ECUACIONES DIFERENCIALES DE COEFICIENTES VARIABLES POR MEDIO DE SERIES DE POTENCIA

$$y = \sum_{n=0}^{\infty} c_n\, x^n$$

$$y = \sum_{n=0}^{\infty} c_n\, x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + \cdots$$

Ejemplo 1:

Resolver la Siguiente Ecuación diferencial Por Medio de Series de Potencias

$$y^{//} + xy^{/} = 0$$

$$y^{/} = \sum_{n=1}^{\infty} c_n n\, x^{n-1}$$

$$y^{//} = \sum_{n=2}^{\infty} c_n n\,(n-1) x^{n-2}$$

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$$\sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2} + x\sum_{n=1}^{\infty} c_n n\, x^{n-1} = 0$$

$$\sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2} + \sum_{n=1}^{\infty} c_n n\, \mathbf{x} \cdot x^{n-1} = 0$$

$$\sum_{n=2}^{\infty} c_n n\,(n-1)x^{n-2} + \sum_{n=1}^{\infty} c_n n\, x^{n} = 0$$

$$k = n - 2 \qquad\qquad k = n$$

$$n = k + 2 \qquad\qquad n = k$$

$$\sum_{k+2=2}^{\infty} c_{k+2}(k+2)(k+2-1)\, x^{k} + \sum_{k=1}^{\infty} c_k k\, x^{k} = 0$$

$$\sum_{k+2=2}^{\infty} c_{k+2}(k+2)(k+2-1)\,x^k + \sum_{k=1}^{\infty} c_k k\,x^k = 0$$

$$\sum_{k=0}^{\infty} c_{k+2}(k+2)(k+1)\,x^k + \sum_{k=1}^{\infty} c_k k\,x^k = 0$$

$$c_2(2)(1) + \sum_{k=1}^{\infty} c_{k+2}(k+2)(k+1)\,x^k + \sum_{k=1}^{\infty} c_k k\,x^k = 0$$

$$\mathbf{2c_2 = 0 \quad \rightarrow \quad c_2 = 0}$$

$$\sum_{k=1}^{\infty} x^k\left[c_{k+2}(k+2)(k+1) + c_k k\right] = 0$$

$$\boldsymbol{c_{k+2}(k+2)(k+1) + c_k k = 0}$$

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$$c_{k+2}(k+2)(k+1) + c_k k = 0 \qquad c_{k+2} = \frac{-c_k k}{(k+2)(k+1)}\ , \qquad k \geq 1$$

$Si\ k = 1$ $\qquad c_{1+2} = \dfrac{-c_1(1)}{(1+2)(1+1)}$ $\qquad \boxed{c_3 = \dfrac{-c_1}{6}}$

$Si\ k = 2$ $\qquad c_{2+2} = \dfrac{-c_2(2)}{(2+2)(2+1)}$ $\qquad c_4 = \dfrac{-2c_2}{12}$ Pero $c_2 = 0$ $\qquad \boxed{c_4 = 0}$

$Si\ k = 3$ $\qquad c_{3+2} = \dfrac{-c_3(3)}{(3+2)(3+1)}$ $\qquad c_5 = \dfrac{-3c_3}{20}$ Pero $c_3 = \dfrac{-c_1}{6}$ $\qquad \boxed{c_5 = \dfrac{c_1}{40}}$

$Si\ k = 4$ $\qquad c_{4+2} = \dfrac{-c_4(4)}{(4+2)(4+1)}$ $\qquad c_6 = \dfrac{-4c_4}{30}$ Pero $c_4 = 0$ $\qquad \boxed{c_6 = 0}$

$Si\ k = 5$ $\qquad c_{5+2} = \dfrac{-c_5(5)}{(5+2)(5+1)}$ $\qquad c_7 = \dfrac{-5c_5}{42}$ Pero $c_5 = \dfrac{c_1}{40}$ $\qquad \boxed{c_7 = -\dfrac{c_1}{336}}$

$$\boldsymbol{y = \sum_{n=0}^{\infty} c_n\, x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 \cdots}$$

$$\boldsymbol{y = c_0 + c_1 x + (0)x^2 + \left(\frac{-c_1}{6}\right) x^3 + (0)x^4 + \left(\frac{c_1}{40}\right) x^5 + (0)x^6 + \left(-\frac{c_1}{336}\right) x^7 \cdots}$$

$$y = c_0 + c_1 \left[x - \frac{1}{6} x^3 + \left(\frac{1}{40}\right) x^5 - \left(\frac{1}{336}\right) x^7 + \cdots\right]$$

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