Electric Energy Conversion

9. Control of switching power electronics

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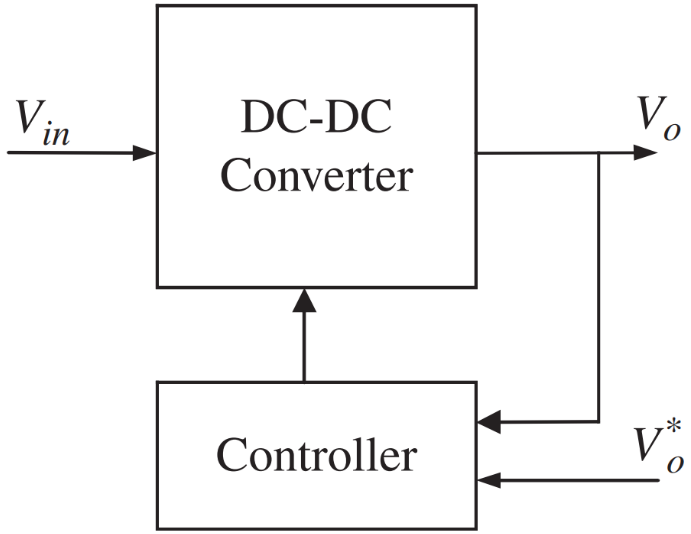
Outline

- Introduction
- System linearization and transfer function
- Feedback controller design
- Simulation

Introduction

- Apart from diode rectifiers, almost every switching power electronics can be controlled.
- The objectives are normally to reach a given voltage, current or power
- These objectives are met by comparing the setpoint with the measured quantity and adapting the switching to reach the setpoint.

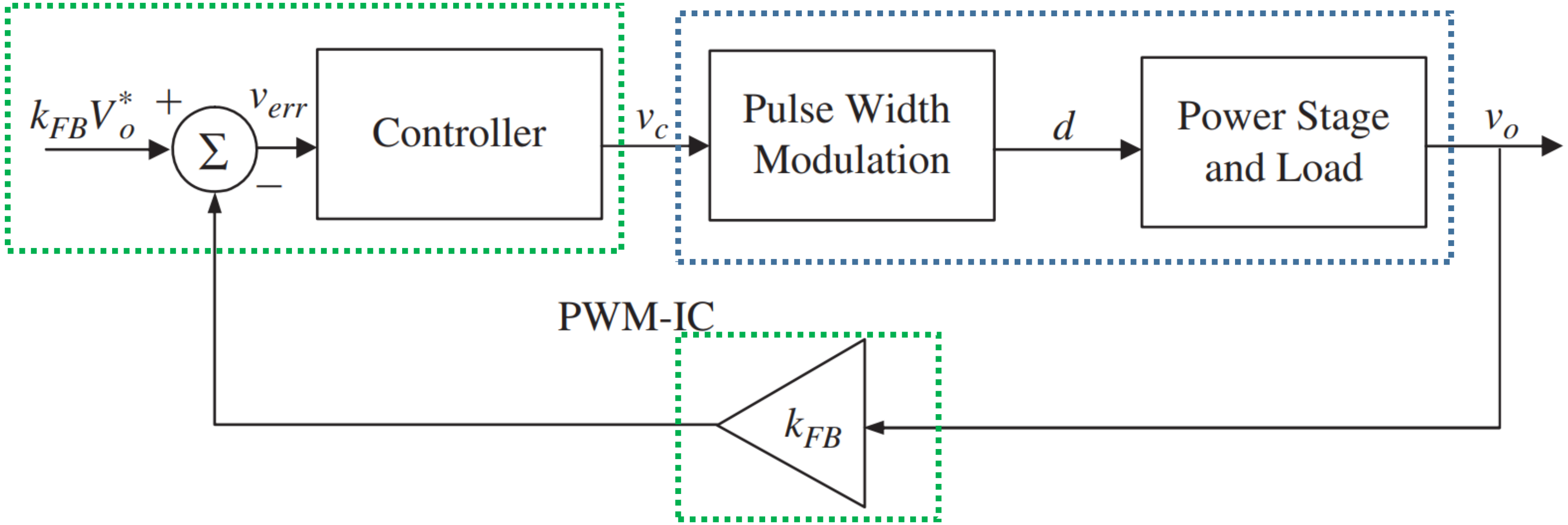
Example of a controlled DC/DC converter



Introduction

- The design of linear feedback controllers normally follows three steps:

1. Derive the system's transfer functions (typically by linearization around an operating point).
2. Design a feedback controller using linear control theory (following a desired performance).
3. Evaluate the system with the controller by simulations for large disturbances.

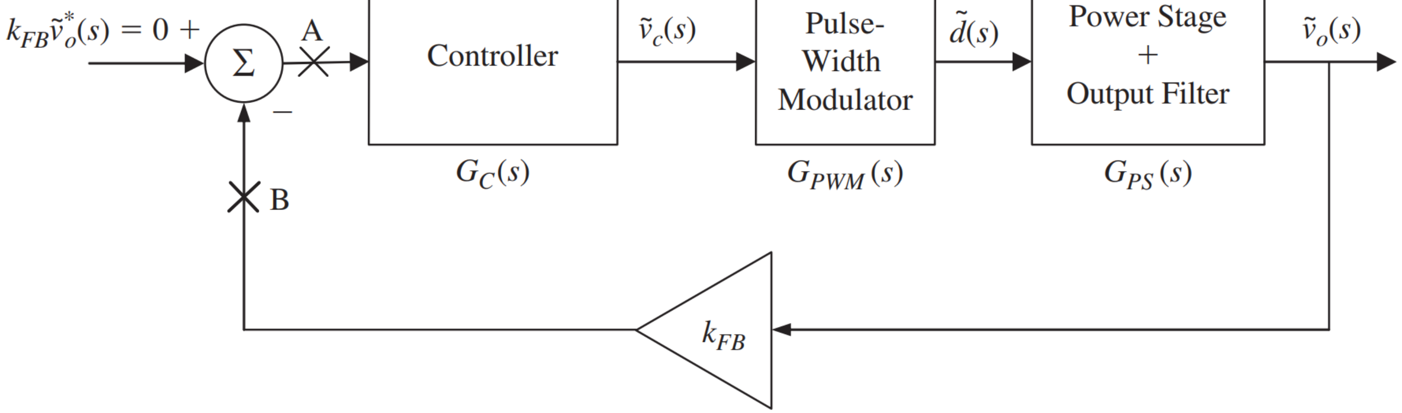


Derive system's transfer functions

- To use linear control theory, we need to **linearize** the system.
- Normally, the linearization is done assuming small-signal perturbations around an operating point.
- We do not consider the switchings in the derivation, only the average value
- We are interested in the **dynamics** of the system
- An example:

Average output — Steady-state value — Small-signal perturbation

$$
\begin{aligned}
\bar{v}_o(t) &= V_o + \tilde{v}_0(t) \\
d(t) &= D + \tilde{d}(t) \\
v_c(t) &= V_c + \tilde{v}_c(t)
\end{aligned}
$$



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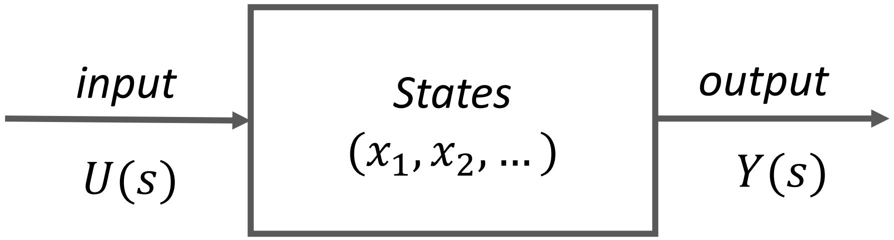


SISO system

Output

$$H(s) = \frac{Y(s)}{U(s)} = \frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{u(t)\}}$$

Input

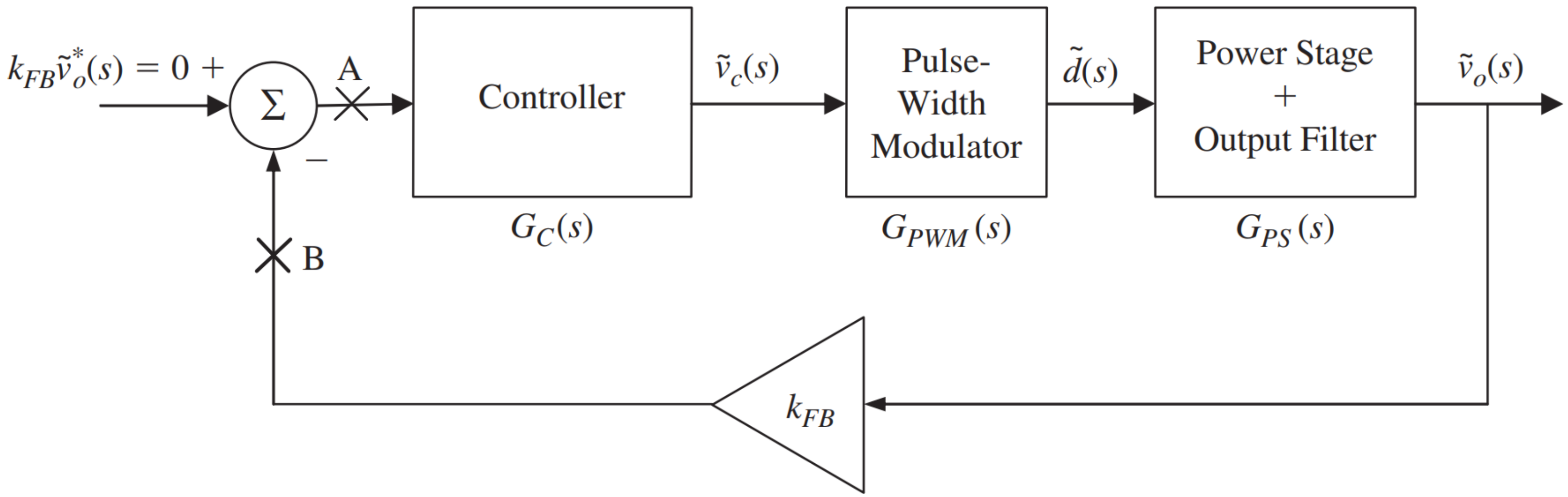
Derive system's transfer functions

- The transfer functions of each element are represented in the Laplace domain.
- $G_c(s)$ is the transfer function of the controller
- $G_{PWM}(s)$ is the transfer function of the PWM
- $G_{PS}(s)$ is the transfer function of the power stage
- $k_{FB}(s)$ is the gain of the measurement (or $F_{FB}(s)$ to be more generic)

Output

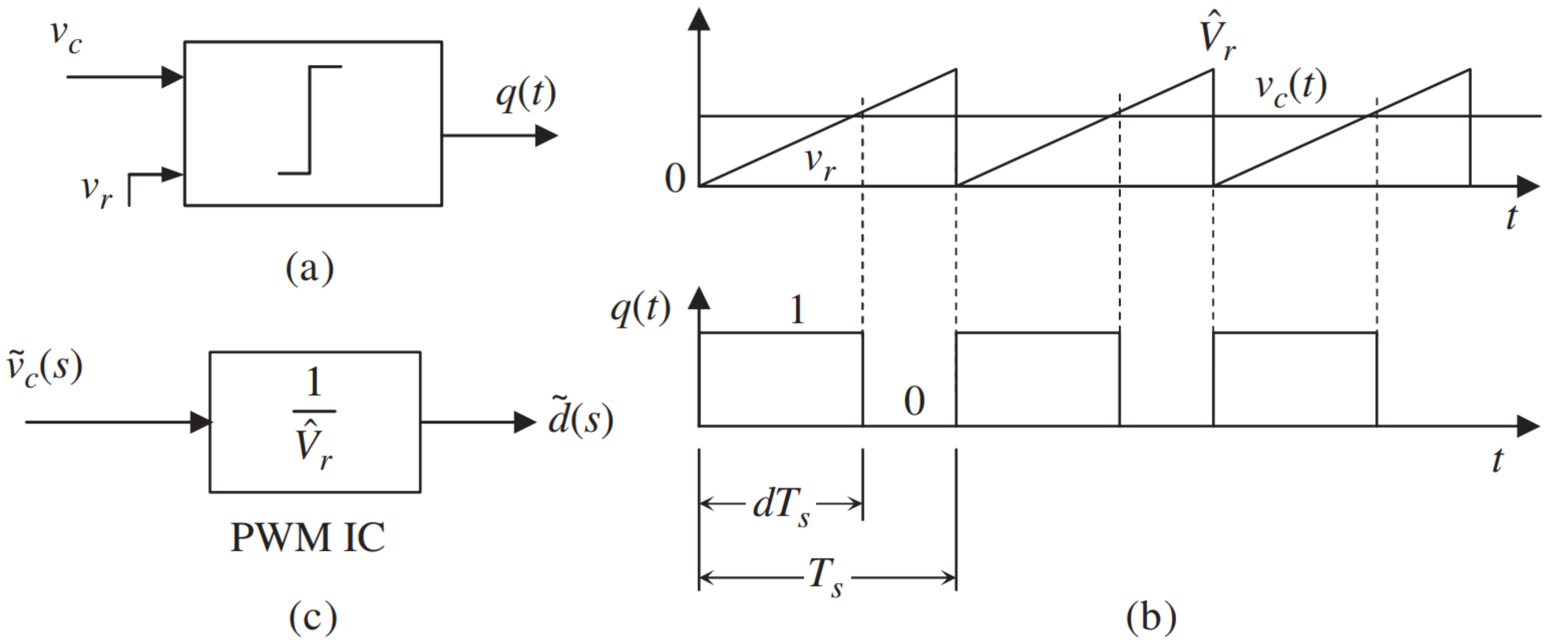
$$H(s) = \frac{Y(s)}{X(s)} = \frac{\mathcal{L}\{y(t)\}}{\mathcal{L}\{x(t)\}}$$

Input



Derive system's transfer functions - PWM

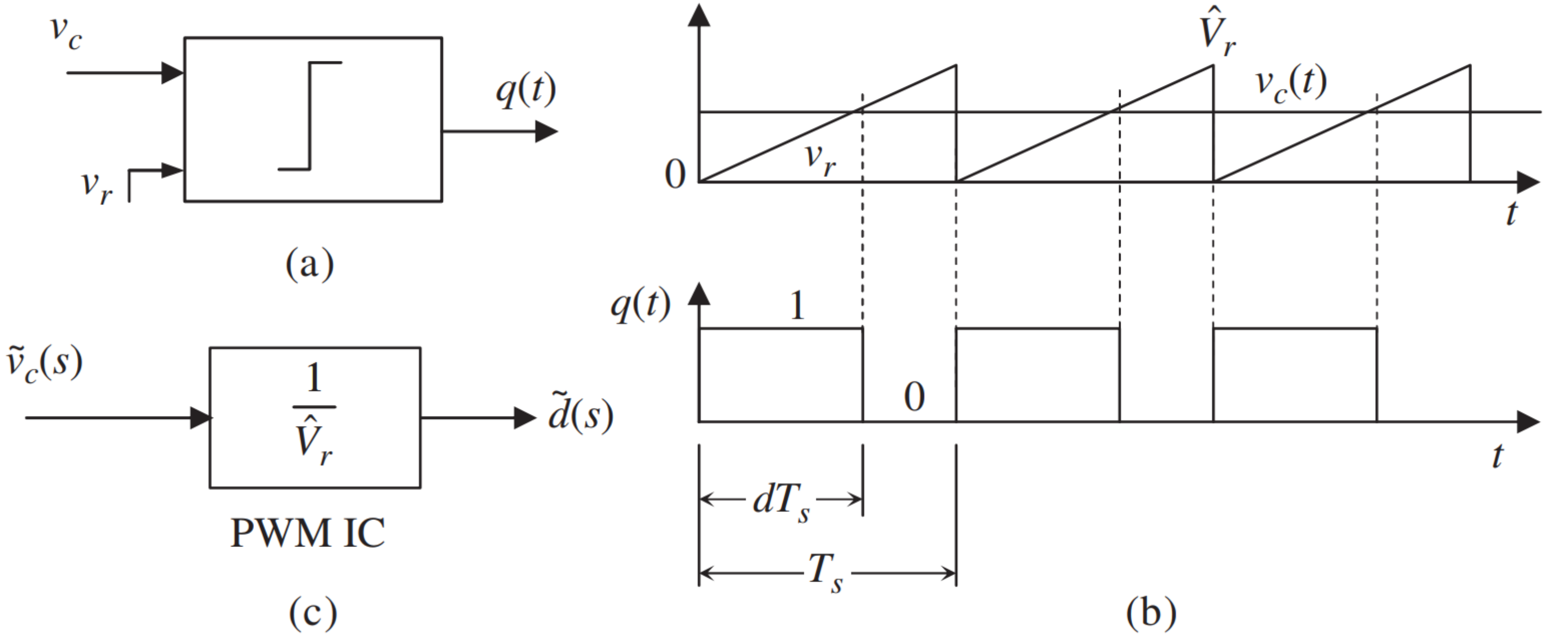
- $G_{PWM}(s)$ is the transfer function of the PWM
- The output of the controller v_c is compared against a carrier wave v_r and generates a switching function $q(t)$
- $q(t)$ is 1 when $v_c > v_r$ or 0 when $v_c < v_r$
- The carrier v_r has an amplitude of V_r and a frequency of f_s.
- The duty cycle $d(t)$ is the average value of the switching function $q(t)$



Derive system's transfer functions - PWM

- The duty cycle is calculated as: $d(t) = \dfrac{v_c(t)}{\hat{V}_r}$
- The controller output is: $v_c(t) = V_c + \tilde{v}_c(t)$

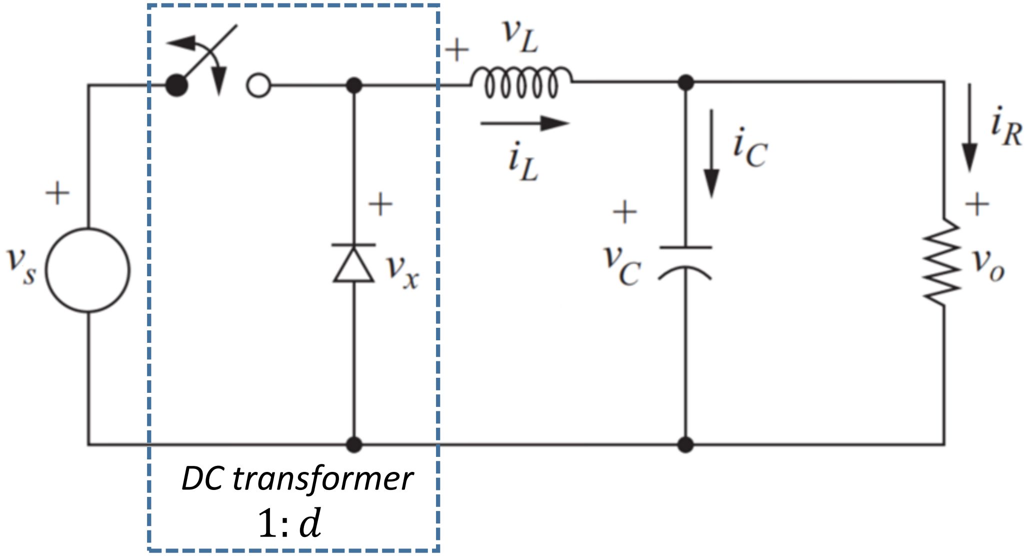
$$d(t) = \underbrace{\frac{V_c}{\hat{V}_r}}_{D} + \underbrace{\frac{\tilde{v}_c(t)}{\hat{V}_r}}_{\tilde{d}(t)} \rightarrow G_{PWM}(s) = \frac{\tilde{d}(s)}{\tilde{v}_c(s)} = \frac{1}{\hat{V}_r}$$



Derive system's transfer functions – power stage

- This process will be illustrated for a buck DC/DC converter
- We want the transfer function $G_{PS} = v_c(s)/d(s)$

input	output	Inductor current	Capacitor voltage
d	$v_o = v_c$	$\frac{di_L}{dt} = \frac{v_s d - v_c}{L}$	$\frac{dv_c}{dt} = \frac{i_L}{C} - \frac{v_c}{RC}$



- Transforming to the Laplace domain we have

Inductor current	Capacitor voltage
$sI_L(s) = \frac{V_s d - V_c(s)}{L}$	$sV_c(s) = \frac{I_L(s)}{C} - \frac{V_c(s)}{RC}$

- We have assumed that v_s is constant ($\tilde{v}_s = 0$).
- If v_s changes, this is a disturbance that needs to be rejected by the control.

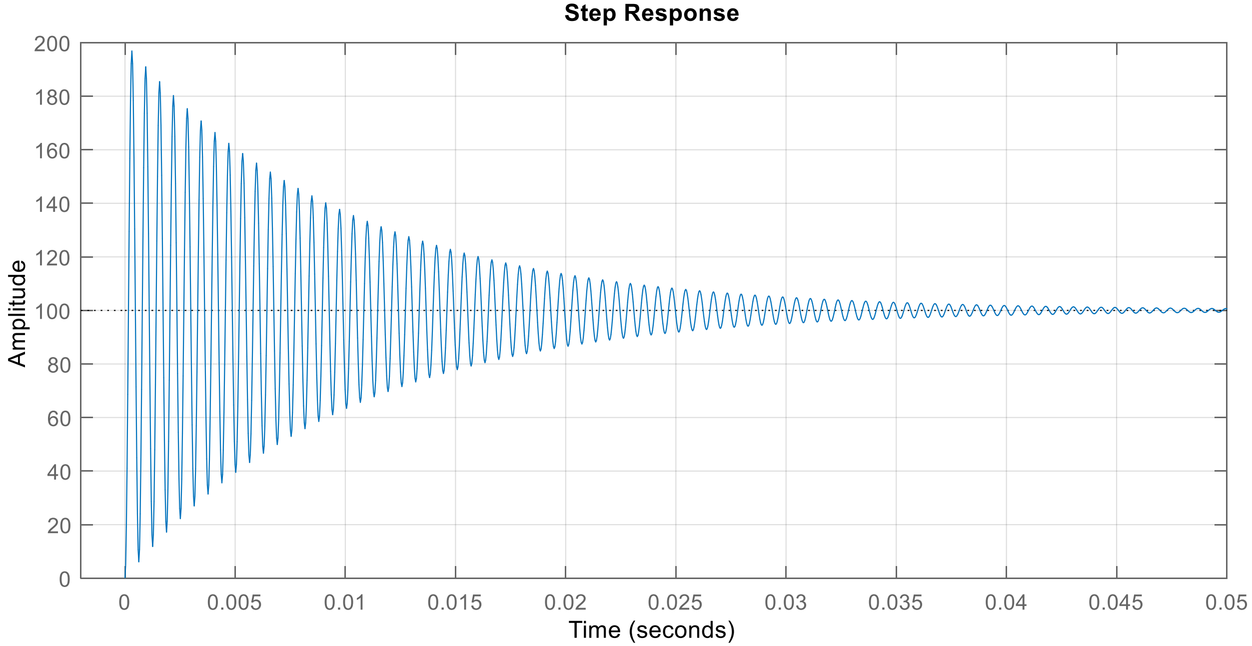
$$\boxed{G_{PS}(s) = \frac{V_s}{LC}\frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}}}$$

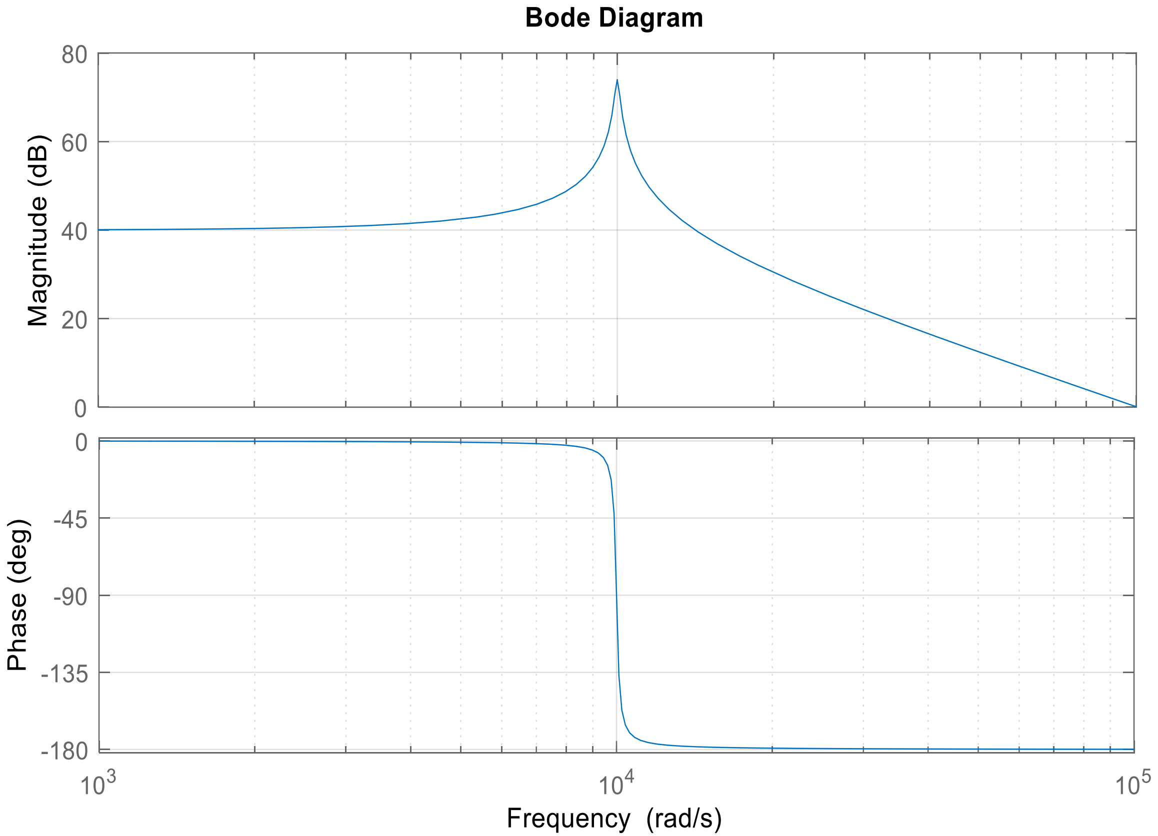
Neglecting losses

Derive system's transfer functions – power stage

- Evaluating $G_{PS} = v_c(s)/d(s)$ for
$L = 200$ µH, $C = 50$ µF, $R = 100\ \Omega$ and $V_s = 100$ V
- This response can also be simulated.
- We will design a controller to damp this response

$$G_{PS}(s) = \frac{V_s}{LC}\frac{1}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$





Feedback controller design

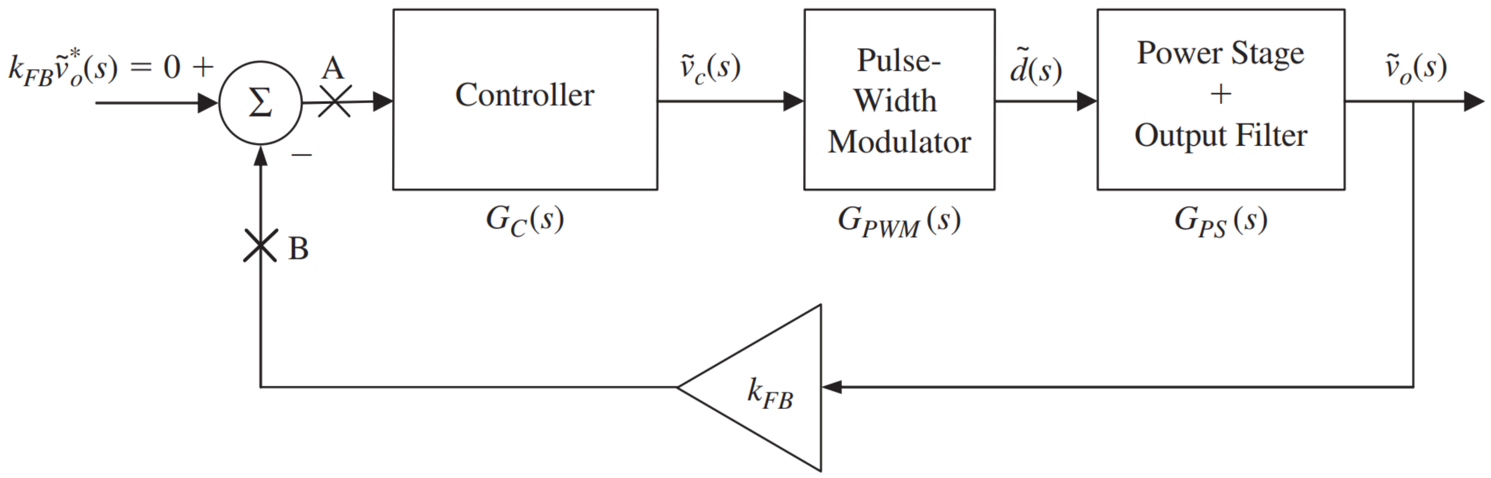
The controller $G_c(s)$ is designed given a set of performance parameters:

- Speed (bandwidth)
- Stability
- Reference tracking
- Disturbance rejection
- Robustness

A few examples of common controllers:

Proportional (P)	*Integral (I)*	*Derivative (D)*	*Lead-lag compensator*
$G_c(s) = K_p$	$G_c(s) = \frac{K_i}{s}$	$G_c(s) = K_D s$	$G_c(s) = K\frac{s+z}{s+p}$

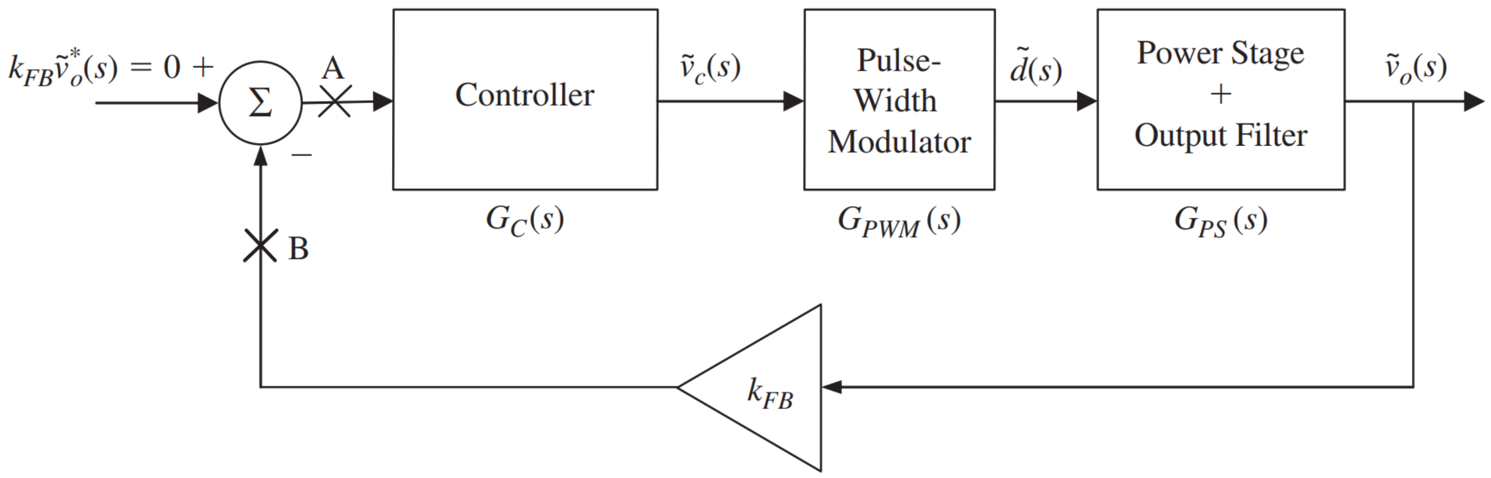
PI	*PD*	*PID*
$G_c(s) = K_p + \frac{K_i}{s}$	$G_c(s) = K_p + K_D s$	$G_c(s) = K_p + \frac{K_i}{s} + K_D s$



Feedback controller design

- The are multiple approaches to design $G_c(s)$: numerical, analytical, optimization, etc. There are several books dedicated only to this topic. Here, we will cover only an introductory example of a vast topic.
- We will design a controller of a simple plant looking at 1) **Speed** (bandwidth) and 2) **Stability** (phase margin).
- We will use the open-loop transfer function (from point A to point B in the figure below):

$$\boxed{G_L(s) = G_C(s)G_{PWM}(s)G_{PS}(s)k_{FB}}$$

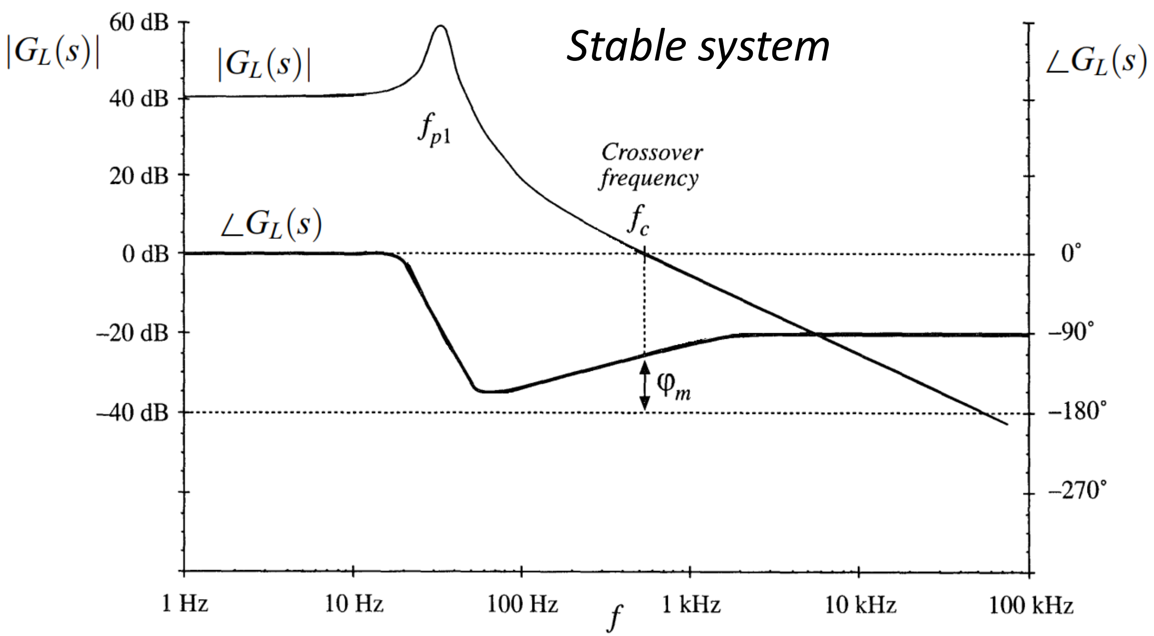


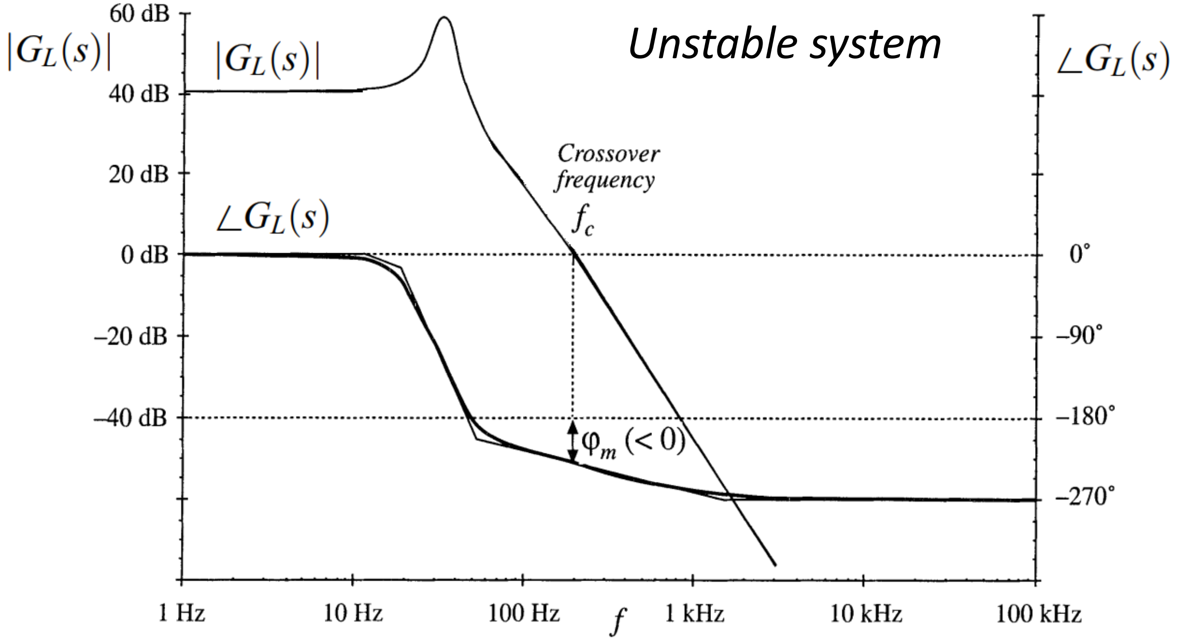
Feedback controller design

$G_L(s) = G_C(s)G_{PWM}(s)G_{PS}(s)k_{FB}$

- The bode plot (frequency domain) can help us identify the speed and stability.
- The cross-over frequency is a good indicator of the speed of the controller. The cross-over frequency (f_c or ω_c) is the frequency at which the gain equals unity ($|G_L(\omega_c)| = 0$ dB).
- The phase-margin is a good indicator of controller stability. The phase delay of $G_L(s)$ at ω_c needs to be less than 180°. The phase margin is the distance with respect to the $-180°$ limit.

$$\phi_{PM} = \angle G_L(s)|_{f_c} - (-180^\circ) = \angle G_L(s)|_{f_c} + 180^\circ \longrightarrow \phi_{PM} = 60^\circ \text{ is a good initial target}$$





Feedback controller design

We can also use **time-domain** performance parameters to design controllers. The step response of the **closed-loop transfer function** is a set of classical time-domain performance criteria:

$$C_{CL}(s) = \frac{G_L(s)}{1 + G_L(s)F_{FB}(s)} \xrightarrow{\textit{Step-response}} y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}C_{CL}(s)\right\}$$

where $G_L(s)$ *Open-loop transfer function*, $F_{FB}(s)$ *Feedback transfer function*

- **Rising time**. The time it takes for the system response to go from 10% to 90% of its final value.
- **Time constant**. It is the inverse of the natural frequency (the frequency the system would oscillate if there was no damping).
- **Settling time**. The time it takes for the system's response to settle within a given range (2% or 5%) of its steady-state value
- **Overshoot**. It is the maximum percentage by which the response overshoots the desired value.
- **Steady-state error**. It is the difference between the desired and actual output
- **Damping ratio**. The exponential decay of the system.

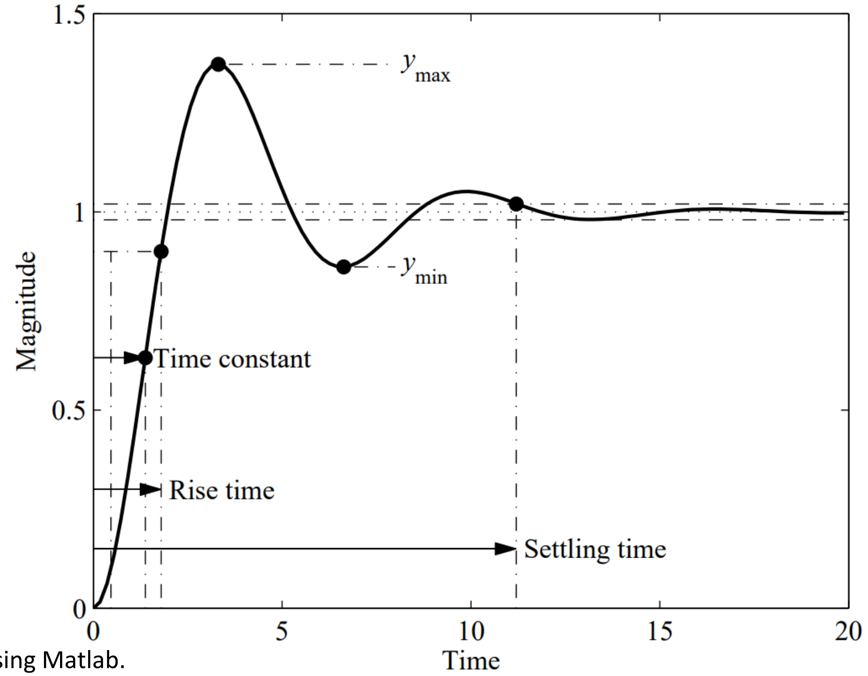


Image source: Pöllänen, R., Koponen, A., Huttunen, M., & Pyrhönen, O. Real-time simulation environment for control loop performance monitoring using Matlab.

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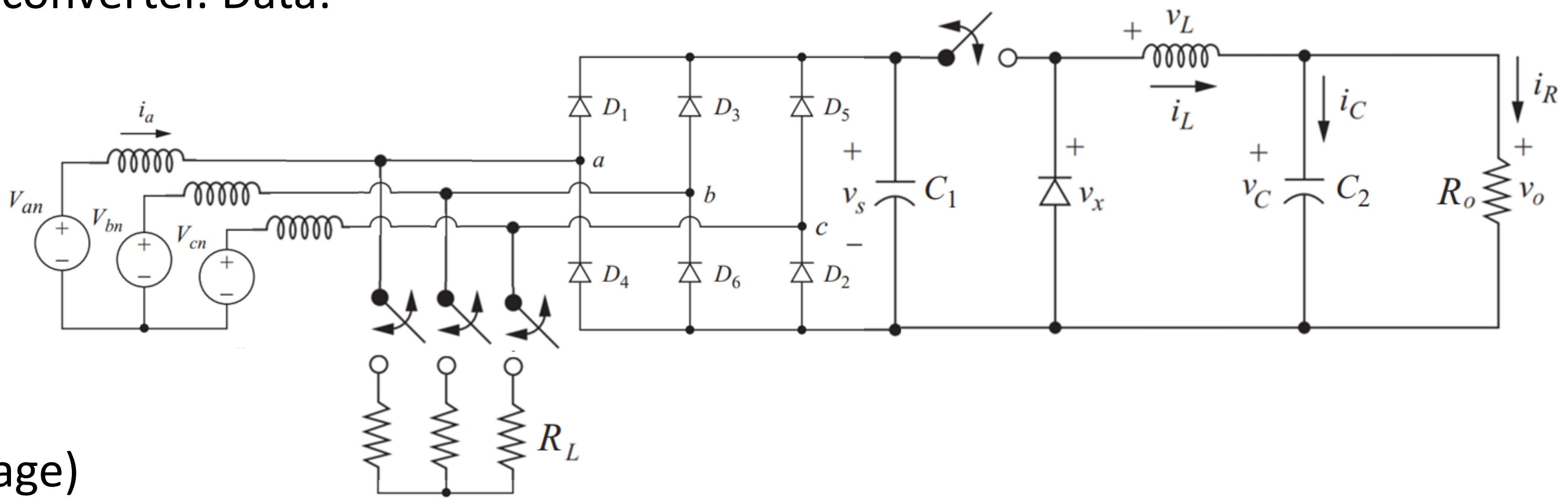
Simulation

Design a controller for the buck converter. Data:

- L = 4 mH (r=1 mΩ)
- $C_1 = 50$ µF , $C_2 = 20$ µF
- R_o = 200 Ω
- Vgrid = 480 V RMS line-to-line
- Lgrid = 100 mH (R = X/30)
- Desired $v_o = 400$ V (DC average)
- Switching frequency = 20 kHz



a) Design a controller that is stable, has less than 10 ms rising time and less than 30% of overshoot.

b) Model the rectifier + buck + controller in Simulink.

c) Apply a step of plus 20% in the reference voltage the performance of your controller.

d) Connect a load ($R_L = 100\ \Omega$) at the converter terminals to have a drop in Vgrid and check the controller's disturbance rejection.

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