8.705	4.3525	0.3505
Block	$b-1 = 3$	71.926	23.9753	1.93072.
Error	$(t-1)(b-1)-1 = 5$	62.089	12.4178	
Total	$bt-2 = 10$	142.72	14.272.	

For treatment :- $F_{0.05}(2,5)$

$F_{cal} \geq F_{0.05}(2,5)$

$0.3505 \geq 5.79.$

$\therefore$ we donot reject H_0 for treatment

Hence α_i's dont differ. significantly

Standard error of difference ...

$SE(\bar{y}_A - \bar{y}_B) = \sqrt{\ldots}$

$SE(\bar{y}_A - \bar{y}_B) = \ldots$

Stand error of difference ...

$SE(\bar{y}_A - \bar{y}_E) = \sqrt{\ldots}$

$SE(\bar{y}_A - \bar{y}_E) = \ldots$

$d^3(r-1)$

$= \frac{(3.86)^2}{d^3(r-1)} = \cdot$

$\lambda = \frac{r(k-1)}{(t-1)}$

(2). LSD.
The missing observations.

$$y_{ijk} = \frac{m(y'_{i..} + y'_{.j.} + y'_{..k}) - 2(y'_{...} + \text{initial estimate})}{(m-1)(m-2)}$$

$$y^{(0)} = \frac{\Sigma y'_{i..}}{m-1}$$

	A	B	C	D	E	F
	92	220	160	149	282	98
	74	213.15	228	168	238	282
	119.5	176	279	188	278	118
	64	222	213	104	295	163
	66	188	187	90	242	96
	124	109	195	79	211	90
$y'_{i..}$	420 (530.5)	915 (1128.15)	1262	778	1546	847

							$y'_{.j.}$
	220	98	149	92	282	160	1001
	74	238	213.15	228	282	168	990 (1203.15)
	188	279	118	278	176	119.5	1039 (1158.5)
	295	222	64	104	213	163	1061
	187	90	242	96	66	188	869
	90	124	195	109	79	211	808
$y'_{..k}$	1054	1051	768 (981.15)	907	1098	890 (1009.5)	

For missing value of K,

$$\hat{y}'_{ijk.} = \frac{6(420+1039+\cancel{768}^{890}) - 2(5768+84)}{5\times 4}$$

$$= \frac{\cancel{11362} - 11704}{20}$$

$$\hat{y}'_{ijk} = \cancel{82.9}\ \frac{2390}{20}$$

$$y^{(0)} = \frac{\sum y'_{i..}}{m-1} = \frac{420}{5} = 84$$

$$\hat{y}'_{A36}\ \hat{y}'_{ijk.} = 119.5$$

For missing value of B,

$$\hat{y}_{ijk.} = \frac{6(915+768+990) - 2(5768+119.5)}{20}$$

$$= \frac{16038 - 11775}{20}$$

$$\hat{y}_{B23} = 213.15$$

A	B	C	D	E	F
92	220	160	149		

$$CF = \frac{y_{..}^2}{m^2} = \frac{(6100.65)^2}{6\times 6} = 1033831.401$$

$$SST = \sum\sum\sum y_{ijk}^2 - CF$$

$$= (220^2 + 98^2 + \cdots 211^2) - CF$$

$$= (461077.9225 + 472808.25 + 274693) - CF$$

$$= 1208579.173 - 1033831.401$$

$$\wedge = \frac{0 \ldots}{(b-1)}$$

classmate

Date ____

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$SST = 174747.7715$

$$SSR = \frac{\sum y_{.j.}^2}{m} - CF$$

$$= \frac{6325439.173}{6} - CF$$

$= 1054239.862 - 1033831.401$

$SSR = 20408.46108$

$$SSc = \frac{\sum y_{..k}^2}{m} - CF$$

$$= \frac{6225515.573}{6} - CF$$

$= 1037585.929 - CF$

$SSC = 3754.5277$

$$SStr = \frac{\sum y_{i..}^2}{m} - CF$$

$$= \frac{6869235.673}{6} - CF$$

$= 1144872.612 - 1033831.401$

$SSt_j = 111041.2111$

$SSE = SST - SSR - SSC - SSt_j$

$= 174747.7715 - 20408.46108 - 3754.5277 - 111041.2111$

$SSE = 39543.57162$

ANOVA TABLE

S.V	d.f	S.S	MSS	F ratio
Treatment	$t-1=5$	111041.2111	22208.24222	70.20182
Row	$r-1=5$	20408.46108	4081.6922	12.9025
Column	$c-1=5$	3754.5277	750.90554	2.37366
Error	$(t-1)(r-1)(c-1)=125$	39543.57162	316.3485	
Total	$trc-2=214$	174747.7715	816.5783	

Row :- Since, $12.9025 > \cancel{8.75}$ 4.39, we ~~fail to~~ reject H_0,

Column :- Since, $2.373 < 4.39$ we don't reject H_0

Treatment :- Since, $70.208 > 4.39$ we reject H_0.

P-3

1) i) The linear model

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i = 1, \ldots, t$$
$$j = 1, \ldots, b$$

where,

Y_{ij} : observations for i^{th} treatment in the j^{th} block of it units.

$\alpha_i =$, $\beta_j =$, ε_{ij}

Assumption :- Same as RBD — " — ".

~~Hypothesis and parameters~~ :-

1. ε_{ij}'s are iid random variables with $N(0, \sigma^2)$ dis^n
2. Treatment & block effects are fixed effects
3. All the observations are independent.

the blocks $(\beta_i = 0)$
$(\beta_i \neq 0)$

H_0^3: there is no sig diff b/w the land $(\gamma_i = 0)$
H_1^3: there is sig diff b/w the land $(\gamma_i \neq 0)$.

P-6

(i) Model :-

$$\mu + \underbrace{\alpha_i + \beta_j + \gamma_k}_{\text{main factors}} + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \delta_l + \varepsilon_{ijkl}$$

where,

$i, j, k = 0, 1 \ldots$
$l = 1, \ldots, r$

y_{ijkl} :- Observation for the l^{th} replicate at i^{th} level of factor A, j^{th} level of factor B & k^{th} level of factor C

α_i : effect due to i^{th} level of factor A
β_j : effect due to j^{th} level of factor B
γ_k :- effect due to k^{th} level of factor C
$(\alpha\beta)_{ij}$:- Interaction effect b/w i^{th} level of factor A & j^{th} level of factor B.
$(\alpha\gamma)_{ik}$: Interaction effect b/w i^{th} level of factor A & k^{th} level of factor C
$(\beta\gamma)_{jk}$:- Interaction effect b/w j^{th} level of factor B & k^{th} level of factor C
$(\alpha\beta\gamma)_{ijk}$:- Interaction effect b/w i^{th} level of factor A & j^{th} level of factor B & k^{th} level of factor C
δ_l :- effect due to l^{th} replicate
ε_{ijkl} :- Random error

Assumption
- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

(ii) Hypothesis
H_0: the main effect & interaction effects are not significant.
H_1: the main effects & interaction effects are significant.

calculation:- $r=2$. Yate's table

	TC	Totals	I	II	III	effect	SS (effect total)^2
$a_0b_0c_0$	(1)	79	141	232	427	G	$\frac{(427)^2}{2^3 r} = \frac{(427)^2}{2^3(2)} = \frac{427^2}{16} = 11395.562$
$a_1b_0c_0$	a	62	91	195	-97	A	$SS_A = 588.0625$
$a_0b_1c_0$	b	63	116	-52	-87	B	$SS_B = 473.0625$
$a_1b_1c_0$	ab	28	79	-45	-35	AB	$SS_{AB} = 76.5625$
$a_0b_0c_1$	c	65	-17	-50	-37	C	$SS_C = 85.5625$
$a_1b_0c_1$	ac	51	-35	-37	7	AC	$SS_{AC} = 3.0625$
$a_0b_1c_1$	bc	55	-14	-18	13	BC	$SS_{BC} = 10.5625$
$a_1b_1c_1$	abc	24	-31	-17	1	ABC	$SS_{ABC} = 0.0625$.

$$\text{Replicate SS} = \frac{R_1^2 + R_2^2}{2^3} - CF$$

$$= \frac{46225 + 44944}{8} - 11395.562$$

$$= 11396.125 - 11395.562.$$

$$\text{Replicate SS} = 0.563$$ //.

$$SST = \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 - CF$$

$$= \cancel{427}\ 12671 - CF \quad \cancel{2) + 82329 - CF}$$

$$SST = \cancel{170933.438} \; 1275.438$$

$\cancel{SSE = SST - RSS}$ $SSE = TSS - SS_A - SS_B \ldots SS_{ABC}$.

$\cancel{SSE = 1274.875}$ $SSE = 37.52$.

ANOVA.

source	d.f	S.S	MSS.	F-ratio
Replicate	$r-1 = 2-1$	0.563	0.563	
A	1	588.0625	588.0625	$F_A = F_{(1,7)}^{(0.05)} = \frac{588.0625}{5.36} = 109.7137$
B	1	473.0625	473.0625	$F_B =$ 88.2579
AB	1	76.5625	76.5625	$F_{AB} =$ 14.2840
C	1	85.5625	85.5625	$F_C =$ 15.9637
AC	1	3.0625	3.0625	$F_{AC} =$ 0.57136
BC	1	10.5625	10.5625	$F_{BC} =$ 1.97061
ABC	1	0.0625	0.0625	$F_{ABC} =$ 0.0116
Error	7	37.52	5.36	
Total	$2^3 r - 1 = 15$			

$F_{tab} \Rightarrow F_{(1,7)} = 5.59$

here. $F_A \Rightarrow 109.7137 > 5.59$, $F_C \Rightarrow 15.9631 > 5.5$

$F_{AB} \Rightarrow 14.2840 > 5.59$

$F_B \Rightarrow 88.2579 > 5.59$

$\therefore F_A; F_B, F_C, F_{AB}$ is greater than F_{tab} va

so we reject H_0

Hence, the main effects & interaction effect a Significant!

(ii) $SE = \sqrt{\frac{MSE}{2r}} \Rightarrow \sqrt{\frac{5.36}{4}} = 1.1575$

H_1: the main effect & Interaction effect are significant.

(2) model :-

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \sigma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\alpha\sigma)_{il} + (\beta\gamma)_{jk} + (\beta\sigma)_{jl} + (\gamma\sigma)_{kl}$$
$$(\alpha\beta\gamma)_{ijk} + (\alpha\beta\sigma)_{ijl} + (\beta\gamma\sigma)_{jkl} +$$
$$(\alpha\gamma\sigma)_{ikl} + (\alpha\beta\gamma\sigma)_{ijkl} + \varepsilon_{ijkl}$$

Assumptions :- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

TC	Total	I	II	III	IV	effect	$\frac{SS(\text{effect total})^2}{2^4 \cdot 8}$
(1)	7	16	105	201	426	G	11342.25 = CF
a	9	-89	96	225	~~115~~ 108	A	~~826.5625~~ 729
b	34	16	112	~~54~~ 47	282	B	4970.25
ab	55	80	113	61	88	AB	484
c	6	21	23	137	-8	C	4
ac	10	91	~~31~~ 24	145	~~5~~ -2	AC	0.25
bc	30	19	32	35	-4	BC	1
abc	50	94	29	53	-10	ABC	~~6.25~~
d	10	2	73	-9	24	D	36
ad	11	21	64	1	14	AD	12.25
bd	30	11	70	81	8	BD	4
abd	61	20	75	-3	18	ABD	~~20.25~~
cd	8	1	19	-9	10	CD	6.25
acd	11	31	16	5	-4	ACD	~~1~~
bcd	34	3	30	-3	14	BCD	~~12.25~~
abcd	60	26	23	-7	-4	ABCD	1

Replicate $SS = \frac{R^2}{2^4}$

$SST = \sum\sum\sum\sum y^2_{ijkl} - CF$

$= 17630 - CF$

$SST = 6287.75$

$SSE = 6287.75 - SS_A - SS_B - \cdots$

$SSE = 40.75$

ANOVA

Source	d.f	S.S	MSS	F-ratio
Treatment	$t-1$			
A	1	729	729	$F_A = F^{(0.05)}_{(1,5)} = \frac{729}{8.15} = 89.44$
B	1	4970.25	4970.25	609.84
AB	1	484	484	59.386
C	1	4	4	0.4907
AC	1	0.25	0.25	0.0306
BC	1	1	1	0.1226
~~ABC~~ D	1	36	36	4.417
AD	1	12.25	12.25	1.5030
~~AD~~ BD	1	4	4	0.4907
~~BD~~ CD	1	6.25	6.25	0.766
~~ABD~~ Error	5	40.75	8.15	
~~CD~~ Total	$2^4-1=15$			

~~ACD~~

~~BCD~~

$F_{tab} = F^{(0.05)}_{(1,5)} = 6.61$

E_A ∴ $F_A, F_B, F_{AB} > 6.61$ ∴ we reject H_0
Hence, the main effects & interaction effects are Significant.

$(a-1)(b-1)(c-1)$
$(ab-a-b+1)(c-1)$
$(ab-a-b+1)(c-1)$
$abc-ab-ac+a-bc+b+c-1$

(1) Conclusion:-
Here the metal used, the amount of primary initiator, and the packing pressure of the explosive and the interaction between the metal used and the amount of primary initiator are not same.

P-7

$ABC = (a-1)(b-1)(c-1)$
$= (ab-a-b+1)(c-1)$
$= abc - bc - ac - ab + a + b + c - 1$

(2).

Rep I		Rep II		Rep III	
(1)	a	(1)	abc	ab	a
ab	b	ab	c	ac	c
bc	c	ac	a	bc	b
ac	abc	bc	b	(1)	abc
−	+	−	+	−	+

abc   abc   abc.

Overall confounding ~~~~ ~~~~ we have same.
interaction confounding

	I		II		III	
(2).	1	k	1	p	p	n
	pk	p	k	nk	npk	k
	nk	n	npk	pk	1	pk
	np	npk	np	n	nk	np.
	npk		np		nk	

Sh (b) Model :-

$$\mu + \alpha_i + \beta_j + \sigma_k + (\alpha\beta)_{ij} + (\alpha\sigma)_{ik} + (\beta\sigma)_{jk} + (\alpha\beta\sigma)_{ijk} + \gamma_l + \varepsilon_{ijkl}$$

Assumption :- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

Hypothesis :-

H_0 :- The main effect & interaction effect are not significant

H_1 :- The main & interaction effect are significant.

$\therefore$ we can say that abc is confounded.

TC	Totals	I	II	III	effect	SS $\frac{(\text{effect tol})^2}{2^3 3} = \overline{24}$
(1)	78	182	428	959	G	38320.04
a	69	246	531	-17	A	12.0416
b	119	240	34	115	B	551.041
ab	127	291	-51	-41 ~~-69~~	AB	70.0416
c	127	26	64	103	C	442.0416
ac	113	8	51	-85	AC	301.0416
bc	164	-14	-18	-13	BC	7.0416
abc	127	-37	-23 ~~-51~~	-5 ~~-23~~	ABC	1.0416

$$SST = \sum\sum\sum\sum y_{ijkl}^2 - CF$$

$= 40621 - 38320.04$

$SST = 2300.96$ //

Incomplete block Sum of Squares.

$BSS = \frac{\Sigma B_i^2}{2^{k-1}} - CF$

$= \frac{154119}{2^{3-1}} - 38320.04$

$= 38529.75 - 38320.04$

$BSS = 209.71$

$ESS = TSS - SS_A - SS_B = - - SS_{ABC}.$

$= 2300.96 -$

$ESS = 708.19566$ //

ANOVA Table

Source	d.f.	S.S	MSS	F ratio
Incomplete block	$b-1=5$	209.71	41.942	$F_I = 0.71068$
A	1	12.0416	12.0416	$F_A = F^{0.05}_{(1,\ )} = \frac{12.0416}{59.0163} = 0.2040$
B	1	551.041	551.041	$F_B = 9.33709$
AB	1	70.0416	70.0416	1.1868
C	1	442.0416	442.0416	7.4901
AC	1	301.0416	301.0416	5.1009
BC	1	7.0416	7.0416	0.1193
Error	$23-6-5=12$	708.1956	59.0163	
Total	$2^3 3-1=23$			

$F^{tab}_{(1,12)} = 4.75$

I.B $\Rightarrow$ $F^{tab}_{(5,12)} = 3.11$

For incomplete block.

$F^{tab}_{(5,12)} \Rightarrow 3.11 > 0.71068$

Since F_{tab} is greater than F_{cal} for incomplete i. block, so we don't reject H_0

Hence the main effect & er

For treatments

$F^{tab}_{(1,12)} = 4.75 < F_B, F_C, F_{AC}$ are greater than tab value.

Hence we reject H_0 for ~~F_A, F_A~~ A, AB, ~~& BC~~ B, C & AC.

(iii) ~~Est~~ To estimate S.E.

a) main effect.

$[A] = \frac{\text{effect total}}{2^3 r} = \frac{-17}{24} = -0.708$

$[B] = \frac{\text{effect total}}{24} = \frac{115}{24} = 4.79$

$[C] = \frac{103}{24} = 4.29$

b) first order interaction

$[AB] = \frac{\text{effect total}}{2^3 r} = \frac{-41}{24} = 1.708$

$[BC] = 0.541$

$[AC] = -3.54$

(2) From the above table we can say that UPK, UP & UK are confounded.

Model :- $\mu + \alpha_i + \beta_j + \sigma_k + (\alpha\beta)_{ij} + (\alpha\sigma)_{ik} + (\beta\sigma)_{jk} + (\alpha\beta\sigma)_{ijk} + \gamma_l + \varepsilon_{ijkl}$.

Assumption :- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

Hypothesis :-

H_0: The main & interaction effect are not significant

H_1: The main & interaction effect are significant.

TC	Totals	I	II	III	effect	SS $2^3 \times 3$
(1)	246	607	1391	2942	G	360640.166
u	361	784	1551	376		5890.6666
p	406	757	87	214		1908.166
up	378	794	289	-344		—
k	256	115	177	160		1066.666
uk	501	-28	37	202		—
pk	375	245	-143	-140		816.666
upk	419	44	-201	-58		—

Date ____
Page ____

adjusted effect total

$NPK = [NPK] - (B_1 - B_2)$ block diff
$= [-58] - (-78)$
$= -58 + 78$ $\Sigma B_1 - \Sigma B_2$
$\boxed{NPK = 20}$

$NP = [NP] - (B_3 - B_4)$
$= (-344) - (-276)$
$\boxed{NP = -68}$

$NK = [NK] - (B_5 - B_6)$
$= [202] - (-184)$
$NK = \cancel{386}\ 386$

$SS_{NPK} = \frac{(\text{effect total})^2}{2^3(r-1)} = \frac{(\cancel{-58}\ 20)^2}{2^3(2)} = \frac{(-58)^2}{16} = \cancel{210.25}\ 25$

$SS_{NP} = \frac{(\cancel{-344}\ 68)^2}{16} = 289$ //

$SS_{NK} = \frac{(\cancel{202}\ 386)^2}{16} = 9312.25$ //

$TSS = 413814 - 360640.166$
$TSS = 53173.834$ //

Incomplete block sum of squares

$BSS = \frac{\Sigma B_i^2}{2^{n-1}} - CF \Rightarrow \frac{1524556}{4} - CF$

$= 381139 - CF$

$BSS = 20498.834$

$$ESS = TSS - BSS - SS_N - SS_P - \cdots - SS_{NPK}$$
$$= 13366.\cancel{586}604$$

ANOVA TABLE

Source	d.f	S.S	MSS.	F-ratio
Treatment	5	20498.834	4099.766	= ~~4.2940~~ 3.3738
N	1	5890.666	5890.66	~~6.6169~~ 4.8477
P	1	1908.166	1908.166	~~1.9985~~ 1.57031
K	1	1066.66	1066.66	~~1.1172~~ 0.87780
PK	1	816.66	816.66	~~0.855~~ 0.67206
~~Error~~	~~23-7-5=14~~	~~13366.~~604	~~954.757~~	
~~Total~~	~~2^3-1=23~~			
NP	1	289	289	0.23783
NK	1	9312.25	9312.25	7.6634
NPK.	1	25	25	0.02057.
Error	23-7-5=11	13366.604	1215.145	
Total	2^3-1=23			

~~$F^{tab}_{(1,14)}$ = 4.60~~

~~$F_{(5,14)}$ = 2.96~~

$F^{tab}_{(1,11)} = \cancel{3.20}\ 4.84$

$F_{(5,11)} = 3.20$

conclusion.
here 4 & 4k is greater than 4.84
Hence we reject H_0
Hence F_N and F_{NK} are significant.

(i.v) Standard Error

a) main effect of $[N] = \frac{(\text{effect total})}{2^2 \cdot r} = \frac{376}{12}$

$= 31.33$

$[P] = \frac{214}{12} = 17.833$

$[K] = \frac{160}{12} = 13.33$

b). Interaction effect

$[NP] = \frac{-68}{12} = -5.66$

$[NK] = \frac{386}{12} = 32.166$

$[PK] = \frac{-140}{12} = -11.666$