Strict submission deadline: 27 May 2024 at 10:00 am.

Exercise #2

Task 2.1

Let the signal

$$x(t) = \sin(2\pi f t)$$

with

$$f = 5\ Hz$$

Write Matlab programs.

a) Sample the signal x(t). The sampling frequency is $f_s = 50\ Hz$. Plot the sampled signal in the timeframe -10 $\leq t \leq 10$ (t in seconds). Use correct axis labels and scaling.
b) Cut out a time frame from x(t) in the range -5 $\leq t \leq 5$. Use a rectangular time window (=1 for -5 $\leq t \leq 5; = 0\ elsewhere$). Hint: multiply the rectangular time window pointwise with the sampled signal. Plot the result from -10 to +10. Use correct axis labels and scaling.
c) Calculate the FFT of the cut-out time frame (hint: 500 sampling points) and plot it. Use correct axis labels and scaling.
d) Once again cut out a time frame from x(t) in the range -5 $\leq t \leq 5$. This time use a Hamming window. Hint: Create a Hamming window using the corresponding Matlab function. Use the same sampling rate that you used for sampling x(t).
e) Calculate the FFT of the cut-out time frame of part d). Use correct axis labels and scaling.

Now please answer the next question without using Matlab

f) Compare the FFTs of c) and e) and discuss the differences.

Submit the solutions: Matlab source code in .m files (no copies or scans), printed plots, and written answers.

Task 2.2

Let a discrete stationary random process x (ζ, t).
The outcomes of the process are the values $x_1 = -1, x_2 = 0\ and\ x_3 = 1$
The probabilities of the occurrence of those outcomes are

$$P(\{x(\zeta, t+\tau) = x_i\} | \{x(\zeta, t) = x_j\}) = \begin{cases} \frac{1}{3}\left(1 + 2\, e^{-|\tau|}\right) & for\ i = j \\ \frac{1}{3}\left(1 - e^{-|\tau|}\right) & for\ i \neq j \end{cases} \qquad i,j = 1,2,3$$

a) Calculate the probabilities

$$P(\{x(\zeta, t) = x_i\}) \quad for\ i = 1,2,3.$$

b) Calculate the ACF $s_{xx}(\tau)$

Submit the calculation path and the solution.