Electric Energy Conversion

4. Controlled rectifiers

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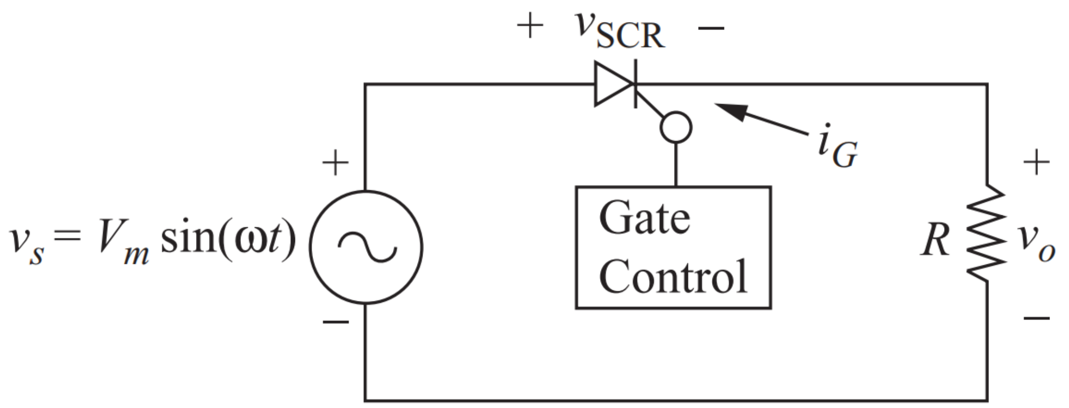
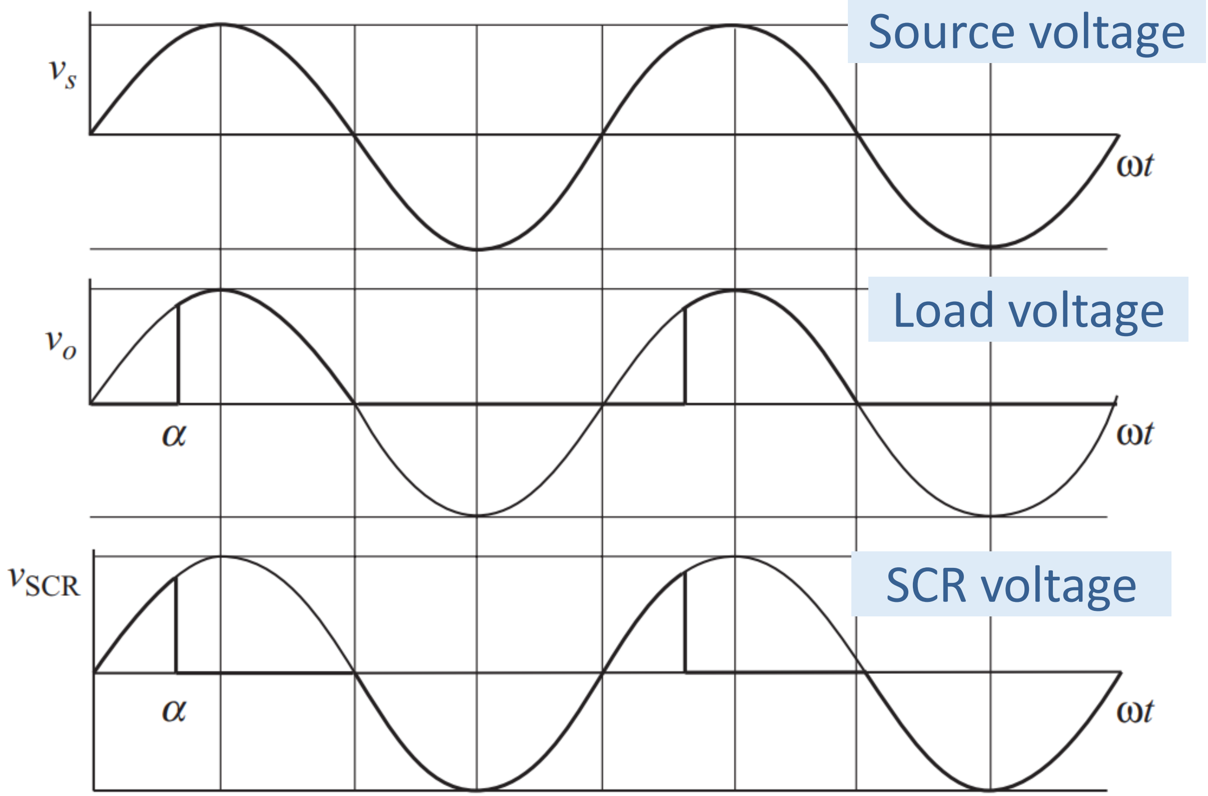
Outline

- Controlled half-wave rectifiers
- Controlled full-wave rectifiers
- Simulation

Controlled half-wave rectifier

- If a thyristor or SCR is used instead of a diode, the ON-state can be controlled.
- The SCR conducts if i) it is forward-biased $v_{SCR} > 0$ and ii) a current is applied to its gate.
- Once the SCR is conducting, the gate current can be removed. It turns OFF only when the current through it goes to zero.



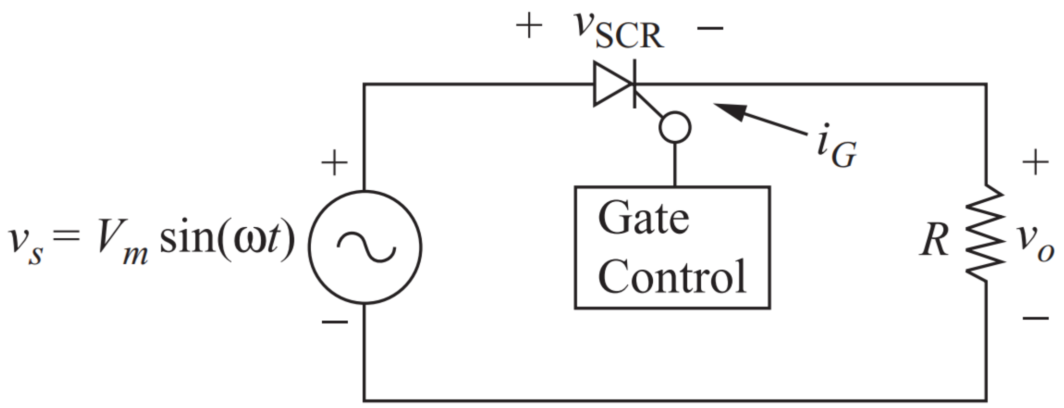


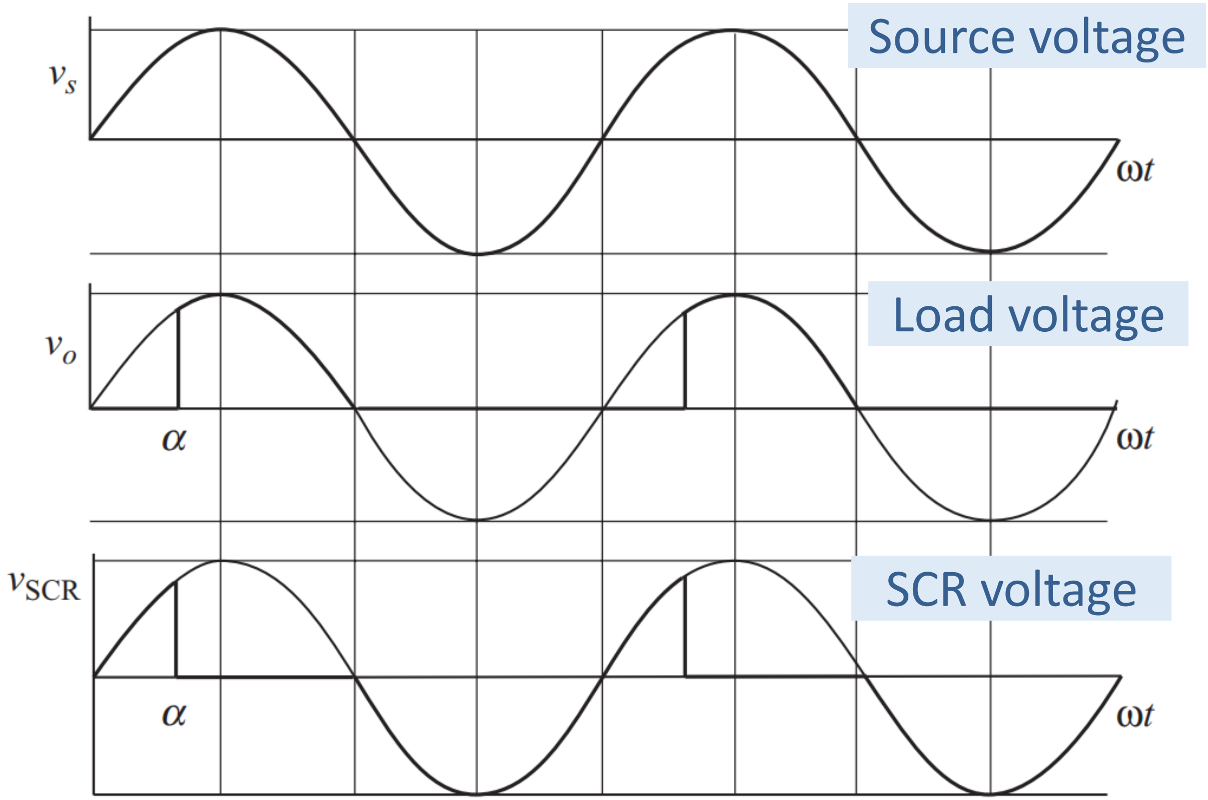
Controlled half-wave rectifier

- The DC voltage across the load is the average of a fraction of the source voltage:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

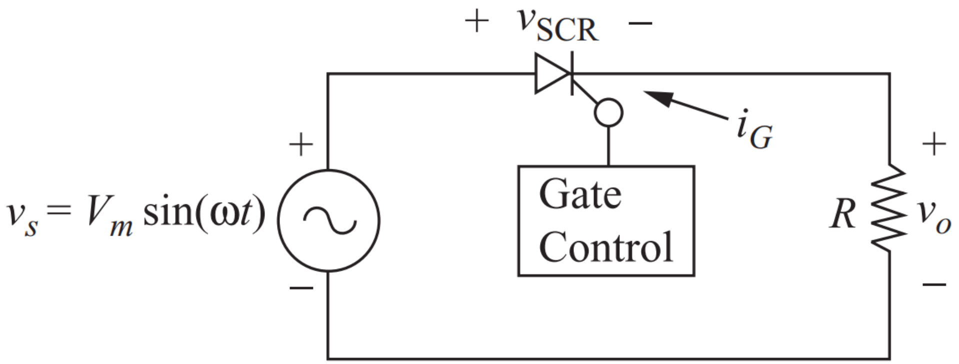
- Where α is the **firing angle** or delay angle





Controlled half-wave rectifier - example

Design a circuit to produce an average voltage of 40 V across a 100-Ω load resistor from a 120-V rms 60-Hz ac source. Determine the power absorbed by the resistance and the power factor.



- Relevant equations

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(1 + \cos\alpha)$$

$$V_{\text{rms}} = \frac{V_m}{2}\sqrt{1 - \frac{\alpha}{\pi} + \frac{\sin(2\alpha)}{2\pi}}$$

Controlled half-wave rectifier - example

- Solution

a)

$$\alpha = \cos^{-1}\left[V_o\left(\frac{2\pi}{V_m}\right) - 1\right]$$

$$= \cos^{-1}\left\{40\left[\frac{2\pi}{\sqrt{2}(120)}\right] - 1\right\} = 61.2° = 1.07 \text{ rad}$$

b)

$$V_{\text{rms}} = \frac{\sqrt{2}(120)}{2}\sqrt{1 - \frac{1.07}{\pi} + \frac{\sin[2(1.07)]}{2\pi}} = 75.6 \text{ V}$$

c)

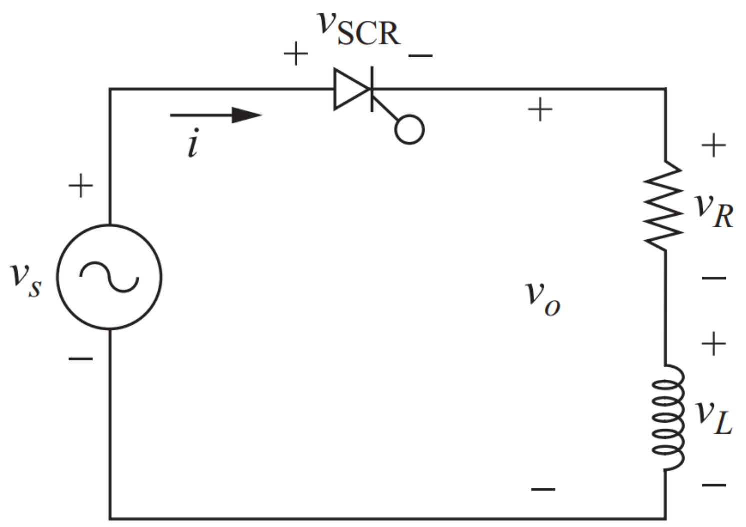
$$P_R = \frac{V_{\text{rms}}^2}{R} = \frac{(75.6)^2}{100} = 57.1 \text{ W}$$

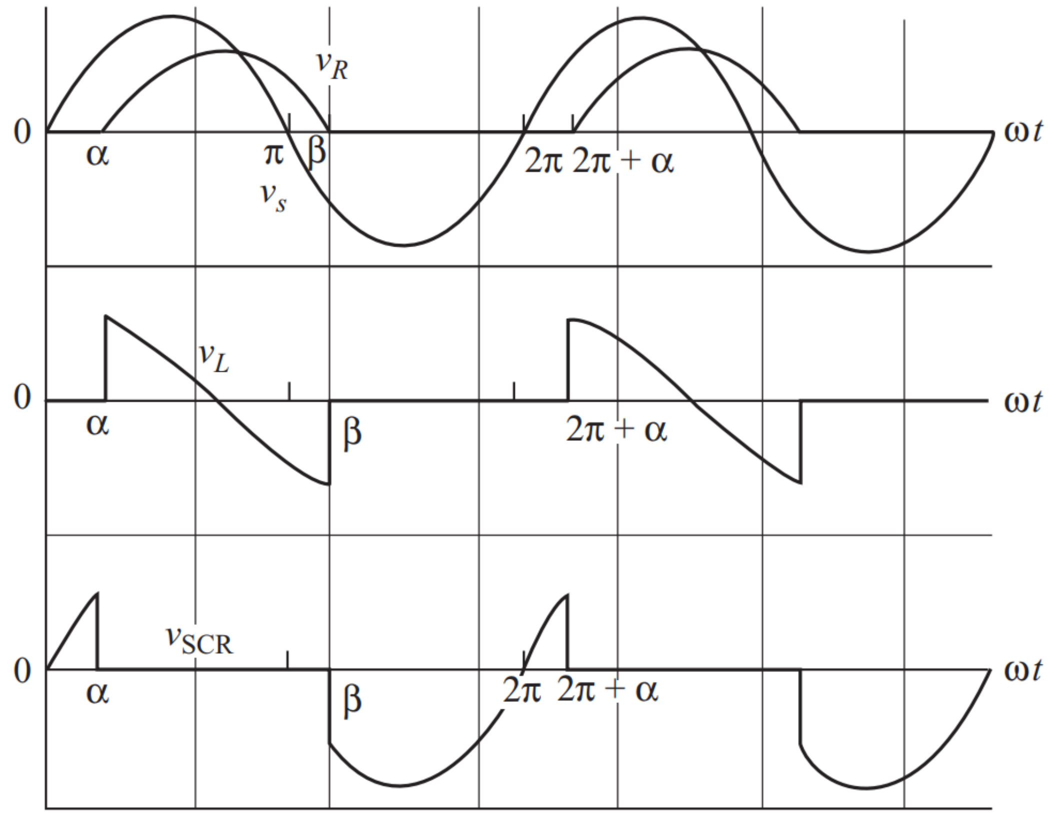
d)

$$\text{pf} = \frac{P}{S} = \frac{P}{V_{S,\,\text{rms}} I_{\text{rms}}} = \frac{57.1}{(120)(75.6/100)} = 0.63$$

Controlled half-wave rectifier with RL load

- Similar to the diode, when the load has an inductive element, the current has a phase with respect to the voltage, so the **thyristor keeps forward-biased** even **after** the voltage has dropped to zero.
- This angle β is called the **extinction angle**.





Controlled half-wave rectifier with RL load

- Similarly to the diode, the inductor introduces transients to the circuit.
- Solving the ODEs similarly to the diode results:

$$i(\omega t) = \begin{cases} \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)e^{(\alpha - \omega t)/\omega\tau}\right] & \text{for } \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

- To calculate the extinction angle, we must equal the previous current to zero and find β numerically.
- The angle $\beta - \alpha$ is called the **conduction angle** γ
- In this case, the DC output voltage is:

$$V_o = \frac{1}{2\pi}\int_{\alpha}^{\beta} V_m \sin(\omega t)\, d(\omega t) = \frac{V_m}{2\pi}(\cos\alpha - \cos\beta)$$

Outline

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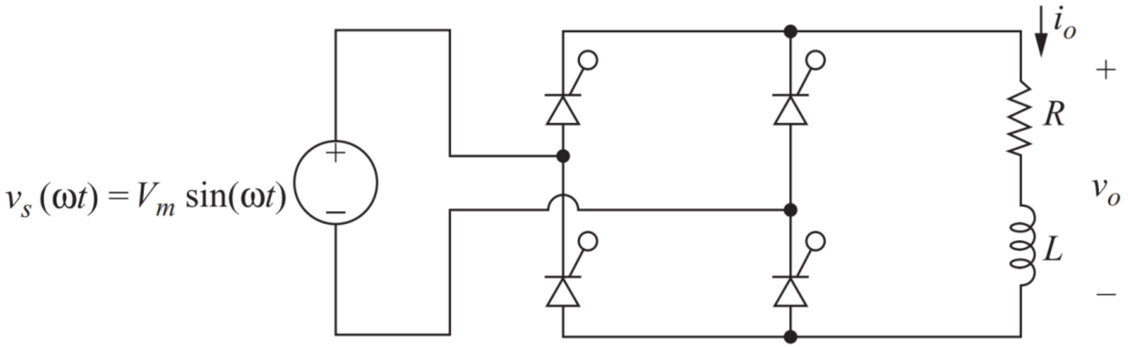
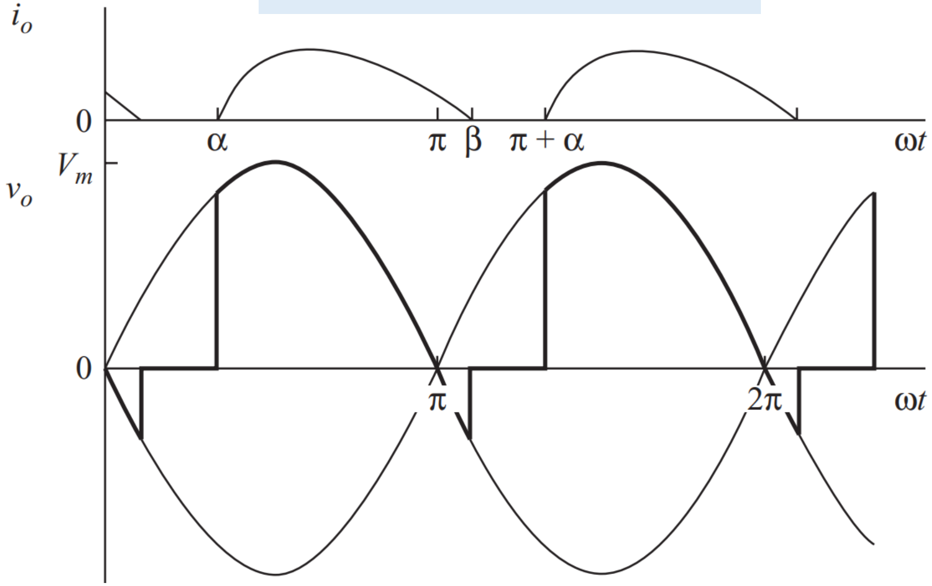
Controlled full-wave rectifier (single phase)

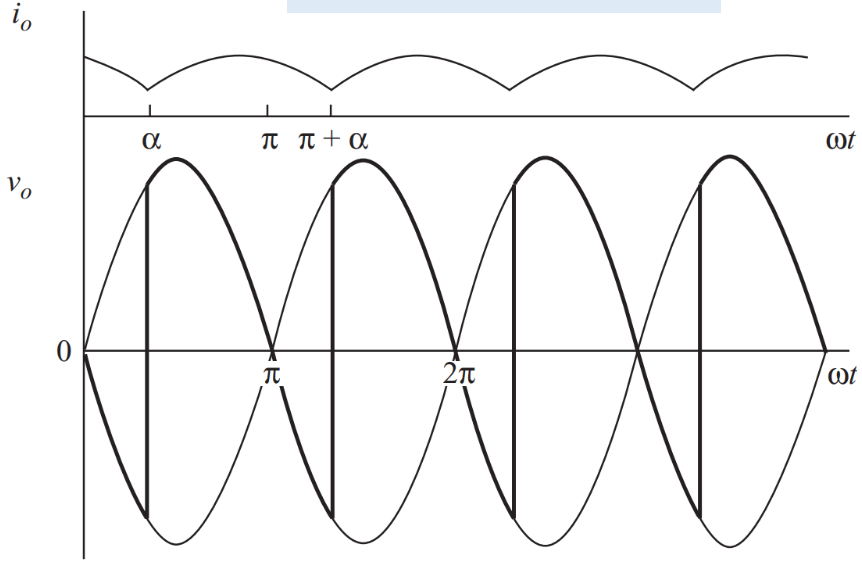
- The controlled full-wave rectifier is similar to the diode bridge, with the difference on the **firing angle** α.



Discontinuous current



Continuous current



Controlled full-wave rectifier (single phase)

- The load current is given by

$$i_o(\omega t) = \frac{V_m}{Z}\left[\sin(\omega t - \theta) - \sin(\alpha - \theta)\, e^{-(\omega t - \alpha)/\omega\tau}\right] \qquad \text{for } \alpha \leq \omega t \leq \beta$$

where

$$Z = \sqrt{R^2 + (\omega L)^2} \qquad \theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad \text{and} \qquad \tau = \frac{L}{R}$$

- The load current becomes zero at $\omega t = \beta$.
- If $\beta < \pi + \alpha$ the current remains at zero until $\omega t = \pi + \alpha$ (**discontinuous current mode**)
- If the current is still positive when $\omega t = \pi + \alpha$, we operate in **continuous current mode**
- We can calculate the α to ensure continuous current mode making the current > 0, which leads to:

$$\alpha \leq \tan^{-1}\left(\frac{\omega L}{R}\right) \leftarrow$$

To ensure continuous current mode, the firing angle depends on the load.

Controlled full-wave rectifier (single phase)

- We can use the Fourier transform to determine the output voltage and current in the continuous mode.
- Giving the Fourier series: $$v_o(\omega t) = V_o + \sum_{n=1}^{\infty} V_n \cos(n\omega_0 t + \theta_n)$$
- The DC term is $$V_o = \frac{1}{\pi}\int_{\alpha}^{\alpha+\pi} V_m \sin(\omega t)\, d(\omega t) = \frac{2V_m}{\pi}\cos\alpha$$
- The amplitude of the AC terms are $$V_n = \sqrt{a_n^2 + b_n^2}$$ where

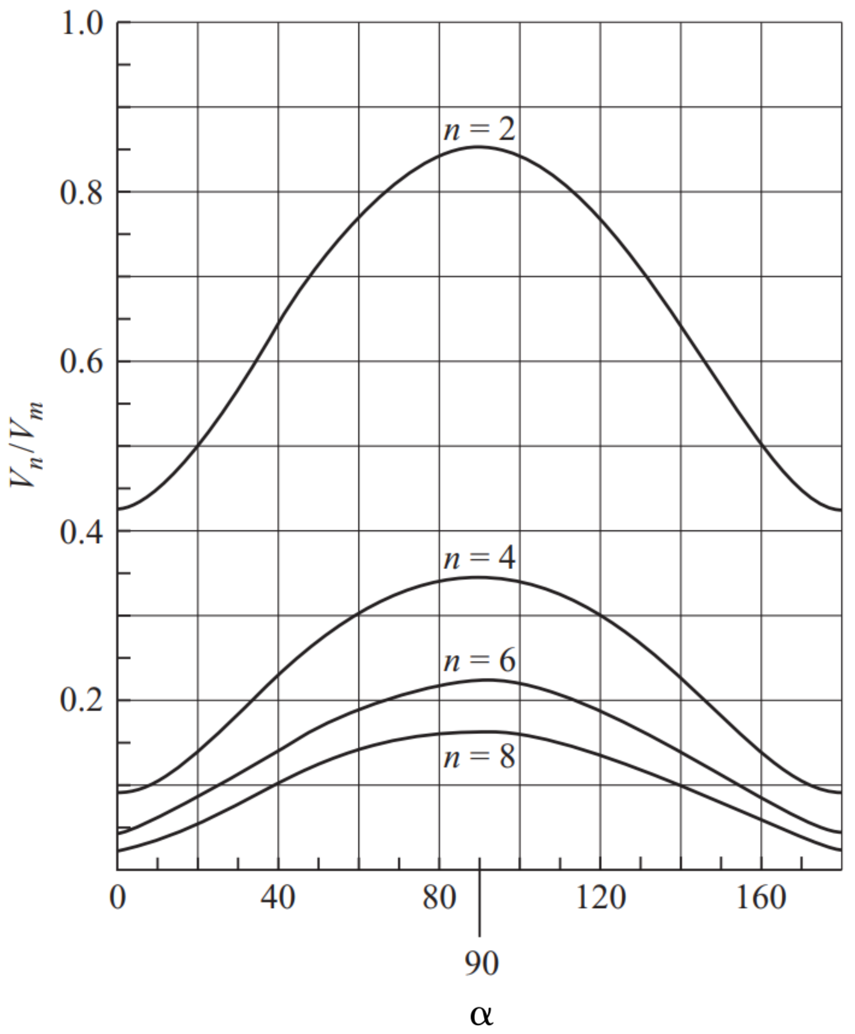
$$a_n = \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{n-1}\right]$$

$$b_n = \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{n+1} - \frac{\sin(n-1)\alpha}{n-1}\right]$$

$$n = 2, 4, 6, \ldots$$

Controlled full-wave rectifier (single phase)

- Normalized harmonics depending on the firing angle:



Controlled rectifier with RL-source Load

- When there is a DC source on the DC side, the thyristors can only be turned ON if they are forward-biased, which limits alpha to:

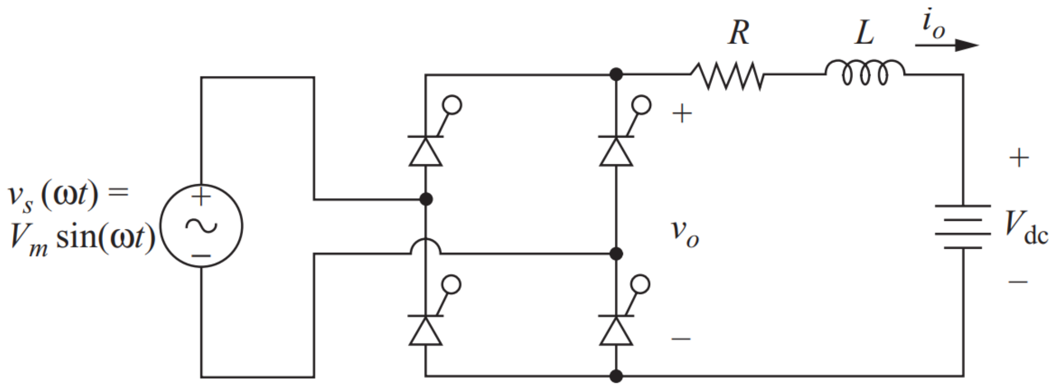
$$\alpha \geq \sin^{-1}\left(\frac{V_{dc}}{V_m}\right)$$

- For the continuous-current mode, the average (DC) output voltage is: $V_o = \frac{2V_m}{\pi}\cos\alpha$

Can be negative

- The average load current is: $I_o = \frac{V_o - V_{dc}}{R}$

- The power absorbed by the DC source is: $P_{dc} = I_o V_{dc}$

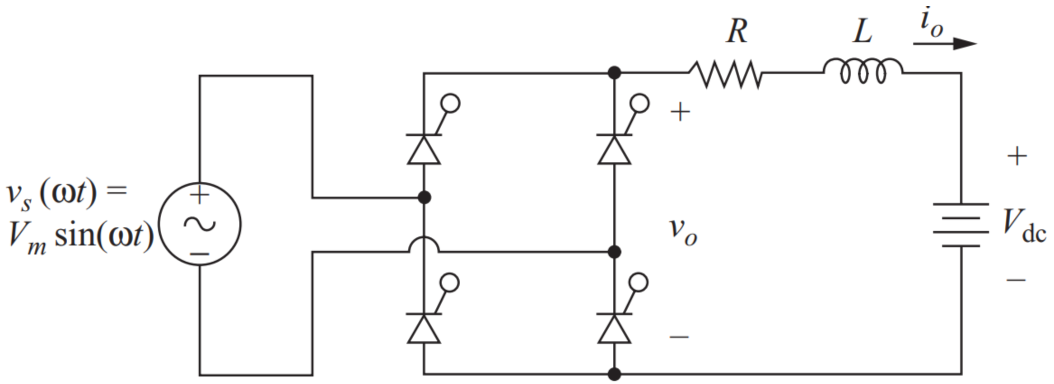


Operating the thyristor bridge as an inverter

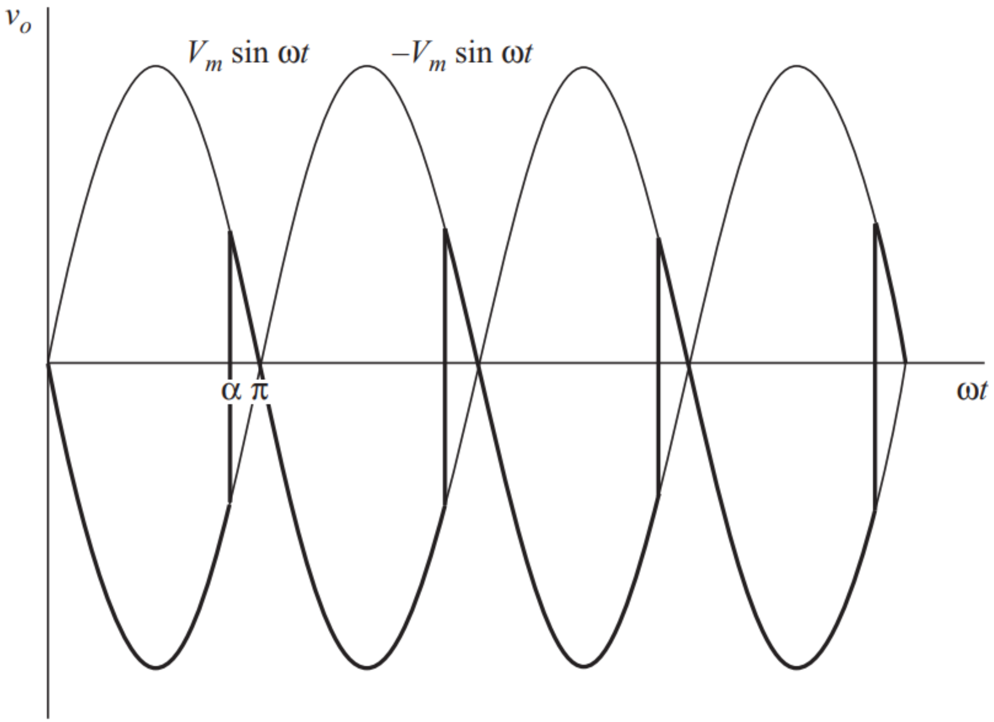
- In the inverter operation, power is fed from the DC-side to the AC-side.
- Given the current in the direction indicated below, the voltage applied to the DC-side must be negative to have a negative power (absorb power).

$$V_o = \frac{2V_m}{\pi} \cos \alpha$$

$0 < \alpha < 90° \rightarrow V_o > 0$	rectifier operation
$90° < \alpha < 180° \rightarrow V_o < 0$	inverter operation



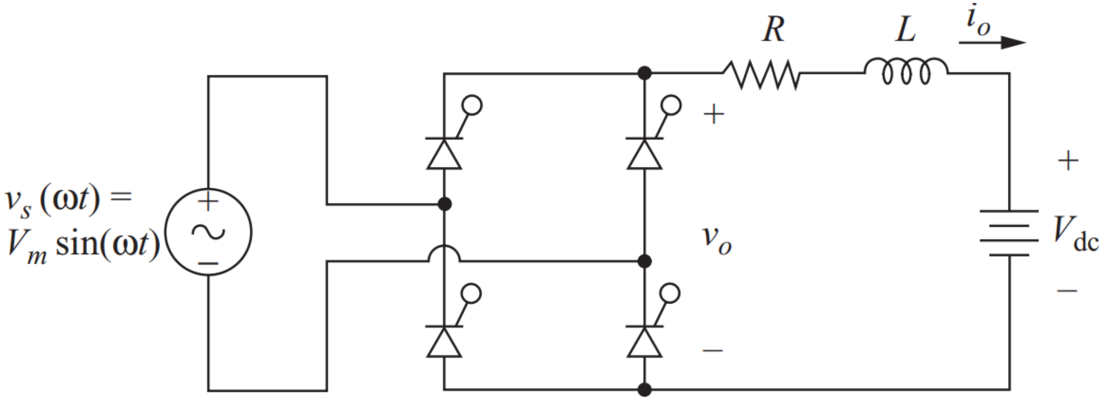
$\alpha = 150°$



Operating the thyristor bridge as an inverter - example

- Daniel Hart, example 4-11

The dc voltage in Fig. 4-14 represents the voltage generated by an array of solar cells and has a value of 110 V, connected such that $V_{dc} = -110$ V. The solar cells are capable of producing 1000 W. The ac source is 120 V rms, $R = 0.5\ \Omega$, and L is large enough to cause the load current to be essentially dc. Determine the delay angle α such that 1000 W is supplied by the solar cell array. Determine the power transferred to the ac system and the losses in the resistance. Assume ideal SCRs.



Operating the thyristor bridge as an inverter - example

- Solution

For the solar cell array to supply 1000 W, the average current must be

$$I_o = \frac{P_{dc}}{V_{dc}} = \frac{1000}{110} = 9.09 \text{ A}$$

The average output voltage of the bridge is determined from Eq. (4-36).

$$V_o = I_o R + V_{dc} = (9.09)(0.5) + (-110) = -105.5 \text{ V}$$

The required delay angle is determined from Eq. (4-35).

$$\alpha = \cos^{-1}\left(\frac{V_o \pi}{2V_m}\right) = \cos^{-1}\left[\frac{-105.5\pi}{2\sqrt{2}(120)}\right] = 165.5°$$

Power absorbed by the bridge and transferred to the ac system is determined from

$$P_{ac} = -V_o I_o = (-9.09)(-105.5) = 959 \text{ W}$$

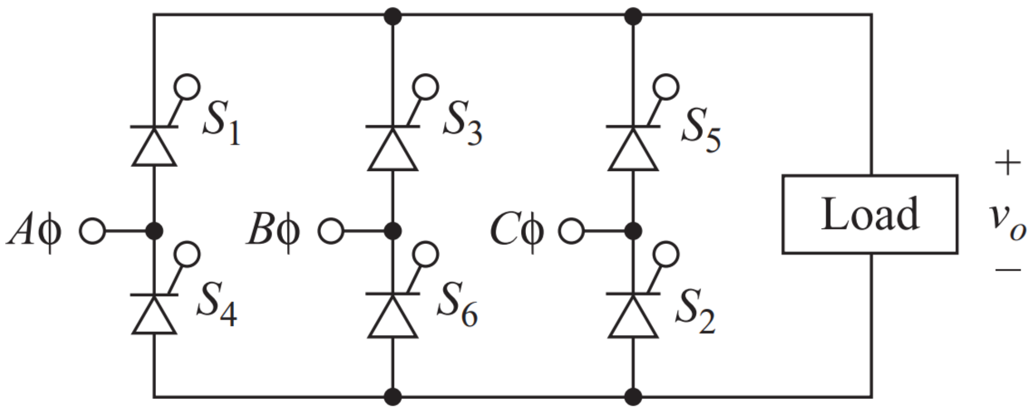
Power absorbed by the resistor is

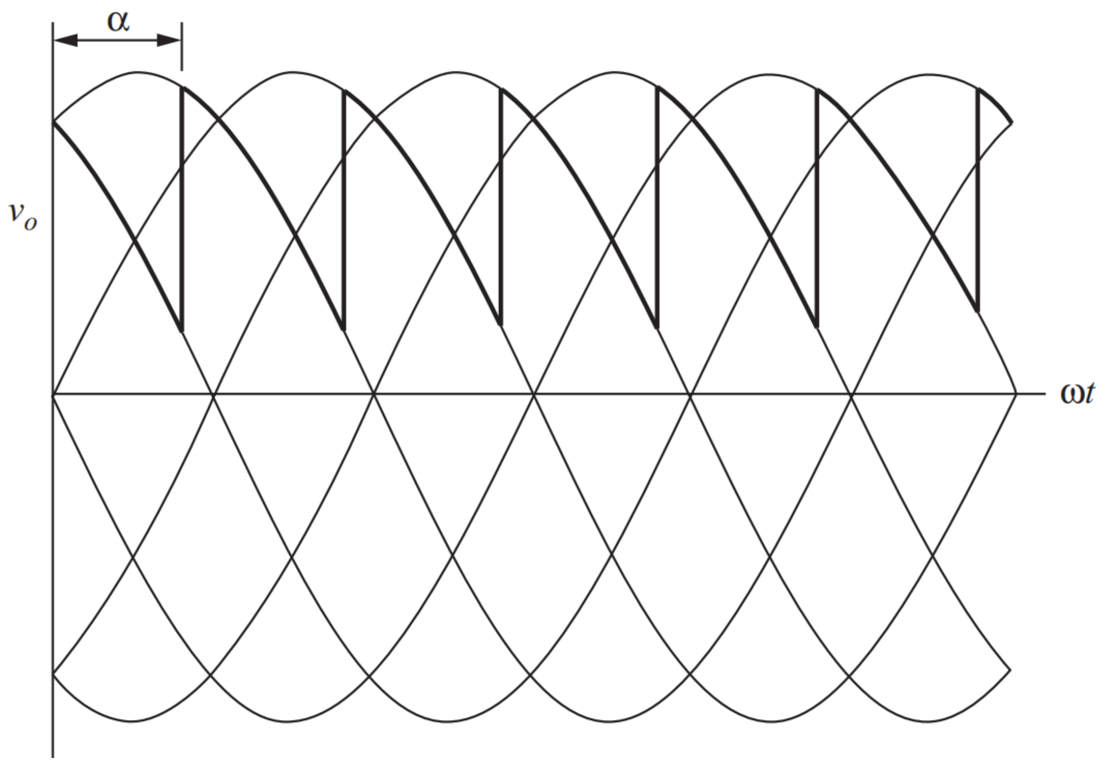
$$P_R = I_{rms}^2 R \approx I_o^2 R = (9.09)^2(0.5) = 41 \text{ W}$$

Controlled three-phase rectifiers

- The controlled three-phase rectifier operates similarly to a three-phase diode bridge. The difference is the control of the firing angle by the thyristor.
- The average (DC) voltage at the load is

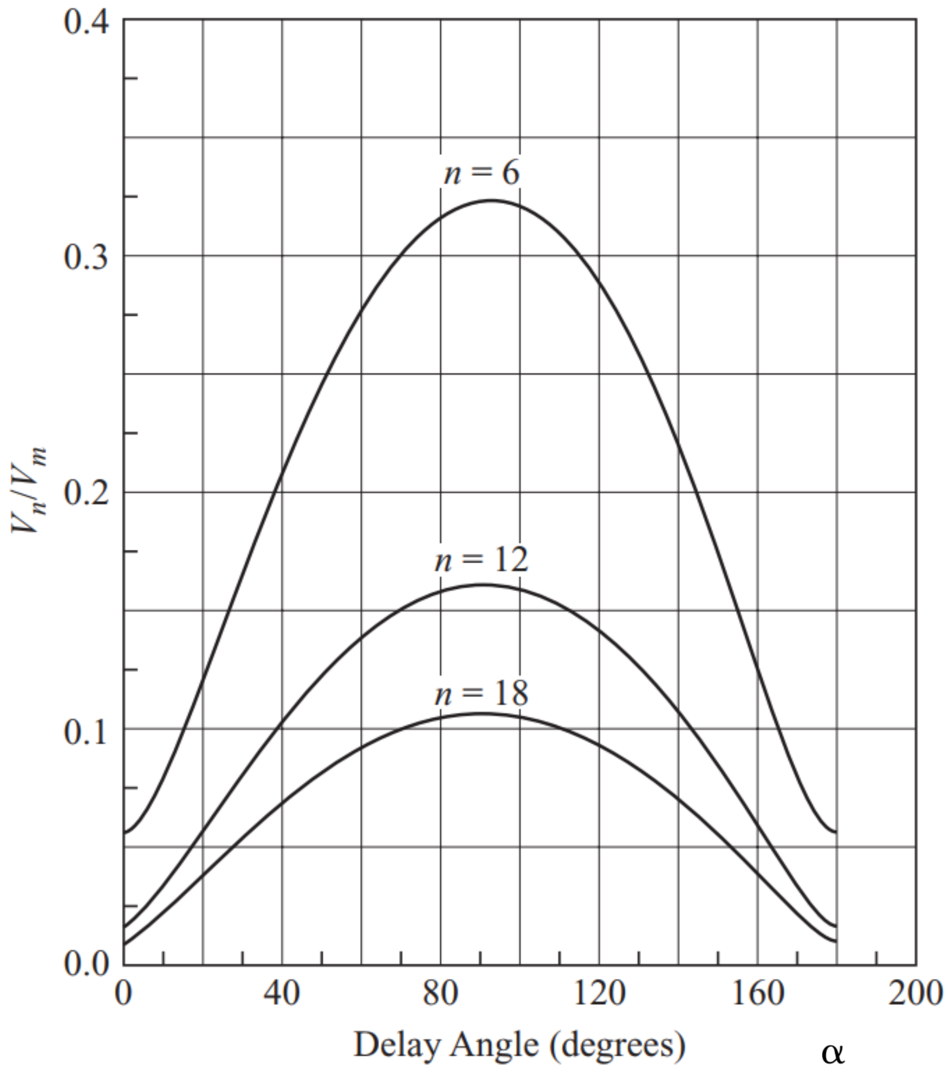
$$V_o = \frac{1}{\pi/3} \int_{\pi/3+\alpha}^{2\pi/3+\alpha} V_{m,L-L} \sin(\omega t)\, d(\omega t) = \frac{3V_{m,L-L}}{\pi} \cos\alpha$$





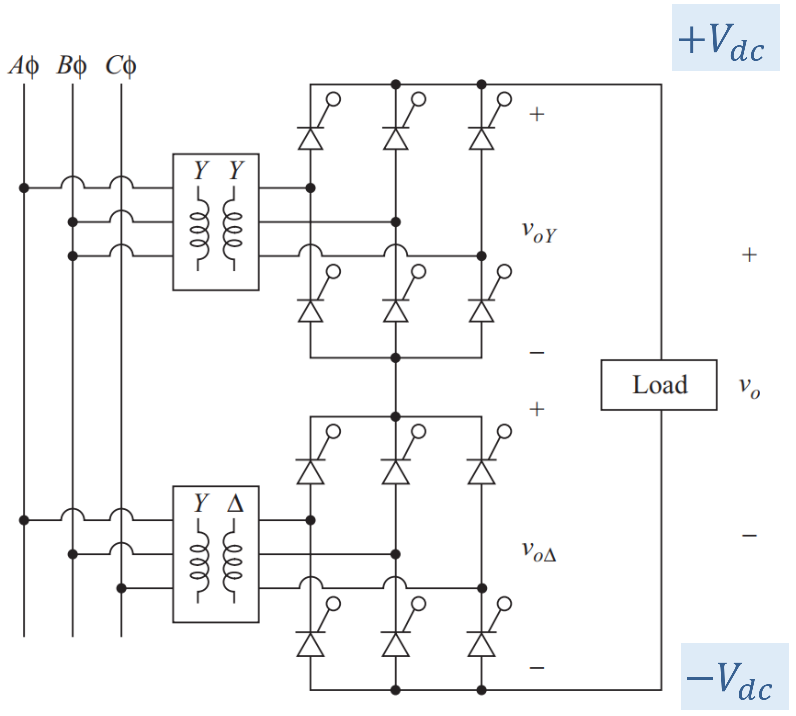
Controlled three-phase rectifiers

- Normalized harmonics depending on the firing angle:



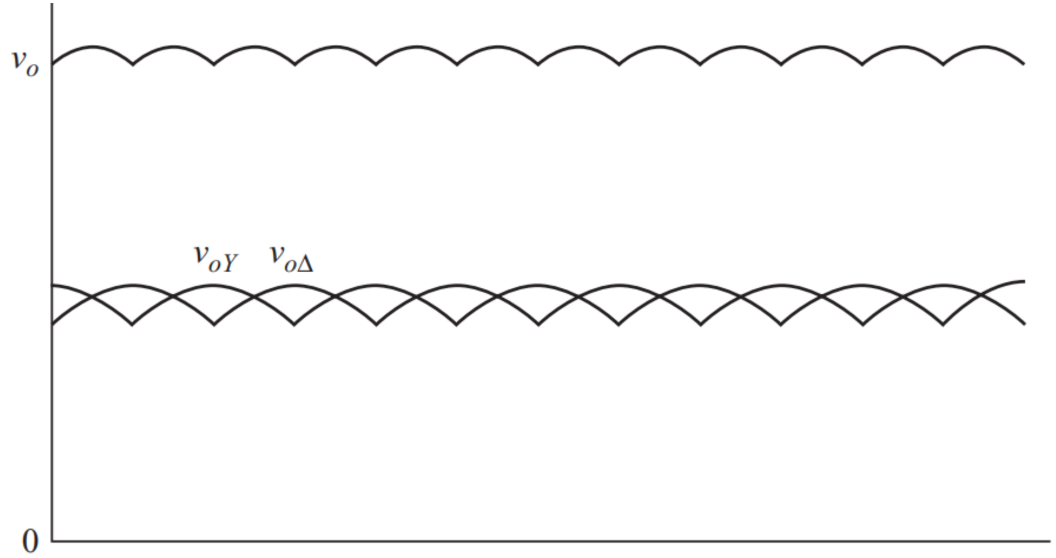
Twelve-pulse rectifier

- The harmonics can be further reduced by using two six-pulse bridges, as shown below.
- One bridge is supplied through a Y-Y transformer and the other by a Y-Δ transformer. This introduces a 30º phase shift between the source and the bridge.
- As the pulses of each bridge alternate every 60º, the sum of both DC voltages has an effective pulse of 30º.
- Another benefit of 12-pulse bridges are the reduced harmonic content. Harmonics are of order $12k \pm 1$



$$V_o = V_{o,Y} + V_{o,\Delta} = \frac{3V_{m,L-L}}{\pi}\cos\alpha + \frac{3V_{m,L-L}}{\pi}\cos\alpha = \frac{6V_{m,L-L}}{\pi}\cos\alpha$$

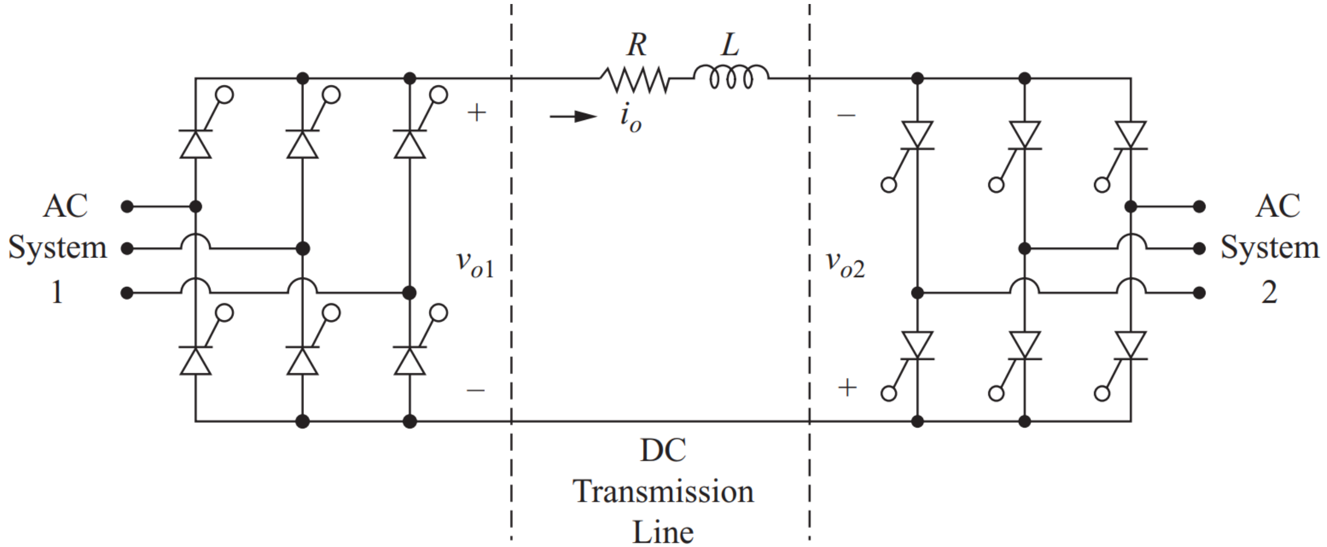
DC term



HVDC transmission using thyristor bridges

- By combining rectifier and inverter modes it is possible to transmit power between two asynchronous AC systems.
- The current has fixed direction defined by the thyristors' orientation, but the sign of the voltage defines the sign of the power flow.
- Both stations must coordinate the firing angles to enable a proper power flow.

$0 < \alpha < 90° \rightarrow V_o > 0$	rectifier operation
$90° < \alpha < 180° \rightarrow V_o < 0$	inverter operation



- DC Current: $I_o = \dfrac{V_{o1} + V_{o2}}{R}$

- DC Voltage:

$$V_{o1} = \frac{3V_{m1,L-L}}{\pi}\cos\alpha_1 \qquad V_{o2} = \frac{3V_{m2,L-L}}{\pi}\cos\alpha_2$$

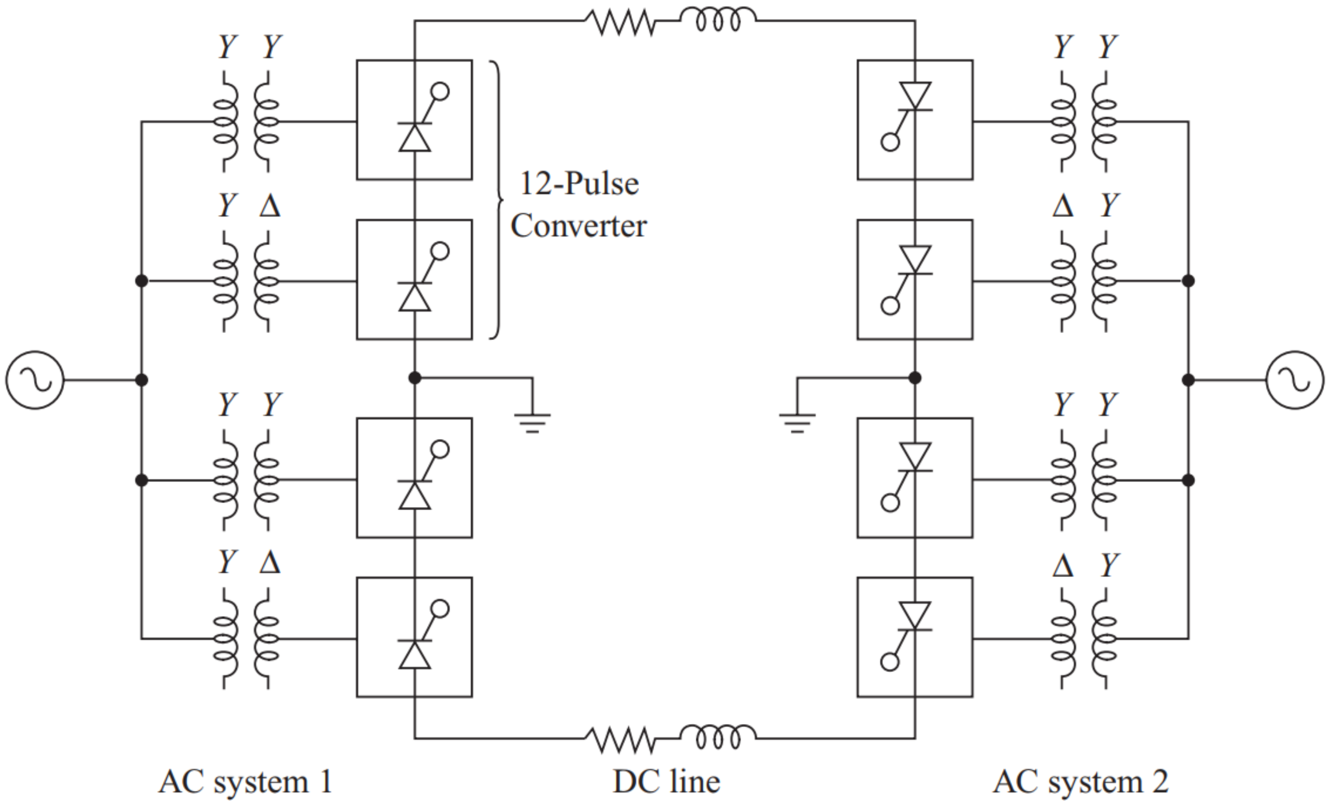
- Power flow

$$P_1 = V_{o1} I_o \qquad P_2 = V_{o2} I_o$$

HVDC transmission using thyristor bridges

- It is common to employ bipolar configuration. In emergency situations the system can operate as monopole, which current return via ground path.

$+V_{dc}$



$-V_{dc}$

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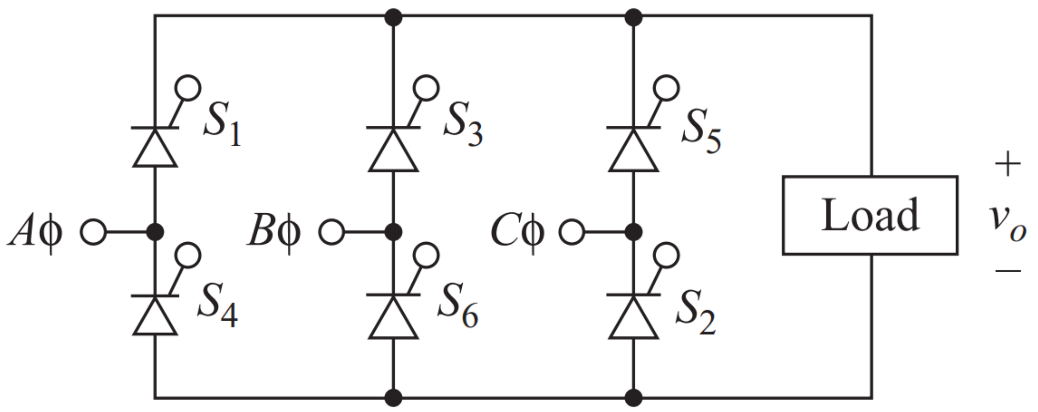
Controlled three-phase rectifier - Simulation

- 1) Simulate the following circuit. The load is composed by $R = 1\ \Omega, L = 0.1\ \mathrm{H}, V_{dc} = 7\ \mathrm{kV}$.
- 2) Measure AC voltages and currents and DC voltage and current.
- 3) Calculate the AC power and DC power.
- 4) Start the converter with $\alpha = 0°$and at $t = 0.4$ s change it to $\alpha = 20°$ and at $t = 0.8$ s change it to $\alpha = 50°$. Explain what happens with the DC current in those cases.



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