Estructuras de Datos

ANÁLISIS DE ALGORITMOS
Pr-1.3- Optimización de algoritmo

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$$T(n) = \begin{cases} T(cond) + T(return) = 2t & si\ n \leq 1 \\ T(cond) + T(return) + T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) = 2t + 2T\left(\frac{n}{2}\right) & si\ n > 1 \end{cases}$$

$$T(n) = 2t + 2T\left(\frac{n}{2}\right)$$

$$= 2t + 2\left(2t + 2T\left(\frac{n}{4}\right)\right) = (2+4)t + 4T\left(\frac{n}{4}\right)$$

$$= (2+4)t + 4\left(2t + 2T\left(\frac{n}{8}\right)\right) = (2+4+8)t + 8T\left(\frac{n}{8}\right)$$

$$= (2+4+8)t + 8\left(2t + 2T\left(\frac{n}{16}\right)\right) = (2+4+8+16)t + 16T\left(\frac{n}{16}\right)$$

$$T(n) = \left(2 + 2^2 + 2^3 + \cdots 2^k\right)t + 2kT\left(\frac{n}{2^k}\right)$$

$$Si\ T(1) = 2t$$

$$\frac{n}{2^k} = 1 \rightarrow n = 2^k \rightarrow \log_2 n = k$$

$$T(n) = \left(2 + 2^2 + 2^3 + \cdots 2^{\log_2 n}\right)t + 2*(\log_2 n)T\left(\frac{n}{2^{\log_2 n}}\right) \rightarrow \left(2 + 2^2 + 2^3 + \cdots 2^{\frac{n}{2}}\right)t + 4(\log_2 n)t$$

$$O\big(T(n)\big) = O((2 + 2^2 + 2^3 + \cdots n) + 4(\log_2 n)\ t)$$

$$O\big(T(n)\big) = \max\left((2 + 2^2 + 2^3 + \cdots n), 4(\log_2 n)\right) = O(n) = n$$

b.

```
int recursiva (int n) {
    if (n <= 1)
        return 23;
    else
        return 2 * recursiva (n / 2);
}
```

$$T(n) = \begin{cases} T(cond) + T(return) = 2t & si\ n \leq 1 \\ T(cond) + T(return) + 2T\left(\frac{n}{2}\right) = 2t + T2\left(\frac{n}{2}\right) & si\ n > 1 \end{cases}$$

$$T(n) = 2t + 2T\left(\frac{n}{2}\right)$$

$$= 2t + 2\left(2t + 2T\left(\frac{n}{4}\right)\right) = (2+4)t + 4T\left(\frac{n}{4}\right)$$

$$= (2+4)t + 4\left(2t + 2T\left(\frac{n}{8}\right)\right) = (2+4+8)t + 8T\left(\frac{n}{8}\right)$$

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$$T(n) = \left(2 + 2^2 + 2^3 + \cdots 2^k\right)t + 2kT\left(\frac{n}{2^k}\right)$$

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$$\frac{n}{2^k} = 1 \to n = 2^k \to \log_2 n = k$$

$$T(n) = \left(2 + 2^2 + 2^3 + \cdots 2^{\log_2 n}\right)t + 2*(\log_2 n)T\left(\frac{n}{2^{\log_2 n}}\right) \to \left(2 + 2^2 + 2^3 + \cdots 2^{\frac{n}{2}}\right)t + 4(\log_2 n)t$$

$$O\big(T(n)\big) = O\big((2 + 2^2 + 2^3 + \cdots n) + 4(\log_2 n)\ t\big)$$

$$O\big(T(n)\big) = \max\big((2 + 2^2 + 2^3 + \cdots n), 4(\log_2 n)\big) = O(n) = \log_2 n$$