TAREA ☒	**No.**	**CARNÉ:**	202100081	**FECHA:**	24/03/2023
HOJA DE TRABAJO ☐	**2**	**NOMBRE:**	Javier Andrés Monjes Solórzano		
EXAMEN CORTO ☐					

EJERCICIOS 7.4

En los problemas 1-8 use el teorema 7.4.1 para evaluar cada una de las transformadas de Laplace.

#1	Pag 315	Ej. 3

3. $\mathscr{L}\{t\cos 2t\}$

$$\mathscr{L}\{t\cos 2t\} \rightarrow -\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{s^2-4}{(s^2-4)^2} \rightarrow \boxed{\frac{s^2-4}{(s^2-4)^2}}$$

5. $\mathscr{L}\{t^2 \operatorname{senh} t\}$

#2	Pag 315	Ej. 5

$$\mathscr{L}\{t^2 \operatorname{senh} t\} = \frac{d^2}{ds^2}\left(\frac{1}{s^2-1}\right) = \frac{6s^2+2}{(s^2-1)^3} \rightarrow \boxed{\frac{6s^2+2}{(S^2-1)^3}}$$

7. $\mathscr{L}\{te^{2t}\operatorname{sen} 6t\}$

#3	Pag 315	Ej. 7

$$\mathscr{L}\{te^{2t}\operatorname{sen} 6t\} = -\frac{d}{ds}\left(\frac{6}{(s-2)^2+36}\right) = \frac{12(s-2)}{[(s-2)^2+36]^2} \rightarrow \boxed{\frac{12(s-2)}{[(s-2)^2+36]^2}}$$

En los problemas 9-14, use la transformada de Laplace para resolver el problema con valores iniciales dado. Use la tabla de transformadas de Laplace del apéndice C cuando sea necesario.

#4	Pag 315	Ej. 9

9. $y' + y = t\operatorname{sen} t, \quad y(0) = 0$

$$s\mathscr{L}\{y\} + \mathscr{L}\{y\} = \frac{2s}{(s^2+1)^2} \rightarrow \mathscr{L}\{y\} = \frac{2s}{(s+1)(s^2+1)^2} = -\frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{1}{s^2+1} + \frac{1}{2}\frac{s}{s^2+1} + \frac{1}{(s^2+1)^2} + \frac{s}{(s^2+1)^2}$$

$$y(t) = -\frac{1}{2}e^{-t} - \frac{1}{2}\operatorname{sen}t + \frac{1}{2}\cos t + \frac{1}{2}(\operatorname{sen}t - t\cos t) + \frac{1}{2}t\operatorname{sen}t = -\frac{1}{2}e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}t\cos t + \frac{1}{2}t\operatorname{sen}t$$

$$\boxed{-\frac{1}{2}e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}t\cos t + \frac{1}{2}t\operatorname{sen}t}$$

11. $y'' + 9y = \cos 3t, \quad y(0) = 2, \quad y'(0) = 5$

#5	Pag 315	Ej. 11

$$s^2\mathscr{L}\{y\} - sy(0) - y'(0) + 9\mathscr{L}\{y\} = \frac{s}{s^2+9}$$

$y(0)=2;\ y'(0)=5$

$$\mathscr{L}\{y\} = \frac{2s^3+5s^2+19s+45}{(s^2+9)^2} = \frac{2s}{s^2+9} + \frac{5}{s^2+9} + \frac{s}{(s^2+9)^2} \rightarrow \boxed{y = 2\cos 3t + \frac{5}{3}\operatorname{sen}3t + \frac{1}{6}t\operatorname{sen}3t}$$

13. $y'' + 16y = f(t), \quad y(0) = 0, \quad y'(0) = 1$, donde

#6	Pag 315	Ej. 13

$$f(t) = \begin{cases} \cos 4t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + 16\mathcal{L}\{y\} = \mathcal{L}\{\cos 4t - \cos 4t\,\mathcal{U}(t-\pi)\}$$

$$(s^2+16)\mathcal{L}\{y\} = 1 + \frac{s}{s^2+16} - e^{-\pi s}\mathcal{L}\{\cos 4(t-\pi)\} = 1 + \frac{s}{s^2+16} - e^{-\pi s}\mathcal{L}\{\cos 4t\} = 1 + \frac{s}{s^2+16} - \frac{s}{s^2+16}e^{-\pi s}$$

$$\mathcal{L}\{y\} = \frac{1}{s^2+16} + \frac{s}{(s^2+16)^2} - \frac{s}{(s^2+16)}e^{-\pi s} \rightarrow \boxed{y = \frac{1}{4}\operatorname{sen} 4t + \frac{1}{8}t\operatorname{sen}4t - \frac{1}{8}(t-\pi)\operatorname{sen}4(t-\pi)\mathfrak{U}(t-\pi)}$$

En los problemas 19-22 proceda como en el ejemplo 3 y encuentre la convolución $f * g$ de las funciones dadas. Después de integrar, encuentre la transformada de Laplace de $f * g$.

#7	Pag 315	Ej. 19

19. $f(t) = 4t, \quad g(t) = 3t^2$ $\rightarrow \mathcal{L}\{1 * t^3\} = \frac{1}{s}\frac{3!}{s^4} = \frac{6}{s^5} \rightarrow \boxed{\frac{6}{s^5}}$

21. $f(t) = e^{-t}, \quad g(t) = e^t$

#8	Pag 315	Ej. 21

$$\mathcal{L}\{e^{-t} * e^t\cos t\} = \frac{s-1}{(s+1)[(s+1)^2+1]} \qquad \boxed{\frac{s-1}{(s+1)[(s-1)^2+1]}}$$

En los problemas 23-34 proceda como en el ejemplo 4 y encuentre la transformada de Laplace de $f * g$ usando teorema 7.4.2. Antes de transformar, no evalúe la integral de convolución.

#9	Pag 316	Ej. 25

25. $\mathscr{L}\{e^{-t} * e^t \cos t\}$

$$\mathcal{L}\left\{\int_0^t e^{-\tau}\cos\tau\,d\tau\right\} \ \frac{1}{2}\mathcal{L}\{e^{-t}\cos t\} = \frac{1}{s}\cdot\frac{s+1}{(s+1)^2+1} \rightarrow \boxed{\frac{s+1}{s(s^2+2s+2)}}$$

27. $\mathscr{L}\left\{\int_0^t e^\tau\,d\tau\right\}$

#10	Pag 316	Ej. 27

$$\mathcal{L}\left\{\int_0^t \tau e^{t-\tau}d\tau\right\} = \{t\}\mathcal{L}\{e^t\} \rightarrow \boxed{\frac{1}{S^2(s-1)}}$$

31. $\mathscr{L}\left\{\int_0^t \tau\, e^{t-\tau}\,d\tau\right\}$

#11	Pag 316	Ej. 31

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{1}{(s-1)}}{s}\right\} = \int_0^t e^\tau d\tau = e^t - 1 \rightarrow \boxed{e^t - 1}$$

#12	Pag 316	Ej. 33

33. $\mathscr{L}\left\{t\int_0^t \operatorname{sen}\tau\,d\tau\right\}$ $\mathcal{L}^{-1}\left\{\frac{1}{s^3(s-1)}\right\} = \mathcal{L}^{-1}\left\{\frac{\frac{1}{s^2(s-1)}}{s}\right\} = \int_0^t (e^\tau - \tau - 1)d\tau = e^t - \frac{1}{2}t^2 - t - 1$

$$\boxed{e^t - \frac{1}{2}t^2 - t - 1}$$

#13	Pag 316	Ej. 37

En los problemas 35-38, use (8) para evaluar cada transformada inversa.

37. $\mathscr{L}^{-1}\left\{\frac{1}{s^3(s-1)}\right\}$ $\mathcal{L}\{y\} + \mathcal{L}\{t\}\,\mathcal{L}\{f\}\mathcal{L}\{t\} \rightarrow \mathcal{L}\{f\} = \frac{1}{s^2+1} \rightarrow f(t) = \operatorname{sen} t \rightarrow \boxed{f(t) = \operatorname{sen} t}$

#14 | Pag 316 | Ej. 43

43. $f(t) = te^t + \int_0^t \tau f(t-\tau)\,d\tau$

$$\mathcal{L}\{f\} = \mathcal{L}\{1\} + \mathcal{L}\{t\} - \mathcal{L}\left\{\frac{8}{3}\int_0^t (t-\tau)^3 f(\tau)\,d\tau\right\} = \frac{1}{s} + \frac{1}{s^2} + \frac{8}{3}\mathcal{L}\{t^3\}\mathcal{L}\{f\} = \frac{1}{s} + \frac{1}{s^2} + \frac{16}{s^4}\mathcal{L}\{f\}$$

$$\mathcal{L}\{f\} = \frac{s^2(s+1)}{s^4-16} = \frac{1}{8}\frac{1}{s+2} + \frac{3}{8}\frac{1}{s-2} + \frac{1}{4}\frac{2}{s^2+4} + \frac{1}{2}\frac{s}{s^2+4} \rightarrow f(t) = \frac{1}{8}e^{-2t} + \frac{3}{8}e^{2t} + \frac{1}{4}\operatorname{sen} 2t + \frac{1}{2}\cos 2t$$

$$\boxed{f(t) = \frac{1}{8}e^{-2t} + \frac{3}{8}e^{2t} + \frac{1}{4}\operatorname{sen} 2t + \frac{1}{2}\cos 2t}$$

#15 | Pag 316 | Ej. 45

45. $f(t) + \int_0^t f(\tau)\,d\tau = 1$

$$s\mathcal{L}\{y\} - y(0) = \mathcal{L}\{1\} - \mathcal{L}\{\operatorname{sen} t\} - \mathcal{L}\{1\}\mathcal{L}\{y\} \rightarrow \mathcal{L}\{y\} = \frac{s^2+s+1}{(s^2-1)^2} = \frac{1}{s^2+1} - \frac{1}{2}\frac{2s}{(s^2+1)^2} \rightarrow \boxed{y = sent - \frac{1}{2}sent}$$

#16 | Pag 316 | Ej. 47

47. $f(t) = 1 + t - \frac{8}{3}\int_0^t (\tau - t)^3 f(\tau)\,d\tau$

$$F(s) = \frac{1}{s} + \frac{1}{s^2}\cdot\frac{16F(s)}{s^4}$$

$$f(t) = 1 + t - \frac{8}{3}\int_0^t (\tau - t)^3 f(\tau)\,d\tau - (t-\tau) \rightarrow f(t) = 1 + \frac{8}{3}(t^3 f(t)) \rightarrow F(s) = \frac{1}{s} + \frac{1}{s^2} + \frac{8}{3}\cdot 6\,\frac{F(s)}{s^4}$$

$$F(s)\left(\frac{s^4-16}{s^4}\right) = \frac{1}{s} + \frac{1}{s^2} \qquad F(s) = \frac{s^3}{(s^4-16)} + \frac{s^2}{(s^4-16)} \rightarrow F(s) = \frac{s^3+s^2}{(s^4-16)} = \frac{s^2(s+1)}{(s^4-16)}$$ F.P en CAS

$$F(s) = \frac{3}{8(s-2)} + \frac{1}{8(s+2)} + \frac{1}{2}\frac{s}{(s^2+4)} - \frac{1}{2(s^2+4)} \rightarrow f(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{2t} + \frac{1}{2}\cos 2t + \frac{1}{2}\operatorname{sen} 2t$$

$$\boxed{f(t) = \frac{3}{8}e^{2t} + \frac{1}{8}e^{-2t} + \frac{1}{2}cos2t\,\frac{1}{2}\operatorname{sen}2t}$$

#17 | Pag 316 | Ej. 49

49. $y'(t) = 1 - \operatorname{sen} t - \int_0^t y(\tau)\,d\tau, \quad y(0) = 0$

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-2as}}\left[\int_0^a e^{-st}dt - \int_a^{2a} e^{-st}dt\right] = \frac{(1-e^{-as})^2}{s(1-e^{-2as})} = \frac{1-e^{-as}}{s(1+e^{-as})} \rightarrow \boxed{\frac{1-e^{-as}}{s(1+c^{-as})}}$$

#18 | Pag 316 | Ej. 51

En los problemas 51 y 52, resuelva la ecuación (10) sujeta a $i(0) = 0$ con L, R, C y $E(t)$ como se dan para cada problema. Use un programa de graficación para trazar la solución en el intervalo $0 \le t < 3$.

51. $L = 0.1$ H, $R = 3\ \Omega$, $C = 0.05$ F, $E(t) = 100[\mathcal{U}(t-1) - \mathcal{U}(t-2)]$

Integración por partes

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-bs}}\int_0^b \frac{a}{b}te^{-st}dt = \boxed{\frac{a}{s}\left(\frac{1}{bs} - \frac{1}{e^{bs}-1}\right)}$$

#19 | Pag 316 | Ej. 53

En los problemas 53-58 use el teorema 7.4.3 para determinar la transformada de Laplace de cada una de las funciones periódicas.

53.

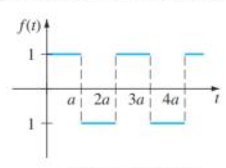

FIGURA 7.4.6 Gráfica para el problema 53.

$$\mathcal{L}\{f(t)\} = \frac{1}{1-e^{-\pi s}}\int_0^\pi e^{-st}\operatorname{sen} t\,dt = \frac{1}{s^2+1}\cdot\frac{e^{\pi s/2}+e^{-\pi s/2}}{e^{\pi s/2}-e^{-\pi s/2}} = \boxed{\frac{1}{s^2+1}\coth\frac{\pi s}{2}}$$

55.

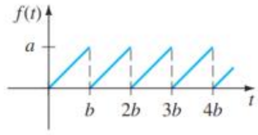

FIGURA 7.4.8 Gráfica para el problema 55.

#20	Pag 316	Ej. 55

$L\frac{di}{dt}+Ri=E(t)$; $i(0)=0$; $Ls\mathcal{L}\{i\}+R\mathcal{L}\{i\}=\mathcal{L}\{E(t)\}$

$$\rightarrow \mathcal{L}\{E(t)\}=\frac{(1-e^{-s})}{s(1+e^{-s})}$$

$$(Ls+R)\mathcal{L}\{i\}=\frac{1-e^{-s}}{s(1+e^{-s})} \rightarrow \mathcal{L}\{i\}=\frac{1}{L}\,\frac{1-e^{-s}}{s(s+\frac{R}{L})(1+e^{-s})}=\frac{1}{L}\,\frac{1-e^{-s}}{s(s+\frac{R}{L})}\,\frac{1}{1+e^{-s}}$$

$$\frac{1}{R}\left(\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right)(1-e^{-s})(1-e^{-s}+e^{-2s}-e^{-3s}+e^{-4s}-\dots)\rightarrow \frac{1}{R}\left(\frac{1}{s}-\frac{1}{s+\frac{R}{L}}\right)(1-2e^{-s}+2e^{-2s}-2e^{-3s}+2e^{-4s}\dots).$$

$$i(t)=\frac{1}{R}(1-e^{-Rt/L})-\frac{2}{R}(1-e^{-R(t-1)/L})\mathcal{U}(t-1)+\frac{2}{R}(1-e^{-R(t-2)/L})\mathcal{U}(t-2)-\frac{2}{R}(1-e^{-R(t-3)/L})\mathcal{U}(t-3)+\dots$$

$$=\frac{1}{R}(1-e^{-Rt/L})+\frac{2}{R}\sum_{n=1}(-1)^n(1-e^{-R(t-n)/L})\mathcal{U}(t-n).$$

$$\rightarrow \boxed{\frac{1}{R}\left(1-e^{\frac{-Rt}{L}}\right)+\frac{2}{R}\sum_{n=1}^{\infty}(-1)^n\left(1-e^{-\frac{-R(t-n)}{L}}\right)\mathfrak{U}(t-n)}$$

57.

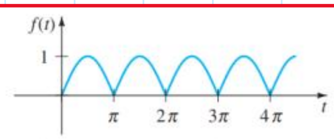

FIGURA 7.4.10 Gráfica para el problema 57.

#21	P.317	Ej. 57

$x''+2x'+10x=20f(t)$; $f(t)\rightarrow a=\pi$; $x(0)=x'(0)=0$

$$(s^2+2s+10)\mathcal{L}\{x(t)\}=\frac{20}{s}(1-e^{-\pi s})\,\frac{1}{1+e^{-\pi s}}$$

$$=\frac{20}{s}(1-e^{-\pi s})(1-e^{-\pi s}+e^{-2\pi s}-e^{-3\pi s}+\dots)\rightarrow\frac{20}{s}(1-2e^{-\pi s}+2e^{-2\pi s}-2e^{-3\pi s}+\dots)\rightarrow \frac{20}{s}+\frac{40}{s}\sum_{n=1}^{\infty}(-1)^n e^{-n\pi s}$$

$$\mathcal{L}\{x(t)\}=\frac{20}{s(s^2+2s+10)}+\frac{40}{s(s^2+2s+10)}\sum_{n=1}^{\infty}(-1)^n e^{-n}\rightarrow \frac{2}{s}-\frac{2s+4}{s^2+2s+10}+\sum_{n=1}^{\infty}(-1)^n\left[\frac{4}{s}-\frac{4s+8}{s^2+2s+10}\right]e^{-n\pi s}$$

$$=\frac{2}{s}\;\frac{2(s+1)+2}{(s+1)^2+9}+4\sum_{n=1}^{\infty}(-1)^n\left[\frac{1}{s}-\frac{(s+1)+1}{(s+1)^2+9}\right]e^{-\pi s}$$

$$\boxed{x(t)=2\left(1-e^{-t}\cos 3t-\frac{1}{3}e^{-t}\,\text{sen}\,3t\right)+4\sum_{n=1}^{\infty}(-1)^n\left[1-e^{(-t-n\pi)}\cos 3(t-n\pi)-\frac{1}{3}e^{-(t-n\pi)}\,\text{sen}3(t-n\pi)\right]\mathfrak{U}(t-n\pi)}$$