Exercise 5 – Bianconi-Barabási model and network failure

(1) Consider a Bianconi-Barabási model with two distinct fitness values $\eta = 1$ and $\eta = a$ with $0 \leq a \leq 1$ resulting in a fitness distribution given by

$$\rho(\eta) = \frac{1}{2}\delta(\eta - 1) + \frac{1}{2}\delta(\eta - a).$$

Calculate the degree dynamics and the leading exponent β for large times using

$$C = \int \rho(\eta)\frac{\eta}{1 - \beta(\eta)} d\eta \quad \text{and}$$
$$\beta(\eta) = \frac{\eta}{C}.$$

Calculate the stationary degree distribution of the network and the degree exponent depending on the parameter a using

$$p(k) = C \int \frac{\rho(\eta)}{\eta}\left(\frac{m}{k}\right)^{\frac{C+\eta}{\eta}} d\eta$$

Examine the terms in your equation, focusing on how each behaves for large degrees. Identify and compare their degree exponents, and determine the leading exponent that dominates as the degree increases. Identify the leading exponent as your final result.

Calculate explicit values for the leading exponent for $a = 0.5$ and the limit cases $a \to 0$ and $a \to 1$.

(10 pts)

(2) The breakdown threshold for random failure in terms of moments of the degree distribution can be written as

$$f_c = 1 - \frac{1}{\kappa - 1} \quad \text{with} \quad \kappa = \frac{\langle k^2 \rangle}{\langle k \rangle}.$$

Calculate explicit expressions for both Erdős-Rényi- and scale-free networks.

Erdős-Rényi: Estimate how the percolation threshold depends on the number of nodes. Find out how many nodes an Erdős-Rényi- network must have so that at least 90% (99%) of the nodes needs to be removed before it falls apart.

Scale-free: Find approximate expressions for large $k_{\max}$ and $2 < \gamma \leq 3$ (bonus for $\gamma > 3$) and by using the natural cutoff a scaling with the number of nodes N.

(8 pts)