Task 5.1)

a) $x(t) = B\cdot[\delta(t) - \delta(t-2T) + \delta(t-3T)]$

$y(t) = x(t) * h(t)$

$y(t) = B[h(t) - h(t-2T) + h(t-3T)]$

$y(t) = \{AB_{[0,T]} - AB_{[2T,3T]} + AB_{[3T,4T]}\}$

$z(t) = x(t) * g(t)$

$= B[g(t) - g(t-2T) + g(t-3T)]$

$z(t) = \{AB_{[0,T/2]} - AB_{[2T,2T+T/2]} + AB_{[3T,3T+T/2]}\}$

- Sketch $\tilde{R}_{yy}(\tau)$, we have

- $y(t)$ has pulses at intervals $[0,T]$, $[2T,3T]$, $[3T,4T]$ of amplitude $AB$.
  Take different values of $\tau$

1) $\tau = 0$

~~Syy~~ $R_{yy}(0) = \int_0^{4T} y(t)\,y(t+\tau)\,dt = \int_0^{4T} (y(t))^2 dt$

$= \int_0^T (y(t))^2 dt + \int_{2T}^{3T} (y(t))^2 dt + \int_{3T}^{4T} (y(t))^2 dt$

$$\boxed{\tilde{R}_{yy}(0) = 3(AB)^2 T}$$

2) $\tau \neq 0$

i) $\tau = T$

$0 \le t < T$

$y(t) = AB, \quad y(t+\tau) = 0$

$2T \le t \le 3T$

$y(t) = -AB, \quad y(t+\tau) = AB$

$S_{yy}(T) = \int_{2T}^{3T} -(AB)^2 dt = -(AB)^2 T$

$3T \le t < 4T$

$y(t) = AB, \quad y(t+\tau) = 0$

ii) $\tau = 2T$

$0 \le t < T$

$y(t) = AB, \quad y(t+\tau) = -AB$

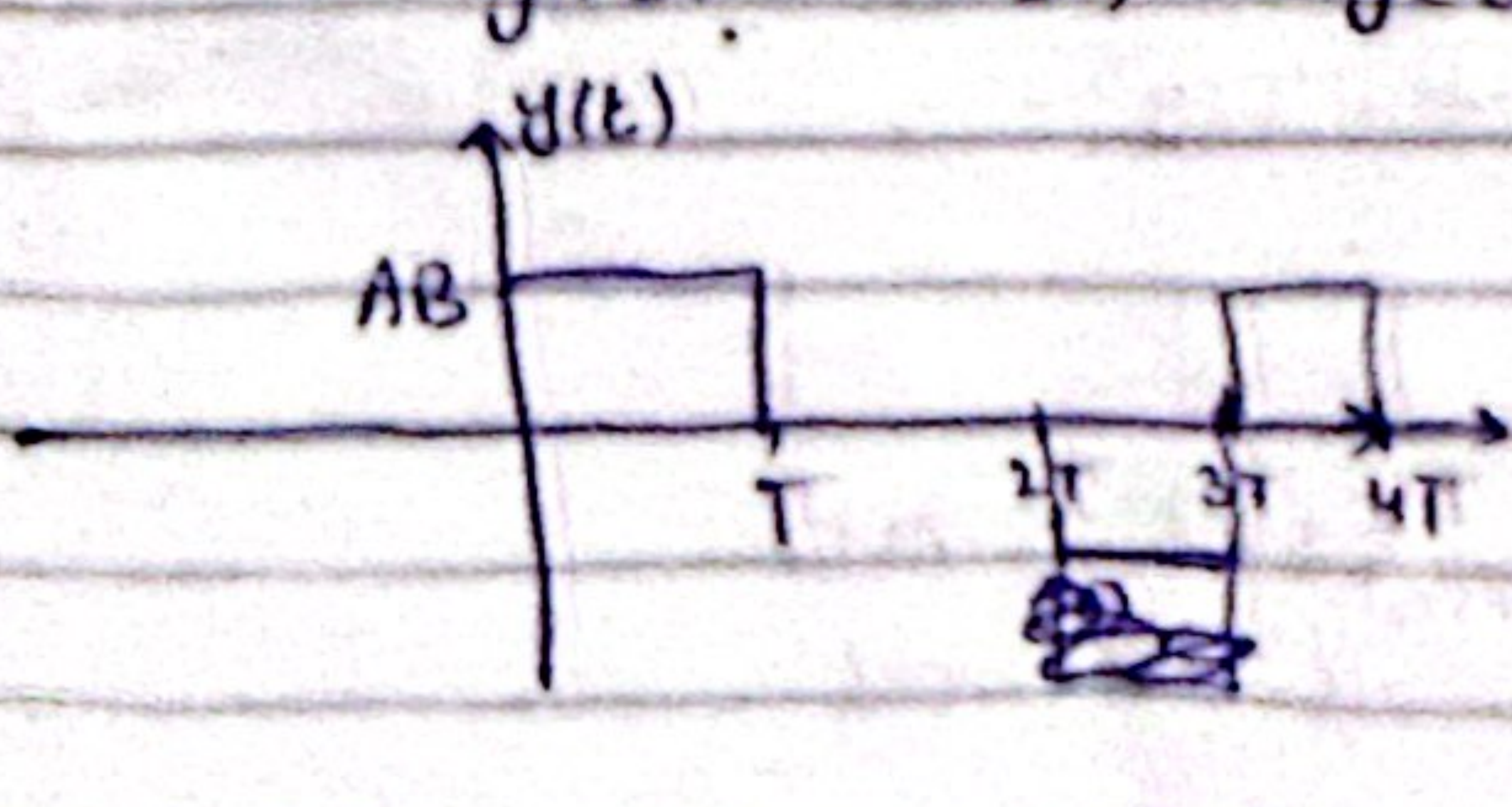
$S_{yy}(2T) = \int_0^T (AB)(-AB)\, dt = -(AB)^2 T$

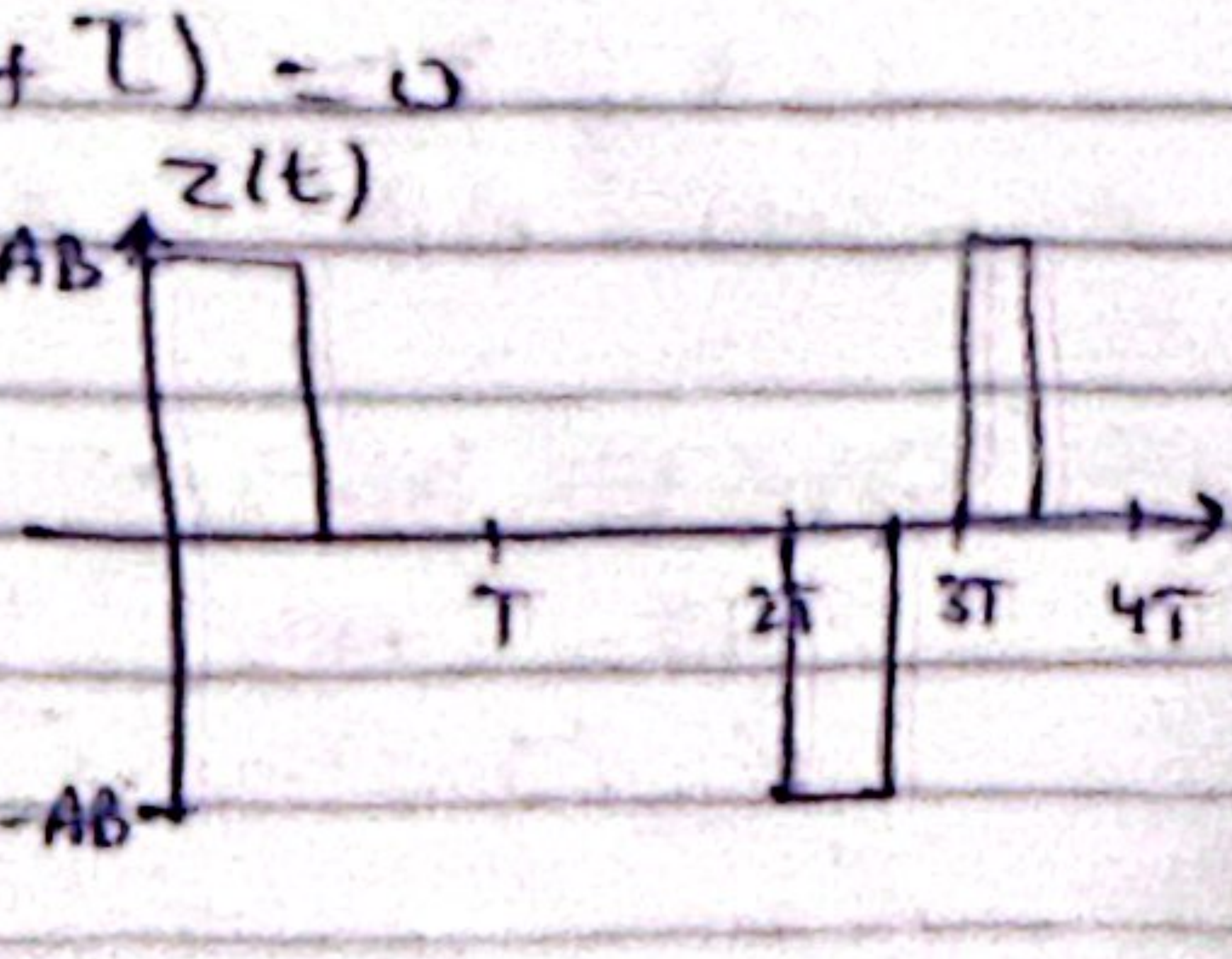
$2T \le t < 3T$

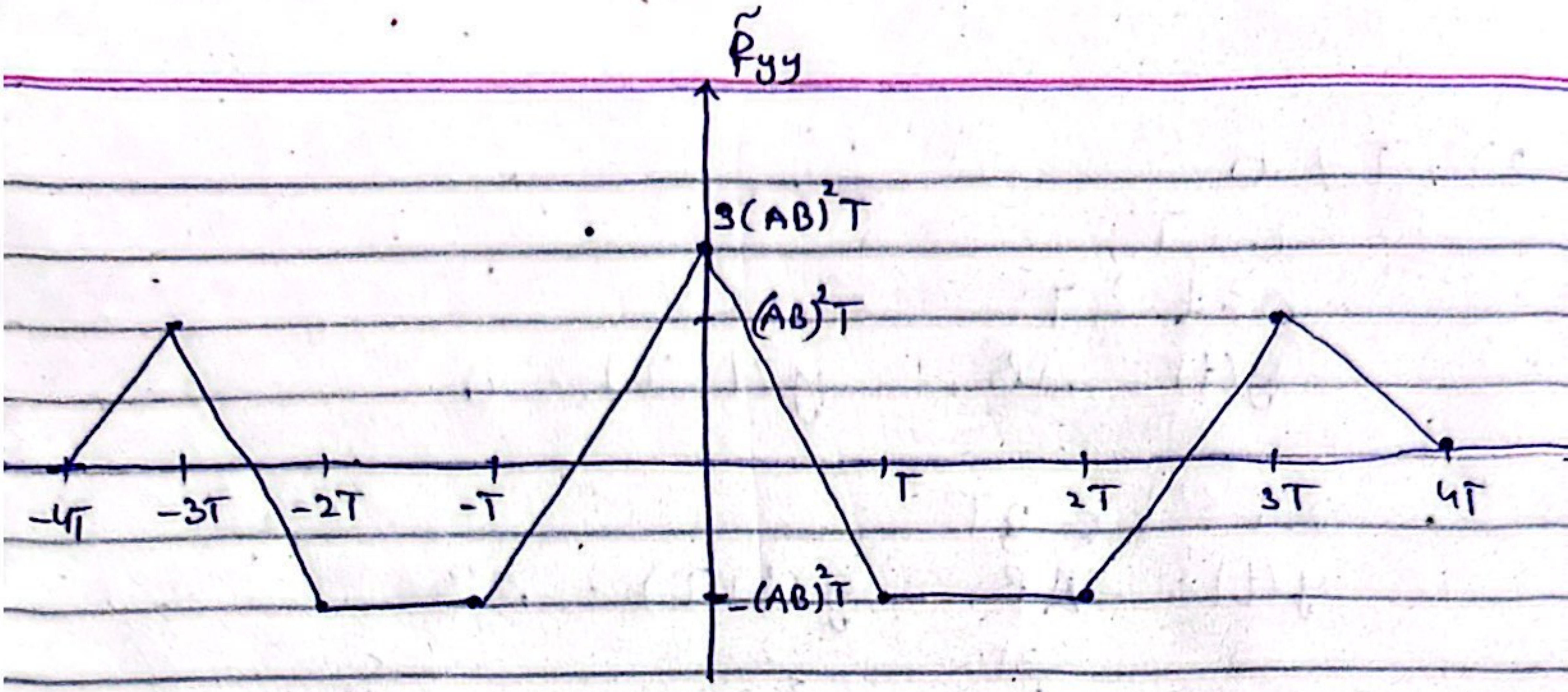
$y(t) = -AB, \quad y(t+\tau) = 0$

$3T \le t < 4T$

$y(t) = AB, \quad y(t+\tau) = 0$






- Sketch of $\tilde{R}_{zz}(\tau)$

① $\tau = 0$

$$\tilde{R}_{zz} = \int_0^{4T} (z(t))^2 \, dt$$

$$\tilde{R}_{zz}(0) = \int_0^{T/2} (z(t))^2 \, dt + \int_{2T}^{2T+T/2} (z(t))^2 \, dt + \int_{3T}^{3T+T/2} (z(t))^2 \, dt$$

$$\boxed{\tilde{R}_{zz}(0) = \frac{3}{2} (AB)^2 T}$$

2) $\tau \neq 0$

$\tau = T/2$

$0 \leq t \leq T/2$

$z(t) = AB, \quad z(t + T/2) = 0$

$2T \leq t < 2T + T/2$

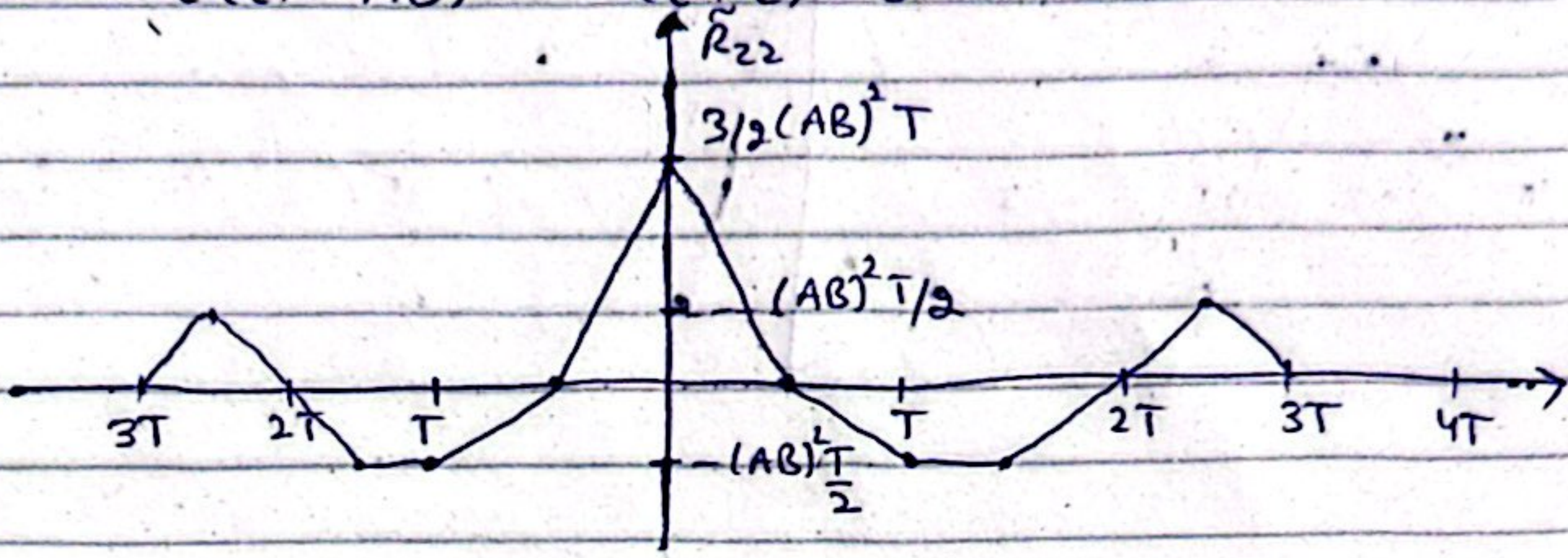
$z(t) = -AB, \quad z(t+\tau) = AB$

$$\tilde{R}_{zz}(T/2) = \int_{2T}^{2T+T/2} -(AB)^2 \, dt$$

$$\boxed{\tilde{R}_{zz}(T/2) = -(AB)^2 . T/2}$$

$3T \le t < 4T$

$z(t) = AB, \quad z(t+\tau) = 0$




- Sketch of $\tilde{R}_{yz}(\tau)$

$$\tilde{R}_{yz}(\tau) = \int_0^{4T} y(t)\, z(t+\tau)\, dt$$

① $\tau = 0$

$$\tilde{R}_{yz}(0) = \int_0^{4T} y(t) \cdot z(t)\, dt$$

$$= \int_0^{T/2} (AB)^2 dt + \int_{2T}^{2T+T/2} (AB)^2 dt + \int_{3T}^{3T+T/2} (AB)^2 dt$$

$$\boxed{\tilde{R}_{yz}(0) = \frac{3}{2} (AB)^2 T}$$

② $\tau \neq 0$

$\tau = T/2$

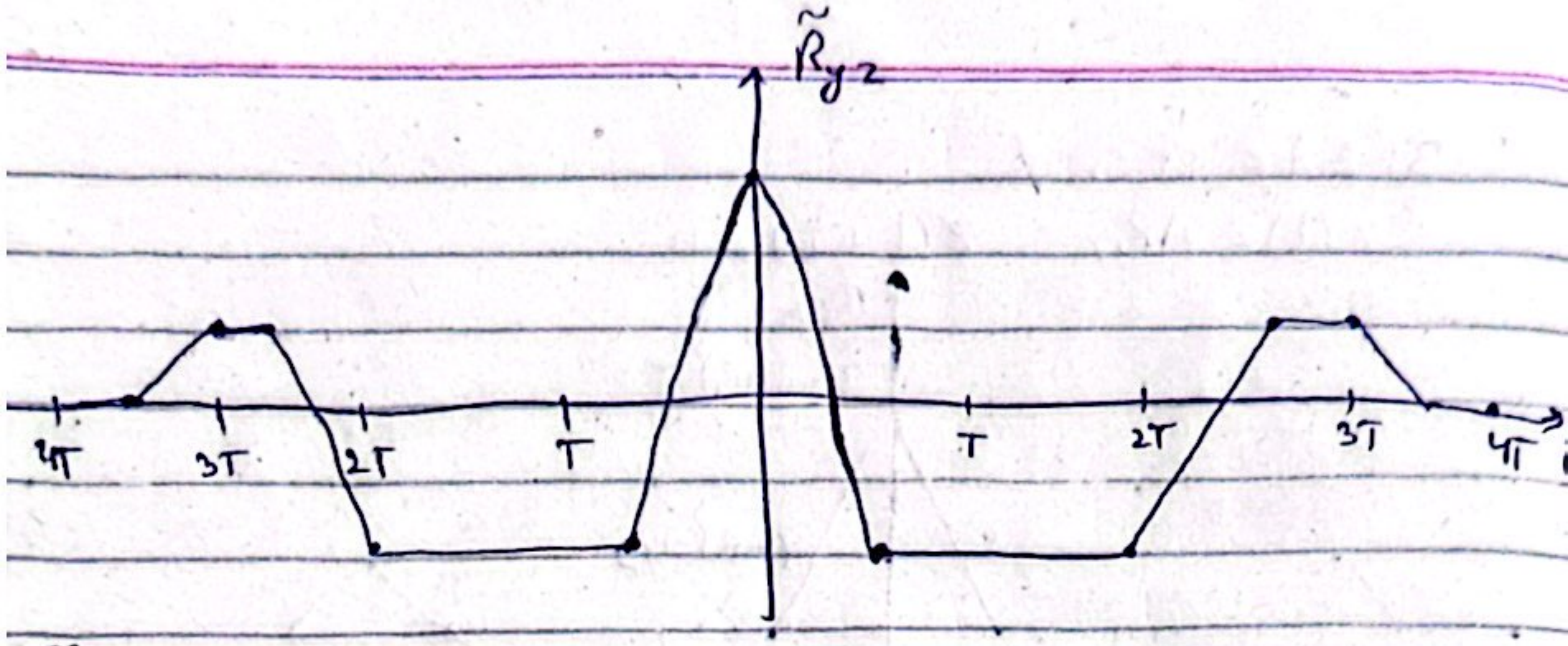
$S_{yz}(T/2) = -(AB)^2 \cdot T/2 \quad ; \; 2T \le t < 2T + T/2$

$\tau = 2T$

$S_{yz}(2T) = -(AB)^2 \cdot T/2 \quad ; \; 0 \le t < T/2$

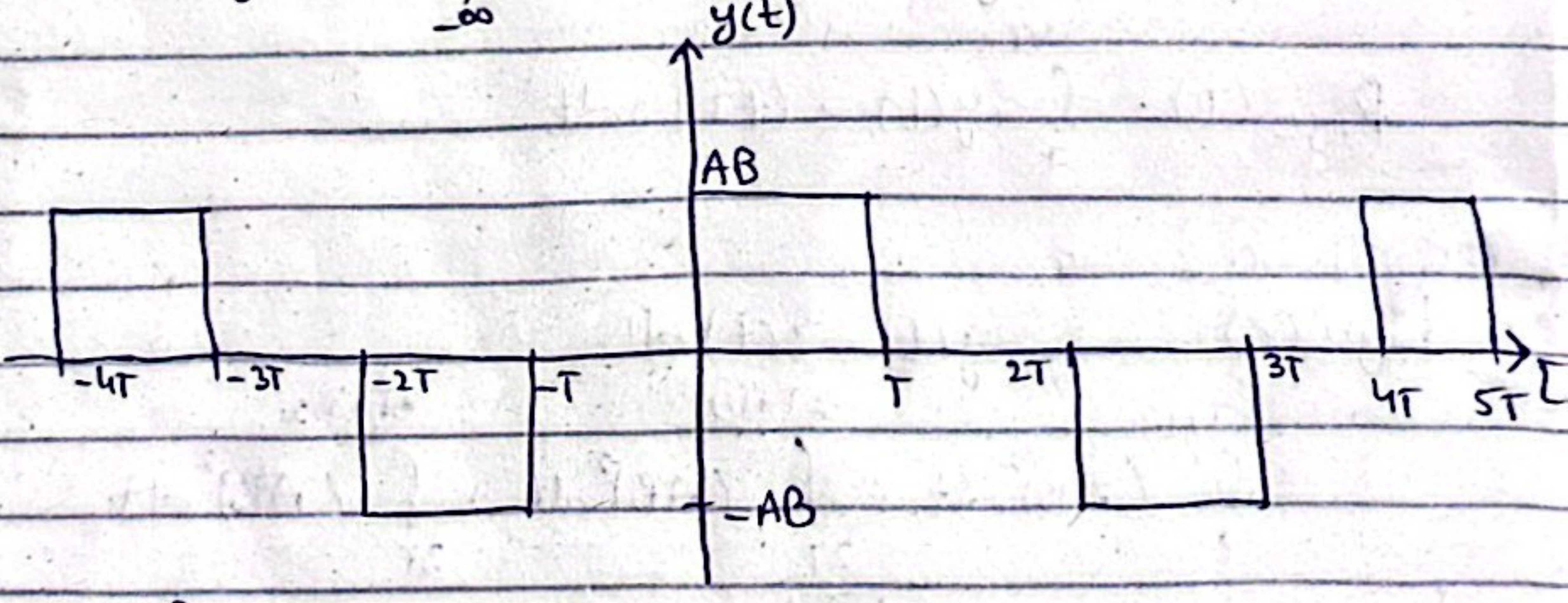
③ $\tau = -T/2$

$\tau = -T$



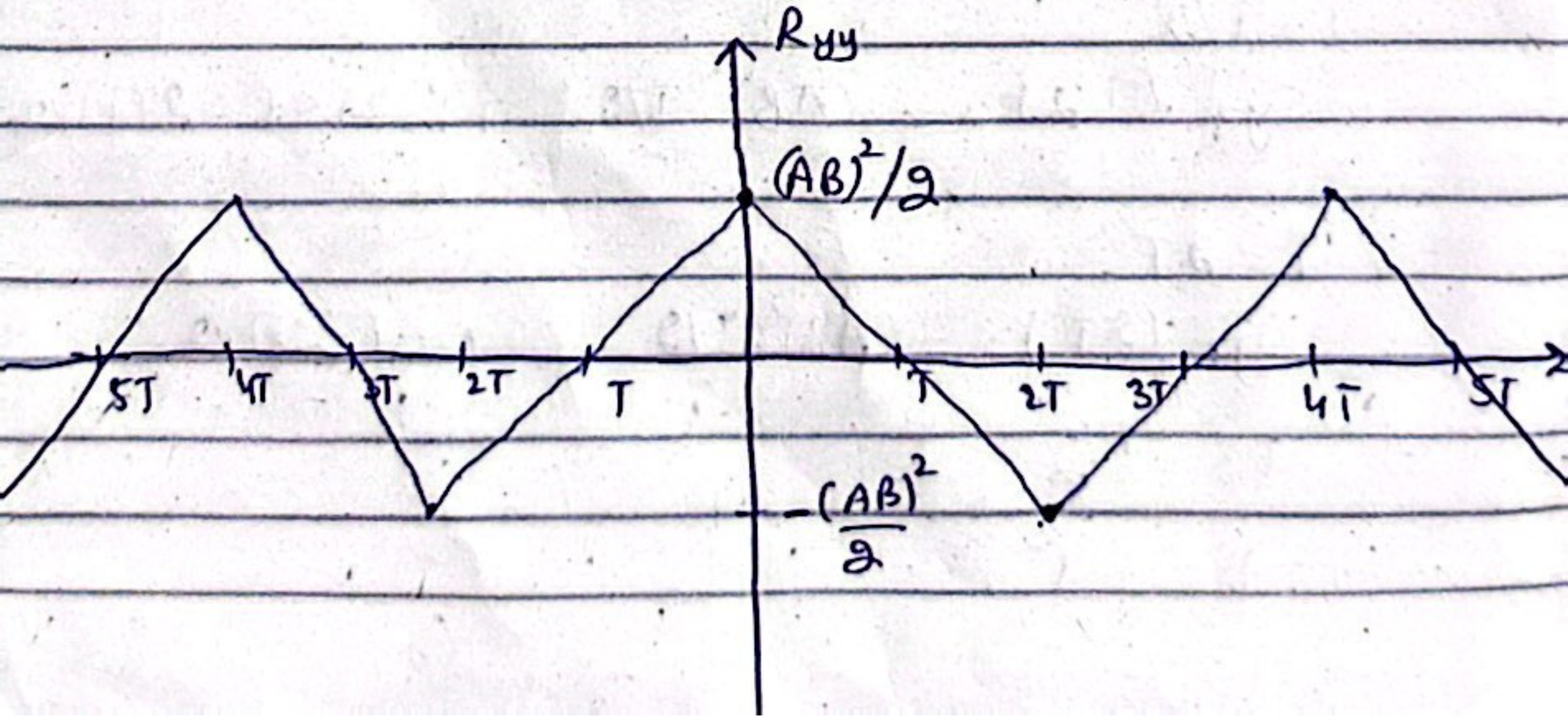

(b)

$$y(t) = \int_{-\infty}^{+\infty} x(t)\, h(t-\tau)\, dt$$




Periodic signal

$$\tilde{R}_{yy}(\tau) = \frac{1}{4T} \int_0^{4T} y(t)\, y(t+\tau)\, dt$$

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Task 5.2)

a) $A(\xi) = 1, \quad A(\xi) = 0$

$P(A=1) = 1/2, \qquad P(A=0) = 1/2$

$P(\phi = 0) = p \quad , \quad P(\phi = -\pi/2) = 1-p$

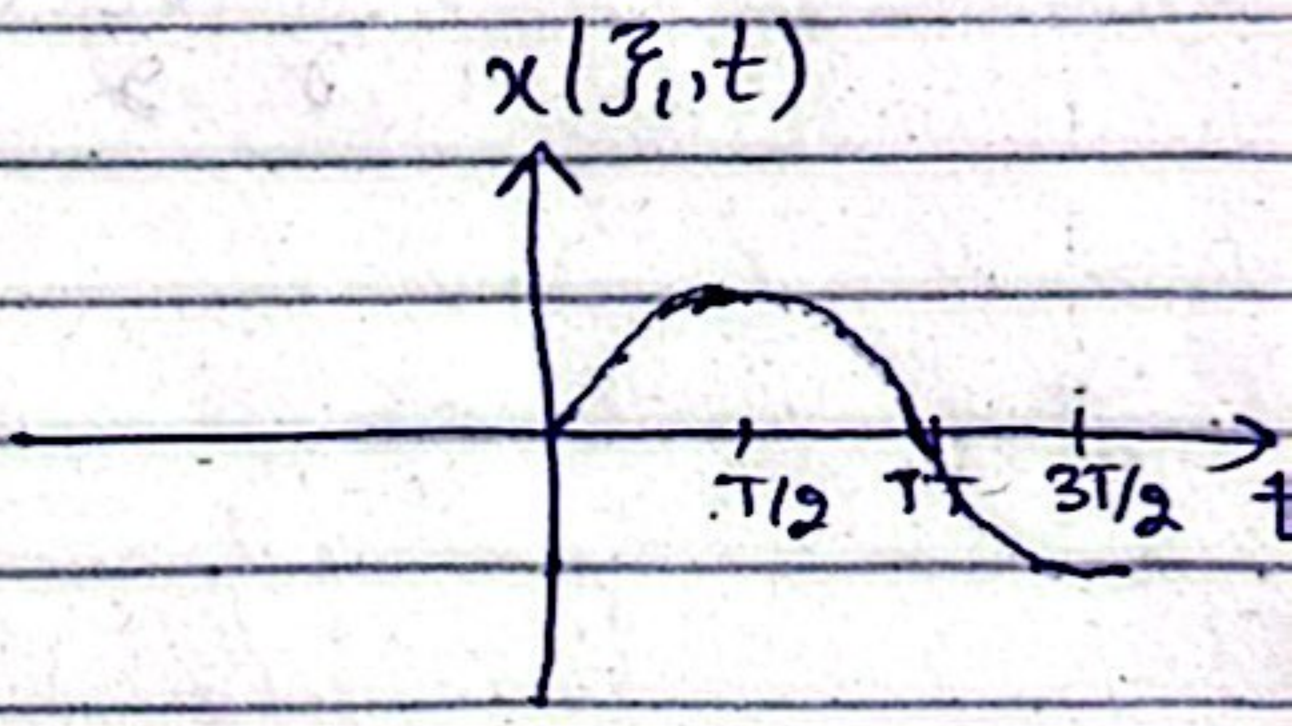
Distant pattern functions.

Case 1) $A = 1$

① $\phi = 0$

$x(\xi_1, t) = \sin\left(\frac{t}{T}\pi\right)$

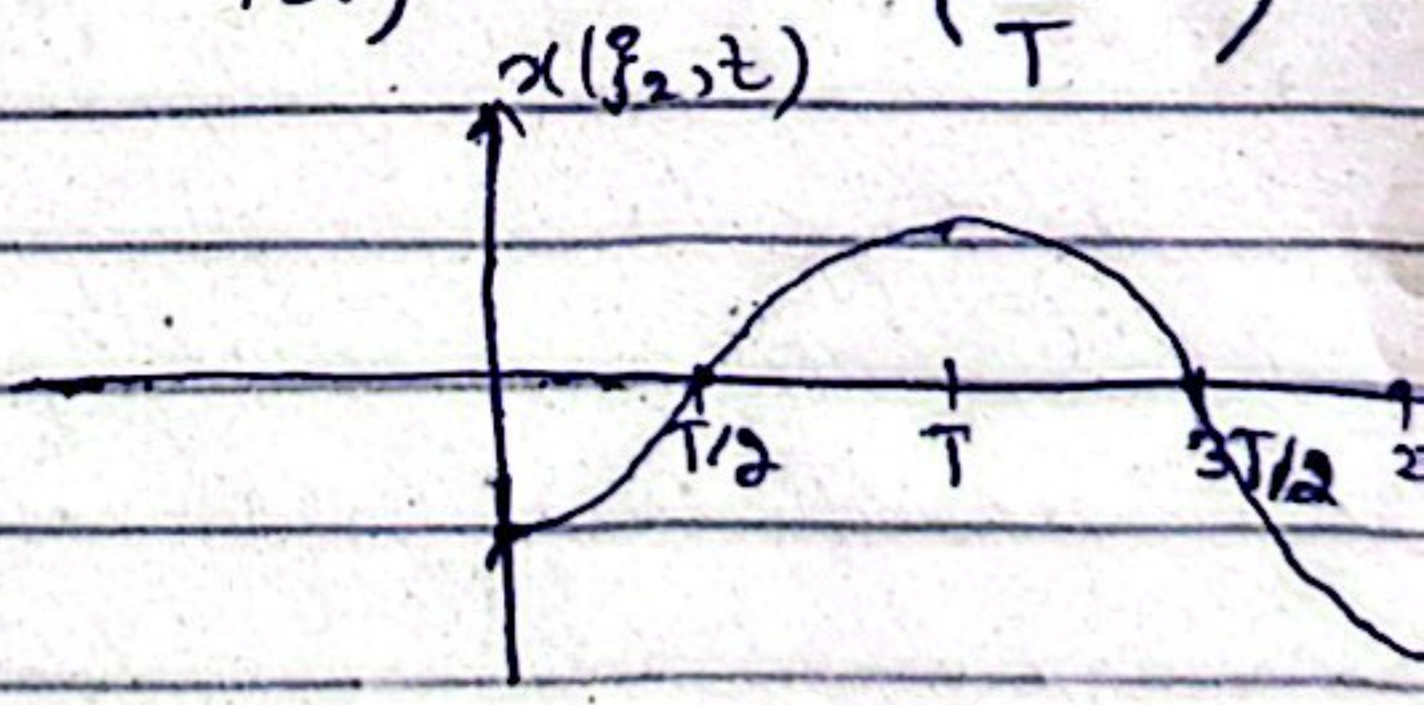
$P(x(\xi_1, t)) = p/2$




② $\phi = -\pi/2$

$x(\xi_2, t) = \sin\left(\frac{t}{T}\pi - \pi/2\right) = -\cos\left(\frac{t}{T}\pi\right)$

$P(x(\xi_2, t)) = \frac{1-p}{2}$

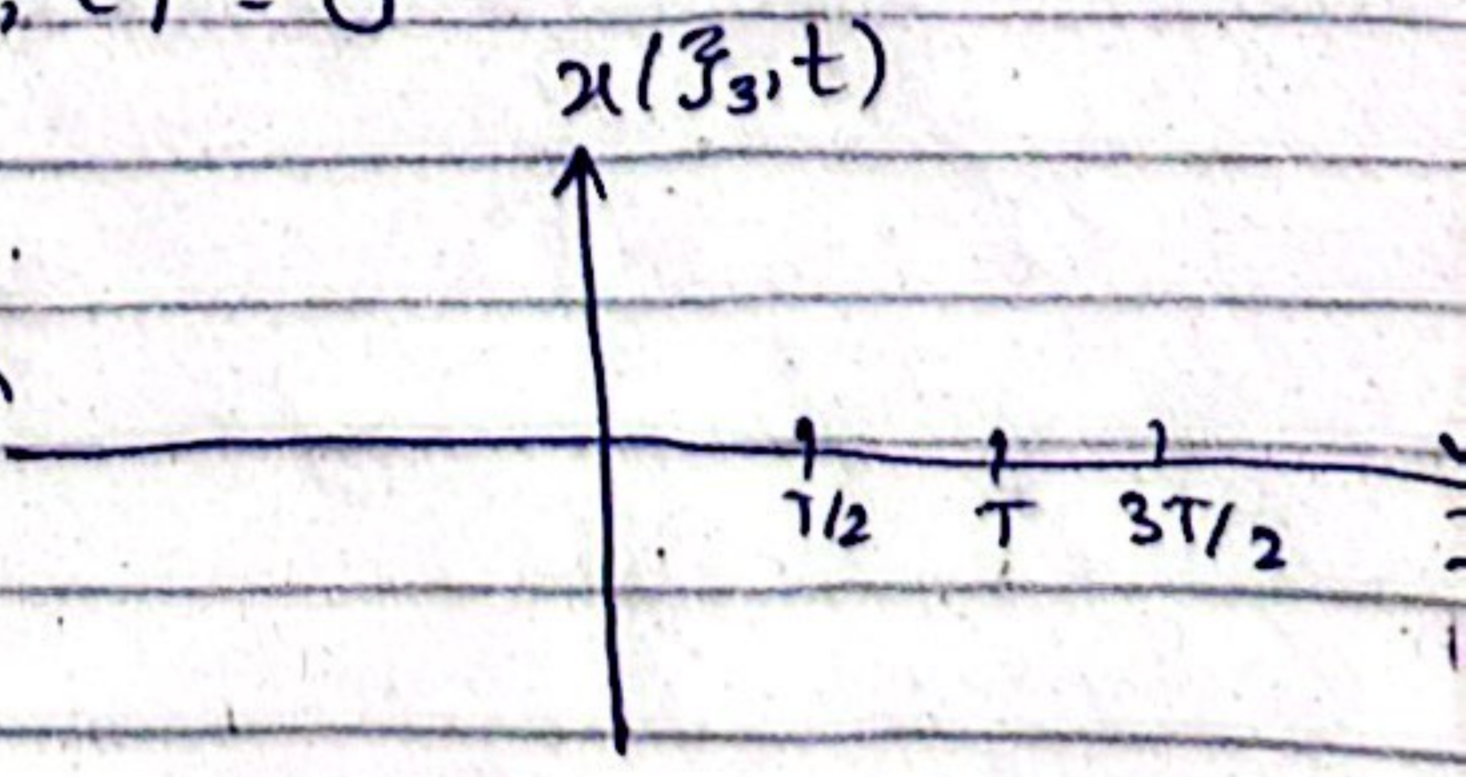



Case 2) $A = 0$

regardless of $\phi$

~~x(t) = 0~~ $x(\xi_3, t) = 0$

$P(x(\xi_3, t)) = 1/2$

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(b) Sketch $F_x(x,t)$ at $t=0$ and $t=T/4$

for $t=0$

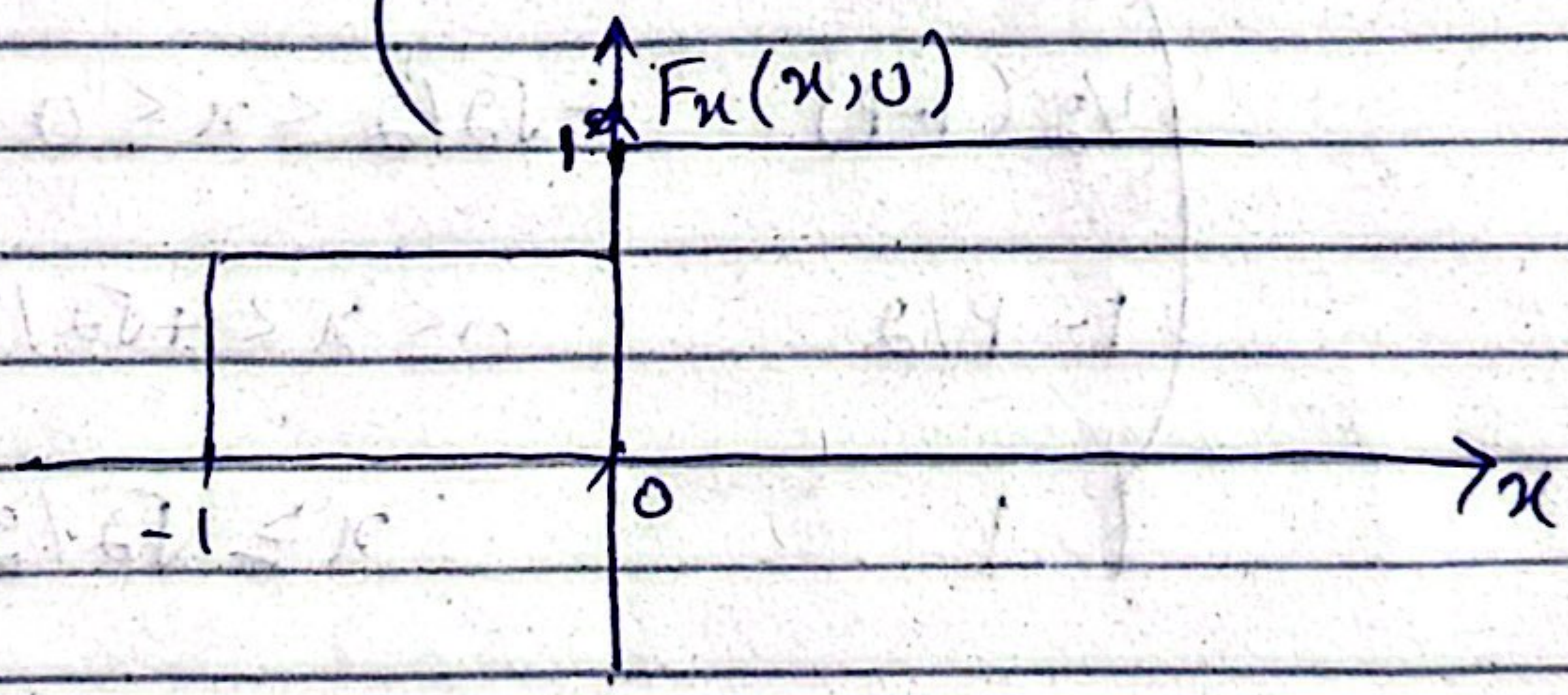
$x(\xi_1,0) = \sin\left(\frac{t}{T}\pi\right) = 0$, with Probability $= P/2$

$x(\xi_2,0) = -\cos\left(\frac{t}{T}\pi\right) = -1$, with Prob $= \frac{(1-P)}{2}$

$x(\xi_3,0) = 0$ with Prob $= 1/2$

CDF is given by

$$F_x(x,0) = \begin{cases} (1-P)/2 & -1 \leq x < 0 \\ 1 & x \geq 0 \end{cases}$$




for $t = T/4$

for $A(\xi) = 1$

$\phi(\xi) = 0$

$x(\xi, T/4) = \sin(\pi/4) = \sqrt{2}/2$ with Prob $= \frac{P}{2}$

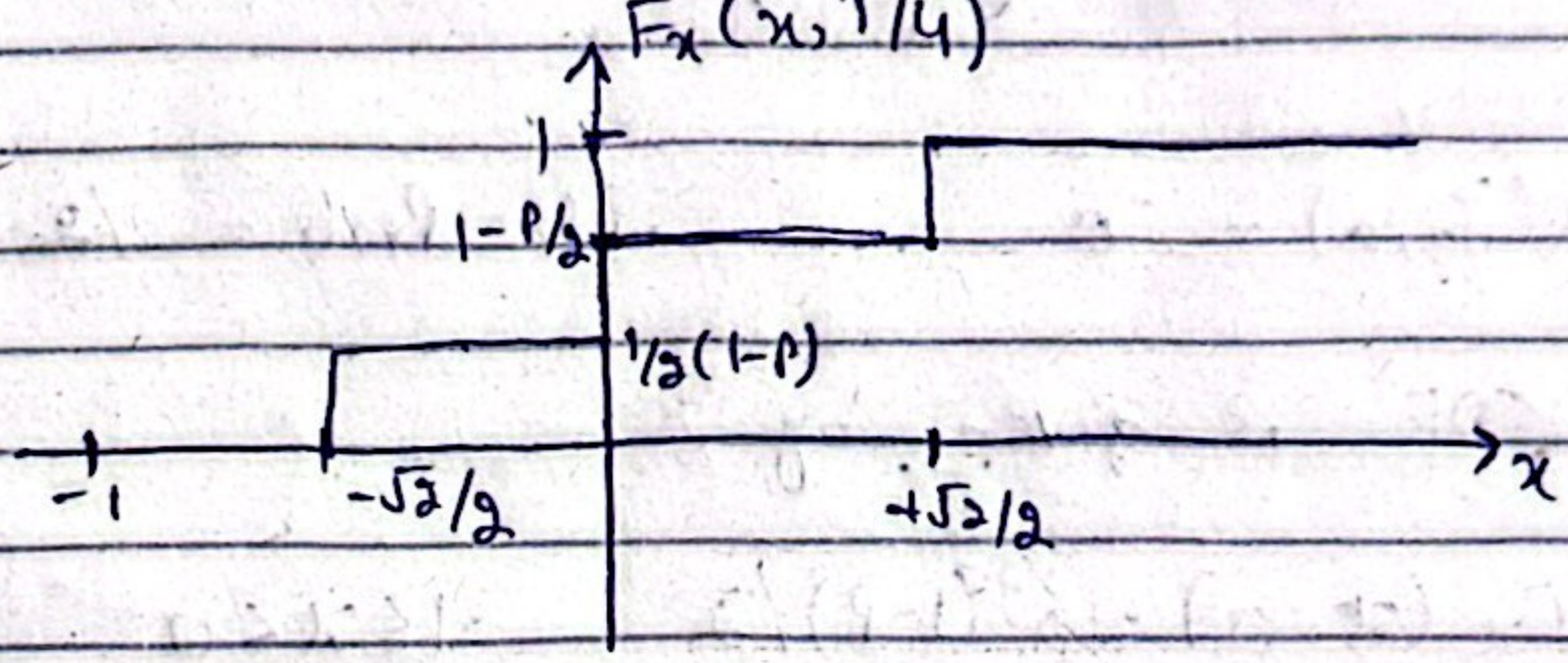
$\phi(\xi) = -\pi/2$

$x(\xi, T/4) = \sin\left(\frac{\pi}{4} - \frac{\pi}{2}\right) = -\sqrt{2}/2$ with Prob $= \frac{1}{2}(1-P)$

$P(x(\xi, T/4)) = 0.5 = 1/2$

$P(x(\xi, T/4 = \sqrt{2}/2)) = P/2$

$P(x(\xi, T/4 = -\sqrt{2}/2)) = (1-P)/2$




$$F_x(x, T/4) = \begin{cases} 0 & x < -\sqrt{2}/2 \\ 1/2(1-P) & -\sqrt{2}/2 \le x < 0 \\ 1 - P/2 & 0 \le x < +\sqrt{2}/2 \\ 1 & x \ge \sqrt{2}/2 \end{cases}$$

(c)

mean = $m_x^{(1)}(t)$ of the process $x(\xi, t)$

$$m_x^{(1)}(t) = \frac{1}{2}\left(p \sin\left(\frac{t}{T}\pi\right) + (1-p)\cos\left(\frac{t}{T}\pi\right)\right)$$

d) Covariance $= C_{xx}(t_1, t_2) =$
$$= E\left((x(\xi, t_1)) \cdot x(\xi, t_2)\right) - m_x^{(1)}(t_1) \cdot m_x^{(1)}(t_2) \rightarrow ①$$

$$\cdot E[x(\xi, t_1) \cdot x(\xi, t_2)] = \tilde{R}_{xx}(t_1, t_2)$$
$$= \frac{1}{2}\left(p \sin\left(\frac{t_1}{T}\pi\right) \sin\left(\frac{t_2}{T}\pi\right) + (1-p) \cos\left(\frac{t_1}{T}\pi\right) \cos\left(\frac{t_2}{T}\pi\right)\right)$$

$$- \; m_x^{(1)}(t_1) = \frac{1}{2}\left[p \sin\left(\frac{t_1}{T}\pi\right) + (1-p) \cos\left(\frac{t_1}{T}\pi\right)\right]$$

$$- \; m_x^{(2)}(t_2) = \frac{1}{2}\left[p \sin\left(\frac{t_2}{T}\pi\right) + (1-p) \cos\left(\frac{t_2}{T}\pi\right)\right]$$

put all the values in equation ①

$C_{xx}(t_1, t_2) =$ ______

e) for each single realization time average is not equal to ensemble average, so it is not ergodic process.