Exercise 5 – Bianconi-Barabási model and network failure

(1) Consider a Bianconi-Barabási model with two distinct fitness values $\eta = 1$ and $\eta = a$ with $0 \leq a \leq 1$ resulting in a fitness distribution given by

$$\rho(\eta) = \frac{1}{2}\delta(\eta - 1) + \frac{1}{2}\delta(\eta - a).$$

Calculate the degree dynamics and the leading exponent for large times.

Calculate the stationary degree distribution of the network and the degree exponent, with its dependence on the parameter a. Consider only the leading β from the degree dynamics for longer times and concentrate on the large degree limit for your final result. Find the numerical values for the exponent in the limit cases $a \to 0$ and $a \to 1$.

(Hint: Use expressions given in section 6.2 from Barabási's book)

(10 pts)

(2) The breakdown threshold of networks in terms of moments of the degree distribution can be written as

$$f_c = 1 - \frac{1}{\kappa - 1} \quad \text{with} \quad \kappa = \frac{\langle k^2 \rangle}{\langle k \rangle}.$$

Calculate explicit expressions for both Erdős-Rényi- and scale-free networks.

Erdős-Rényi: Estimate how the percolation threshold depends on the number of nodes. Find out how many nodes an Erdős-Rényi- network must have so that at least 90% (99%) of the nodes needs to be removed before it falls apart.

Scale-free: Find approximate expressions for large $k_{\max}$ and $2 < \gamma \leq 3$ (bonus for $\gamma > 3$) and by using the natural cutoff a scaling with the number of nodes N.

(10 pts)