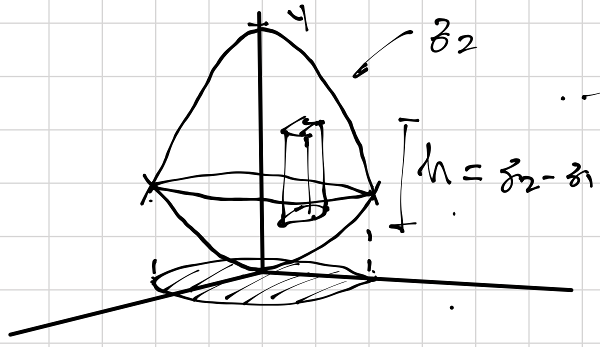
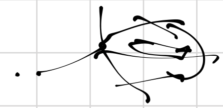
Ej. Determine el volumen limitado por la intersección de los paraboloides

$z = x^2 + y^2$ y $z = 4 - x^2 - y^2$





$V = \iint (z_2 - z_1)\, dA$

$V = \iint (4 - x^2 - y^2) - (x^2 + y^2)\, da$

$V = \iint (4 - 2x^2 - 2y^2)\, dA$

$V = \iint 2(2 - (x^2 + y^2))\, dA$

$V = 2\iint (2 - r^2)\, r\, dr\, d\theta$

$V = 2\int_0^{2\pi}\int_0^{\sqrt{2}} (2 - r^2)\, r\, dr\, d\theta$

$V = 2\int_0^{2\pi} r^2 - \frac{1}{4} r^4 \Big|_0^{\sqrt{2}}\, d\theta$

$V = 2\int_0^{2\pi} (2 - 1)\, d\theta = 2(1)\int_0^{2\pi} d\theta$

$V = 2\theta\Big|_0^{2\pi} = 2(2\pi) = 4\pi\ u^3$

$(\sqrt{2})^4$

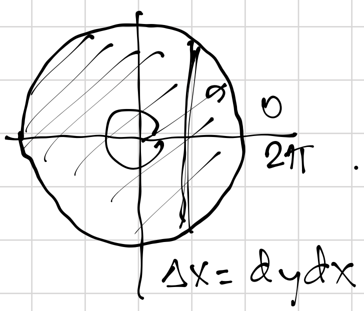
$(2)^2 = 4$

$z = z$

$x^2 + y^2 = 4 - x^2 - y^2$

$[2x^2 + 2y^2 = 4] * \frac{1}{2}$

$x^2 + y^2 = 2$



$\sqrt{r^2} = \sqrt{2} \to r = \sqrt{2}$

Ej. Use coordenadas polares para combinar la integral.

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\, dy\, dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\, dy\, dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\, dy\, dx$$

R_1 $\quad$ R_2 $\quad$ R_3

$\Delta x = dydx$

R_1 Para y

$y = x$, $(y)^2 = (\sqrt{1-x^2})^2 \to y^2 = 1 - x^2$

$x^2 + y^2 = 1$

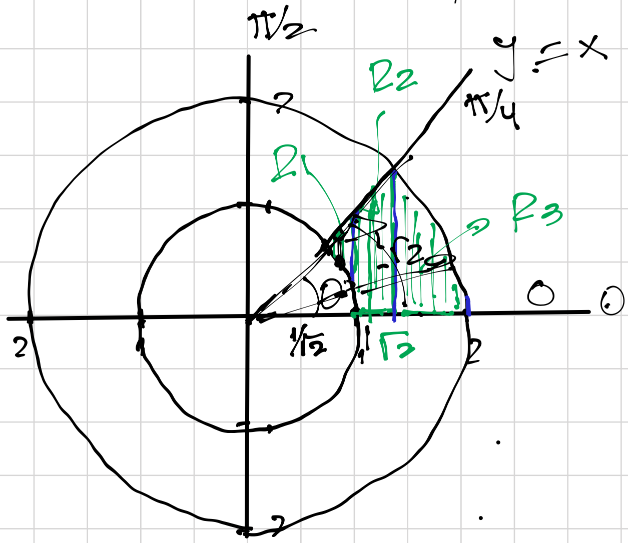
Para x $\quad x = 1$, $x = \frac{1}{\sqrt{2}}$

R_2 Para y $\quad y = x \quad , \quad y = 0$

Para x $\quad x = \sqrt{2} \quad , \quad x = 1$

R_3 Para y $\quad (y)^2 = (\sqrt{4-x^2})^2 \quad , \quad y = 0 \qquad y^2 = 4 - x^2 \rightarrow x^2 + y^2 = 4$

Para x $\quad x = 2 \quad , \quad x = \sqrt{2}$



$$\int_0^{\pi/4}\int_1^2 (r\cos\theta)(r\operatorname{sen}\theta)\, r\, \boxed{dr\,d\theta}$$

$x^2 + y^2 = 1$

$\sqrt{r^2} = \sqrt{1} \rightarrow r = 1$

$x^2 + y^2 = 4$

$\sqrt{r^2} = \sqrt{4} \rightarrow r = 2$

$y = x$

$m = 1$

$\theta = tg^{-1} m = tg^{-1} 1 = \frac{\pi}{4}$

$$\int_0^{\pi/4} \frac{1}{4} r^4 \Big|_1^2 \cos\theta \operatorname{sen}\theta \, d\theta$$

$u = \operatorname{sen}\theta$

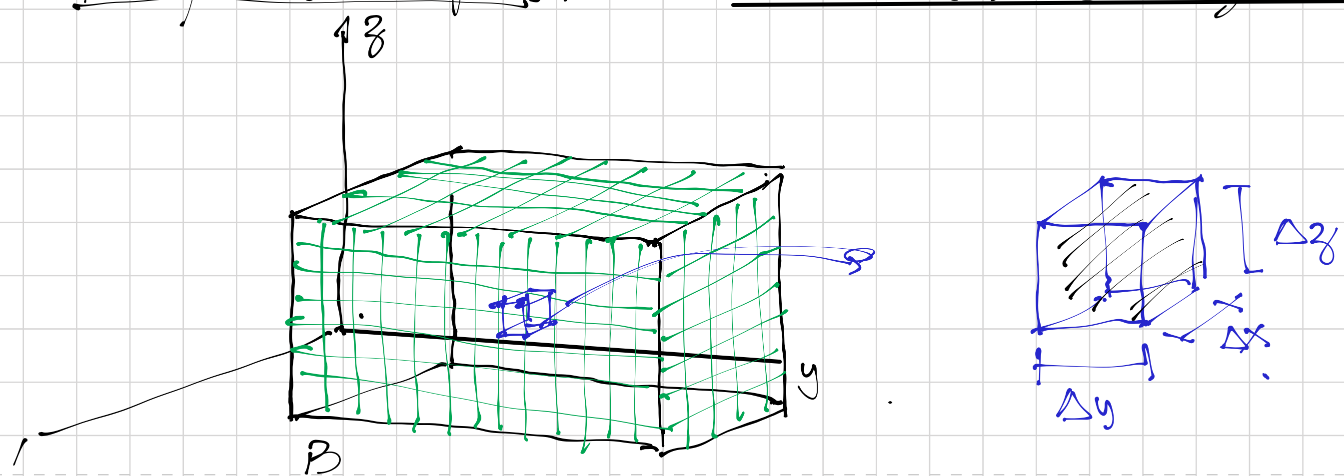
$du = \cos\theta d\theta$

$$\frac{1}{4}(2^4 - 1^4)\int_0^{\pi/4} \cos\theta \operatorname{sen}\theta \, d\theta$$

$$\frac{15}{4}\int_0^{\pi/4} u\,du = \frac{15}{4}\left(\frac{1}{2}\right) u^2 \Big|_0^{\pi/4} = \frac{15}{8} \operatorname{sen}^2\theta \Big|_0^{\pi/4}$$

$$= \frac{15}{8}\left[(\operatorname{sen}(\pi/4))^2 - (\operatorname{sen} 0)^2\right] = \frac{15}{16}$$

Integrales triples → coordenadas rectangulares



$$V = \Delta x_i \, \Delta y_j \, \Delta z_k$$

$$V = \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} \Delta x_i \, \Delta y_j \, \Delta z_k$$

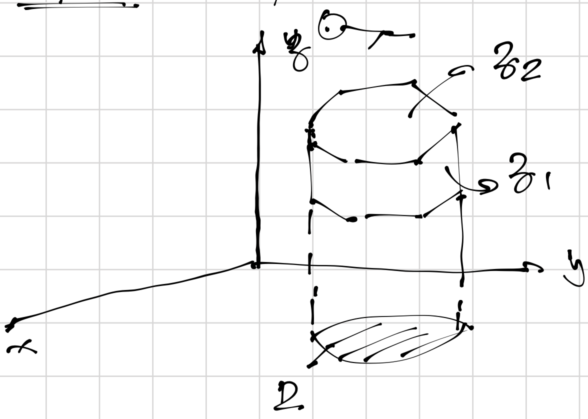
$$V = \lim_{l,m,n \to \infty} \sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} \Delta x_i \, \Delta y_j \, \Delta z_k$$

$$V = \iiint dx\,dy\,dz = \iiint dV$$

<u>Definición</u> La integral triple de f en la caja B es

$$\iiint f(x,y,z)\,dV = \lim_{l,m,n \to \infty} \sum\sum\sum f(x_i, y_j, z_k)\,\Delta x_i\,\Delta y_j\,\Delta z_k$$

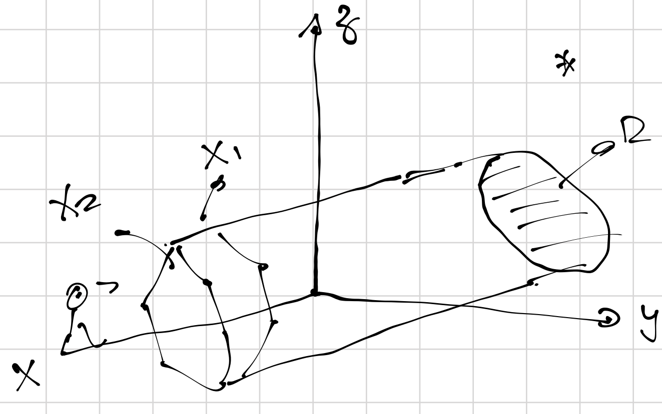
<u>Tipo I</u> plano xy



$$V = \iint_R \int_{z_1}^{z_2} dz\,dx\,dy$$

$$V = \iint_R \int_{z_1}^{z_2} dz\,dy\,dx$$

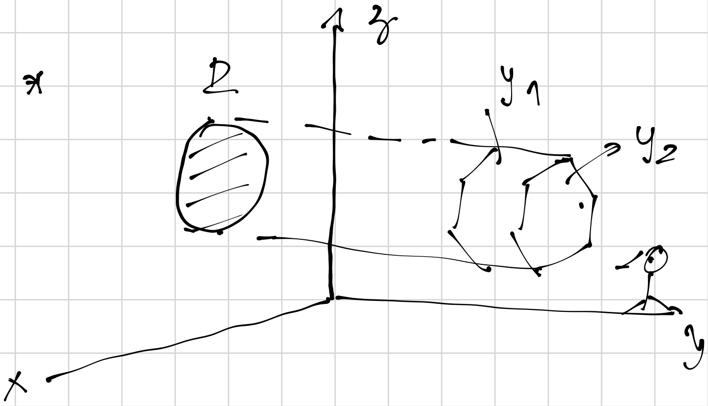
<u>Tipo II</u> plano yz



$$V = \iint_R \int_{x_1}^{x_2} dx\,dy\,dz$$

$$V = \iint_R \int_{x_1}^{x_2} dx\,dz\,dy$$

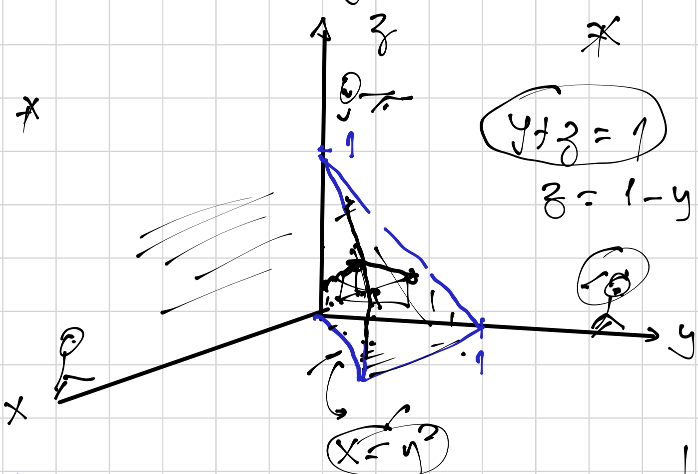
Tipo III plano xz

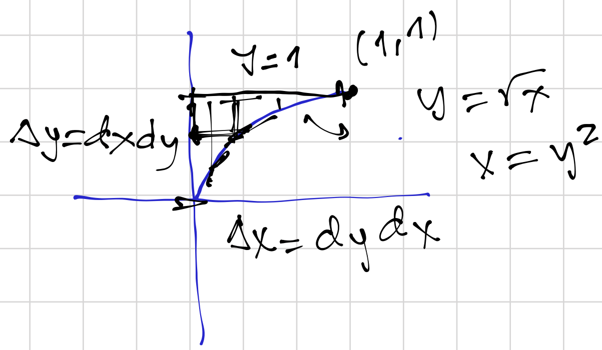


$$V = \iint_R \int_{y_1}^{y_2} dy\, dx\, dz$$

$$V = \iint_R \int_{y_1}^{y_2} dy\, dz\, dx$$

Ej. plantear 6 ordenes de integración para el volumen del sólido limitado por el plano $y+z=1$ y el cilindro $y=\sqrt{x}$





Tipo I xy

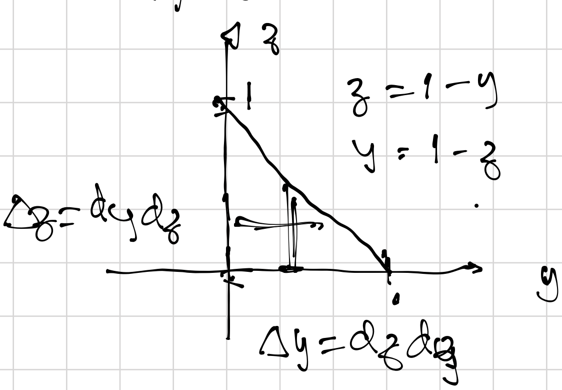
z-simples.

$$V = \int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} dz\, dy\, dx = \frac{1}{12}$$

$$V = \int_0^1 \int_0^{y^2} \int_0^{1-y} dz\, dx\, dy = \frac{1}{12}$$

Tipo II yz

x-simples.



$$V = \int_0^1 \int_0^{1-y} \int_0^{y^2} dx\, dz\, dy = \frac{1}{12}$$

$$V = \int_0^1 \int_0^{1-z} \int_0^{y^2} dx\, dy\, dz = \frac{1}{12}$$

Tipo III xz

y-simples.

$y + z = 1$ $\qquad \sqrt{x} = \sqrt{y^2}$

$y = 1 - z$ $\qquad y = \sqrt{x}$

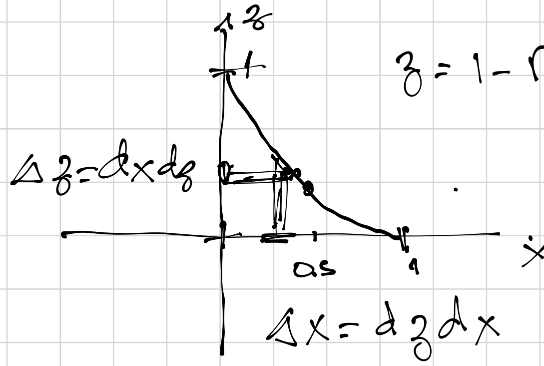
$y = y$

$1 - z = \sqrt{x}$

$z = 1 - \sqrt{x}$

$(\sqrt{x})^2 = (1-z)^2$

$x = (1-z)^2$



$z = 1 - \sqrt{x}$

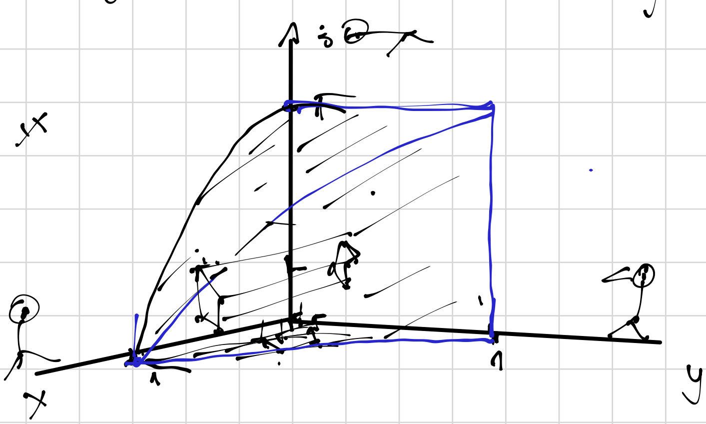
$$V = \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} dy\,dz\,dx = \frac{1}{12}$$

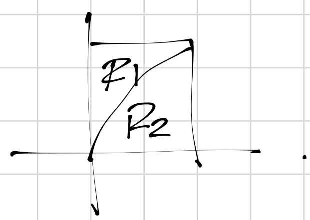
$$V = \int \int \int dy\,dx\,dz$$

$$V = \int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} dy\,dx\,dz = \frac{1}{12}$$

Ej volumen del sólido limitado por la intersección.

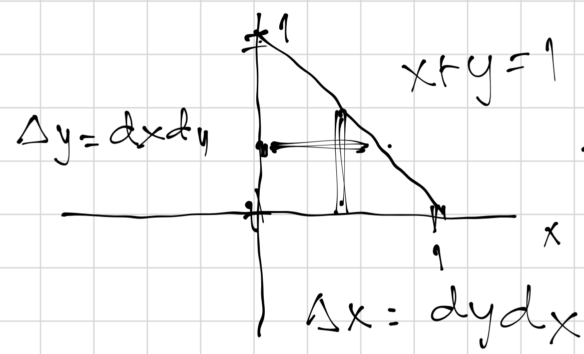
$z = 1 - x^2$ y $x + y = 1$





$x = 1 - y$

$y = 1 - x$

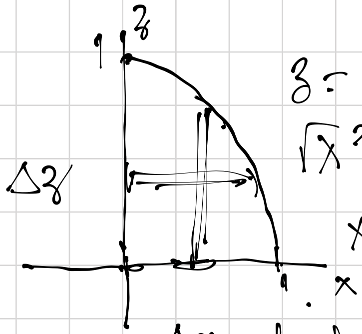


z-simples

$$V = \int_0^1 \int_0^{1-x} \int_0^{1-x^2} dz\,dy\,dx$$

$$V = \int_0^1 \int_0^{1-y} \int_0^{1-x^2} dz\,dx\,dy$$

y-simples



$$z = 1 - x^2$$

$$\sqrt{x^2} = \sqrt{1-z}$$

$$x = \sqrt{1-z}$$

$$\Delta x = dz\,dx$$

$$V = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} dy\, dz\, dx$$

$$V = \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} dy\, dx\, dz$$