Strict submission deadline: 16 June 2025 at 08:00 am.

Exercise #6

Task 6.1

Solve Task 1.3.15 of the "Exercises 2025 V4" provided on the course page.

Submit the calculation method and the final result

Task 6.2

a) Explain the difference between a random process, a random variable, a pattern function, and a process variable.
b) What is the fundamental prerequisite for a Matched Filter construction?
c) Assume a perfectly working Matched Filter. What is the output of that filter if the shape of the transmitted signal is a rectangular impulse?
d) Write down the Wiener-Hopf equation and explain its parts.
e) Assume you should implement a Wiener-Kolmogorov filter. Give a reasoned decision, if a causal Wiener-Kolmogorov filter should be used, or if a non-causal filter should be applied.

Submit the calculation method and the final result

Task 6.3

To monitor a measured value, it is sampled equidistantly over time. The expected value is a constant. The measurement is disturbed by additive noise. Write a Matlab program for a Kalman filter for noise suppression.

For the Kalman filter you may use the code [z(k) is the measured signal]

```
%% Kalman Filter Initialization
x_est = zeros(1, N);  % Estimated value of the state
P = zeros(1, N);      % Estimated error covariance

% Initial guesses
x_est(1) = 0;         % Initial estimate
P(1) = 1;             % Initial error covariance

Q = 0.0;              % Process noise covariance (0 because state is constant)
R = 1.0;              % Measurement noise covariance

%% Kalman Filter Loop
for k = 2:N
  % Prediction Step
  x_pred = x_est(k-1);    % Predicted state
  P_pred = P(k-1) + Q;    % Predicted error covariance

  % Update Step
  K = P_pred / (P_pred + R);               % Kalman Gain
  x_est(k) = x_pred + K * (z(k) - x_pred);  % Updated state estimate
  P(k) = (1 - K) * P_pred;                 % Updated error covariance
end
```

Assume the constant value to be 3.5. Create an input signal with a length of 300 sampling points. The nature of the noise signal changes over time as follows:
from sampling point 1 to 100 the noise is Gaussian noise with a maximum amplitude of 0.5,
from sampling point 101 to 200 the noise is Gaussian noise with a maximum amplitude of 1.0,
from sampling point 201 to 250 the noise signal is equal to zero,
from sampling point 251 to 300 the noise is uniformly distributed with a maximum amplitude of 2.0.
Plot the constant value, the signal, the prediction and the error signal in one diagram.

Submit the solution (Matlab source code, all plots, and written answers)