# Electric Energy Conversion

## 3. Diode rectifiers

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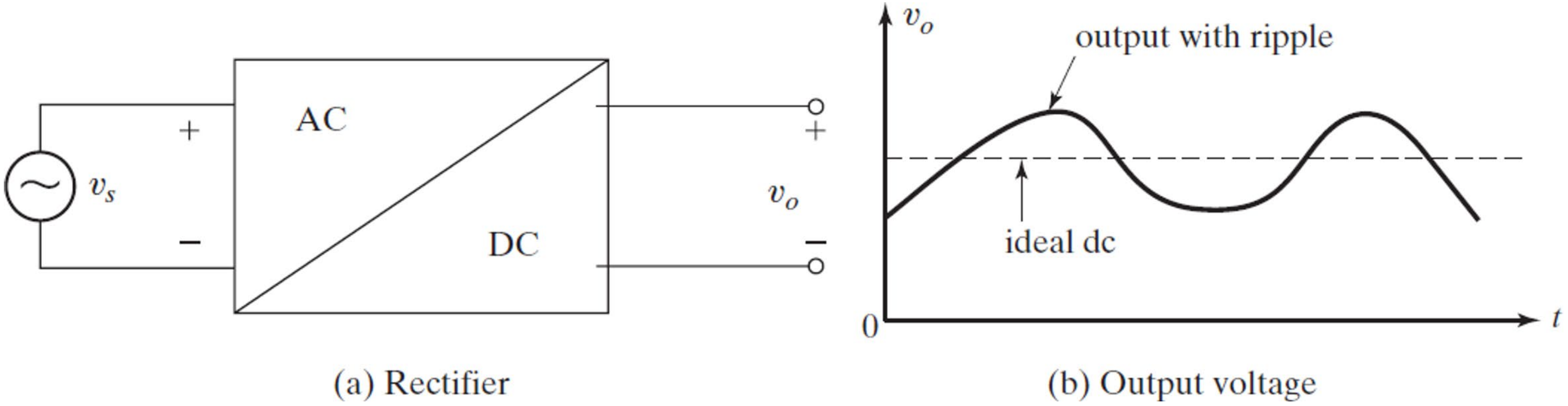
# Outline

- Half-wave rectifiers
- Full-wave rectifiers
- Simulation

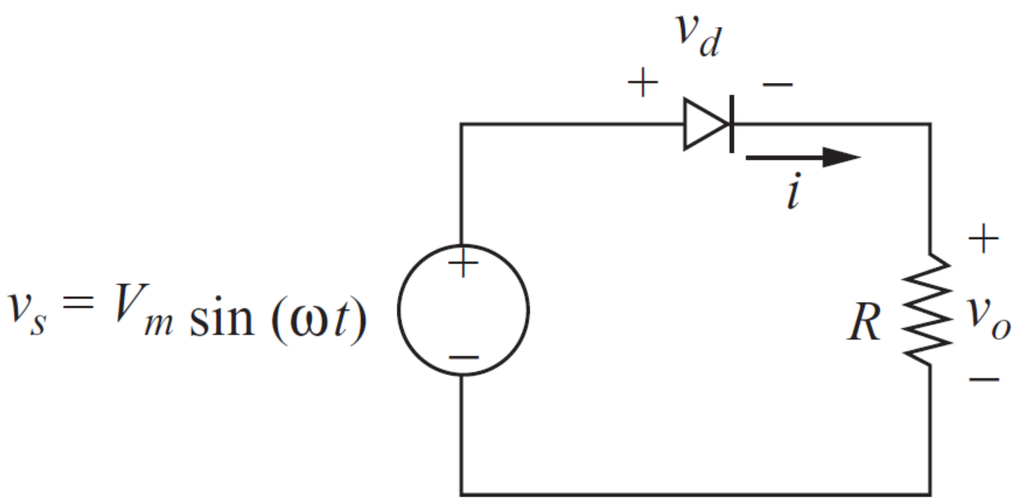
# Introduction to rectifiers

- A rectifier is a circuit that converts an AC signal into a DC signal (AC/DC converter).
- They can be single-phase/three-phase and half/full wave.
- Ideally we would like to have a perfect DC wave, but in practice the rectifiers provide a DC wave with harmonics or ripple.

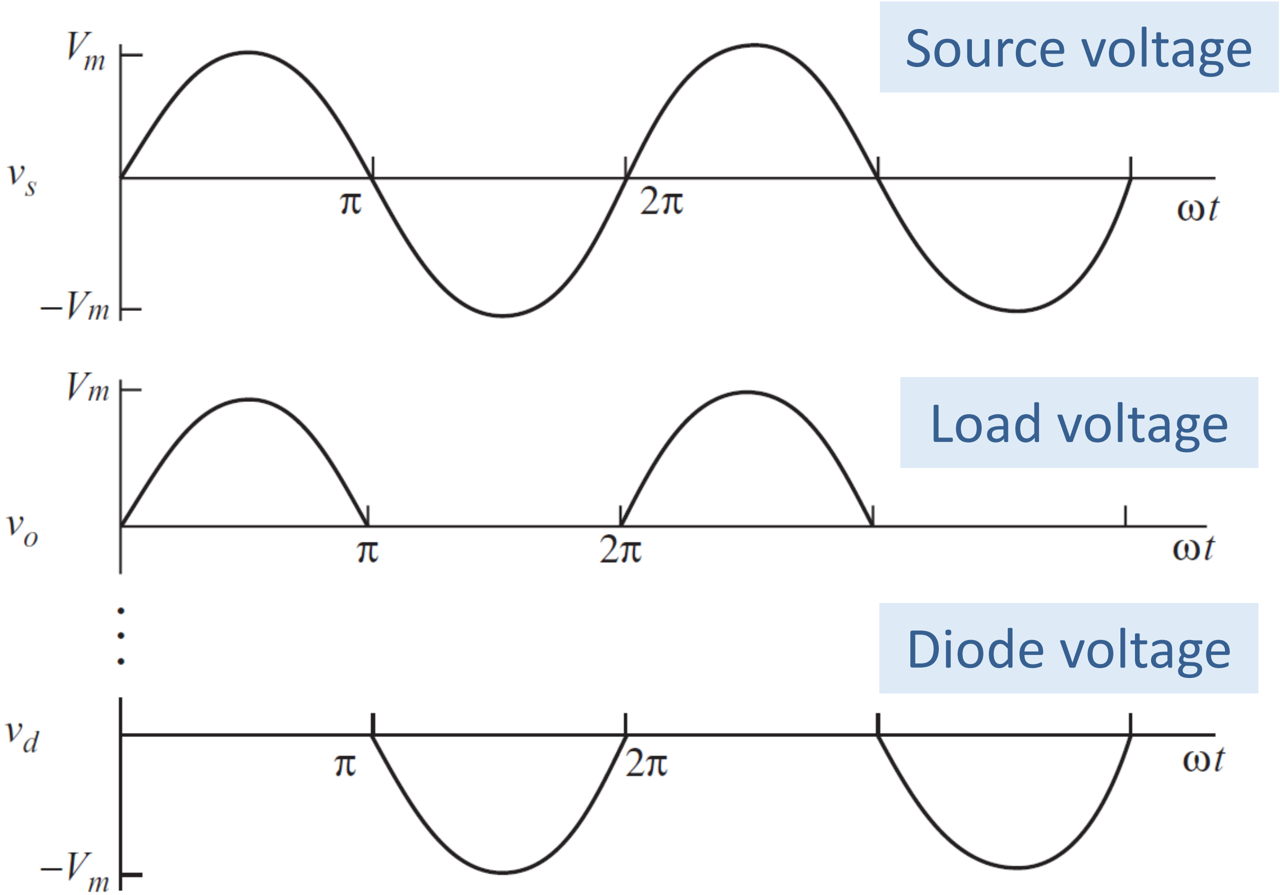


# Half-wave rectifier

- The half-wave diode rectifier is the simplest rectifier configuration.
- The diode  does not conduct when its reverse biased, ’’holding’’ ‘ the negative voltage




(a)




(b)

# Half-wave rectifier

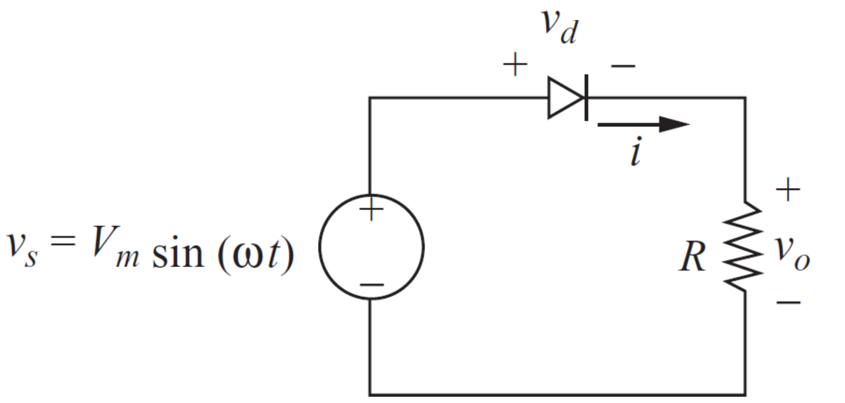
- The DC component of vo can be calculated as the average value during the complete period:

$$V_o = V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)\, d\omega t = \frac{V_m}{\pi}$$

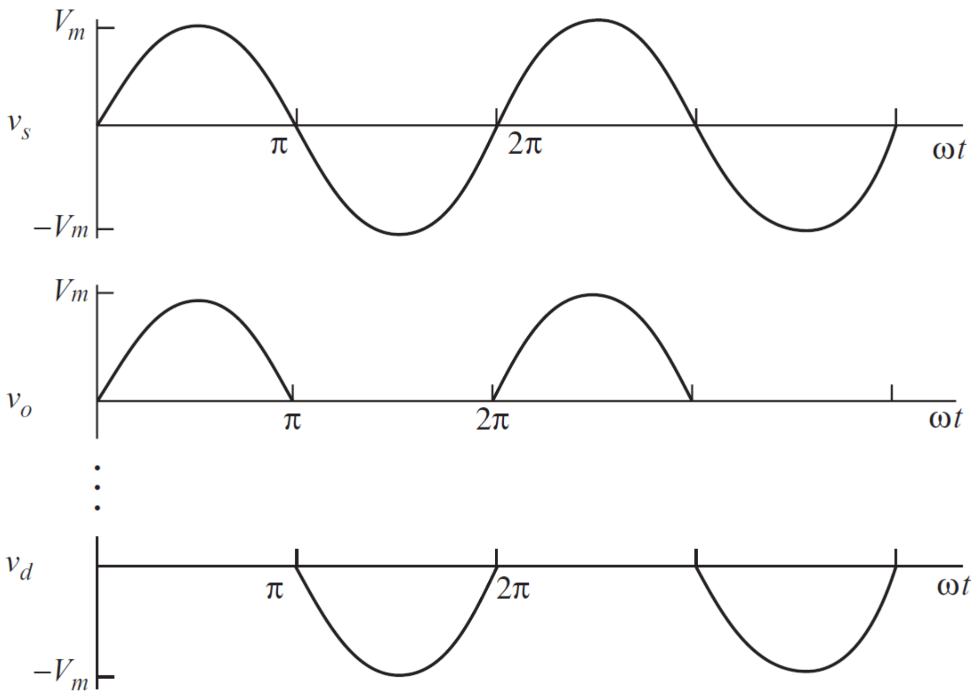
- The DC component of the current of a purely resistive load: $I_o = \dfrac{V_o}{R} = \dfrac{V_m}{\pi R}$

- The power absorbed by the load can be computed from the rms values:

$$P = I_{rms}^2 R = V_{rms}^2 / R$$



(a)

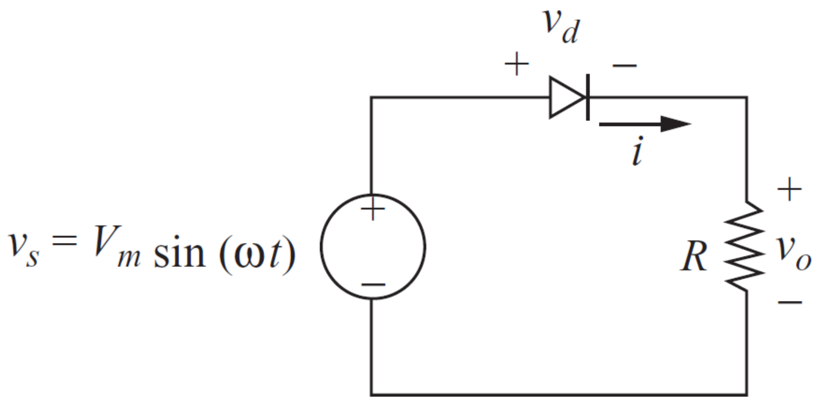


(b)

# Example

- From D. Hart page 67:

For the half-wave rectifier of Fig. 3-1$a$, the source is a sinusoid of 120 V rms at a frequency of 60 Hz. The load resistor is 5 Ω. Determine ($a$) the average load current, ($b$) the average power absorbed by the load and ($c$) the power factor of the circuit.




($a$)

# Example

- Solution

(*a*) The voltage across the resistor is a half-wave rectified sine wave with peak value $V_m = 120\sqrt{2} = 169.7$ V. From Eq. (3-2), the average voltage is $V_m/\pi$, and average current is

$$I_o = \frac{V_o}{R} = \frac{V_m}{\pi R} = \frac{\sqrt{2}(120)}{5\pi} = 10.8 \text{ A}$$

(*b*) From Eq. (3-3), the rms voltage across the resistor for a half-wave rectified sinusoid is

$$V_{\text{rms}} = \frac{V_m}{2} = \frac{\sqrt{2}(120)}{2} = 84.9 \text{ V}$$

The power absorbed by the resistor is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{84.9^2}{4} = 1440 \text{ W}$$

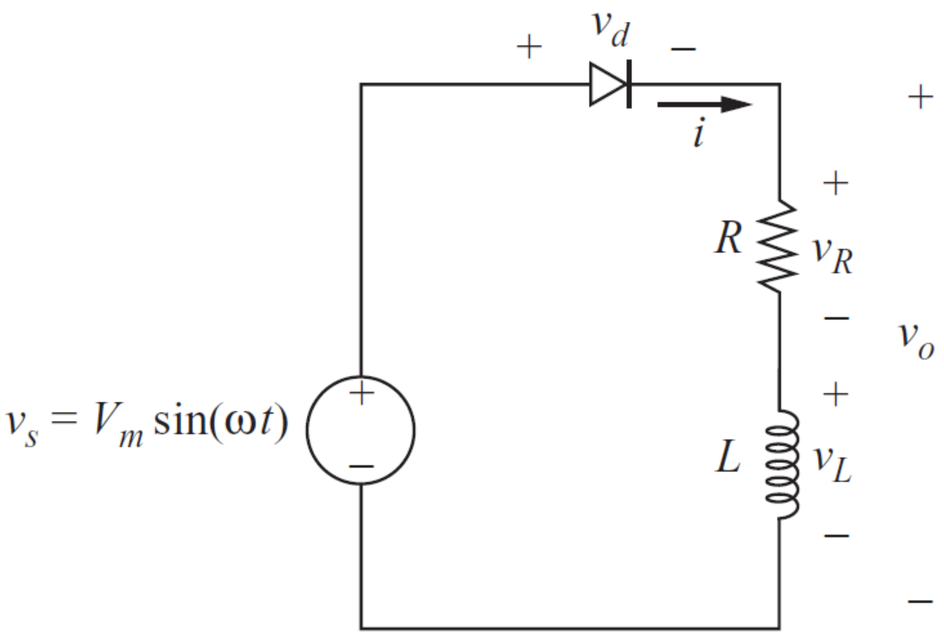
The rms current in the resistor is $V_m/(2R) = 17.0$ A, and the power could also be calculated from $I_{\text{rms}}^2 R = (17.0)^2(5) = 1440$ W.

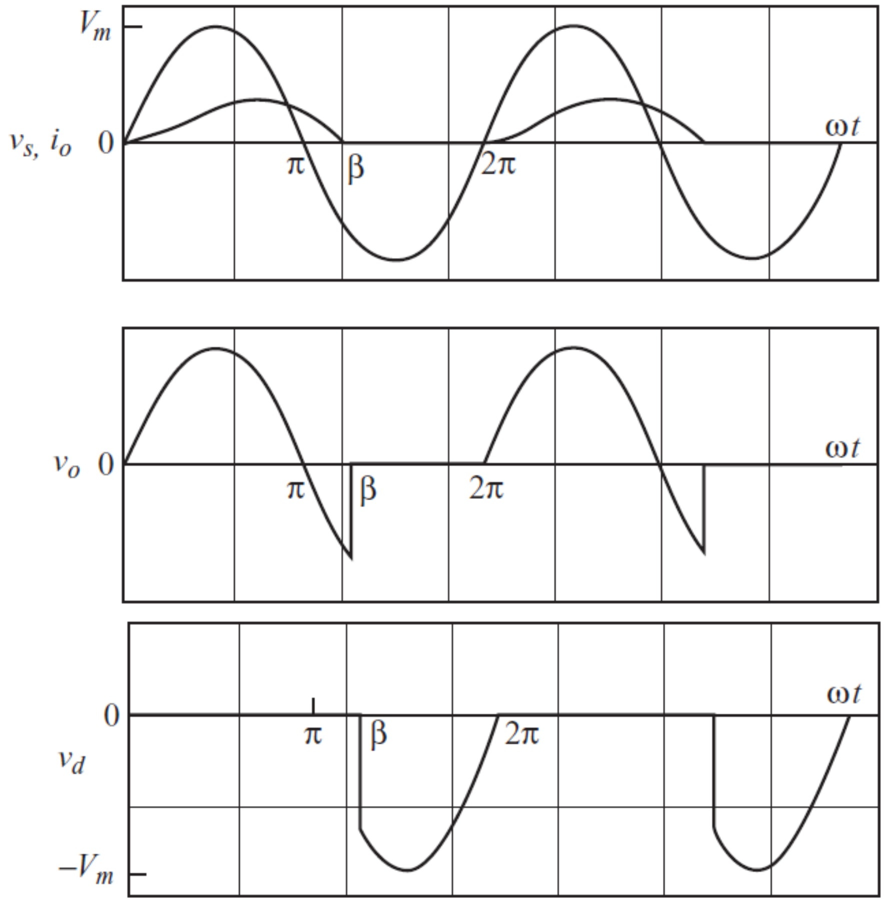
(*c*) The power factor is

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{s,\text{rms}} I_{s,\text{rms}}} = \frac{1440}{(120)(17)} = 0.707$$

# Half-wave rectifier with RL load

- When the load has an inductive element, the current has a phase with respect to the voltage, so the **diode keeps forward-biased** even **after** the voltage has drop to zero.



# Half-wave rectifier with RL load

- The inductor introduces new dynamics to the circuit. When the diode is forward-biased the KVL becomes:

$$V_m \sin(\omega t) = Ri(t) + L\frac{di(t)}{dt}$$

- The current is expressed by the sum of the transient response ($i_t$) and the steady-state ($i_{ss}$) response:

$$v(t) = Ri(t) + L\frac{di(t)}{dt} \longrightarrow i(t) = i_t(t) + i_{ss}(t)$$

- We can calcualte the steady-state current using phasor analysis:

$$i_{ss}(t) = \frac{V_m}{Z}\sin(\omega t - \theta) \quad \text{where} \quad Z = \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

- And the transient current is calculated by solving the ODE from the KVL:

$$i_t(t) = \frac{V_m}{Z}\sin(\theta)\, e^{-t/\tau} \quad \text{where} \quad \tau = L/R$$

# Half-wave rectifier with RL load

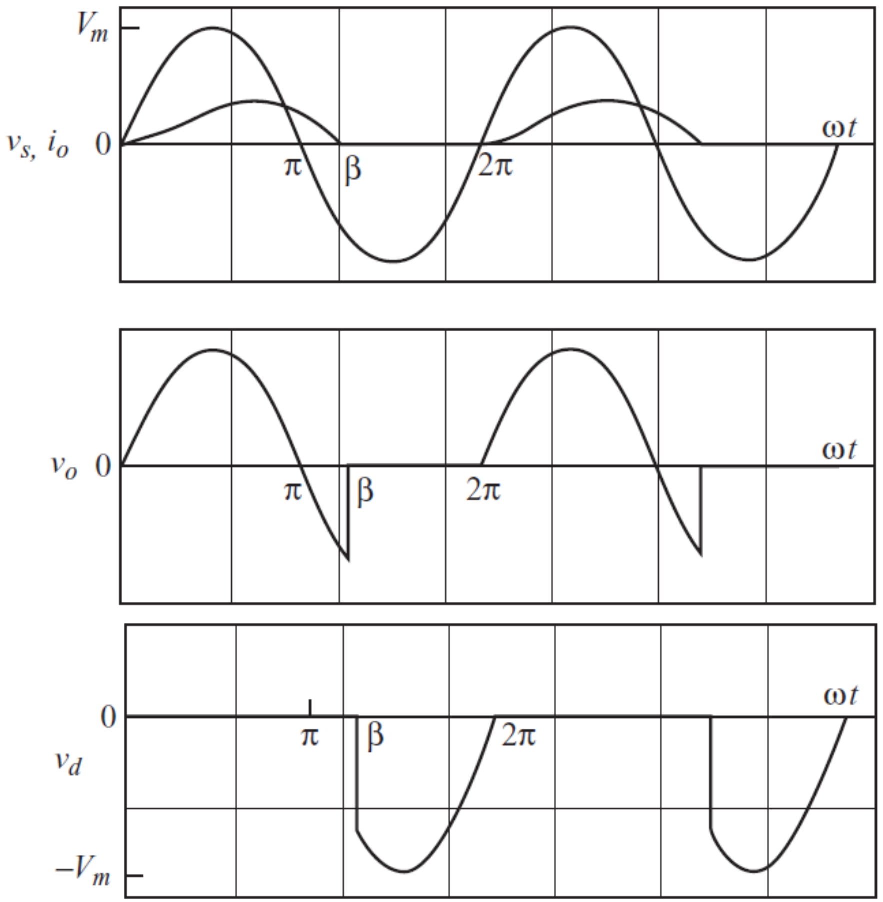
- From the previous equations we can find the **extinction angle** ($\beta = \omega t$) that represents the moment where the current crosses zero and the diode gets reverse-biased.

$$i(\beta) = \frac{V_m}{Z}\left[\sin(\beta - \theta) + \sin(\theta) e^{-\beta/\omega\tau}\right] = 0$$

- Which reduces to:

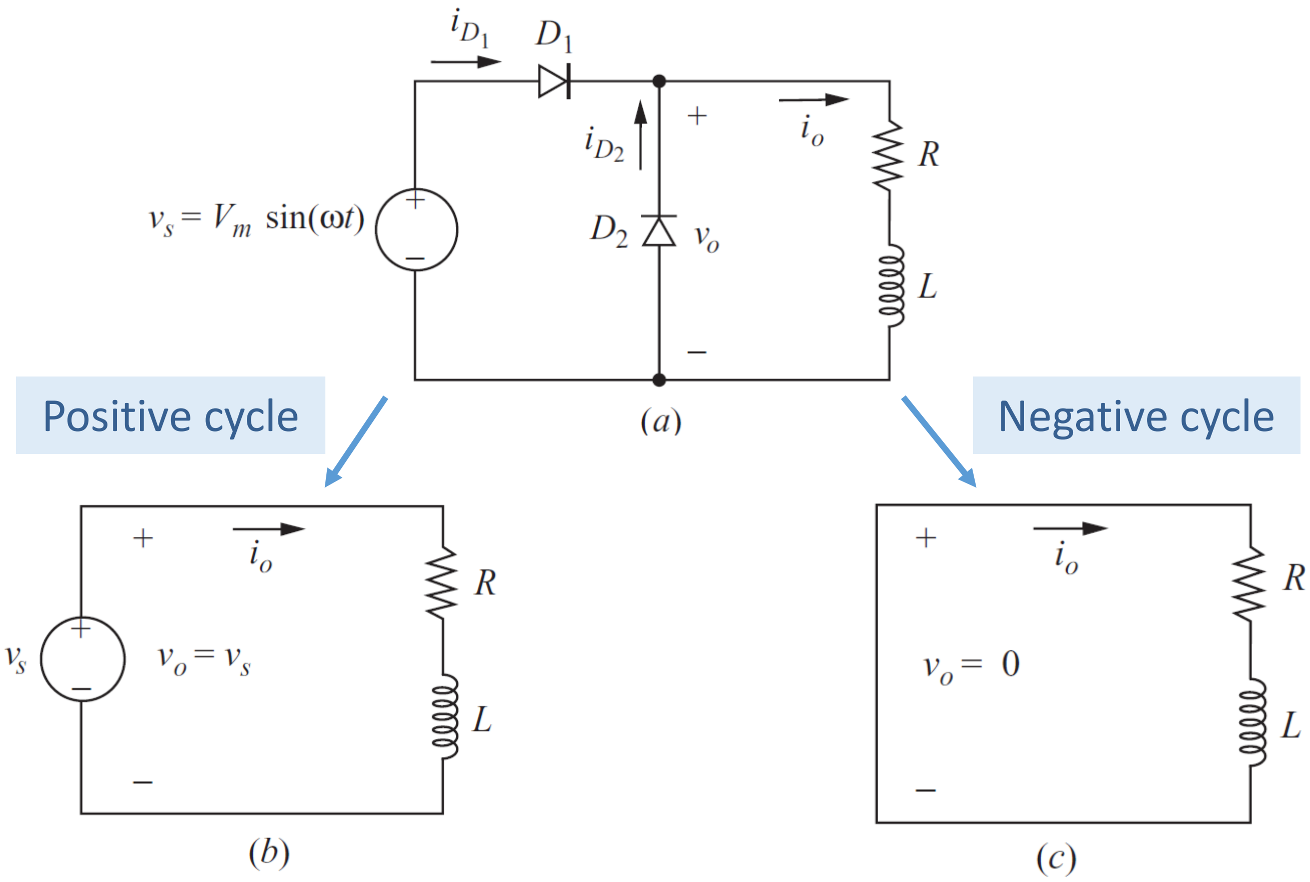
$$\sin(\beta - \theta) + \sin(\theta)\, e^{-\beta/\omega\tau} = 0$$

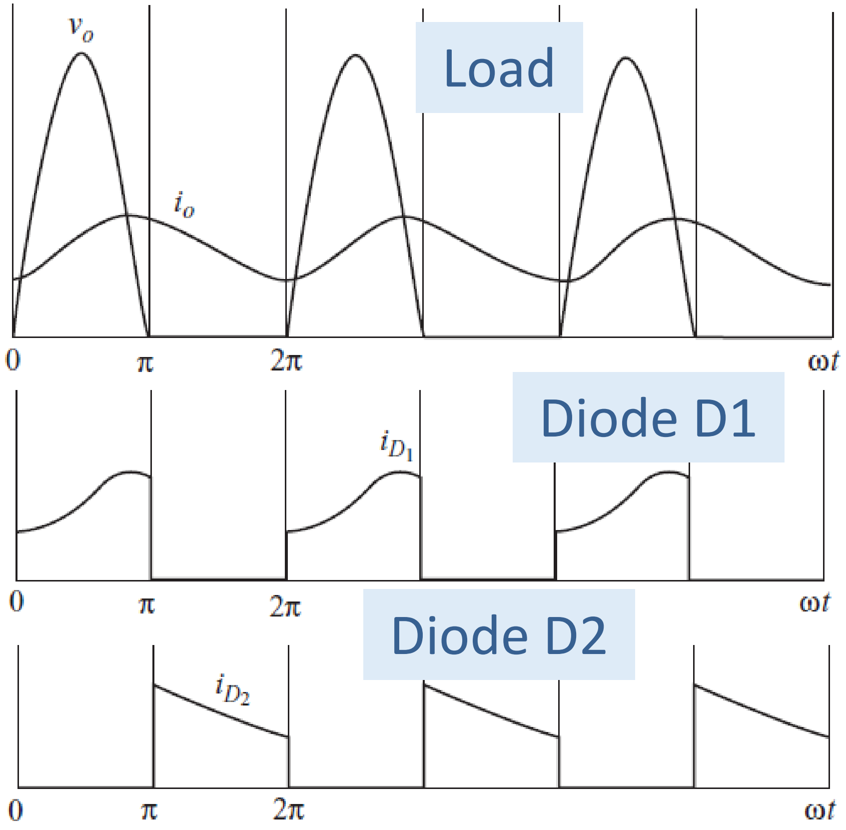
- The previous equation is solved numerically to find $\beta$.



# Half-wave rectifier with freewheeling diode

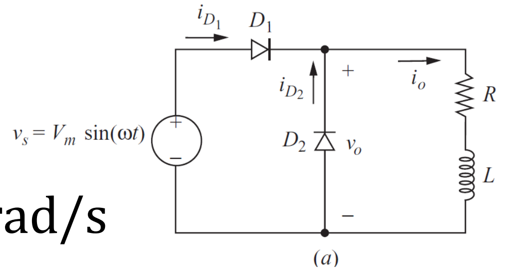
- In the previous circuit, the current goes to zero every cycle because the load voltage goes negative after half-cycle.
- But if we add a freewheeling diode, we can block the negative voltage.



# Half-wave rectifier with freewheeling diode

- The Fourier series of the half-wave rectified voltage across the load is:




$$v(t) = \frac{V_m}{\pi} + \frac{V_m}{2}\sin(\omega_1 t) - \sum_{n=2,4,6\ldots}^{\infty} \frac{2V_m}{(n^2-1)\pi}\cos(n\omega_1 t) \quad \text{where} \quad \omega_1 = 2\pi 50 \text{ rad/s}$$

DC

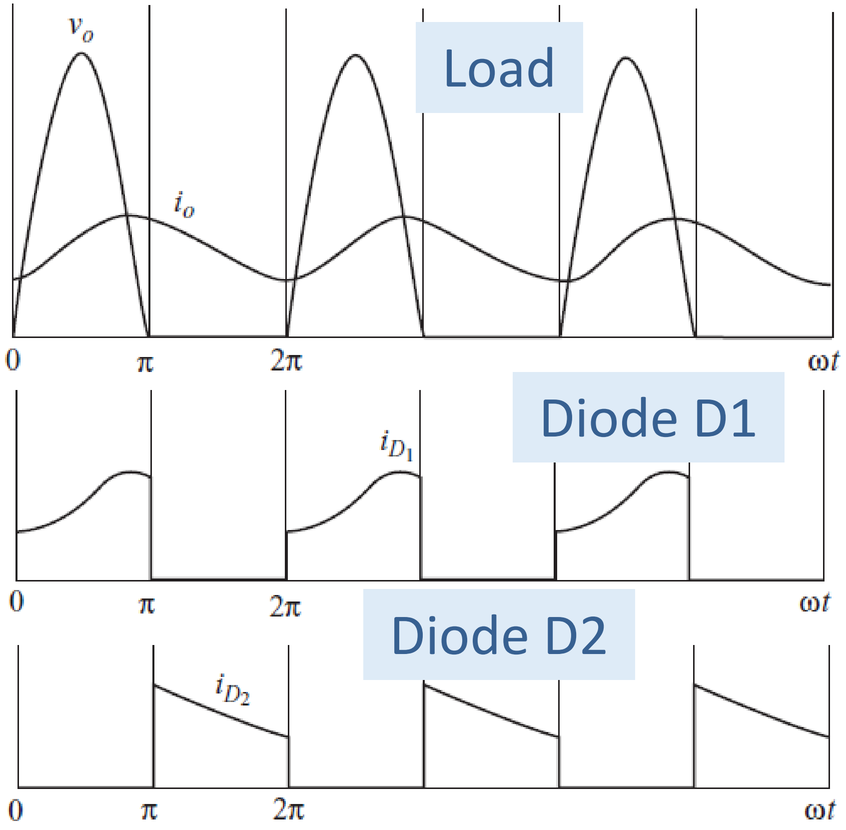
1-st harmonic

even harmonics

- The steady-state (phasor) current is

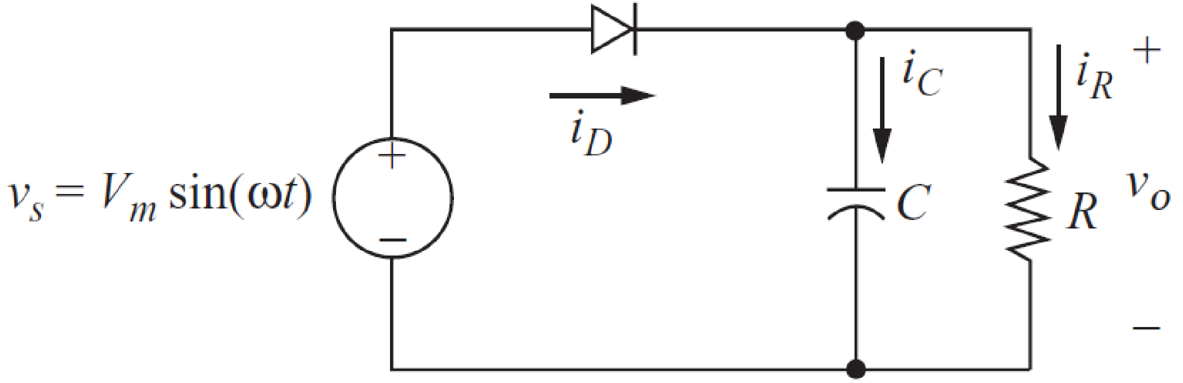
$$I_n = \frac{V_n}{Z_n} \quad \text{where} \quad Z_n = |R + jn\omega_1 L|$$

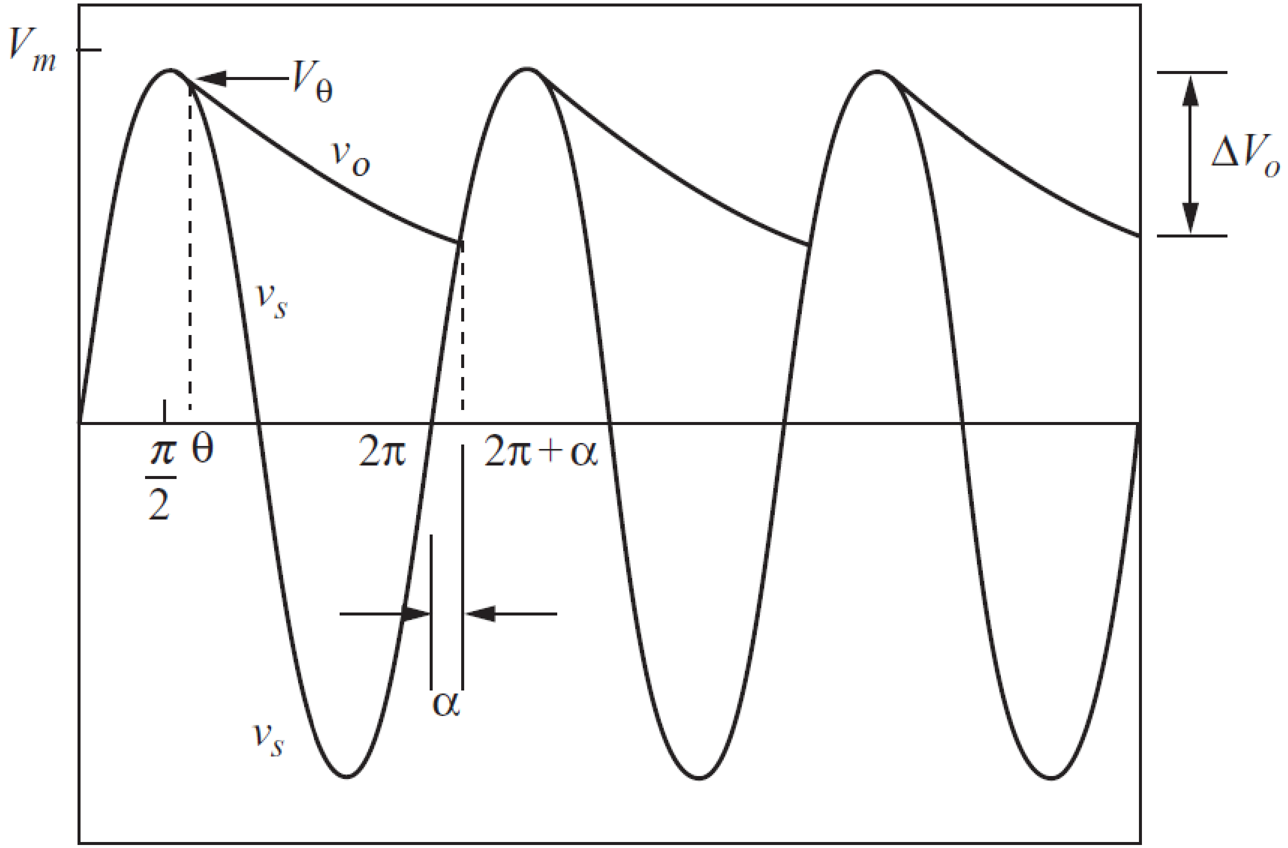
- Conclusion: the bigger the $L$, the more DC-like (less harmonics) the current will be.



# Half-wave rectifier with capacitor filter

- If a capacitor is added in parallel with the load, it helps to keep the voltage more like DC.
- During the charging, the diode is forward-biased. When the source voltage becomes lower than the capacitor voltage, the diode blocks and the voltage at the load decreases with a time constant $RC$.



# Half-wave rectifier with capacitor filter

- The output voltage (load voltage) of this rectifier can be defined by these two moments:

$$v_o = \begin{cases} V_m \sin \omega t & \text{Diode ON} \\ V_m \sin(\theta) e^{-\frac{\omega t - \theta}{\omega RC}} & \text{Diode OFF} \end{cases} \qquad \text{where} \qquad \theta = \tan^{-1}(\omega RC) \approx \frac{\pi}{2}$$

- When the source voltage exceeds the capacitor voltage, the diode starts conducting again.
- This angle $\alpha$ can be found numerically by solving

$$\sin \alpha = \sin(\theta)\, e^{-(2\pi + \alpha - \theta)/\omega RC}$$

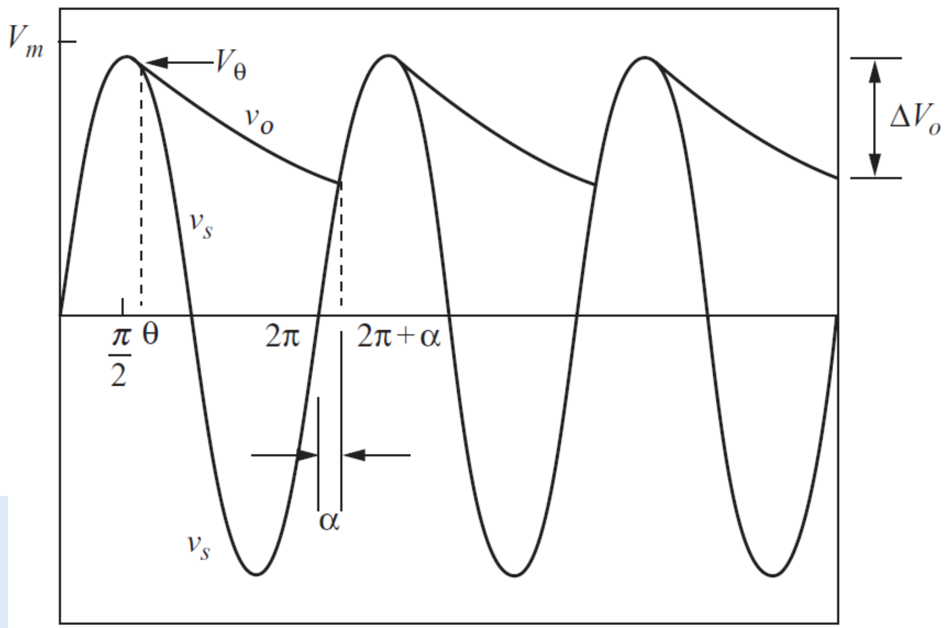
- The ripple on the DC voltage ($\Delta V_o$) is calculated as

$$\Delta V_o = V_m(1 - \sin \alpha)$$

- $\Delta V_o$ can be approximated as

$$\Delta V_o \approx V_m \left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

ripple decreases with $C$



# Outline

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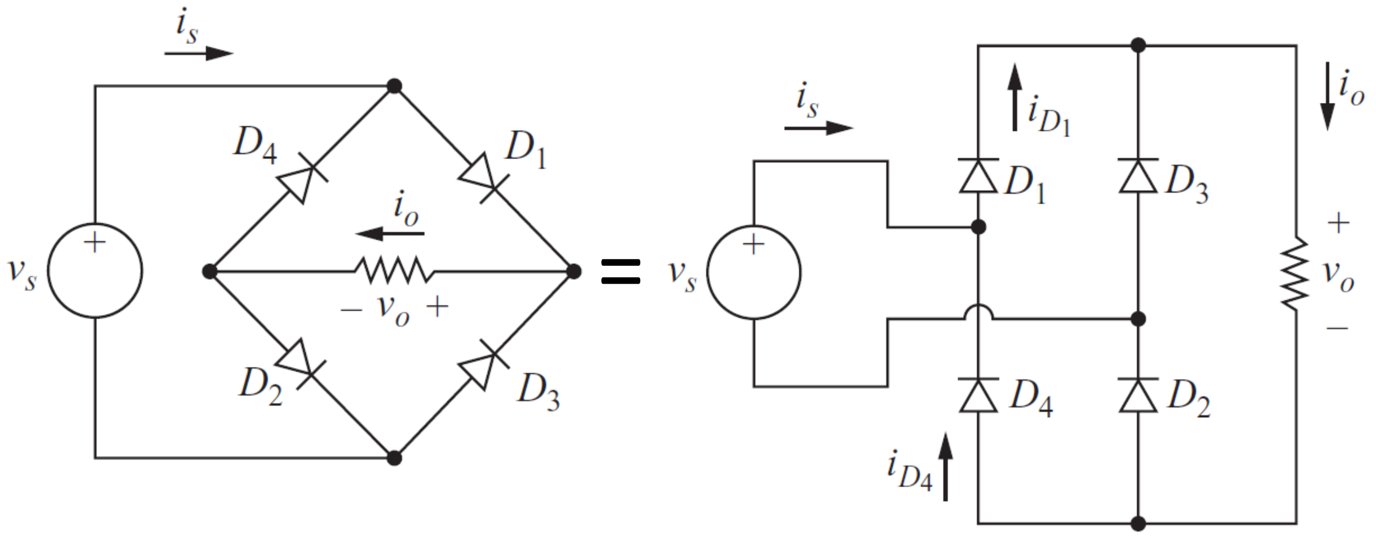
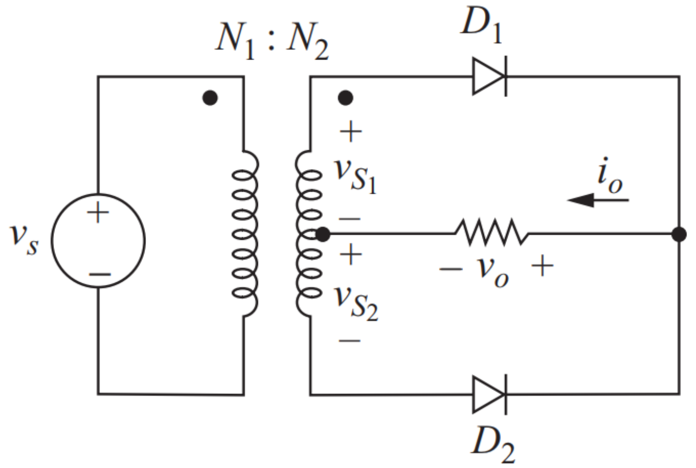
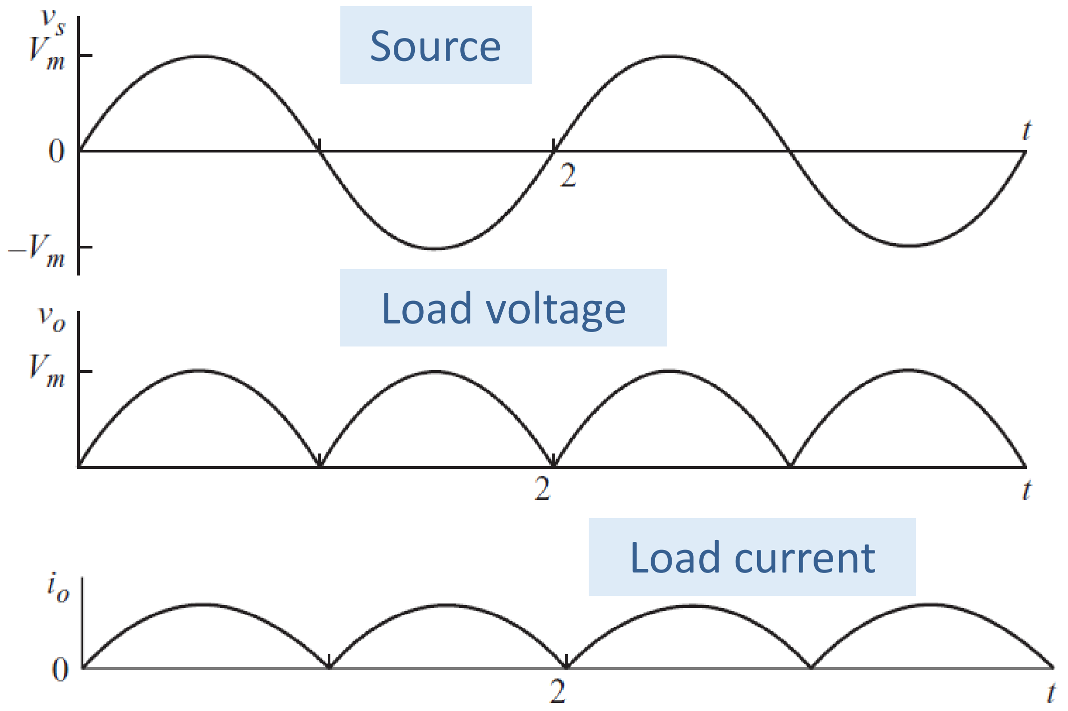
# Single-phase full-wave rectifier

- The full-wave rectifier offers more advantages compared to the half-wave rectifier, such as symmetry for the currents and less ripple.



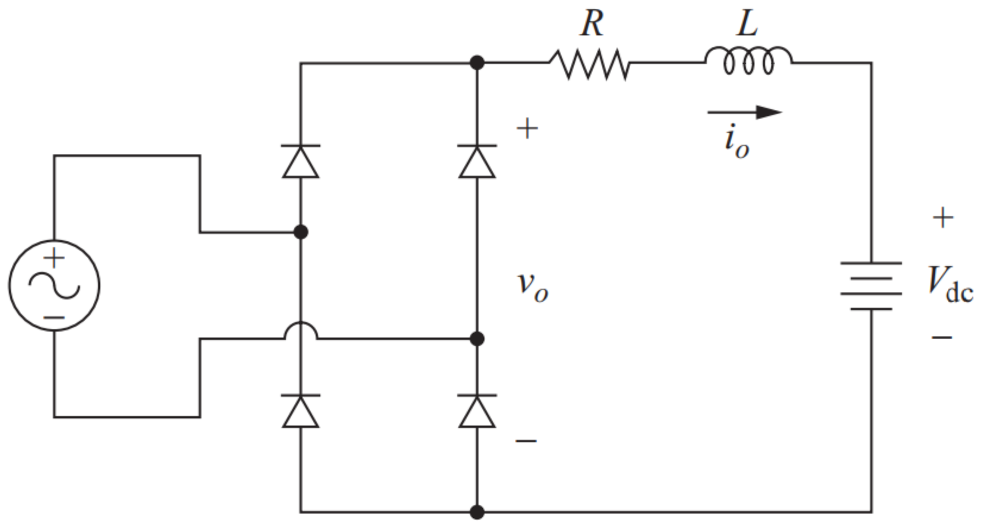



# Single-phase full-wave rectifier with RL-source load

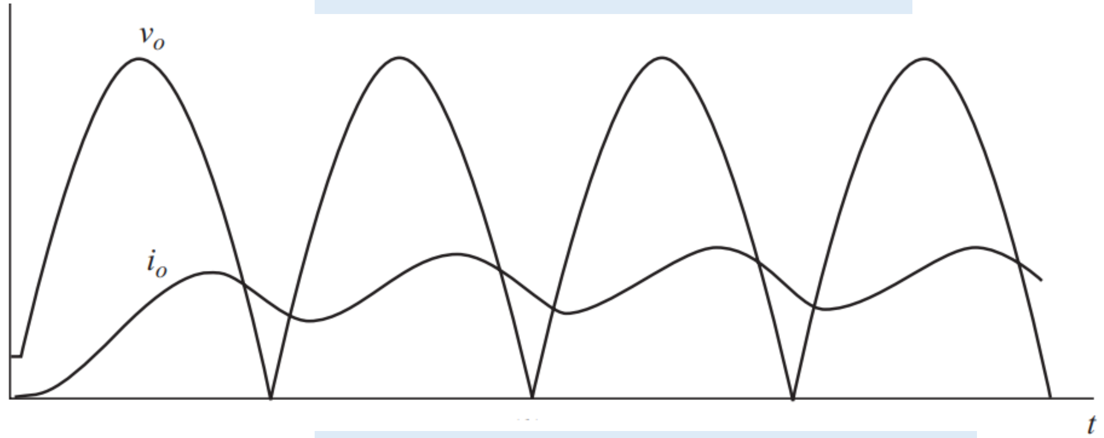
- DC motor drives and battery chargers are examples of RL-source loads.
- In continuous mode, the current does not drop to zero:

$$I_o = \frac{V_o - V_{dc}}{R} = \frac{\frac{2V_m}{\pi} - V_{dc}}{R}$$

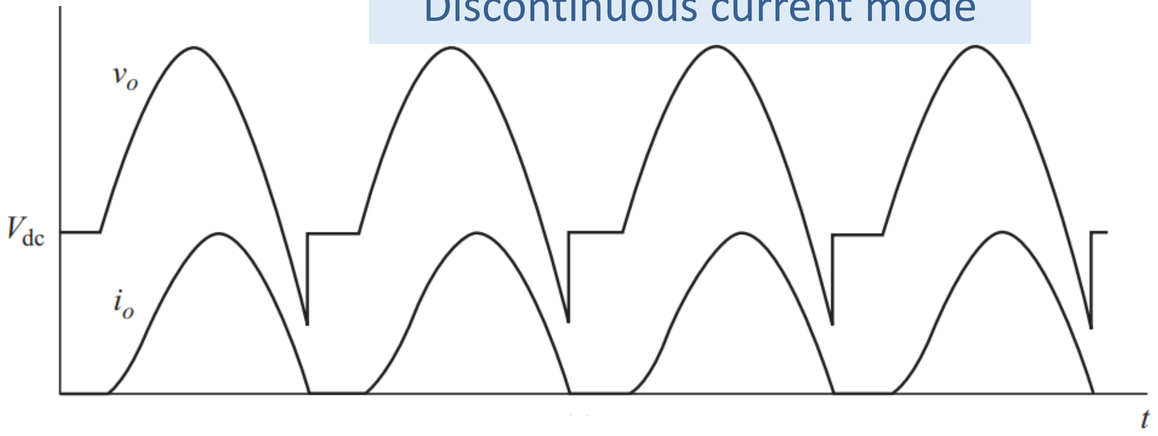
- In discontinuos mode, the load current can be analysed like the half-wave rectifier.




Continuous current mode

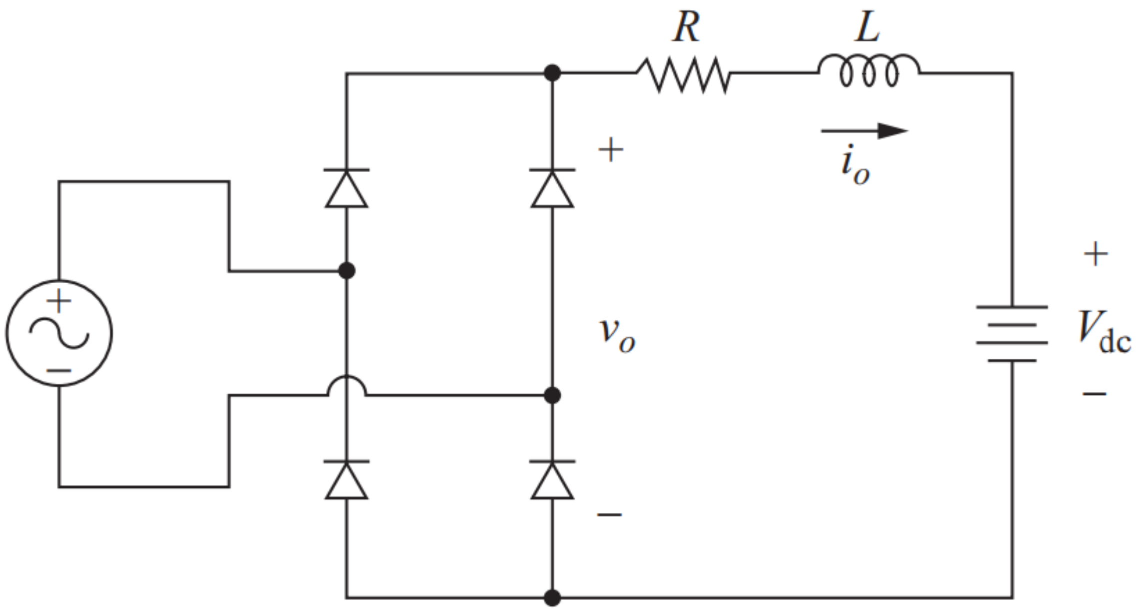



Discontinuous current mode



# Single-phase full-wave rectifier with RL-source load - example

- For the full-wave bridge rectifier circuit of figure below, the AC source is 120 V rms at 60 Hz, $R = 2\ \Omega, L = 10\ \text{mH}, \text{and}\ V_{dc} = 80\ V$ . Determine the power absorbed by the DC voltage source and the power absorbed by the load resistor.




- Equations

$$v_o(t) = V_o + \sum_{n=2,4\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

where

$$V_o = \frac{2V_m}{\pi} \quad \text{and} \quad V_n = \frac{2V_m}{\pi}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

and

$$I_o = \frac{V_o - V_{dc}}{R} = \frac{\frac{2V_m}{\pi} - V_{dc}}{R}$$

# Single-phase full-wave rectifier with RL-source load - example

- Solution

For continuous current, the voltage across the load is a full-wave rectified sine wave which has the Fourier series given by Eq. (4-4). Equation (4-7) is used to compute the average current, which is used to compute power absorbed by the dc source,

$$I_0 = \frac{\dfrac{2V_m}{\pi} - V_{dc}}{R} = \frac{\dfrac{2\sqrt{2}(120)}{\pi} - 80}{2} = 14.0 \text{ A}$$

$$P_{dc} = I_0 V_{dc} = (14)(80) = 1120 \text{ W}$$

The first few terms of the Fourier series using Eqs. (4-4) and (4-5) are shown in Table 4-1.

**Table 4-1** Fourier series components

| $n$ | $V_n$ | $Z_n$ | $I_n$ |
|---|---|---|---|
| 0 | 108 | 2.0 | 14.0 |
| 2 | 72.0 | 7.80 | 9.23 |
| 4 | 14.4 | 15.2 | 0.90 |

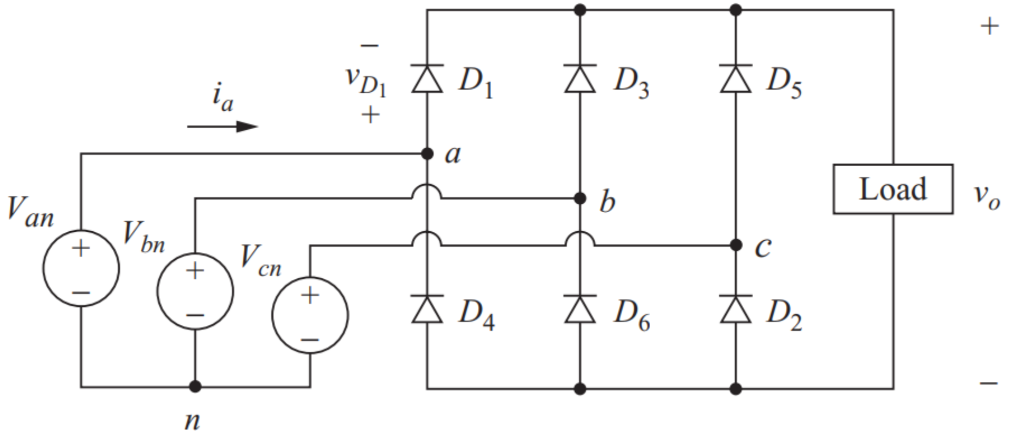
$$I_{rms} = \sqrt{14^2 + \left(\frac{9.23}{\sqrt{2}}\right)^2 + \left(\frac{0.90}{\sqrt{2}}\right)^2 + \cdots} \approx 15.46 \text{ A} \longrightarrow$$

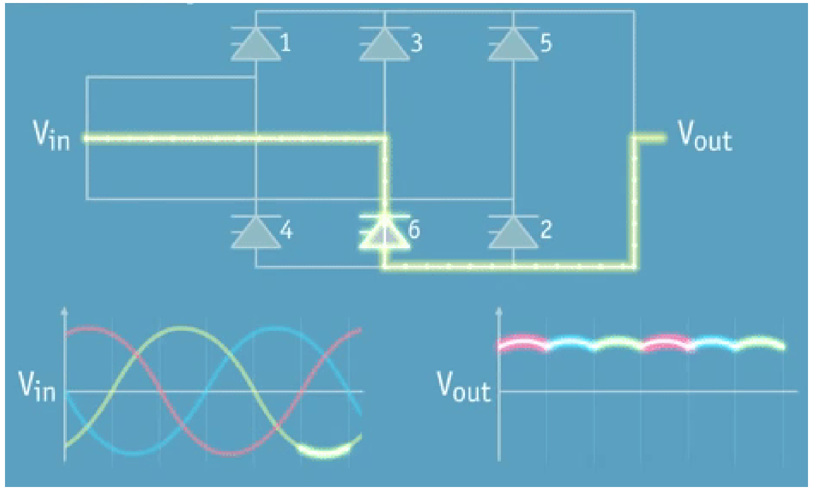
Power absorbed by the resistor is

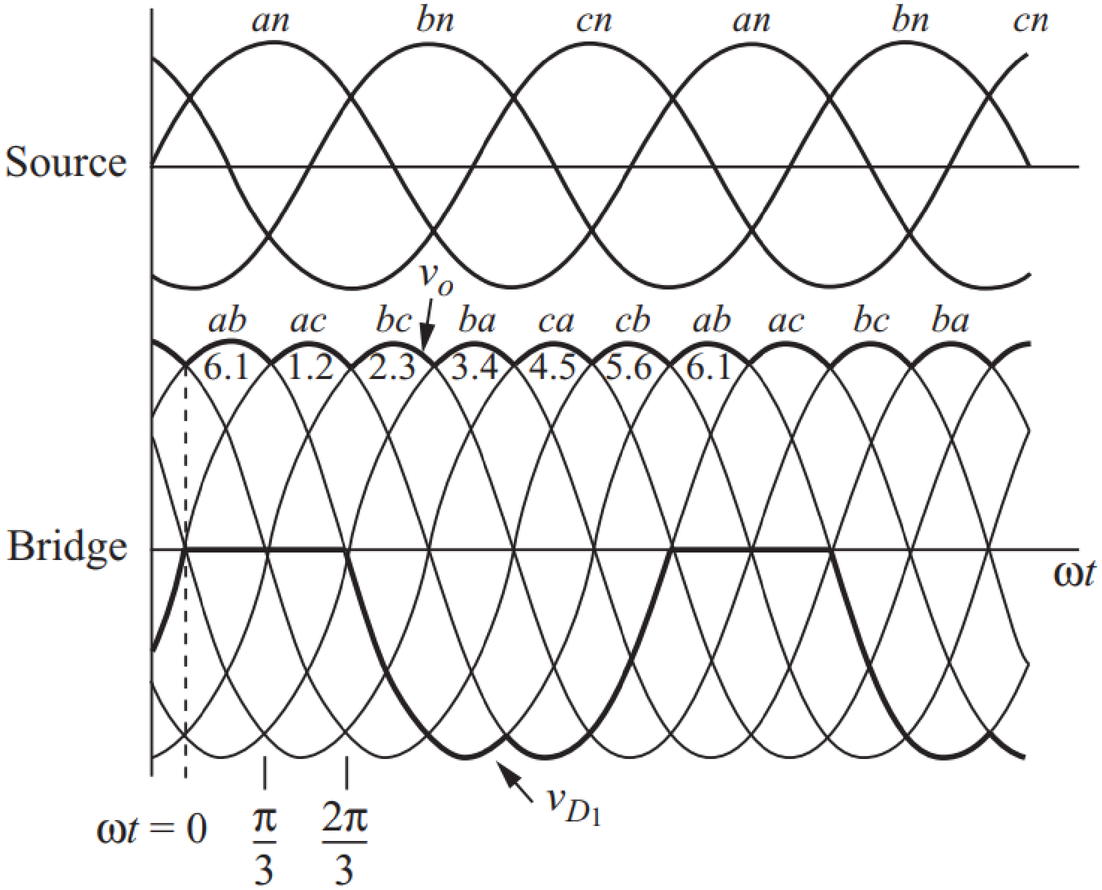
$$\boxed{P_R = I_{rms}^2 R = (15.46)^2(2) = 478 \text{ W}}$$

# Three-phase rectifier

- A three-phase rectifier is commonly found in industry. It is used to create a DC voltage for large loads.




Source: HVDC Concepts, TranspowerNZ https://www.youtube.com/user/TranspowerNZ

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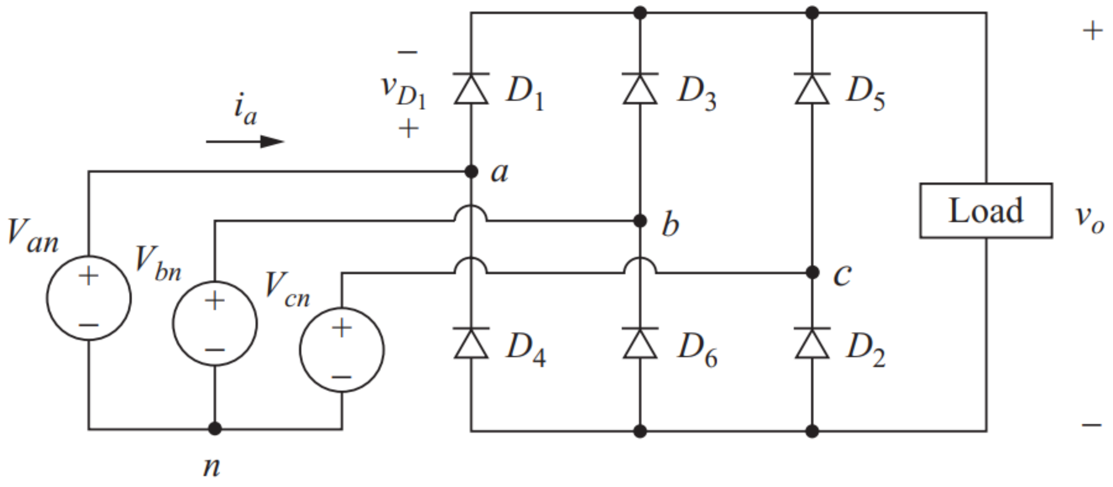
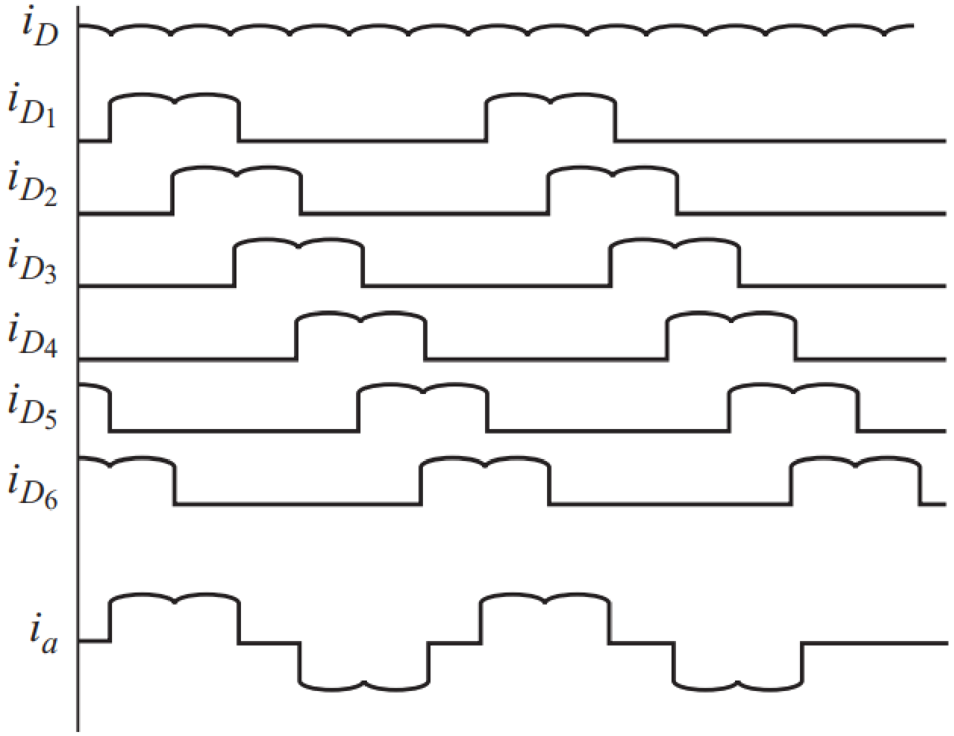



- Load current and source current

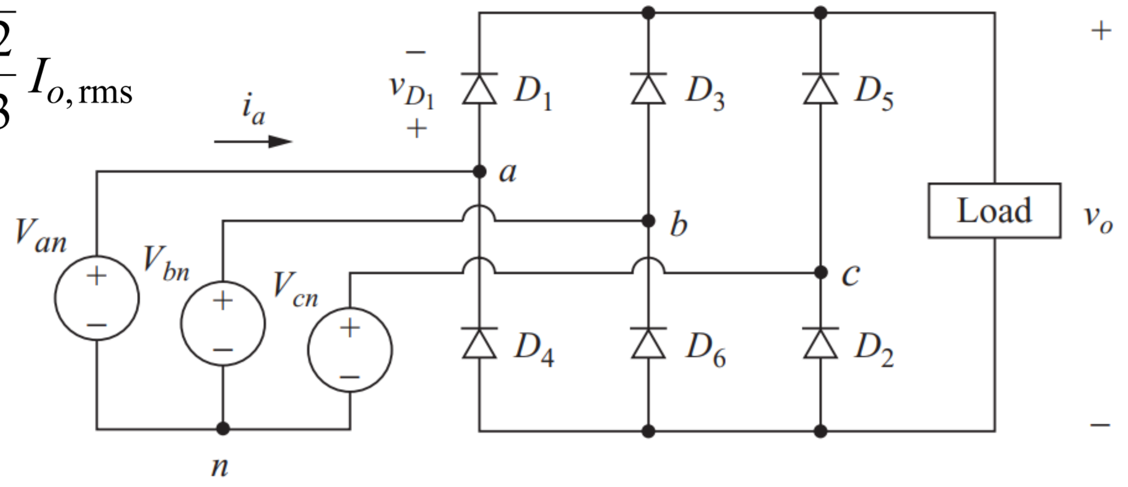



- AC current has harmonics

# Three-phase rectifier

- Since each diode conducts one-third of the time (6-pairs conducting), the load current can be calculated from the source current as:

$$I_{D,\text{avg}} = \frac{1}{3} I_{o,\text{avg}} \rightarrow I_{D,\text{rms}} = \frac{1}{\sqrt{3}} I_{o,\text{rms}} \rightarrow I_{s,\text{rms}} = \sqrt{\frac{2}{3}} I_{o,\text{rms}}$$




- The voltage at the load is:

$$v_o(t) = V_o + \sum_{n=6,12,18,\ldots}^{\infty} V_n \cos(n\omega_0 t + \pi)$$

Where $V_o = \dfrac{3V_{m,\text{L-L}}}{\pi}$ $\qquad V_n = \dfrac{6V_{m,\text{L-L}}}{\pi(n^2-1)}$

The average voltage at the load is $V_0 = \dfrac{1}{\pi/3}\displaystyle\int_{\pi/3}^{2\pi/3} V_{m,L-L}\sin(\omega t)\,d(\omega t) = \dfrac{3V_{m,L-L}}{\pi} = 0.955\, V_{m,L-L}$

peak line-to-line voltage of the source

# Outline

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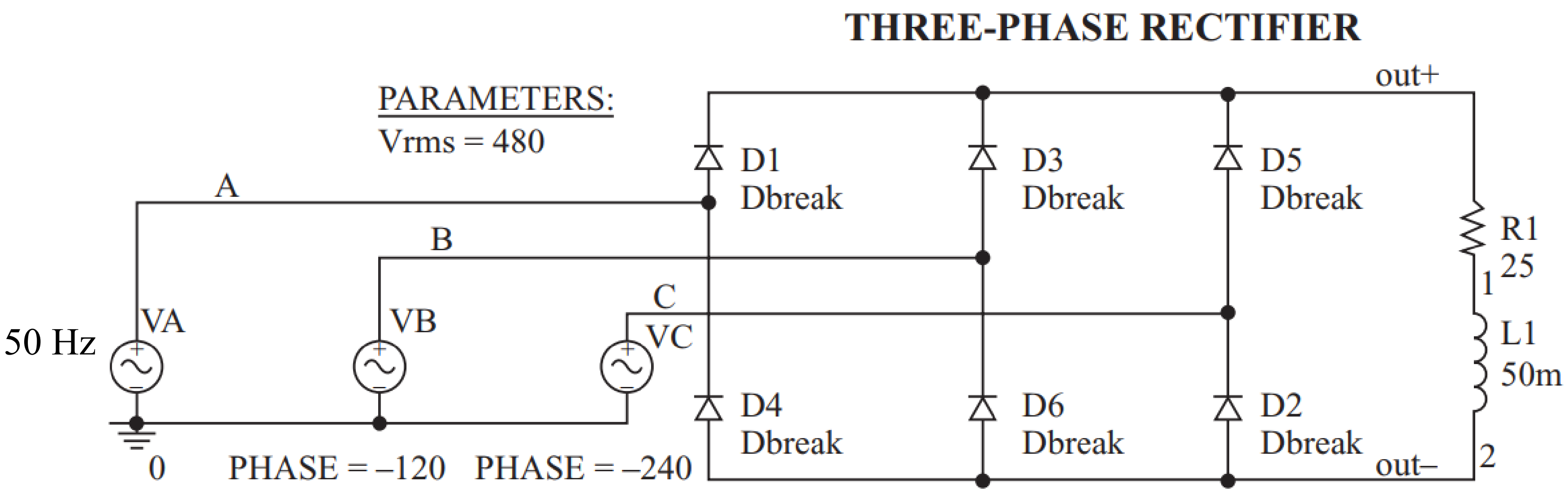
# Three-phase rectifier - Simulation

- 1) Simulate the following circuit.
- 2) Calculate the RMS value of source and load currents, and the ripple of the load voltage. Plot the load and source currents.
- 3) Plot the harmonics of the source current and load voltage
- 4) Design a capacitor to connect in parallel to the load to reduce DC voltage ripple to 5%.



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