

$$a) \quad P(A) = \sum_{i=1}^n P(A/B_i) \cdot P(B_i)$$

$$\text{let } P(\{X(\xi, t) = x_i\}) = P_i$$

$$\frac{1}{3} (1 + 2e^{-|\tau|}) = a$$

$$\frac{1}{3} (1 - e^{-|\tau|}) = b$$

$$P_1 = a P_1 + b P_2 + b P_3 \rightarrow \textcircled{1}$$

$$P_2 = a P_2 + b P_1 + b P_3 \rightarrow \textcircled{2}$$

$$P_3 = a P_3 + b P_1 + b P_2 \rightarrow \textcircled{3}$$

$$\Rightarrow P_1 - P_2 = a(P_1 - P_2) + b(P_2 - P_1)$$

$$\Rightarrow P_1 - a P_1 + b P_1 = P_2 - a P_2 + b P_2$$

$$\Rightarrow \boxed{P_1 = P_2}$$

$$\textcircled{3} \Rightarrow P_3 - a P_3 = 2b P_1$$

$$P_3 - \frac{1}{3} P_3 - \frac{2}{3} e^{-|\tau|} P_3 = \frac{2}{3} P_1 - \frac{2}{3} e^{-|\tau|} P_1$$

$$\frac{2}{3} P_3 (1 - e^{-|\tau|}) = \frac{2}{3} P_1 (1 - e^{-|\tau|})$$

$$\boxed{P_1 = P_2 = P_3}$$

$$\text{As } P_1 + P_2 + P_3 = 1$$

$$\boxed{P_1 = P_2 = P_3 = 1/3}$$

$$\textcircled{b) } R_{XX}(\tau) = E\{X(\xi, t+\tau), X(\xi, t)\}$$

$$= \sum_{i=1}^3 \sum_{j=1}^3 X_i \cdot X_j P(\{X(\xi, t+\tau) = x_i \mid X(\xi, t) = x_j\})$$

$$P(\{X(\xi, t) = x_j\})$$

$$\begin{aligned}
 &= 1 \cdot 1 \cdot \frac{1}{3} (1 + 2 e^{-|\tau|}) \cdot \frac{1}{3} + (-1)(-1) \cdot \frac{1}{3} (1 + 2 e^{-|\tau|}) \cdot \frac{1}{3} \\
 &\quad + (-1)(1) \cdot \frac{1}{3} (1 - e^{-|\tau|}) \cdot \frac{1}{3} + (1)(-1) \cdot \frac{1}{3} (1 - e^{-|\tau|}) \cdot \frac{1}{3} \\
 &= \frac{2}{9} (1 + 2 e^{-|\tau|}) - \frac{2}{9} (1 - e^{-|\tau|})
 \end{aligned}$$

$$= \frac{6}{9} e^{-|\tau|} = \frac{2}{3} e^{-|\tau|}$$

x_j