Task 6.2

a) The mean of the process is related to the long term behaviour of the auto correlation function $\tau \to \infty$ and mean $m_z^{(1)} = 0$ —— (given)

$$m_z^{(1)} = \lim_{\tau \to \infty} S_{zz}(\tau) = 0$$

$$\lim_{\tau \to \infty} k e^{-\alpha|\tau|} + c = 0$$

$$\Rightarrow \quad 0 + c = 0 \qquad \boxed{\therefore \; c = 0}$$

The standard deviation is related to the auto correlation function at $\tau = 0$ and standard deviation $(\sigma_z) = 1$ —— (given)

$$\underset{\text{(varience)}}{\sigma_z^2} = m_z^{(2)} - [m_z^{(1)}]^2$$

$$(1)^2 = m_z^{(2)} - [0]^2$$

$$\therefore \; m_z^{(2)} = 1$$

$$\therefore \; m_z^{(2)} = \lim_{\tau \to 0} S_{zz}(\tau) = 1$$

$$\Rightarrow \lim_{\tau \to 0} k e^{-\alpha|\tau|} + c = 1$$

put $c = 0$

$$\lim_{\tau \to 0} k \cdot e^{-\alpha|\tau|} = 1$$

$$\Rightarrow k \cdot e^{0} = 1 \Rightarrow \boxed{k = 1}$$

b) $S_{zy}(t_1, t_2) = E\{z(\xi, t_1)\, y(\xi, t_2)\}$ — (given)

and,

$$y(\xi, t) = \int_{t_0}^{t} z(\xi, \lambda)\, d\lambda \quad \text{for } t > t_0$$

..... (given)

$$\therefore S_{zy}(t_1, t_2) = E\left\{ z(\xi, t_1) \int_{t_0}^{t} z(\xi, \lambda)\, d\lambda \right\}$$

$$= \int_{t_0}^{t} E\{ z(\xi, t_1) \cdot z(\xi, \lambda) \}\, d\lambda$$

Here, $E\{z(\xi, t_1) \cdot z(\xi, \lambda)\} = S_{zz}(\tau)$

$$\therefore S_{zz}(t_1, t_2) = \int_{t_0}^{t} S_{zz}(\tau)\, d\lambda$$

$$= \int_{t_0}^{t} k \cdot e^{-\alpha|\tau|} + c\, d\lambda$$

Put the values $c = 0$ and $k = 1$

$$\therefore S_{zy}(t_1, t_2) = \int_{t_0}^{t} e^{-\alpha|\tau|} \, d\lambda$$

Here τ is nothing but the time difference which can be shown as $\tau = t_2 - t_1$

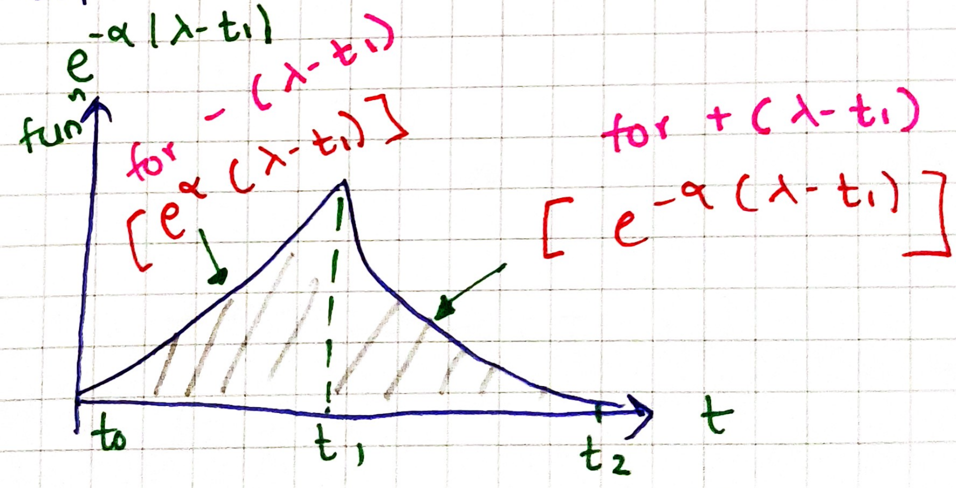
- Here in this case, $\tau = \lambda - t_1$

$$\therefore S_{zy}(t_1, t_2) = \int_{t_0}^{t} e^{-\alpha|\lambda - t_1|} \, d\lambda$$

Here $|\lambda - t_1| \rightarrow$ this will get vary

Case I :- for $t_0 < t_1 < t_2$ and

$|\lambda - t_1| = -(\lambda - t_1)$ and $+(\lambda - t_1)$



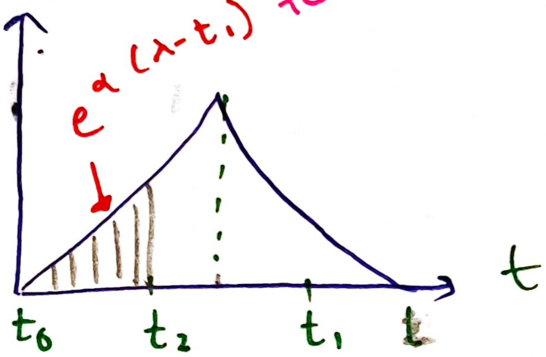
$$\therefore S_{zy}(t_1, t_2) = \int_{t_0}^{t_1} e^{\alpha(\lambda - t_1)} \, d\lambda + \int_{t_1}^{t_2} e^{-\alpha(\lambda - t_1)} \, d\lambda$$

$$\Rightarrow \frac{1}{\alpha}\left[2 - e^{\alpha(t_0 - t_1)} - e^{-\alpha(t_2 - t_1)}\right]$$ ④

Case II :- for $t_0 < t_2 < t_1$

Here in this case integral will apply only on negative part.

fun^n $(e^{-\alpha|\lambda - t_1|})$ $e^{\alpha(\lambda - t_1)}$ for $-(\lambda - t_1)$



$$\therefore S_{zy}(t_1, t_2) = \int_{t_0}^{t_2} e^{\alpha(\lambda - t_1)}\, d\lambda$$

$$\therefore S_{zy}(t_1, t_2) = \frac{1}{\alpha}\left[e^{\alpha(t_2 - t_1)} - e^{\alpha(t_0 - t_1)}\right]$$

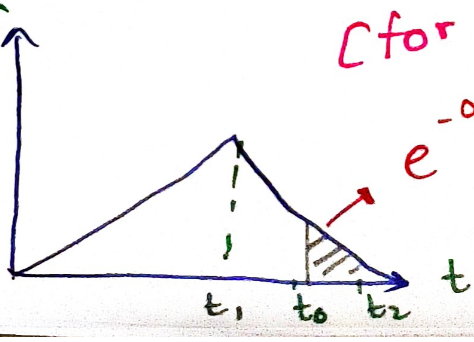
Case III :- for $t_1 < t_0 < t_2$

Now in this case integral will apply only on positive part.

fun

[for $+(\lambda - t_1)$]

$e^{-\alpha(\lambda - t_1)}$



$$\therefore \quad S_{zy}(t_1,t_2) = \int_{t_0}^{t_2} e^{-\alpha(\lambda - t_1)} \, d\lambda$$

$$\therefore \quad S_{zy}(t_1,t_2) = \frac{1}{\alpha}\left[e^{-\alpha(t_0-t_1)} - e^{-\alpha(t_2-t_1)}\right]$$

Case IV :- $\quad 0 \quad ; \quad$ elsewhere

c) $y(\zeta, t)$ is non stationary process as the function $S_{zy}(t_1,t_2)$ depends on time t.