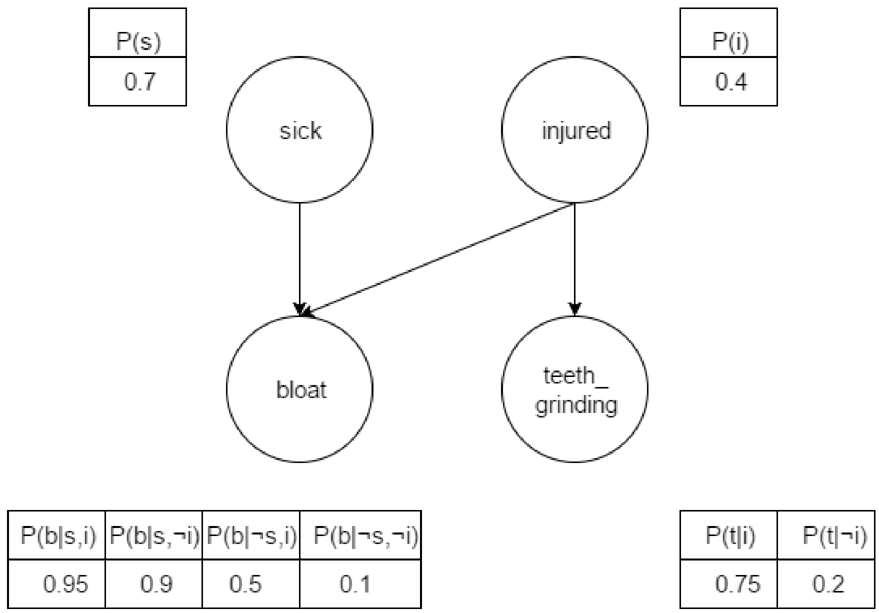
HOMEWORK 4

QUESTION 1

a. Bayesian Network



(s – sick, i – injured, t – teeth_grinding, b – bloat)

b. Full Joint Probability Table

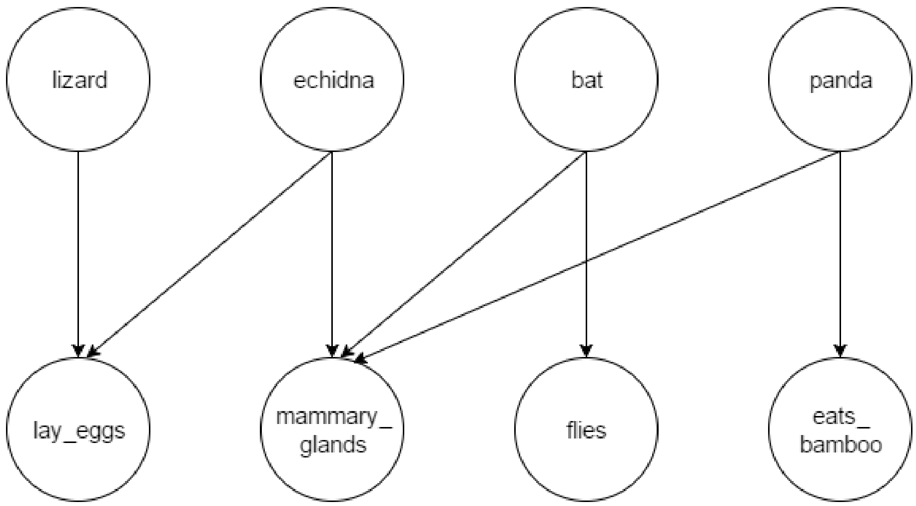
		injured		¬injured	
		teeth_grinding	¬teeth_grinding	teeth_grinding	¬teeth_grinding
Sick	bloat	0.1995	0.0665	0.0756	0.3024
	¬bloat	0.0105	0.0035	0.0084	0.0336
¬sick	bloat	0.045	0.015	0.0036	0.0144
	¬bloat	0.045	0.015	0.0324	0.1296

Calculations:

1. P(sick) * P(boat | sick) * P(injured) * P(teeth_grinding | injured)
   = 0.7 * 0.4 * 0.95 * 0.75 = 0.1995
2. P(sick) * P(boat | sick) * P(injured) * P(¬teeth_grinding | injured)
   = 0.7 * 0.4 * 0.95 * 0.25 = 0.0665
3. P(sick) * P(boat | sick) * P(¬injured) * P(teeth_grinding | ¬injured)
   = 0.7 * 0.6 * 0.9 * 0.2 = 0.0756
4. P(sick) * P(boat | sick) * P(¬injured) * P(¬teeth_grinding | ¬injured)
   = 0.7 * 0.6 * 0.9 * 0.8 = 0.3024
5. P(sick) * P(¬boat | sick) * P(injured) * P(teeth_grinding | injured)
   = 0.7 * 0.4 * 0.05 * 0.75 = 0.0105
6. P(sick) * P(¬boat | sick) * P(injured) * P(¬teeth_grinding | injured)
   = 0.7 * 0.4 * 0.05 * 0.25 = 0.0035
7. P(sick) * P(¬boat | sick) * P(¬injured) * P(teeth_grinding | ¬injured)
   = 0.7 * 0.6 * 0.1 * 0.2 = 0.0084
8. P(sick) * P(¬boat | sick) * P(¬injured) * P(¬teeth_grinding | ¬injured)
   = 0.7 * 0.6 * 0.1 * 0.8 = 0.0336
9. P(¬sick) * P(boat | ¬sick) * P(injured) * P(teeth_grinding | injured)
   = 0.3 * 0.4 * 0.5 * 0.75 = 0.045
10. P(¬sick) * P(boat | ¬sick) * P(injured) * P(¬teeth_grinding | injured)
    = 0.3 * 0.4 * 0.5 * 0.25 = 0.015
11. P(¬sick) * P(boat | ¬sick) * P(¬injured) * P(teeth_grinding | ¬injured)
    = 0.3 * 0.6 * 0.1 * 0.2 = 0.0036
12. P(¬sick) * P(boat | ¬sick) * P(¬injured) * P(¬teeth_grinding | ¬injured)
    = 0.3 * 0.6 * 0.1 * 0.8 = 0.0144
13. P(¬sick) * P(¬boat | ¬sick) * P(injured) * P(teeth_grinding | injured)
    = 0.3 * 0.4 * 0.5 * 0.75 = 0.045
14. P(¬sick) * P(¬boat | ¬sick) * P(injured) * P(¬teeth_grinding | injured)
    = 0.3 * 0.4 * 0.5 * 0.25 = 0.015
15. P(¬sick) * P(¬boat | ¬sick) * P(¬injured) * P(teeth_grinding | ¬injured)
    = 0.3 * 0.6 * 0.9 * 0.2 = 0.0324
16. P(¬sick) * P(¬boat | ¬sick) * P(¬injured) * P(¬teeth_grinding | ¬injured)
    = 0.3 * 0.6 * 0.9 * 0.8 = 0.1296

QUESTION 2

a. Bayes Net



l – lizard, e – echidna, b – bat, p – panda,

le – lay_eggs, mg – mammary_glands, f – flies, eb – eats_bamboo

b. Conditional Independence:

Conditional independence among the nodes in the Bayesian network can be determined using the following rules:

Rule 1: Markov Blanket says that the node X is conditionally independent of the rest of the network given:

a. X's parents
b. X's children
c. X's spouses (X's children's parents)

Rule 2: Children are conditionally independent of each other, given their common parents.
Rule 3: A node is conditionally independent of its non-descendants, given its parents.

Markov Blankets of all the nodes in the above network:

- lizard: {lay_eggs, echidna}
- echidna: {lizard, bat, panda, lay_eggs, mammary_glands}
- bat: {echidna, panda, mammary_glands, flies}
- panda: {echidna, bat, mammary_glands, eats_bamboo}
- lay_eggs: {lizard, echidna}
- mammary_glands: {echidna, bat, panda}
- flies: {bat}
- eats_bamboo: {panda}

The following conditional independencies exist in the above Bayesian network:

- 'lizard' is conditionally independent of the rest of the network, given 'lay_eggs' and 'echidna' (Rule 1, Markov Blanket)
- 'echidna' is conditionally independent of 'flies' and 'eats_bamboo, given 'lizard', 'lay_eggs', 'bat', 'mammary_glands' and 'panda' (Rule 1, Markov Blanket)
- 'bat' is conditionally independent of the rest of the network, given 'echidna', 'mammary_glands' and 'flies' (Rule 1, Markov Blanket)
- 'panda' is conditionally independent of 'lizard', 'lay_eggs' and 'flies' given 'echidna', 'mammary_glands', 'bat' and 'eat_bamboo' (Rule 1, Markov Blanket)
- 'mammary_glands' is conditionally independent of 'lay_eggs', given 'echidna' (Rule 2)
- 'mammary_glands' is conditionally independent of 'flies', given 'bat' (Rule 2)
- 'mammary_glands' is conditionally independent of 'eats_bamboo', given 'panda' (Rule 2)
- 'lay_eggs' is conditionally independent of 'flies' and 'eats_bamboo', given 'lizard' and 'echidna' (Rule 3)
- 'flies' is conditionally independent of 'eats_bamboo', given 'bat' (Rule 3)

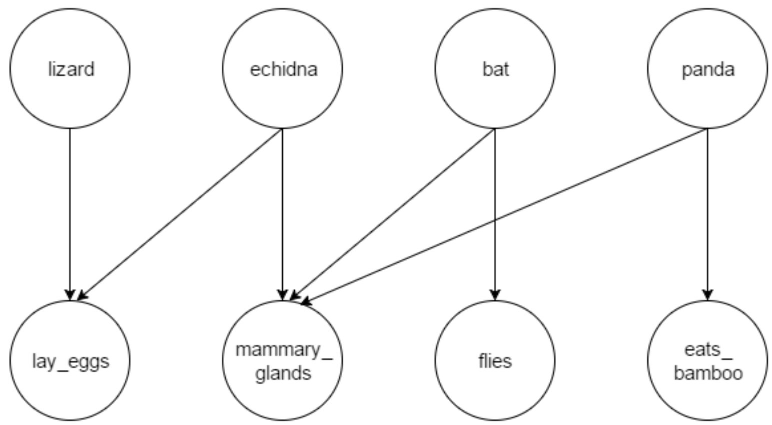
c. Probability Table:

lizard	P(lizard)
T	0.05
F	0.95

bat	P(bat)
T	0.03
F	0.97

echidna	P(echidna)
T	0.01
F	0.99

panda	P(panda)
T	0.02
F	0.98



lizard	echidna	lays_eggs	P(lays_eggs | lizard, echidna)
T	T	T	0.7
T	F	T	0.57
F	T	T	0.03
F	F	T	0.04
T	T	F	0.3
T	F	F	0.43
F	T	F	0.97
F	F	F	0.96

echidna	bat	panda	mammary_ glands	P(mammary_ glands | echidna, bat, panda)
T	T	T	T	0.9
T	T	F	T	0.75
T	F	T	T	0.4
T	F	F	T	0.02
F	T	T	T	0.5
F	T	F	T	0.08
F	F	T	T	0.5
F	F	F	T	0.01
T	T	T	F	0.1
T	T	F	F	0.25
T	F	T	F	0.6
T	F	F	F	0.98
F	T	T	F	0.5
F	T	F	F	0.92
F	F	T	F	0.5
F	F	F	F	0.99

bat	flies	P(flies | bat)
T	T	0.85
F	T	0.05
T	F	0.15
F	F	0.95

panda	eats_bamboo	P(eats_bamboo | panda)
T	T	0.6
F	T	0.03
T	F	0.4
F	F	0.97

1) What is the probability that the animal has mammary glands?

mg – the animal has mammary_glands

$$P(mg) = \sum_{L,E,B,P,LE,F,EB} P(L,E,B,P,LE,mg,F,EB)$$
$$= \sum_{L,E,B,P,LE,F,EB} P(L)P(E)P(B)P(P)P(LE|L,E)P(mg|E,B,P)P(F|B)P(EB|P)$$
$$= \sum_{E,B,P,L,LE,F,EB} P(E)P(B)P(P)P(mg|E,B,P)P(L)P(LE|L,E)P(F|B)P(EB|P)$$
$$= \sum_{E,B,P,L,LE,F} P(E)P(B)P(P)P(mg|E,B,P)P(L)P(LE|L,E)P(F|B) \sum_{\boldsymbol{EB}} \boldsymbol{P(EB|P)}$$

Since, $\sum_{EB} P(EB|P) = 1$
$$= \sum_{E,B,P,L,LE} P(E)P(B)P(P)P(mg|E,B,P)P(L)P(LE|L,E) \sum_{\boldsymbol{F}} \boldsymbol{P(F|B)}$$

Since, $\sum_{F} P(F|B) = 1$
$$= \sum_{E,B,P,L} P(E)P(B)P(P)P(mg|E,B,P)P(L) \sum_{\boldsymbol{LE}} \boldsymbol{P(LE|L,E)}$$

Since, $\sum_{LE} P(LE|L,E) = 1$
$$= \sum_{E,B,P} P(E)P(B)P(P)P(mg|E,B,P) \sum_{\boldsymbol{L}} \boldsymbol{P(L)}$$

Since, $\sum_{L} P(L) = 1$
$$= \sum_{E,B,P} P(E)P(B)P(P)P(mg|E,B,P)$$

As there are three binary variables E, B and P, there are 8 possible combinations as follows:

$$\begin{aligned}
= \; & P(E)P(B)P(P)P(mg|E,B,P) && + \\
& P(E)P(B)P(\neg P)P(mg|E,B,\neg P) && + \\
& P(E)P(\neg B)P(P)P(mg|E,\neg B,P) && + \\
& P(E)P(\neg B)P(\neg P)P(mg|E,\neg B,\neg P) && + \\
& P(\neg E)P(B)P(P)P(mg|\neg E,B,P) && + \\
& P(\neg E)P(B)P(\neg P)P(mg|\neg E,B,\neg P) && + \\
& P(\neg E)P(\neg B)P(P)P(mg|\neg E,\neg B,P) && + \\
& P(\neg E)P(\neg B)P(\neg P)P(mg|\neg E,\neg B,\neg P)
\end{aligned}$$

= 0.01 * 0.03 * 0.02 * 0.9 + 0.01 * 0.03 * 0.98 * 0.75 +
0.01 * 0.97 * 0.02 * 0.4 + 0.01 * 0.97 * 0.98 * 0.02 +
0.99 * 0.03 * 0.02 * 0.5 + 0.99 * 0.03 * 0.98 * 0.08 +
0.99 * 0.97 * 0.02 * 0.5 + 0.99 * 0.97 * 0.98 * 0.01

= 0.0000054 + 0.0002205 + 0.0000776 + 0.00019012 +
0.000297 + 0.00232848 + 0.009603 + 0.00941094

= 0.02213304

Thus, the probability that the animal has mammary glands is 0.02213304.

2) What is the probability that the animal is echidna?

e – the animal is an echidna

$$P(e) = \sum_{L,B,P,LE,MG,F,EB} P(L)P(e)P(B)P(P)P(LE|L,e)P(MG|e,B,P)P(F|B)P(EB|P)$$
$$= \sum_{L,B,P,LE,MG,F} P(L)P(e)P(B)P(P)P(LE|L,e)P(MG|e,B,P)P(F|B) \sum_{\boldsymbol{EB}} \boldsymbol{P(EB|P)}$$

Since, $\sum_{EB} P(EB|P) = 1$
$$= \sum_{L,B,P,LE,MG} P(L)P(e)P(B)P(P)P(LE|L,e)P(MG|e,B,P)P(F|B) \sum_{\boldsymbol{F}} \boldsymbol{P(F|B)}$$

Since, $\sum_{F} P(F|B) = 1$
$$= \sum_{L,B,P,LE} P(L)P(e)P(B)P(P)P(LE|L,e) \sum_{\boldsymbol{MG}} \boldsymbol{P(MG|e,B,P)}$$

Since, $\sum_{MG} P(MG|e,B,P) = 1$
$$= \sum_{L,B,P} P(L)P(e)P(B)P(P) \sum_{\boldsymbol{LE}} \boldsymbol{P(LE|L,e)}$$

Since, $\sum_{LE} P(LE|L,e) = 1$
$$= \sum_{L,B} P(L)P(e)P(B) \sum_{\boldsymbol{P}} \boldsymbol{P(P)}$$

Since, $\sum_{P} P(P) = 1$
$$= \sum_{L} P(L)P(e) \sum_{\boldsymbol{B}} \boldsymbol{P(B)}$$

Since, $\sum_{B} P(B) = 1$
$$= P(e) \sum_{\boldsymbol{L}} \boldsymbol{P(L)}$$

Since, $\sum_{L} P(L) = 1$
$$= P(e)$$

This result shows that 'echidna' has nobody as its parent, so its probability will remain as it is provided in the table if nothing is given.

Thus, from the probability distribution table, the probability that the animal is an echidna is 0.01.

3) What is the probability that the animal is a bat, given that it eats bamboo?

b – the animal is a bat, eb – the animal eats bamboo.

$$P(b|eb) = \frac{P(b,eb)}{P(eb)}$$

$$= \frac{\sum_{L,E,P,LE,MG,F} P(L,E,b,P,LE,MG,F,eb)}{P(eb)}$$

$$= \frac{\left(\sum_{L,E,P,LE,MG,F} P(L)P(E)P(b)P(P)P(LE|L,E)P(MG|E,b,P)P(F|b)P(eb|P)\right)}{P(eb)}$$

$$= \frac{\left(\sum_{P,L,E,LE,MG,F} P(P)P(eb|P)P(b)P(L)P(E)P(LE|L,E)P(MG|E,b,P)P(F|b)\right)}{P(eb)}$$

$$= \frac{\sum_{P,L,E,LE,MG} P(P)P(eb|P)P(b)P(L)P(E)P(LE|L,E)P(MG|E,b,P)\sum_{F} \boldsymbol{P(F|b)}}{P(eb)}$$

$$= \frac{\sum_{P,L,E,LE} P(P)P(eb|P)P(b)P(L)P(E)P(LE|L,E)\sum_{MG} \boldsymbol{P(MG|E,b,P)}}{P(eb)}$$

$$= \frac{\sum_{P,L,E} P(P)P(eb|P)P(b)P(L)P(E)\sum_{LE} \boldsymbol{P(LE|L,E)}}{P(eb)}$$

$$= \frac{\sum_{P,L} P(P)P(eb|P)P(b)P(L)\sum_{E} \boldsymbol{P(E)}}{P(eb)}$$

$$= \frac{\sum_{P} P(P)P(eb|P)P(b)\sum_{L} \boldsymbol{P(L)}}{P(eb)}$$

$$= \frac{P(b)\sum_{P} \boldsymbol{P(eb|P)P(P)}}{P(eb)}$$

$$= \frac{P(b)\sum_{P} \boldsymbol{P(eb,P)}}{P(eb)}$$

$$= \frac{P(b)\boldsymbol{P(eb)}}{P(eb)}$$

$$= P(b)$$

$$= 0.03$$

Thus, the probability that the animal is a bat, given that it eats bamboo is 0.03

Why is P(bat | eats_bamboo) = P(bat)? Because, bat is independent of eats_bamboo as 'eats_bamboo' doesn't come under bat's Markov Blanket.