Calcular $\mathcal{L}\{\ \}$ de :

$$\mathcal{L}\left\{e^t \cdot \int_0^t \tau f(\tau)\cdot g(t-\tau)d\tau\right\} = \mathcal{L}\left\{\int_0^t \underbrace{\tau f(\tau)}_{\substack{\tau\to t\\ \downarrow\\ t f(t)}} \cdot \underbrace{g(t-\tau)}_{\substack{t-\tau\to t\\ \downarrow\\ g(t)}} d\tau\right\}_{s\to s-1}$$

$$\mathcal{L}\{t f(t) * g(t)\}_{s\to s-1}$$

$$\mathcal{L}\{t\cdot f(t)\}\cdot\mathcal{L}\{g(t)\}\Big|_{s\to s-1}$$

$$\left((-1)^1\frac{d}{ds}[\mathcal{L}\{f(t)\}]\right)\cdot(G(s))\Big|_{s\to s-1}$$

$$-F'(s)G(s)\Big|_{s\to s-1} \longrightarrow -F'(s-1)G(s-1)$$

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TRANSFORMADA INVERSA POR TEOREMA DE CONVOLUCION

$$\mathcal{L}^{-1}\{F(S)\cdot G(S)\}$$
$$= \int_0^t \mathcal{L}^{-1}\{F(S)\}_{\underbrace{t\to\theta}} \cdot \mathcal{L}^{-1}\{G(S)\}_{\underbrace{t\to t-\theta}} d\theta$$

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Calcular $\mathcal{L}^{-1}\{\ \}$ de :

1) $H(s) = \dfrac{1}{s^2(s-1)}$ $= \underbrace{\left(\dfrac{1}{s^2}\right)}_{F(s)}\cdot\underbrace{\left(\dfrac{1}{s-1}\right)}_{G(s)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s-1)}\right\} = \int_0^t \underbrace{\mathcal{L}^{-1}\left\{\cancel{\frac{1}{s^2}}\right\}^{t}_{t\to t-\theta}}\ \underbrace{\mathcal{L}^{-1}\left\{\cancel{\frac{1}{s-1}}\right\}^{e^t}_{t\to\theta}}\ d\theta$$

$$\mathcal{L}^{-1}\{H(s)\} = \int_0^t \underbrace{(t-\theta)\cdot e^{\theta}}\, d\theta = t\int_0^t e^{\theta}\,d\theta - \int_0^t \theta\cdot e^{\theta}\,d\theta$$

d	$\int$
$-\theta$	e^{θ}
-1	e^{θ}
0	e^{θ}

$$h(t) = t\left[e^{\theta}\Big|_0^t\right] + \left[-\theta e^{\theta} + e^{\theta}\Big|_0^t\right]$$

$$h(t) = \cancel{t}\,[\cancel{e^t} - 1] + [\cancel{-te^t} + e^t - 1]$$

$$h(t) = -t + e^t - 1$$

Calcular $\mathcal{L}^{-1}\{\ \}$ de :

$$2)\ H(s) = \frac{1}{(s-a)^2 \cdot s} = \left(\frac{1}{(s-a)^2}\right)\cdot\left(\frac{1}{s}\right) ; \quad \left(\frac{1}{s-a}\right)\left(\frac{1}{s(s-a)}\right)$$

$$\mathcal{L}^{-1}\{H(s)\} = \int_0^t \mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2}\right\}_{t\to\theta}^{\to e^{at}t} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}_{t\to t-\theta}^{\to 1} d\theta$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-a)^2}\right\} \quad s-a\to s$$

$$e^{at}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$

$$e^{at}t$$

$$h(t) = \int_0^t e^{a\theta}\cdot\theta\, d\theta = \left[\frac{\theta}{a}e^{a\theta} - \frac{1}{a^2}e^{a\theta}\Big|_0^t\right]$$

d	$\int$
θ (+)	$e^{a\theta}$
1 (−)	$\frac{1}{a}e^{a\theta}$
0	$\frac{1}{a^2}e^{a\theta}$

$$h(t) = \frac{t}{a}e^{at} - \frac{1}{a^2}e^{at} + \frac{1}{a^2}$$

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