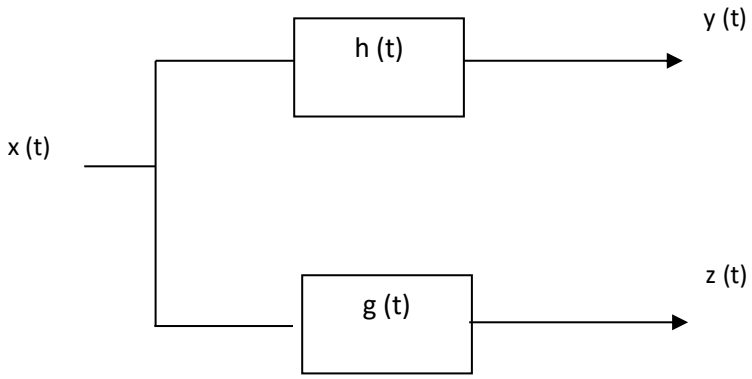
Strict submission deadline: 24 June 2024 at 10:00 am.
Submit a PDF or Word document for calculations and plots. Submit Matlab source code for Matlab programs.

## Exercise #5

### Task 5.1 Signal and Systems

Given is the following block diagram:




The two impulse responses h (t) and g (t) are:

$$h(t) = \begin{cases} A & 0 \le t < T \\ 0 & otherwise \end{cases} \qquad g(t) = \begin{cases} A & 0 \le t < \frac{T}{2} \\ 0 & otherwise \end{cases}$$

The input of the system is the deterministic signal x (t).

a) Assuming that

$$x(t) = B \cdot (\delta(t) - \delta(t - 2T) + \delta(t - 3T))$$

sketch the functions,

$$s_{yy}(\tau) = \lim_{u \to \infty} \int_{-u}^{+u} y(t) y\,(t + T) dt$$

$$s_{zz}(\tau) = \lim_{u \to \infty} \int_{-u}^{+u} z(t) z(t + T) dt \qquad and$$

$$s_{yz}(\tau) = \lim_{u \to \infty} \int_{-u}^{+u} y(t) z(t + T) dt$$

b) Assume that

$$x(t) = B \sum_{k=-\infty}^{+\infty} (-1)^k \, \delta(t - 2kT)$$

and sketch the function

$$s_{yy}(\tau) = \lim_{u \to \infty} \frac{1}{2u} \int_{-u}^{+u} y(t) y(t + T) dt$$

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**Task 5.2 Random processes and moments**

A random process x (ζ, t) is given by

$$x(\zeta, t) = A(\zeta)\sin(\frac{t}{T}\pi + \varphi(\zeta)) \quad with\, T > 0.$$

The random variable A (ζ) takes the value +1 and 0 with equal probability. A random variable ϕ (ζ) which is statistically independent from the random variable A (ζ) is described by its probability density function:

$$f_{\varphi}(\varphi) = p\,\delta(\varphi) + (1-p)\delta\left(\varphi + \frac{\pi}{2}\right) \quad with\, 0 \le p \le 1$$

a) Sketch all distinct pattern functions of the random process x (ζ, t) and specify the probabilities with which the sample functions occur.
b) Sketch the cumulative distribution function $F_x(x,t)$ at the time t = 0 and t = T / 4
c) Calculate the mean $m_x^{(1)}(t)$ of the process $x(\zeta, t)$.
d) Determine the covariance function $c_{xx}(t_1, t_2)$.
e) Why is the random process is not ergodic?

**Task 5.3 Cross-correlation function**

Let a stationary random process x(ζ, t)

$$s_{xx}(\tau) = a\, e^{-\alpha|\tau|} + b$$

Known are the mean (=0), the standard deviation (=1), and the autocorrelation function.

Let a random process y (ζ, t),

$$y(\zeta, t) = \begin{cases} 0 & for\, t \le t_0 \\ \int_{t_0}^{t} x(\zeta, \lambda)\, d\lambda & for\, t > t_0 \end{cases}$$

a) Determine the constants a and b.
b) Determine the cross-correlation function $s_{xy}(t_1, t_2) = E\{x(\zeta, t_1)\, y(\zeta, t_2)\}$
c) Is y (ζ, t) a stationary process?