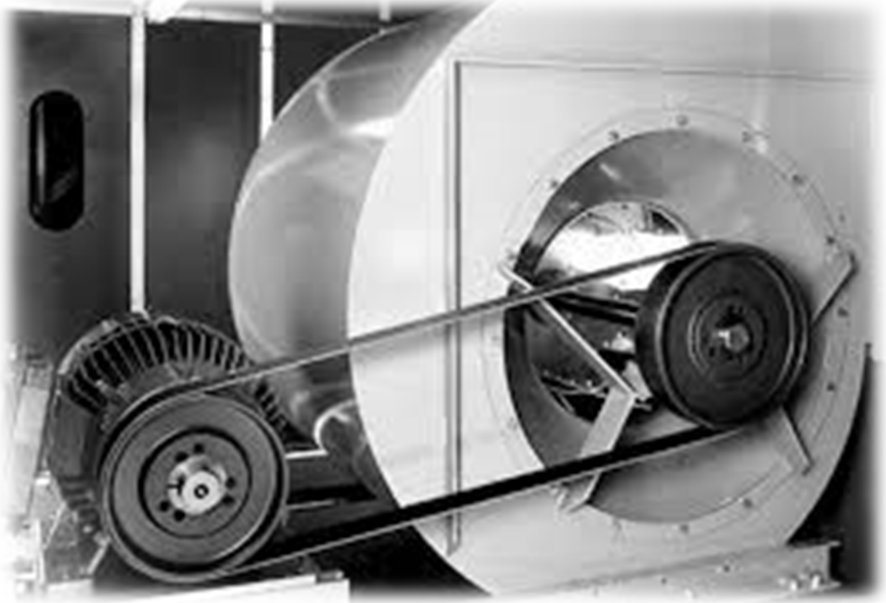
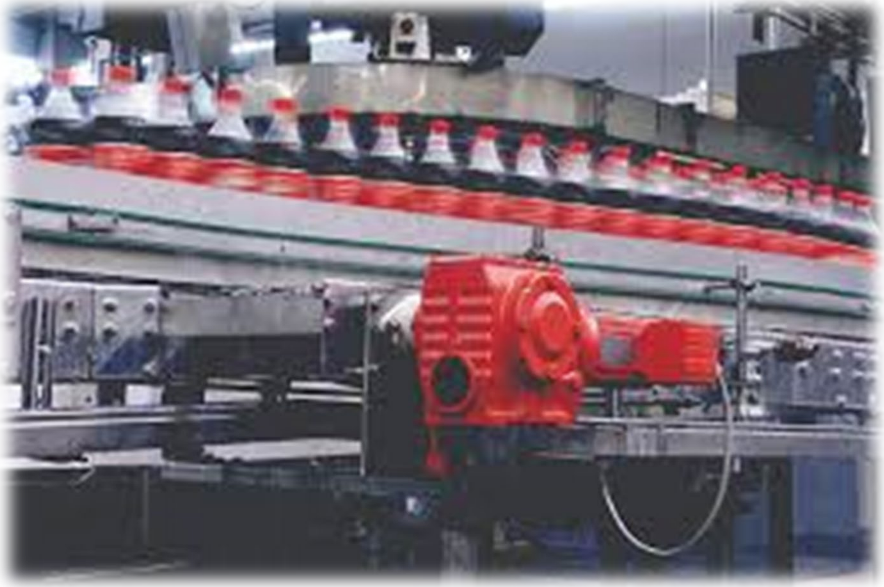
Mechatronic applications
Motor selection





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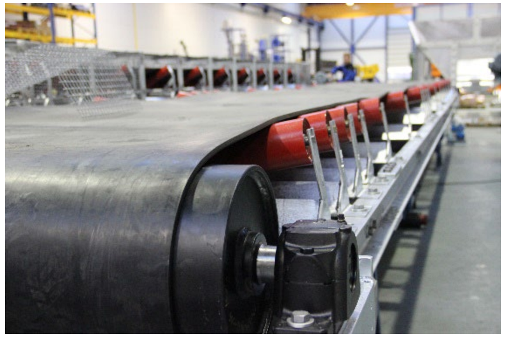
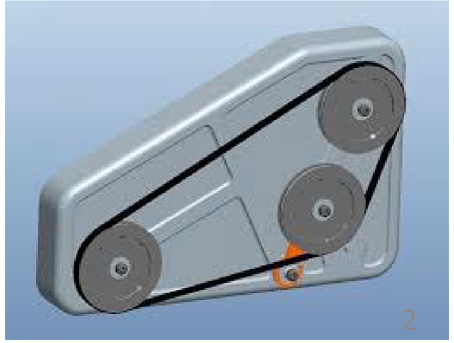
What is an axis in industrial applications?

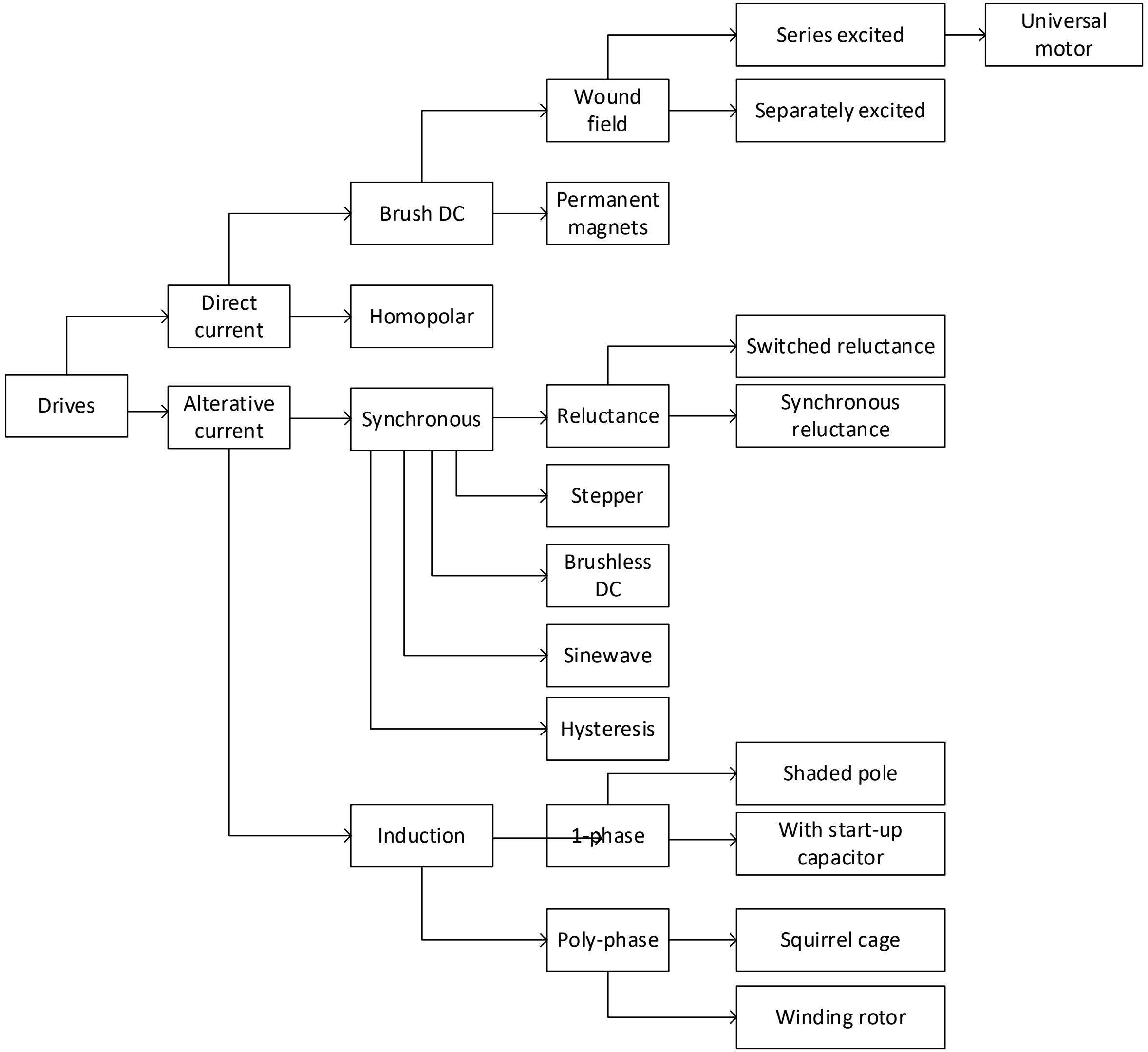
- A mechanical axis (mechanism) is a set of several linked mechanical components.
  - An actuator, usually an electric motor, is the element which drives the movement.
  - A motion profile is usually requested to satisfy a final application fulfilling certain kinematic and dynamic specifications.



The relevant question we want to address, how do we select the motor?

Which motor types exists?



Several options, but **are we addressing the issue from a correct starting point**????

WE NEED THE REQUIREMENTS THAT THE MOTOR SHOULD SATISFY FIRST !

What is the typical way to face motor related exercises in engineering?

Exercise: Study of an induction motor

We consider a squirrel cage induction motor powered by a network with a frequency of **50 Hz** and voltages **between phases** equal to **380 V**.
It was subjected to the following tests:

At no-load:

- **No-load absorbed power :** $P_{a0} = 360\ W$
- **No-load current :** $I_0 = 3.6\ A$
- **Rotation speed :** $N_0 = 2995\ rpm$

With load:

- **Absorbed power :** $P_a = 4560\ W$
- **Current :** $I = 8.1\ A$
- **Rotation speed :** $N = 2880\ rpm$

The windings of the stator are **coupled in star**; The resistance of **each of them** is $r = 0.75\Omega$.
Iron losses in the stator P_{fs} are estimated at **130 W**.

1- What is the synchronism speed **Ns** ? Deduce the slip **g** under load.

2- **For no-load operation:**

a) Calculate the Joule losses P_{j0s} at the stator.
b) Justify that in addition to rotor iron losses, the Joule losses in the rotor are negligible.
c) Deduce the mechanical losses p_{mec}.

3- **Under load operation :**

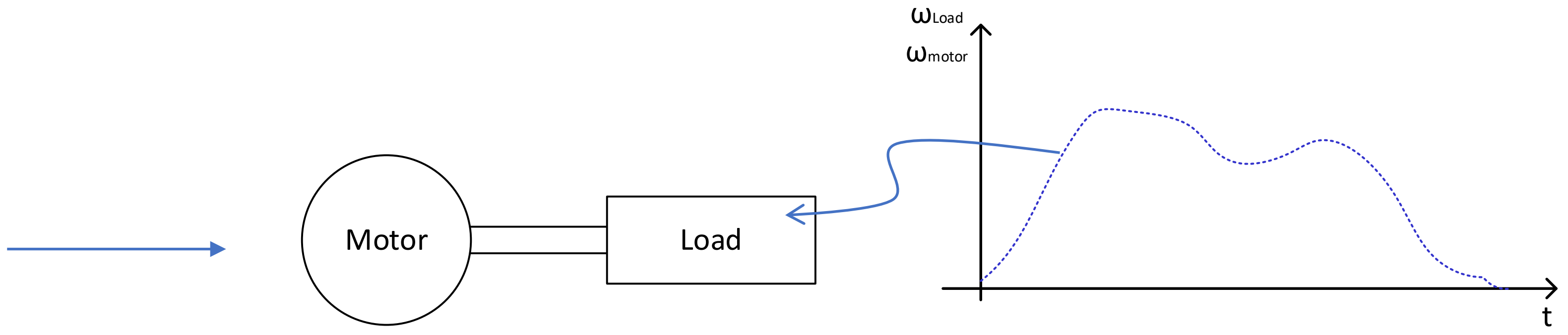
a) Calculate the stator joule losses P_{js} and the joule losses in the rotor P_{jr}
b) Calculate the useful power P_u and torque C_u of the motor
c) Calculate the efficiency of the motor η

This talks about steady state and just discrete operation points.

What about real machine's cycles?

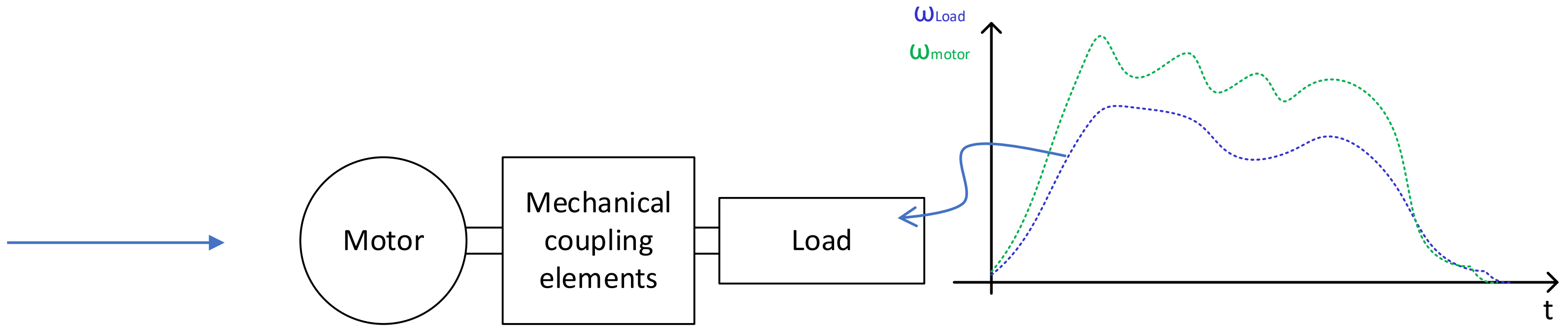
What do we want and what do we really have!

This is what we want



We want the kinematics and dynamics from the motor perspective

This is what we have



We can have a totally different profile movement at motor side. It depends on the coupling elements type and their linearity

So, from where can we start?

Try to make questions to yourself

What elements can be interconnected in an axis chain and how to model them?

Where can the profile be connected?

Which are the characteristic curves of a motor?

Which kind of movement profiles entries can we have?

How to propagate kinematics and dynamics in between elements?

How to judge if a motor is valid or not?

Mechanical elements?

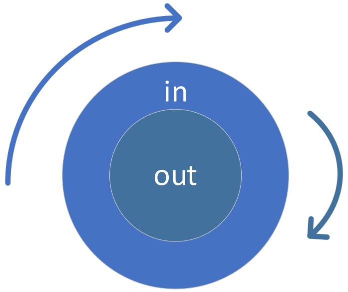
- Four main families (depending on input/output type of movement)

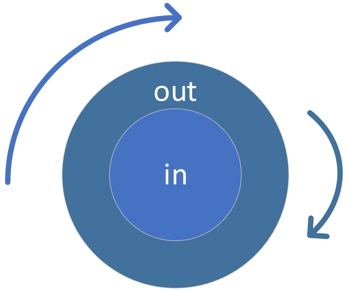
  - **Rotary – Rotary**
  - **Linear – Linear**
  - **Linear – Rotary**
  - **Rotary – Linear**

Non linear element such as Winder, Crank are out of scope

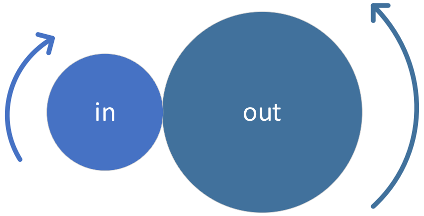
Rotary - Rotary

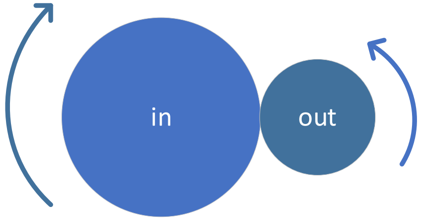
Reduction Gear



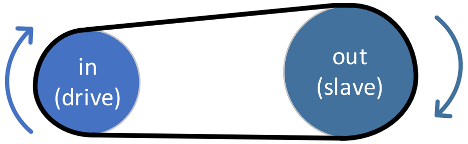


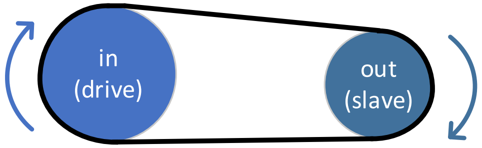
Gear



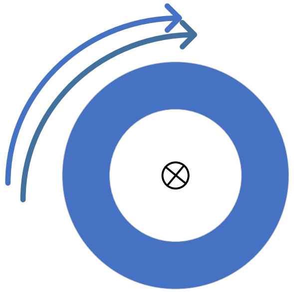


Belt transmission

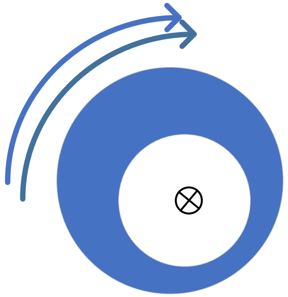




Load

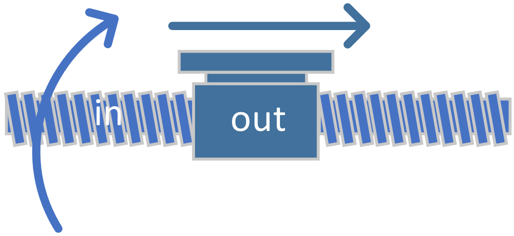


Eccentric load

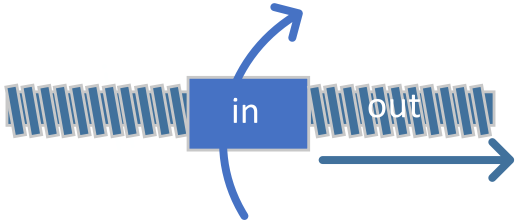


Rotary - Linear

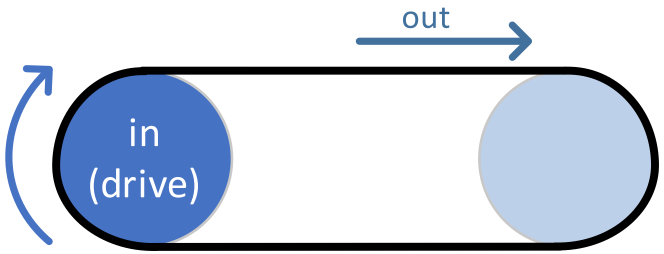
Ball screw



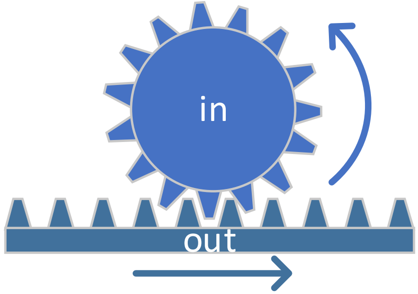
Nut rotating



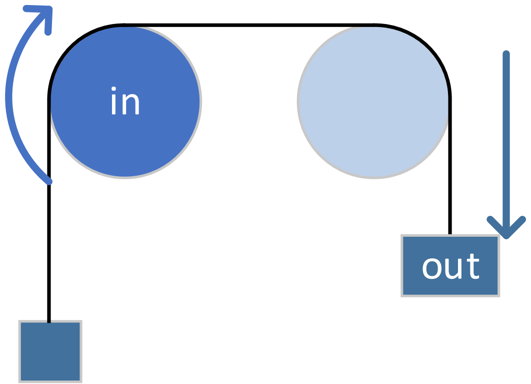
Belt conveyor



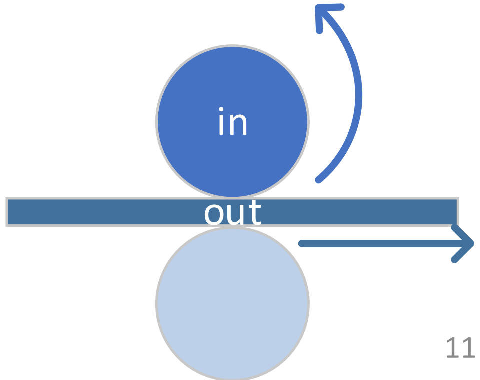
Pinion Rack



Suspension

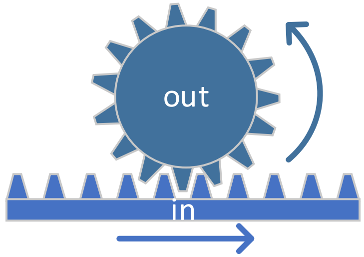


Roll feed



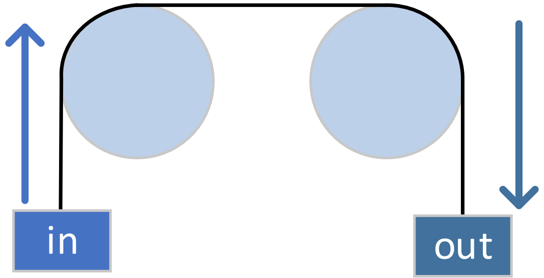
Linear - Rotary

Rack pinion

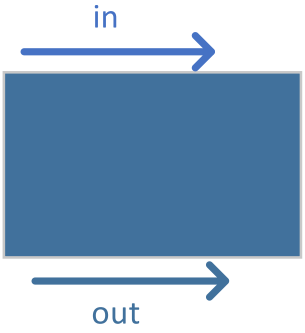


Linear - Linear

Balancer



Mass

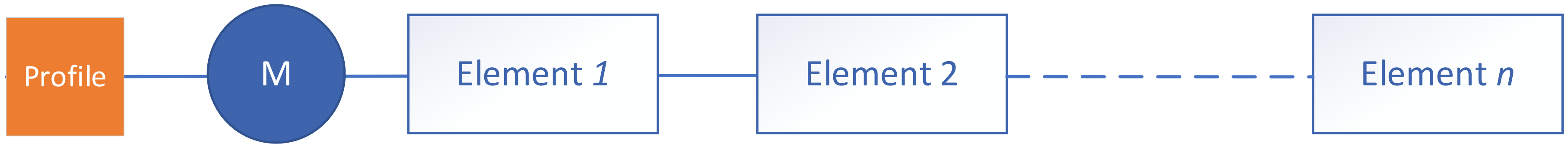


Lever

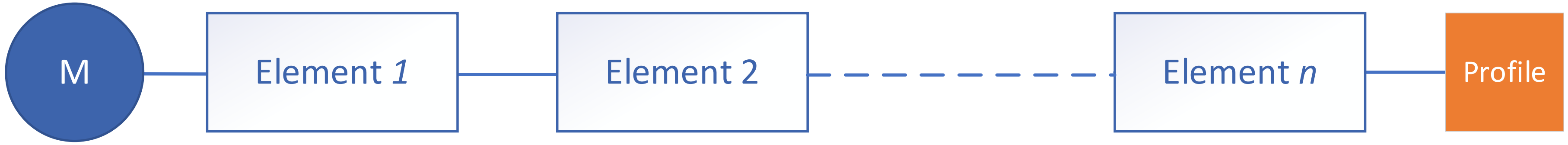
Where can the profile be connected?

- There are two typical options according to the final selection objective.

  - Profile connected upstream motor (*first element*)



  - Profile connected downstream axis (*last element*)



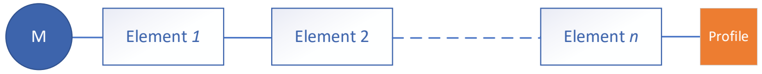
- Profile connected upstream motor

Aim → fix motor movement.

Allows to evaluate how each mechanical element reacts in terms of kinematics (position, speed, acceleration) and dynamics (torque/force).

The motor can be directly related to Profile's kinematics inputs but NOT dynamics

- Profile connected downstream axis

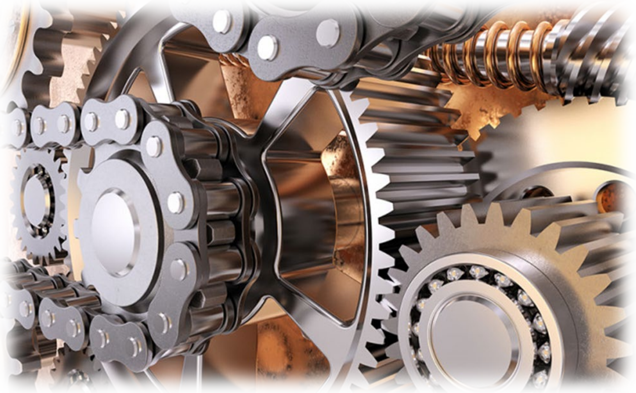


Aim → fix final movement.

By calculus propagation of Element *n* to Motor item the process allows to obtain the kinematics (position, speed, acceleration) and dynamics (torque/force) specifications of the motor.

The motor can not be directly related to any kind of Profile's inputs

How to propagate kinematics and dynamics in between elements?



- In industrial applications is typical to define the kinematics, i.e., the profile movement from speed
  - Position is simply obtained by integration
  - Acceleration is simply obtained by derivation

Calculations can be done analytically but are impractical to automate computations easily and it results complicated to move values from one element to another. This fact is accentuated under complex movement obtained by combination of different segments

At profile

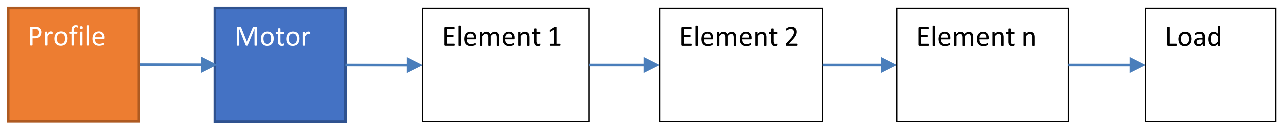
1. Consider
   1. A resolution (**res** $\in \mathbb{Z}^+$) –> number of points for calculations
   2. A time for each instruction, i.e., segment duration (**time** $\in \mathbb{R}^+$)
2. Split
   1. The desired speed ($\omega(t)$) profile into **res**. → ω_k, where k is the k-th discrete value
   2. The segment duration into time deltas of **time** over **res** → $\Delta t =$ **time/res**
3. To obtain:
   1. Position (θ_k) → $\theta_k = \theta_{k-1} + \omega_k \cdot \Delta t$
   2. Acceleration (α_k) → $\alpha_k = (\omega_k - \omega_{k-1})/\Delta t$

Note that θ_k, ω_k and α_k are rotational position, speed and acceleration but are directly usable in linear case just replacing by x_k, v_k and a_k

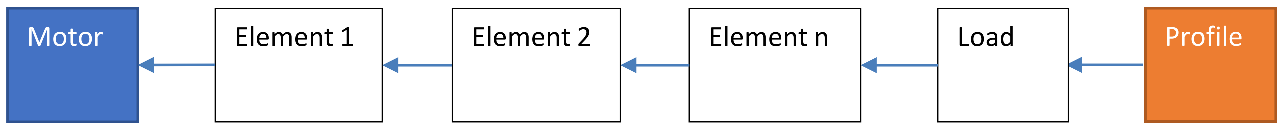
Note, that the propagation of kinematics calculus (“flow”) are dependent on where the Profile is connected to.

kinematics

Motor side

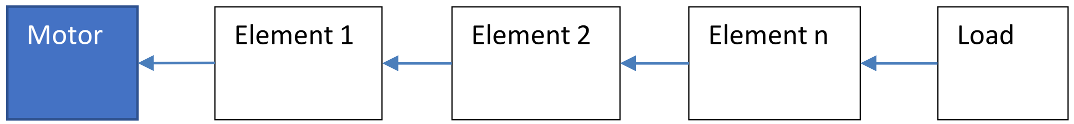


Load side



But the dynamics is more reasonable to be obtained always from Load side and propagate calculations from Load to motor side**, independently on where the profile is defined**

dynamics



Ok, but how to propagate input/output kinematics at same element level?

$Input_{n-1}$ —— Element *n-1* —— $Output_{n-1} = Input_n$ —— Element *n* —— $Output_n$

Assuming linear transformation input/output

The generation of the position tables in **from motor to load** direction are done by the general expressions:

$$Speed_{Output_n} = \boldsymbol{k_p} \cdot Speed_{Input_n}$$

$$Speed_{Input_n} = Speed_{Output_{n-1}}$$

And in **from load to motor** direction:

$$Speed_{Input_n} = \boldsymbol{k_p^{-1}} \cdot Speed_{Output_n}$$

$$Speed_{Output_{n-1}} = Speed_{Input_n}$$

$\boldsymbol{k_p}$ **is the key**

↓

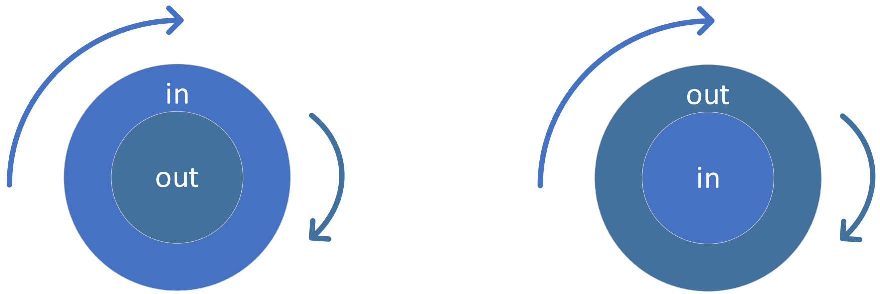
Transformation ratio

Just to define a criteria it has been selected that $\boldsymbol{k_p}$ is defined as the kinematics' output/input ratio

Transformation ratios Rotary - Rotary

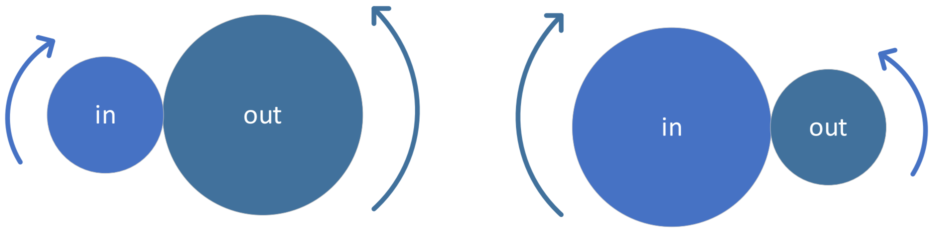
Reduction Gear

$$k_p = \frac{\emptyset_{in}}{\emptyset_{out}}$$



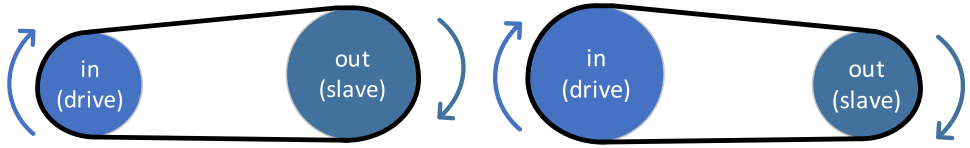
Gear

$$k_p = -\frac{\emptyset_{in}}{\emptyset_{out}}$$



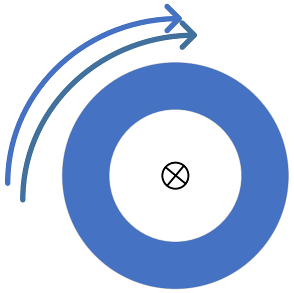
Belt transmission

$$k_p = \frac{\emptyset_{in}}{\emptyset_{out}}$$



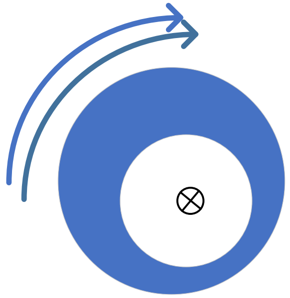
Cylinder load

$$k_p = 1$$



Eccentric load

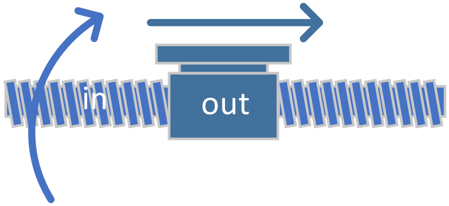
$$k_p = 1$$



Transformation ratios Rotary - Linear

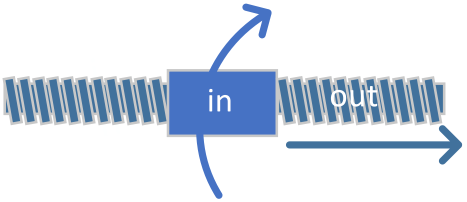
Ball screw

$$k_p = \frac{\text{Screw Pitch}}{2\pi}$$



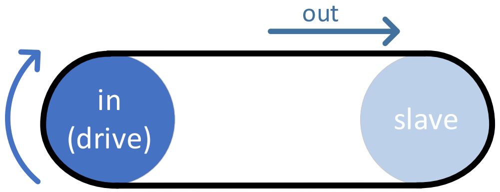
Nut rotating

$$k_p = \frac{\text{Screw Pitch}}{2\pi}$$



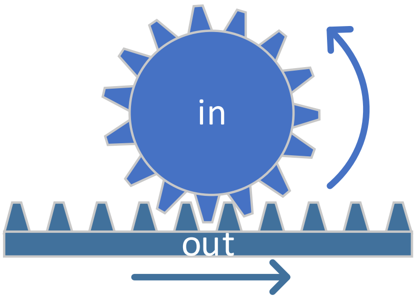
Belt conveyor

$$k_p = \frac{\emptyset_{DrivePulley}}{2}$$



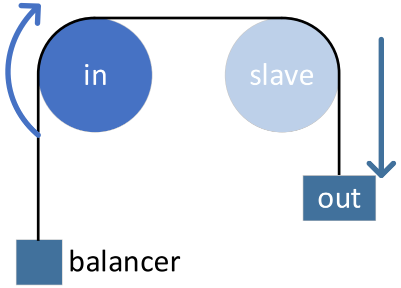
Pinion Rack

$$k_p = \frac{\emptyset_{Pinion}}{2}$$



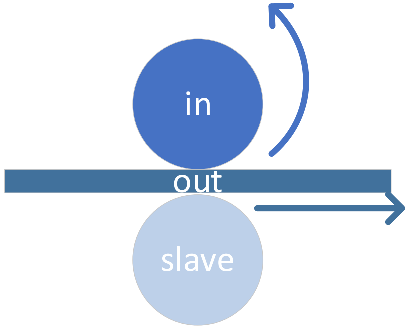
Suspension

$$k_p = \frac{\emptyset_{DriveRoller}}{2}$$



Roll feed

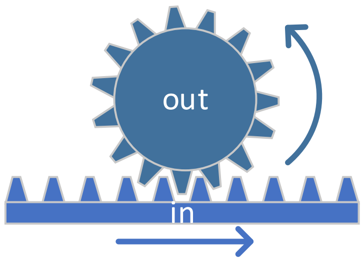
$$k_p = \frac{\emptyset_{DriveRoller}}{2}$$



Transformation ratios Linear - Rotary

Rack pinion

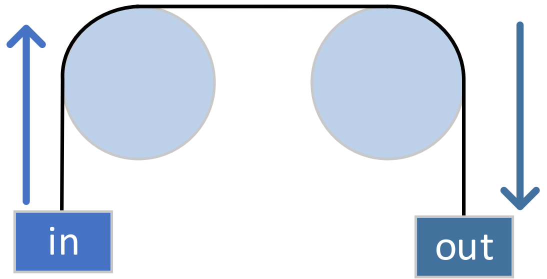
$$\boldsymbol{k_p} = \frac{2}{\emptyset_{Pinion}}$$



Transformation ratios Linear - Linear

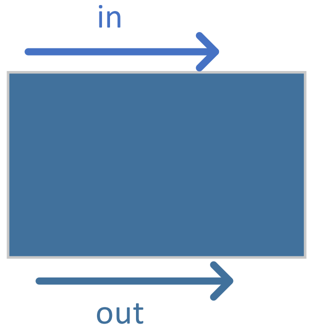
Balancer

$k_p = 1$



Mass

$k_p = 1$



Lever

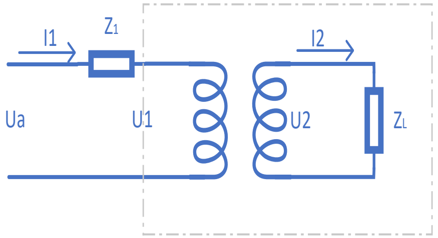
$k_p = \frac{l_{out}}{l_{in}}$

Ok, but how to reflect inertia to motor side?

Reduction to primary side (from load to powered side) → Let's try to find a duality electricity - mechanics

Electricity

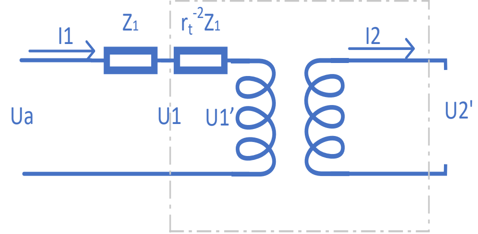
$$r_t = u_2/u_1$$



$$P = K = U_1 I_1 = U_2 I_2$$

$$Z_L = \frac{u_2}{i_2},\ u_2 = r_t u_1,\quad i_1 = i_2 r_t$$

$$Z_L' = \frac{u_1}{i_1} = \frac{\frac{u_2}{r_t}}{i_2 r_t} = r_t^{-2}\frac{u_2}{i_2} = r_t^{-2} Z_L$$

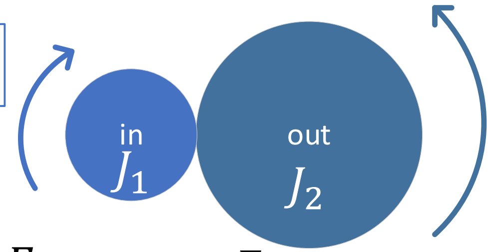


$$Z_{1eq} = Z_1 + Z_L'$$

In the electric case the Z is the opposition to current

Mechanics

$$k_p = \omega_2/\omega_1$$

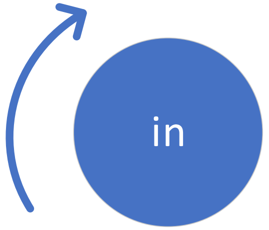


$\Gamma_1,\ \omega_1,\ \alpha_1$ $\Gamma_2,\ \omega_2,\ \alpha_2$

$$P = K = \Gamma_1 \omega_1 = \Gamma_2 \omega_2$$

$$J_2 = \frac{\Gamma_2}{\alpha_2},\ \alpha_2 = k_p \alpha_1,\ \Gamma_1 = \Gamma_2 k_p$$

$$J_2' = \frac{\Gamma_1}{\alpha_1} = \frac{\Gamma_2 k_p}{\frac{\alpha_2}{k_p}} = \mathrm{k}_\mathrm{p}^2 \frac{\alpha_2}{\Gamma_2} = \mathrm{k}_\mathrm{p}^2 J_2$$



$$J_{1eq} = J_1 + J_2'$$

In the mechanical case the J is the opposition to speed

What is the practical implication of this result?

$$J_2' = \frac{\Gamma_2 k_p}{\frac{\alpha_2}{k_p}} = \mathrm{k_p^2}\ \frac{\alpha_2}{\Gamma_2} = \mathrm{k_p^2} J_2$$

The practical implication is that if we want to add more inertia, in other words, more difficulty to a change in speed, elements such as flywheels are better to be fitted at the side where the $\boldsymbol{k_p}$ is not affecting quadratically the reflected results.

Usually, if we consider a gear with a reduction speed at its output this means that the flywheel is connected at the motor side.

$$\mathrm{Inertia(Mass)_{Input}} = \mathrm{k_p^2 Inertia(Mass)_{Output}}$$

$$Cumulative\ Inertia\ to\ load\ side_N = (Cumulative\ Inertia\ to\ load\ side_{N-1} + Inertia\ (or\ Mass)_N)k_{\mathrm{p}}^{-2}$$

$$\boxed{Cumulative\ Inertia\ to\ motor\ side_N = Cumulative\ Inertia\ to\ motor\ side_{N+1} \cdot k_{\mathrm{p}}^2 + Inertia\ (or\ Mass)_N}$$

$Cumulative\ Inertia\ to\ motor\ side_N|_{N=motor}$ is the total inertia that “sees” the motor.

A motor should be able to move its own inertia plus all other inertias reduced to the motor’s axis.

Rigidity criteria

Final considerations

In each intermediate element the corresponding force and torque calculations will be done on the input movement frame (**Newton's Law**):

Rotary - Rotary

$$\Gamma_{Input_n} = Inertia_n \cdot \frac{d}{dt}\,\omega_{Input_n} + k_{\mathrm{p}}\,\Gamma_{Output_n} + \Gamma_{External_n}$$

Rotary - Linear

$$\Gamma_{Input_n} = Inertia_n \cdot \frac{d}{dt}\,\omega_{Input_n} + k_{\mathrm{p}}\,F_{Output_n} + \Gamma_{External_n}$$

Linear - Linear

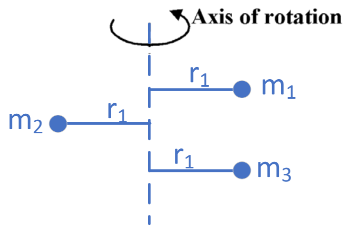
$$F_{Input_n} = Mass_n \cdot \frac{d}{dt}\,v_{Input_n} + k_{\mathrm{p}}\,F_{Output_n} + F_{External_n}$$

Linear - Rotary

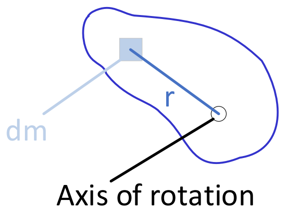
$$F_{Input_n} = Mass_n \cdot \frac{d}{dt}\,v_{Input_n} + k_{\mathrm{p}}\,\Gamma_{Output_n} + F_{External_n}$$

Where $\Gamma_{External_n}$ or $F_{External_n}$is, if proceed, the external torque or force applied at element n seen from the input side.

My inertia

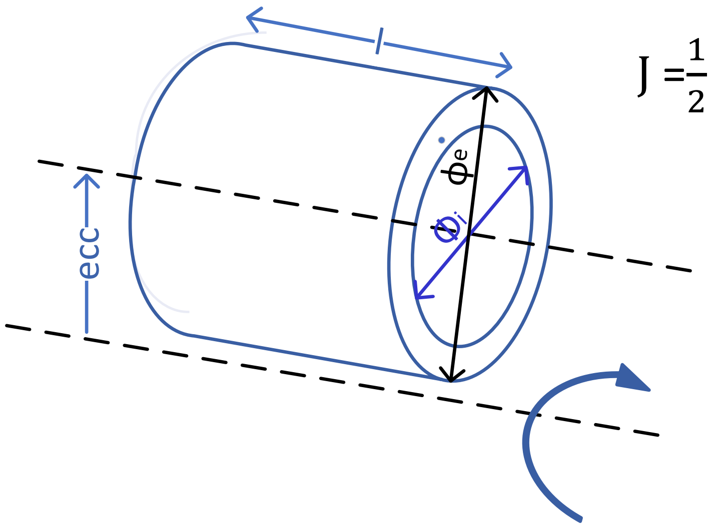


$$J = \sum_i m_i \cdot r_i^2$$



$$J = \int_0^M r^2 dm$$

Cylindrical Geometry



$$J = \frac{1}{2} mR^2 \text{ (solid cilinder)}$$

$$m = \frac{\rho \pi l}{4}\left(\phi_e^2 - \phi_i^2\right)$$

$$J = \frac{\rho \pi l}{32}\left(\phi_e^4 - \phi_i^4\right) + m \cdot ecc^2 = \frac{m}{8}\left(\phi_e^2 + \phi_i^2\right) + m \cdot ecc^2$$

Being

ρ: density of material [kg/m^3]

l: length [m]

m: mass [kg]

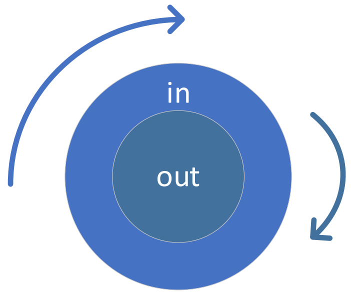
ecc: eccentricity [m]

r: distance [m]

My inertia Rotary - Rotary

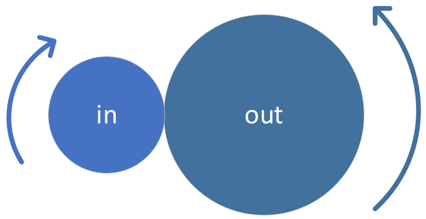
Reduction Gear (Considered as a unique element in terms of inertia)

$$J = \frac{\rho\pi}{32}\left(l_{in}\varnothing_{in}^4 + l_{out}\varnothing_{out}^4\right)$$



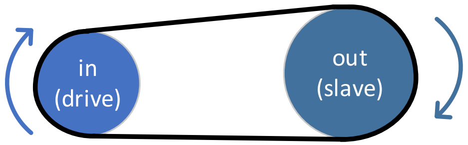
Gear

$$J_{in} = \frac{\rho\pi}{32} l_{in}\varnothing_{in}^4$$
$$J_{out} = \frac{\rho\pi}{32} l_{out}\varnothing_{out}^4$$
$$J = J_{in} + J_{out} \cdot \mathrm{k}_{\mathrm{p}_{\mathrm{gear}}}^2$$



Belt transmission

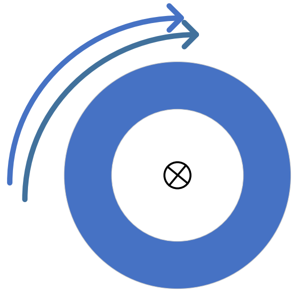
$$J_{in} = \frac{\rho\pi}{32} l_{drive}\varnothing_{drive}^4$$
$$J_{out} = \frac{\rho\pi}{32} l_{slave}\varnothing_{slave}^4$$



$$J = J_{in} + J_{out} \cdot \mathrm{k}_{\mathrm{p}_{\mathrm{beltTransmission}}}^2 + \frac{\varnothing_{drive}^2}{4} m_{belt}$$

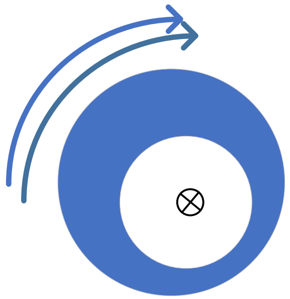
Cylinder load

$$J = \frac{\rho\pi l}{32}\left(\varnothing^4 - \varnothing_{hole}^4\right)$$



Eccentric load

$$m = \frac{\rho\pi l}{4}\left(\varnothing^2 - \varnothing_{hole}^2\right)$$
$$J = \frac{\rho\pi l}{32}\left(\varnothing^4 - \varnothing_{hole}^4\right) + m \cdot ecc^2$$

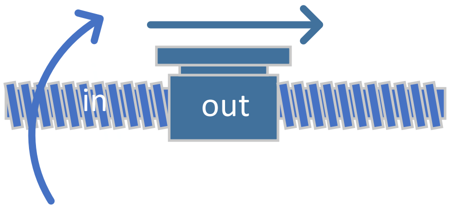


My inertia Rotary - Linear

We could think of mass loads as if the mass was removed and replaced by a flywheel fitted to the pulley (or sheave) shaft, the value of which is J.

Ball screw

$$J_{in} = \frac{\rho\pi}{32} l_{screw} \varnothing_{screw}^4$$

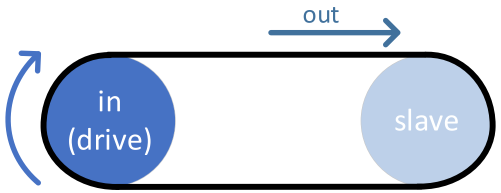


$$J = J_{in} + m_{table} \cdot \left(\frac{\text{Screw Pitch}}{2\pi}\right)^2$$

Belt conveyor

$$J_{in} = \frac{\rho\pi}{32} l_{drive} \varnothing_{drive}^4$$

$$J_{slave} = \frac{\rho\pi}{32} l_{slave} \varnothing_{slave}^4$$

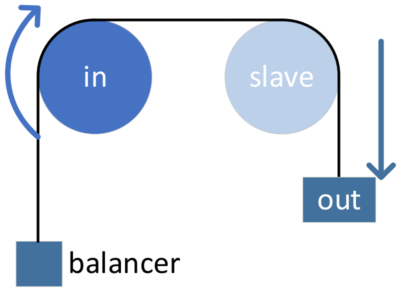


$$J = J_{in} + \Big(m_{belt} + \overbrace{J_{slave} \cdot \frac{4}{\varnothing_{slave}^2}}^{Equivalent\ Mass} \Big) \frac{\varnothing_{drive}^2}{4}$$

Suspension

$$J_{in} = \frac{\rho\pi}{32} l_{drive} \varnothing_{drive}^4$$

$$J_{slave} = \frac{\rho\pi}{32} l_{slave} \varnothing_{slave}^4$$



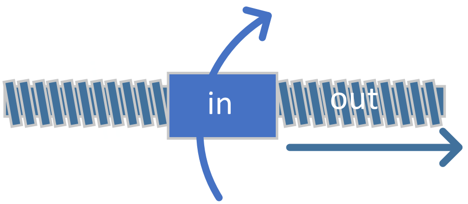
$$J = J_{in} + \Big(m_{balancer} + m_{workpiece} + \overbrace{J_{slave} \cdot \frac{4}{\varnothing_{slave}^2}}^{Equivalent\ Mass} \Big) \frac{\varnothing_{drive}^2}{4}$$

My inertia Rotary - Linear

We could think of mass loads as if the mass was removed and replaced by a flywheel fitted to the pulley (or sheave) shaft, the value of which is J.

Nut rotating

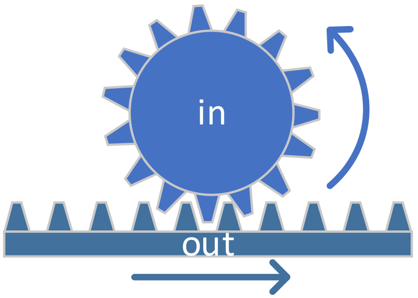
$$J_{in} = \frac{\rho\pi}{32} l_{Nut} \emptyset_{Nut}^4$$



$$J = J_{in} + m_{Screw} \cdot \left(\frac{\text{Screw Pitch}}{2\pi}\right)^2$$

Pinion Rack

$$J_{in} = \frac{\rho\pi}{32} l_{gear} \emptyset_{gear}^4$$

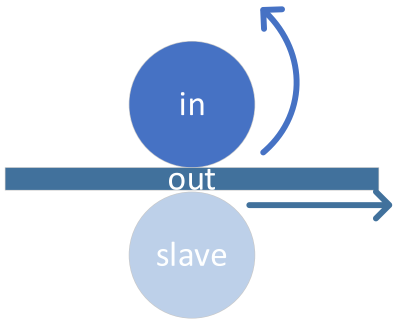


$$J = J_{in} + m_{rack} \cdot \frac{\emptyset_{gear}^2}{4}$$

Roll feed

$$J_{in} = \frac{\rho\pi}{32} l_{in} \emptyset_{in}^4$$

$$J_{slave} = \frac{\rho\pi}{32} l_{slave} \emptyset_{slave}^4$$

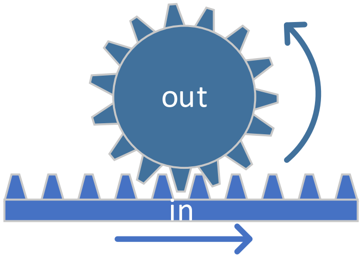


$$J = J_{in} + J_{slave} \cdot \mathrm{k}_{\mathrm{p}_{RollFeed}}^2 + m_{feed} \cdot \frac{\emptyset_{drive}^2}{4}$$

My equivalent mass Linear - Rotary

Rack pinion

$$\mathbf{J_{out}} = \frac{\rho\pi}{32} l_{gear} \emptyset_{gear}^4$$



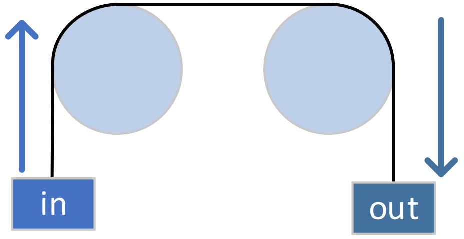
$$\mathbf{m} = \overbrace{J_{out} \cdot \frac{4}{\emptyset_{gear}^2}}^{Equivalent\ Mass} + m_{rack}$$

My equivalent mass Linear - Linear

Balancer

$$J_{\mathbf{roller1}} = \frac{\rho\pi}{32} l_{roller1} \emptyset^4_{roller1}$$

$$J_{\mathbf{roller2}} = \frac{\rho\pi}{32} l_{roller2} \emptyset^4_{roller2}$$

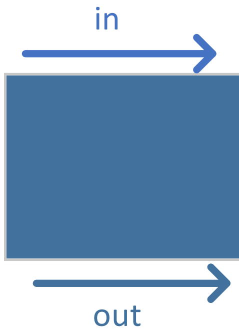


$$\mathbf{m} = \overbrace{J_{Roller1} \cdot \frac{4}{\emptyset^2_{roller1}} + J_{Roller2} \cdot \frac{4}{\emptyset^2_{roller3}}}^{Equivalent\ Mass} + m_{in} + m_{out}$$

Lever

$$\mathbf{m} = m_{out} \cdot \left(\frac{l_{out}}{\mathrm{l}_{in}}\right)^2$$

Mass



$$\mathbf{m} = m$$

Ok, but the component efficiency to which variables affects?

Let's deduce it from Reduction gear or gear box case by denoting the efficiency of the gearbox as η_g

$$P_{out} = \eta_g \, P_{in}$$

$$T_{out}\omega_{out} = \eta_g T_{in}\omega_{in}$$

By definition the relationship between input and output speeds applied can be defined as

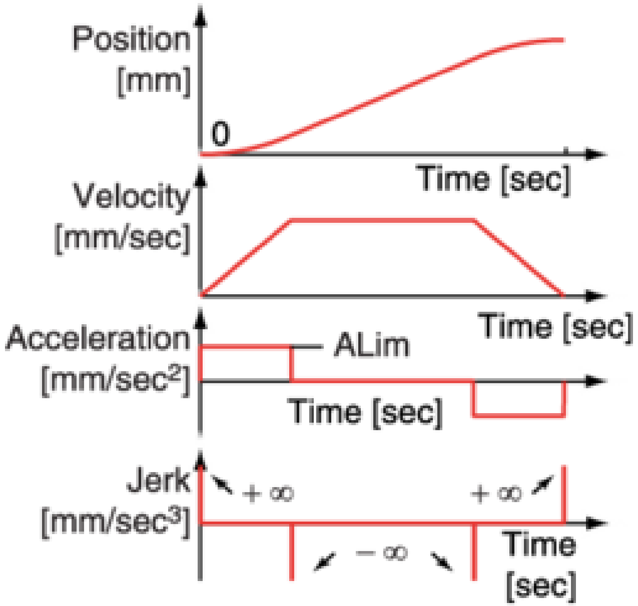
$$k_p = \frac{\emptyset_{in}}{\emptyset_{out}} = \frac{\omega_{out}}{\omega_{in}}$$

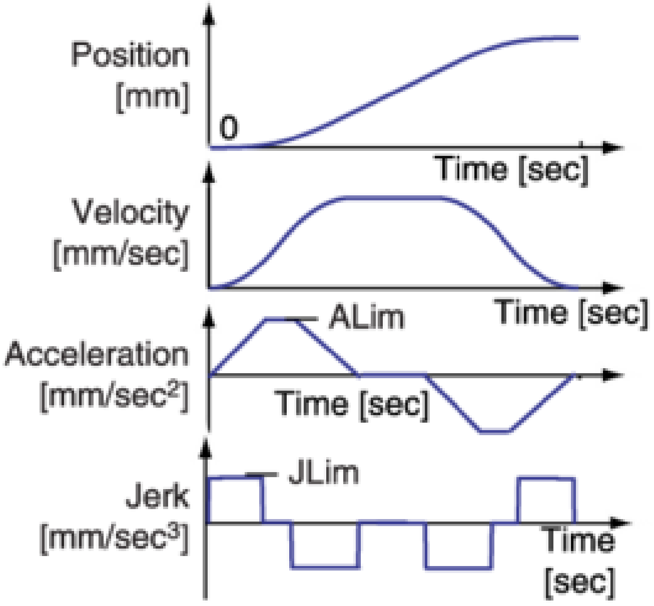
Then, it can be deduced that the relationship between the speeds keeps unchanged. In other words, the efficiency only affects the Torque or the power but not kinematics

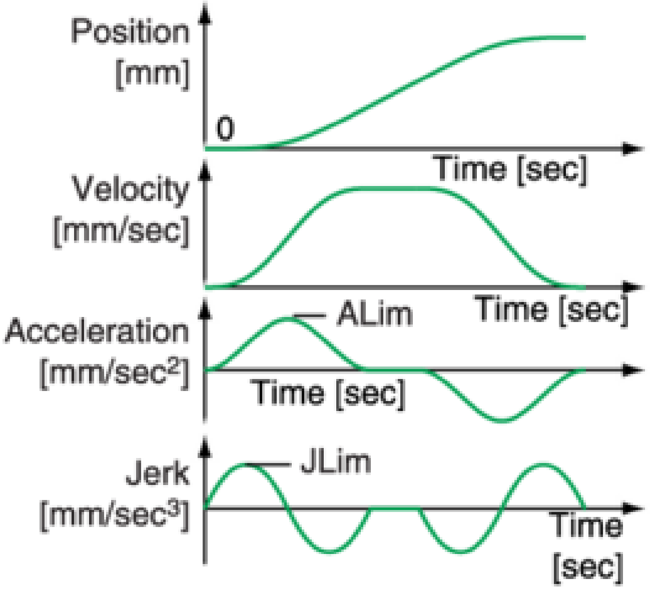
$$k_p = \frac{\omega_{out}}{\omega_{in}} = \frac{\eta_g T_{in}}{T_{out}}$$

$$T_{out} = \frac{\eta_g T_{in}}{k_p}$$

Which kind of movement profiles entries can we have?



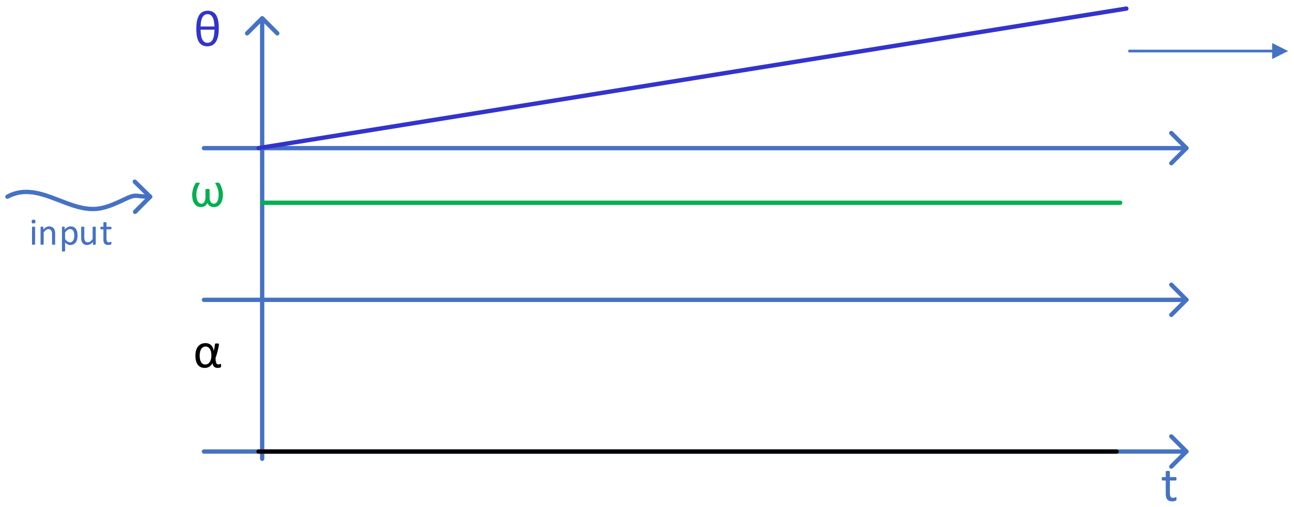




Typical profiles entries (when apply)

- Previous position, speed and acceleration
- Increment of position (desired trajectory)
- Time to position
- Dwell time (rest time after movement)
- External Torque or Force (for directly applied torques or forces to the profile's connected component)

Constant speed

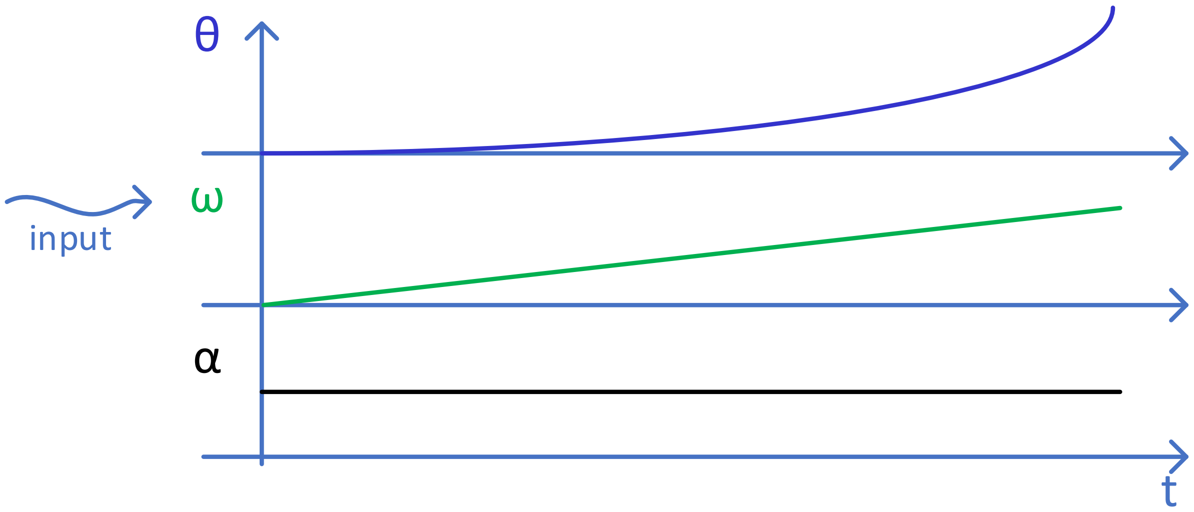


First order (Uniform Rectilinear Movement)

$$\omega = \boldsymbol{\omega_0}$$
$$\theta = \boldsymbol{\theta_0} + \omega t$$

Bold indicates known or entry parameter

Ramp speed



Second order (Uniform Accelerated Movement)

$$\theta = \theta_0 + \omega t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{\Delta\omega}{t}$$

In this case it is typical to entry by (we always know $\boldsymbol{\omega_0}$, $\boldsymbol{\theta_0}$,$\boldsymbol{t}$)

- Final position ($\boldsymbol{\theta_f}$)

$$\alpha \boldsymbol{t} = \omega_f - \boldsymbol{\omega_0}$$

$$\omega_0 \boldsymbol{t} + \frac{(\omega_f - \boldsymbol{\omega_0})\boldsymbol{t}}{2} = \boldsymbol{\theta_f} - \boldsymbol{\theta_0} \longrightarrow \alpha = 2\frac{\boldsymbol{\theta_f} - \boldsymbol{\theta_0} - \boldsymbol{\omega_0 t}}{\boldsymbol{t^2}} \rightarrow \theta, \omega$$

- Final speed ($\boldsymbol{\omega_f}$)

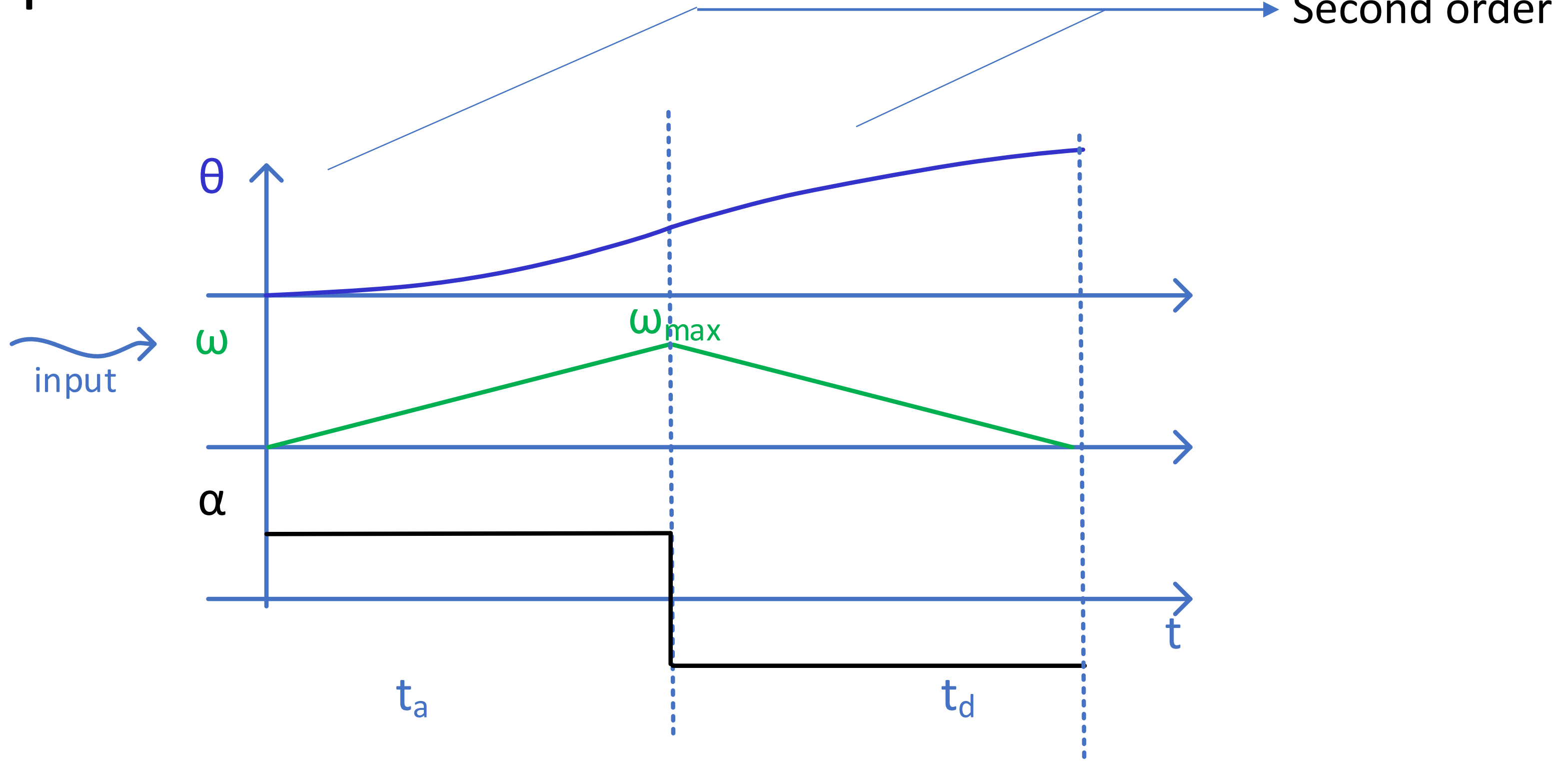
$$\alpha = \frac{\boldsymbol{\omega_f} - \boldsymbol{\omega_0}}{\boldsymbol{t}} \longrightarrow \theta, \omega$$

- Acceleration ($\boldsymbol{\alpha}$)

$$\alpha = \boldsymbol{\alpha} \longrightarrow \theta, \omega$$

Bold indicates known or entry parameter

Triangular speed



$$\boldsymbol{\theta_{total}} = \theta_a + \theta_d$$

$$\boldsymbol{\theta_{total}} = \left(\frac{1}{2} t_a \omega_{max}\right) + \left(\frac{1}{2} t_d \omega_{max}\right)$$

$$\boldsymbol{\theta_{total}} = \left(\frac{1}{2} \omega_{max}\right)(t_a + t_d)$$

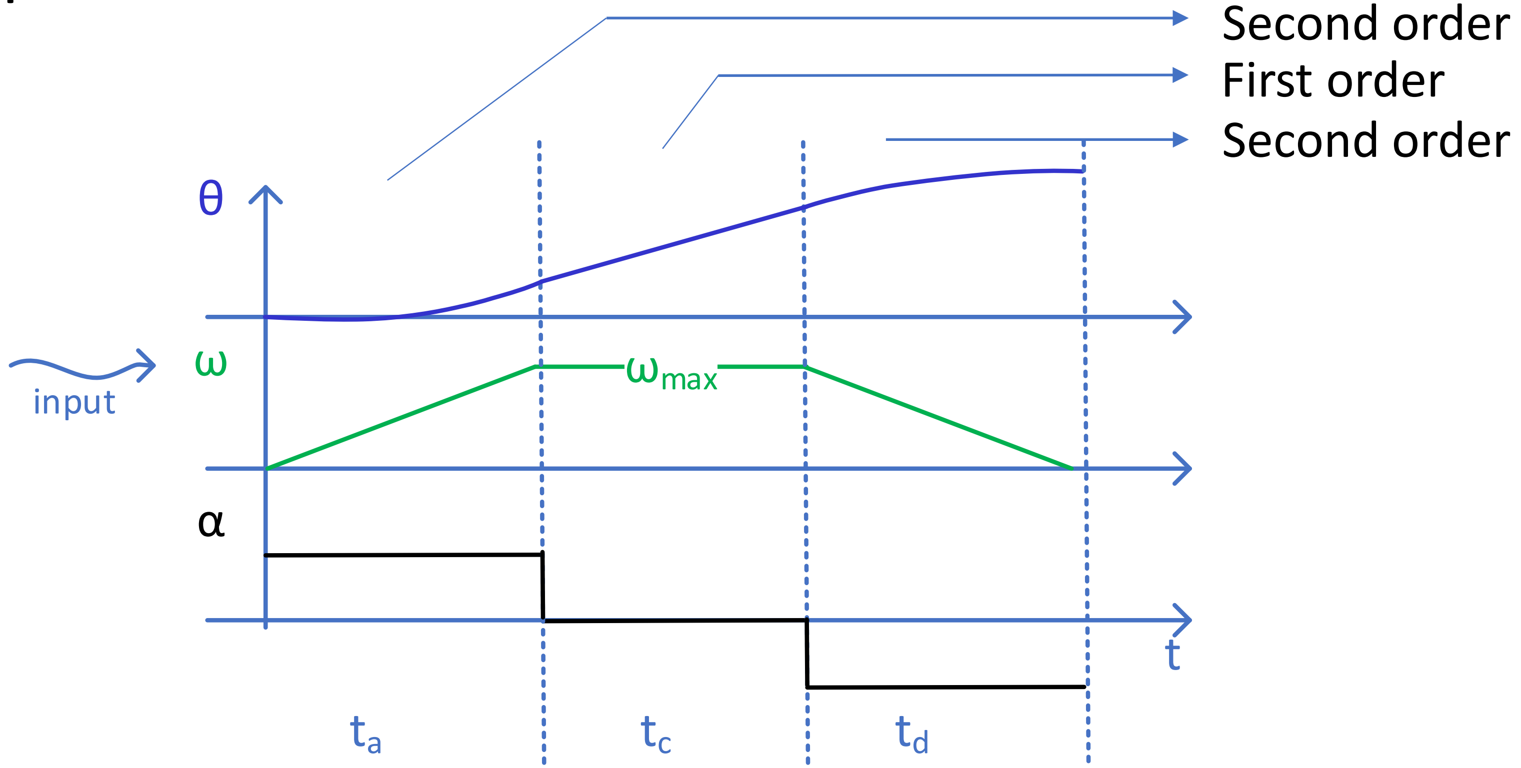
For symmetrical profiles (1/2 each segment)

$$\boldsymbol{\theta_{total}} = \frac{1}{2} \boldsymbol{t_{total}} \omega_{max}$$

$$\begin{cases} \omega_{max} = 2 \dfrac{\boldsymbol{\theta_{total}}}{\boldsymbol{t_{total}}} \\ |\alpha| = \dfrac{\omega_{max}}{t_a} = \dfrac{\omega_{max}}{t_d} \equiv 4 \dfrac{\boldsymbol{\theta_{total}}}{\boldsymbol{t_{total}^2}} \end{cases}$$

Bold indicates known or entry parameter

Trapezoidal speed



$$\boldsymbol{\theta_{total}} = \theta_a + \theta_c + \theta_d$$

$$\boldsymbol{\theta_{total}} = \left(\frac{1}{2} t_a \omega_{max}\right) + (t_c \omega_{max}) + \left(\frac{1}{2} t_d \omega_{max}\right)$$

$$\boldsymbol{\theta_{total}} = \left(\frac{1}{2} \omega_{max}\right)(t_a + t_d) + (t_c \omega_{max})$$

For symmetrical profiles (1/3 each segment)

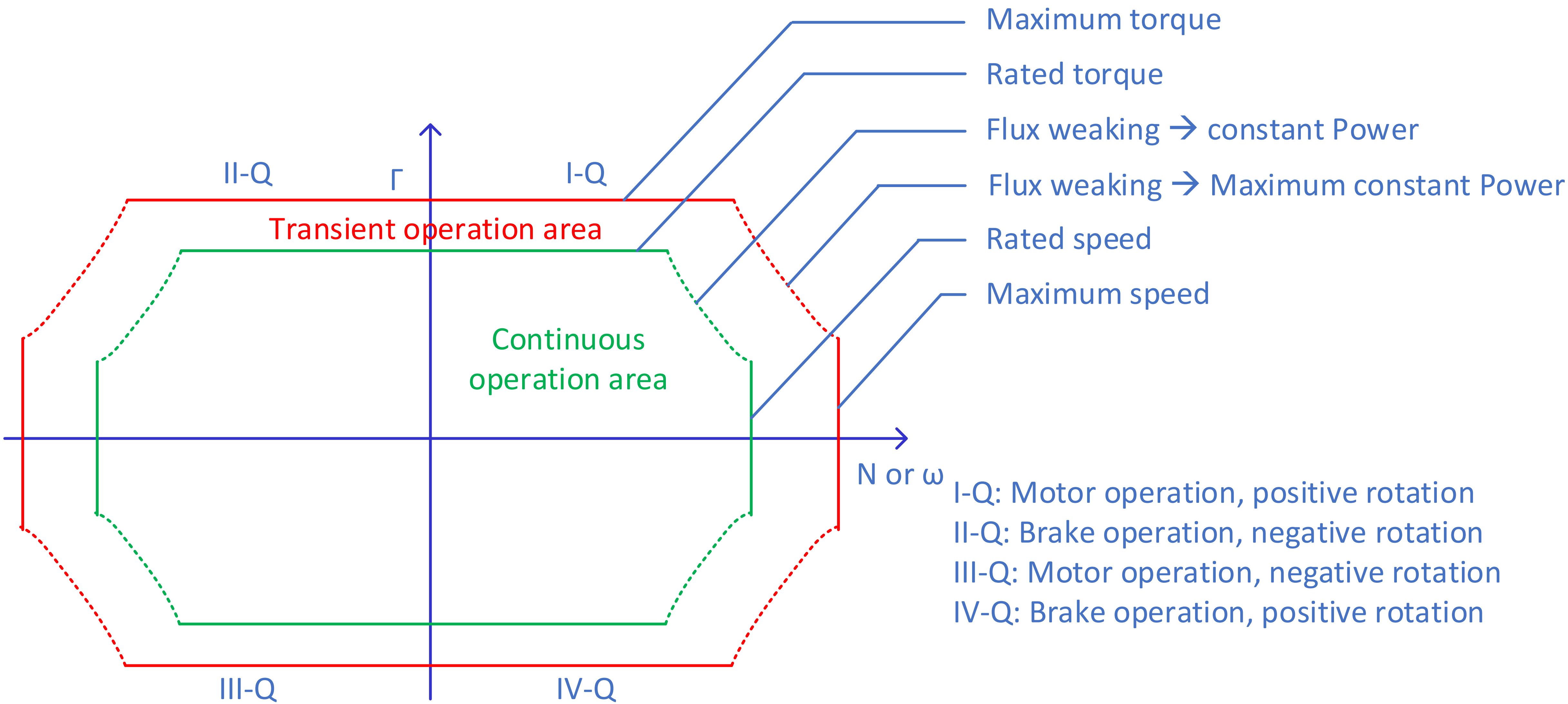
$$\boldsymbol{\theta_{total}} = \frac{2}{3} \boldsymbol{t_{total}} \omega_{max}$$

$$\omega_{max} = \frac{3}{2} \frac{\boldsymbol{\theta_{total}}}{\boldsymbol{t_{total}}}$$

$$|\alpha| = \frac{\omega_{max}}{t_a} = \frac{\omega_{max}}{t_d} \equiv 4.5 \frac{\boldsymbol{\theta_{total}}}{\boldsymbol{t_{total}^2}}$$

Bold indicates known or entry parameter

Which are the characteristic curves of a motor?

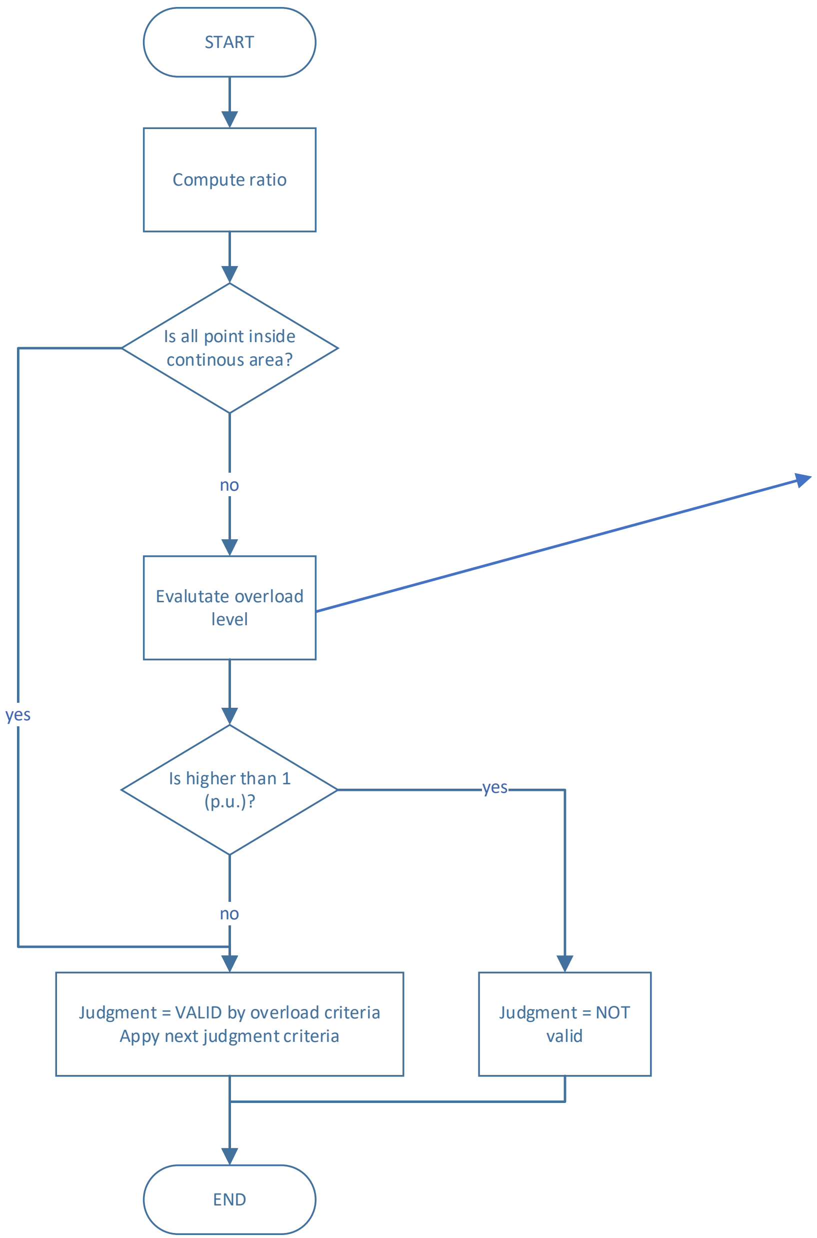


I-Q: Motor operation, positive rotation
II-Q: Brake operation, negative rotation
III-Q: Motor operation, negative rotation
IV-Q: Brake operation, positive rotation

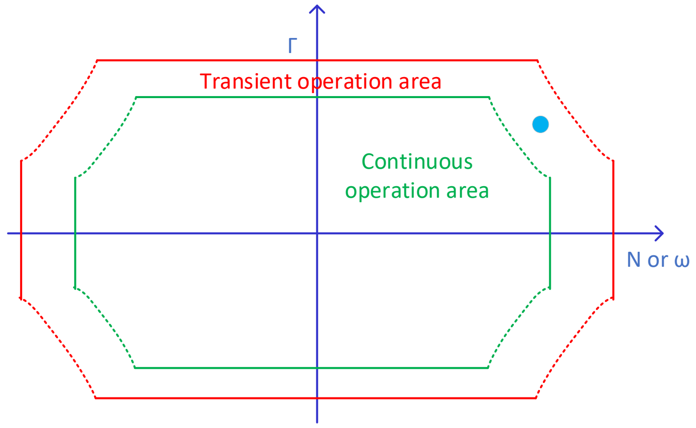
Motor judgment?

Not everything is black or white in engineering & the same applies to the industry → Usually is typical to define levels, at least 2 or 3 (Good, Not recommended (orange), Not good (red))

By Mn Graph



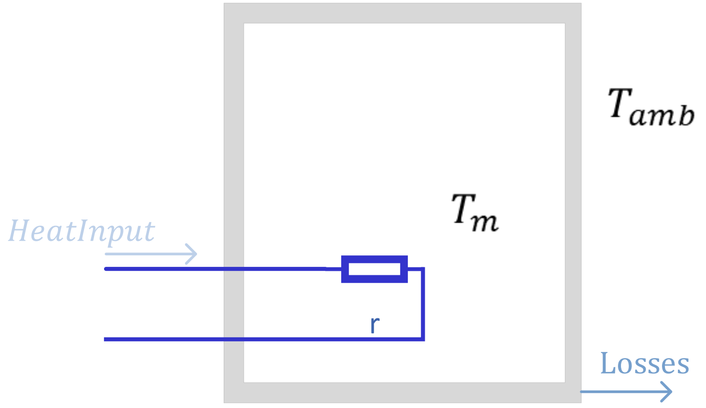
How much time can we be in between continuous and transient operation area?



Overload evaluation (I/IV)

A first order thermal model can be used for this purpose. The model starts defining

$$T = T_m - T_{amb} \qquad (1)$$



being

T: Temperature rise above ambient [ºC]

T_m: Motor temperature (windings) [ºC]

T_{amb}: Ambient temperature [ºC]

Then, the rate of increase temperature that defines the thermal equilibrium yields to

$$HeatInput - Losses = C_s m \frac{dT_m}{dt} = C_s m \frac{dT}{dt} \qquad (2)$$

being

C_s: specific heat of windings [W·s/kg·ºC]

m: mass [kg]

Accumulated heat in W

Overload evaluation (II/IV)

The quantity of heat transferred to the surrounding ambient (losses) can be modelled as

$$Losses = \frac{T_m - T_{amb}}{R} = \frac{T}{R} \qquad (3)$$

being

R: thermal resistance [ºC/W]

By using the windings current, I, in (2) for the $HeatInput$ term and by also applying (3), it is possible to obtain

$$I^2 r - \frac{T}{R} = C_s m \frac{dT}{dt}$$

$$I^2 r R = mRC_s \frac{dT}{dt} + T \qquad (4)$$

being

r: winding resistance

And defining a thermal constant as

$$\tau = mRC_s$$

Overload evaluation (III/IV)

It is possible to rearrange (4) as

$$I^2 = \tau\left(\frac{1}{rR}\frac{dT}{dt}\right) + \frac{T}{rR} \qquad (5)$$

Let's define, for simplicity, a virtual equivalent temperature

$$T_v = \frac{T}{rR}$$

Thus, (5) yields to

$$\boxed{I^2 = \tau\,\frac{dT_v}{dt} + T_v} \qquad (6)$$

By solving the ODE (6)

$$\boxed{T_v = I^2 \cdot \left(1 - e^{-t/\tau}\right) + I_0^2 \cdot e^{-t/\tau}} \qquad (7)$$

Usually corresponds to T_{amb}

where

I_0: initial current [A]

Overload evaluation (IV/IV)

The incremental form of (7) is

$$\boxed{T_v(k) = I^2 \cdot \left(1 - e^{-(t_k - t_{k-1})/\tau}\right) + T_v(k\text{–}1)\cdot e^{-(t_k - t_{k-1})/\tau}} \quad (8)$$

But results more interesting to use (8) in p.u.

$$\boxed{t_v(k) = i^2 \cdot \left(1 - e^{-t/\tau}\right) + t_v(k\text{–}1)\cdot e^{-t/\tau}} \quad (9)$$

where

i: is the ratio I over the rated current I_r

t_v : virtual temperature in p.u

Note

- If $t_k - t_{k-1}$ =Δt is small, then $e^{-\Delta t/\tau} \approx (1 - \Delta t/\tau)$
- If (9) is used, as i is a relative magnitude it can be supplied by p.u. torque (γ) under motor operation where $\Gamma \propto I$. Thus, the proportional factor does not affect.

$$\boxed{t_v(k) = \gamma^2 \cdot \left(1 - e^{-t/\tau}\right) + t_v(k\text{–}1)\cdot e^{-t/\tau}} \quad (10)$$

$\boldsymbol{safety\ margin}$: Percentage of extra torque that we are assuming due to simplifications on calculus procedures or lack of information. Usually, at industrial level, this margin is about 10÷20%.

Overload level. In this case it has been conceptualizes overload as a temperature issue

$$OL = t_v$$

$OL \leq 1 (with\ safety\ margin)$	Good
$OL \leq 1\ (without\ safety\ margin)$	Not recommended
$OL > 1\ (w/o\ safety\ margin)$	Not good

By Ratios

Inertia ratio. It is related with axis mechanical rigidity (to avoid torsional issues and stability problems (this is capability to control kinematic variables without fatigue))

$$J_i = \frac{J'_{all\ axis}}{J_{motor}}$$

A conservative criteria is

$J_i \leq 1$	Good
$1 < J_i \leq 2$	Not recommended
$J_i > 2$	Not good

Torque ratios.

$$\Gamma_{i_{Rated}} = \frac{\Gamma_{TotalRequiredRated}}{\Gamma_{MotorRated}}$$

Including motor

$\Gamma_{i_{Rated}} \leq 1\ (with\ safety\ margin)$	Good
$\Gamma_{i_{Rated}} \leq 1\ (without\ safety\ margin)$	Not recommended
$\Gamma_{i_{Rated}} > 1\ (w/o\ safety\ margin)$	Not good

$$\Gamma_{i_{Max}} = \frac{\Gamma_{TotalRequiredMax}}{\Gamma_{MotorMax}}$$

Including motor

$\Gamma_{i_{Max}} \leq 1\ (with\ safety\ margin)$	Good
$\Gamma_{i_{Max}} \leq 1\ (without\ safety\ margin)$	Not recommended
$\Gamma_{i_{Max}} > 1\ (w/o\ safety\ margin)$	Not good

By Ratios

Speed ratios.

$$\omega_{i_{Rated}} = \frac{\omega_{Rated@Motor}}{\omega_{MotorRated}}$$

$\omega_{i_{Rated}} \leq 1$ Good

$\omega_{i_{Rated}} > 1$ Not good

$$\omega_{i_{Max}} = \frac{\omega_{Max@Motor}}{\Gamma_{MotorMax}}$$

$\omega_{i_{Max}} \leq 1$ Good

$\omega_{i_{Max}} > 1$ Not good

Brake?

How to brake the motor?

The fundamentals of braking calculations are:

- There is sufficient torque to stop and hold the machine at no speed
- For dynamic stops, the power dissipation is acceptable for the contact area of the brake
- The operating temperature of the brake is controlled (avoid thermal stress and loss of service)

There exists many options to brake an electric motor.

- External brake mechanism — (mechanical, magnetic,…)
- Bidirectional capability of the drive — Regenerative brake
- Use of an electric mechanims (breaker) to shortcircuit the motor

How to brake the motor?

- By external brake mechanism (mechanical, magnetic,…)

This is the most easy and extended mechanism to brake a motor.
We simply need to know the external braking torque and the brake own inertia (to consider when coupled to axis)

$$\Gamma_b = \Gamma_J + \Gamma_{extL} - \Gamma_f \longrightarrow \Gamma_b - \Gamma_{extL} + \Gamma_f = J_{reducedBrake} \cdot \alpha$$

Being

Γ_b: brake torque [N·m]

Γ_J: inertia torque [N·m]

Γ_{extL}: external torque [N·m]

Γ_f: friction torque [N·m]

Note: All inertia must be referred to the brake shaft

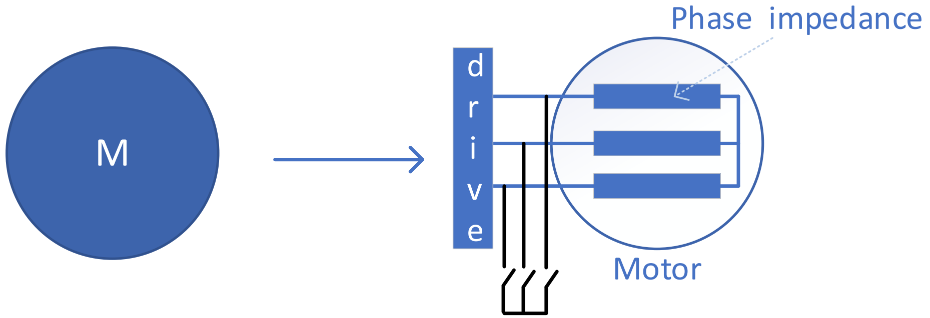
Usually, the brake torque is constant and computed by the kinematic energy (E_k) balance and the total distance (θ_b) to rest the machine at no speed

$$E_{k0} = \frac{1}{2} J_{reducedBrake} \omega_0^2 \qquad E_{kf} = 0 \qquad \Delta E_k = \Gamma_b \theta_b \text{ (work done on inertia(mass))}$$

How to brake the motor?

- By using some electric mechanims (breaker) to shortcircuit the motor

This strategy looks to take profit of Joule's losses effect and provide a secure way to move the motor to rest state.



It is required to know the motor model somehow. It is common that manufacturers offers

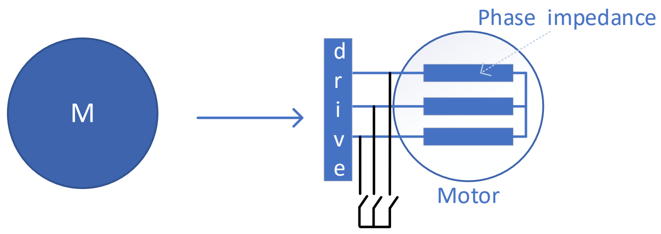
- k_e: ratio voltage over speed [V/rad/s]
- R: equivalent series resistance of phase impedance [Ω]
- L: equivalent series inductance of phase impedance [H]

Note: Remember that the inductive impedance $X_L = X_L(f)$, being f the electric frequency [Hz] is also related with the mechanical angular speed and pair of poles (p) of the Machine.

$$X_L = 2\pi f$$

$$\omega = \frac{2\pi f}{p}$$

How to brake the motor?



- By using some electric mechanims (breaker) to shortcircuit the motor

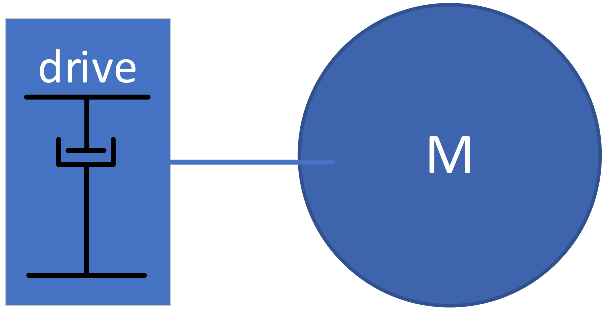
  Then, the braking torque and the speed or distance evolution can be obtained from an initial speed ω_0 as follows

1. Phase voltatge → $V_{ph} = k_e \omega_0$
2. Motor frequency → $\mathrm{f} = \frac{\omega_0 \cdot p}{2\pi}$
3. The equivalent motor's impedace per phase → $Z_{eq} = \sqrt{R^2 + (2\pi f L)^2}$
4. Line current → $I_l = \frac{V_{ph}}{Z_{eq}}$
5. Total Losses → $P_{losses} = 3 \cdot I_l^2 \cdot R$
6. Braking torque → $\Gamma_b = \frac{P_{losses}}{\omega_0}$
7. Deceleration → $\alpha = -\frac{(\Gamma_b + \Gamma_f)}{J}$
8. Compute new speed (ω_0 for next iteration) from deceleration and iterate until null speed.

How to brake the motor?

- By bidirectional capability of the drive, regenerative brake

Usually, the motor is driven by a power electronics based inverter.



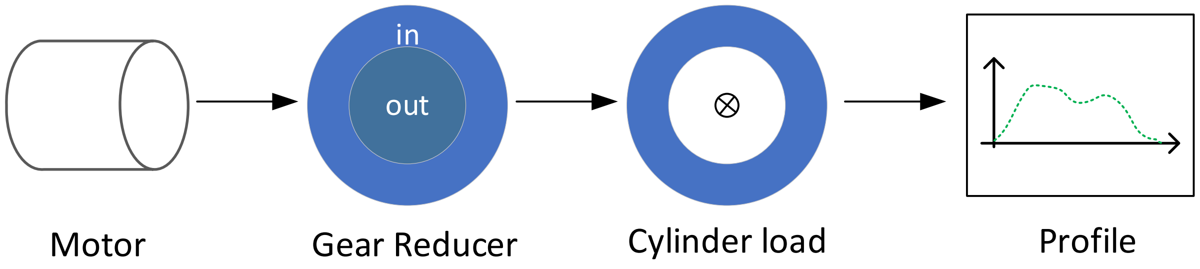
This inverter is in charge of being capable (according to used topology and selected switching devices) to be bidirectional in current, so in machine’s torque. This enables regenerative braking if the current is change in direction and it can be, for example, storaged into a battery.

From this point the braking analysis can be similar to previous one presented by starting from point 5 and instead of consider P_{loss} it is required to consider regenerative capability in power P_{reg}

1. Regenerative power → P_{reg}
2. Braking torque → $\Gamma_b = \frac{P_{reg}}{\omega_0}$
3. Deceleration → $\alpha = -\frac{(\Gamma_b + \Gamma_f)}{J}$
4. Compute new speed from deceleration and iterate until null speed.

Let’s see one example

Problem statement



Safety margin [%]: 20

Input motor	
Rated speed [rpm]	1000
Max. speed [rpm]	2000
Rated Torque [N·m]	28,7
Max Torque [N·m]	71,7
Max Inertia ratio	10
Inertia [kg·m^2]	0,00492
Brake Inertia [kg·m^2]	0,00047
Power [W]	3000
Mass [kg]	23,5
Thermal constant [s]	28

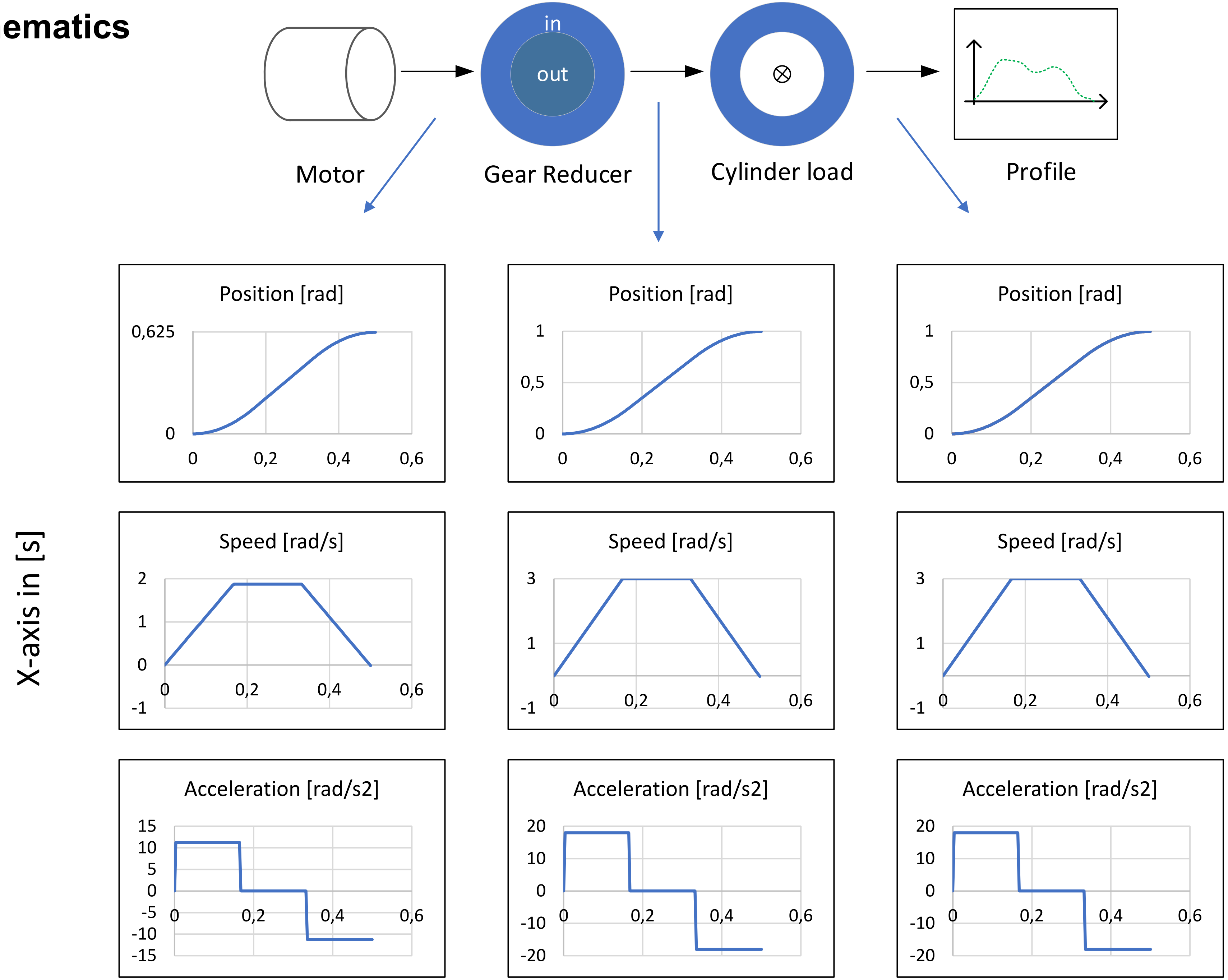
Input item set-up	
Convertion ratio	1,6
Inclination [deg]	0
Efficency [%]	100
Friction Torque [N·m]	1
Inertia [kg·m^2]	0,015

Input item set-up	
Convertion ratio	1
Inclination [deg]	0
Efficency [%]	100
Friction Torque [N·m]	1
Inertia [kg·m^2]	0,01

Input profile set-up	
Type	Trapezoidal
Initial Speed [m/s]	0
Increment of position [rad]	1
Time to position [s]	0,5
Wait time [s]	0
External Torque	1
Resolution	150

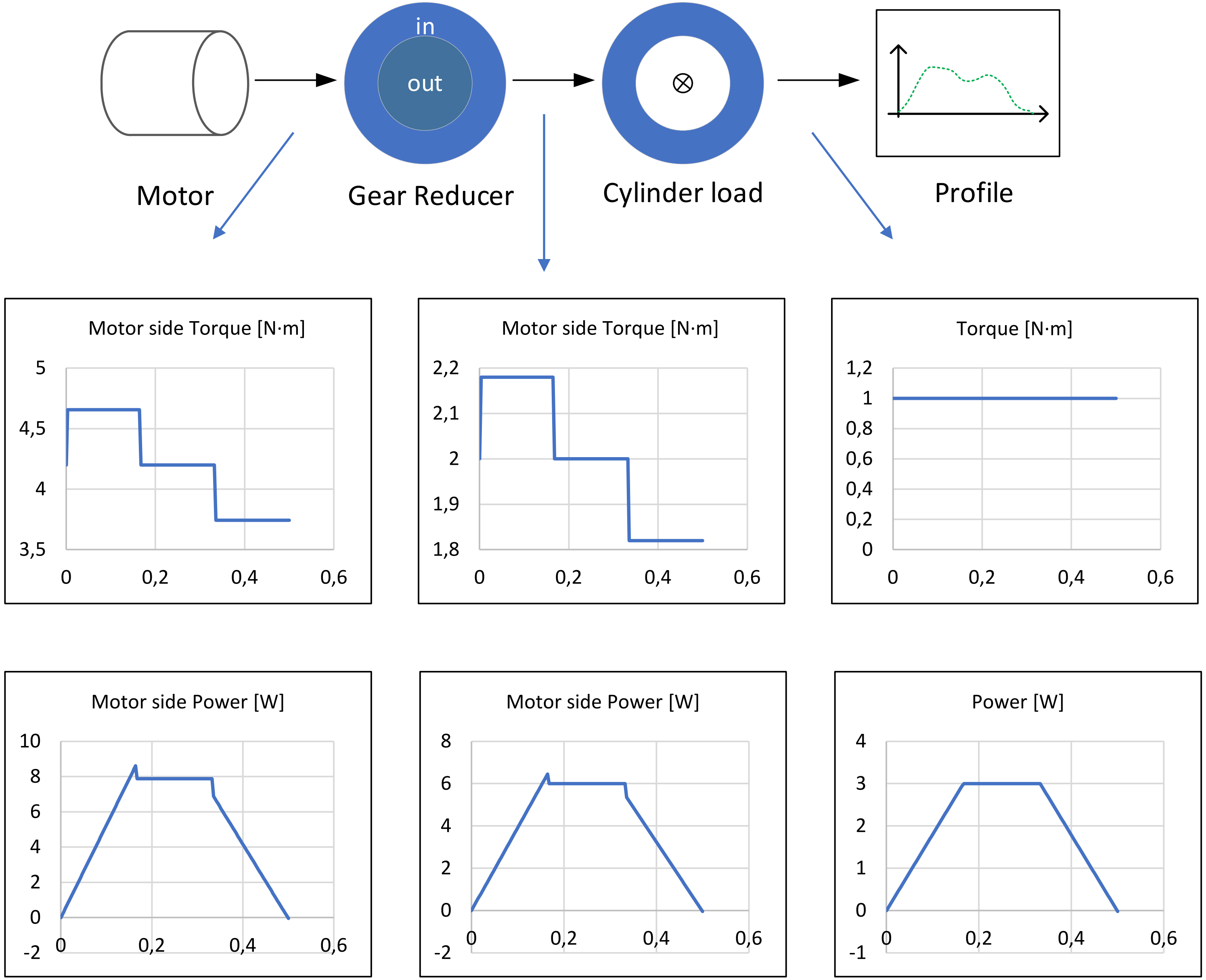
This profile is repeated for 54 s and then rest time until 200 s

Kinematics



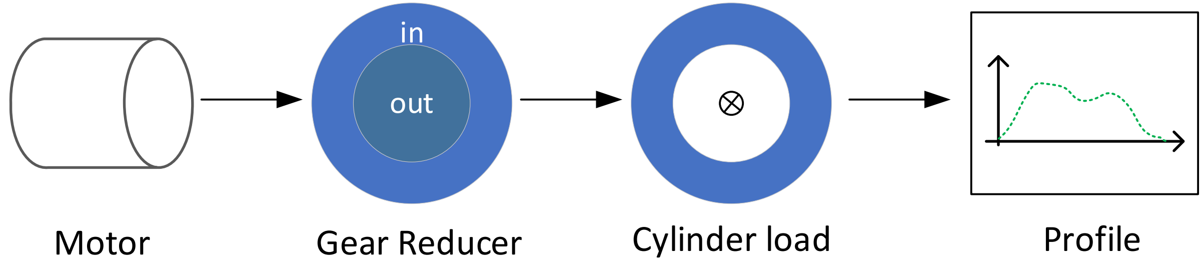
Motor
in
out
Gear Reducer
⊗
Cylinder load
Profile
X-axis in [s]
Position [rad]
0,625
0
0
0,2
0,4
0,6
Position [rad]
1
0,5
0
0
0,2
0,4
0,6
Position [rad]
1
0,5
0
0
0,2
0,4
0,6
Speed [rad/s]
2
1
0
-1
0
0,2
0,4
0,6
Speed [rad/s]
3
1
-1
0
0,2
0,4
0,6
Speed [rad/s]
3
1
-1
0
0,2
0,4
0,6
Acceleration [rad/s2]
15
10
5
0
-5
-10
-15
0
0,2
0,4
0,6
Acceleration [rad/s2]
20
10
0
-10
-20
0
0,2
0,4
0,6
Acceleration [rad/s2]
20
10
0
-10
-20
0
0,2
0,4
0,6

Dynamics



X-axis in [s]

Inertia reflection



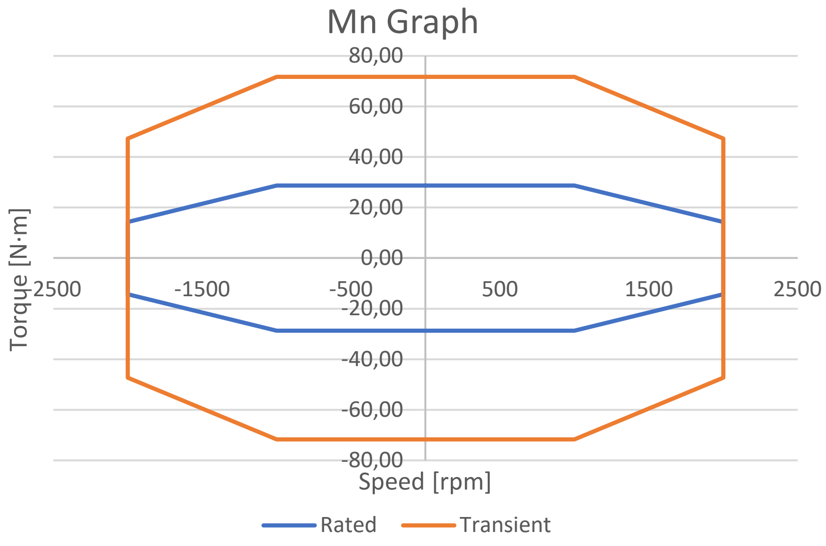
My inertia → J = 0,00492 kg·m2 J = 0,015 kg·m2 J = 0,01 kg·m2

Reflected inertia to motor → J’ = 0,0455 kg·m2 J’ = 0,0406 kg·m2

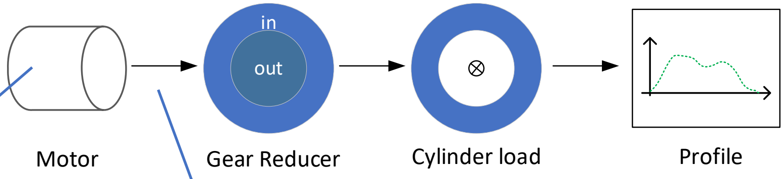
Motor analysis: requirements

Mn characteristic curve

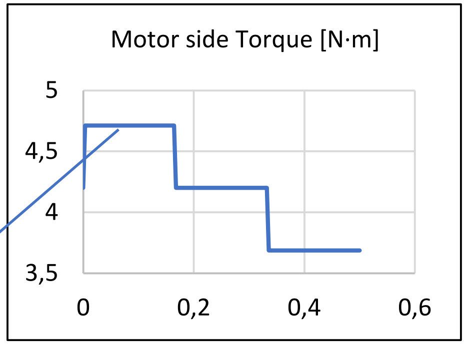
Mn rated		Mn max	
Speed [rpm]	Torque [N·m]	Speed [rpm]	Torque [N·m]
0	28,70	0	71,70
1000	28,70	1000	71,70
2000	14,32	2000	47,26
2000	0,00	2000	0,00
2000	-14,32	2000	-47,26
1000	-28,70	1000	-71,70
0	-28,70	0	-71,70
-1000	-28,70	-1000	-71,70
-2000	-14,32	-2000	-47,26
-2000	0,00	-2000	0,00
-2000	14,32	-2000	47,26
-1000	28,70	-1000	71,70
0	28,70	0	71,70



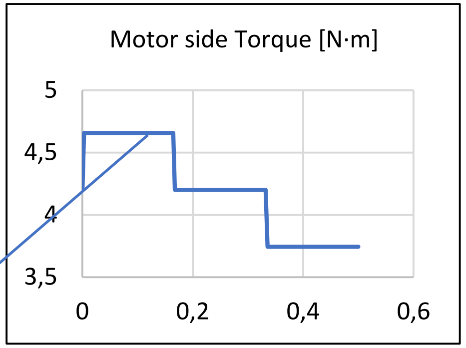
X axis in [s]

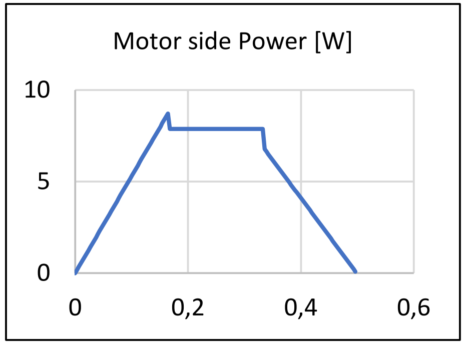


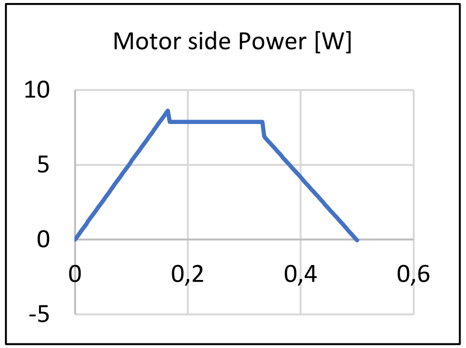
4,7121

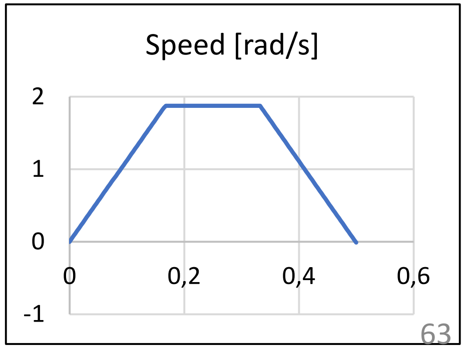


4,6568





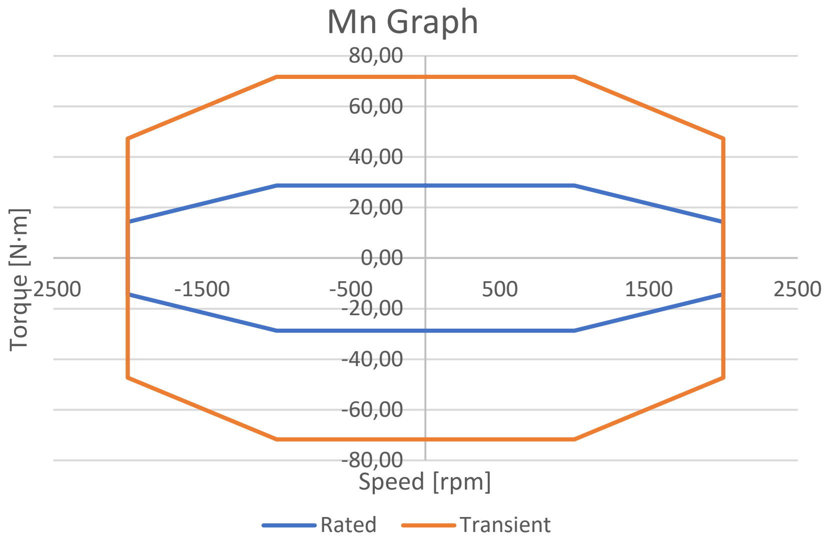


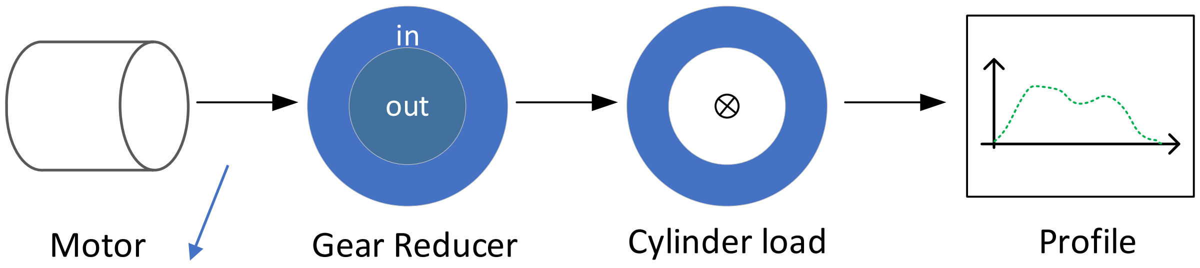


Motor analysis: requirements

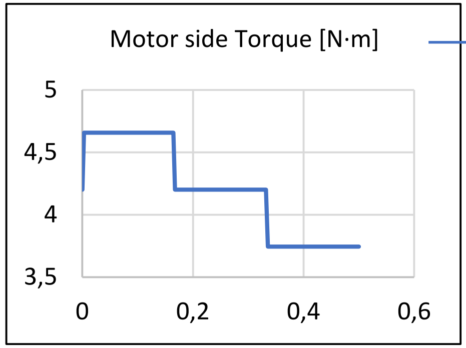
Mn characteristic curve

Mn rated		Mn max	
Speed [rpm]	Torque [N·m]	Speed [rpm]	Torque [N·m]
0	28,70	0	71,70
1000	28,70	1000	71,70
2000	14,32	2000	47,26
2000	0,00	2000	0,00
2000	-14,32	2000	-47,26
1000	-28,70	1000	-71,70
0	-28,70	0	-71,70
-1000	-28,70	-1000	-71,70
-2000	-14,32	-2000	-47,26
-2000	0,00	-2000	0,00
-2000	14,32	-2000	47,26
-1000	28,70	-1000	71,70
0	28,70	0	71,70

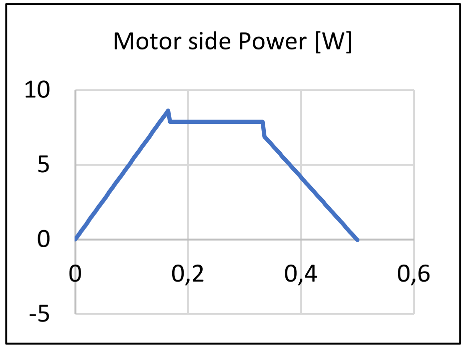


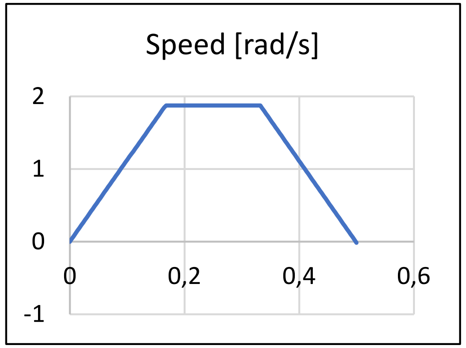


X axis in [s]



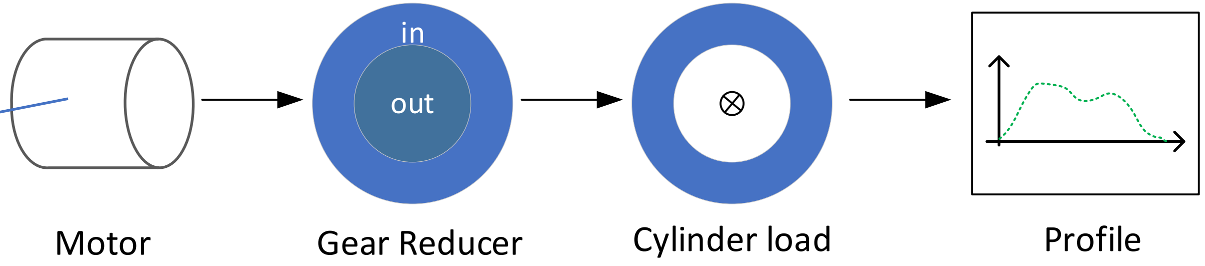
+ safety margin of 20%





	Without safety margin		With safety margin	
	Without motor	With motor	Without motor	With motor
Max Speed [rad/s]	1,88			
Effective Speed [rad/s]	1,39			
Max Torque [N·m]	4,66	4,71	**5,59**	**5,65**
Effective Torque [N·m]	4,21	4,22	**5,06**	**5,06**
Inertia [kg·m^2]	0,04	0,05	0,04	0,05

Motor analysis II: Judgment

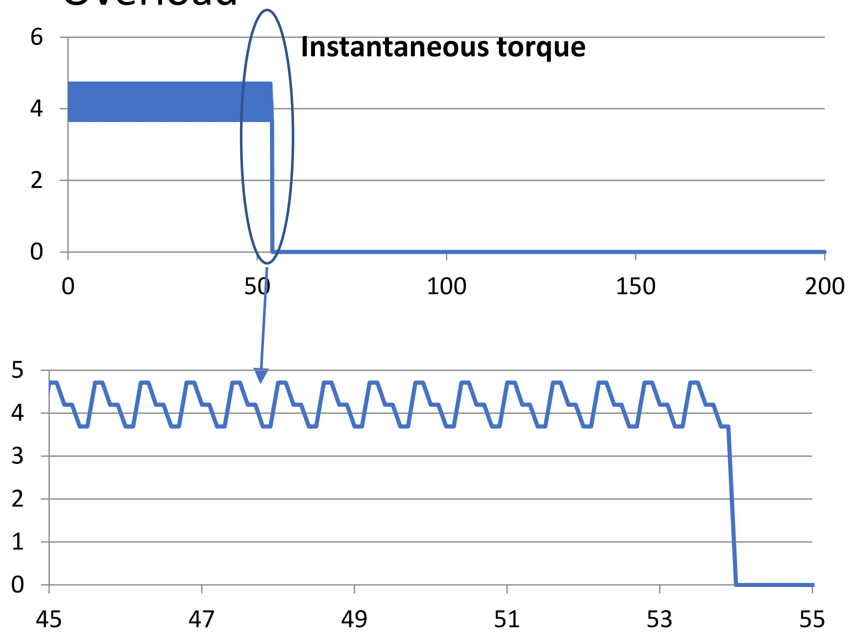


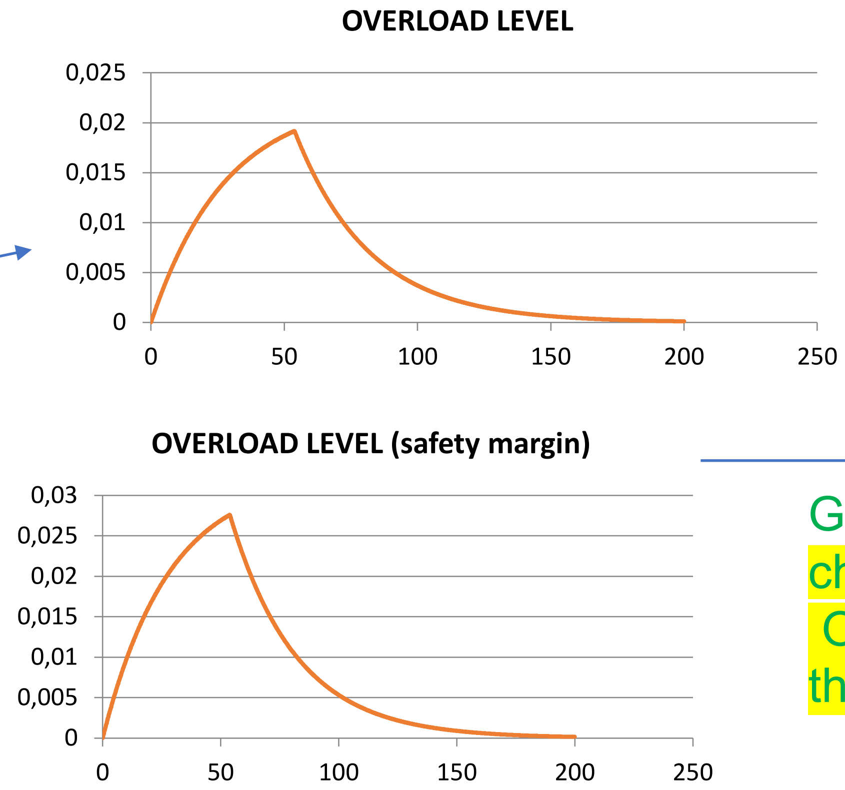
Ratios

		Ratios [%]	
		[%]	[ad]
Motor max speed [rad/s]	209,43	0,90	--
Motor effective speed [rad/s]	104,72	1,33	--
Motor max torque [N·m]	71,70	7,79	--
Motor effective torque [N·m]	28,70	17,62	--
Motor inertia capability [ad]	10,00	--	8,25

Good but not well chosen.
Oversized in speed and torque

Overload





Good but not well chosen.
Oversized in terms of thermal specifications