The parameter :-

$t=7,\ b=7,\ r=3,\ K=3$

$\lambda =$ no of blocks in which a pair of treatment occurs together.

$$\lambda = \frac{r(K-1)}{t-1} \Rightarrow \frac{3(3-1)}{7-1} = \frac{6}{6} = 1$$

Hypothesis :-

$H_0^1$ :

$H_1^1$ :

$H_0^2$ :

$H_1^2$ :

$$CF = \frac{G^2}{n} = \frac{Y_{..}^2}{bK} \Rightarrow \frac{(2715)^2}{7 \times 3} = \frac{7371225}{21}$$

$$CF = 351010.7143$$

Total sum of squares TSS

$$TSS = \sum_i \sum_j Y_{ij}^2 - CF$$

$$= 353611 - 351010.7143$$

$$TSS = 2600.2857$$

Blocks s.s (unadjusted)

$$BSS = \frac{\sum Y_{.j}^2}{K} - CF$$

$$= \frac{1056375}{3} - CF$$

BSS $=$ ~~3~~ $1114.2857$ //.

Adjusted treatment totals

$$Q_i = Y_{i.} - \frac{1}{K}\left(\sum_{j=1}^{b} I_{ij} Y_{.j}\right)$$

where $I_{ij} = \begin{cases} 1, & \text{if } i^{th} \text{ treatment appears in } j^{th} \text{ block} \\ 0, & \text{otherwise} \end{cases}$

$$Q_1 = 351 - \frac{1}{3}(1 \times 381 + 381 + 378)$$

$$Q = -29$$

$$Q_2 = 365 - \frac{1}{3}(1 \times 381 + 402 + 372)$$

$$Q_2 = -20$$

$$Q_3 = 388 - \frac{1}{3}(1 \times 402 + 366 + 378)$$

$$= 6$$

$$Q_4 = 419 - \frac{1}{3}(1 \times 381 + 366 + 435)$$

$$Q_4 = 25$$

$$Q_5 = 438 - \frac{1}{3}(1 \times 402 + 435 + 381)$$

$$Q_5 = 32$$

$$Q_6 = 361 - \frac{1}{3}(1 \times 366 + 381 + 372)$$

$Q_6 = -12$

$Q_7 = -2$

$\therefore$ adjusted treatments

$$Tr.ss\ adj = \frac{K \sum Q_i^2}{\lambda t}$$

$$= \frac{3 \times 2963074}{1 \times 7}$$

$$= \frac{888}{7} = 1317.43$$

undjusted T.SS. $= \frac{\sum Y_{i.}^2}{r} - CF$

$$= \frac{1059145}{3} - 35100.7143$$

undjusted $\cancel{BSS} = 2037.6190$

$ESS = TSS - BSS(\text{undjusted}) - Trss(\text{adjusted})$

$= 2600.2857 - 1114.2857 - 1317.43$

$ESS = 168.58$

ANOVA.

| | d.f | S.S | MSS | F-ratio |
|---|---|---|---|---|
| treatment (adj) | t-1=6 | 1317.43 | 219.571 | 10.4197 |
| block (unadj) | b-1=6 | 1114.2857 | 185.7142 | 80 |

(Error) $(bK-1)-(t-1)-(b-1)=8$   168.58   21.0725

(Total) $bK-1=20$   2600.2857   130.0142

$F_{cal} > F_{(0.05)}(6,8)$

$F_{cal} > \cancel{10.4197}\ 3.58$

$10.4197 \geq 3.58$

$\therefore$ we reject $H_0$, since all concentrations are not same.

$BSS(adj) = Trss(adj) + BSS(unadjusted) - Trss(unadj) = 394.0967$

ANOVA.

| S.V | d.f | S.S | MSS | F-ratio |
|---|---|---|---|---|
| Trt (unadj) | 6 | 2037.6190 | 339.6031 | |
| Blocks (adj) | 6 | 394.08 | 65.68 | 3.11 |
| Error | 8 | 168.58 | 21.0725 | |
| Total | 20 | 2600.2857 | 130.014 | |

$F_{cal} > F_{0.05}(6,8)$

$F_{cal} > 3.58$

$3.11 < 3.58$

$\therefore$ we donot reject $H_0$.

$\therefore$ all days are same.

$H_0^1$: all concentration (treatments) are same

$H_1^1$: all concentration are not same.

$H_0^2$: all days (blocks) are same

$H_1^2$: all days (blocks) are not same

(iv) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\dfrac{2k\hat{\sigma}^2}{\lambda t}} = \sqrt{\dfrac{2K\,MSSE}{\lambda t}}$

i – bloc
k = rows.

$$\sqrt{\frac{2 \times 3 \times 21.0725}{1 \times 7}} = \sqrt{18.0621}$$

$$S.E(Q_i - Q_i) = 4.2$$

(3). Model :-

$$y_{ijk} = \mu + \alpha_i + \beta_j + \delta_k$$

where, $\alpha_i =$ , $\beta_j =$

$i = 1, ---, t$
$j = 1, ---, b$
$k = 1, ---, r$

Assumptions:-

i) $\alpha_i$'s, $\beta_j$'s & $\delta_k$'s are fixed effect

ii) $\epsilon_{ijk} \overset{iid}{\sim} N(0, \sigma^2)$

Hypothesis :-

$H_0^1$ : The four manorial treatment are same

$H_1^1$ : The four manorial treatment are not same.

$H_0^2$

$H_1^2$ ;

$$\lambda = \frac{r(k-1)}{t-1} = \frac{3(3-1)}{4-1} = \frac{6}{3} = 2$$

$H_0^3$

$H_1^3$ ;

$t = 4$, $b = 4$, $r = 3$, $K = 3$, $n = b \times k = 12$

Treatm.

| | A | B | C | D |
|---|---|---|---|---|
| | 432 | 442 | 540 | 340 |
| | 468 | 490 | 562 | 384 |
| | 568 | 584 | 556 | 526. |
| $y_{i..}$ | 1468 | 1516 | 1658 | 1250 |

$$CF = \frac{G^2}{n} = \frac{34715664}{12} = 2892972$$

$$TSS = \sum\sum y_{ijk}^2 - CF$$

$$= 2961104 - CF$$

$$TSS = 68132.$$

$$\cancel{BSS(}\ RSS = \frac{\sum y_{..k}^2}{b} - CF$$

$$RSS = 30150$$

$$BSS(\text{unadj}) = \frac{\sum y_{.j.}^2}{k} - CF$$

$$BSS(\text{un}) = 11150.666$$

1445.333

adjusted treatment totals

$$Q_i = y_{i..} - \frac{\sum n_{ij} y_{.j.}}{K}, \quad i = 1, \dots, t$$

where $n_{ij} = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ treatment appears in block } j \\ 0, & \text{o.w} \end{cases}$

$$Q_1 = 1468 - \left(\frac{1372 + 1572 + 1392}{3}\right)$$

$Q_1 = 22.67$

$Q_2 = 9.33$

$Q_3 = 1658 - \frac{(1372+1562+1556)}{3}$

$Q_3 = 158$

$Q_4 = 1250 - \frac{(1372+1556+1392)}{3}$

$Q_4 = -190$

adjusted Tr. ss i.e.,

$$Trss(adj) = \frac{k}{\lambda t} \sum_{i=1}^{t} Q_i^2$$

$$= \frac{3}{2 \times 4} \times 61664.9778$$

$$Trss(adj) = 23124.3666$$

$$ESS = TSS - \underset{unadj}{(BSS)} - \underset{(adj)}{Tr.ss} - RSS.$$

$$= 68132 - 11150.666 - 23124.366 - 30150$$

$$ESS = 3706.974$$

ANOVA.

| SV | d.f | S.S. | MSS. | F ratio |
|---|---|---|---|---|
| treat (adj) | 4-1=3 | 23124.366 | 7708.122 | 6.23 |
| row | 3-1=2 | 30150 | 15075 | |
| Block (und | 4-1=3 | 11150.66 | 3716.88 | |
| error | (3+2+3-11) = 3 | 3706.974 | 1235.658 | |
| total | n-1=12-1 11 | 68132 | | |

$F_{cal} \geq F_{\alpha}(3,3)$
$6.23 \geq 9.28$
$\therefore$ we donot reject $H_0$.
The ~~no~~ four manorial treatments are same.

(iii) $S.E(\hat{\alpha}_i - \hat{\alpha}_j) = \sqrt{\frac{2K\, MSSE}{\lambda t}}$

$= \sqrt{\frac{2 \times 3 \times 1235.658}{2 \times 4}}$

$= \sqrt{926.7435}$

$S.E(\hat{\alpha}_i - \hat{\alpha}_j) = 30.44$ //.

(2) ANOVA TABLE

| S.V. | d.f. | S.S | M.SS | F value | P-value |
|---|---|---|---|---|---|
| Treat | 4 | 7.6133 | 1.9033 | 3.1263 | 0.04445 |
| Block | 9 | 3.5124 | 0.39027 | 0.6410 | 0.74734 |
| Residuals | 16 | 9.7409 | 0.60881 | | |

Hypothesis
$H_0$: Treatment effects are same V/S
$H_1$: Treatment effects are not same.

conclusion :- $F_1 = 3.1263$
$F_1 \geq F_{cal}$

$$3.1263 \geq F_\alpha(4,16)$$
$$3.1263 \geq 3.01$$

Hence we reject $H_0$, Hence all the treatment effect are not same.

$$F_\alpha \geq F_{cal}$$
$$0.6410 \geq F(9,16)$$
$$0.6410 \geq 2.54$$

we do not reject $H_0$, Hence all the treatment effect are same.

## P-4

(1) (i) the model is

$$y_{ij} = \mu + \alpha_i + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij} \quad , i=1,\dots,t \quad j=1,\dots,r$$

where

$y_{ij}$ :- value of observation for the $j$th response variable for the $i$th treatment

$\mu$ :- overall mean.

$\alpha_i$ :- $i$th Treatment effect

$\beta$ :- linear regression coefficient of Y on X

$x_{ij}$ :- $j$th observation on the covariate for the $i$th treatment.

$\bar{x}_{..}$ - sample mean of x observation

$\varepsilon_{ij}$ :- random errors.

Assumptions
$\varepsilon_{ij} \sim N(0, \sigma^2)$
$\Sigma \alpha_i = 0$

Calculation :-
$r = 5, \; t = 3, \; n = r \times t = 5 \times 3 = 15$

$X_{..} = 185 \qquad Y_{..} = 714.9$

$C.F_{XX} = \frac{X_{..}^2}{n} = \frac{185^2}{15} = 2281.666$

$CF_{YY} = \frac{Y_{..}^2}{n} = \frac{714.9^2}{15} = 34072.134$

$CF_{XY} = \frac{X_{..} Y_{..}}{n} = \frac{185 \times 714.9}{15} = 8817.1$

$TSS_{XX} = \Sigma\Sigma X_{ij}^2 - CF_{XX} = 2317 - 2281.666$
$TSS_{XX} = 35.334$
$TSS_{YY} = \Sigma\Sigma Y_{ij}^2 - CF_{YY} =$
$TSS_{YY} = 49.296$
$TSS_{XY} = \Sigma\Sigma X_{ij} Y_{ij} - CF_{XY}$
$= 8783.4 - CF_{XY} = -33.7$

$Tr.SS_{YY} = \frac{\Sigma Y_{ij}^2}{t} - CF_{YY} =$
$= \frac{102263.52}{3} - 34072.134 = 15.706$

$$Tr.SS_{xx} = \frac{\Sigma X_{i.}^2}{r} - CF_{xx}$$

$$= \frac{11427}{5} - 2281.666$$

$$= \cancel{1527.334}\ 3.734$$

$$Tr.SS_{xy} = \frac{\Sigma X_{i.} Y_{i.}}{r} - CF_{xy}$$

$$= \frac{44056.3}{5} - CF_{xy}$$

$$= -5.84.$$

$$E_{yy} = TSS_{yy} - Tr.SS_{yy}$$

$$= 49.296 - 15.706$$

$$= 33.59$$

$$E_{xx} = TSS_{xx} - Tr.SS_{xx}$$

$$= 35.334 - 3.734$$

$$= 31.6$$

$$E_{xy} = TSS_{xy} - Tr.S_{xy}$$

$$= -33.7 - (-5.84)$$

$$= -27.86$$

(ii). Hypothesis

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

$H_0$: there is no effect of covariate.

$H_1$: the effect of covariate is there

$$SSE = E_{yy} - \frac{E_{xy}^2}{E_{xx}} \Rightarrow 33.159 - \frac{(-27.86)^2}{31.6}$$

$$= 33.15 - 24.5626$$

$$SSE = 8.5874$$

$$MSE = \frac{SSE}{n-t-1} = \frac{8.5874}{15-3-1} = 0.78$$

$$F_{cal} = \frac{E_{xy}^2 / E_{xx}}{MSE} \sim F_{(\beta, n-t-1)}$$

$$= \frac{24.5626}{0.78} \sim F^{(0.05)}_{(1,11)}$$

$$= 31.49$$

$$\therefore F^{(0.05)}_{(1,11)} = 4.84$$

$$\therefore F_{cal} > F_{tab}$$

Conclusion $31.49 > 4.84$

$\therefore$ we reject $H_0$

$\therefore$ the effect of covariate is there.

(ii) $H_0 : \alpha_1 = 0 \quad \forall i$

~~$H_0$ the glue formulations are~~ not ~~significant.~~

$H_0$: there is no significant difference in the glue formation.

$H_1 : \alpha_1 \neq 0$

There is a significant difference in glue formation.

$$SSE' = TSS_{YY} - \frac{TSS_{XY}^2}{TSS_{XX}}$$

$$= 49.296 - \frac{(-33.7)^2}{35.334}$$

$$= 49.296 - 32.1415$$

$$SSE' = 17.1545$$

$$F_0 \frac{(SSE' - SSE)/t-1}{MSSE} \sim F(t-1, n-t-1)$$

$$F_0 = \frac{(17.1545 - 8.5874)/3-1}{0.78}$$

$$F_0 = \frac{4.28355}{0.78} = 5.491$$

$F(t-1, n-t-1) \Rightarrow F(3-1, 15-3-1) \Rightarrow F(2,11) = 3.98$

~~ANOVA Table~~

$\therefore F_{cal} > F_{tab}$

$\therefore 5.491 > 3.98$

$\therefore$ we reject $H_0$

$\therefore$ there is a significant difference in glue formation

ANCOVA Table.

| S.V. | Sum of Squares & Cross products: $x$ | $xy$ | $y$ | d.f | adjusted treatment SS for Regression S.S | MSS | |
|---|---|---|---|---|---|---|---|
| Treatment | $T_{xx}=3.734$ | $-5.84$ | $15.706$ | . | $SSE=$ $8.5874$ | | $F_{cal}$ |
| Error | $E_{xx}=31.6$ | $-27.86$ | $33.15$ | $n-t-1=11$ | $SSE'=$ $17.1545$ | $0.78$ | $F_0=$ $5.49$ |
| Total | $TSS=35.334$ | $-33.7$ | $49.296$ | $n-2=13$ | $Tr.SS=SSE'-SSE$ $4.28355$ | | |
| Treatment | | | | $t-1=2$ | | | |

(iv) SE of adj $(\bar{y}_{i.}-\bar{y}_{j.}) = \sqrt{MSE\left(\frac{2}{r}+\frac{\bar{x}_{i.}-\bar{x}_{j.}}{E_{xx}}\right)}$

$\bar{x}_{i.} \rightarrow$ 1 treatment mean.

$\bar{x}_{j.} \rightarrow$ 2 treatment mean. 0.67

$$= \sqrt{MSE\left(\frac{2}{5}+\frac{6}{31.6}\right)}$$

$= 0.68$

(i). The linear model is,

$$Y_{ij} = \mu + \alpha_i + \xi_j + \beta(x_{ij} - \bar{x}_{..}) + \varepsilon_{ij}$$

$i = 1, \ldots, t$

$j = 1, \ldots, b$

where, $Y_{ij} =$ , $\mu =$ , $\alpha_i =$ , $\xi_j =$ , $\beta =$ , $\varepsilon_{ij} =$

Hypothesis:

$H_0 : \beta = 0$ v/s $H_1 : \beta \neq 0$

1

~~Hole~~ Soln: $t=3,\ b=4,\ n=tb=12.$

$CF_{xx} = \frac{X_{..}^2}{n} = \frac{290^2}{12} = 7008.33$

$CF_{yy} = \frac{Y_{..}^2}{n} = \frac{483^2}{12} = \frac{233289}{12} = 19440.75$

$CF_{xy} = \frac{X_{..}Y_{..}}{n} = \frac{\cancel{11949}\ 290\times 483}{12} = \cancel{995.75}\ 11672.5$

$TSS_{xx} = \sum\sum X_{ij}^2 - CF_{xx}$

$= 7262 - CF_{xx} = 253.67$

$TSS_{yy} = \sum\sum Y_{ij}^2 - CF_{yy}$

$TSS_{yy} = 19781 - 19440.75 = 340.25$

$TSS_{xy} = \cancel{11949}\ \sum\sum X_{ij} Y_{ij} - CF_{xy}$

$= 11949 - 11672.5$

$TSS_{xy} = 276.5$

$Tr_{yy} = \frac{\sum Y_{i.}^2}{b} - CF_{yy}$

$= \frac{78597}{4} - 19440.75$

$= 19649.25 - 19440.75$

$Tr_{yy} = 208.5$

$Tr_{xx} = \frac{\sum X_{i.}^2}{b} - CF_{xx} \Rightarrow$

$Tr_{xx} = \frac{28468}{4} - CF_{xx} = 7117 - CF_{xx} = 108.67$

$$Tr_{xy} = \frac{\sum X_{i.} Y_{i.}}{b} - CF_{xy} = \frac{47292}{4} - CF_{xy}$$

$$Tr_{xy} = 11823 - CF_{xy} \Rightarrow 150.5$$

$$B_{yy} = \frac{\sum Y_{.j}^2}{t} - CF_{yy} = \frac{58463}{3} - CF_{yy}$$

$$= 19487.666 - CF_{yy}$$

$$B_{yy} = 46.9166$$

$$B_{xx} = \frac{\sum X_{.j}^2}{t} - CF_{xx} = \frac{21146}{3} - CF_{xx} = 40.33$$

$$B_{xy} = \frac{\sum X_{.j} Y_{.j}}{t} - CF_{xy}$$

$$= \frac{35140}{3} - 11672.5 = 11713.33 - CF_{xy}$$

$$B_{xy} = 40.833$$

$$E_{yy} = TSS_{yy} - TrSS_{yy} - B_{yy}$$

$$= 340.25 - 208.5 - 46.9166$$

$$= 84.8334$$

$$E_{xx} = TSS_{xx} - TrSS_{xx} - B_{xx}$$

$$= 253.67 - 108.67 - 40.33$$

$$= 104.67$$

$$E_{xy} = TSS_{xy} - TrSS_{xy} - B_{xy}$$

$$= 276.5 - 150.5 - 40.833$$

$$= 85.167$$

$$\hat{\beta} = \frac{E_{xy}}{E_{xx}} = \frac{85.167}{104.67} = 0.8136$$

P 2

The model.,

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i = 1, \ldots t$$
$$j = 1, \ldots b$$

$\alpha_i =$ , $\beta_j =$ , $\varepsilon_{ij}$

Assumptions

— " —

Hypothesis :-

$H_0^1 = \alpha_1 = \alpha_2 = \ldots = \alpha_t = 0$

$H_1^1 =$ (overall)

$H_0^2 = \beta_1 = \beta_2 = \ldots = \beta_b = 0$

$H_1^2 =$

estimate the missing observation

$$\hat{y}_{ij} = \frac{t\,y'_{i.} + b\,y'_{.j} - y'_{..}}{(b-1)(t-1)}$$

where,

$y'_{i.}$ = sum of known observations for $i^{th}$.

$y'_{.j}$ = sum of known observation for $j^{th}$.

$y_{..}$ = sum of all known observations

Given, $b = 4$ , $t = 3$

$$\hat{y}_{22} = \frac{3(109.4) + 4(58.7) - 377}{3 \times 2}$$

$$\hat{y}'_{22} = \frac{186}{6}$$

$$\hat{y}'_{22} = 31$$

| | A | B | C | $Y_{i.}$ |
|---|---|---|---|---|
| 1 | 32.4 | 35.6 | 38.7 | 106.7 |
| 2 | 28.8 | 31 | 29.9 | 89.7 |
| 3 | 36.5 | 37.6 | 29.1 | 103.2 |
| 4 | 34.4 | 36.2 | 37.8 | 108.4 |
| $Y_{.j}$ | 132.1 | 140.4 | 135.5 | |

$$C.F = \frac{Y_{..}^2}{b \times t} = \frac{(408)^2}{4 \times 3} = \frac{166464}{12} = 13872$$

$$SST_r = \frac{\sum Y_{i.}^2}{b} - CF$$

$$= \frac{132.1^2 + 140.4^2 + 135.5^2}{4} - CF$$

$$= 13880.705 - 13872$$

$$SST_r = 8.705$$

$$SSB = \frac{\sum Y_{.j}^2}{t} - CF$$

$$= \frac{(106.7^2 + 89.7^2 + \dots - 108.4^2)}{3} - CF$$

TSS =

$= 13943.9266 - 13872$

$SSB = 71.926$

$SST = \sum_i \sum_j y_{ij}^2 - CF$

$= 14014.72 - 13872$

$SST = 142.72$

$SSE = TSS - (SST_r + SSB)$

$= 142.72 - 8.705 - 71.926$

$SSE = 62.089$

ANOVA TABLE

| SV | d.f | S.S. | MSS | F-ratio |
|---|---|---|---|---|
| Treatmt | $t-1 = 2$ | 8.705 | 4.3525 | 0.3505 |
| Block | $b-1 = 3$ | 71.926 | 23.9753 | 1.93072. |
| Error | $(t-1)(b-1)-1 = 5$ | 62.089 | 12.4178 | |
| Total | $bt-2 = 10$ | 142.72 | 14.272. | |

For treatment :- $F_{0.05}(2,5)$

$F_{cal} \geq F_{0.05}(2,5)$

$0.3505 \geq 5.79.$

$\therefore$ we donot reject $H_0$ for treatment

Hence $\alpha_i$'s don't differ. significantly

Standard error of difference

$SE(\bar{y}_A - \bar{y}_B) = $ [illegible]

$=$ [illegible]

$=$ [illegible]

$=$ [illegible]

$SE(\bar{y}_A - \bar{y}_B) = $ [illegible]

Stand error of difference [illegible]

$SE(\bar{y}_A - \bar{y}_C) = $ [illegible]

$=$ [illegible]

$SE(\bar{y}_A - \bar{y}_C) = $ [illegible]

$\lambda = \frac{r(k-1)}{(t-1)}$

(2). LSD.

The missing observations.

$$y_{ijk} = \frac{m(y'_{i..} + y'_{.j.} + y'_{..k}) - 2(y'_{...} + \text{initial estimate})}{(m-1)(m-2)}$$

$$y^{(0)} = \frac{\Sigma y'_{i..}}{m-1}$$

| | A | B | C | D | E | F | |
|---|---|---|---|---|---|---|---|
| | 92 | 220 | 160 | 149 | 282 | 98 | |
| | 74 | 213.15 | 228 | 168 | 238 | 282 | ~~4...~~ |
| | 119.5 | 176 | 279 | 188 | 278 | 118 | |
| | 64 | 222 | 213 | 104 | 295 | 163 | |
| | 66 | 188 | 187 | 90 | 242 | 96 | |
| | 124 | 109 | 195 | 79 | 211 | 90 | |
| $y'_{i..}$ | 420 | 915 | 1262 | 778 | 1546 | 847 | |
| | 530.5 | 1128.15 | | | | | |

| | | | | | | | $y'_{.j.}$ |
|---|---|---|---|---|---|---|---|
| | 220 | 98 | 149 | 92 | 282 | 160 | ~~45~~, 1001 |
| | 74 | 238 | 213.15 | 228 | 282 | 168 | 990 1203.15 |
| | 188 | 279 | 118 | 278 | 176 | 119.5 | 1039 1158.5 |
| | 295 | 222 | 64 | 104 | 213 | 163 | 1061 |
| | 187 | 90 | 242 | 96 | 66 | 188 | 869 |
| | 90 | 124 | 195 | 109 | 79 | 211 | 808 |
| $y'_{..k}$ | 1054 | 1051 | 768 981.15 | 907 | 1098 | 890 1009.5 | |

For missing value of A,

$$\hat{y}'_{ijk.} = \frac{6(420+1039+\cancel{768}^{890}) - 2(5768+84)}{5\times4}$$

$$= \frac{\cancel{11562} - 11704}{20}$$

$$\hat{y}'_{ijk} = \frac{\cancel{82.9}\ 2390}{20}$$

$y^{(0)} = \frac{\Sigma y'_{i..}}{m-1}$

$= \frac{420}{5} = 84$

$\hat{y}'_{A36}$ $\hat{y}'_{ijk.} = 119.5$

For missing value of B,

$$\hat{y}_{ijk.} = \frac{6(915+768+990) - 2(5768+119.5)}{20}$$

$$= \frac{16038 - 11775}{20}$$

$\hat{y}_{B23} = 213.15$ //

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| 92 | 220 | 160 | 149 | 2 | |

$CF = \frac{y^2_{..}}{m^2} = \frac{(6100.65)^2}{6\times6} = 1033831.401$

$SST = \Sigma\Sigma\Sigma y^2_{ijk} - CF$

$= (220^2 + 98^2 + \cdots 211^2) - CF$

$= (461077.9225 + 472808.25 + 274693) - CF$

$= 1208579.173 - 1033831.401$

$\Lambda = \frac{01\ldots}{(6-1)}$

classmate

Date

Page

$SST = 174747.7715$

$$SSR = \frac{\sum y_{.j.}^2}{m} - CF$$

$$= \frac{6325439.173}{6} - CF$$

$$= 1054239.862 - 1033831.401$$

$SSR = 20408.46108$

$$SSc = \frac{\sum y_{..k}^2}{m} - CF$$

$$= \frac{6225515.573}{6} - CF$$

$$= 1037585.929 - CF$$

$SSC = 3754.5277$

$$SStr = \frac{\sum y_{i..}^2}{m} - CF$$

$$= \frac{6869235.673}{6} - CF$$

$$= 1144872.612 - 1033831.401$$

$SSt_r = 111041.2111$

$SSE = SST - SSR - SSC - SSt_r$

$= 174747.7715 - 20408.46108 - 3754.5277 - 111041.2111$

$SSe = 39543.57162$

ANOVA TABLE

| S.V | d.f | S.S | MSS | F ratio |
|---|---|---|---|---|
| Treatment | $t-1=5$ | 111041.2111 | 22208.24222 | 70.20182 |
| Row | $r-1=5$ | 20408.46108 | 4081.6922 | 12.9025 |
| Column | $c-1=5$ | 3754.5277 | 750.90554 | 2.37366 |
| Error | $(t-1)(r-1)(c-1)=125$ | 39543.57162 | 316.3485 | |
| Total | $trc-2=214$ | 174747.7715 | 816.5783 | |

Row:- Since, $12.9025 > \cancel{8.75}$ 4.39, we ~~fail to~~ reject $H_0$,

Column:- Since, $2.373 < 4.39$ we don't reject $H_0$

Treatment:- Since, $70.208 > 4.39$ we reject $H_0$.

P-3

i) The linear model

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i=1,\dots,t \quad j=1,\dots,b$$

where,

$Y_{ij}$: observation for $i$th treatment in the $j$th block of $i$t units.

$\alpha_i$ = , $\beta_j$ = , $\varepsilon_{ij}$

Assumption:- Same as RBD — " — ".

~~Hypothesis~~ ~~and parameters~~:-

1. $\varepsilon_{ij}$'s are iid random variables with $N(0,\sigma^2)$ dist$^n$
2. Treatment & block effects are fixed effects
3. All the observations are independent.

... the blocks ($\beta_i = 0$)
... ($\beta_i \neq 0$)

$H_0^3$: there is no sig diff b/w the land ($\gamma_i = 0$)

$H_1^3$: there is sig diff b/w the land ($\gamma_i \neq 0$).

## P – 6

(i) Model :-

$$\mu + \alpha_i + \overbrace{\beta_j}^{\text{main factors}} + \gamma_k + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\beta\gamma)_{jk} + (\alpha\beta\gamma)_{ijk} + \delta_l + \epsilon_{ijkl}$$

where, $i, j, k = 0, 1, \ldots$ $l = 1, \ldots, r$

$y_{ijkl}$ :- observation for the $l^{th}$ replicate at $i^{th}$ level of factor A, $j^{th}$ level of factor B & $k^{th}$ ~~ith~~ level of factor C

$\alpha_i$: effect due to $i^{th}$ level of factor A
$\beta_j$: effect due to $j^{th}$ level of factor B
$\gamma_k$: effect due to $k^{th}$ level of factor C
$(\alpha\beta)_{ij}$: Interaction effect b/w $i^{th}$ level of factor A & $j^{th}$ level of factor B.
$(\alpha\gamma)_{ik}$: Interaction effect b/w $i^{th}$ level of factor A & $k^{th}$ level of factor C
$(\beta\gamma)_{jk}$: Interaction effect b/w $j^{th}$ level of factor B & $k^{th}$ level of factor C
$(\alpha\beta\gamma)_{ijk}$: Interaction effect b/w $i^{th}$ level of factor A & $j^{th}$ level of factor B & $k^{th}$ level of factor C
$\delta_l$: effect due to $l^{th}$ replicate
$\epsilon_{ijkl}$ :- Random error

Assumption
$\epsilon_{ijkl} \sim N(0, \sigma^2)$

(ii) Hypothesis
$H_0$: The main effect & interaction effects are not significant.
$H_1$: The main effects & interaction effects are significant.

calculation :- $r = 2$. Yate's table

| | TC | Totals | I | II | III | effect | SS $\frac{(\text{effect total})^2}{2^3 r}$ |
|---|---|---|---|---|---|---|---|
| $a_0b_0c_0$ | (1) | 79 | 141 | 232 | 427 | G | $\frac{(427)^2}{2^3(2)} = \frac{427^2}{16} = 11395.562$ |
| $a_1b_0c_0$ | a | 62 | 91 | 195 | -97 | A | $SS_A = 588.0625$ |
| $a_0b_1c_0$ | b | 63 | 116 | -52 | -87 | B | $SS_B = 473.0625$ |
| $a_1b_1c_0$ | ab | 28 | 79 | -45 | -35 | AB | $SS_{AB} = 76.5625$ |
| $a_0b_0c_1$ | c | 85 | -17 | -50 | -37 | C | $SS_C = 85.5625$ |
| $a_1b_0c_1$ | ac | 51 | -35 | -37 | 7 | AC | $SS_{AC} = 3.0625$ |
| $a_0b_1c_1$ | bc | 55 | -14 | -18 | 13 | BC | $SS_{BC} = 10.5625$ |
| $a_1b_1c_1$ | abc | 24 | -31 | -17 | 1 | ABC | $SS_{ABC} = 0.0625$. |

$$\text{Replicate SS} = \frac{R_1^2 + R_2^2}{2^3} - CF$$

$$= \frac{46225 + 44944}{8} - 11395.562$$

$$= 11396.125 - 11395.562.$$

$$\text{Replicate SS} = 0.563 \,//.$$

$$SST = \sum_i \sum_j \sum_k \sum_l y_{ijkl}^2 - CF$$

$$= \cancel{427}\ 12671 - CF \quad \cancel{2)+82329 - CF}$$

$$SST = \cancel{170933.438}\ 1275.438$$

$$\cancel{SSE = SST - RSS} \quad SSE = TSS - SS_A - SS_B \ldots SS_{ABC}.$$

$$\cancel{SSE = 1274.875} \quad SSE = 37.52.$$

ANOVA.

| Source | d.f | S.S | MSS. | F-ratio |
|---|---|---|---|---|
| Replicate | $r-1 = 2-1$ | 0.563 | 0.563 | |
| Treatment | | | | |
| A | 1 | 588.0625 | 588.0625 | $F_A = F_{(1,7)}^{(0.05)} = \frac{588.0625}{5.36} = 109.7137$ |
| B | 1 | 473.0625 | 473.0625 | $F_B =$ 88.2579 |
| AB | 1 | 76.5625 | 76.5625 | $F_{AB} =$ 14.2840 |
| C | 1 | 85.5625 | 85.5625 | $F_C =$ 15.9631 |
| AC | 1 | 3.0625 | 3.0625 | $F_{AC} =$ 0.57136 |
| BC | 1 | 10.5625 | 10.5625 | $F_{BC} =$ 1.97061 |
| ABC | 1 | 0.0625 | 0.0625 | $F_{ABC} =$ 0.0116 |
| Error | 7 | 37.52 | 5.36 | |
| Total | $2^3 r - 1 = 15$ | | | |

$F_{tab} \Rightarrow F_{(1,7)} = 5.59$

Here. $F_A \Rightarrow 109.7137 > 5.59$ , $F_C \Rightarrow 15.9631 > 5.5$

$F_{AB} \Rightarrow 14.2840 > 5.59$

$F_B \Rightarrow 88.2579 > 5.59$

$\therefore F_A; F_B, F_C, F_{AB}$ is greater than $F_{tab}$ va

so we reject $H_0$

Hence, the main effects & interaction effect a

Significant!

(ii) $SE = \sqrt{\frac{MSE}{2r}} \Rightarrow \sqrt{\frac{5.36}{4}} = 1.1575$

$H_1$: the main effect & interaction effect are significant.

2) model:-

$$y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + \sigma_l + (\alpha\beta)_{ij} + (\alpha\gamma)_{ik} + (\alpha\sigma)_{il} + (\beta\gamma)_{jk} + (\beta\sigma)_{jl} + (\gamma\sigma)_{kl}$$
$$(\alpha\beta\gamma)_{ijk} + (\alpha\beta\sigma)_{ijl} + (\beta\gamma\sigma)_{jkl} +$$
$$(\alpha\gamma\sigma)_{ikl} + (\alpha\beta\gamma\sigma)_{ijkl} + \varepsilon_{ijkl}$$

'37

Assumptions :- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

Yates table

| TC | Total | I | II | III | IV | effect | $\frac{SS(\text{effect total})^2}{2^4 r}$ |
|---|---|---|---|---|---|---|---|
| (1) | 7 | 16 | 105 | 201 | 426 | G | 11342.25 = CF |
| a | 9 | 89 | 96 | 225 | 108 ~~115~~ | A | 729 ~~826.5625~~ |
| b | 34 | 16 | 112 | 47 ~~54~~ | 282 | B | 4970.25 |
| ab | 55 | 80 | 113 | 61 | 88 | AB | 484 |
| c | 6 | 21 | 23 | 137 | -8 | C | 4 |
| ac | 10 | 91 | 24 ~~31~~ | 145 | -2 ~~5~~ | AC | 0.25 |
| 75.59 bc | 30 | 19 | 32 | 35 | -4 | BC | 1 |
| abc | 50 | 94 | 29 | 53 | -10 | ABC | ~~6.25~~ |
| d | 10 | 2 | 73 | -9 | 24 | D | 36 |
| ad | 11 | 21 | 64 | -1 | 14 | AD | 12.25 |
| value bd | 30 | 11 | 70 | 81 | 8 | BD | 4 |
| abd | 61 | 20 | 75 | -3 | 18 | ABD | ~~20.25~~ |
| are cd | 8 | 1 | 19 | -9 | 10 | CD | 6.25 |
| acd | 11 | 31 | 16 | 5 | -4 | ACD | ~~1~~ |
| bcd | 34 | 3 | 30 | -3 | 14 | BCD | ~~12.25~~ |
| abcd | 60 | 26 | 23 | -7 | -4 | ABCD | 1 |

Replicate $SS = \frac{R^2}{2^4}$

$$SST = \sum\sum\sum\sum y_{ijkl}^2 - CF$$

$$= 17630 - CF$$

$$SST = 6287.75$$

$$SSE = 6287.75 - SS_A - SS_B - \dots$$

$$SSE = 40.75$$

ANOVA

| Source | d.f | S.S | MSS | F-ratio |
|---|---|---|---|---|
| Treatment | | | | |
| A | 1 | 729 | 729 | $F_A = F_{(1,5)}^{(0.05)} = \frac{729}{8.15} = 89.44$ |
| B | 1 | 4970.25 | 4970.25 | 609.84 |
| AB | 1 | 484 | 484 | 59.386 |
| C | 1 | 4 | 4 | 0.4907 |
| AC | 1 | 0.25 | 0.25 | 0.0306 |
| BC | 1 | 1 | 1 | 0.1226 |
| ~~ABC~~ D | 1 | 36 | 36 | 4.417 |
| ~~D~~ AD | 1 | 12.25 | 12.25 | 1.5030 |
| ~~AD~~ BD | 1 | 4 | 4 | 0.4907 |
| ~~BD~~ CD | 1 | 6.25 | 6.25 | 0.766 |
| ~~ABD~~ Error | 5 | 40.75 | 8.15 | |
| ~~CD~~ Total | $2^4 - 1 = 15$ | | | |

~~ACD~~

~~BCD~~

$$F_{tab} = F_{(1,5)}^{(0.05)} = 6.61$$

$E_A$ :. $F_A, F_B, F_{AB} > 6.61$ :. we reject $H_0$
Hence, the main effects & interaction effects are significant.

$(a-1)(b-1)(c-1)$
$(ab-a-b+1)(c-1)$
$abc-ab-ac+a-bc+b+c-1$

(1) Conclusion:-
Here the metal used, the amount of primary initiator, and the packing pressure of the explosive and the interaction between the metal used and the amount of primary initiator are not same.

P-7

$ABC = (a-1)(b-1)(c-1)$
$= (ab-a-b+1)(c-1)$
$= abc - bc - ac - ab + a + b + c - 1$

(2).

| Rep I. | | Rep II | | Rep III. | |
|---|---|---|---|---|---|
| (1) | a | (1) | abc | ab | a |
| ab | b | ab | c | ac | c |
| bc | c | ac | a | bc | b |
| ac | abc | bc | b | (1) | abc |
| − | + | − | + | − | + |
| abc | | abc | | abc. | |

Would confounding ~~BCA~~ ~~any~~ we have same. interaction confounding

| | I | | II | | III | |
|---|---|---|---|---|---|---|
| (2). | 1 | k | 1 | p | p | n |
| | pk | p | k | nk | npk | k |
| | nk | n | npk | pk | 1 | pk |
| | np | npk | np | n | nk | np |
| | npk | | np | | nk | |

Sh (1) **model** :-

$$\mu + \alpha_i + \beta_j + \sigma_k + (\alpha\beta)_{ij} + (\alpha\sigma)_{ik} + (\beta\sigma)_{jk} + (\alpha\beta\sigma)_{ijk} + \gamma_l + \varepsilon_{ijkl}$$

**Assumptions** :- $\varepsilon_{ijkl} \sim N(0, \sigma^2)$

**hypothesis** :-

$H_0$ :- The main effect & interaction effect are not significant

$H_1$ :- the main & interaction effect are significant.

$\therefore$ we can say that abc is confounded.

| TC | Total | I | II | III | effect | SS $\frac{(\text{effect tot})^2}{2^3 \cdot 3} = 24$ |
|---|---|---|---|---|---|---|
| (1) | 78 | 182 | 428 | 959 | G | 38320.04 |
| a | 64 | 246 | 531 | -17 | A | 12.0416 |
| b | 119 | 240 | 34 | 115 | B | 551.041 |
| ab | 127 | 291 | -51 | -41 ~~-69~~ | AB | 70.0416 |
| c | 127 | 26 | 64 | 103 | C | 442.0416 |
| ac | 113 | 8 | 51 | -85 | AC | 301.0416 |
| bc | 164 | -14 | -18 | -13 | BC | 7.0416 |
| abc | 127 | -37 | -23 ~~-51~~ | -5 ~~-33~~ | ABC | 1.0416 |

$$SST = \sum\sum\sum y_{ijkl}^2 - CF$$

$= 40621 - 38320.04$

$SST = 2300.96$ //

**Incomplete block Sum of Squares.**

$BSS = \frac{\sum B_i^2}{2^{k-1}} - CF$

$= \frac{154119}{2^{3-1}} - 38320.04$

$= 38529.75 - 38320.04$

$BSS = 209.71$

$ESS = TSS - SS_A - SS_B - \dots - SS_{ABC}.$

$= 2300.96 -$

$ESS = 708.19566$ //

**ANOVA Table**

| Source | d.f. | S.S | MSS | F ratio |
|---|---|---|---|---|
| Incomplete block | $b-1=5$ | 209.71 | 41.942 | $F_1 = 0.71068$ |
| A | 1 | 12.0416 | 12.0416 | $F_A = F^{0.05}_{(1,)} = \frac{12.0416}{59.0163} = 0.2040$ |
| B | 1 | 551.041 | 551.041 | $F_B = 9.33709$ |
| AB | 1 | 70.0416 | 70.0416 | 1.1868 |
| C | 1 | 442.0416 | 442.0416 | 7.4901 |
| AC | 1 | 301.0416 | 301.0416 | 5.1009 |
| BC | 1 | 7.0416 | 7.0416 | 0.1193 |
| Error | $23-6-5=12$ | 708.1956 | 59.0163 | |
| Total | $2^3 \cdot 3 - 1 = 23$ | | | |

$F^{tab}_{(1,12)} = 4.75$

I.B $\Rightarrow$ $F^{tab}_{(5,12)} = 3.11$

For incomplete block.

$F^{tab}_{(5,12)} \Rightarrow 3.11 > 0.71068$

Since $F_{tab}$ is greater than $F_{cal}$ for incomplete block, so we do not reject $H_0$

Hence the main effect is ...

For treatments

$F^{tab}_{(1,12)} = 4.75 < F_B, F_C, F_{AC}$ are greater than tab value.

Hence we reject $H_0$ for ~~$F_A, F_A$~~ A, AB, ~~& BC~~ B, C & AC.

(iii) ~~Est~~ To estimate S.E.

a) main effect.

$$[A] = \frac{\text{effect total}}{2^{3}r} = \frac{-17}{24} = -0.708$$

$$[B] = \frac{\text{effect total}}{24} = \frac{115}{24} = 4.79$$

$$[C] = \frac{103}{24} = 4.29$$

b) first order interaction

$[AB] = \frac{\text{effect total}}{2^3 r} = \frac{-41}{24} = 1.708$

$[BC] = 0.541$

$[AC] = -3.54$

(2) From the above table we can say that upk, up & uk are confounded.

Model:- $\mu + \alpha_i + \beta_j + \sigma_k + (\alpha\beta)_{ij} + (\alpha\sigma)_{ik} + (\beta\sigma)_{jk} + (\alpha\beta\sigma)_{ijk} + \gamma_l + \epsilon_{ijkl}$.

Assumption :- $\epsilon_{ijkl} \sim N(0, \sigma^2)$

Hypothesis :-

$H_0$: The main & interaction effect are not significant

$H_1$: The main & interaction effect are significant.

| TC | Totals | I | II | III | effect | SS $2^3 \times 3$ |
|---|---|---|---|---|---|---|
| (1) | 246 | 607 | 1391 | 2942 | G | 360640.166 |
| u | 361 | 784 | 1551 | 376 | | 5890.6666 |
| p | 406 | 757 | 87 | 214 | | 1908.166 |
| up | 378 | 794 | 289 | -344 | | — |
| k | 256 | 115 | 177 | 160 | | 1066.666 |
| uk | 501 | -28 | 37 | 202 | | — |
| pk | 375 | 245 | -143 | -140 | | 816.666 |
| upk | 419 | 44 | -201 | -58 | | — |

adjusted effect total

$$NPK = [NPK] - (B_1 - B_2)$$
$$= [-58] - (-78)$$
$$= -58 + 78 \quad \Sigma B_1 - \Sigma B_2$$
$$\boxed{NPK = 20}$$

$$NP = [NP] - (B_3 - B_4)$$
$$= (-344) - (-276)$$
$$\boxed{NP = -68}$$

$$NK = [NK] - (B_5 - B_6)$$
$$= [202] - (-184)$$
$$NK = \cancel{386}\ 386$$

$$SS_{NPK} = \frac{(\text{effect total})^2}{2^3(r-1)} = \frac{(\overset{20}{\cancel{-58}})^2}{2^3(2)} = \frac{(-58)^2}{16} = \overset{25}{\cancel{210.25}}$$

$$SS_{NP} = \frac{(\overset{68}{\cancel{-344}})^2}{16} = 289$$

$$SS_{NK} = \frac{(\overset{386}{\cancel{202}})^2}{16} = 9312.25$$

$$TSS = 413814 - 360640.166$$
$$TSS = 53173.834$$

Incomplete block sum of squares

$$BSS = \frac{\Sigma B_i^2}{2^{k-1}} - CF \Rightarrow \frac{1524556}{4} - CF$$
$$= 381139 - CF$$

$$BSS = 20498.834$$

$$ESS = TSS - BSS - SS_N - SS_P - \cdots SS_{NPK}$$
$$= 13366.\cancel{586}604$$

## ANOVA TABLE

| Source | d.f | S.S | MSS | F-ratio |
|---|---|---|---|---|
| Treatment | 5 | 20498.834 | 4099.766 | = ~~4.2940~~ 3.3738 |
| N | 1 | 5890.666 | 5890.66 | ~~6.6169~~ 4.8477 |
| P | 1 | 1908.166 | 1908.166 | ~~1.9985~~ 1.57031 |
| K | 1 | 1066.66 | 1066.66 | ~~1.1172~~ 0.87780 |
| PK | 1 | 816.66 | 816.66 | ~~0.855~~ 0.67206 |
| NP | 1 | 289 | 289 | 0.23783 |
| NK | 1 | 9312.25 | 9312.25 | 7.6634 |
| NPK | 1 | 25 | 25 | 0.02057 |
| Error | 23-7-5=11 | 13366.604 | 1215.145 | |
| Total | $2^3-1=23$ | | | |

~~$F^{tab}_{(1,14)} = 4.60$~~

~~$F_{(5,14)} = 2.96$~~

$$F^{tab}_{(1,11)} = \cancel{3.20}\ 4.84$$

$$F_{(5,11)} = 3.20$$

conclusion.
here 4 & 4k is greater than 4.84
Hence we reject $H_0$
Hence $F_N$ and $F_{NK}$ are significant.

(iv) Standard Error

a) main effect of $[N] = \frac{(\text{effect total})}{2^2 \cdot r} = \frac{376}{12}$
$= 31.33$

$[P] = \frac{214}{12} = 17.833$

$[K] = \frac{160}{12} = 13.33$

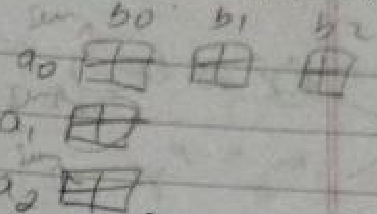
b). Interaction effect

$\{NP\} = \frac{-68}{12} = -5.66$

$\{NK\} = \frac{386}{12} = 32.166$

$\{PK\} = \frac{-140}{12} = -11.666$

# P-8




$i = 1, \ldots, 3^2$

$j = 1, \ldots, 3^2$

(1) The model is

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$$

Main effects

A

B

where, $y_{ij}$ = observation of $i^{th}$ factor A and $j^{th}$ factor B

$\mu$ = overall mean effect.

$\alpha_i$ = effect of A

$\beta_j$ = effect of B

$\therefore r = 4$

$\varepsilon_{ij}$ = random error.

Hypothesis :-

$H_0$ : Effects $A_L, A_Q, B_L, B_Q, A_LB_L, A_LB_Q, A_QB_L, A_QB_Q$ are insignificant

$H_1$ : Effects $A_L, A_Q, B_L, B_Q, A_LB_L, A_LB_Q, A_QB_L, A_QB_Q$ are significant.

| T.C. | Total | I | II | effects | d | SS (effect)²/d |
|---|---|---|---|---|---|---|
| $a_0b_0$ | 11 [1] | [1]+[2]+[3] = 65 | 226 | $G \to 3^2 r$ | = 36 | $(226)^2/36 = 1418.777$ |
| $a_1b_0$ | 22 [2] | = 73 | ~~+~~68 | $A_L \to 2^1 3^{2-1} r$ | = 24 | 192.666 |
| $a_2b_0$ | 32 [3] | = 88 | -20, 23 | $A_B \to 2^1 3^2 r$ | = 72 | 5.555 |
| $a_0b_1$ | 10 [4] | [3]-[1] = 21 | [illegible] | $B_L \to 2^1 3^{2-1} r$ | = 24 | 22.041 |
| $a_1b_1$ | 28 [5] | = 25 | 1 | $A_LB_L \to 2^2 3^{2-2} r$ | = 16 | 0.062 |
| $a_2b_1$ | 35 [6] | = 22 | -7 | $A_QB_L \to 2^2 3^{2-1} r$ | = 48 | 1.020 |
| $a_0b_2$ | 17 [7] | [1]-2[2]+[3] = -1 | 7 | $B_Q \to 2^1 3^2 r$ | = 72 | 0.680 |
| $a_1b_2$ | 32 [8] | = -11 | -7 | $A_LB_Q \to 2^2 3^{2-1} r$ | = 48 | 1.020 |
| $a_2b_2$ | 39 [9] | = -8 | 13 | $A_QB_Q \to 2^2 3^2 r$ | = 144 | 1.173 |

$d$ — $2^m 3^{n-p} r$.

m = no of factors in effect
n = no of factors in experiment
p = no of linear factors in effect.

(NO ANOVA Table)

$TSS = 1714 - 1418.77 = 295.23$

$ESS = TSS - (224.215)$
$= 71.015$

ii) Splitting the treatment S.S.

| | | B | | | |
|---|---|---|---|---|---|
| | | $b_0$ | $b_1$ | $b_2$ | effects |
| A | $a_0$ | $a_0b_0$ | $a_0b_1$ | $a_0b_2$ | $[A_0]$ |
| | $a_1$ | $a_1b_0$ | $a_1b_1$ | $a_1b_2$ | $[A_1]$ |
| | $a_2$ | $a_2b_0$ | $a_2b_1$ | $a_2b_2$ | $[A_2]$ |
| | $[AB]_2$ | $a_0b_0$ | $a_0b_1$ | $a_0b_2$ | $[AB^2]_0$ |
| | $[AB]_0$ | $a_1b_0$ | $a_1b_1$ | $a_1b_2$ | $[AB^2]_2$ |
| | $[AB]_1$ | | | | $[AB^2]_1$ |

$a_i b_j = 0,1,2$
$0,1,2$

$A : i = 0,1,2$, mod

$B : j = 0,1,2$, mod

$AB : i+j = 0,1,2$, m

$AB^2 : i+2j = 0,1,2$, m

| | | B | | | | |
|---|---|---|---|---|---|---|
| | | $b_0$ | $b_1$ | $b_2$ | effects | |
| A | $a_0$ | 11 | 10 | 17 | 38 | $[A_0]$ |
| | $a_1$ | 22 | 28 | 32 | 82 | $[A_1]$ |
| | $a_2$ | 32 | 35 | 39 | 106 | $[A_2]$ |
| | $[AB]_2$ 77 | 11 | 10 | 17 | 78 | $[AB^2]_0$ |
| | $[AB]_0$ 78 | 22 | 28 | 32 | 74 | $[AB^2]_2$ |
| | $[AB]_1$ 71 | | | | 74 | $[AB^2]_1$ |

$[B_0]$ 65 $[B_1] = 73$ $[B_2] = 88$

~~$b_0 = b_1 = b_2$~~

go
SS due to $A = \dfrac{[A_0]^2 + [A_1]^2 + [A_2]^2}{3r} - CF$

$$= \frac{(38)^2 + (82)^2 + (106)^2}{3 \times 4} - 1418.77$$

$$= 198.23$$

SS due to $B = \dfrac{[B_0]^2 + [B_1]^2 + [B_2]^2}{3r} - CF$

$$= \frac{65^2 + 73^2 + 88^2}{3 \times 4} - 1418.77$$

$$= 22.73$$

SS due to $AB^2 = \dfrac{[AB^2]_0^2 + [AB^2]_1^2 + [AB^2]_2^2}{3r} - CF$

$$= \frac{78^2 + 74^2 + 74^2}{3 \times 4} - CF$$

$$= 0.89 \,//$$

~~ANOVA~~ TABLE

~~Source~~ d.f

A

B

AB

$AB^2$

Error

Total

SS due to $AB = \dfrac{[AB]_0^2 + [AB]_1^2 + [AB]_2^2}{3 \times 4} - CF$

$$= 2.396$$

$$TSS = 1714 - 1418.77 = 295.23$$

$$ESS = TSS - SS_A - SS_B - SS_{AB} - SS_{AB^2}$$

$$= 295.23 -$$

$$= 71.015 \,//$$

## ANOVA TABLE

| Source | d.f | S.S | MSS | F-ratio |
|---|---|---|---|---|
| A | 2 | 198.23 | 99.115 | 37.70 |
| B | 2 | 22.73 | 11.365 | 4.32 |
| AB | 2 | 2.396 | 1.198 | 0.455 |
| $AB^2$ | 2 | 0.8966 | 0.4483 | 0.1705 |
| Error | 35-8=27. | 71 | 2.629 | |
| Total | $3^2r-1=35$ | 295.23 | | |

$$F_{tab} = F_{(2,27)}^{(0.05)} = 3.35$$

Conclusion:-

Hence AB & $AB^2$ are less than 3.35
Hence we reject $H_0$ for A & B
∴

(2) Model:-

$$y_{ijk} = \mu + \alpha_i + \beta_j + \sigma_k + \varepsilon_{ijk}.$$

$\mu$ = overall, $\alpha_i$ = , $\beta_j$ = , $\sigma_k$ =

$i = 1, \dots 3^3$
$j = 1, \dots 3^3$
$k = 1, \dots 3^3$

Hypothesis.

$H_0$: Effect

$H_1$:

$\cancel{\frac{(522)^2}{3r} = 272484}$

$$CF = \frac{G^2}{3^3 \cdot r} = \frac{(5119)^2}{54} = \cancel{1637760.063} \quad 485262.2407$$

B

| | | $b_0$ | $b_1$ | $b_2$ | |
|---|---|---|---|---|---|
| A | $a_0$ | 520 | 573 | 617 | $[A_0] = 1710$ |
| | $a_1$ | 501 | 577 | 643 | $[A_1] = 1721$ |
| | $a_2$ | 490 | 566 | 632 | $[A_2] = 1688$ |
| $1684 = [AB]_2$ | | 520 | 573 | 617 | $[AB^2]_0 = 1729$ |
| $1729 = [AB]_0$ | | 501 | 577 | 643 | $[AB^2]_2 = 1684$ |
| $1706 = [AB]_1$ | | $[B_0] = 1511$ | $[B_1] = 1716$ | $[B_2] = 1892$ | $[AB^2]_1 = 1706$ |

$$SS_A = \frac{[A_0]^2 + [A_1]^2 + [A_2]^2}{3^2 8} - CF \Rightarrow 1455880.833 - CF$$

$$= 970618.5923$$

$$SS_B = \cancel{1420123.000}\ 982644.5923$$

$$SS_{AB} = 970693.25$$

$$SS_{AB^2} = 970693.25$$

C

| | | $c_0$ | $c_1$ | $c_2$ | |
|---|---|---|---|---|---|
| A | $a_0$ | 575 | 575 | 560 | 1710 $[A_0]$ |
| | $a_1$ | 571 | 579 | 571 | 1721 $[A_1]$ |
| | $a_2$ | 561 | 567 | 560 | 1688 $[A_2]$ |
| $[AC]_2$ 1700 | | 575 | 575 | 560 | 1714 $[AC^2]_0$ |
| $[AC]_0$ 1713 | | 571 | 579 | 571 | 1698 $[AC^2]_2$ |
| $[AC]_1$ 1706 | | | | | 1707 $[AC^2]_1$ |

$[C_0] = 1707$ $[C_1] = 1721$ $[C_2] = 1691$

$\boxed{Err = 413}$

$SS_A = 970618.5926$ 970599.59

$SS_C =$ ~~9706599.5926~~ ~~970538.5926~~ ~~9706599.59~~

$SS_{AC} =$ ~~970693.2593~~ 970538.5926

$SS_{AC^2} = 970545.96$

C

| | $c_0$ | $c_1$ | $c_2$ | |
|---|---|---|---|---|
| $b_0$ | 512 | 498 | 501 | $[B_0] = 1511$ |
| $b_1$ | 564 | 580 | 566 | $[B_1] = 1716$ |
| $b_2$ | 631 | 637 | 624 | $[B_2] = 1892$ |
| $[BC]_2 = 1718$ | 512 | 498 | 501 | $[BC^2]_0 =$ ~~1729~~ 1657 |
| $[BC]_0 = 1715$ | 564 | 586 | 566 | $[BC^2]_2 = 1702$ |
| $[BC]_1 = 1686$ | | | | $[BC^2]_1 = 1695$ |

$SS_B =$ ~~970644.59~~ 4040.07

$SS_C =$ ~~970599.593~~ 25.07

$SS_{BC} =$ ~~970628.593~~ 34.74

$SS_{BC^2} =$ ~~974616.093~~ 21.81

$TSS = 489953 - CF(485262.2)$

$TSS = 4690.7593$

$ESS = TSS - SSA - SSB - \cdots = 413.45$

| Source | S.S | d.f | MSS | F-ratio |
|---|---|---|---|---|
| A | 31.41 | 2 | 15.685 | 1.327 |
| B | 4040.07 | 2 | 2020.215 | 171.03 |
| AB | 56.3 | 2 | 28.13 | 2.381 |
| C | 25.07 | 2 | 12.518 | 1.059 |
| AC | 4.74 | 2 | 2.352 | 0.199 |
| BC | 34.74 | 2 | 17.382 | 1.471 |
| $AB^2$ | 56.3 | 2 | 28.13 | 2.381 |
| $AC^2$ | 4.74 | 2 | 3.575 | 0.302 |
| $BC^2$ | 21.811 | 2 | 10.2075 | 0.923 |
| Error | 413.43 | 35 | 11.812 | |
| Total | 4690.73 | $3 \times 3^2 - 1 = 53$ | | |

$$F_{tab} = F_{(2,35)}^{(0.05)} = 3.27$$

6/3/25

# P-10

1. Model:-

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \qquad i = 1 \ldots 3^2, \quad j = 1, \ldots 3^2$$

where

$\mu$ = overall effect.

$\alpha_i$ = effect of $i^{th}$ treatment for factor A

$\beta_j$ = effect of $j^{th}$ treatm for factor B

$\varepsilon_{ij}$ = random error.

Hypothesis :-

$H_0$: Effects $A_L, A_Q, B_L, B_Q, A_LB_L, A_LB_Q, A_QB_L, A_QB_Q$ are ~~sig~~ insignificant

$H_1$ :- Efferts are significant.

2).

| | $b_0$ | $b_1$ | $b_2$ | |
|---|---|---|---|---|
| $a_0$ | 52.7 | 47.8 | 44.6 | $[A_0] = 145.1$ |
| $a_1$ | 52.2 | 46 | 51.4 | $[A_1] = 149.6$ |
| $a_2$ | 49.3 | 44.1 | 52.5 | $[A_2] = 145.9$ |
| $139.9 = [AB]_2$ | 52.7 | 47.8 | 44.6 | $[AB^2]_0 = 151.2$ |
| $148.2\,[AB]_0$ | 52.2 | 46 | 51.4 | $[AB^2]_2 = 140.9$ |
| $152.5\,[AB]_1$ | | | | $[AB^2]_1 = 148.5$ |
| | $[B_0] =$ | $[B_1] =$ | $[B_2] =$ | |

| | $b_0$ | $b_1$ | $b_2$ |
|---|---|---|---|
| $a_0$ | 87.9 | 95 | 92.4 |
| $a_1$ | 97.6 | 102 | 101.9 |
| $a_2$ | 93.9 | 87.2 | 102.2 |
| | 87.9 | 95 | 92.4 |
| | 97.6 | 102 | ~~101.9~~ |

$[AB]_2 = 288.3$

$[AB]_0 = 277.0$

$[AB]_1 = 294.8$

$[A_0] = 275.3$

$[A_1] = 301.5$

$[A_2] = 283.3$

$[AB^2]_0 = 292.1$

$[AB^2]_2 = 277.2$

$[AB^2]_1 = 290.8$

$[B_0] = 279.4$ $[B_1] = 284.2$ $[B_2] = 296.5$

$$CF = \frac{(860.1)^2}{3^2 r} \Rightarrow 41098.4$$

$$SS \text{ due to } A = \frac{(275.3)^2 + (301.5)^2 + (283.3)^2}{\cancel{3(2-1)}\ 3(r)} - CF \quad \frac{246951.2}{3(2)} - CF$$

$= \cancel{82317.1}\ \cancel{41159.2}\ 60.1$

SS due to $B = 26.0$

$$SS \text{ due to } AB = \frac{288.3^2 + 277^2 + 294.8^2}{3(r)} = \cancel{41158.6}\ 27.1$$

SS due to $AB^2 = \cancel{41143.9}\ 22.7$

$$BSS = \frac{\sum B_i^2}{3^{n-1}} - CF$$

$$B_1^2 = \frac{(52.2 + 44.6 + 44.1)^2}{3^{n-1}} = \frac{19852.8}{3} = 6617.6$$

$B_2^2 = 7620.5$

$B_3^2 = \frac{(47.8 + 51.4 + 49.3)^2}{\cancel{3(2-1)} 3^{r-1}} = 7350.8$

$B_4^2 = 5529.8$

$B_5^2 = 6749.8$

$B_6^2 = 7340.9$

$BSS = \frac{\cancel{40809.4}\ 41209.4}{\cancel{6}} - CF = 111$

$\cancel{0.126030t}$

$TSS = \Sigma\Sigma y_{ij}^2 - CF$

$= [211665.64 + 19815.99] - CF$

$TSS = 383.19$

$ESS = TSS - SS_A - SS_B - \ldots$

$= \cancel{247.3}\ 136.60$

ANOVA Table

| Source | d.f. | S.S | MSS | F-ratio |
|---|---|---|---|---|
| A | 2 | 60.1 | 30.1 | 0.9 |
| B | 2 | 26 | 13 | 0.4 |
| AB | 2 | 27.1 | 13.6 | 0.4 |
| $AB^2$ | 2 | 22.7 | 11.4 | 0.3 |
| BSS ~~ESW~~ | 6-1=5 | 111 | 22.2 | $\boxed{1}$ |
| Error | 17-13=4 | 136.6 | 34.2 | |
| Total | $3^2 r - 1 = 17$ | 383.19 | 22.5 | |

compare

$F_{(5,4)}^{(0.05)} = 6.26$ $\therefore 6.26 > 1$

$\cancel{F_{(2,4)}}$ $\therefore$ Hence we don't reject $H_0$

2 - CF

(2)

~~Hypothesis~~ & Model :-

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ijk} \qquad i = 1, \ldots 3, \quad j = 1, \ldots 3, \quad k = 1, \ldots 3$$

where.

$\mu =$ $\alpha_i =$ , $\beta_j =$

Hypothesis.

$H_0$: Effect are not Significant

$H_1$ :- effects are significant.

$(a_0b_0c_0 + a_0b_0c_1 + a_0b_0c_2) = 172$

$(a_0b_1c_0 + a_0b_1c_1 + a_0b_1c_1)$

| | $b_0$ | $b_1$ | $b_2$ | |
|---|---|---|---|---|
| $a_0$ | 171 | 155 | 159 | 485 |
| $a_1$ | 176 | 203 | 175 | 554 |
| $a_2$ | 149 | 183 | 157 | 489 |
| $[AB]_2 = 511$ | 171 | 155 | 159 | $531 = [AB^2]_0$ |
| $[AB]_0 = 529$ | 176 | 203 | 175 | $518 = [AB^2]_2$ |
| $[AB]_1 = 488$ | | | | $479 = [AB^2]_1$ |

$[B_0] =$ ~~526~~ 496 $[B_1] = 541$ $[B_2] = 491$

$$CF = \frac{2334784}{3^3 r} = 86473.5$$

$$SS_A = \frac{(485)^2 + (554)^2 + (489)^2}{3^2 r} - CF = 333.4$$

$SS_B = 168.5$

$SS_{AB} = 93.8$

$SS_{AB^2} = 162.7$

| | $c_0$ | $c_1$ | $c_2$ | | |
|---|---|---|---|---|---|
| $b_0$ | 149 | 179 | 168 | 496 | $[B_0]$ |
| $b_1$ | 163 | 188 | 190 | 541 | $[B_1]$ |
| $b_2$ | 142 | 187 | 162 | 491 | $[B_2]$ |
| $[BC]_2 = 497$ | 149 | 179 | 168 | 499 | $[BC^2]_0 =$ |
| $[BC]_0 = 526$ | 163 | 188 | 190 | 518 | $[BC^2]_2 =$ |
| $[BC]_1 = 504$ | | | | 511 | $[BC^2]_1 =$ |

$[C_0] = 454$ $[C_1] = 554$ $[C_2] = 520$

$SS_B = 168.52$

$SS_C = 574.52$

$SS_{BC} = 48.297$

$SS_{BC^2} = 20.52$.

| | $c_0$ | $c_1$ | $c_2$ | |
|---|---|---|---|---|
| $a_0$ | 126 | 168 | 191 | 485 |
| $a_1$ | 167 | 209 | 178 | 554 |
| $a_2$ | 161 | 177 | 151 | 489 |
| $[AC]_2 = 561$ | 126 | 168 | 191 | $486 = [AC^2]_0$ |
| $[AC]_0 = 481$ | 167 | 209 | 178 | $535 = [AC^2]_2$ |
| $[AC]_1 = 486$ | | | | $507 = [AC^2]_1$ |

$[C_0] = 454$ $[C_1] = 554$ $[C_2] = 520$

$SS_A = 333.4074$ $SS_{AC^2} = 134.2963$

$SS_C = 574.5186$

$SS_{AC} = 446.29637$

8

$$BSS = \frac{\sum B_i^2}{3^{4-1}} - CF \quad 2) \; \frac{\sum B_i^2}{3^{3-1}} - CF$$

$$B_1^2 = \frac{(40+52+55+\dots)^2}{3^2} = 275560$$

$B_2^2 = 28561$
$B_3^2 = 30392.1$

$BSS = (\underline{275560 + 28561 + 30392.1}) - CF$
$BSS = \cancel{2480390}\ 35.63$

$TSS = \sum\sum y_{ijk}^2 - CF = 2378.52$
$ESS = TSS - SS_A - SS_B \dots = 357.48$

## ANOVA Table

| Source | d.f | MSS | F-ratio |
|---|---|---|---|
| A | 2 | 168.21 | 2.82 |
| B | 2 | 84.26 | 1.41 |
| AB | 2 | 46.92 | 0.79 |
| $AB^2$ | 2 | 81.37 | 1.37 |
| C | 2 | 287.26 | 4.82 |
| AC | 2 | 223.15 | 3.75 |
| $AC^2$ | 2 | 67.115 | 1.13 |
| BC | 2 | 24.15 | 0.041 |
| $BC^2$ | 2 | 10.26 | 0.17 |

[20 - mark]

| | | | |
|---|---|---|---|
| Block | 2 | 17.815 | 0.3 |
| Error | 6 | 59.58 | |
| Total | 36 | 2378.52. | |

~~tcal~~ $F_{(2,6)}^{(0.05)} = 5.14$

Conclusion :-

Since the table value is greater we donot reject Ho .