☒ TAREA		
HOJA DE TRABAJO		
EXAMEN CORTO		

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EJERCICIOS 7.6

#1	P. 325	Ej.5

5. $2\frac{dx}{dt} + \frac{dy}{dt} - 2x = 1$

$\frac{dx}{dt} + \frac{dy}{dt} - 3x - 3y = 2$

$x(0) = 0, \quad y(0) = 0$

$(2s-2)\mathcal{L}\{x\} + s\mathcal{L}\{y\} = \frac{1}{s}$; $(s-3)\mathcal{L}\{x\} + (s-3)\mathcal{L} = \frac{2}{s}$

$\mathcal{L}\{x\} = \frac{-s-3}{s(s-2)(s-3)} = -\frac{1}{2}\frac{1}{s} + \frac{5}{2}\frac{1}{s-2} - \frac{2}{s-3} \rightarrow \mathcal{L}\{y\} = \frac{3s-7}{s(s-2)(s-3)} = -\frac{1}{6}\frac{1}{s} - \frac{5}{5}\frac{1}{s-2} + \frac{8}{3}\frac{1}{s-3}$

$$\boxed{x = -\frac{1}{2} + \frac{5}{2}e^{2t} - 2e^{3t} \;;\; y = -\frac{1}{6} - \frac{5}{2}e^{2t} + \frac{8}{3}e^{3t}}$$

#2	P. 325	Ej.7

7. $\frac{d^2x}{dt^2} + x - y = 0$

$\frac{d^2y}{dt^2} + y - x = 0$

$x(0) = 0, \quad x'(0) = -2,$

$y(0) = 0, \quad y'(0) = 1$

$(s^2+1)\mathcal{L}\{x\} - \mathcal{L}\{y\} = -2$

$-\mathcal{L}\{x\} + (s^2+1)\mathcal{L}\{y\} = 1 \rightarrow \mathcal{L}\{x\} = \frac{-2s^2-1}{s^4+2s^2} = -\frac{1}{2}\frac{1}{s^2} - \frac{3}{2}\frac{1}{s^2+2} \rightarrow x = -\frac{1}{2}t - \frac{3}{2\sqrt{2}}\operatorname{sen}\sqrt{2}t$

$$\boxed{y = x'' + x = \frac{1}{2}t + \frac{3}{2\sqrt{2}}\operatorname{sen}\sqrt{2}t}$$

#3	P. 325	Ej.9

9. $\frac{d^2x}{dt^2} + \frac{d^2y}{dt^2} = t^2$

$\frac{d^2x}{dt^2} - \frac{d^2y}{dt^2} = 4t$

$x(0) = 8, \quad x'(0) = 0,$

$y(0) = 0, \quad y'(0) = 0$

$\frac{d^2x}{dt^2} = \frac{1}{2}t^2 + 2t$; $\frac{d^2y}{dt^2} = \frac{1}{2}t^2 - 2t$

$\mathcal{L}\{x\} = 8\frac{1}{s} + \frac{1}{24}\frac{4!}{s^5} + \frac{1}{3}\frac{3!}{s^4}$; $\mathcal{L}\{y\} = \frac{1}{24}\frac{4!}{s^5} - \frac{1}{3}\frac{3!}{s^4}$

$$\boxed{x = 8 + \frac{1}{24}t^4 + \frac{1}{3}t^3 yy = \frac{1}{24}t^4 - \frac{1}{3}t^3}$$

#4	P. 325	Ej.10

10. $\frac{dx}{dt} - 4x + \frac{d^3y}{dt^3} = 6\operatorname{sen} t$

$\frac{dx}{dt} + 2x - 2\frac{d^3y}{dt^3} = 0$

$x(0) = 0, \quad y(0) = 0,$

$y'(0) = 0, \quad y''(0) = 0$

$(s-4)\mathcal{L}\{x\} + s^3\mathcal{L}\{y\} = \frac{6}{s^2+1} \rightarrow (s+2)\mathcal{L}\{x\} - 2s^3\mathcal{L}\{y\} = 0$

$\mathcal{L}\{x\} = \frac{4}{(s-2)(s^2+1)} = \frac{4}{5}\frac{s}{s^2+1} - \frac{8}{5}\frac{1}{s^2+1}$; $\mathcal{L}\{y\} = \frac{2s+4}{s^3(s-2)(s^2+1)} = \frac{1}{s} - \frac{2}{s^2} - 2\frac{2}{s^3} + \frac{1}{5}\frac{1}{s-2} - \frac{6}{5}\frac{s}{s^2+1} + \frac{8}{5}\frac{1}{s^2+1}$

$$\boxed{x = \frac{4}{5}e^{2t} - \frac{4}{5}\cos t - \frac{8}{s}\text{sent} \;;\; y = 1 - 2t - 2t^2 + \frac{1}{5}e^{2t} - \frac{6}{5}\cos t + \frac{8}{5}\text{sent}}$$

12. $\dfrac{dx}{dt} = 4x - 2y + 2\mathcal{U}(t-1)$

$\dfrac{dy}{dt} = 3x - y + \mathcal{U}(t-1)$

$x(0) = 0, \quad y(0) = \frac{1}{2}$

#5	P. 325	Ej.12

$$(s-4)\mathcal{L}\{x\} + 2\mathcal{L}\{y\} = \frac{2e^{-s}}{s} \;;\; -3\mathcal{L}\{x\} + (s+1)\mathcal{L}\{y\} = \frac{1}{2} + \frac{e^{-s}}{s}$$

$$\mathcal{L}\{x\} = \frac{-\frac{1}{2}}{(s-1)(s-2)} + e^{-s}\frac{1}{(s-1)(s-2)} = \frac{1}{2}\frac{1}{s-1} - \frac{1}{2}\frac{1}{s-2} + e^{-s}\left[-\frac{1}{s-1} + \frac{1}{s-2}\right]$$

$$\mathcal{L}\{y\} = \frac{e^{-s}}{s} + \frac{\frac{s}{4}-1}{(s-1)(s-2)} + e^{-s}\frac{-\frac{s}{2}+2}{(s-1)(s-2)} = \frac{3}{4}\frac{1}{s-1} - \frac{1}{2}\frac{1}{s-2} + e^{-s}\left[\frac{1}{s} - \frac{3}{2}\frac{1}{s-1} + \frac{1}{s-2}\right]$$

$$\boxed{x = \frac{1}{2}e^{t} - \frac{1}{2}e^{2t} + \left[-e^{-t} + e^{2(t-1)}\right]\mathfrak{A}(t-1) \quad y = \frac{3}{4}e^{t} - \frac{1}{2}e^{2t} + \left[1 - \frac{3}{2}e^{t-1} + e^{2(t-1)}\right]\mathfrak{A}(t-1)}$$

13. Resuelva el sistema (1) cuando $k_1 = 3$, $k_2 = 2$, $m_1 = 1$, $m_2 = 1$ y $x_1(0) = 0$, $x_1'(0) = 1$, $x_2(0) = 1$, $x_2'(0) = 0$.

#6	P. 325	Ej.13

$x_1'' = -3x_1 + 2(x_2 - x_1)$

$x_2'' = -2(x_2 - x_1)$

$x_1(0) = 0 \;\cdot\; x_2(0) = 1$

$x_1'(0) = 1 \;;\; x_2'(0) = 0$

$$(s^2+5)\mathcal{L}\{x_1\} - 2\mathcal{L}\{x_2\} = 1 \to -2\mathcal{L}\{x_1\} + (s^2+2)\mathcal{L}\{x_2\} = s$$

$$\mathcal{L}\{x_1\} = \frac{s^2+2s+2}{s^4+7s^2+6} = \frac{2}{5}\frac{s}{s^2+1} + \frac{1}{5}\frac{1}{s^2+1} - \frac{2}{5}\frac{s}{s^2+6} + \frac{4}{5\sqrt{6}}\frac{\sqrt{6}}{s^2\sqrt{6}} \;;\; \mathcal{L}\{x_2\} = \frac{s^2+5s+2}{(s^2+1)(s^2+6)} = \frac{4}{5}\frac{s}{s^2+1} + \frac{2}{5}\frac{1}{s^2+1} + \frac{1}{5}\frac{s}{s^2+6} - \frac{s}{5\sqrt{6}} - \frac{2}{5\sqrt{6}}\frac{\sqrt{6}}{s^2+6}$$

$$\boxed{x_1 = \frac{2}{5}\cos t + \frac{1}{5}\sin t - \frac{2}{5}t - \frac{2}{5}\cos\sqrt{6}t + \frac{4}{5\sqrt{6}}\sin\sqrt{6}t \;; \quad x_2 = \frac{4}{5}\cos t + \frac{2}{5}\sin t + \frac{1}{5}\cos\sqrt{6}t - \frac{2}{5\sqrt{6}}\sin\sqrt{6}t}$$

15. a) Demuestre que el sistema de ecuaciones diferenciales para las corrientes $i_2(t)$ e $i_3(t)$ en la red eléctrica que se muestra en la figura 7.6.7 es

$$L_1\frac{di_2}{dt} + Ri_2 + Ri_3 = E(t)$$

$$L_2\frac{di_3}{dt} + Ri_2 + Ri_3 = E(t).$$

b) Resuelva el sistema del inciso a) si $R = 5\ \Omega$, $L_1 = 0.01$ H, $L_2 = 0.0125$ H, $E = 100$ V, $i_2(0) = 0$ e $i_3(0) = 0$.

c) Determine la corriente $i_1(t)$.

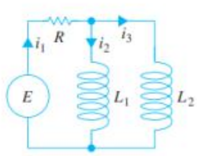

FIGURA 7.6.7 Red del problema 15.

#7	P. 325	Ej.9

a) $i_1 = i_2 + i_3 \to E(t) = Ri_1 + L_1 i_2' \;;\; E(t) = Ri_1 + L_2 i_3'$

$L_1 i_2' + Ri_2 + Ri_3 = E(t) \qquad L_2 i_3' + Ri_2 + Ri_3 = E(t)$

b) $0.01 i_2' + 5i_2 + 5i_3 = 100$

$0.0125 i_3' + 5i_2 + 5i_3 = 100$

$\to (s+500)\mathcal{L}\{i_2\} + 500\mathcal{L}\{i_3\} = \dfrac{10{,}000}{s}$

$400\mathcal{L}\{i_2\} + (s+900)\mathcal{L}\{i_3\} = \dfrac{8{,}000}{s}$

$$\mathcal{L}\{i_3\} = \frac{8{,}000}{s^2+900s} = \frac{80}{9}\frac{1}{s} - \frac{80}{9}\frac{1}{s+900}$$

$$i_3 = \frac{80}{9} - \frac{80}{9}e^{-900t} \;:\; i_2 = 20 - 0.0025i'_3 - i_3 = \frac{100}{9} - \frac{100}{9}e^{-900t}$$

c) $i_1 = i_2 + i_3 = 20 - 20e^{-900t}$