FIX SPEED (SQUIRREL CAGE) WIND TURBINES

PROBLEM 1. MECHANICAL POWER COMPUTATION FOR FIX SPEED WIND TURBINES.

A 3.2 MW wind turbine with rotor diameter of *D*=100 m and gearbox ratio of N_{gb}=80 based on an induction generator (with 2 pole pairs) is connected to a 960 V 50 Hz grid . It can be assumed an air density of ρ=1.225 kg/m³ and a power coefficient expression of

$$C_p = 0.0045\left(100 - (\lambda - 10)^2\right)$$

Neglecting slip (assuming s=0). Calculate the mechanical power generated for wind speeds of 5, 8, 11 and 14 m/s.

The mechanical power generated can be found using

$$P_{mech} = \frac{1}{2}\rho C_p A v_w^3$$

with air density ρ=1.225 kg/m³, areas swept by the rotor $A = \pi D^2/4$ with diameter *D*=100 m and power coefficient $C_p = 0.0045\left(100 - (\lambda - 10)^2\right)$, where the tip speed ratio λ can be computed as

$$\lambda = \frac{\omega_t D}{2v_w} = \frac{\omega_g D}{2N_{gb} v_w}$$

Where ω_t is wind turbine speed (slow axis), ω_g is the generator speed (fast axis), and N_{gb} is the gearbox ratio.

The previous expressions can be combined as

$$P_{mech} = \frac{0.0045}{2}\rho\left(100 - (\lambda - 10)^2\right) A v_w^3$$

$$P_{mech} = \frac{0.0045}{2}\rho\left(100 - \left(\frac{\omega_g D}{2N_{gb} v_w} - 10\right)^2\right) v_w^3 \pi D^2/4$$

where all the quantities are known except the generator speed ω_g.

The generator speed depends mainly on the slip *s* and the synchronous speed ω_s :

$$\omega_g = (1-s)\omega_s = (1-s)\frac{2\pi f}{p}$$

where *p* are pairs of poles. Assuming s=0, $\omega_g = \omega_s = 2\pi 50/p = \pi 50$ rad/s

Therefore, the mechanical power can be computed as

$$P_{mech} = \frac{0.0045}{2}\rho\left(100-\left(\frac{\omega_s D}{2N_{gb}v_w}-10\right)^2\right)v_w^3\pi D^2/4$$

$$P_{mech} = \frac{0.0045}{2}1.225\left(100-\left(\frac{\pi 50\times 100}{2\times 80\times v_w}-10\right)^2\right)v_w^3\pi 100^2/4$$

The following resultants are obtained:

ω_e = 3.141592653589793e+002 rad/s
ω_s = 1.570796326794897e+002 rad/s
ω_g = 1.570796326794897e+002 rad/s
ω_t = 1.963495408493621 rad/s
λ = 19.634954084936204
C_p = 0.032254469015270
P = 1.939527243080785e+004 W

The calculations can be extended to the other wind speeds, obtaining:

v_w=5	8	11	14	m/s
ω_s = 157.0796	157.0796	157.0796	157.0796	rad/s
ω_g = 157.0796	157.0796	157.0796	157.0796	rad/s
ω_t = 1.9635	1.9635	1.9635	1.9635	rad/s
λ = 19.6350	12.2718	8.9250	7.0125	
C_p = 0.0323	0.4268	0.4448	0.4098	
P = 1.9395e+004	1.0511e+006	2.8480e+006	5.4099e+006	W
			P=3.2 MW Pitch!!	

where it can be noted that when the mechanical power exceeds the nominal power (3.2 MW), the pitch system must reduce the mechanical power to its nominal value.

Develop a Matlab program to solve the previous exercise.

Code:

```
clear;clc;
vw=[5 8 11 14]
ss=0;rho=1.225;N=80;D=100;poles=2;Ugrid=960;f=50;

we=2*pi*f;
ws=we/poles;
wg=ws*(1-ss);
wt=wg/N;

lam= wt*(D/2) ./ vw
Cpp= 0.0045 * (100 - (lam-10).^2)
P1=0.5*rho*Cpp*(pi*(D^2)/4).*vw.^3
```

Results:

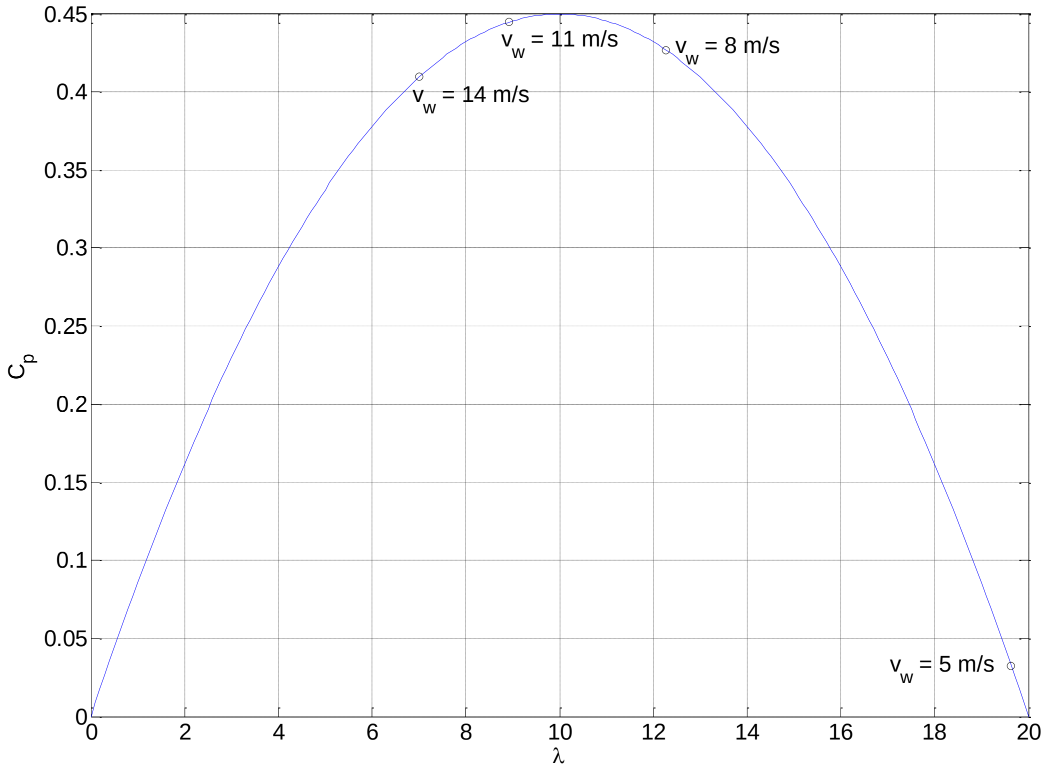
```
vw =      5      8     11     14
lam =   19.634954084936204  12.271846303085129   8.924979129516457
7.012483601762931
Cpp =    0.032254469015270   0.426774214688213   0.444799485576112
0.409836355966191
P1 =   1.0e+006 *
   0.019395272430808   1.051148578283138   2.847988990930852
5.409916510373950
```

Develop a Matlab program to locate the operating points in the Cp- λ curve.

Code:

```
%% Develop a Matlab program to locate the  operating points in the Cp-
lambda curve.

lam1=0:.1:20;
Cpp1= 0.0045 * (100 - (lam1-10).^2);
plot(lam1,Cpp1);hold on;grid on;
plot(lam,Cpp,'ko');
for ii=1:1:4
    txt{ii}=['v_w = ' num2str(vw(ii)) ' m/s'];
    text(lam(ii),Cpp(ii),txt{ii},'FontSize',18);
end;
xlabel('\lambda','FontSize',18);
ylabel('C_p','FontSize',18);
```

Results:

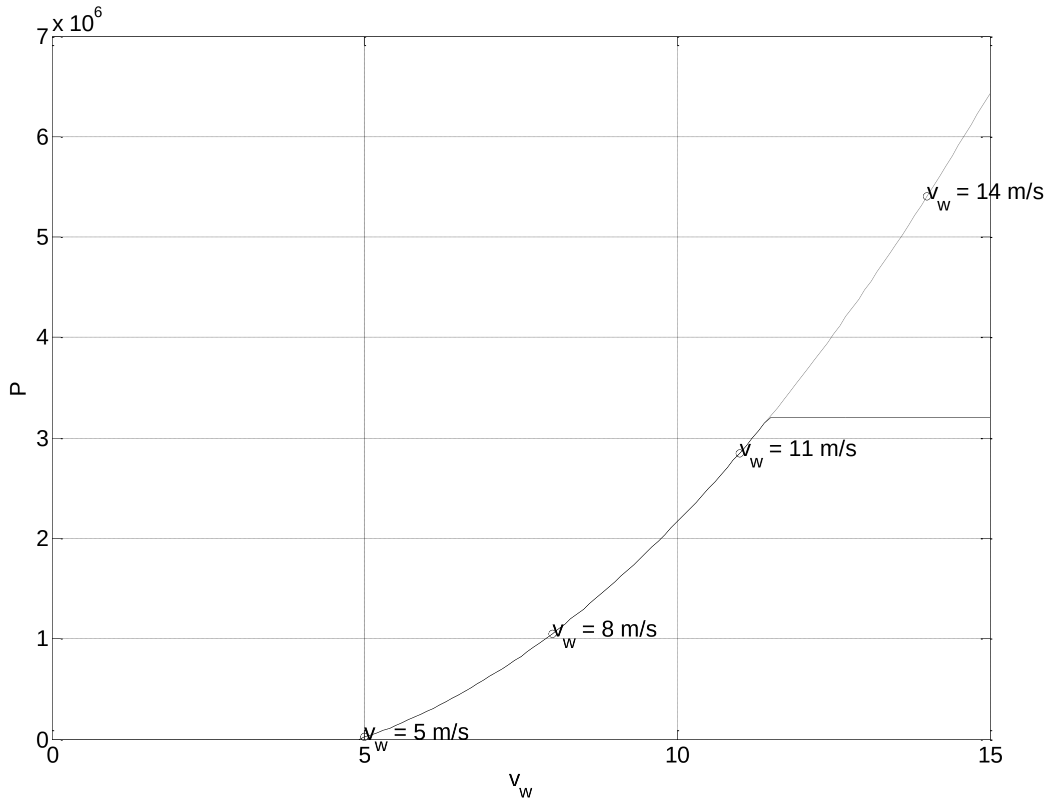


Develop a Matlab program to plot the power generated for different wind speeds.

Code:

```
%% Develop a Matlab program to plot the power generated for different wind
%% speeds. Include a comparison between the obtained power and the maximum
available power.
figure(2);
vw2=0:.1:15
lam2= min(20,max(wt*(D/2) ./ vw2,0));
Cpp2= 0.0045 * (100 - (lam2-10).^2)
P2=0.5*rho*Cpp2*(pi*(D^2)/4).*vw2.^3;
P2a=min(0.5*rho*Cpp2*(pi*(D^2)/4).*vw2.^3,Pn);

h=subplot(1,1,1);
plot(vw2,P2,':k');hold on;grid on;
plot(vw2,P2a,'k');
plot(vw,P1,'ko');
for ii=1:1:4
    txt{ii}=['v_w = ' num2str(vw(ii)) ' m/s'];
    text(vw(ii),P1(ii),txt{ii},'FontSize',18);
end;
xlabel('v_w','FontSize',18);
ylabel('P','FontSize',18);
set(h,'FontSize',18);
```

Results:

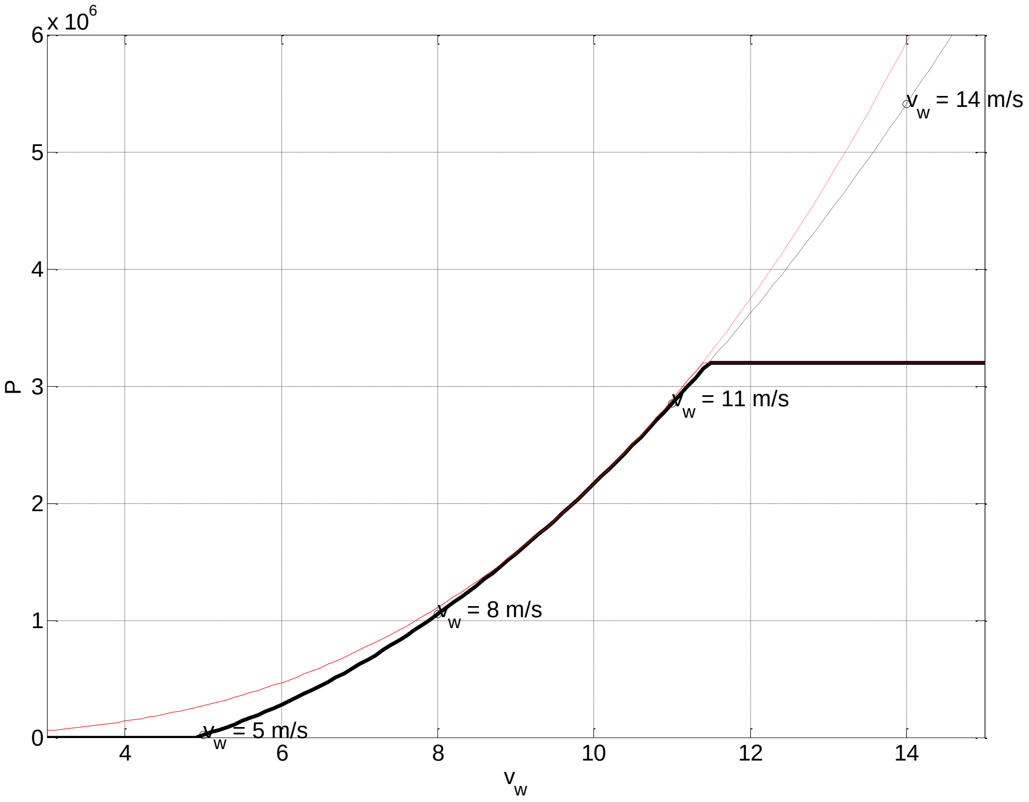


Include a comparison between the obtained power and the maximum available power.

Code:

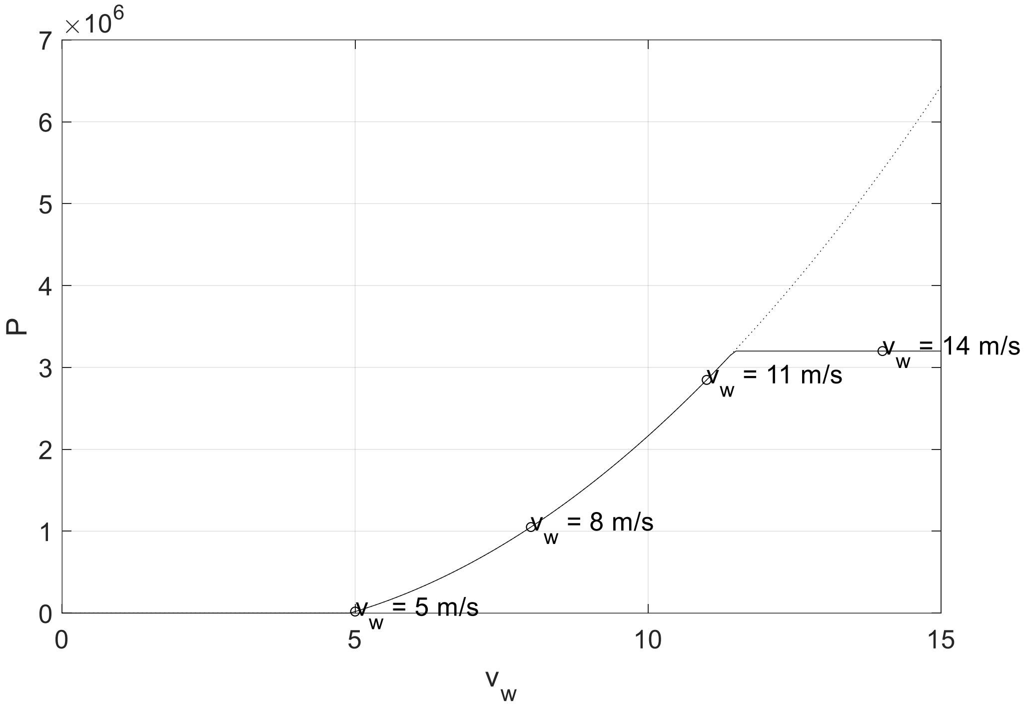
```
figure(3);
lam2b= 10;
Cpp2b= 0.45;
P3=0.5*rho*Cpp2b*(pi*(D^2)/4).*vw2.^3;
P3a=min(0.5*rho*Cpp2b*(pi*(D^2)/4).*vw2.^3,Pn);

h=subplot(1,1,1);
plot(vw2,P2,':k');hold on;grid on;
plot(vw2,P2a,'k','LineWidth',3);
plot(vw,P1,'ko');
plot(vw2,P3,':r');
plot(vw2,P3a,'r');

for ii=1:1:4
    txt{ii}=['v_w = ' num2str(vw(ii)) ' m/s'];
    text(vw(ii),P1(ii),txt{ii},'FontSize',18);
end;
xlabel('v_w','FontSize',18);
ylabel('P','FontSize',18);
set(h,'FontSize',18);
axis([3 15 0 6e6]);
```

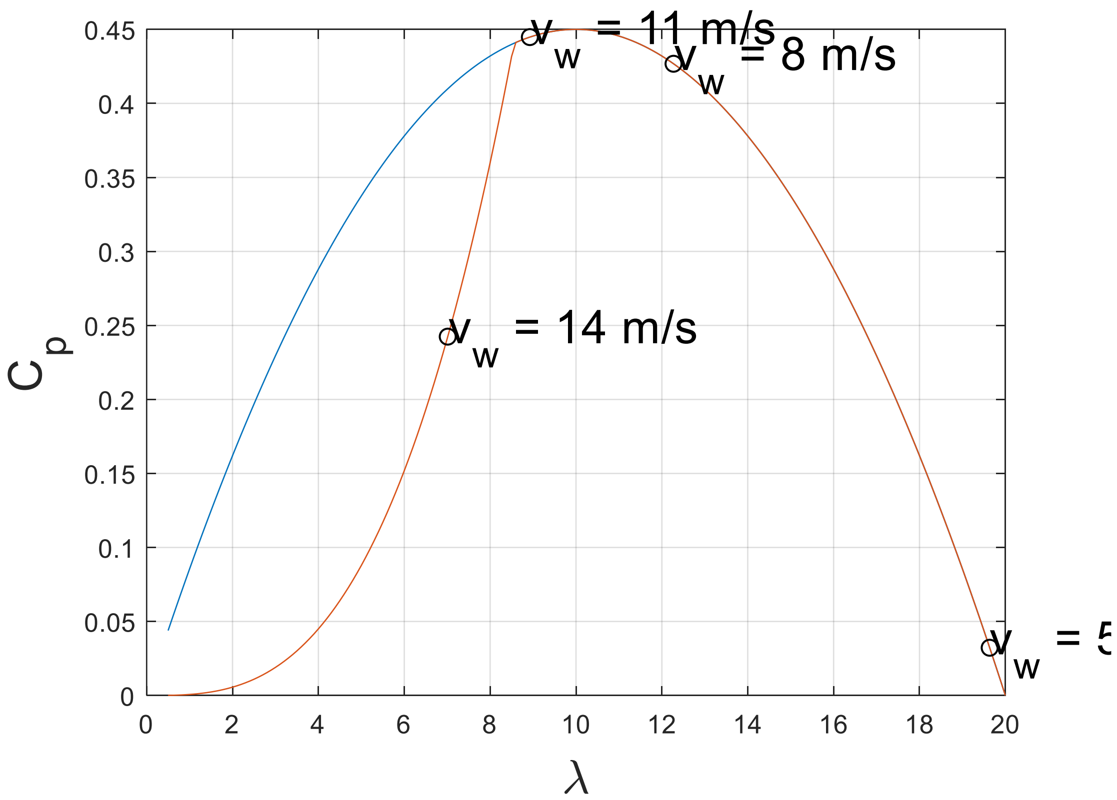
Results:



How should we modify the scripts in order to consider appropriately the maximum power limitation (with implied pitch control to limit the power)?

Results





PROBLEM 2. MECHANICAL AND ELECTRICAL ANALYSIS OF FIX SPEED WIND TURBINES.

A 3.2 MW wind turbine with rotor diameter of *D*=100 m and gearbox ratio of N_{gb}=80 based on an induction generator (with the parameters of the table and 2 pairs of poles) is connected to a 960 V 50 Hz grid .

R_s=0.015 Ω	X_s=0.1 Ω	R_m=50 Ω	X_m=8 Ω	R_r=0.01 Ω	X_r=0.1 Ω

It can be assumed an air density of *ρ*=1.225 kg/m³ and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

Calculate the mechanical power generated for wind speed 8 m/s and the corresponding electrical active and reactive power exchanged with the grid.

Mechanical power calculation

The rationale of Problem 1 can be followed with the only difference of not assuming s=0. The mechanical power can be expressed as

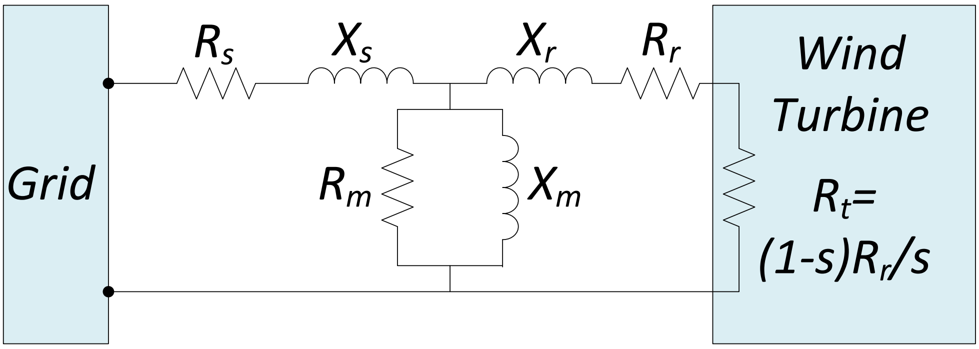
$$P_{mech} = \frac{0.0045}{2}\rho\left(100-\left(\frac{\omega_g D}{2N_{gb}v_w}-10\right)^2\right)v_w^3\pi D^2/4$$

$$P_{mech} = \frac{0.0045}{2}\rho\left(100-\left(\frac{(1-s)\frac{2\pi f}{p}D}{2N_{gb}v_w}-10\right)^2\right)v_w^3\pi D^2/4$$

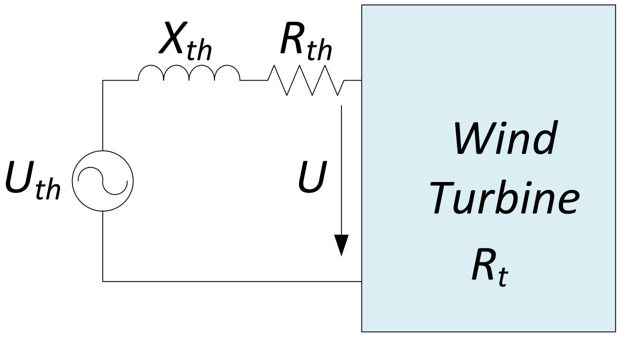
where it is clearly shown that for each wind speed a different generated power will be obtained depending on the slip *s*.

Electrical analysis

The equivalent one-phase model of an induction generator shown in the Figure can be used for the electrical analysis.



Neglecting mechanical losses, the power generated in the wind turbine is known from the previous analysis and corresponds to the power “generated” by the equivalent resistance R_t. The grid voltage is known, but the current must be calculated from the circuit analysis. This is a typical problem of two node power flow which can be solved using the Thevenin equivalent seen from the resistance R_t.



Using $\underline{Z}_{sm} = R_m // \mathrm{j} X_m = \dfrac{\mathrm{j} R_m X_m}{R_m + \mathrm{j} X_m}$, the Thevenin equivalent impedance can be found:

$$\underline{Z}_{th} = R_{th} + \mathrm{j} X_{th} = \left(R_s + \mathrm{j} X_s\right) // \underline{Z}_{sm} + \left(R_r + \mathrm{j} X_r\right)$$

The Thevenin equivalent voltage can be found:

$$\underline{U}_{th} = \frac{\underline{U}_{pn-grid}}{\left(R_s + \mathrm{j} X_s\right) + \underline{Z}_{sm}} \underline{Z}_{sm}$$

Where $\underline{U}_{pn-grid}$ is the phase to neutral voltage. Once the Thevenin equivalent is known, the voltage U can be found from

$$U^4 + U^2\left(2R_{th}P + 2X_{th}Q - U_{th}^2\right) + \left(R_{th}^2 + X_{th}^2\right)\left(P^2 + Q^2\right) = 0$$

Where the power P and Q are the **one-phase** active and reactive power exchanged between the resistance R_t and the equivalent circuit using the load convention (P>0 load, P<0 generation). It can be noted that P will be the mechanical power generated (negative, one phase) $P_{1phase} = -P_{mech}/3$ and Q will be 0 since the model is a pure resistance.

$$U^4 + U^2\left(2R_{th}P_{1phase} - U_{th}^2\right) + \left(R_{th}^2 + X_{th}^2\right)P_{1phase}^2 = 0$$

Once the voltage U is known, the resistance R_t will be found

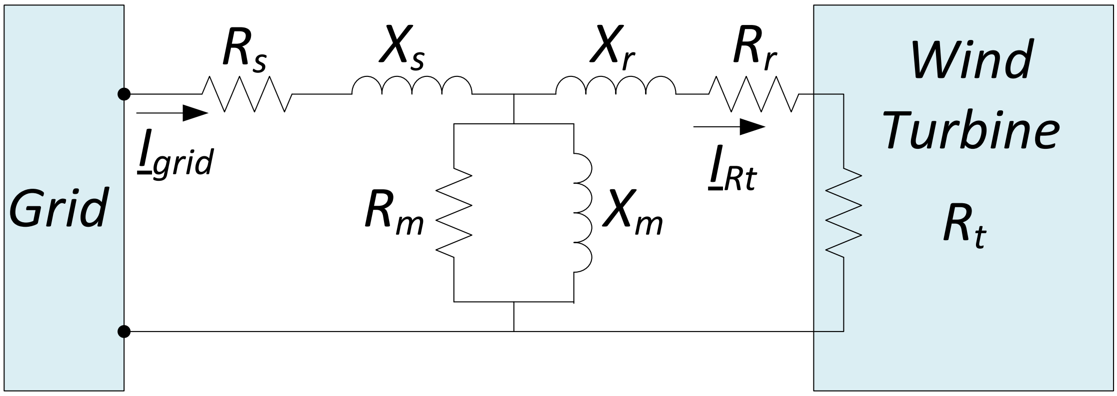
$$R_t = \frac{U^2}{P_{1phase}}$$

And the slip

$$s = \frac{R_r}{R_r + R_t}$$

The resistance R_t current can be calculated

$$\underline{I}_{Rt} = \frac{\underline{U}_{\text{th}}}{\underline{Z}_{\text{th}} + R_t}$$



The grid current can be calculated using different approaches. For example

$$\underline{I}_{grid} = \frac{\underline{U}_{pn-grid}}{\left(R_s + \mathrm{j}X_s\right) + \underline{Z}_{sm} // \left(R_t + R_r + \mathrm{j}X_r\right)}$$

The power exchanged with the grid yields:

$$\underline{S}_{grid} = 3\underline{U}_{pn-grid}\underline{I}^*_{grid}$$

<u>Iterative solution</u>

In the mechanical analysis it has been shown that for given wind conditions the mechanical power depends on the slip. In the electrical analysis it has been shown that for each generated power, the equivalent resistance and slip can be calculated.

An iterative solution procedure can be applied to the case of wind speed 8 m/s.

Implementation in Matlab for vw=8 m/s

```
clc;clear;
% Parameters
rho=1.225;N=80;D=100;poles=2;Ugrid=960;f=50;
Rs=0.015;Xs=0.1;Rm=50;Xm=8;Rr=0.01;Xr=0.1;
```

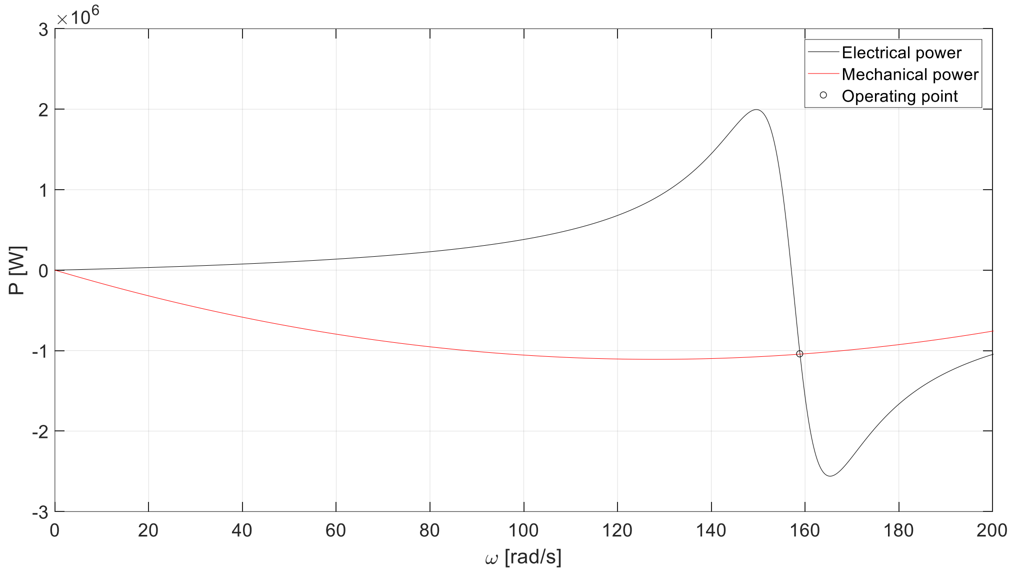
```
% Equivalent model
Zs=Rs+j*Xs;Zm=Rm*(j*Xm)/(Rm+j*Xm);Zr=Rr+j*Xr;
Zsm=Zs*Zm/(Zs+Zm);
Zth=Zr+Zsm;
V=Ugrid/sqrt(3);
Uth= Zm*V/(Zm+Zs);
Uu=abs(Uth);

vw=8;

% Initialization
ss=0;kk=0;err=1;P1=0;

while ((kk<100)&&(err>1e-6))

    kk=kk+1;

    we=2*pi*f;ws=we/poles;wg=ws*(1-ss);wt=wg/N;
    lam= wt*(D/2) / vw;
    Cpp= 0.0045 * (100 - (lam-10)^2);
    P1a=0.5*rho*Cpp*(pi*(D^2)/4)*vw^3;
    err=abs(P1a-P1);
    P1=P1a;
    Pmech=-P1/3;
    a1=1;a2=2*real(Zth)*Pmech-Uu^2;a3=abs(Zth)^2*(Pmech^2);
    sol=roots([a1 0 a2 0 a3]);
    Rt=sol.^2/Pmech;
    s=Rr./(Rr+Rt);
    ss=s(2);
end;

P1
ss
Igrid = V/(Zs+Zm*(Zr+Rt(2))/(Zm+(Zr+Rt(2))))
Sgrid= 3*V*conj(Igrid)
losses=real(Sgrid)+P1
```

Results:

```
P1 =   1.0437e+06

ss =   -0.0117

Igrid =  -5.9875e+02 - 2.1262e+02i

Sgrid =   -9.9558e+05 + 3.5354e+05i

losses =   4.8108e+04
```

```
ssaux=-1:.00001:1;
Iaux=Uth./(Zth+Rr*(1-ssaux)./ssaux);
Paux=3*abs(Iaux).^2.*Rr.*(1-ssaux)./ssaux;
wgaux=ws*(1-ssaux);
wtaux=wgaux/N;
lamaux= wtaux*(D/2) / vw;
Cppaux= 0.0045 * (100 - (lamaux-10).^2);
Pauxmec=0.5*rho*Cppaux*(pi*(D^2)/4)*vw^3;

figure(1);
```

```
h=subplot(1,1,1);
plot(wgaux,Paux,'k');hold on;grid on;
plot(wgaux,-Pauxmec,'r');
plot(wg,-P1,'ok');
xlabel('\omega [rad/s]','FontSize',18);
ylabel('P [W]','FontSize',18);
legend('Electrical power','Mechanical power','Operating point');
set(h,'FontSize',18);
axis([0 200 -3e6 3e6]);
```



PROBLEM 3. MECHANICAL AND ELECTRICAL ANALYSIS OF FIX SPEED WIND TURBINES.

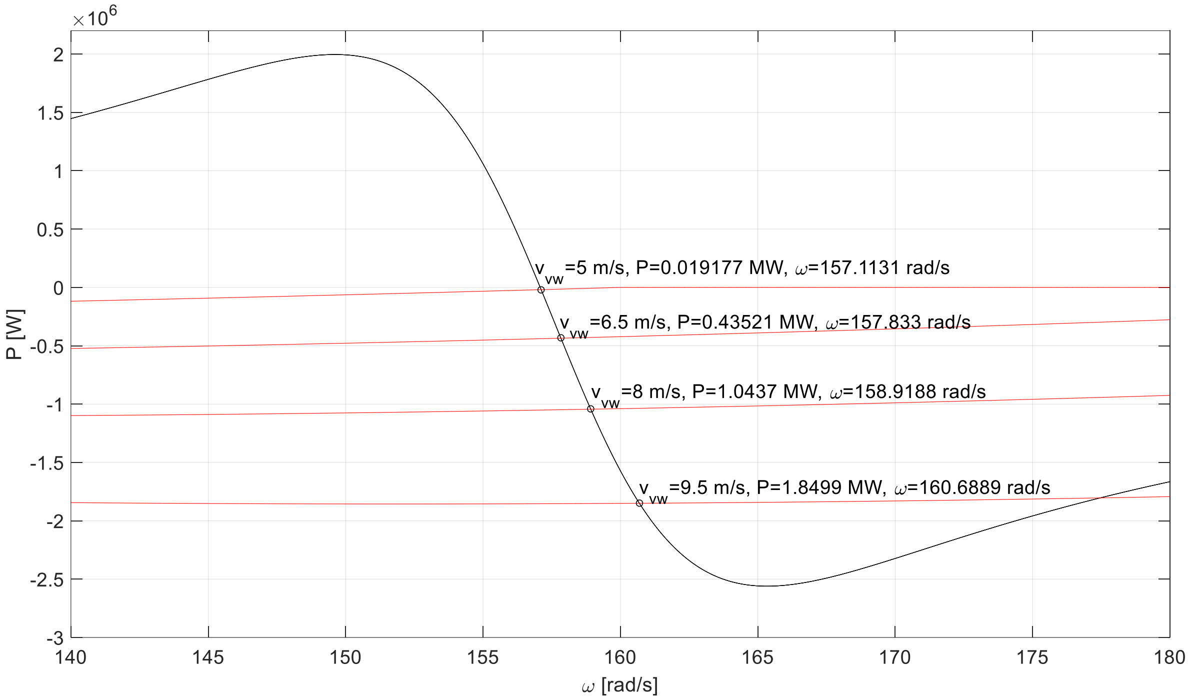
A 3.2 MW wind turbine with rotor diameter of D=100 m and gearbox ratio of N_{gb}=80 based on an induction generator (with the parameters of the table and 2 pairs of poles) is connected to a 960 V 50 Hz grid .

R_s=0.015 Ω	X_s=0.1 Ω	R_m=50 Ω	X_m=8 Ω	R_r=0.01 Ω	X_r=0.1 Ω

It can be assumed an air density of ρ=1.225 kg/m^3 and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

Calculate the equibrium point for wind speeds 5, 6.5, 8 and 9.5 m/s.



PROBLEM 4. MECHANICAL AND ELECTRICAL ANALYSIS OF FIX SPEED WIND TURBINES.

A 3.2 MW wind turbine with rotor diameter of D=100 m and gearbox ratio of N_{gb}=80 based on an induction generator (with the parameters of the table and 2 pairs of poles) is connected to a 960 V 50 Hz grid .

R_s=0.015 Ω	X_s=0.1 Ω	R_m=50 Ω	X_m=8 Ω	R_r=0.01 Ω	X_r=0.1 Ω

It can be assumed an air density of ρ=1.225 kg/m^3 and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

Calculate the mechanical power generated for wind speed of 9 m/s and the corresponding electrical active and reactive power exchanged with the grid.

```
P1 =    1.5587e+06
ss =   -0.0184
Igrid =   -8.8453e+02 - 4.0470e+02i
Sgrid =   -1.4708e+06 + 6.7293e+05i
losses =    8.7936e+04
```

In the previous problem, recalculate everything for a voltage increase to 1100 V.

```
P1 =    1.5605e+06
ss =   -0.0135
wg =  159.1960
Igrid =  -7.7956e+02 - 2.9458e+02i
Sgrid =  -1.4853e+06 + 5.6126e+05i
losses =    7.5270e+04
```

In the previous problem, recalculate everything for a voltage of 960 V and frequency of 55 Hz.

```
P1 =    1.5010e+06
ss =  -0.0176
wg =   175.8247
Igrid =   -8.5316e+02 - 3.7809e+02i
Sgrid =   -1.4186e+06 + 6.2867e+05i
losses =    8.2418e+04
```

Try with other voltages and frequencies:

- **A frequency increase implies a power increase or decrease?**
- **A voltage increase implies a reactive power increase or decrease?**
- **A frequency increase implies a reactive power increase or decrease?**

VARIABLE SPEED WIND TURBINES

PROBLEM 5. OPTIMUM POWER CALCULATION.

A 3.2 MW variable speed wind turbine with rotor diameter of D=100 m and gearbox ratio of N_{gb}=80 based on an induction generator is connected to a 960 V 50 Hz grid . It can be assumed an air density of ρ=1.225 kg/m^3 and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

Calculate the mechanical power generated for wind speeds of 5, 8 and 11 m/s.

The mechanical power generated can be found using

$$P_{mech} = \frac{1}{2}\rho C_p A v_w^3$$

with air density ρ=1.225 kg/m^3, areas swept by the rotor $A=\pi D^2/4$ with diameter D=100 m and power coefficient $C_p = 0.0045\left(100-(\lambda-10)^2\right)$, where the tip speed ratio λ can be computed as

$$\lambda = \frac{\omega_t D}{2v_w} = \frac{\omega_g D}{2N_{gb}v_w}$$

Where ω_t is wind turbine speed (slow axis), ω_g is the generator speed (fast axis), and N_{gb} is the gearbox ratio.

The previous expressions can be combined as

$$P_{mech} = \frac{0.0045}{2}\rho\left(100-(\lambda-10)^2\right)Av_w^3$$

$$P_{mech} = \frac{0.0045}{2}\rho\left(100-\left(\frac{\omega_g D}{2N_{gb}v_w}-10\right)^2\right)v_w^3\pi D^2/4$$

where all the quantities are known except the generator speed ω_g.

The main difference with Problem 1 is that in the presence exercise a variable speed wind turbine is considered. Therefore, the wind turbines speed depends on the optimal operation point and not in the electrical grid frequency.

The optimal operation point can be found by differentiating the C_p expression:

$$\frac{dC_p}{d\lambda}=\frac{d\left[0.0045\left(100-(\lambda-10)^2\right)\right]}{d\lambda}=-0.0045\times 2(\lambda-10)=-0.009(\lambda-10)$$

$$\frac{dC_p}{d\lambda}=0\rightarrow\lambda_{opt}=10$$

$$\frac{d^2C_p}{d\lambda^2}=-0.009<0\rightarrow\lambda_{opt}=10 \text{ Maximum}$$

The wind turbine speed can be calculated as

$$\omega_t=\frac{2\lambda_{opt}v_w}{D}$$

The generator speed:

$$\omega_g=N\omega_t$$

The following resultants are obtained:

v_w	5	8	11	m/s
λ_{opt}	10	10	10	
C_p	0.4500	0.4500	0.4500	
ω_t	1	1.6000	2.2000	rad/s
P_{mech}	.27059	1.1084	2.8813	MW

What is the wind speed that reaches nominal power? (Assuming two operation regions, partial power and maximum power)

The wind speed can be calculated using

$$P_{mech-nom}=\frac{1}{2}\rho C_{p-\max}Av_w^3$$

$$v_w=\sqrt[3]{\frac{2P_{mech-nom}}{\rho C_{p-\max}A}}=11.391489041664725 \text{ m/s}$$

Above this wind speed, how is the wind turbine operated? Calculate the turbine speed, the power coefficient, tip speed ratio and mechanical power generated for wind speeds of 14 m/s.

The wind turbine speed is maintained constant at the nominal value and the pitch system is used to maintain the power at the nominal value.

It can be assumed that the nominal speed is the turbine speed for the wind speed of the previous section. Therefore

$$\omega_t = \frac{2\lambda_{opt} v_w}{D} = 2.278297808332945 \text{ rad/s}$$

For a wind speed above 11.39, the mechanical power will be the nominal power of 3.2 MW. The Cp for a wind speed of 14 m/s can be calculated from

$$P_{mech} = \frac{1}{2}\rho C_p A v_w^3$$

$$C_p = \frac{2P_{mech}}{\rho A v_w^3} = 0.242420809374220$$

The tip speed ratio cannot be obtained from $C_p = 0.0045\left(100-(\lambda-10)^2\right)$, since this expression is not considering the pitch angle. As the wind turbine rotational speed has been previously calculated, the tip speed ratio can be also calculated as

$$\lambda = \frac{\omega_t D}{2v_w} = 8.136777886903376$$

PROBLEM 6. WIND TURBINE BASED ON DOUBLY FED INDUCTION GENERATOR

A 4.7 MW wind turbine with rotor diameter of D=110 m and gearbox ratio of N_{gb}=87 based on a doubly fed induction generator (2 pairs of poles) is connected to a 960 V 50 Hz grid . The DFIG power converter is rated to 1.33 MW both in the rotor and grid sides.

It can be assumed an air density of ρ=1.225 kg/m^3 and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

For a wind speed 12 m/s, calculate the mechanical power generated, the generator speed and slip, and the approximate power in the stator and rotor of the machine.

The optimal operation point can be found by differentiating the C_p expression:

$$\frac{dC_p}{d\lambda} = \frac{d\left[0.0045\left(100-(\lambda-10)^2\right)\right]}{d\lambda} = -0.0045\times 2(\lambda-10) = -0.009(\lambda-10)$$

$$\frac{dC_p}{d\lambda} = 0 \rightarrow \lambda_{opt} = 10$$

$$\frac{d^2C_p}{d\lambda^2} = -0.009 < 0 \rightarrow \lambda_{opt} = 10 \text{ Maximum}$$

The power coefficient

$$C_p = 0.0045\left(100-(\lambda_{opt}-10)^2\right) = 0.45$$

As it is a variable speed wind turbine, for wind speed of 12 m/s, the wind turbine speed can be calculated as

$$\omega_t = \frac{2\lambda_{opt}v_w}{D} = 2.1818181818 \text{ rad/s}$$

The generator speed

$$\omega_g = N_{gb}\omega_t = 189.81818 \text{ rad/s}$$

The generator slip

$$s = \frac{\omega_s-\omega_g}{\omega_s} = \frac{\frac{2\pi f}{p}-\omega_g}{\frac{2\pi f}{p}} = \text{-0.20842}$$

The generated mechanical power yields

$$P_{mech} = \frac{1}{2}\rho C_p A v_w^3 = 4.52624 \text{ MW}$$

The power in stator can be approximated as

$$P_s = \frac{P_{mech}}{1-s} = 3.7456\ \text{MW}$$

The power in the rotor can be approximated as

$$P_r = \frac{-sP_{mech}}{1-s} = 780.65\ \text{kW}$$

PROBLEM 7. WIND TURBINE BASED ON FULL POWER CONVERTER

A PMSG direct drive 7 MW wind turbine with rotor diameter of *D*=120 m is connected to a 960 V 50 Hz grid . The power converter is rated to 8 MVA in the machine and grid sides.

R_g=0.002 Ω	**L_g=80 uH**	**60 Pole pairs**	**E(phase-neutral)=200 ω_g**	**Grid inductance X=.0175 Ω R=.005 Ω**

It can be assumed an air density of *ρ*=1.225 kg/m^3 and a power coefficient expression of

$$C_p = 0.0045\left(100-(\lambda-10)^2\right)$$

Calculate the mechanical power generated for a wind speed 12 m/s and the electrical frequency, active and reactive power exchanged with the machine side converter.

The optimal operation point can be found by differentiating the C_p expression:

$$\frac{dC_p}{d\lambda} = \frac{d\left[0.0045\left(100-(\lambda-10)^2\right)\right]}{d\lambda} = -0.0045\times 2(\lambda-10) = -0.009(\lambda-10)$$

$$\frac{dC_p}{d\lambda} = 0 \rightarrow \lambda_{opt} = 10$$

$$\frac{d^2C_p}{d\lambda^2} = -0.009 < 0 \rightarrow \lambda_{opt} = 10\ \text{Maximum}$$

The power coefficient

$$C_p = 0.0045\left(100-(\lambda_{opt}-10)^2\right) = 0.45$$

As it is a variable speed wind turbine, for wind speed of 12 m/s, the wind turbine (and generator) speed can be calculated as

$$\omega_g = \omega_t = \frac{2\lambda_{opt}v_w}{D} = 2\ \text{rad/s}$$

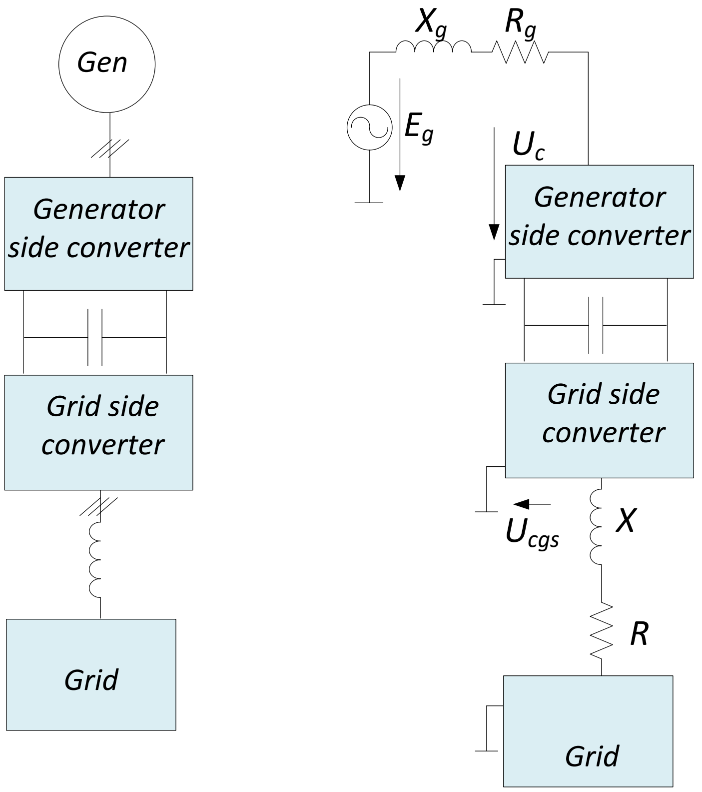
The generated mechanical power yields

$$P_{mech} = \frac{1}{2}\rho C_p A v_w^3 = 5.3866 \text{ MW}$$

The electrical frequency can be calculated as

$$f_e = p\frac{\omega_g}{2\pi} = 19.1 \text{ Hz}$$

The system under analysis is shown in the figure below. It can be noted that several energy conversions will be produced before injecting the power to the main grid.



The generator internal voltage

$$\underline{E}_g = 200\omega_g = 400 \text{ V}$$

Assuming the generator current is in phase with the internal generator voltage

$$P_{mech} = 3\,\text{Re}\left\{\underline{E}_g \underline{I}_c^*\right\} = 3E_g I_c$$

$$I_c = \frac{P_{mech}}{3E_g} = 4488.833 \text{ A}$$

The converter voltage can be calculated as

$$\underline{U}_c = \underline{E}_g - \left(R_g + \text{j}\omega_e L_g\right)\underline{I}_c = 393.3897\ \angle\text{-}6.29^{\text{o}} \text{ V}$$

The converter power:

$$\underline{S}_c = 3\underline{U}_c \underline{I}_c^* = 5.2657 \text{ -j } 0.58031 \text{ MVA}$$

The generator losses are 120.8977 kW.

Assuming a converter efficiency of 97 % and unity power factor operation (in the converter side) calculate the active and reactive power exchanged with the grid.

The power injected by the grid-side converter can be calculated as

$$\underline{S}_{gs} = \eta \operatorname{Re}\left(\underline{S}_c\right) = 5.10773 + 0\text{j MVA}$$

The grid voltage $\underline{U}_{pn-grid} = 960/\sqrt{3}\ \text{V}$ and the active and reactive power provided by the converter are known. The grid-side converter can be found from

$$U^4 + U^2\left(2R\,P + 2X\,Q - U_{pn-grid}^2\right) + \left(R^2 + X^2\right)\left(P^2 + Q^2\right) = 0$$

Where the power *P* and *Q* are the **one-phase** active and reactive power exchanged between the converter and the grid (P>0 load, P<0 generation). It can be noted that P will be the converter power (negative, one phase) $P_{1phase} = -P_{mech}/3$ and Q=0.

The following voltages are obtained:

-566.7774196585969 V

566.7774196585964 V

-54.6729079164701 V

54.6729079164702 V

Resulting in voltage of 981.6872 in the grid-side converter.

The converter equivalent resistance can be calculated from:

$$R_{ceq} = \frac{U^2}{P_{1phase}} = -0.1886767\ \Omega$$

The converter current can be calculated as:

$$\underline{I}_{conv} = \frac{\underline{U}_{pn-grid}}{R_{ceq} + R + \text{j}X} = 3003.96 \angle\text{-}174.55^{\circ}\ A$$

The power exchanged with the grid

$$\underline{S}_{grid} = -3\underline{U}_{pn-grid}\,\underline{I}_{conv}^* = 4.97237 \text{ - j } 0.4737 \text{ MVA}$$

The total system losses is of 414.22 kW and the efficiency

$$\eta_{total} = \frac{P_{grid}}{P_{mech}} = 92.31\ \%$$