Abstract

Given the matrices, explore the relationship between the exponentiating of the matrices and the rotation matrices.

Given three matrices A, B,C .

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathrm{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathrm{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

Then we did a scalar multiplication of matrices by a variable `t`. We then, take the exponential map of the resulting matrices by expanding the power series. Using power series, we are able to compute the exponential map by just using matrix multiplication and addition.

$$exp(M) = I_n + M + \tfrac{1}{2!}M^2 + \tfrac{1}{3!}M^3 + \cdots + \tfrac{1}{m!}M^m + \cdots$$

$$exp(At) = \begin{bmatrix} -0.00138t^6 + 0.0416t^4 - 0.5t^2 + 1 & -0.0083t^5 + 0.16t^3 - t & 0 \\ 0.0083t^5 - 0.16t^3 + t & -0.00138t^6 + 0.0416t^4 - 0.5t^2 + 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$exp(Bt) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.0013t^6 + 0.0416t^4 - 0.5t^2 + 1 & -0.0083t^5 + 0.16t^3 - t \\ 0 & 0.0083t^5 - 0.16t^3 + t & -0.00138t^6 + 0.0416t^4 - 0.5t^2 + 1 \end{bmatrix}$$

$$exp(Ct) = \begin{bmatrix} -0.00138t^6 + 0.0416t^4 - 0.5t^2 + 1 & 0 & 0.0083t^5 - 0.16t^3 + t \\ 0 & 1 & 0 \\ -0.0083t^5 + 0.16t^3 - t & 0 & -0.00138t^6 + 0.0416t^4 - 0.5t^2 + 1 \end{bmatrix}$$

We have three exponentials of the three matrices. We observe that the resulting matrices are the matrices for rotation.so we get three different types of rotations.Then, we take the derivatives of these matrices.

$$\frac{d}{dt}exp(tA) = \gamma_a = \begin{bmatrix} -\sin(t) & -\cos(t) & 0 \\ \cos(t) & -\sin(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{d}{dt}exp(tB) = \gamma_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin(t) & -\cos(t) \\ 0 & -\sin(t) & \cos(t) \end{bmatrix}$$

$$\frac{d}{dt}exp(tc) = \gamma_c = \begin{bmatrix} -\sin(t) & -\sin(t) & 0 \\ 0 & 0 & 0 \\ \sin(t) & 0 & \cos(t) \end{bmatrix}$$

$$\frac{d}{dt}exp(tQ) = \begin{bmatrix} -\frac{1}{4}\cos(t) - 2\cos(2t) + \frac{9}{4}\cos(3t) & -\frac{1}{4}\sin(t) + 2\sin(2t) - \frac{9}{4}\sin(3t) \\ 7\sin(t) - \sin(2t) - 2\sin(4t) - \frac{9\sqrt{2}}{2}\sin\left(t + \frac{\pi}{4}\right) + \frac{9\sqrt{2}}{4}\sin\left(3t + \frac{\pi}{4}\right) + 2\cos(t) + \frac{9\sqrt{2}}{4}\cos\left(t + \frac{\pi}{4}\right) & -4(-\cos(2t) + 1)^2 - \frac{\sqrt{2}}{4}\sin\left(t + \frac{\pi}{4}\right) - 10\cos(2t) + \frac{9\sqrt{2}}{4}\cos\left(3t + \frac{\pi}{4}\right) + 6 \\ 7\sin(t) - \sin(2t) - 2\sin(4t) - \frac{9\sqrt{2}}{4}\sin\left(t + \frac{\pi}{4}\right) - 2\cos(t) + \frac{9\sqrt{2}}{2}\cos\left(t + \frac{\pi}{4}\right) - \frac{9\sqrt{2}}{4}\cos\left(3t + \frac{\pi}{4}\right) & -4(-\cos(2t) + 1)^2 + \frac{9\sqrt{2}}{4}\sin\left(3t + \frac{\pi}{4}\right) - 10\cos(2t) - \frac{\sqrt{2}}{4}\cos\left(t + \frac{\pi}{4}\right) + 6 \end{bmatrix}$$

The derivative of the exponentials evaluated t= 0 gives the original matrices. When we multiply the negative exponentials and the postive exponentials of the matrices, we get the Q matrix which is a rotation of 90 degree with respect to x-axis.

We take the derivatives of the rotation matrices γ_a , γ_b , γ_c and Q. These evaluated at t= 0 gives us the tangent vectors of the specific rotations

Lastly, we verified that the product of [A,B] = AB-BA = C

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

Unexpected result: The rotation matrices when taking the exponent of the given matrices.

Challenge : To take the derivatives in sympy and interpret what it meant on the matrices transformation