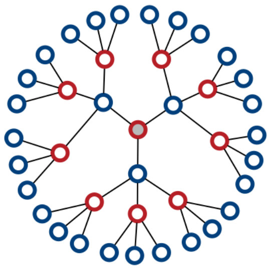
Exercise sheet 3 – Random networks

(1) Tree

Consider a balanced (or Cayley-) tree which is a symmetric tree, constructed starting from central a node with r neighbors. Each neighboring node branches into r more neighbors until nodes are reached of distance or height h ($h > 0$) from the central node. These nodes only have degree $= 1$ (leaf nodes).



Cayley tree with $r = 3$ and $h = 3$

(i) Calculate a general expression of the total number of nodes and edges such a tree possesses given r and h. (3pts)

(ii) Calculate only the number of leaf nodes. For which values of r and h does the number of inner nodes exceeds the number of leaf nodes? (2pts)

(iii) Find an expression for the diameter d of a Cayley tree in terms of the total number of nodes N. How does it connect to random networks? (2pts)

(iv) What is the clustering coefficient of Cayley trees. (1pts)

(2) Random network

Consider an Erdős-Rényi-network with $N = 2 \cdot 10^4$ nodes and a probability $p = 10^{-3}$.

(i) Calculate the expected number of links and the average degree. (2pts)

(ii) In which regime is the network and why? (1pts)

(iii) What would be the critical probability p_c? (1pts)

(iv) From which critical number of nodes N_c we would expect to only find one connected component (for $p = 10^{-3}$)? (2pts)

(3) Random vs real networks

Suppose the following table contains measurements about 2 real world networks and one Erdős-Rényi reference network. The first column gives the number of nodes N, the second the number of edges L and the third refers to the average clustering coefficient C:

	N	L	C
G1	4941	6594	0.08
G2	125	560	0.07
G3	256985	7778954	0.009

Unfortunately, the actual labels are lost and therefore also the identification of the random network among them. Please help deduce from the measurements which network is most likely to be the random one! (3pts)