$$③\; T(n) = 1 + \sum_{i=\cancel{x}}^{n} 1 + \max\left(2, \sum_{j=\cancel{3x}}^{\cancel{3}y} \left(\sum_{j=3x}^{3y} 1\right) + 1 + T\left(\frac{n}{2}\right) + 1\right)$$

$$T(n) = 4 + n + 3n^2 + T\left(\frac{n}{2}\right)$$

$n = 2^k \quad T(2^k) = 4 + 2^k + 2^{2k} \cdot 3 + T(2^{k-1})$

$T(2^k) \equiv x^k \quad x^k = 4 + 2^k + \cancel{4}^k \cdot 3 + x^{k-1}$

HOMOGÉNEA

$x^k - x^{k-1} = 0$

$x^{k-1}(x-1) = 0$

$x = 1$

$x^H = A \cdot 1^n = A$

PARTICULAR

$x^{k-1}(x-1) = \cancel{4}\, 4 + 2^k + \cancel{4}^k \cdot 3$

$x = 2 \quad x = 4$

$x^P = B \cdot 2^k + C \cdot 4^k$

$x = A + 2^k B + 4^k C$

$x = A + Bn + Cn^2$

$$\Downarrow$$

$$\boxed{O(n^2)}$$

$⑨\; T(n) = 1$

$⑨\; T(n) = \max(1, 1 + T(n-1))$

$T(n) = 1 + T(n-1)$

HOMOGÉNEA

$T(n) \equiv x^n \quad x^n - x^{n-1} = 0$

$x^{n-1}(x-1) = 0$

$x = 1$

$x^H = A \cdot 1^n = A$

PARTICULAR

$x^n = 1 + x^{n-1} \qquad x = 1$

$x^n - x^{n-1} = 1 * 1^n \qquad x^P = B$

Al ser la raíz uno, habrá que multiplicar a la particular por n

$x^P = B \cdot n$

$x = x^H + x^P = A + Bn$

$\boxed{\text{Complejidad}: O(n)}$