Strict submission deadline: 13 May 2024 at 10:00 am.

Exercise #1

Task 1.1

Given the joint density function

$$f_{xy}(x,y) = \begin{cases} 2 & for\ x \geq 0\ , y \geq 0\ and\ x + y\ \leq 1 \\ 0 & elsewhere \end{cases}$$

Calculate $f_x(x)$, $f_y(y)$, $F_{xy}(x,y)$, $F_x(x)$, $F_y(y)$

Don't submit the solution! It's just for practicing. Try to solve it without looking up the solution in the script. No submission!

Task 1.2

Let $a(\zeta)$ be a random variable taking the value 0 with probability $p_0 = \frac{1}{4}$ and the value 1 with probability $p_1 = \frac{3}{4}$. A random process x (ζ, t) is defined as:

$$x(\zeta, t) = \begin{cases} 1 - \frac{4}{T}\, t\, a(\zeta) & for\ 0 \leq t < \frac{T}{2} \\ -1 + \left(\frac{4}{T}\, t - 2\right) a(\zeta) & for \frac{T}{2} \leq t < T \\ 0 & otherwise \end{cases}$$

a) Sketch all pattern functions of the random process x (ζ, t).
b) Calculate the mean $m_x^{(1)}(t)$.
c) Calculate the autocorrelation function $s_{xx}(t_1, t_2)$.
d) Calculate the variance $\sigma_x^2(t)$.

Submit the calculation path and the solution.

Task 1.3

Given the following signal

$$x(t) = \sin(2\pi f t)$$

with

$$f = 1Hz$$

use Matlab for

a) Plotting the signal in the timeframe $-10 \leq t \leq 10$. Use correct axis labels and scaling.
b) Calculating the ACF of x(t) using a rectangular time window (=1 for $-5 \leq t \leq 5; = 0\ elsewhere$)
c) Plotting the ACF calculated in part b)

Now please answer the next question without using Matlab

d) What would be the difference if you could calculate the ACF in the range $-\infty \leq t \leq +\infty$?

And

e) Sketch the ACF from part d) in the timeframe $-10 \leq t \leq 10$

Submit the solutions (Matlab source code, printed plots, written answers, and sketches)