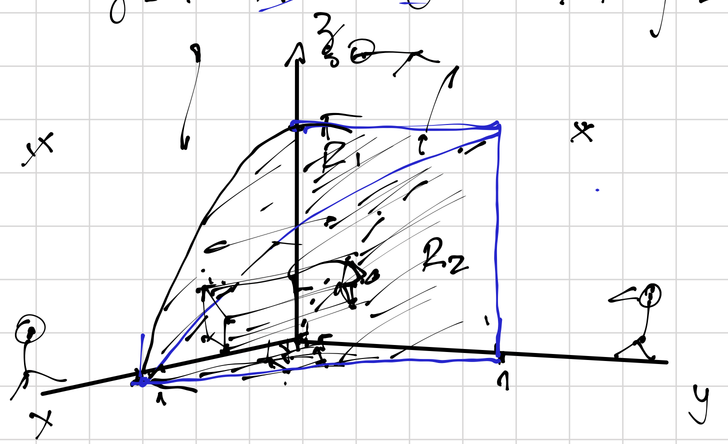
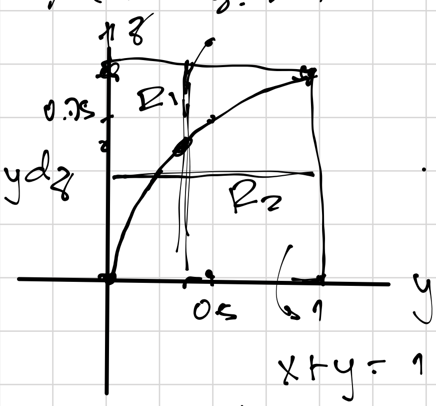
$z = 1 - x^2$ & $x + y = 1$



x-simples $\quad z = 1 - x^2$



$\Delta z = dy\,dz$

$x + y = 1$

$\Delta y = dz\,dy$

$z = 1 - x^2$

$\sqrt{x^2} = \sqrt{1 - z}$

$x = \sqrt{1 - z}$

$x + y = 1$

$x = 1 - y$

$x = x$

$(\sqrt{1-z})^2 = (1-y)^2 \rightarrow \sqrt{1-z} = 1 - y$

$1 - z = (1-y)^2 \rightarrow \quad y = 1 - \sqrt{1-z}$

$z = 1 - (1-y)^2$

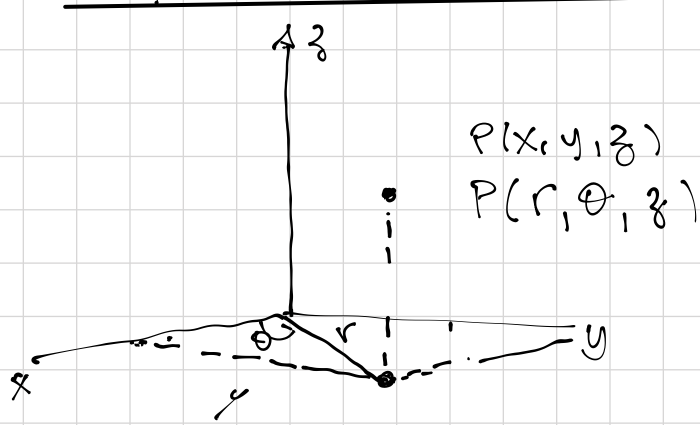
$$V = \int_0^1 \int_{1-(1-y)^2}^{1} \int_0^{\sqrt{1-z}} dx\,dz\,dy + \int_0^1 \int_0^{1-(1-y)^2} \int_0^{1-y} dx\,dz\,dy$$

R1 $\qquad$ R2

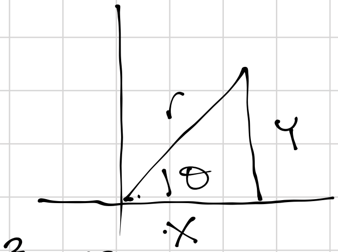
$$V = \int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1-z}} dx\,dy\,dz + \int_0^1 \int_{1-\sqrt{1-z}}^{1} \int_0^{1-y} dx\,dy\,dz$$

R1 $\qquad$ R2

Integrales triples en coordenadas cilíndricas



$z = z$

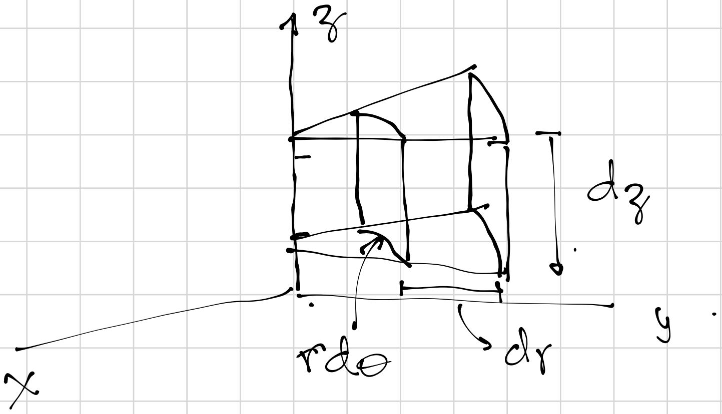


$x^2 + y^2 = r^2$

$\operatorname{sen}\theta = \dfrac{y}{r}$

$y = r\,\text{sen}\theta$ $\qquad \cos\theta = \dfrac{x}{r} \rightarrow x = r\cos\theta.$

<u>diferencial del volumen</u>



$dV = r\,d\theta \cdot dr \cdot dz$

$\boxed{dV = r\,dz\,dr\,d\theta}$

$$\iiint f(x,y,z)\,dz\,dx\,dy = \iiint f(r\cos\theta, r\,\text{sen}\theta, z)\,r\,dz\,dr\,d\theta$$

Ej. Evalue la integral cambiando a coordenadas cilindricas

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{9-x^2-y^2} \sqrt{x^2+y^2}\;dz\,dy\,dx$$

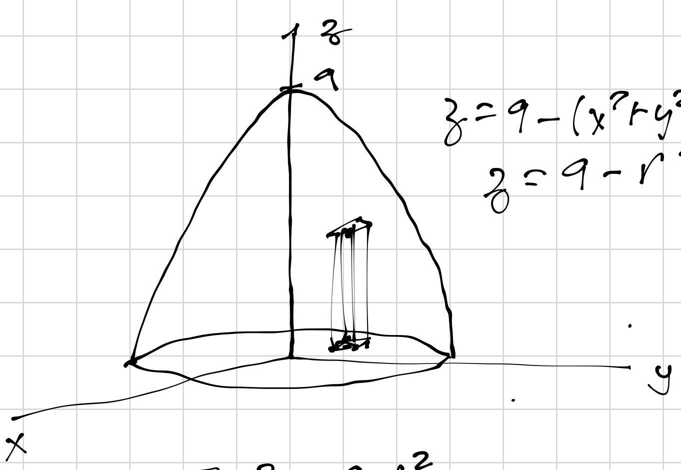
Para z: $\quad z = 0 \qquad z = 9 - x^2 - y^2$ – paraboloide

Para y: $\quad y = 0 \qquad (y)^2 = (\sqrt{9-x^2})^2 \rightarrow y^2 = 9 - x^2 \rightarrow x^2 + y^2 = 9$

Para x: $\quad x = -3 \qquad x = 3$

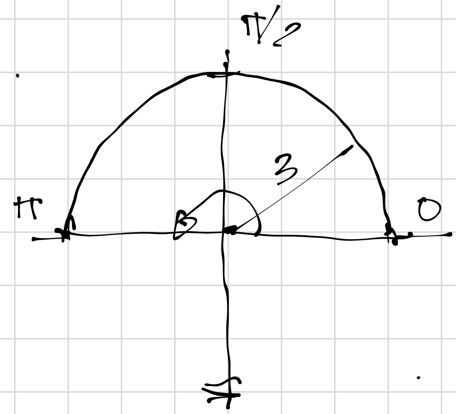
$\sqrt{r^2} = \sqrt{9}$

$r = 3$



$z = 9 - (x^2 + y^2)$

$z = 9 - r^2$



$x^2 + y^2 = r^2$

$x = r\cos\theta$

$y = r\,\text{sen}\theta$

$$\int_0^{\pi}\int_0^{3}\int_0^{9-r^2} \sqrt{r^2}\;r\,dz\,dr\,d\theta = \int_0^{\pi}\int_0^{3} z\Big|_0^{9-r^2} r^2\,dr\,d\theta$$

$$\int_0^{\pi}\int_0^{3} (9-r^2)\,r^2\,dr\,d\theta = \int_0^{\pi} 3r^3 - \frac{1}{5}r^5\Big|_0^3\,d\theta$$

$9r^2 - r^4$

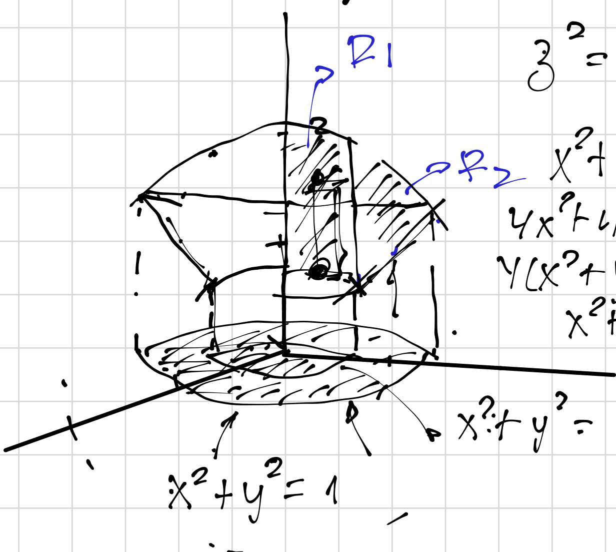
$\theta\Big|_0^{\pi}$

$$\int_0^{\pi}\left(3(3)^3-\frac{1}{5}(3)^5\right)d\theta = (3)(3)^3-\frac{1}{5}(3)^5(\pi)$$

Ej. Determine el volumen del solido limitado por las esferas $x^2+y^2+z^2=4$, $x^2+y^2+z^2=1$ y el cono $z=\sqrt{3x^2+3y^2}$

$$x^2+y^2+z^2=4$$

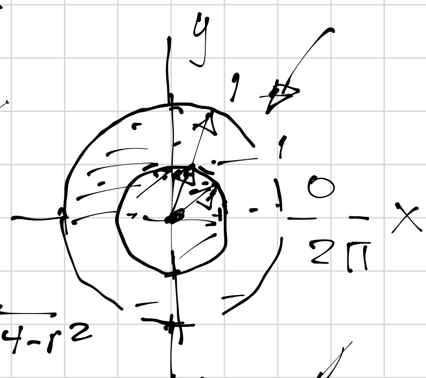
$$z^2=3x^2+$$



$$x^2+y^2+3x^2+3y^2=1$$
$$4x^2+4y^2=1$$
$$4(x^2+y^2)=1$$
$$x^2+y^2=\frac{1}{4}$$

$$x^2+y^2=1 \rightarrow \sqrt{r^2}=\sqrt{1}$$
$$\boxed{r=1}$$

$$x^2+y^2=\frac{1}{4}$$
$$\sqrt{r^2}=\sqrt{\frac{1}{4}}$$
$$\boxed{r=\frac{1}{2}}$$



$$V=\iiint r\,dz\,dr\,d\theta$$

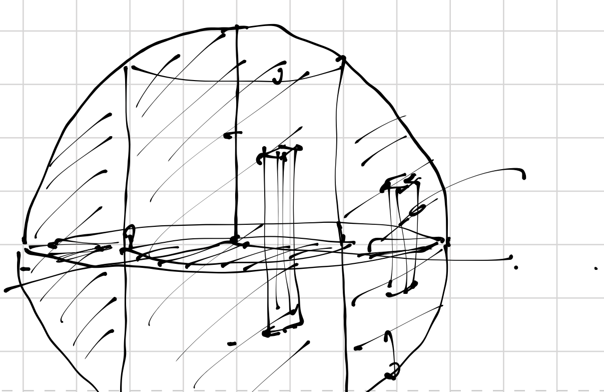
$$V=\underbrace{\int_0^{2\pi}\int_0^{1/2}\int_{\sqrt{1-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}_{R_1}+\underbrace{\int_0^{2\pi}\int_{1/2}^{1}\int_{\sqrt{3}r}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta}_{R_2}$$

$$x^2+y^2+z^2=4 \rightarrow \sqrt{z^2}=\sqrt{4-(x^2+y^2)} \rightarrow z=\sqrt{4-r^2}$$

$$x^2+y^2+z^2=1 \rightarrow \sqrt{z^2}=\sqrt{1-(x^2+y^2)} \rightarrow z=\sqrt{1-r^2}$$

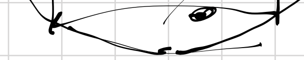
$$z=\sqrt{3(x^2+y^2)} \rightarrow z=\sqrt{3r^2}=\sqrt{3}\,r$$

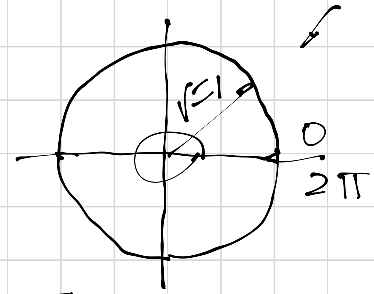
Ej. Plantear el volumen del solido dentro de la esfera $x^2+y^2+z^2=4$ y el cilindro $x^2+y^2=1$



$$V=\iiint r\,dz\,dr\,d\theta$$

$$x^2+y^2=1$$
$$\sqrt{r^2}=\sqrt{1} \rightarrow r=1$$





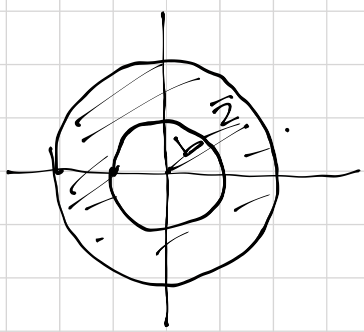
$$z^2 = \sqrt{4 - x^2 - y^2}$$

$$z = \pm\sqrt{4 - (x^2 + y^2)}$$

$$z = \pm\sqrt{4 - r^2}$$

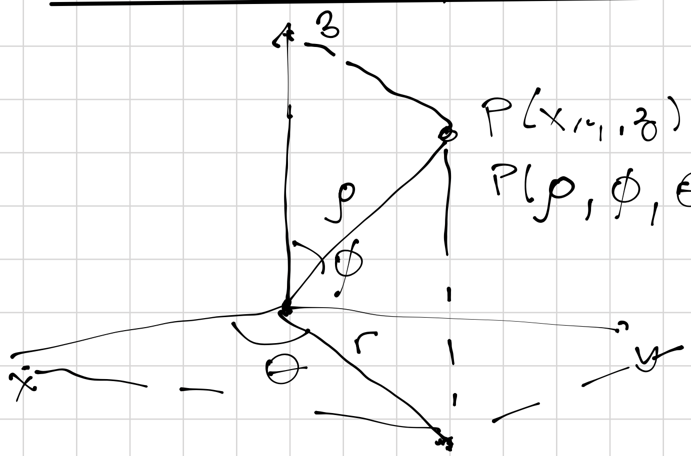
$$V = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$$

$$x^2 + y^2 + \cancel{z^2}^{\,0} = 4 \to x^2 + y^2 = 4 \to \sqrt{r^2} = \sqrt{4} \to r = 2$$



$$V = \int_0^{2\pi} \int_1^2 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$$

Integrales triples en coordenadas esfericas.



$$y = r\,\text{sen}\,\theta$$
$$x = r\cos\theta$$

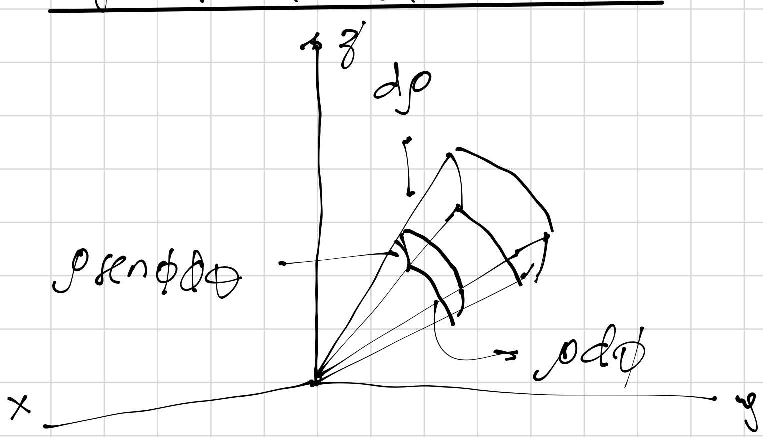
$$z = \rho\cos\phi$$

$$x = \rho\,\text{sen}\,\phi\,\cos\theta$$

$$y = \rho\,\text{sen}\,\phi\,\text{sen}\,\theta$$

$$\rho^2 = x^2 + y^2 + z^2$$

diferencial del volumen

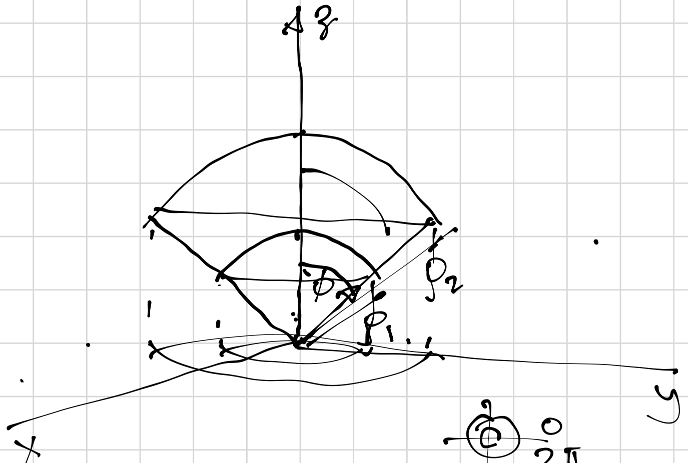


$$dV = (\rho\,\text{sen}\,\phi\, d\theta)(\rho\, d\phi)\cdot d\rho$$

$$dV = \rho^2\,\text{sen}\,\phi\; d\rho\, d\phi\, d\theta$$

$$\iiint f(x,y,z)\,dz\,dx\,dy = \iiint f(\rho,\phi,\theta)\,\rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

Ej. $x^2+y^2+z^2=4$, $x^2+y^2+z^2=1$, $z=\sqrt{3x^2+3y^2}$



$$V=\int_0^{2\pi}\int_0^{\pi/6}\int_1^2 \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$

$x^2+y^2+z^2=\rho^2=4 \qquad \sqrt{\rho^2}=\sqrt{4}$

$\rho=2$

$x^2+y^2+z^2=\rho^2=1 \qquad \sqrt{\rho^2}=\sqrt{1}$

$\rho=1$

$z^2=3x^2+3y^2$

$(\rho\cos\phi)^2 = 3(\rho\operatorname{sen}\phi\cos\theta)^2 + 3(\rho\operatorname{sen}\phi\operatorname{sen}\theta)^2$

$\rho^2\cos^2\phi = 3\rho^2\operatorname{sen}^2\phi\cos^2\theta + 3\rho^2\operatorname{sen}^2\phi\operatorname{sen}^2\theta$

$\cancel{\rho^2}\cos^2\phi = 3\cancel{\rho^2}\operatorname{sen}^2\phi\,\underbrace{\cancel{(\cos^2\theta+\operatorname{sen}^2\theta)}}_{1}$

$\cos^2\phi = 3\operatorname{sen}^2\phi$

$\dfrac{1}{3} = \dfrac{\operatorname{sen}^2\phi}{\cos^2\phi} = tg^2\phi \qquad \sqrt{tg^2\phi}=\sqrt{\dfrac{1}{3}}$

$\cancel{tg^{-1}}\, tg\phi = \sqrt{\dfrac{1}{3}}$ tg^{-1} $\rightarrow$ $\phi = tg^{-1}\sqrt{\dfrac{1}{3}} = \dfrac{\pi}{6}$

$$\int_0^{2\pi}\int_0^{\pi/6} \frac{1}{3}\rho^3\Big|_1^2 \operatorname{sen}\phi\, d\phi\, d\theta$$

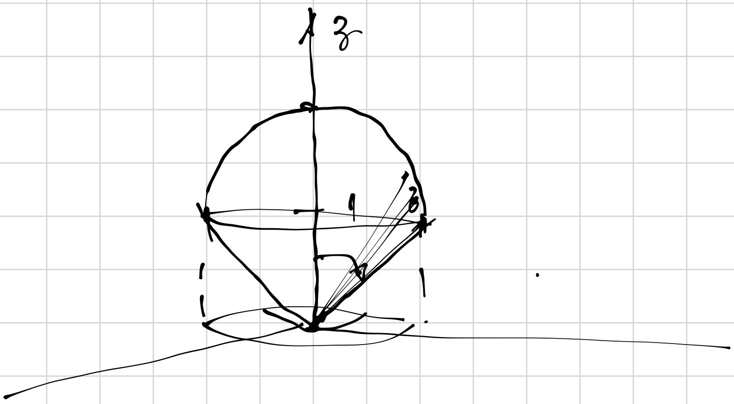
$$\frac{7}{3}\int_0^{2\pi} -\cos\phi\Big|_0^{\pi/6} d\theta = -\frac{7}{3}\left[\cos\frac{\pi}{6}-\cos 0\right]\int_0^{2\pi} d\theta$$

$$-\frac{7}{3}\left[\cos\frac{\pi}{6}-\cos 0\right]2\pi$$

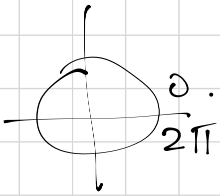
Ej. Determine el volumen del sólido limitado por el cono $z = \sqrt{x^2+y^2}$ y la esfera $x^2+y^2+z^2 = 2z$

$x^2+y^2+z^2-2z = 0 \qquad x^2+y^2+(z^2-2z+1) = 1$

$x^2+y^2+(z-1)^2 = 1 \rightarrow C(0,0,1)$



$$V = \int_0^{2\pi}\int_0^{\pi/4}\int_0^{2\cos\phi} \rho^2 \operatorname{sen}\phi \, d\rho \, d\phi \, d\theta$$



$x^2+y^2+z^2 = 2z$

$\rho^{\cancel{2}} = 2\cancel{\rho}\cos\phi$

$\rho = 2\cos\phi$

$z^2 = x^2+y^2$

$(\rho\cos\phi)^2 = (\rho\operatorname{sen}\phi\cos\theta)^2 + (\rho\operatorname{sen}\phi\operatorname{sen}\theta)^2$

$\rho^2\cos^2\phi = \rho^2\operatorname{sen}^2\phi\cos^2\theta + \rho^2\operatorname{sen}^2\phi\operatorname{sen}^2\theta$

$\cancel{\rho^2}\cos^2\phi = \cancel{\rho^2}\operatorname{sen}^2\phi\,\underbrace{(\cos^2\theta+\operatorname{sen}^2\theta)}_{1}$

$\cos^2\phi = \operatorname{sen}^2\phi \qquad 1 = \dfrac{\operatorname{sen}^2\phi}{\cos^2\phi} = tg^2\phi$

$\sqrt{tg^2\phi} = \sqrt{1} \rightarrow \cancel{tg^{-1}}\, tg\,\phi = tg^{-1}\, 1 \rightarrow \phi = \dfrac{\pi}{4}$