$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \operatorname{sen} x\,dx = -\cos x + C \qquad \int \cos x\,dx = \operatorname{sen} x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1} x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{sen}^{-1} x + C$$

$$\int \operatorname{senh} x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \operatorname{senh} x + C$$

Regla de sustitución

PROBLEMA ÁREA $\rightarrow \int_a^b f(x)\,dx \rightarrow \int_a^b f(x) = F(b) - F(a)$

TFC 2

$F(x) = \int f(x)\,dx \leftarrow$ INTEGRAL INDEFINIDA

4 **Regla de sustitución** Si $u = g(x)$ es una función derivable cuyo rango es un intervalo I y f es continua sobre I, entonces

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du$$

REGLA CADENA

Evalúe cada una de las integrales indefinidas siguientes.

$$\int x\sqrt{1-x^2}\,dx$$

$$u = 1 - x^2$$

$$\frac{du}{dx} = -2x$$

$$\frac{du}{-2x} = dx$$

$$\int \cancel{x}\sqrt{u}\,\frac{du}{-2\cancel{x}} = \int \sqrt{u}\left(-\tfrac{1}{2}du\right)$$

$$-\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\int u^{1/2}du = -\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)+C$$

$$= -\frac{1}{3}(1-x^2)^{3/2}+C$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int x^2 e^{x^3}\,dx$$

A) $u = x^2 \rightarrow \frac{du}{dx} = 2x$ NO

B) $u = e^{x^3} \rightarrow \frac{du}{dx} = e^{x^3} 3x^2$ NO

C) $u = x^3 \rightarrow \frac{du}{dx} = 3x^2$ SÍ

D) $u = x^2 \cdot e^{x^3} \rightarrow \frac{du}{dx} = ?$ NO

$u = x^3$ $\quad \frac{du}{dx} = 3x^2,$ $\quad dx = \frac{du}{3x^2}$

$$\int \cancel{x^2} e^u \frac{du}{3\cancel{x^2}} = \frac{1}{3}\int e^u du = \frac{1}{3} e^u + C$$

$$= \frac{1}{3} e^{x^3} + C$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \underbrace{(3x-2)}^{20}\,dx$$

$u = 3x - 2$

$\frac{du}{dx} = 3$, $dx = \frac{du}{3}$

$$\int u^{20}\frac{du}{3} = \frac{1}{3}\int u^{20}\,du = \frac{1}{3}\,\frac{u^{21}}{21} + C$$

$$= \frac{1}{63}(3x-2)^{21} + C$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \operatorname{sen} \pi t \, dt$$

$$z = \pi t$$

$$\frac{dz}{dt} = \pi, \quad dt = \frac{dz}{\pi}$$

$$\int \operatorname{sen} z \cdot \frac{dz}{\pi} = \frac{1}{\pi} \int \operatorname{sen} z \, dz = \frac{1}{\pi}(-\cos z) + C$$

$$\int \operatorname{sen} \pi t \, dt = -\frac{1}{\pi} \cos(\pi t) + C.$$

$$(\operatorname{sen} \pi t)' = \cos(\pi t) \cdot \pi$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \cos^3\theta \,\text{sen}\,\theta\, d\theta$$

A) $w = \cos^3\theta$ NO

B) $w = \text{sen}\,\theta$ NO

C) $w = \cos^3\theta\,\text{sen}\,\theta$ NO

D) $w = \cos\theta$ SÍ

$w = \cos\theta$

$\frac{dw}{d\theta} = -\text{sen}\,\theta, \quad d\theta = \frac{dw}{-\text{sen}\,\theta}$

$$-\int w^3\,dw = -\frac{1}{4}w^4 + C$$

$$\int \cos^3\theta\,\text{sen}\,\theta\,d\theta = -\frac{1}{4}\cos^4\theta + C$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \operatorname{sen} x \operatorname{sen}(\underset{x}{\cos x})\, dx$$

$u = \cos x$

$\frac{du}{dx} = -\operatorname{sen} x$

$dx = -\frac{du}{\operatorname{sen} x}$

$$\int \cancel{\operatorname{sen} x} \operatorname{sen}(u)\left(-\frac{du}{\cancel{\operatorname{sen} x}}\right)$$

$$-\int \operatorname{sen}(u)\, du = \cos(u) + C.$$

$$\int \operatorname{sen} x \operatorname{sen}(\cos x)\, dx = \cos(\cos x) + C.$$ //

Evalúe cada una de las integrales indefinidas siguientes.

$$\int x\sqrt{x+2}\,dx$$

$u = x + 2 \rightarrow x = u - 2$

$\frac{du}{dx} = 1 = x^0$, $dx = du$

$$\int x\sqrt{u}\,du = \int (u-2)\sqrt{u}\,du$$

$$\int (u^{3/2} - 2u^{1/2})\,du = \int u^{3/2}\,du - 2\int u^{1/2}\,du$$

$$\frac{2}{5}u^{5/2} - 2\left(\frac{2}{3}u^{3/2}\right) + C$$

$$\int x\sqrt{x+2}\,dx = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \frac{\operatorname{sen} 2x}{1+\cos^2 x}\,dx = \int \frac{2\cos x \operatorname{sen} x}{1+\cos^2 x}\,dx$$

$$w = 1+\cos^2 x$$

$$\frac{dw}{dx} = 2\cos x(-\operatorname{sen} x) = -2\cos x \operatorname{sen} x$$

$$-\int \frac{1}{w}\,dw = -\int w^{-1}\,dw$$

$$-\int \frac{1}{w}\,dw = -\ln|w| = -\ln w + C.$$

$$\int \frac{\operatorname{sen} 2x}{1+\cos^2 x}\,dx = -\ln(1+\cos^2 x) + C \;\#$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \frac{x}{x^2+4}\,dx$$

$$z = x^2+4$$

$$\frac{dz}{dx} = 2x, \qquad dx = \frac{dz}{2x}$$

$$\int \frac{x}{z}\,\frac{dz}{2x} = \frac{1}{2}\int \frac{1}{z}\,dz = \frac{1}{2}\ln z + C.$$

$$\int \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln(x^2+4) + C.$$

Evalúe cada una de las integrales indefinidas siguientes.

$$\int \frac{x}{1+x^4}\,dx$$

$w = 1 + x^4 \rightarrow x^2 = ?$

$\frac{dw}{dx} = 4x^3 \; ; \; dx = \frac{dw}{4x^3}$

$$\int \frac{\cancel{x}}{w} \frac{dw}{4x^{\cancel{3}}} = \frac{1}{4}\int \frac{1}{w x^2}\,dw \rightarrow ?$$

OPCIÓN 2.

$$\int \frac{x}{1+(x^2)^2}\,dx$$

$w = x^2$

$\frac{dw}{dx} = 2x \; , \; dx = \frac{dw}{2x}$

$$\int \frac{x}{1+w^2} \frac{dw}{2x} = \frac{1}{2}\int \frac{1}{1+w^2}\,dw$$

$$= \frac{1}{2}\,\text{TAN}^{-1}(w) + C$$

$$\int \frac{x}{1+x^4}\,dx = \frac{1}{2}\,\text{TAN}^{-1}(x^2) + C$$

Evalúe cada una de las integrales definidas siguientes.

$$\int_0^{\pi/2} \cos x \,\operatorname{sen}(\operatorname{sen} x)\, dx = F\left(\tfrac{\pi}{2}\right) - F(0)$$

OPCIÓN 1

$$\int \cos x \,\operatorname{SEN}(\operatorname{SEN} x)\, dx = F(x)$$

$w = \operatorname{SEN} x$

$\frac{dw}{dx} = \cos x, \qquad dx = \frac{dw}{\cos x}$

$$\int \operatorname{SEN} w \, dw = -\cos w + C$$

$$= -\cos(\operatorname{SEN} x) + C = F(x)$$

OPCIÓN 2

$w = \operatorname{SEN} x$

$$\int_0^{\pi/2} \cos x \,\operatorname{sen}(\operatorname{sen} x)\, dx$$

$1 = \operatorname{SEN}(\tfrac{\pi}{2})$

$$\int_0^1 \operatorname{SEN} w \, dw = \int_0^1 \operatorname{SEN}(w)\, dw$$

$$\int_0^1 \operatorname{SEN} w\, dw = -\cos w \Big|_0^1 = -\cos(1) - (-\cos(0))$$

$$= -\cos(1) + 1 \simeq 0.46$$

\#

Evalúe cada una de las integrales definidas siguientes.

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx$$

$$z = \frac{1}{x^2}$$

$$x = 2 \qquad z = \frac{1}{2}$$

$$\frac{dz}{dx} = -\frac{1}{x^2}$$

OPCIÓN 2

$$-\int_1^{\frac{1}{2}} e^z\,dz = -e^z\Big|_1^{1/2} = -e^{1/2} - (-e^1)$$

$$= -e^{1/2} + e = 1.07$$

Un tanque de almacenamiento de petróleo se rompe en $t = 0$, y el petróleo se fuga del tanque con una rapidez de $r(t) = 100e^{-0.01t}$ litros por minuto. ¿Cuánto petróleo se escapa durante la primera hora?

$$r(t) = 100e^{-0.01t} = \frac{dy}{dt}$$

VOL INICIAL

$$V(1) - V(0) = \int_0^{60} V'(t) = \int_0^{60} 100e^{-0.01t}\,dt$$

TEOREMA CAMBIO NETO

$$100\int_0^{60} e^{-0.01t}\,dt$$

$$z = -0.01t$$

$$\frac{dz}{dt} = -0.01$$

$$dt = \frac{dz}{-0.01}$$

$$\frac{100}{-0.01}\int_0^{-0.6} e^z\,dz = ?$$