6.1 →

a) $m_x^{(1)}(t) = \sqrt{\lim_{\tau \to \infty} S_{xx}(\tau)}$

$m_x^{(2)}(t) = \sqrt{\lim_{\tau \to 0} S_{xx}(\tau)}$

Find a and b.

$m_x^{(1)}(t) = \sqrt{\lim_{\tau \to \infty} a e^{-\alpha|\tau|} + b}$

$0 = \sqrt{0+b}$

$\therefore \boxed{b = 0}$

$\sigma_x^2 = m_x^{(2)}(t) - [m_x^{(1)}(t)]^2$

$m_x^{(2)}(t) = \lim_{\tau \to 0} S_{xx}(\tau) = a + b$

$1 = a + b \qquad \because b = 0$

$\boxed{\therefore \quad a = 1 \quad ; \quad b = 0}$

b) $S_{xy}(t_1, t_2) = E\{x(\zeta, t_1)\, y(\zeta, t_2)\}$

$= E\{x(\zeta, t_1)\} E\{y(\zeta, t_2)\}$

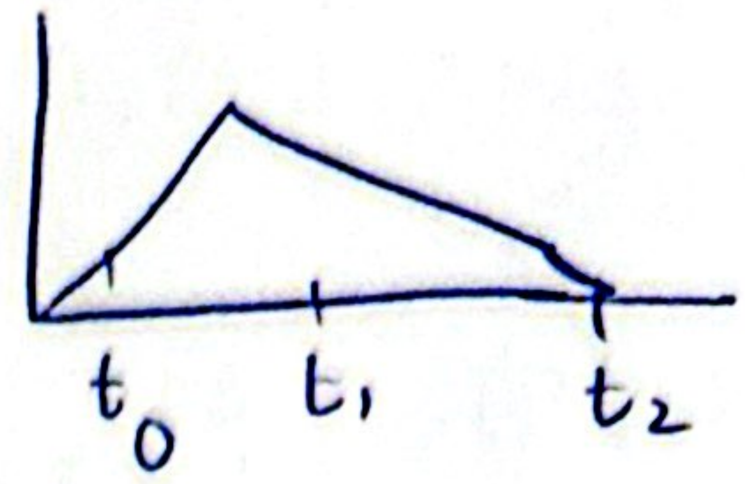
$= E\{x(\zeta, t_1)\} \int_{t_0}^{t_2} x(\zeta, \lambda)\, d\lambda\}$

$= \int_{t_0}^{t_2} E\{x(\zeta, t_1) \cdot x(\zeta, \lambda)\}\, d\lambda$

$= \int_{t_0}^{t_2} S_{xx}(t_1 - \lambda)\, d\lambda$

$= \int_{t_0}^{t_2} e^{-\alpha|t_1 - \lambda|}\, d\lambda$

Case I → $t_0 < t_1 < t_2$



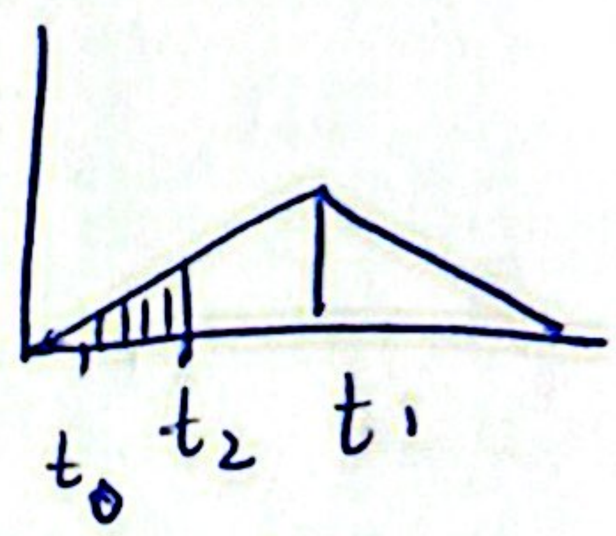
$$\int_{t_0}^{t_1} e^{\alpha(\lambda - t_1)} \, d\lambda + \int_{t_1}^{t_2} e^{-\alpha(\lambda - t_1)} \, d\lambda$$

$$= \left. \frac{e^{\alpha(\lambda - t_1)}}{\alpha} \right|_{t_0}^{t_1} + \left. \frac{e^{-\alpha(\lambda - t_1)}}{-\alpha} \right|_{t_1}^{t_2}$$

$$= \frac{1}{\alpha} \left[1 - e^{\alpha(t_0 - t_1)} - \left[e^{-\alpha(t_2 - t_1)} - 1 \right] \right]$$

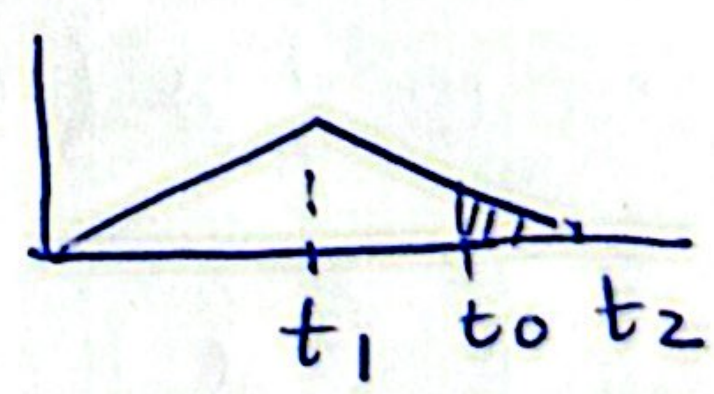
$$= \frac{1}{\alpha} \left[2 - e^{-\alpha(t_0 - t_1)} - e^{-\alpha(t_2 - t_1)} \right]$$

Case II → $t_0 < t_2 < t_1$



$$\int_{t_0}^{t_2} e^{\alpha(\lambda - t_1)} \, d\lambda = \frac{1}{\alpha} \left[e^{\alpha(t_2 - t_1)} - e^{\alpha(t_0 - t_1)} \right]$$

~~Case~~ Case III → $t_1 < t_0 < t_2$



$$\int_{t_0}^{t_2} e^{-\alpha(\lambda - t_1)} \, d\lambda = -\frac{1}{\alpha} \left[e^{-\alpha(t_2 - t_1)} - e^{-\alpha(t_0 - t_1)} \right]$$

~~Case~~ Case IV →

0 elsewhere

c) → yes, this is weekly Stationary random Process as it's ACF depends on τ.

Info.

1. The process is stationary if the ACF does not depend on t or τ.

2. The process is weakly stationary if its ACF depends on τ.

3. The process is non-stationary if it's ACF depends on t.