## ***Ejemplo 2:***

Resolver la Siguiente Ecuación diferencial Por Medio de Series de Potencias

$$(x^2-1)y'' + 4xy' + 2y = 0$$

$$y' = \sum_{n=1}^{\infty} c_n n\, x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2}$$

$$(x^2-1)\sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2} + 4x\sum_{n=1}^{\infty} c_n n\, x^{n-1} + 2\sum_{n=0}^{\infty} c_n\, x^n = 0$$

$$\sum_{n=2}^{\infty} c_n n\,(n-1)x^{n} - \sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2} + 4\sum_{n=1}^{\infty} c_n n\, x^{n} + 2\sum_{n=0}^{\infty} c_n\, x^n = 0$$

$k = n \qquad k = n-2 \qquad k = n \qquad k = n$

$k = n \qquad n = k+2 \qquad k = n \qquad k = n$

$$\sum_{k=2}^{\infty} c_k k\,(k-1)x^{k} - \sum_{k+2=2}^{\infty} c_{k+2}(k+2)\,(k+2-1)x^{k} + 4\sum_{k=1}^{\infty} c_k k\, x^{k} + 2\sum_{k=0}^{\infty} c_k\, x^k = 0$$

$$\sum_{k=2}^{\infty} c_k k(k-1)x^k - \sum_{k=\cancel{0}\,\cancel{1}\,2}^{\infty} c_{k+2}(k+2)(k+1)x^k + 4\sum_{k=\cancel{1}\,2}^{\infty} c_k k\, x^k + 2\sum_{k=\cancel{0}\,\cancel{1}\,2}^{\infty} c_k\, x^k = 0$$

$$-c_2(2)(1) - c_3(3)(2)x + 4c_1(1)x \qquad + 2c_0 + 2c_1 x$$

$$-c_2(2) + 2c_0 = 0 \quad \rightarrow \qquad \boxed{c_0 = c_2}$$

$$-c_3(3)(2)x + 4c_1(1)x + 2c_1 x = 0 \quad \rightarrow \qquad \boxed{c_1 = c_3}$$

$$\sum_{k=2}^{\infty} c_k k(k-1)x^k - \sum_{k=2}^{\infty} c_{k+2}(k+2)(k+1)x^k + 4\sum_{k=2}^{\infty} c_k k\, x^k + 2\sum_{k=2}^{\infty} c_k\, x^k = 0$$

$$\sum_{\boxed{k=2}}^{\infty} x^k \left[c_k k(k-1) - c_{k+2}(k+2)(k+1) + 4c_k k + 2c_k\right] = 0$$

$$c_k k(k-1) - c_{k+2}(k+2)(k+1) + 4c_k k + 2c_k = 0$$

$$c_{k+2} = \frac{c_k k(k-1) + 4c_k k + 2c_k}{(k+2)(k+1)}$$

$$c_{k+2} = \frac{c_k k^2 - c_k k + 4c_k k + 2c_k}{(k+2)(k+1)}$$

$$c_{k+2} = \frac{c_k\left(k^2 + 3k + 2\right)}{(k+2)(k+1)} = \frac{c_k\cancel{(k+2)}\cancel{(k+1)}}{\cancel{(k+2)}\cancel{(k+1)}}$$

$$\boxed{c_{k+2} = c_k \quad , \qquad k \geq 2}$$


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| | | |
|---|---|---|
| $Si\ k = 2$ | $c_{2+2} = c_2$ | $c_4 = c_2$ **Pero** $c_0 = c_2$<br>$c_4 = c_0$ |
| $Si\ k = 3$ | $c_{3+2} = c_3$ | $c_5 = c_3$ Pero $c_1 = c_3$<br>$c_5 = c_1$ |
| $Si\ k = 4$ | $c_{4+2} = c_4$ | $c_6 = c_4$ Pero $c_4 = c_0$<br>$c_6 = c_0$ |
| $Si\ k = 5$ | $c_{5+2} = c_5$ | $c_7 = c_5$ Pero $c_5 = c_1$<br>$c_7 = c_1$ |
| $Si\ k = 6$ | $c_{6+2} = c_6$ | $c_8 = c_6$ Pero $c_6 = c_0$<br>$c_8 = c_0$ |

$$y=\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + \overset{c_0}{\cancel{c_2}}x^2 + \overset{c_1}{\cancel{c_3}}x^3 + \overset{c_0}{\cancel{c_4}}x^4 + \overset{c_1}{\cancel{c_5}}x^5 + \overset{c_0}{\cancel{c_6}}x^6 + \overset{c_1}{\cancel{c_7}}x^7 + \overset{c_0}{\cancel{c_8}}x^8 \cdots$$

$$y = c_0 + c_1 x + c_0 x^2 + c_1 x^3 + c_0 x^4 + c_1 x^5 + c_0 x^6 + c_1 x^7 + c_0 x^8 \cdots$$

$$y = c_0\left[\underbrace{1 + x^2 + x^4 + x^6 + x^8 +}\cdots\right] + c_1\left[\underbrace{x + x^3 + x^5 + x^7 \cdots}\right]$$


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