Task 2.3 $\longrightarrow$ Task 1.3.9

Random Process:

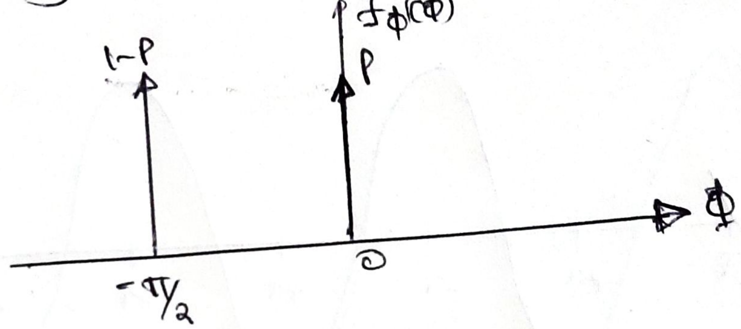
$X(\zeta,t) = A(\zeta)\ \sin\left(\frac{t}{T}\pi + \Phi(\zeta)\right)$ with $T>0$

$A(\zeta)$ takes $+1$ and 0 $\rightarrow$ Equal Probability.

$\Phi(\zeta)$ pdf:

$f_\Phi(\phi) = p\,\delta(\phi) + (1-p)\,\delta(\phi+\pi/2)$ with $0 \le p \le 1$

a) Sketching pdf of $\Phi(\zeta)$



When $A(\zeta)=1$ and $\Phi(\zeta)=0$

$P_A = 1/2$ $\qquad$ $P_\Phi = p$ $\rightarrow$ statistically independent

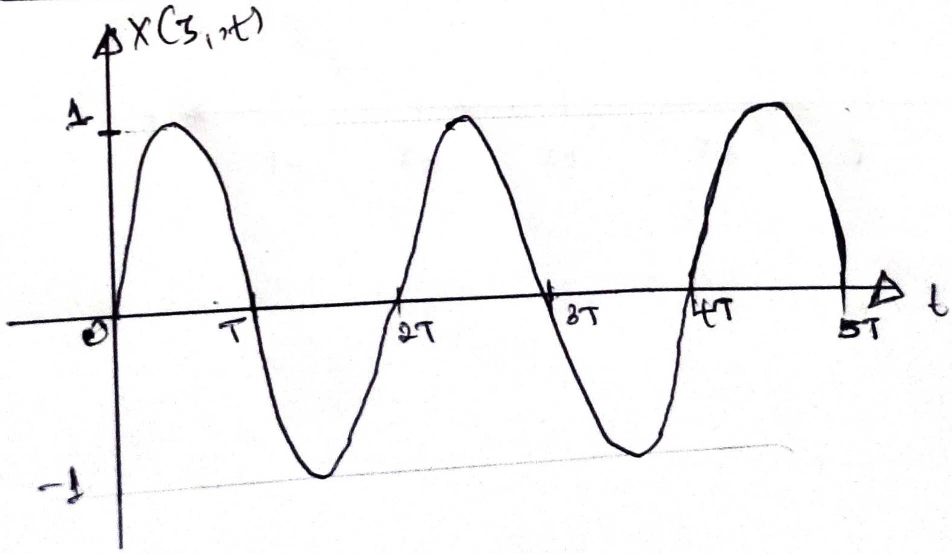
$P(A(\zeta))$ and $\Phi(\zeta) = \frac{1}{2}p$

$= \frac{1}{2}p$

$X(\zeta,t) = \sin\left(\frac{t}{T}\pi\right)$

probability $= \frac{1}{2}p$ $\qquad T>0$

Sketch.



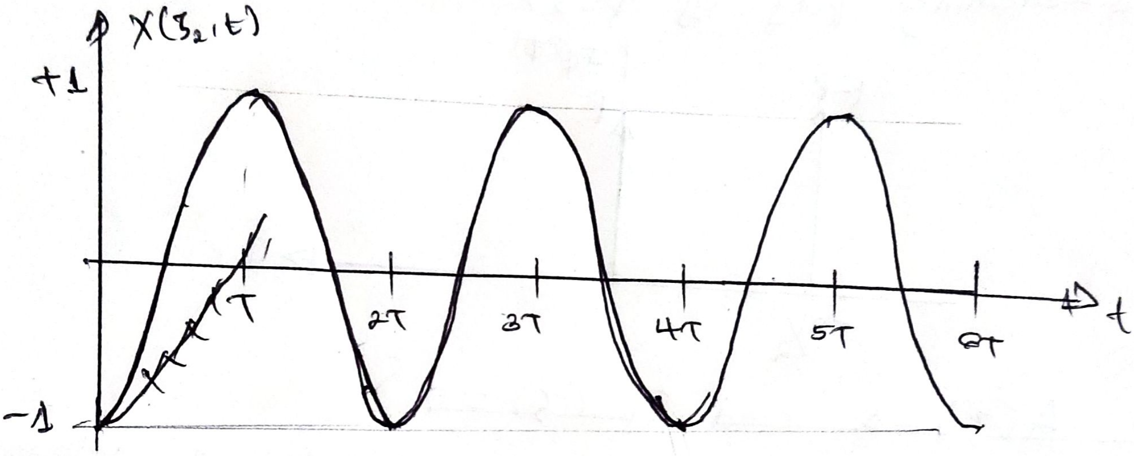
When $A(\zeta) = 1$ and $\Phi(\zeta) = -\pi/2$

$$X(\zeta_2, t) = \sin\left(\frac{t}{T}\pi - \frac{\pi}{2}\right)$$

Probability $= \frac{1}{2}(1-p)$

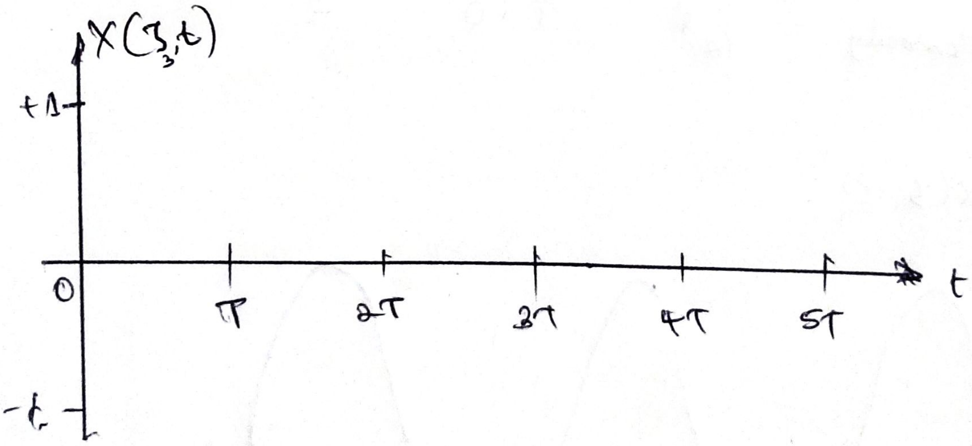
$$\therefore X(\zeta_2, t) = -\cos\left(\frac{t}{T}\pi\right)$$

Sketch.



When $A(\zeta) = 0$

$X(\zeta_3, t) = 0 \Rightarrow$ Probability $= \frac{1}{2}$



b) Cumulative distribution function → Three pattern function

$F_X(x,t)$ at $t=0$ & $t=T/4$

at $t=0$

$X(\zeta_1,t) = \sin(t/T\,\pi)$

$X(\zeta_1,t=0) = 0$, $P_1 = P/2$

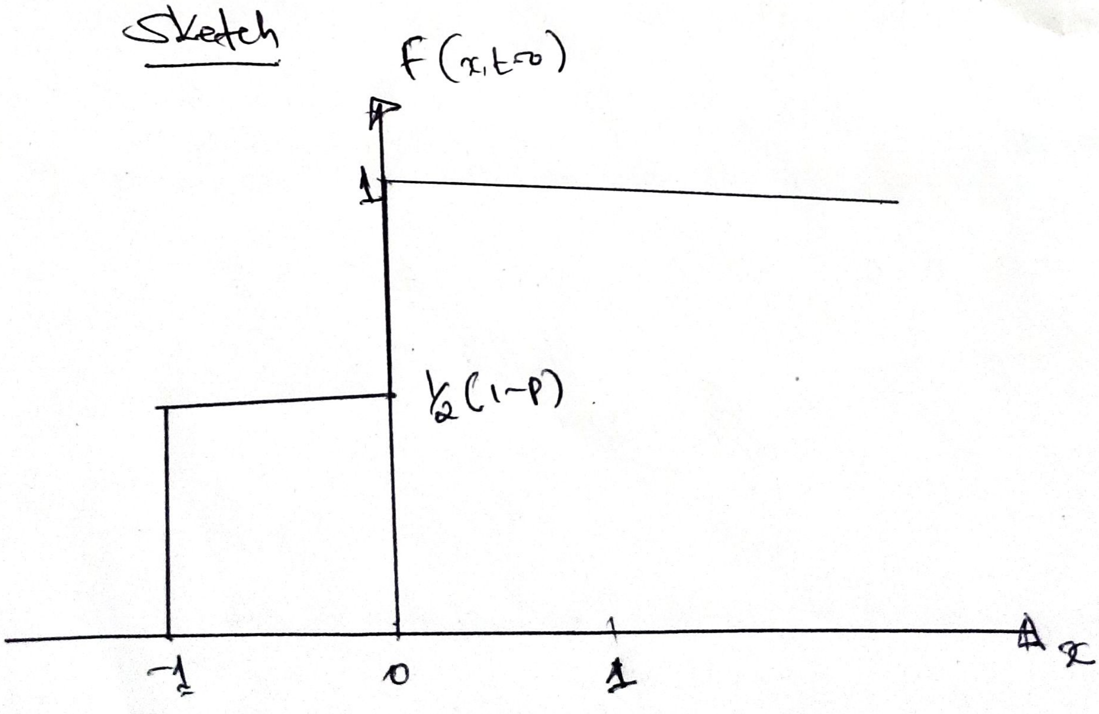
$X(\zeta_2,t) = -\cos(t/T\,\pi)$

$X(\zeta_2,t=0) = -\cos(0) = -1$, $P_2 = 1/2(1-P)$

$X(\zeta_3,t) = 0$, $P_3 = 1/2$

$$F(x,t=0) = \begin{cases} 0 & x < -1 \\ 1/2(1-P) & -1 \le x < 0 \\ 1 & x \ge 0 \end{cases}$$

Sketch



(at) $t = T/4$

$$X(\zeta_1, t = T/4) = \sin\left(\tfrac{T}{4} \cdot \tfrac{1}{T} \cdot \pi\right) = \sin(\pi/4) = \frac{\sqrt{2}}{2}$$

Probability, $P_1 = P/2$

$$X(\zeta_2, t = T/4) = -\cos\left(\tfrac{T}{4} \cdot \tfrac{1}{T} \cdot \pi\right) = -\cos(\pi/4) = -\frac{\sqrt{2}}{2}$$

Probability, $P_2 = \frac{1}{2}(1-P)$

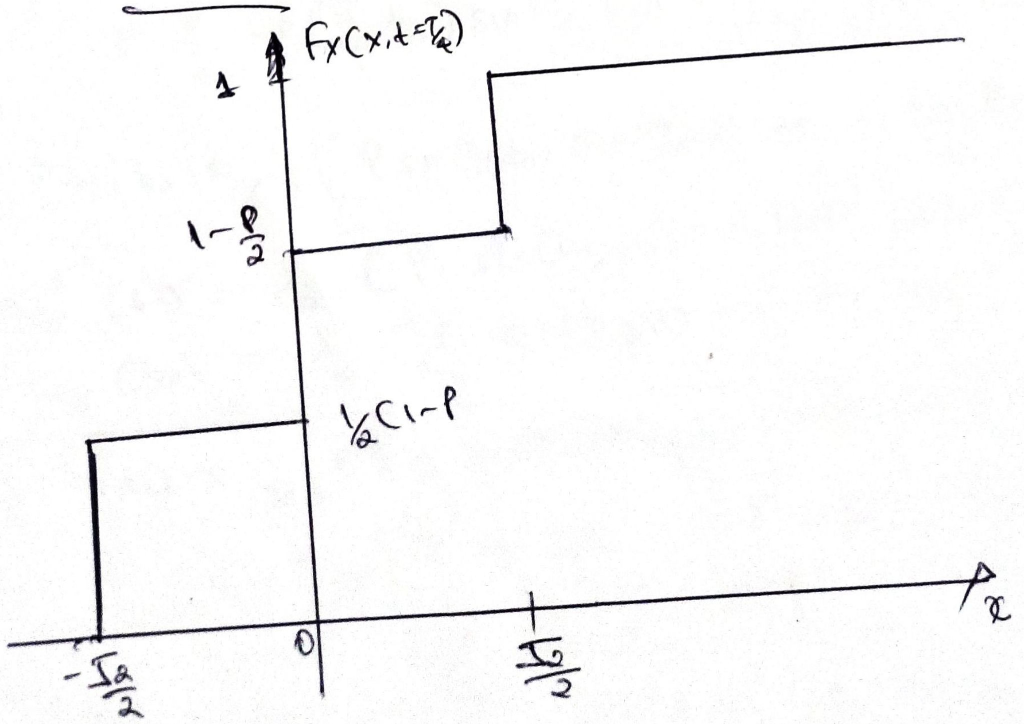
$$X(\zeta_3, t = T/4) = 0$$

Probability, $P_3 = 1/2$

$$F_x(x, t = T/4) = \begin{cases} 0 & x < -\frac{\sqrt{2}}{2} \\ P/2 & -\frac{\sqrt{2}}{2} \le x < 0 \\ \frac{1}{2}(1+P) & 0 \le x < \frac{\sqrt{2}}{2} \\ 1 & x \ge \frac{\sqrt{2}}{2} \end{cases}$$

(*) $P/2 + 1/2 = \frac{1}{2}(1+P)$ for $0 \le x < \sqrt{2}/2$

Sketch



c) Mean $M_x^{(1)}$ of the Process

$$M_x^{(1)} = E\{x(\zeta,t)\}$$
$$= \sum_v x_v P_v$$
$$= \sin(t/_T \pi)\cdot P/_2 + (-\cos t/_T\pi)\cdot \frac{1-P}{2} + 1/_2 (0)$$
$$= P/_2 \sin(t/_T\pi) - \left(\frac{1-P}{2}\right)\cos(t/_T\pi)$$

$$\boxed{M_x^{(1)} = \frac{1}{2}\left[P\sin(t/_T\pi) - (1-P)\cos(t/_T\pi)\right]}$$

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d) Covariance $C_{xx}(t_1,t_2)$ Pg the eqn (4.16)

$$C_{xx}(t_1,t_2) = S_{xx}(t_1,t_2) - M_x^{(1)}(t_1)\, M_x^{(1)}(t_2)$$

$$S_{xx}(t_1,t_2) = \sum_v x_i(\zeta,t_1)\, x_i(\zeta,t_2)\, P_v$$

$$= \sin(\pi/_T t_1)\cdot\sin\left(\frac{\pi}{T}t_2\right)\cdot P/_2 + (-\cos(\pi/_T t_1) - \cos\pi/_T t_2)\cdot \frac{1-P}{2} + 0\cdot 1/_2$$

$$= \frac{P}{2}\sin\left(\frac{\pi}{T}t_1\right)\sin(\pi/_T t_2) + \left(\frac{1-P}{2}\right)\cos(\pi/_T t_1)\cos(\pi/_T t_2)$$

$$S_{xx}(t_1,t_2) = \frac{1}{2}\left[P\sin(\pi/_T t_1)\sin(\pi/_T t_2) + (1-P)\cos(\pi/_T t_1)\cos(\pi/_T t_2)\right]$$

$$M_x^{(1)}(t_1) = \frac{1}{2}\left[P\sin(t_1/_T\pi) - (1-P)\cos(t_1/_T\pi)\right]$$
$$M_x(t_2) = \frac{1}{2}\left[P\sin\left(\frac{t_2}{T}\pi\right) - (1-P)\cos(t_2/_T\pi)\right]$$

$$\boxed{\begin{aligned}C_{xx}(t_1,t_2) = {} & \frac{1}{2}\left[P\sin\left(\frac{t_1}{T}\pi\right)\sin\left(\frac{t_2}{T}\pi\right) + (1-P)\cos\left(\frac{t_1}{T}\pi\right)\cos\left(\frac{t_2}{T}\pi\right)\right] \\ & - \frac{1}{4}\left[\left(P\sin\left(\frac{t_1}{T}\pi\right) - (1-P)\cos\left(\frac{t_1}{T}\pi\right)\right)\cdot\left(P\sin\frac{t_2}{T}\pi - (1-P)\cos(t_2/_T\pi)\right)\right]\end{aligned}}$$

e) Not Ergodic, since the process is not stationary as the ACF depends on time. Non-stationary process cannot be ergodic.