

# Electric Energy Conversion

## 7. DC/AC converters – part 1

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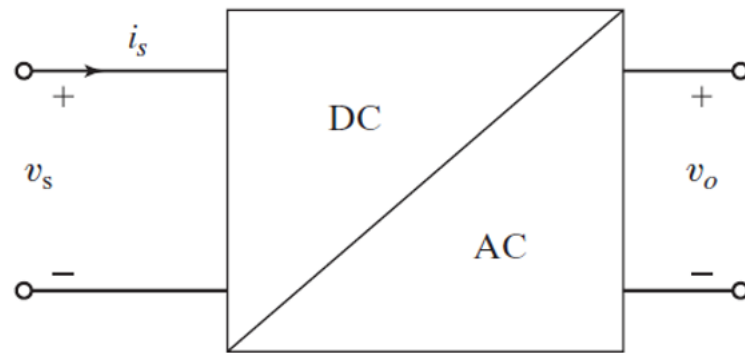


# Outline

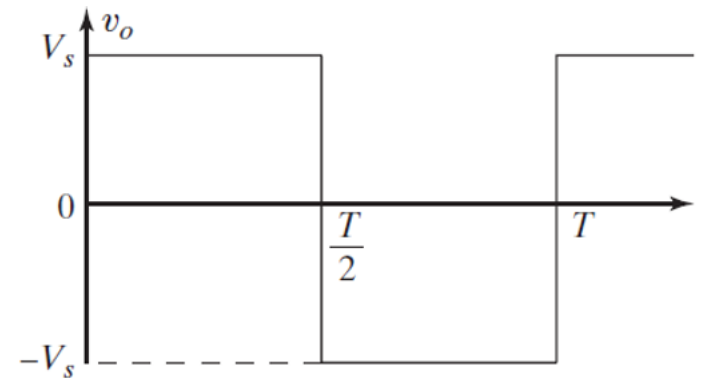
- Single-phase inverters
- Three-phase inverters
- Simulation

# Introduction

- DC/AC converters are also called **inverters** because they can synthesise AC waveforms from a DC source.
- The AC waveforms are created by switching ON and OFF the converter transistors.



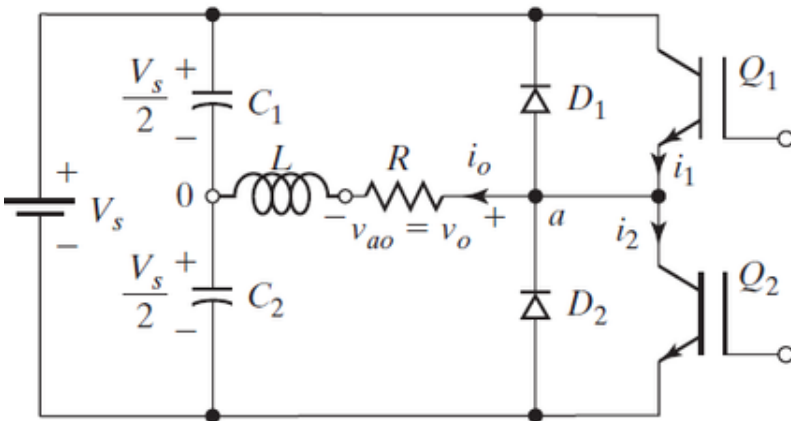
(a) Block diagram



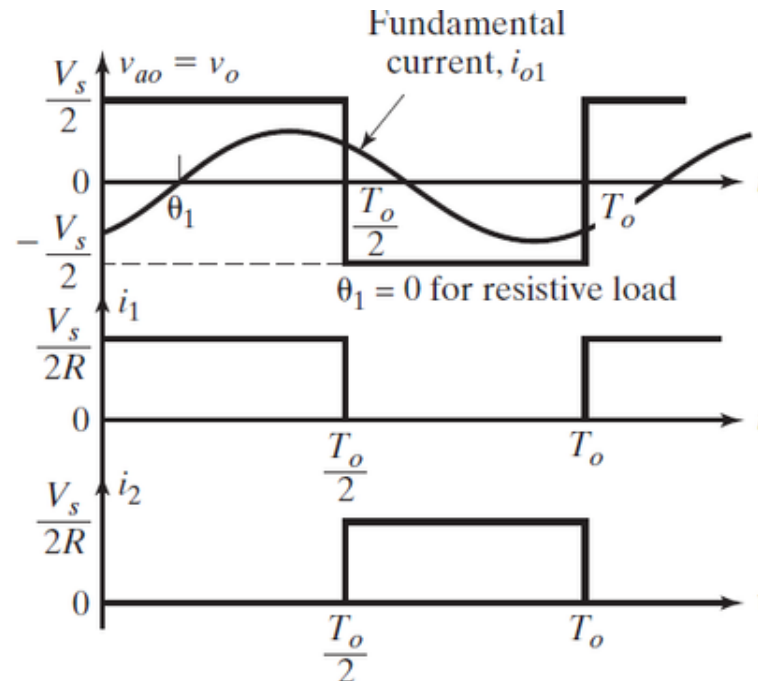
(b) Output voltage

# Principle of operation – half-bridge

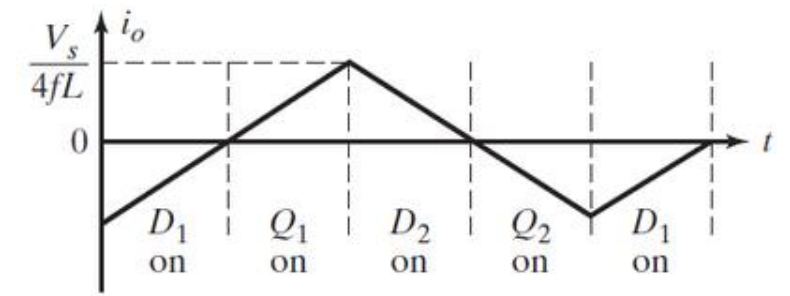
- The principle of operation of inverters can be illustrated by analysing a single phase-inverter.
- When  $Q_1$  is *ON* ( $Q_2$  *OFF*) the instantaneous voltage on the load is  $V_s/2$ .
- When  $Q_2$  is *ON* ( $Q_1$  *OFF*) for the instantaneous voltage on the load is  $-V_s/2$ .
- The commutation diodes  $D_1$  and  $D_2$  create a path for the current that cannot return through  $Q_1$  and  $Q_2$ . So each pair  $Q_1/D_1$  and  $Q_2/D_2$  conducts for  $T_0/2$ .



(a) Circuit



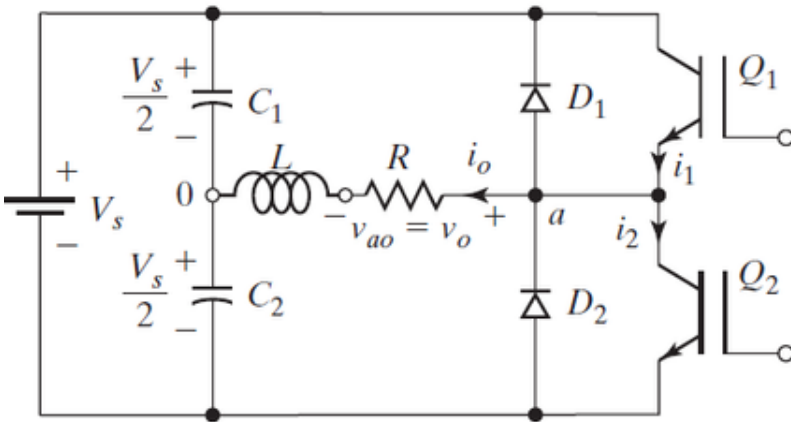
(b) Waveforms with resistive load



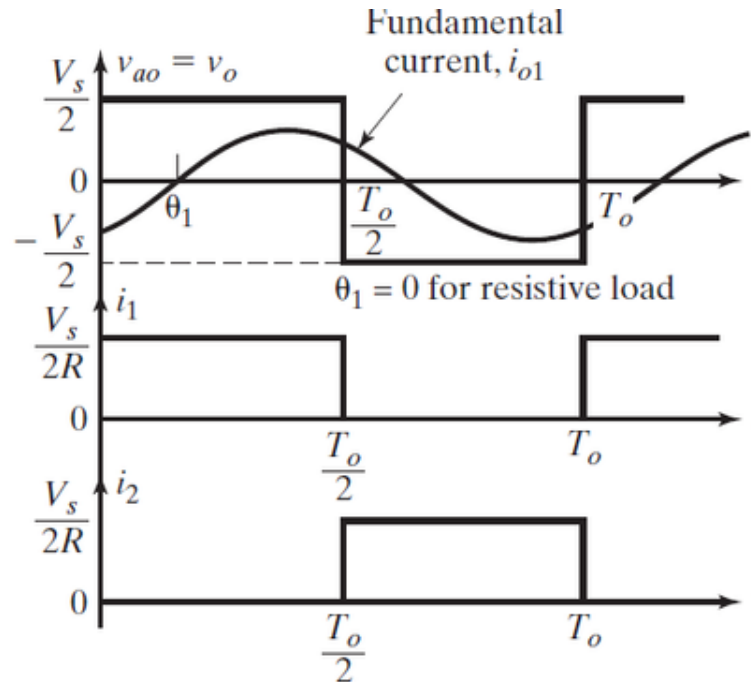
(c) Load current with highly inductive load

# Principle of operation – half-bridge

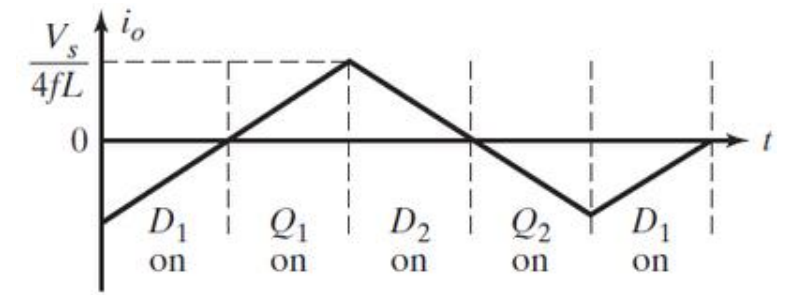
- The output voltage is a square wave with amplitude  $V_s/2$ .
- The RMS voltage is  $V_o = \sqrt{\frac{2}{T_o} \int_0^{T_o/2} \left(\frac{V_s}{2}\right)^2 dt} = V_s/2$
- The Fourier series of the instantaneous output voltage gives  $v_o = \sum_{n=1,3,5,\dots}^{\infty} \frac{2V_s}{(n\pi)} \sin n\omega t$
- The greater the angle between voltage and current, the more the diodes  $D_1$  and  $D_2$  conduct (from 0 to 90°)



(a) Circuit



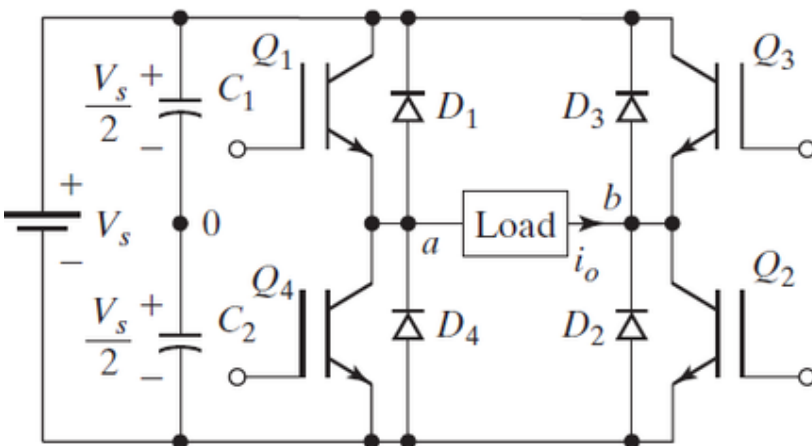
(b) Waveforms with resistive load



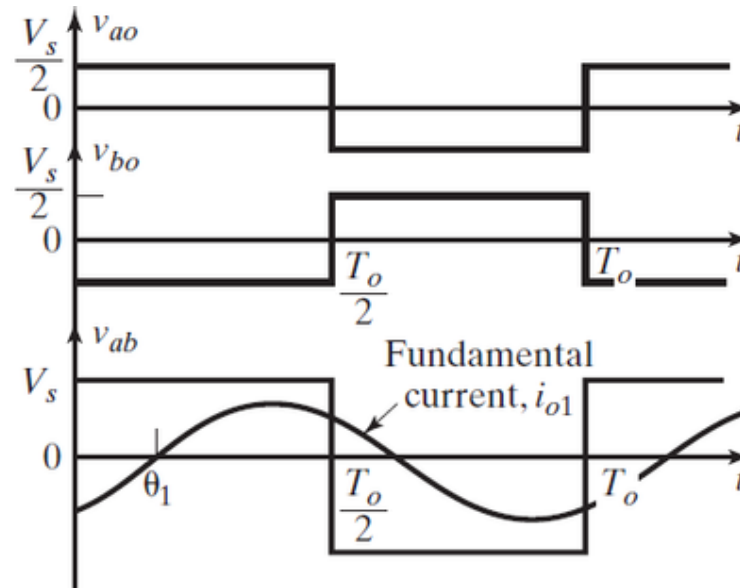
(c) Load current with highly inductive load

# Principle of operation – full bridge

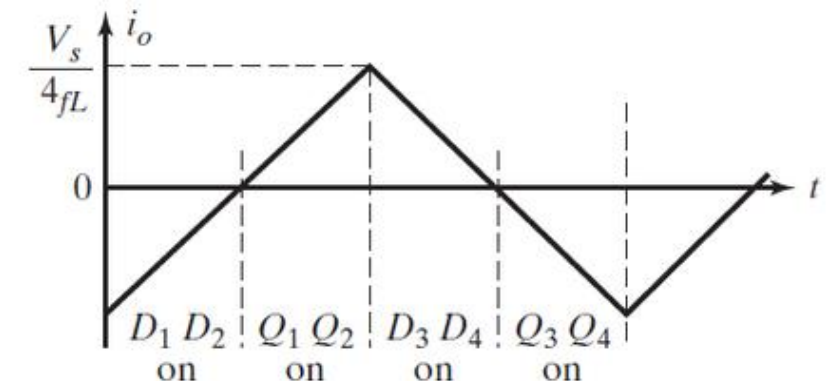
- A full-bridge inverter is built with 4 transistors.
- When  $Q_1$  and  $Q_2$  are *ON* for  $T_o/2$  the instantaneous voltage on the load is  $V_s$ .
- When  $Q_3$  and  $Q_4$  are *ON* for  $T_o/2$  the instantaneous voltage on the load is  $-V_s$ .
- The full-bridge can apply the full voltage to the load and also apply zero voltage, creating a third voltage level.



(a) Circuit



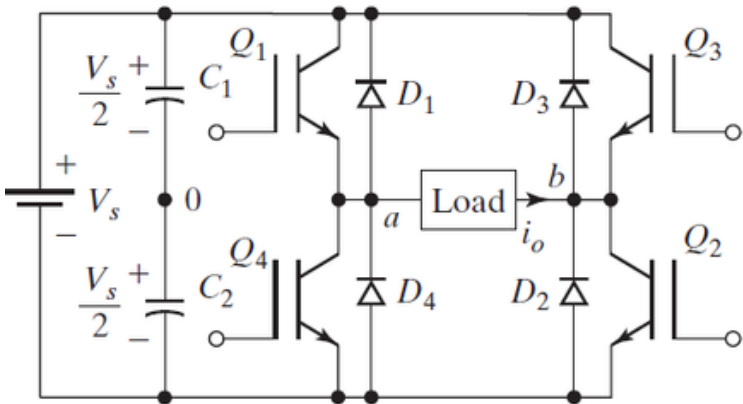
(b) Waveforms



(c) Load current with highly inductive load

# Principle of operation – full bridge

- Switch states: 1 if an upper switch is ON and 0 if a lower switch is OFF.

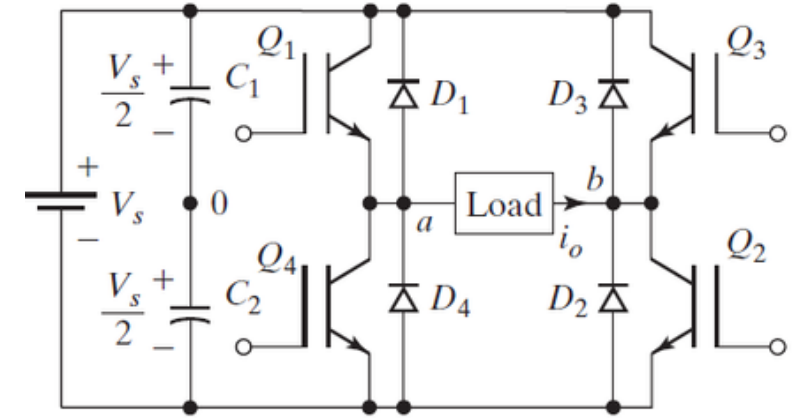


State	State No.	Switch State*	$v_{ao}$	$v_{bo}$	$v_o$	Components Conducting
$S_1$ and $S_2$ are on and $S_4$ and $S_3$ are off	1	10	$V_S/2$	$-V_S/2$	$V_S$	$S_1$ and $S_2$ if $i_o > 0$ $D_1$ and $D_2$ if $i_o < 0$
$S_4$ and $S_3$ are on and $S_1$ and $S_2$ are off	2	01	$-V_S/2$	$V_S/2$	$-V_S$	$D_4$ and $D_3$ if $i_o > 0$ $S_4$ and $S_3$ if $i_o < 0$
$S_1$ and $S_3$ are on and $S_4$ and $S_2$ are off	3	11	$V_S/2$	$V_S/2$	0	$S_1$ and $D_3$ if $i_o > 0$ $D_1$ and $S_3$ if $i_o < 0$
$S_4$ and $S_2$ are on and $S_1$ and $S_3$ are off	4	00	$-V_S/2$	$-V_S/2$	0	$D_4$ and $S_2$ if $i_o > 0$ $S_4$ and $D_2$ if $i_o < 0$
$S_1, S_2, S_3,$ and $S_4$ are all off	5	off	$-V_S/2$ $V_S/2$	$V_S/2$ $-V_S/2$	$-V_S$ $V_S$	$D_4$ and $D_3$ if $i_o > 0$ $D_1$ and $D_2$ if $i_o < 0$

- The RMS voltage is  $V_o = \sqrt{\frac{2}{T_0} \int_0^{T_0/2} V_s^2 dt} = V_s$
- The Fourier series of the instantaneous output voltage gives  $v_o = \sum_{n=1,3,5,\dots}^{\infty} 4V_s/(n\pi) \sin n\omega t$

# Exercise

- The full-bridge inverter of the figure is connected to an RLC load with  $R = 10 \Omega$ ,  $L = 31.5 \text{ mH}$  and  $C = 112 \mu\text{F}$ . The fundamental AC frequency is 60 Hz.
- a) Express the instantaneous current up to the ninth harmonic.
- b) Calculate the RMS load current at the fundamental frequency.



- Key equations

The RMS voltage is 
$$V_o = \sqrt{\frac{2}{T_0} \int_0^{T_0/2} V_s^2 dt} = V_s$$

The Fourier series of the instantaneous output voltage is 
$$v_o = \sum_{n=1,3,5,\dots}^{\infty} 4V_s/(n\pi) \sin n\omega t$$



# Exercise

- The load impedance is:

$$X_L = j_n \omega L = j 2 n \pi \times 60 \times 31.5 \times 10^{-3} = j 11.87 n \, \Omega$$

$$X_c = \frac{j}{n \omega C} = -\frac{j 10^6}{2 n \pi \times 60 \times 112} = \frac{-j 23.68}{n} \, \Omega$$

$$Z_n = R + j(11.87 n - 23.68/n)$$

- The output voltage is: 
$$v_o(t) = \sum_{n=1,3,5,\dots}^{\infty} \frac{4V_s}{(n\pi)} \sin n\omega t$$
  

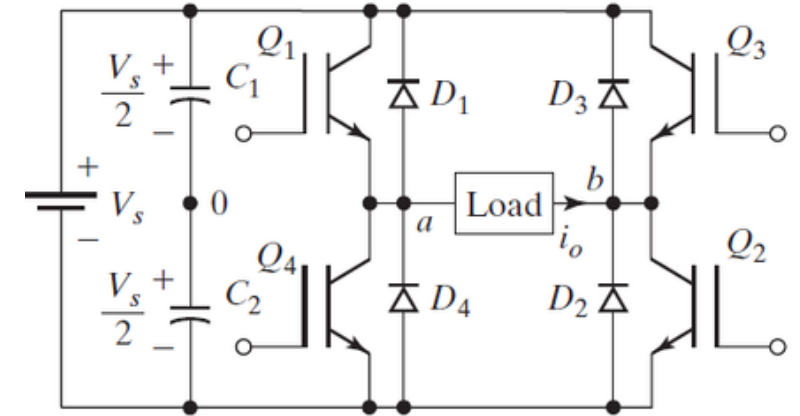
$$= 280.1 \sin(\omega t) + 93.4 \sin(3\omega t) + 56 \sin(5\omega t) + 40 \sin(7\omega t) + 31.1 \sin(9\omega t)$$

- The output current is obtained by dividing each voltage harmonic by its impedance:

$$i_o(t) = 18.1 \sin(\omega t + 49.7^\circ) + 3.2 \sin(3\omega t - 70.2^\circ) + \sin(5\omega t - 79.6^\circ) + 0.5 \sin(7\omega t - 82.9^\circ) + 0.3 \sin(9\omega t - 84.5^\circ)$$

- The the RMS laod current at fundamental frequency is:

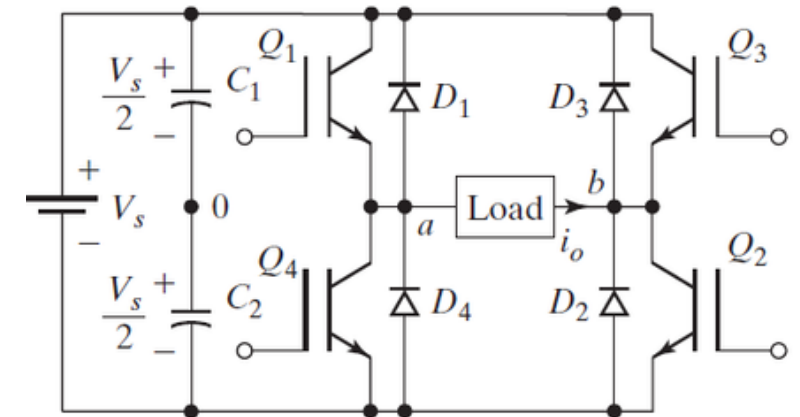
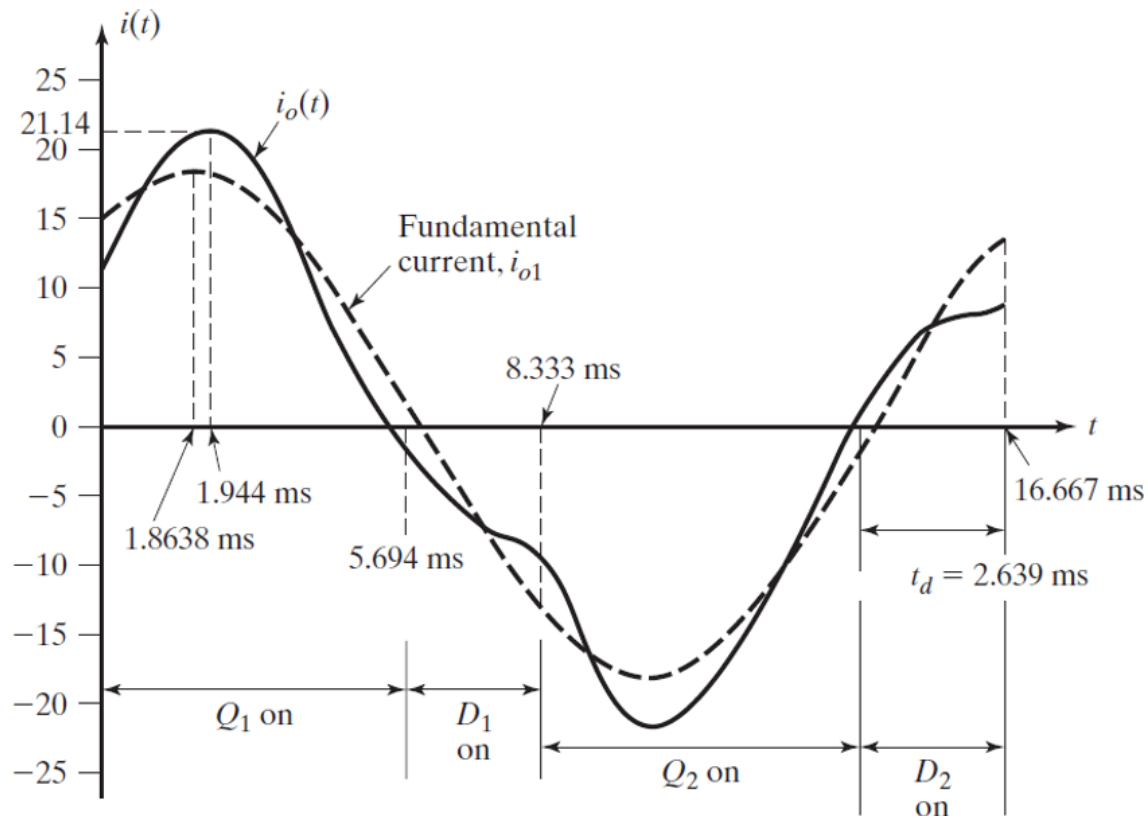
$$I_{o1} = \frac{18.1}{\sqrt{2}} = 12.8 \, \text{A}$$



# Exercise

- The current waveform is shown below

$$i_o(t) = 18.1 \sin(\omega t + 49.7^\circ) + 3.2 \sin(3\omega t - 70.2^\circ) + \sin(5\omega t - 79.6^\circ) + 0.5 \sin(7\omega t - 82.9^\circ) + 0.3 \sin(9\omega t - 84.5^\circ)$$



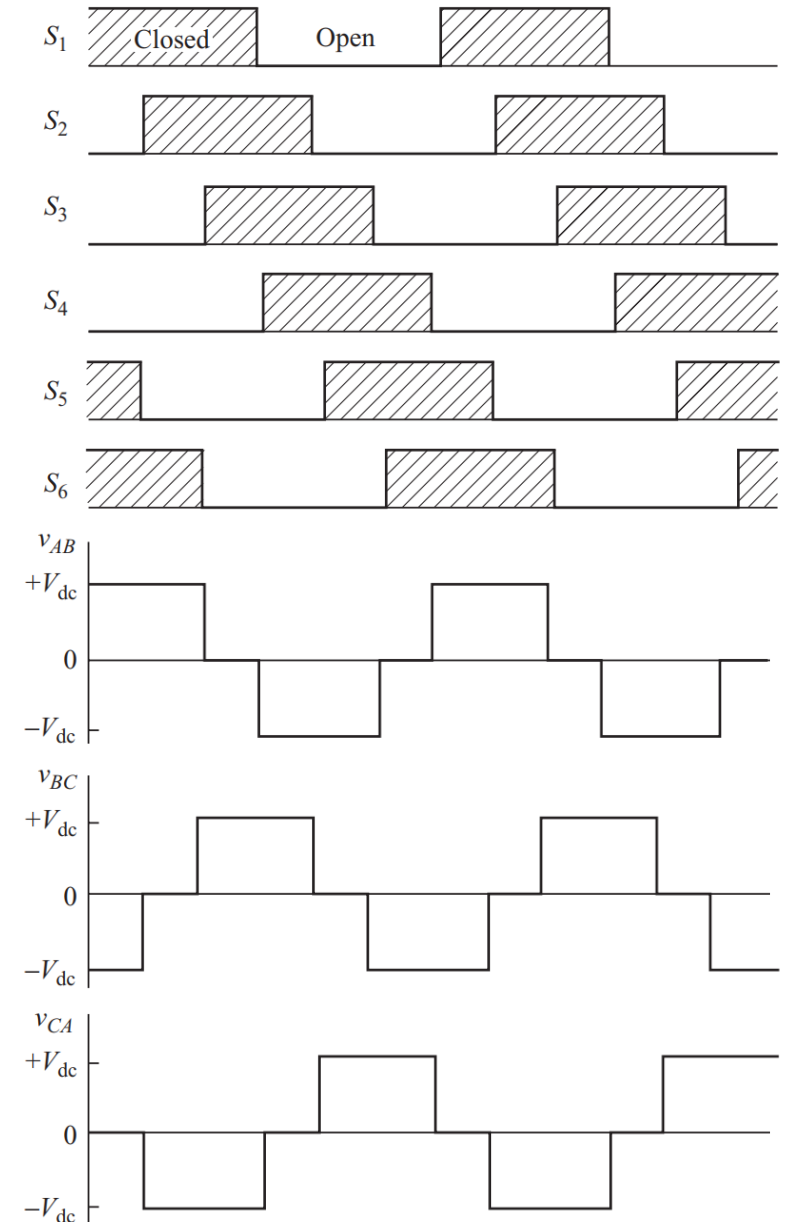
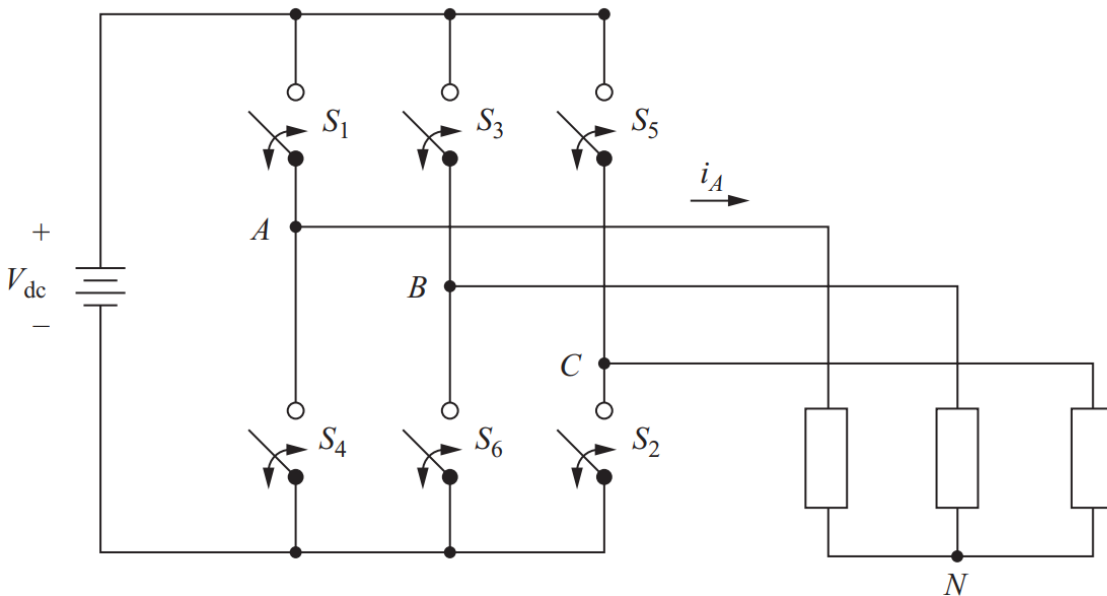
# Outline

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- Single-phase inverters
- **Three-phase inverters**
- Simulation

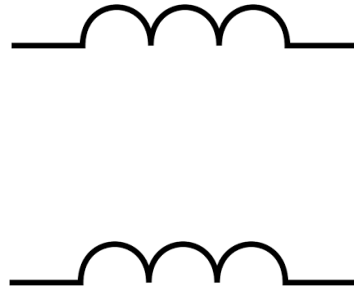
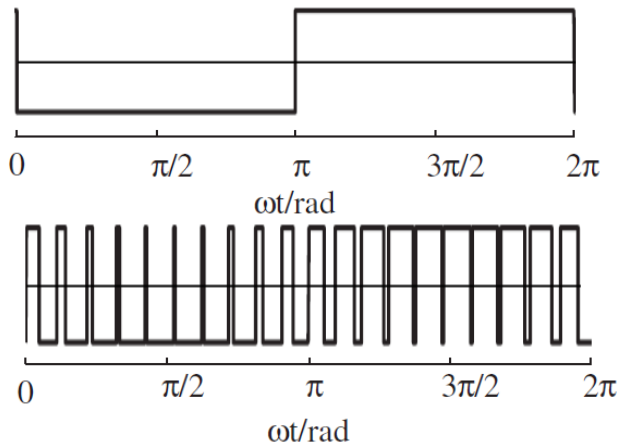
# Three-phase inverters

- The simplest operation of a three-phase inverter is the  $180^\circ$  conduction method.
- The switches  $(S_1, S_4), (S_2, S_5), (S_3, S_6)$  are opposed to each other.
- A switching action occurs every  $T/6$  time interval or every  $60^\circ$
- For a Y-connected load:  $V_{n,L-L} = \left| \frac{4V_{dc}}{n\pi} \cos\left(n\frac{\pi}{6}\right) \right| \quad n = 1, 5, 7, 11, 13, \dots$



# Three-phase inverters

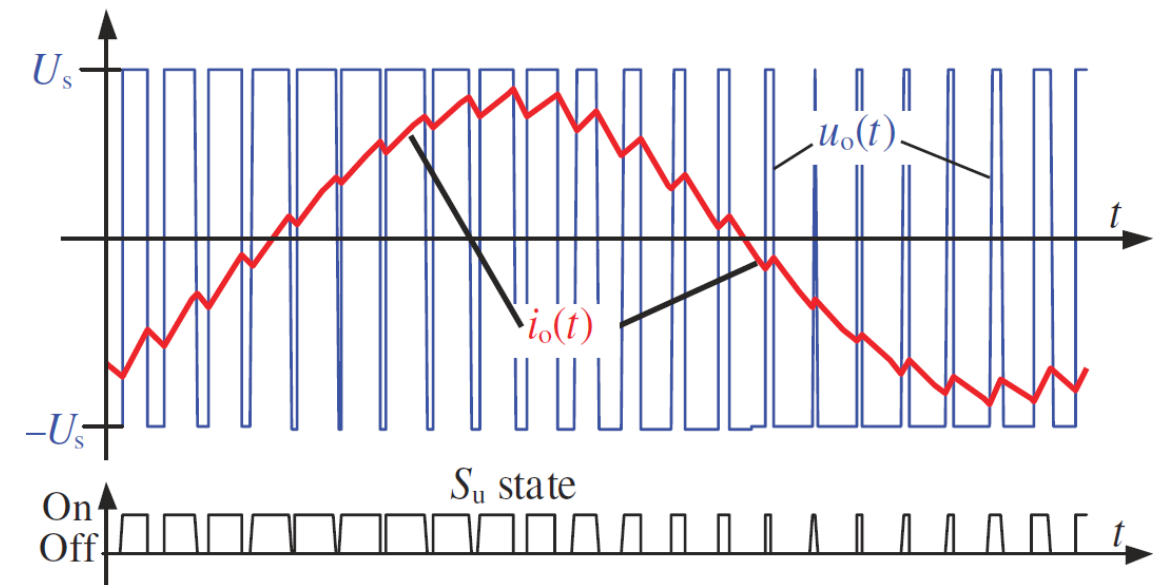
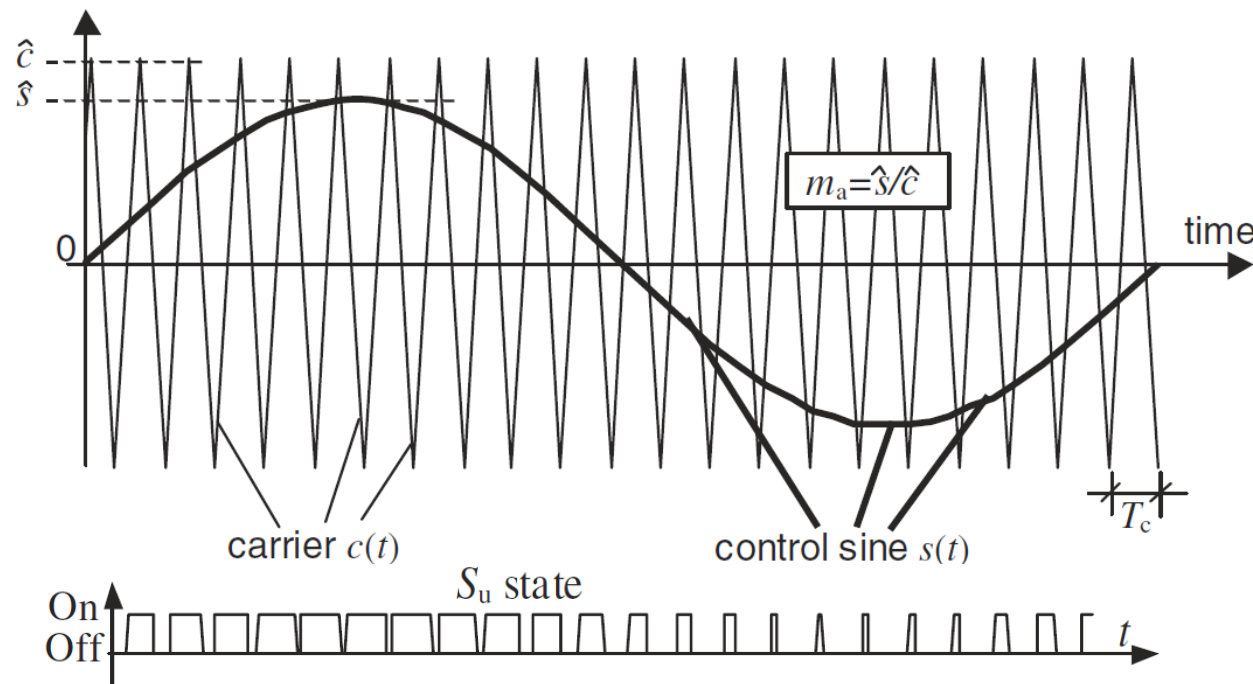
Which type of switching will produce a better sinusoidal current?



- To have good quality on the waveforms, high switching frequency is needed. Losses increase with switching frequency → Trade-off between power quality and losses.

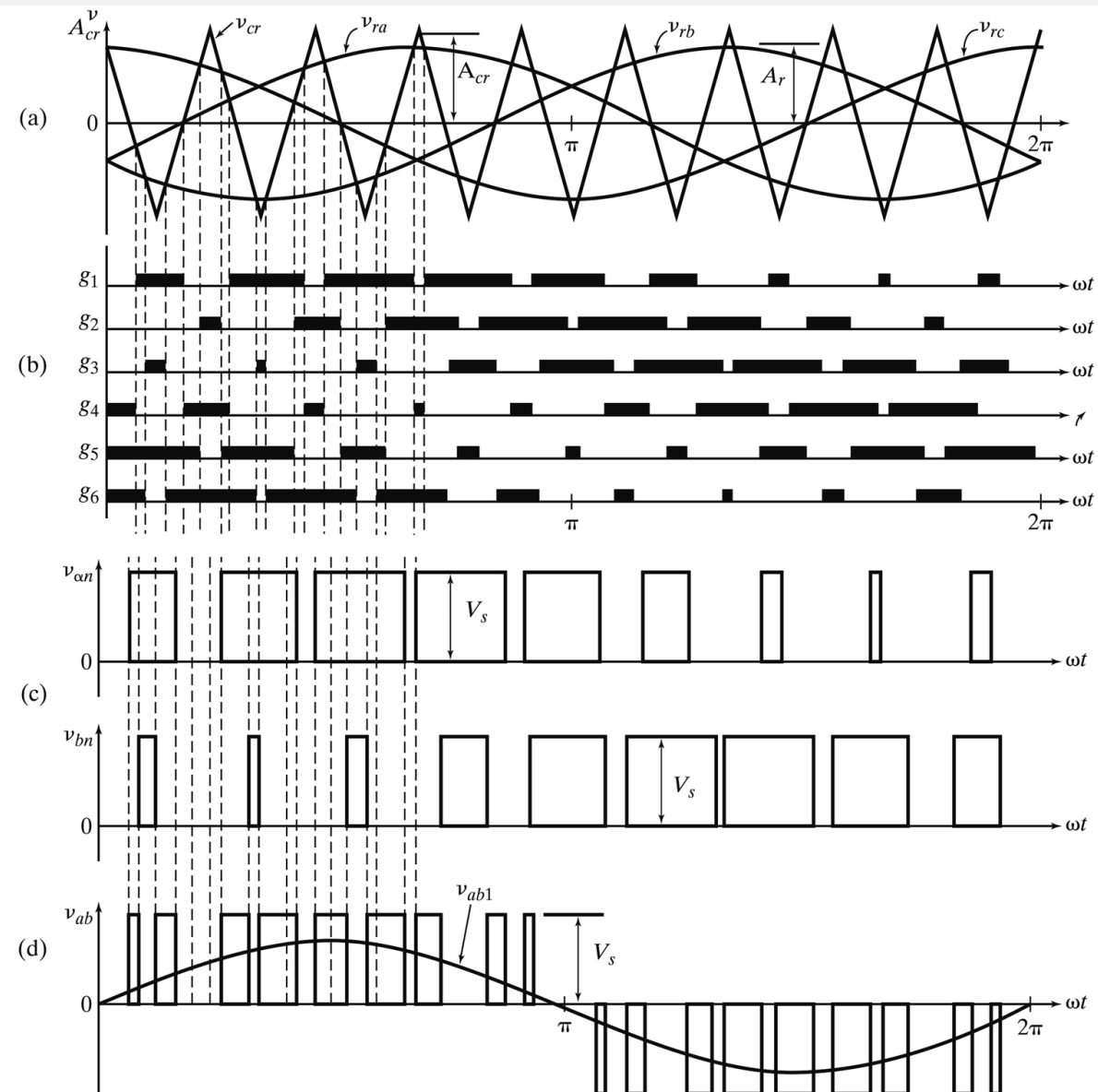
# Three-phase inverters

- The previous method generates an AC voltage of fixed amplitude and high harmonic content.
- **Pulse-Width Modulation (PWM)** is used to allow control of AC voltage amplitude and reduce filtering requirements. Among several methods, the sinusoidal PWM (SPWM) is widely used in industry.
- In this method, the gates of each switch are generated by comparing a sinusoidal **reference wave** with a **triangular carrier wave**. The switches are turned ON when they are greater than the carrier.



Source: M. Ceraolo and D. Poli (2014), Fundamentals of Electric Power Engineering: From Electromagnetics to Power Systems, IET, John Wiley & Sons

# Three-phase sinusoidal PWM

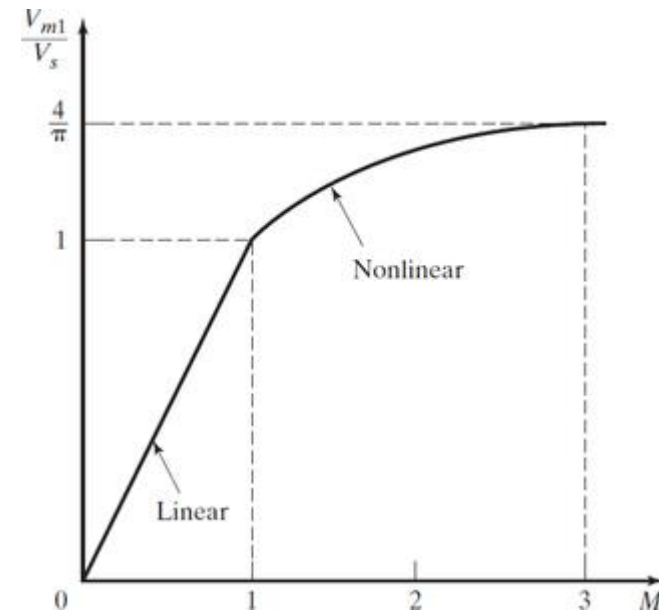
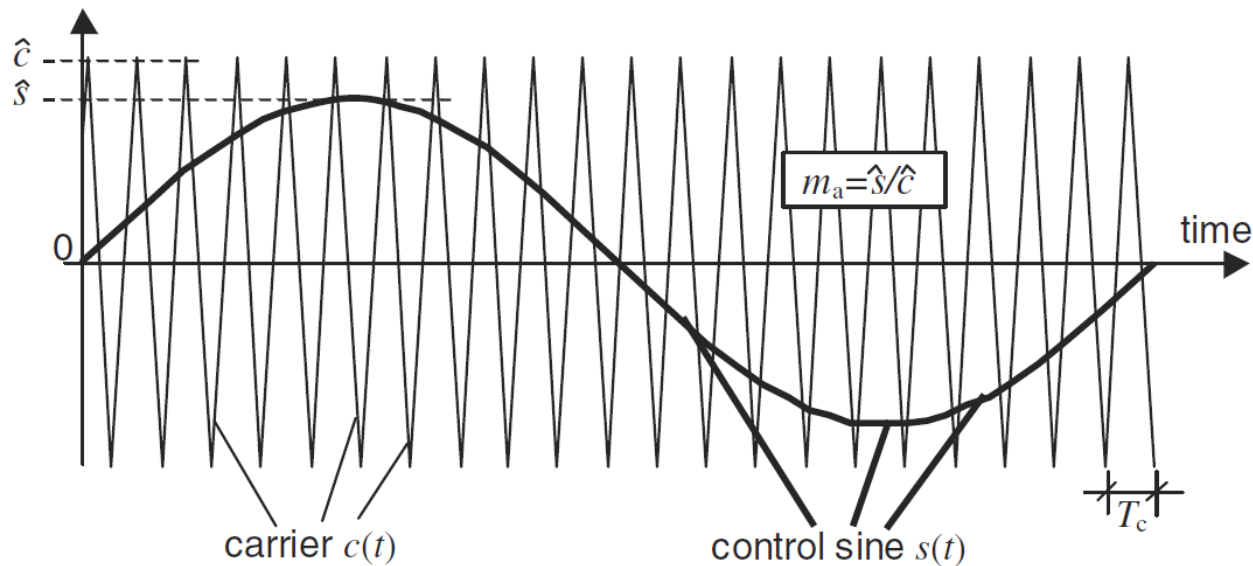


# Three-phase sinusoidal PWM

- The relation between line-to-line AC voltage and DC voltage is

$$\hat{v}_{ab1} = M\sqrt{3}\frac{V_s}{2} \quad \text{for } 0 < M \leq 1$$

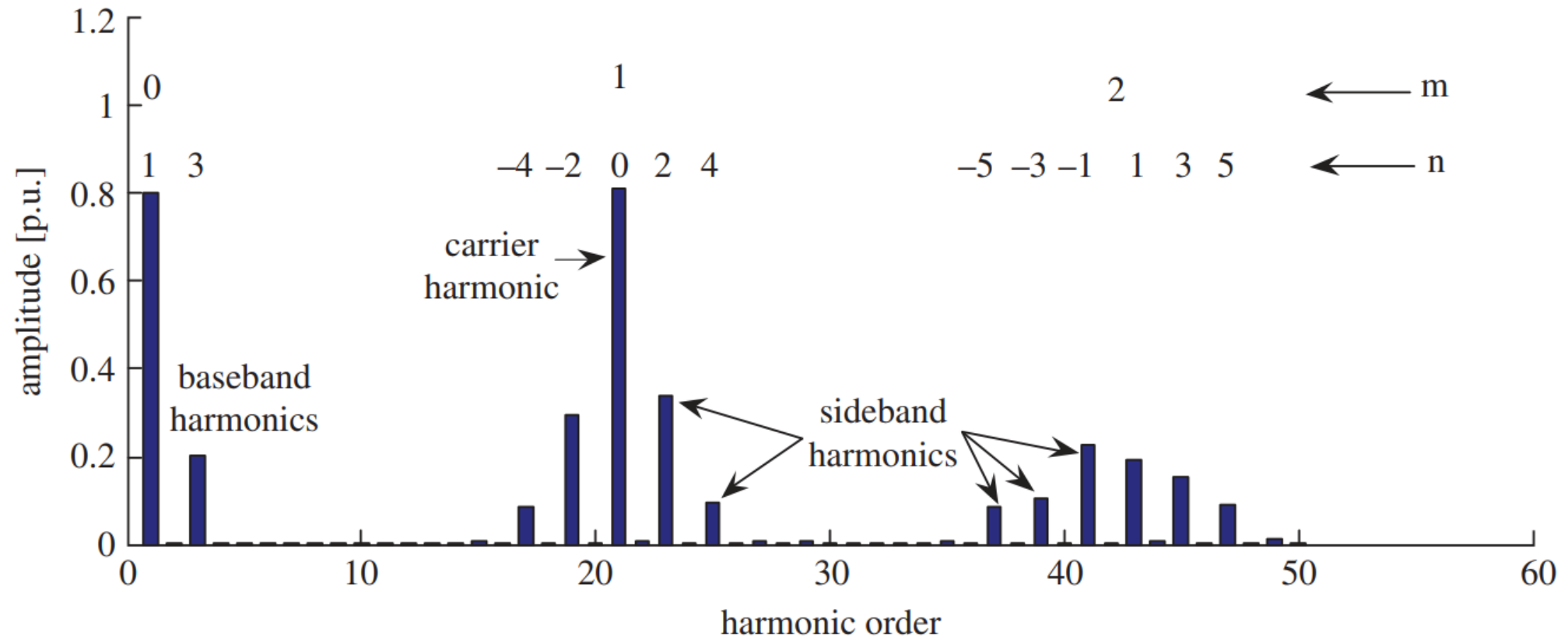
- The AC voltage grows linearly with the modulation. But if  $M > 1$  the relationship becomes nonlinear due to **overmodulation**.
- It is important to define the correct DC and AC voltage magnitudes to avoid overmodulation.





# Three-phase sinusoidal PWM

- The SPWM shifts the harmonics towards the high-frequency spectrum, facilitating the filtering of the AC voltages and currents.



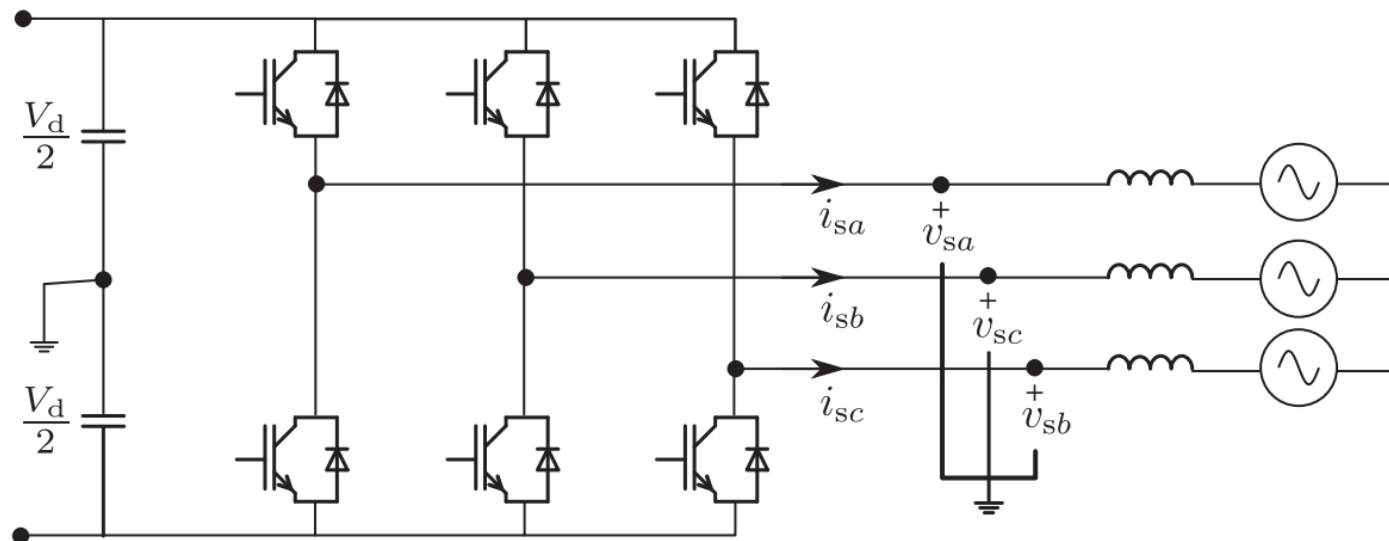
Source: Sharifabadi, K., Harnefors, L., Nee, H. P., Norrga, S., & Teodorescu, R. (2016). Design, control, and application of modular multilevel converters for HVDC transmission systems. John Wiley & Sons.

# Three-phase sinusoidal PWM

- When a three-phase inverter is connected to the grid or load, the power imported or exported will depend on the voltage difference with respect to the grid.
- Neglecting losses, the power exported to the AC side is equal to the power imported from the DC side:

$$\frac{3}{2} \hat{v}_s \hat{i}_s \cos(\varphi) = V_d I_d$$

- The AC voltage is function of the reference voltage:  $\hat{v}_s = m_a \frac{V_d}{2} \longrightarrow 3m_a \hat{i}_s \cos(\varphi) = 2I_d$



# Outline

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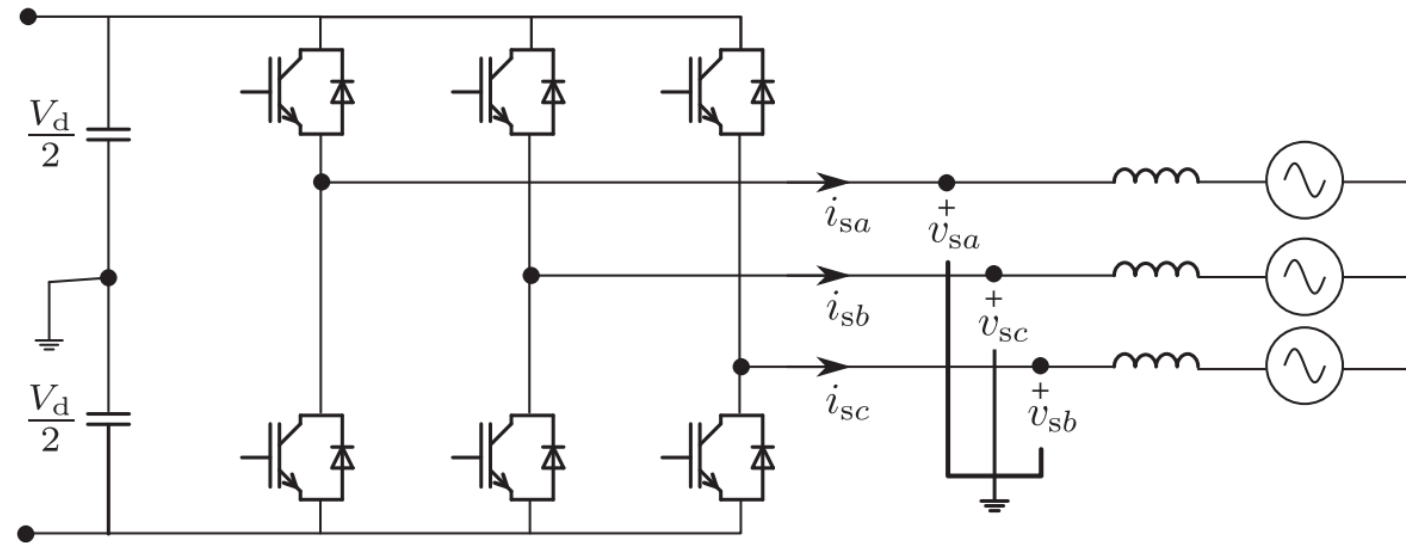
- Single-phase inverters
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- **Simulation**

# Simulation

- Simulate a three-phase DC/AC converter.

Data:

- $V_{dc} = 1000 \text{ V}$
- $L = 10 \text{ mH}$  ( $R = X/30$ )
- $V_{grid} = XXXX \text{ RMS}$
- Carrier freq =  $33 \cdot 50 \text{ Hz}$



- Define a carrier amplitude to generate YY V in the output
- Define the angle to export XXX MW to the grid

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