Electric Energy Conversion

5. DC/DC converters

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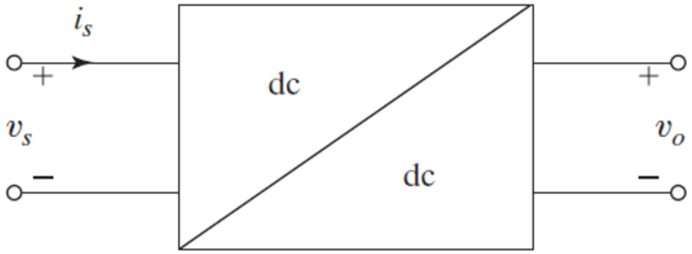
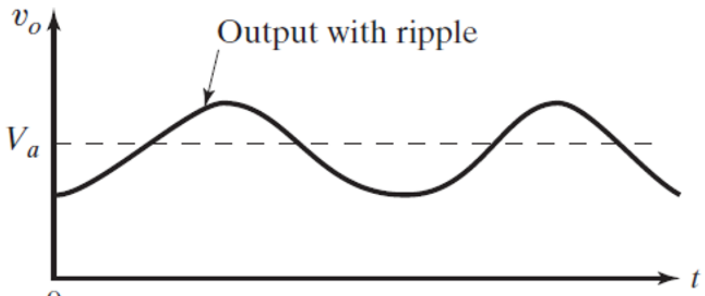
Outline

Introduction

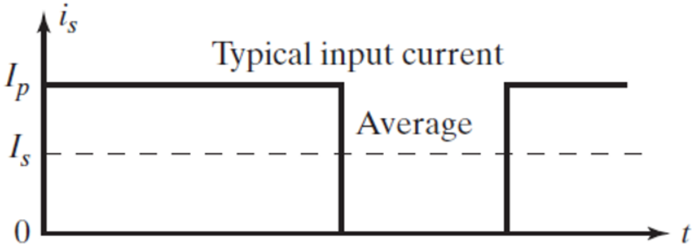
- In many industrial applications, it is required to generate a fixed or variable DC voltage from another DC voltage source.
- A DC/DC converter is the DC equivalent of an AC transformer with a controllable turns ratio. It can be used to step up or step down a DC voltage source.



(a) Block diagram



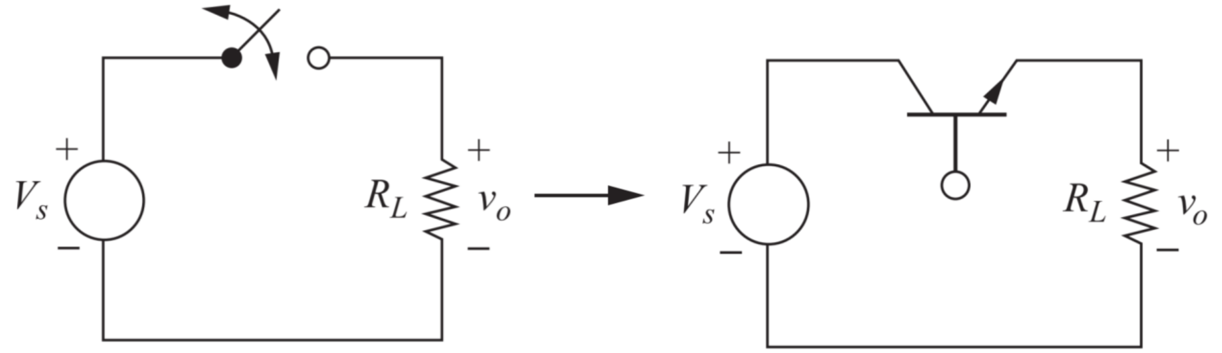
(b) Output voltage

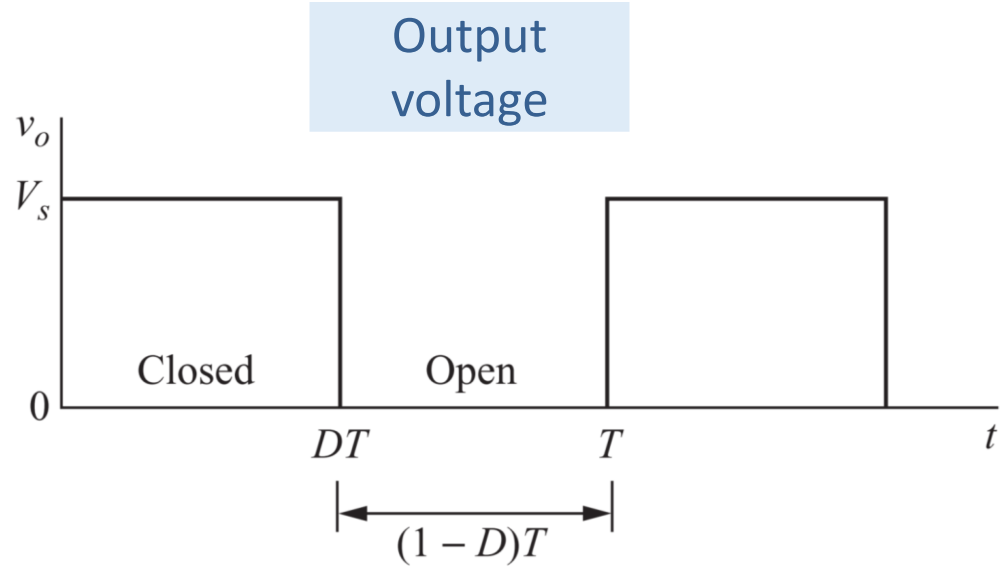


(c) Input current

Basic switching converter

- A basic DC/DC converter can be built with a single switch.
- The DC component of the output voltage is: $V_o = \frac{1}{T}\int_0^T v_o(t)\,dt = \frac{1}{T}\int_0^{DT} V_s\,dt = V_s D$
- Where D is the **duty cycle** or duty ratio: $D \equiv \frac{t_{\text{on}}}{t_{\text{on}} + t_{\text{off}}} = \frac{t_{\text{on}}}{T} = t_{\text{on}} f$



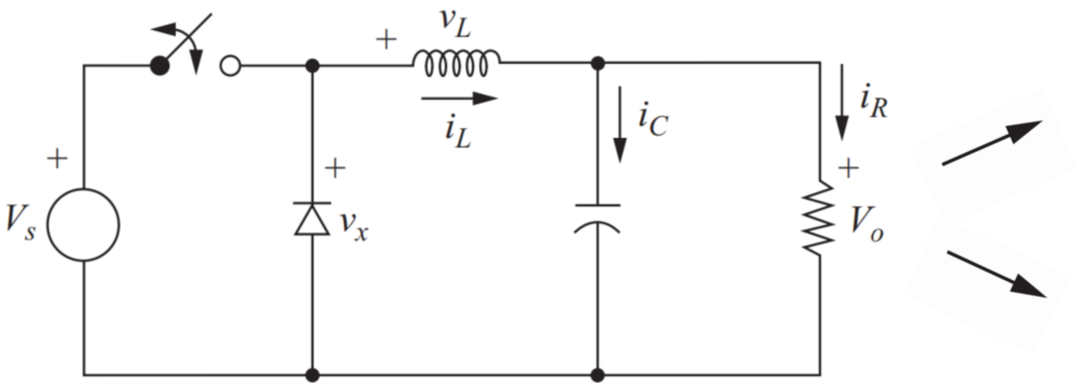


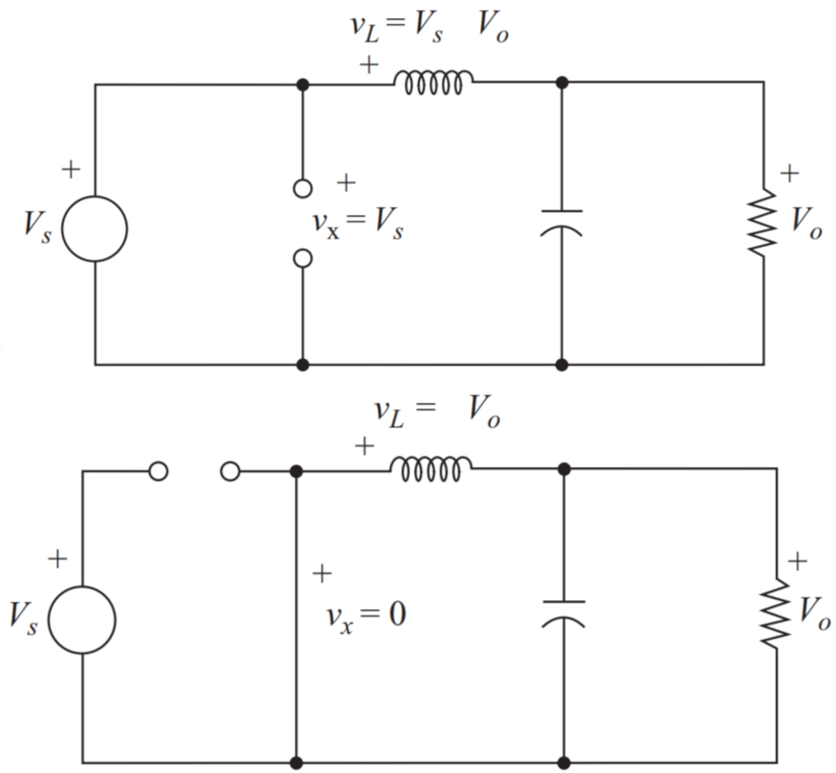
Outline

- Introduction
- Buck converter
- Boost converter
- Simulation

Buck (step-down) converter

- If a filter is added to the previous circuit, the output voltage will have less ripple.
- This converter is called **buck** or **step-down** because the output voltage is always less than the input.
- If the inductor current remains positive when the switch is open, the diode remains forward-biased the entire time (**continous mode**) and the average output voltage is $V_0 = V_S D$.





Switch closed

Switch open

Buck (step-down) converter

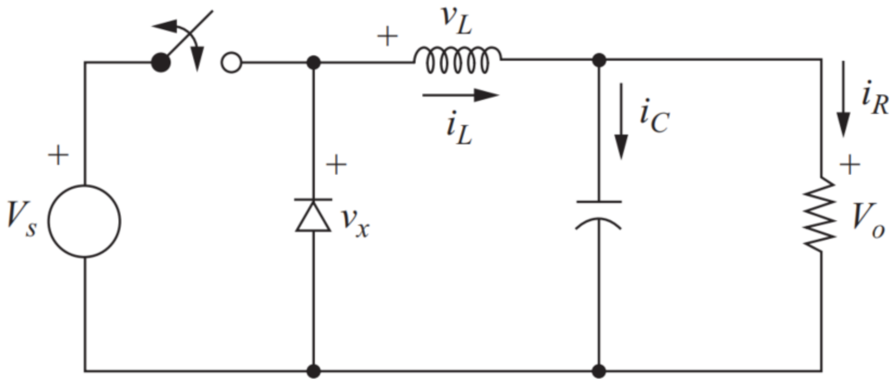
Assuming that the circuit is in the steady state, the input and output powers are equal and no energy accumulates in the inductor (average voltage is zero) or capacitor (average current is zero).

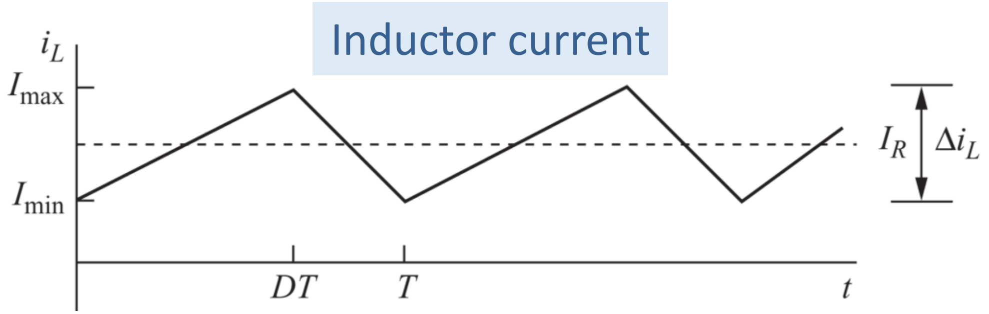
- **When the switch is closed:**

$$v_L = V_s - V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L} \xrightarrow{\text{Increases linearly}} (\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right)DT$$

- **When the switch is open:**

$$v_L = -V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{-V_o}{L} \xrightarrow{\text{Decreases linearly}} (\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$





Buck (step-down) converter

- **When the switch is closed:**

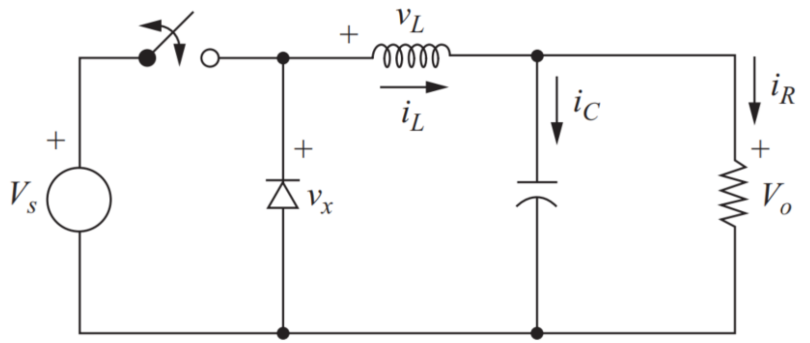
$$v_L = V_s - V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L} \xrightarrow{\text{Increases linearly}} (\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right)DT$$

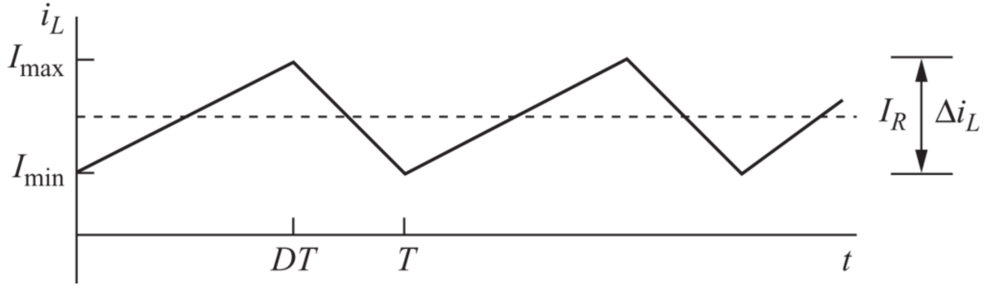
- **When the switch is open:**

$$v_L = -V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{-V_o}{L} \xrightarrow{\text{Decreases linearly}} (\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$

- As no energy accumulates in the inductor:

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0 \longrightarrow \left(\frac{V_s - V_o}{L}\right)(DT) - \left(\frac{V_o}{L}\right)(1-D)T = 0 \longrightarrow \boxed{V_o = V_s D} \longrightarrow V_o < V_s$$





Buck (step-down) converter

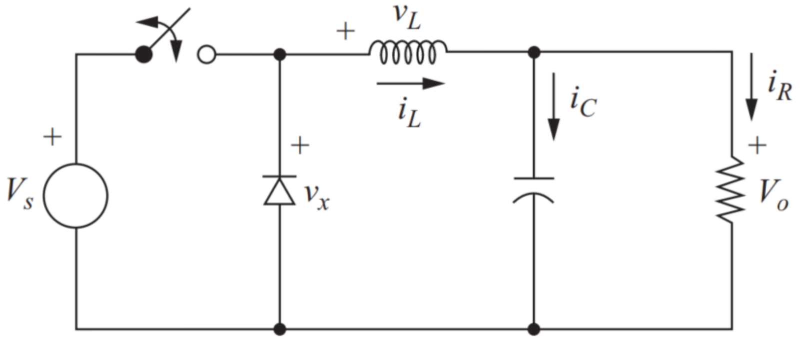
- The inductor is designed to provide a DC current with a given amount of ripple:

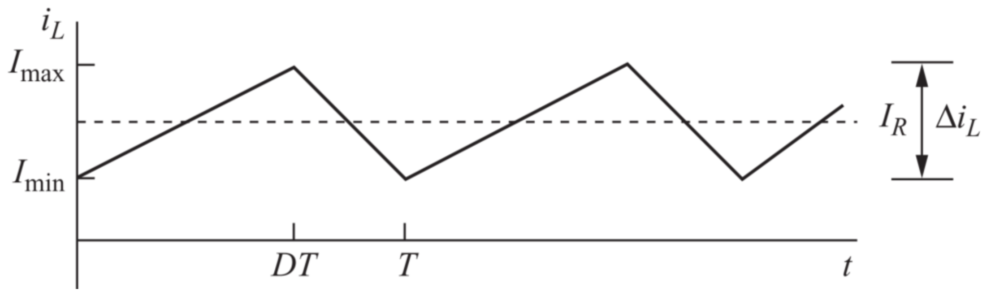
$$(\Delta i_L)_{\text{closed}} = \left(\frac{V_s - V_o}{L}\right) DT$$

$$\boxed{V_o = V_s D}$$

$$\Delta i_L = \left(\frac{V_s - V_o}{Lf}\right) D = \frac{V_o(1-D)}{Lf}$$

$$\boxed{L = \left(\frac{V_s - V_o}{\Delta i_L f}\right) D = \frac{V_o(1-D)}{\Delta i_L f}}$$





Buck (step-down) converter

- The ripple can also be calculated for the voltage at the capacitor using its variation of charge:

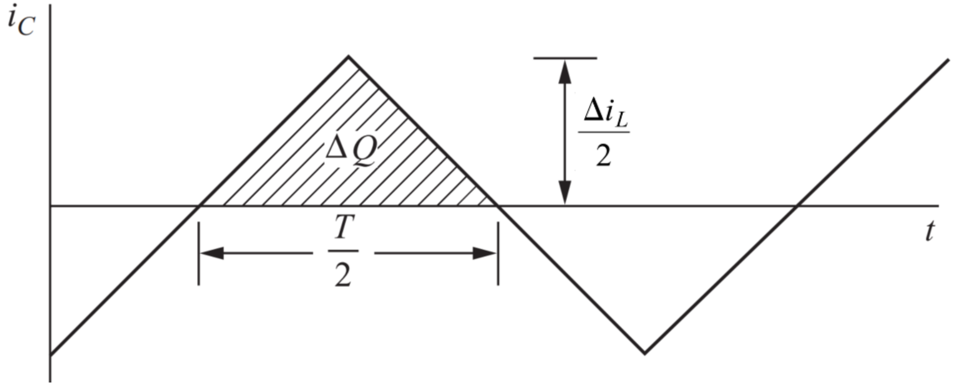
$$Q = CV_o \longrightarrow \Delta Q = C\Delta V_o \longrightarrow \Delta V_o = \frac{\Delta Q}{C}$$

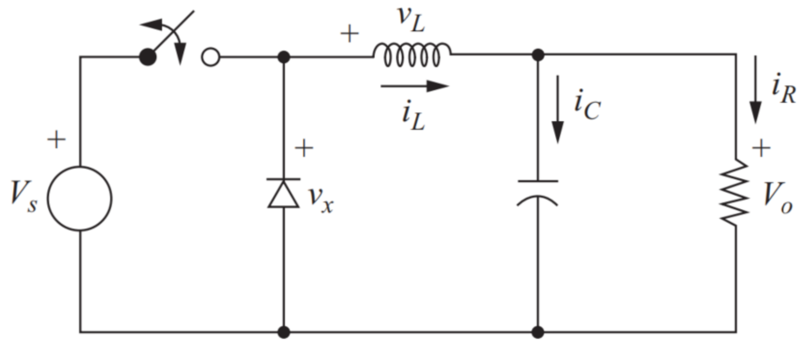
$$(\Delta i_L)_{\text{open}} = -\left(\frac{V_o}{L}\right)(1-D)T$$

$$\Delta Q = \frac{1}{2}\left(\frac{T}{2}\right)\left(\frac{\Delta i_L}{2}\right) = \frac{T\Delta i_L}{8} \longrightarrow \Delta V_o = \frac{T\Delta i_L}{8C} \longrightarrow \Delta V_o = \frac{TV_o}{8CL}(1-D)T = \frac{V_o(1-D)}{8LCf^2}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2}} \longrightarrow \boxed{C = \frac{1-D}{8L(\Delta V_o/V_o)f^2}}$$

Area of the triangle above positive axis

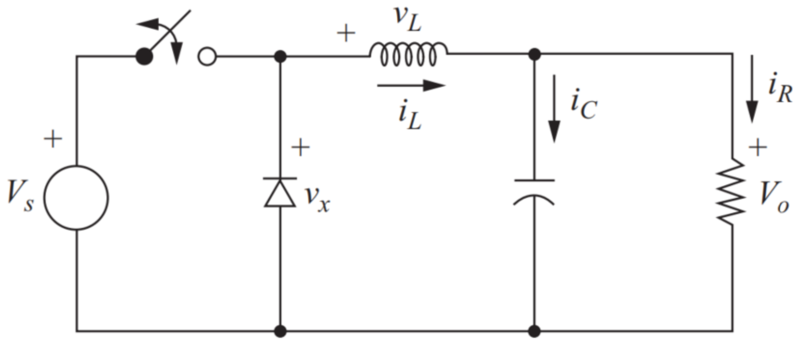




Buck (step-down) converter - example

The DC/DC converter below has the following parameters.

$$
\begin{aligned}
V_s &= 50\ \text{V} \\
D &= 0.4 \\
L &= 400\ \mu\text{H} \\
C &= 100\ \mu\text{F} \\
f &= 20\ \text{kHz} \\
R &= 20\ \Omega
\end{aligned}
$$



Assuming ideal components, calculate:

a) Output voltage V_o

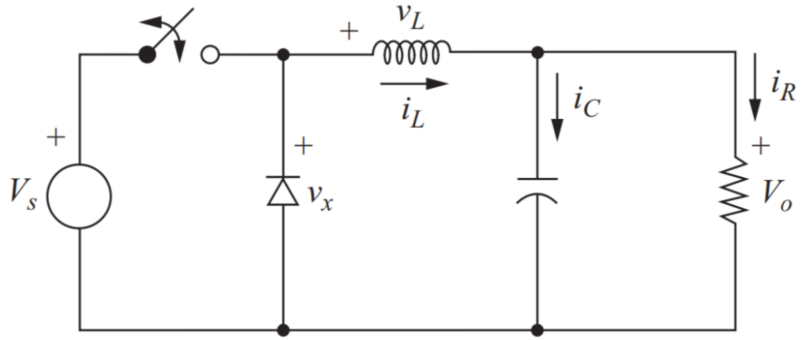
b) Maximum and Minimum inductor current

c) Output voltage ripple

$$\Delta i_L = \left(\frac{V_s - V_o}{Lf}\right) D = \frac{V_o(1-D)}{Lf}$$

Buck (step-down) converter - example

Solution



(a) The inductor current is assumed to be continuous, and the output voltage is computed from Eq. (6-9),

$$V_o = V_s D = (50)(0.4) = 20 \text{ V}$$

(b) Maximum and minimum inductor currents are computed from Eqs. (6-11) and (6-12).

$$I_{\max} = V_o\left(\frac{1}{R} + \frac{1-D}{2Lf}\right)$$

$$= 20\left[\frac{1}{20} + \frac{1-0.4}{2(400)(10)^{-6}(20)(10)^3}\right]$$

$$= 1 + \frac{1.5}{2} = 1.75 \text{ A}$$

$$I_{\min} = V_o\left(\frac{1}{R} - \frac{1-D}{2Lf}\right)$$

$$= 1 - \frac{1.5}{2} = 0.25 \text{ A}$$

The average inductor current is 1 A, and $\Delta i_L = 1.5$ A. Note that the minimum inductor current is positive, verifying that the assumption of continuous current was valid.

(c) The output voltage ripple is computed from Eq. (6-19).

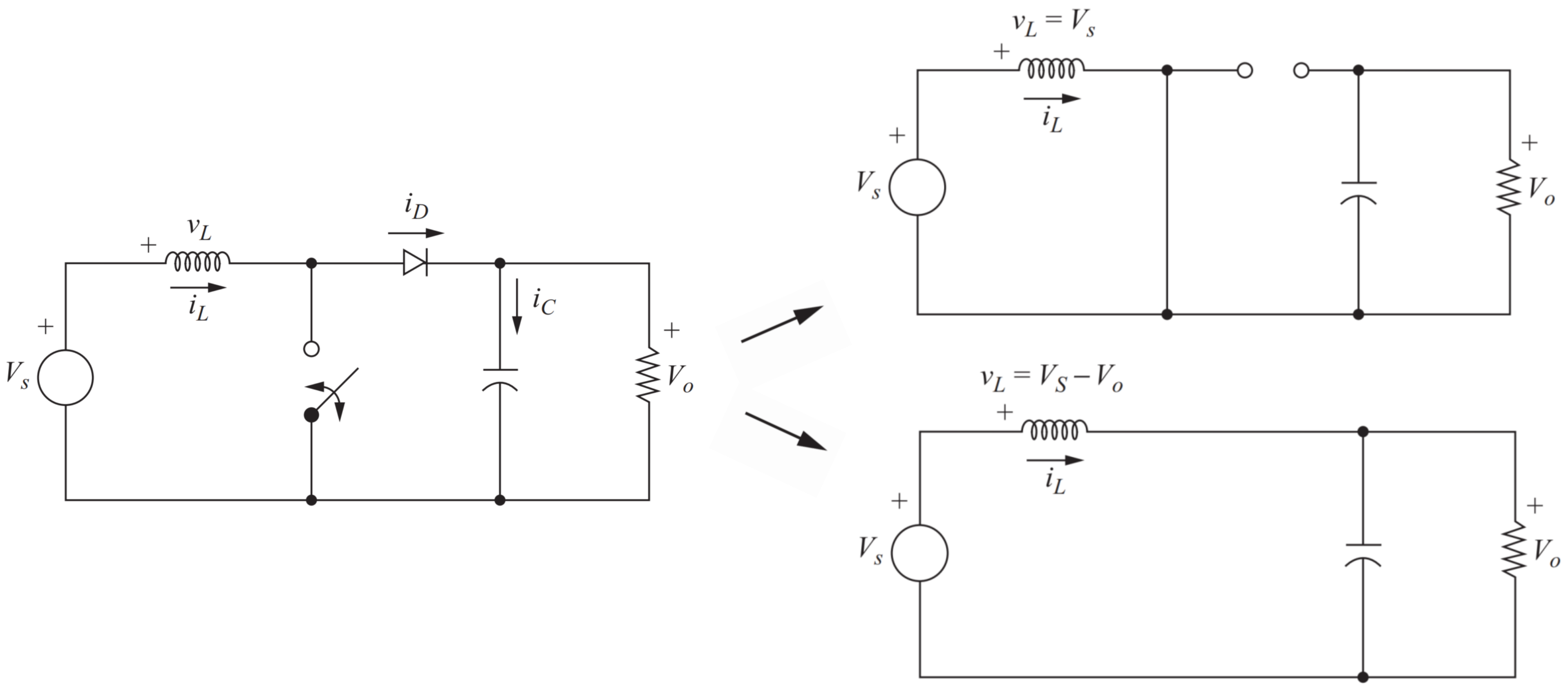
$$\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf^2} = \frac{1-0.4}{8(400)(10)^{-6}(100)(10)^{-6}(20{,}000)^2}$$

$$= 0.00469 = 0.469\%$$

Outline

Boost (step up) converter

- The boost (step up) converter produces a higher output voltage than the input.
- It can be analysed using the same assumptions as for the buck converter.



Switch closed

Switch open

Boost (step up) converter

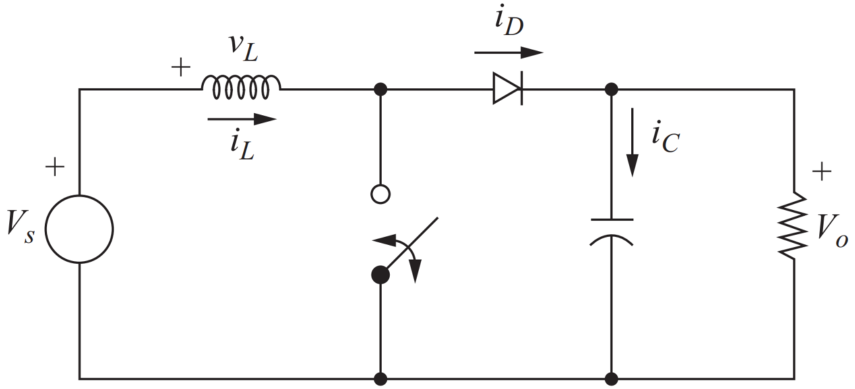
Assuming that the components are ideal, the circuit is in the steady-state, the inductor current is continuous (always positive) and that the capacitor is very large and can hold the output voltage constant at V_o.

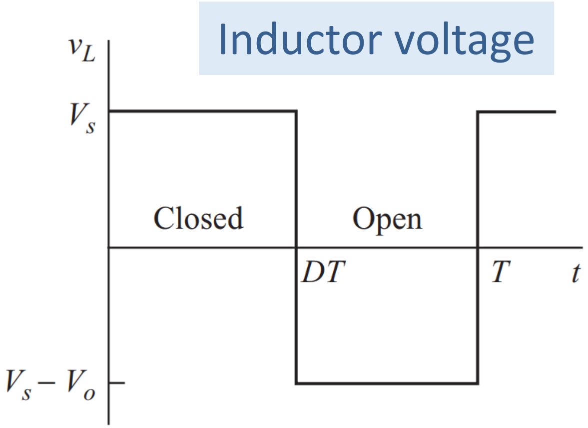
- **When the switch is closed:**

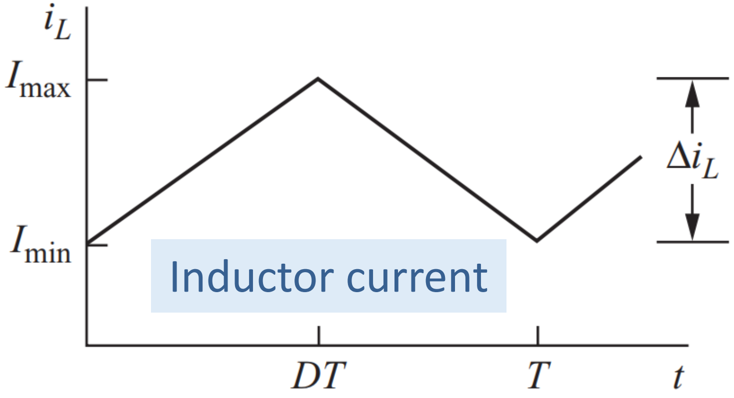
$$v_L = V_s = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s}{L} \xrightarrow{\text{Increases linearly}} \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \xrightarrow{\text{Solving for } \Delta i_L} (\Delta i_L)_{\text{closed}} = \frac{V_s DT}{L}$$

- **When the switch is open:**

$$v_L = V_s - V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L} \xrightarrow{\text{Decreases linearly}} \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \xrightarrow{\text{Solving for } \Delta i_L} (\Delta i_L)_{\text{open}} = \frac{(V_s - V_o)(1-D)T}{L}$$







Boost (step up) converter

- **When the switch is closed:**

$$v_L = V_s = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s}{L} \xrightarrow{\text{Increases linearly}} \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{DT} = \frac{V_s}{L} \xrightarrow{\text{Solving for } \Delta i_L} (\Delta i_L)_{\text{closed}} = \frac{V_s DT}{L}$$

- **When the switch is open:**

$$v_L = V_s - V_o = L\frac{di_L}{dt} \longrightarrow \frac{di_L}{dt} = \frac{V_s - V_o}{L} \xrightarrow{\text{Decreases linearly}} \frac{\Delta i_L}{\Delta t} = \frac{\Delta i_L}{(1-D)T} = \frac{V_s - V_o}{L} \xrightarrow{\text{Solving for } \Delta i_L} (\Delta i_L)_{\text{open}} = \frac{(V_s - V_o)(1-D)T}{L}$$

- As no energy accumulates in the inductor:

$$(\Delta i_L)_{\text{closed}} + (\Delta i_L)_{\text{open}} = 0 \longrightarrow \frac{V_s DT}{L} + \frac{(V_s - V_o)(1-D)T}{L} = 0 \longrightarrow \boxed{V_o = \frac{V_s}{1-D}} \longrightarrow V_o > V_s$$

As D approaches to 1, V_o approaches infinity (ideal components).
This will not happen with real devices.

Boost (step up) converter

- The average current at the inductor can be calculated assuming that the power delivered by the source is the same as the power consumed by the load (neglecting losses):

$$P_o = \frac{V_o^2}{R} = V_o I_o \xrightarrow{\text{Conservation of power}} V_s I_L = \frac{V_o^2}{R} = \frac{[V_s/(1-D)]^2}{R} = \frac{V_s^2}{(1-D)^2 R} \xrightarrow{\text{Solving for } i_L} \boxed{I_L = \frac{V_s}{(1-D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s}}$$

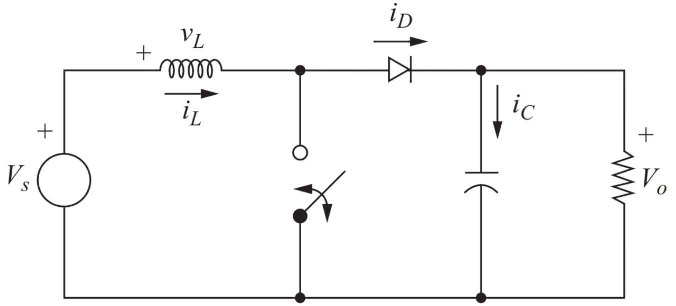
- If the current drops to much the inductor can leave the continous operating mode. To calculate this mode, we can combine two previous equations:

$$I_{\max} = I_L + \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} + \frac{V_s DT}{2L}$$

$$I_{\min} = I_L - \frac{\Delta i_L}{2} = \frac{V_s}{(1-D)^2 R} - \frac{V_s DT}{2L}$$

$$\xrightarrow{\text{Solving for } I_{min} = 0} \boxed{L_{\min} = \frac{D(1-D)^2 R}{2f}}$$

- We can also design the inductor for a given max ripple: $L = \dfrac{V_s DT}{\Delta i_L} = \dfrac{V_s D}{\Delta i_L f}$

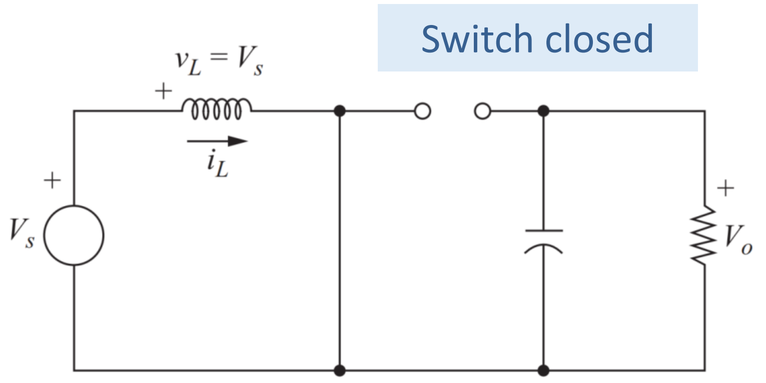


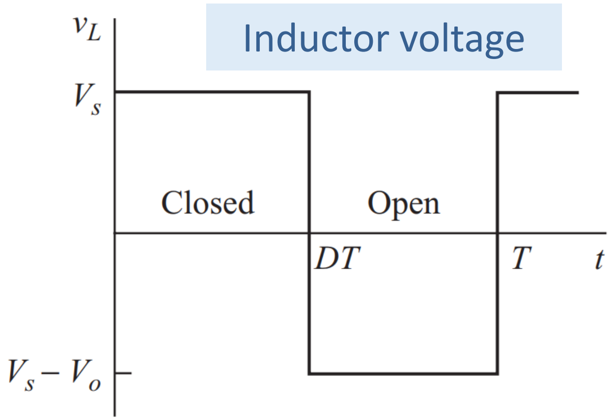
Boost (step up) converter

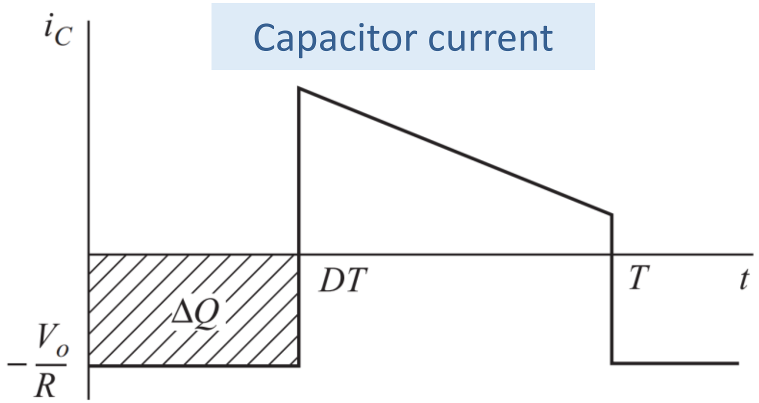
- If the capacitor is not infinite, a voltage ripple will appear on the load side. This ripple can be calculated from the energy stored in the capacitor during the charge and discharge periods.

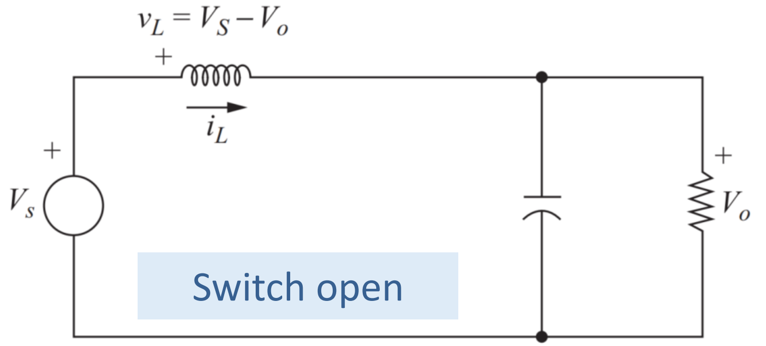
$$|\Delta Q| = \left(\frac{V_o}{R}\right)DT = C\Delta V_o \longrightarrow \boxed{\frac{\Delta V_o}{V_o} = \frac{D}{RCf}}$$

Load current









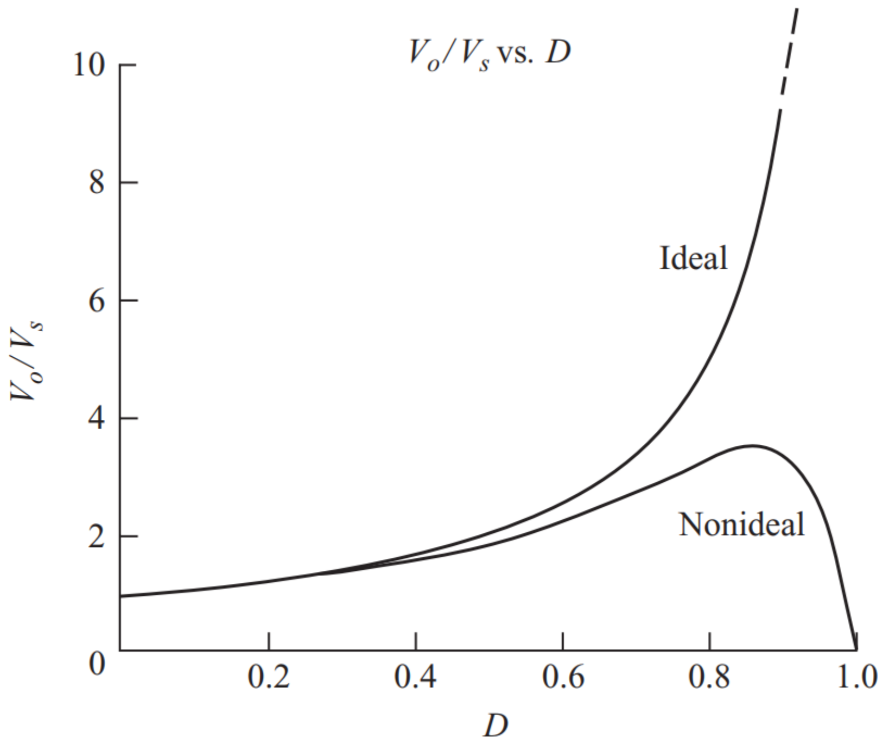
Boost (step up) converter

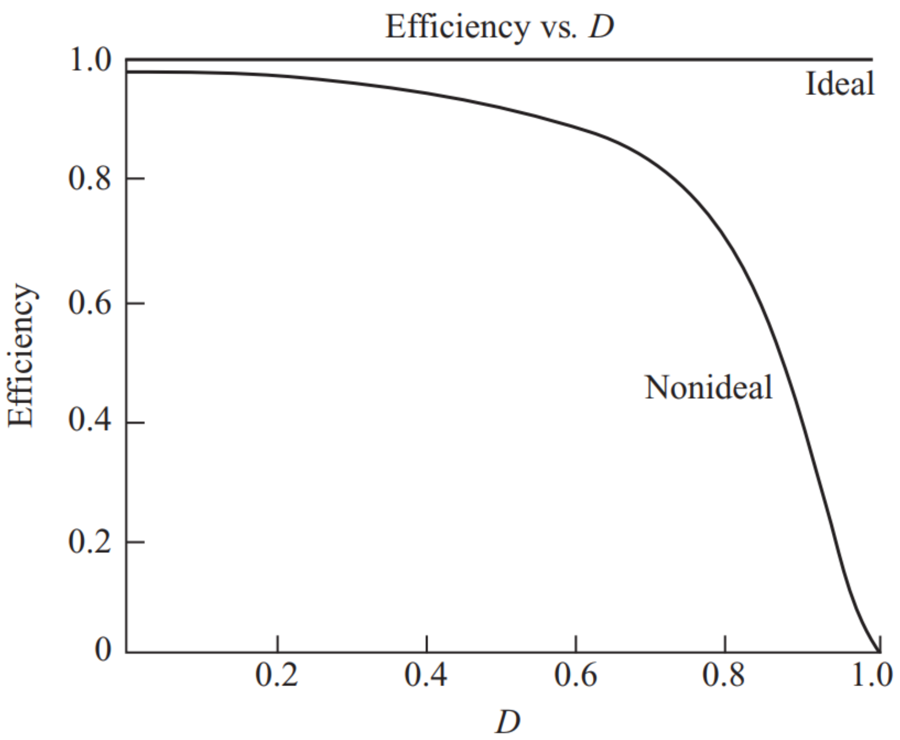
- If we consider the non-ideal inductor, its resistance will affect the boost converter for high values of duty cycles.

$$V_o = \frac{V_s}{1-D}$$

Considering losses

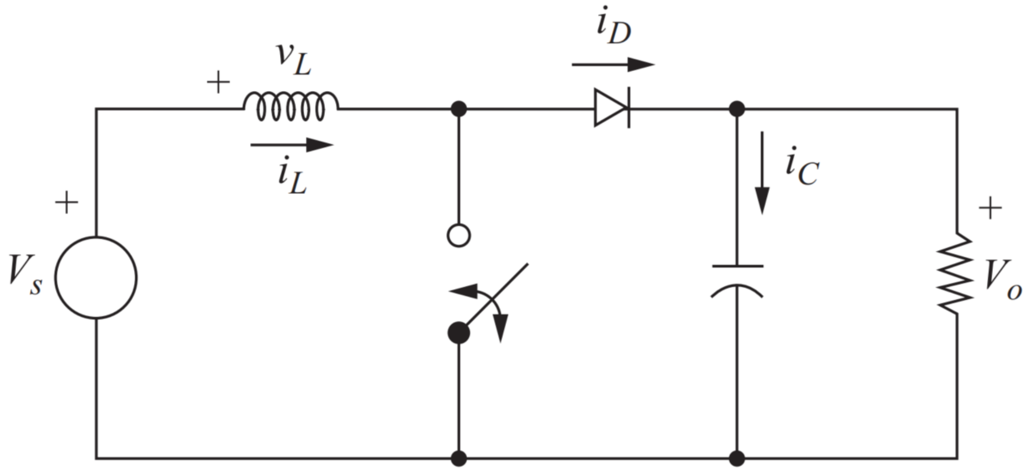
$$V_o = \left(\frac{V_s}{1-D}\right)\left(\frac{1}{1 + r_L/[R(1-D)^2]}\right)$$





Boost (step up) converter - example

Design a boost converter that will have an output of 30 V from a 12-V source. Design for continuous inductor current and an output ripple voltage of less than one percent. The load is a resistance of 50 Ω. Assume ideal components for this design.
the switching frequency is selected at 25 kHz to be above the audio range



$$\boxed{V_o = \frac{V_s}{1 - D}}$$

$$\boxed{L_{\min} = \frac{D(1 - D)^2 R}{2f}}$$

$$\boxed{I_L = \frac{V_s}{(1 - D)^2 R} = \frac{V_o^2}{V_s R} = \frac{V_o I_o}{V_s}}$$

$$L = \frac{V_s D T}{\Delta i_L} = \frac{V_s D}{\Delta i_L f}$$

$$\boxed{\frac{\Delta V_o}{V_o} = \frac{D}{RCf}}$$

Boost (step up) converter - example

- Solution

First, determine the duty ratio from Eq. (6-27),

$$D = 1 - \frac{V_s}{V_o} = 1 - \frac{12}{30} = 0.6$$

If the switching frequency is selected at 25 kHz to be above the audio range, then the minimum inductance for continuous current is determined from Eq. (6-32).

$$L_{\min} = \frac{D(1-D)^2(R)}{2f} = \frac{0.6(1-0.6)^2(50)}{2(25{,}000)} = 96\ \mu\text{H}$$

To provide a margin to ensure continuous current, let $L = 120\ \mu\text{H}$. Note that L and f are selected somewhat arbitrarily and that other combinations will also give continuous current.

Using Eqs. (6-28) and (6-25),

$$I_L = \frac{V_s}{(1-D)^2(R)} = \frac{12}{(1-0.6)^2(50)} = 1.5\text{ A}$$

$$\frac{\Delta i_L}{2} = \frac{V_s DT}{2L} = \frac{(12)(0.6)}{(2)(120)(10)^{-6}(25{,}000)} = 1.2\text{ A}$$

$$I_{\max} = 1.5 + 1.2 = 2.7\text{ A}$$

$$I_{\min} = 1.5 - 1.2 = 0.3\text{ A}$$

The minimum capacitance required to limit the output ripple voltage to 1 percent is determined from Eq. (6-35).

$$C \geq \frac{D}{R(\Delta V_o/V_o)f} = \frac{0.6}{(50)(0.01)(25{,}000)} = 48\ \mu\text{F}$$

Outline

- Introduction
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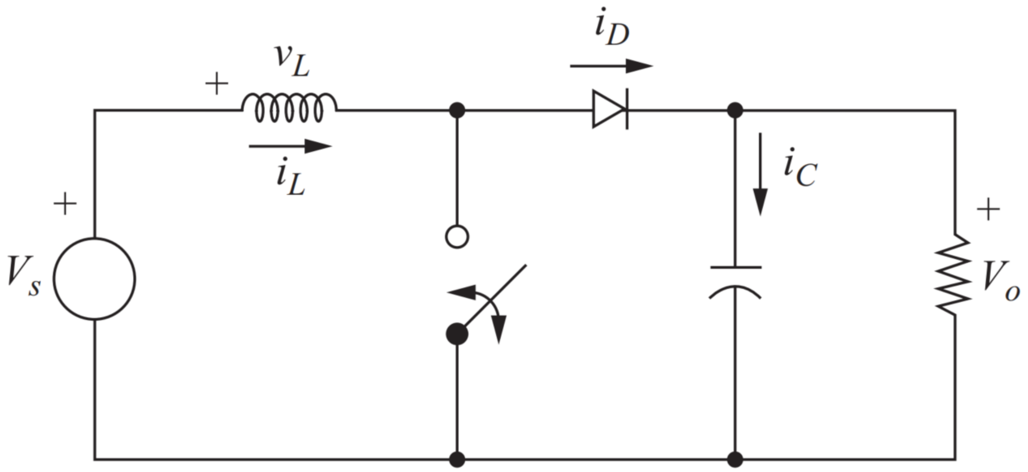
- Simulation

Simulation

- Simulate a boost converter.

Data:

- Vs = 100 V (DC)
- L = 100 uH
- C = 10 uF
- R = 100 Ohm
- f = 20 kHz



a) Verify if the converter is in continuous or discontinuous mode.

b) Define the minimum inductor to operate in continuous mode

c) Design a capacitor to achieve 1% voltage ripple when D = 0.7

d) Raise the curve between Vo and D

e) Raise the curve between efficiency and D

Electric Energy Conversion

4. Controlled rectifiers

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