Solución de un Sistema de Ecuaciones lineales Diferenciales

$$a_n y^{n} + a_{n-1} y^{n-1} + \dots + a_1 y' + a_0 y = g(t)$$

$$[a_n D^n + a_{n-1} D^{n-1} + \dots + a_1 D + a_0] y = g(t) \quad (1)$$

$$b_n x^{n} + b_{n-1} x^{n-1} + \dots + b_1 x' + b_0 x = f(t)$$

$$[b_n D^n + b_{n-1} D^{n-1} + \dots + b_1 D + b_0] x = f(t) \quad (2)$$

Ej

Resolver

$$\frac{dx}{dt} = 4x + 7y \rightarrow Dx = 4x + 7y \quad (1)$$

$$\frac{dy}{dt} = x - 2y \rightarrow Dy = x - 2y \quad (2)$$

$$Dx - 4x - 7y = 0 \rightarrow (D-4)x - 7y = 0 \quad (1)$$

$$-x + Dy + 2y = 0 \rightarrow -x + (D+2)y = 0 \quad (2)$$

$$(D-4)x - 7y = 0 \rightarrow (D-4)x - 7y = 0 +$$

$$[-x + (D+2)y = 0](D-4) \quad -(D-4)x + (D+2)(D-4)y = 0$$

$$[(D+2)(D-4) - 7]y = 0$$

$$[D^2 - 4D + 2D - 8 - 7]y = 0$$

$$(D^2 - 2D - 15)y = 0 \rightarrow \frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 15y = 0$$

$$\rightarrow D^2 - 2D - 15 = 0$$

$$\begin{array}{c|c} D \times -5 & -5D \\ D \times +3 & +3D \\ & \hline & -2D \end{array} \rightarrow (D-5)(D+3) = 0$$

$$D = 5$$

$$D = -3$$

$$y(t) = c_1 e^{5t} + c_2 e^{-3t}$$

$$X(t) = C_3 e^{5t} + C_4 e^{-3t}$$

$$\frac{dy}{dx} = x - 2y$$

$$x = \frac{dy}{dt} + 2y$$

$$y'(t) = 5C_1 e^{5t} - C_2 e^{-3t}$$

$$x(t) = 5C_1 e^{5t} - C_2 e^{-3t} + 2(C_1 e^{5t} + C_2 e^{-3t})$$

$$\boxed{x(t) = 7C_1 e^{5t} + C_2 e^{-3t}}$$

Regla de Cramer.

$$ax + by = c$$
$$dx + cy = f$$

$$\rightarrow \quad x = \frac{\begin{vmatrix} c & b \\ f & e \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \frac{ce - bf}{ae - db}$$

$$y = \frac{\begin{vmatrix} a & c \\ d & f \end{vmatrix}}{\begin{vmatrix} a & b \\ d & e \end{vmatrix}} = \frac{af - cd}{ae - bd}$$

$$(D-4)x - 7y = 0$$

$$-x + (D+2)y = 0$$

$$x = \frac{\begin{vmatrix} 0 & -7 \\ 0 & D+2 \end{vmatrix}}{\begin{vmatrix} D-4 & -7 \\ -1 & D+2 \end{vmatrix}} = \frac{0-0}{(D-4)(D+2) - (-7)(-1)} = \frac{0}{D^2+2D-4D-8-7}$$

$$x = \frac{0}{D^2 - 2D - 15}$$

$$(D^2-2D-15)\,x = 0$$

$$x(t) = c_1 e^{5t} + c_2 e^{-3t}$$

$$y = \frac{\begin{vmatrix} D-4 & 0 \\ -1 & 0 \end{vmatrix}}{D^2-2D-15} = 0$$

$$(D^2-2D-15)\,y = 0 \rightarrow D=5 \quad D=-3$$

$$y(t) = c_3 e^{5t} + c_4 e^{-3t}$$

$$\frac{dy}{dt} = x - 2y$$

$$5c_3 e^{5t} - 3c_4 e^{-3t} = c_1 e^{5t} + c_2 e^{-3t} - 2(c_3 e^{5t} + c_4 e^{-3t})$$

$$5c_3 e^{5t} - 3c_4 e^{-3t} = c_1 e^{5t} + c_2 e^{-3t} - 2c_3 e^{5t} - 2c_4 e^{-3t}$$

$$7c_3 e^{5t} - c_4 e^{-3t} = c_1 e^{5t} + c_2 e^{-3t}$$

$$7c_3 e^{5t} = c_1 e^{5t} \rightarrow c_1 = 7c_3 \quad c_3 = \frac{1}{7} c_1$$

$$-c_4 e^{-3t} = c_2 e^{-3t} \rightarrow c_2 = -c_4$$

$$y = \frac{1}{3} c_1 e^{5t} - c_2 e^{-3t}$$

Ej. Resolver $(D^2+5)x - 2y = 0$

$$-2x + (D^2+2)y = 0$$

$$x = \frac{\begin{vmatrix} 0 & -2 \\ 0 & D^2+2 \end{vmatrix}}{\begin{vmatrix} D^2+5 & -2 \\ -2 & D^2+2 \end{vmatrix}} = \frac{0-0}{(D^2+5)(D^2+2)-(-2)(-2)}$$

$$x = \frac{0}{D^4+7D^2+10-4} = \frac{1}{D^4+7D^2+6}$$

$$(D^4+7D^2+6)\,x=0$$

$$\begin{array}{c|c|c} D^2 & 6 & 6D^2 \\ D^2 & 1 & D^2 \\ \hline & & 7D^2 \end{array} \quad \rightarrow \quad (D^2+6)(D^2+1)=0$$

$$D=\pm\sqrt{6}\,i$$

$$D=\pm i$$

$$x(t)=C_1\cos t+C_2\operatorname{sen} t+C_3\cos\sqrt{6}t+C_4\operatorname{sen}\sqrt{6}t$$

$$y=\frac{\begin{vmatrix} D^2+5 & 0 \\ -2 & 0 \end{vmatrix}}{\begin{vmatrix} D^2+5 & -2 \\ -2 & D^2+2 \end{vmatrix}}=\frac{0}{D^4+7D^2+6}$$

$$(D^4+7D^2+6)\,y=0 \quad \rightarrow \quad D=\pm i \qquad D=\pm\sqrt{6}\,i$$

$$y(t)=C_5\cos t+C_6\operatorname{sen} t+C_7\cos\sqrt{6}t+C_8\operatorname{sen}\sqrt{6}t$$

$$(D^2+5)\,x=2y$$

$$\dot{x}\rightarrow -C_1\operatorname{sen}x+C_2\cos t-\sqrt{6}C_3\operatorname{sen}\sqrt{6}t+\sqrt{6}C_4\cos\sqrt{6}t$$

$$D^2x\rightarrow -C_1\cos x-C_2\operatorname{sen}t-6C_3\cos\sqrt{6}t-6C_4\operatorname{sen}\sqrt{6}t$$

$$-C_1\cos t-C_2\operatorname{sen}t-6C_3\cos\sqrt{6}t-6C_4\operatorname{sen}\sqrt{6}t+5C_1\cos t+5C_2\operatorname{sen}t$$
$$+5C_3\cos\sqrt{6}t+5C_4\operatorname{sen}\sqrt{6}t=2C_5\cos t+2C_6\operatorname{sen}t+2C_7\cos\sqrt{6}t$$
$$+2C_8\operatorname{sen}\sqrt{6}t$$

$$\cos t(-C_1+5C_1)+\operatorname{sen}t(-C_2+5C_2)+\cos\sqrt{6}t(-6C_3+5C_3)+\operatorname{sen}\sqrt{6}t(-6C_4+5C_4)=$$

$$2C_5\cos t+2C_6\operatorname{sen}t+2C_7\cos\sqrt{6}t+2C_8\operatorname{sen}\sqrt{6}t$$

$4C_1 = 2C_5 \qquad \boxed{C_5 = 2C_1}$

$4C_2 = 2C_6 \qquad \boxed{C_6 = 2C_2}$

$-C_3 = 2C_7 \qquad \boxed{C_7 = -\frac{1}{2}C_3}$

$-C_4 = 2C_8 \qquad \boxed{C_8 = -\frac{1}{2}C_4}$

$x(t) = C_1\cos t + C_2 \operatorname{sen} t + C_3 \cos\sqrt{6}\,t + C_4 \operatorname{sen}\sqrt{6}\,t$

$y(t) = 2C_1\cos t + 2C_2 \operatorname{sen} t - \frac{1}{2}C_3\cos\sqrt{6}\,t - \frac{1}{2}C_4 \operatorname{sen}\sqrt{6}\,t$

Ej. Resolver

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = e^t \quad (1)$$

$$\frac{dx}{dt} - x + \frac{dy}{dt} = 5e^t \quad (2)$$

(1) $2Dx - 5x + Dy = e^t \rightarrow (2D-5)x + Dy = e^t$

(2) $Dx - x + Dy = 5e^t \rightarrow (D-1)x + Dy = 5e^t$

$$x = \frac{\begin{vmatrix} e^t & D \\ 5e^t & D \end{vmatrix}}{\begin{vmatrix} (2D-5) & D \\ D-1 & D \end{vmatrix}} = \frac{De^t - D(5e^t)}{(2D-5)(D) - D(D-1)} = \frac{e^t - 5e^t}{2D^2 - 5D - D^2 + D}$$

$$x = \frac{-4e^t}{D^2 - 4D}$$

$$(D^2 - 4D)x = -4e^t$$

$$\left(\frac{D^2x}{dt^2} - 4\frac{dx}{dt}\right) = -4e^t$$

$$x = x_c + x_p$$

$D(D-4)=0 \rightarrow D=0,\ D=4$

$x_c = c_1 e^{0x} + c_2 e^{4t} = c_1 + c_2 e^{4t}$

$x_p \quad g(t) = -4e^t \rightarrow \alpha = 1$

$(D-1)(-4e^t) = 0 \rightarrow D = 1$

$x_p = c_3 e^t$

$x' = c_3 e^t$

$x'' = c_3 e^t$

$c_3 e^t - 4c_3 e^t = -4e^t$

$-3c_3 e^t = -4e^t \Rightarrow \boxed{c_3 = \frac{4}{3}}$

$x_p = \frac{4}{3} e^t$

$x(t) = c_1 + c_2 e^{4t} + \frac{4}{3} e^t$

$$y = \frac{\begin{vmatrix} 2D-5 & e^t \\ D-1 & 5e^t \end{vmatrix}}{\begin{vmatrix} 2D-5 & D \\ D-1 & D \end{vmatrix}} = \frac{(2D-5)(5e^t) - (D-1)(e^t)}{(D^2-5)D - D(D-1)}$$

$$= \frac{10D(e^t) - 25e^t - De^t + e^t}{D(D-4)}$$

$$y = \frac{10e^t - 25e^t - e^t + e^t}{D(D-4)}$$

$D(D-4)y = -15e^t$

$y = y_c + y_p$

$y_c \quad D(D-4) = 0 \rightarrow D = 0,\ D = 4$

$$y_c = C_4 e^{0t} + C_5 e^{4t} = C_4 + C_5 e^{4t}$$

y_p $\quad g(t) = -15e^t \rightarrow \alpha = 1$

$$(D-1)(-15e^t) = 0$$

$$p = 1$$

$$y_p = C_6 e^t$$

$$y' = C_6 e^t$$

$$y'' = C_6 e^t$$

$$C_6 e^t - 4C_6 e^t = -15e^t$$

$$-3C_6 e^t = -15e^t \rightarrow C_6 = \frac{-15}{-3} = 5$$

$$y_p = 5e^t$$

$$y(t) = C_3 + C_4 e^{4t} + 5e^t$$

$$2\frac{dx}{dt} - 5x + \frac{dy}{dt} = e^t$$

$$2\left[4C_2 e^{4t} + \frac{4}{3}e^t\right] - 5\left(C_1 + C_2 e^{4t} + \frac{4}{3}e^t\right) + 4C_4 e^{4t} + 5e^t = e^t$$

$$8C_2 e^{4t} + \frac{8}{3}e^t - 5C_1 - 5C_2 e^{4t} - \frac{20}{3}e^t + 4C_4 e^{4t} + 5e^t = e^t$$

$$e^{4t}(8C_2 - 5C_2 + 4C_4) = 0$$

$$3C_2 + 4C_4 = 0 \rightarrow \quad C_4 = -\frac{3}{4}C_2$$

$$-5C_1 = 0 \rightarrow \quad C_1 = 0$$

$$x(t) = C_2 e^{4t} + \frac{4}{3}e^t$$

$$y(t) = C_3 - \frac{3}{4} C_2 e^{4t} + 5e^{t} \frac{1}{6} r$$