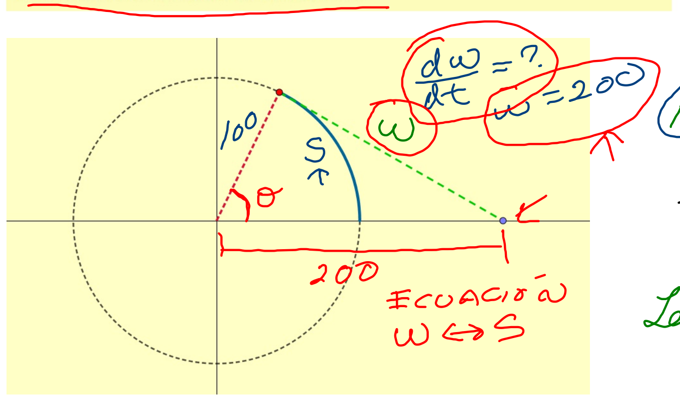
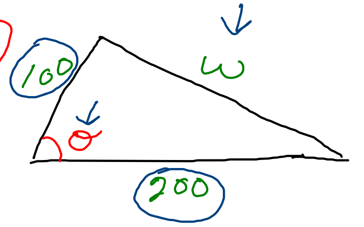
Un individuo corre por una pista circular de 100 m de radio a una rapidez constante de 7 m/s. Un amigo del corredor está parado a una distancia de 200 m del centro de la pista. ¿Qué tan rápido cambia la distancia entre los amigos cuando la distancia entre ellos es de 200 m?





Ley Cosenos.

$$\omega^2 = 100^2 + 200^2 - 2(100)(200)\cos\theta$$

$$\omega^2 = 50{,}000 - 40{,}000\cos\theta$$

$(\omega \leftrightarrow \theta)$

$s = r\theta$

$$2\omega\frac{d\omega}{dt} = -40{,}000\left(-\operatorname{sen}\theta\,\frac{d\theta}{dt}\right)$$

$$2\omega\frac{d\omega}{dt} = 40{,}000\operatorname{sen}\theta\,\frac{d\theta}{dt}$$

$$\frac{d\omega}{dt} = \frac{40{,}000\operatorname{sen}\theta\,\frac{d\theta}{dt}}{2\omega}$$

$$\frac{ds}{dt} = 7.$$

$$s = r\theta \leftarrow$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$\omega^2 = 50{,}000 - 40{,}000\cos\theta$$

$\omega = 200$

$$200^2 = 50{,}000 - 40{,}000\cos\theta$$

$$\theta = 75.52^\circ$$

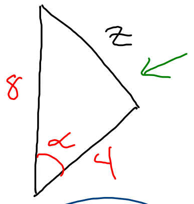
$$\frac{ds}{dt} = 100\frac{d\theta}{dt}$$

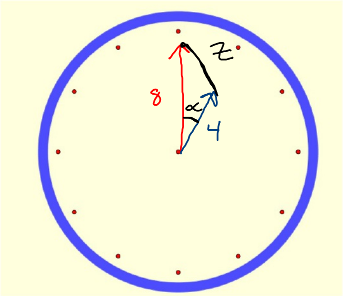
$$\rightarrow \boxed{\frac{7}{100} = \frac{d\theta}{dt}}$$

$$\frac{d\omega}{dt} = \frac{40{,}000\operatorname{sen}(75.52^\circ)\frac{7}{100}}{2(200)} = 6.78\ m/s$$

$$\theta = \cos^{-1}\left(\frac{200^2 - 50{,}000}{-40{,}000}\right)$$

La manecilla de los minutos de un reloj mide 8 mm de largo y la manecilla de las horas mide 4 mm de largo. ¿Qué tan rápido cambia la distancia entre las puntas de las manecillas cuando es 13:00?





$\frac{dz}{dt} = ?$

$\boxed{\frac{d\alpha}{dt} = ?}$

$$z^2 = 8^2 + 4^2 - 2(8)(4)\cos\alpha$$

$$\boxed{z^2 = 80 - 64\cos\alpha}$$

$$2z\frac{dz}{dt} = -64(-\operatorname{sen}\alpha)\frac{d\alpha}{dt}$$

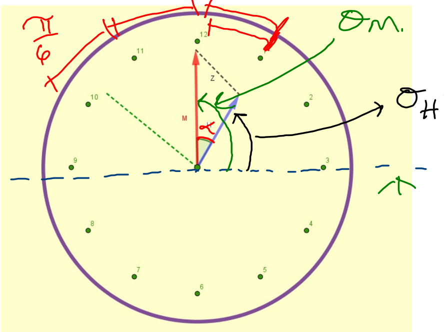
$$2z\frac{dz}{dt} = 64\operatorname{sen}\alpha\frac{d\alpha}{dt}$$

$$\frac{dz}{dt} = \frac{64\operatorname{sen}\alpha\frac{d\alpha}{dt}}{2z}$$

θ_M, θ_H

$\alpha = \theta_M - \theta_H$

$$\frac{d\alpha}{dt} = \underbrace{\frac{d\theta_M}{dt}}_{\text{VEL AGUJA MIN'}} - \underbrace{\frac{d\theta_H}{dt}}_{\text{VEL AGUJA HORAS}}$$



$$\frac{d\theta_M}{dt} = \frac{2\pi}{1} = -2\pi\ \text{RAD}/h, \quad \frac{d\theta_H}{dt} = -\frac{\frac{\pi}{6}}{1} = -\frac{\pi}{6}\frac{\text{RAD}}{h}$$

$$\boxed{\frac{d\alpha}{dt}} = -2\pi - \left(-\frac{\pi}{6}\right) = \boxed{-\frac{11}{6}\pi\ \text{RAD}/h.}$$

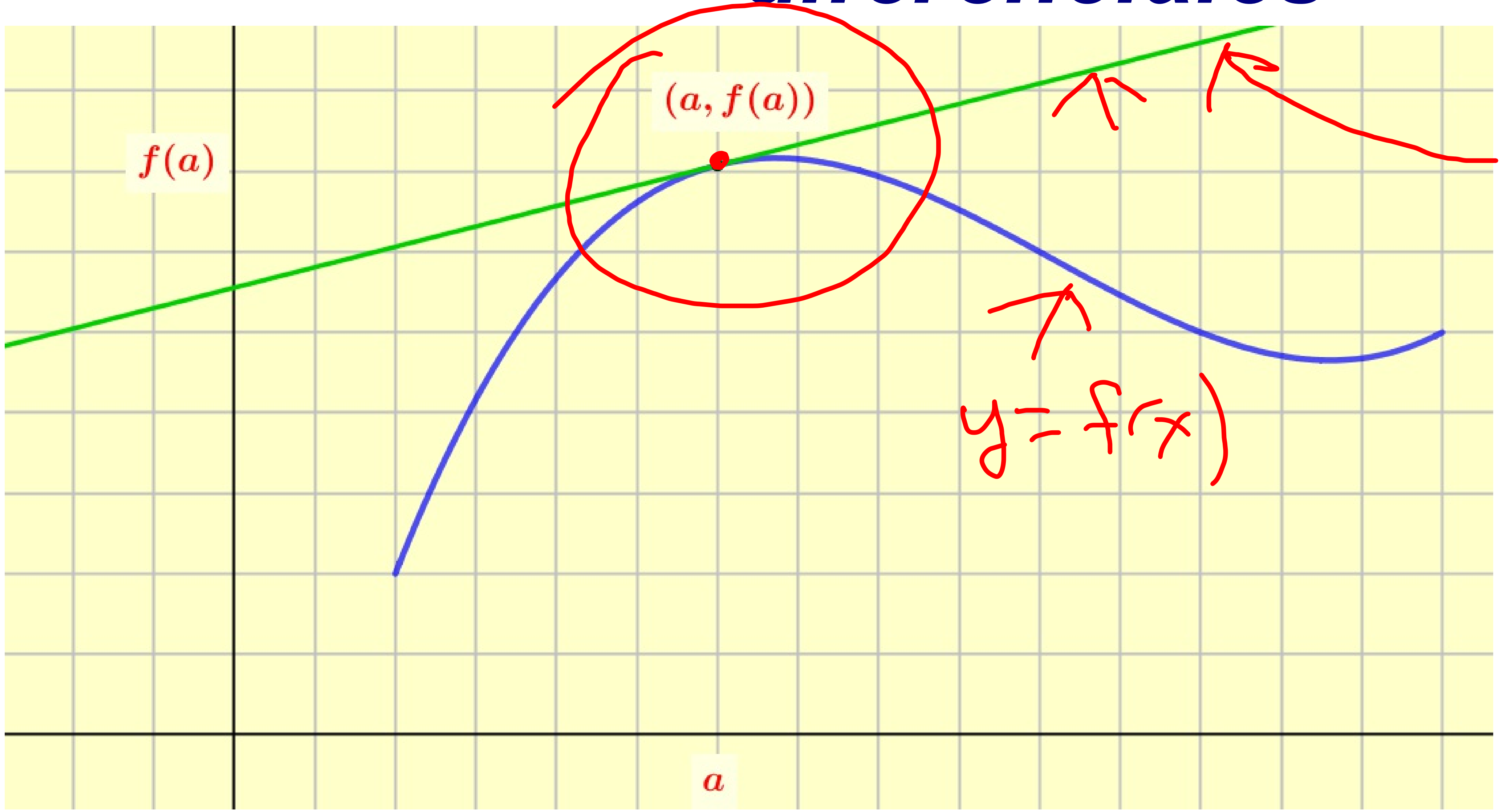
$\alpha = \frac{\pi}{6}$, $\quad z^2 = 80 - 64\cos\alpha$

$$\boxed{z = \sqrt{80 - 64\underbrace{\cos\left(\frac{\pi}{6}\right)}_{\text{RAD.}}}}$$

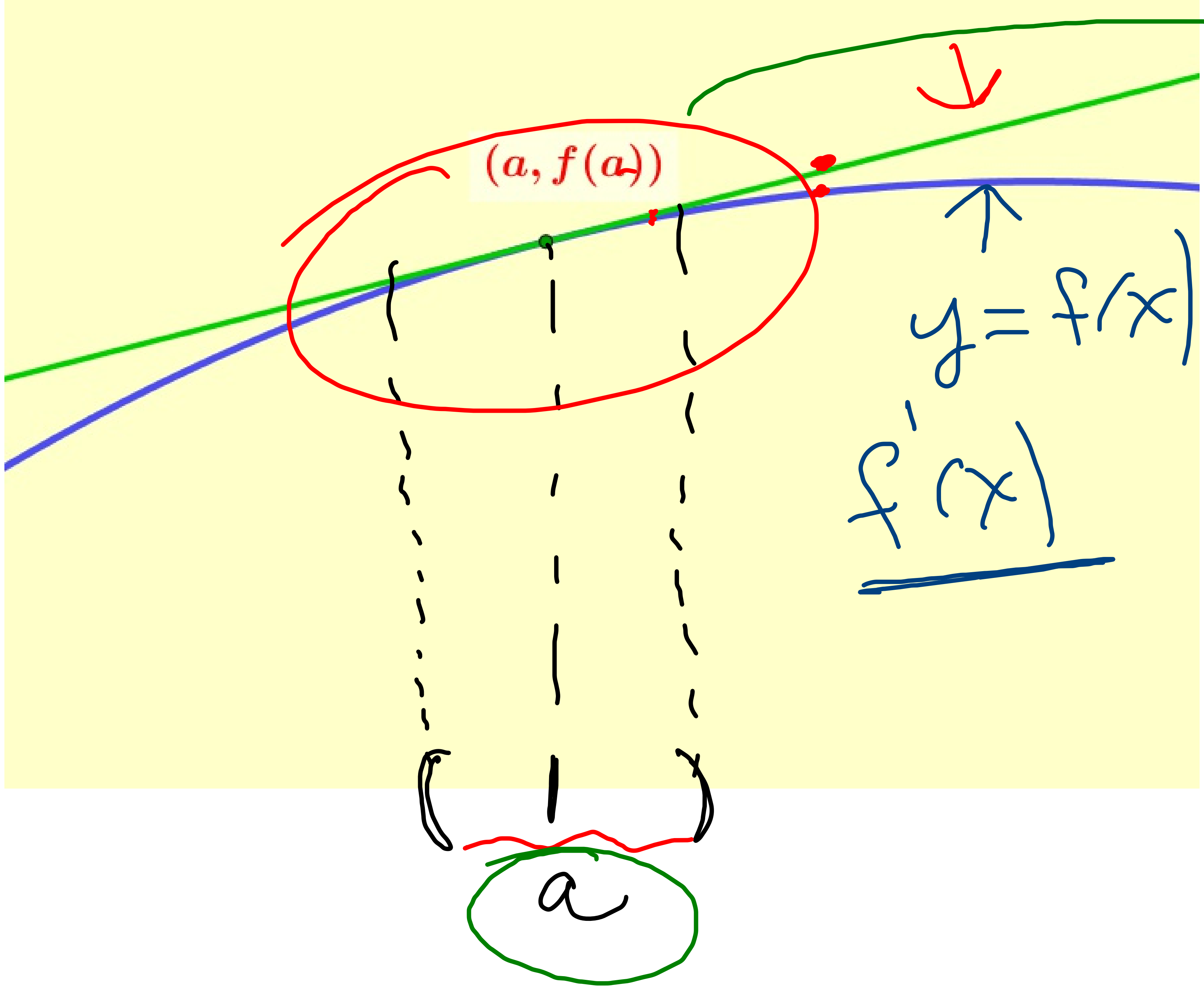
$$\frac{dz}{dt} = \frac{64\operatorname{sen}\alpha\frac{d\alpha}{dt}}{2z}$$

$$\frac{dz}{dt} = -18.59\ \text{mm}/h.$$

Aproximaciones lineales y diferenciales



$$y = f(a) + f'(a)(x - a)$$



$$f(x) \approx f(a) + f'(a)(x-a)$$

APROX. IGUAL

$$L(x) = f(a) + f'(a)(x-a)$$

LINEALIZACIÓN DE f(x)

(RECTA TANGENTE)

Encuentre la linealización $L(x)$ de la función en a.

$f(x) = x^3 - x^2 + 3, \quad a = -2$

$\mathcal{L}(x) = f(a) + f'(a)(x-a)$

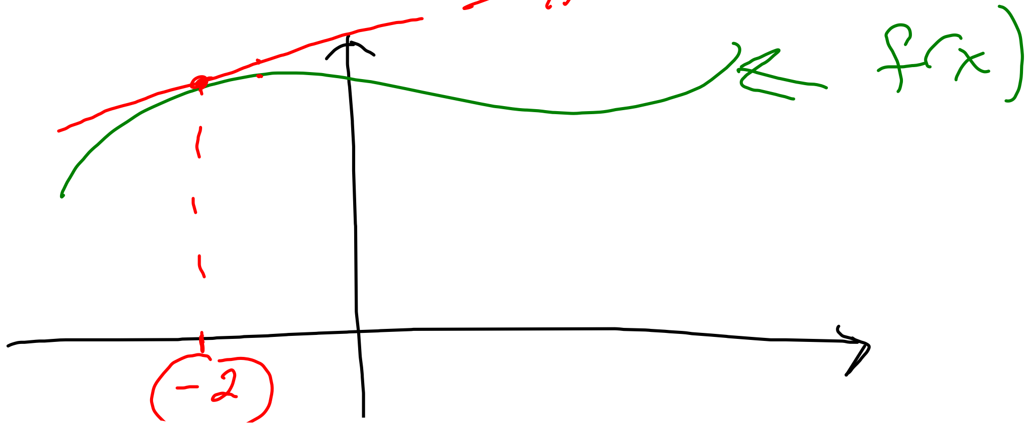
$f(x) = x^3 - x^2 + 3, \rightarrow f(a) = f(-2) = -9$

$f'(x) = 3x^2 - 2x \rightarrow f'(a) = f'(-2) = 16$

$\mathcal{L}(x) = -9 + 16(x+2)$

$\mathcal{L}(x) = -9 + 16x + 32.$

$\mathcal{L}(x) = 16x + 23$



$f(-2.1) = (-2.1)^3 - (-2.1)^2 + 3 \doteq -10.671$

$\mathcal{L}(-2.1) = 16(-2.1) + 23 = -10.6$

Encuentre la linealización $L(x)$ de la función en a.

$f(x) = \sqrt{x}, \quad a = 4$

$f'(x) = \frac{1}{2\sqrt{x}}$,

$\overbrace{f(4)}^{f(a)} = 2$

$\underbrace{f'(4)}_{f'(a)} = \frac{1}{4}$

$L(x) = 2 + \frac{1}{4}(x-4)$

$L(x) = 2 + \frac{1}{4}x - 1$

$L(x) = \frac{1}{4}x + 1$ ~~1~~

$\underbrace{\sqrt{x}}_{f(x)} \approx \underbrace{\frac{1}{4}\cdot x + 1}_{L(x)}$, $\quad a=4$

$\sqrt{4.2} = 2.0494$, $\quad L(4.2) = \underbrace{\frac{4.2}{4} + 1}$ $= 2.05$