Tabla de Identidades Trigonométricas

Funciones Trigonométricas

$$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}, \quad \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Funciones Trigonométricas en función de las Otras Cinco

	$\boldsymbol{\sin x}$	$\boldsymbol{\cos x}$	$\boldsymbol{\tan x}$
$\sin x =$	$\sin x$	$\pm\sqrt{1-\cos^2 x}$	$\pm\frac{\tan x}{\sqrt{1+\tan^2 x}}$
$\cos x =$	$\pm\sqrt{1-\sin^2 x}$	$\cos x$	$\pm\frac{1}{\sqrt{1+\tan^2 x}}$
$\tan x =$	$\pm\frac{\sin x}{\sqrt{1-\sin^2 x}}$	$\pm\frac{\sqrt{1-\cos^2 x}}{\cos x}$	$\tan x$
$\csc x =$	$\frac{1}{\sin x}$	$\pm\frac{1}{\sqrt{1-\cos^2 x}}$	$\pm\frac{\sqrt{1+\tan^2 x}}{\tan x}$
$\sec x =$	$\pm\frac{1}{\sqrt{1-\sin^2 x}}$	$\frac{1}{\cos x}$	$\pm\sqrt{1+\tan^2 x}$
$\cot x =$	$\pm\frac{\sqrt{1-\sin^2 x}}{\sin x}$	$\pm\frac{\cos x}{\sqrt{1-\cos^2 x}}$	$\frac{1}{\tan x}$
	$\boldsymbol{\csc x}$	$\boldsymbol{\sec x}$	$\boldsymbol{\cot x}$
$\sin x =$	$\frac{1}{\csc x}$	$\pm\frac{\sqrt{\sec^2 x-1}}{\sec x}$	$\pm\frac{1}{\sqrt{1+\cot^2 x}}$
$\cos x =$	$\pm\frac{\sqrt{\csc^2 x-1}}{\csc x}$	$\frac{1}{\sec x}$	$\pm\frac{\cot x}{\sqrt{1+\cot^2 x}}$
$\tan x =$	$\pm\frac{1}{\sqrt{\csc^2 x-1}}$	$\pm\sqrt{\sec^2 x-1}$	$\frac{1}{\cot x}$
$\csc x =$	$\csc x$	$\pm\frac{\sec x}{\sqrt{\sec^2 x-1}}$	$\pm\sqrt{1+\cot^2 x}$
$\sec x =$	$\pm\frac{\csc x}{\sqrt{\csc^2 x-1}}$	$\sec x$	$\pm\frac{\sqrt{1+\cot^2 x}}{\cot x}$
$\cot x =$	$\pm\sqrt{\csc^2 x-1}$	$\pm\frac{1}{\sqrt{\sec^2 x-1}}$	$\cot x$

Algunos Valores Especiales

Función	$0(0^\circ)$	$\frac{\pi}{12}(15^\circ)$	$\frac{\pi}{6}(30^\circ)$	$\frac{\pi}{4}(45^\circ)$	$\frac{\pi}{3}(60^\circ)$	$\frac{5\pi}{12}(75^\circ)$	$\frac{\pi}{2}(90^\circ)$
sin	0	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	1
cos	1	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	0
tan	0	$2-\sqrt{3}$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$2+\sqrt{3}$	$\nexists(\pm\infty)$
csc	$\nexists(\pm\infty)$	$\sqrt{6}+\sqrt{2}$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{6}-\sqrt{2}$	1
sec	1	$\sqrt{6}-\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\sqrt{6}+\sqrt{2}$	$\nexists(\pm\infty)$
cot	$\nexists(\pm\infty)$	$2+\sqrt{3}$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	$2-\sqrt{3}$	0

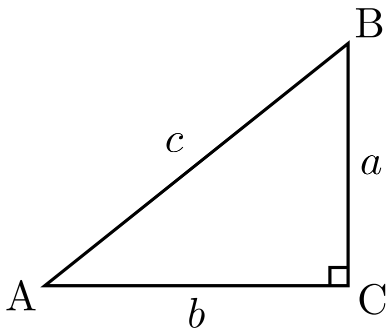
Identidades por Simetría, Periodicidad o Desplazamiento

$-x$ *o* $360^\circ - x$	$90^\circ - x$	$180^\circ - x$
$\sin(-x) = -\sin x$	$\sin(\frac{\pi}{2}-x) = +\cos x$	$\sin(\pi - x) = +\sin x$
$\cos(-x) = +\cos x$	$\cos(\frac{\pi}{2}-x) = +\sin x$	$\cos(\pi - x) = -\cos x$
$\tan(-x) = -\tan x$	$\tan(\frac{\pi}{2}-x) = +\cot x$	$\tan(\pi - x) = -\tan x$
$\csc(-x) = -\csc x$	$\csc(\frac{\pi}{2}-x) = +\sec x$	$\csc(\pi - x) = +\csc x$
$\sec(-x) = +\sec x$	$\sec(\frac{\pi}{2}-x) = +\csc x$	$\sec(\pi - x) = -\sec x$
$\cot(-x) = -\cot x$	$\cot(\frac{\pi}{2}-x) = +\tan x$	$\cot(\pi - x) = -\cot x$
$x + 90^\circ$	$x + 180^\circ$	$x + 360^\circ$
$\sin(x+\frac{\pi}{2}) = +\cos x$	$\sin(x+\pi) = -\sin x$	$\sin(x+2\pi) = +\sin x$
$\cos(x+\frac{\pi}{2}) = -\sin x$	$\cos(x+\pi) = -\cos x$	$\cos(x+2\pi) = +\cos x$
$\tan(x+\frac{\pi}{2}) = -\cot x$	$\tan(x+\pi) = +\tan x$	$\tan(x+2\pi) = +\tan x$
$\csc(x+\frac{\pi}{2}) = +\sec x$	$\csc(x+\pi) = -\csc x$	$\csc(x+2\pi) = +\csc x$
$\sec(x+\frac{\pi}{2}) = -\csc x$	$\sec(x+\pi) = -\sec x$	$\sec(x+2\pi) = +\sec x$
$\cot(x+\frac{\pi}{2}) = -\tan x$	$\cot(x+\pi) = +\cot x$	$\cot(x+2\pi) = +\cot x$

Cálculo de Funciones Trigonométricas

Función	Derivada	Integral
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x = 1 + \tan^2 x$	$-\ln\lvert\cos x\rvert + C$
$\csc x$	$-\csc x \cot x$	$-\ln\lvert\csc x + \cot x\rvert + C$
$\sec x$	$\sec x \tan x$	$\ln\lvert\sec x + \tan x\rvert + C$
$\cot x$	$-\csc^2 x = -(1+\cot^2 x)$	$\ln\lvert\sin x\rvert + C$

Ley de Senos



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Ley de Cosenos

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}$$

Ley de Tangentes

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

$$\frac{b-c}{b+c} = \frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$$

$$\frac{a-c}{a+c} = \frac{\tan\left(\frac{A-C}{2}\right)}{\tan\left(\frac{A+C}{2}\right)}$$

Identidades Pitagóricas

$$\begin{aligned} \cos^2 x + \sin^2 x &= 1 \\ \sec^2 x - \tan^2 x &= 1 \\ \csc^2 x - \cot^2 x &= 1 \end{aligned}$$

$$\sin x = \pm\sqrt{1-\cos^2 x}$$

$$\cos x = \pm\sqrt{1-\sin^2 x}$$

Suma y Diferencia de Ángulos

$$\begin{aligned} \sin(x \pm y) &= \sin x\cos y \pm \cos x\sin y \\ \cos(x \pm y) &= \cos x\cos y \mp \sin x\sin y \\ \tan(x \pm y) &= \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y} \\ \csc(x \pm y) &= \frac{1}{\sin(x \pm y)} \\ \sec(x \pm y) &= \frac{1}{\cos(x \pm y)} \\ \cot(x \pm y) &= \frac{\cot x\cot y \mp 1}{\cot y \pm \cot x} \end{aligned}$$

Producto a Suma

$$\begin{aligned} \cos x\cos y &= \frac{\cos(x-y)+\cos(x+y)}{2} \\ \sin x\sin y &= \frac{\cos(x-y)-\cos(x+y)}{2} \\ \sin x\cos y &= \frac{\sin(x+y)+\sin(x-y)}{2} \\ \cos x\sin y &= \frac{\sin(x+y)-\sin(x-y)}{2} \end{aligned}$$

Suma a Producto

$$\begin{aligned} \sin x \pm \sin y &= 2\sin\left(\frac{x \pm y}{2}\right)\cos\left(\frac{x \mp y}{2}\right) \\ \cos x + \cos y &= 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) \\ \cos x - \cos y &= -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right) \\ \tan x \pm \tan y &= \frac{\sin(x \pm y)}{\cos x\cos y} \end{aligned}$$

Identidades de Ángulo Doble

$$\begin{aligned} \sin 2x &= \frac{2\tan x}{1+\tan^2 x} = 2\sin x\cos x \\ \cos 2x &= \frac{1-\tan^2 x}{1+\tan^2 x} = \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 = 1 - 2\sin^2 x \\ \tan 2x &= \frac{2\tan x}{1-\tan^2 x} \\ \cot 2x &= \frac{\cot^2 x - 1}{2\cot x} \end{aligned}$$

Identidades de Ángulo Triple

$$\begin{aligned}\sin 3x &= 3\cos^2 x \sin x - \sin^3 x\\ &= 3\sin x - 4\sin^3 x\\ \cos 3x &= \cos^3 x - 3\sin^2 x \cos x\\ &= 4\cos^3 x - 3\cos x\\ \tan 3x &= \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\\ \cot 3x &= \frac{3\cot x - \cot^3 x}{1 - 3\cot^2 x}\end{aligned}$$

Identidades de Ángulo Medio

$$\begin{aligned}\sin\tfrac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{2}}\\ \cos\tfrac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{2}}\\ \tan\tfrac{x}{2} &= \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \csc x - \cot x\\ &= \frac{\sin x}{1+\cos x}\\ \cot\tfrac{x}{2} &= \pm\sqrt{\frac{1+\cos x}{1-\cos x}} = \csc x + \cot x\\ &= \frac{\sin x}{1-\cos x} = \frac{1+\cos x}{\sin x}\\ \tan\left(\frac{x+y}{2}\right) &= \frac{\sin x + \sin y}{\cos x + \cos y} = -\frac{\cos x - \cos y}{\sin x - \sin y}\end{aligned}$$

Reducción de Exponentes

$$\begin{aligned}\sin^2 x &= \frac{1-\cos 2x}{2}\\ \sin^3 x &= \frac{3\sin x - \sin 3x}{4}\\ \sin^4 x &= \frac{3 - 4\cos 2x + \cos 4x}{8}\\ \sin^5 x &= \frac{10\sin x - 5\sin 3x + \sin 5x}{16}\end{aligned}$$

$$\begin{aligned}\cos^2 x &= \frac{1+\cos 2x}{2}\\ \cos^3 x &= \frac{3\cos x + \cos 3x}{4}\\ \cos^4 x &= \frac{3 + 4\cos 2x + \cos 4x}{8}\\ \cos^5 x &= \frac{10\cos x + 5\cos 3x + \cos 5x}{16}\end{aligned}$$

$$\begin{aligned}\sin^2 x\cos^2 x &= \frac{1-\cos 4x}{8}\\ \sin^3 x\cos^3 x &= \frac{3\sin 2x - \sin 6x}{32}\\ \sin^4 x\cos^4 x &= \frac{3 - 4\cos 4x + \cos 8x}{128}\\ \sin^5 x\cos^5 x &= \frac{10\sin 2x - 5\sin 6x + \sin 10x}{512}\end{aligned}$$

Cuadrados a Producto

$$\begin{aligned}\sin^2(x) - \sin^2(y) &= \sin(x+y)\sin(x-y)\\ \cos^2(x) - \sin^2(y) &= \cos(x+y)\cos(x-y)\end{aligned}$$

Composición de Funciones

$$\begin{aligned}\sin(\arccos x) &= \sqrt{1-x^2}\\ \tan(\arcsin x) &= \frac{x}{\sqrt{1-x^2}}\\ \sin(\arctan x) &= \frac{x}{\sqrt{1+x^2}}\\ \tan(\arccos x) &= \frac{\sqrt{1-x^2}}{x}\\ \cos(\arctan x) &= \frac{1}{\sqrt{1+x^2}}\\ \cot(\arcsin x) &= \frac{\sqrt{1-x^2}}{x}\\ \cos(\arcsin x) &= \sqrt{1-x^2}\\ \cot(\arccos x) &= \frac{x}{\sqrt{1-x^2}}\end{aligned}$$

Suma y Diferencia de Inversas

$$\begin{aligned}\arcsin x + \arccos x &= \frac{\pi}{2}\\ \arctan x + \operatorname{arccot} x &= \frac{\pi}{2}\\ \arctan x + \arctan\frac{1}{x} &= \begin{cases}\frac{\pi}{2}, & \text{si } x > 0\\ -\frac{\pi}{2}, & \text{si } x < 0\end{cases}\end{aligned}$$

$$\begin{aligned}&\arcsin x \pm \arcsin y\\ &\qquad = \arcsin(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})\\ &\arccos x \pm \arccos y\\ &\qquad = \arccos(xy \mp \sqrt{(1-x^2)(1-y^2)})\\ &\arctan x \pm \arctan y\\ &\qquad = \arctan\left(\frac{x \pm y}{1 \mp xy}\right)\end{aligned}$$

Fórmulas de Límites

Límites y límites laterales

$$\lim_{x\to c^+} f(x) = \lim_{x\to c^-} f(x) = L \iff \lim_{x\to c} f(x) = L$$

$$\lim_{x\to c^+} f(x) \neq \lim_{x\to c^-} f(x) \implies \lim_{x\to c} f(x) \text{ no existe}$$

Límites de funciones simples

$$\lim_{x\to c} a = a$$

$$\lim_{x\to c} x = c$$

$$\lim_{x\to c} ax + b = ac + b$$

$$\lim_{x\to c} x^r = c^r \qquad \text{si } r \text{ es entero positivo}$$

$$\lim_{x\to 0^+} \frac{1}{x^r} = +\infty$$

$$\lim_{x\to 0^-} \frac{1}{x^r} = \begin{cases} -\infty, & \text{si } r \text{ es impar} \\ +\infty, & \text{si } r \text{ es par} \end{cases}$$

Hechos sobre $\pm\infty$

Si $a \neq 0$ y $a < \infty$:

$$0 + \infty = \infty$$

$$a + \infty = \infty$$

$$\frac{a}{\infty} = 0$$

$$\frac{a}{0} = \begin{cases} \infty, & a > 0 \\ -\infty, & a < 0 \end{cases}$$

$$a \cdot \infty = \begin{cases} \infty, & a > 0 \\ -\infty, & a < 0 \end{cases}$$

Hecho sobre funciones

$$\lim_{x\to 0} \sin(x) = \sin(0) = 0$$

$$\lim_{x\to 0} \cos(x) = \cos(0) = 1$$

$$\lim_{x\to a} \sin(x) = \sin(a)$$

$$\lim_{x\to a} \cos(x) = \cos(a)$$

$$\lim_{x\to 0} e^x = e^0 = 1$$

$$\lim_{x\to a} \log_a(x) = \log_a(a) = 1$$

Si $a > 1$:

$$\lim_{x\to 0^+} \log_a x = \lim_{x\to 0^+} \ln x = \lim_{x\to 0^+} \log_{10} x = -\infty$$

$$\lim_{x\to \infty} \log_a x = \lim_{x\to \infty} \ln x = \lim_{x\to \infty} \log_{10} x = \infty$$

Si $a < 1$:

$$\lim_{x\to 0^+} \log_a x = \infty$$

$$\lim_{x\to \infty} \log_a x = -\infty$$

Formas Indeterminadas

$$\frac{0}{0},\ \frac{\infty}{\infty},\ 0 \times \infty,\ 1^\infty,\ \infty - \infty,\ 0^0 \text{ y } \infty^0$$

Formas no Indeterminadas

Si $\lim_{x\to c} \frac{f(x)}{g(x)}$ tiene la forma $\left[\frac{1}{0}\right]$ entonces

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \begin{cases} -\infty, \\ +\infty, \\ \text{no existe} \end{cases}$$

Si $\lim_{x\to c} f(x)^{g(x)}$ tiene la forma $[0^\infty]$ entonces

$$\lim_{x\to c} f(x)^{g(x)} = 0$$

Límites cerca de Infinito

$$\lim_{x\to\infty} a/x = 0, \qquad \text{para todo real } a$$

$$\lim_{x\to\infty} \sqrt[x]{x} = 1$$

$$\lim_{x\to\infty} \sqrt[a]{x} = \infty \qquad \text{para todo } a > 0$$

$$\lim_{x\to\infty} x/a = \begin{cases} \infty, & a > 0 \\ \text{no existe}, & a = 0 \\ -\infty, & a < 0 \end{cases}$$

$$\lim_{x\to\infty} x^a = \begin{cases} \infty, & a > 0 \\ 1, & a = 0 \\ 0, & a < 0 \end{cases}$$

$$\lim_{x\to\infty} a^x = \begin{cases} \infty, & a > 1 \\ 1, & a = 1 \\ 0, & 0 < a < 1 \end{cases}$$

$$\lim_{x\to\infty} a^{-x} = \begin{cases} 0, & a > 1 \\ 1, & a = 1 \\ \infty, & 0 < a < 1 \end{cases}$$

Límites de Polinomios

$$\lim_{x\to\infty}[a_n x^n + ... + a_1] = \lim_{x\to\infty} a_n x^n \qquad \text{máxima potencia}$$

$$\lim_{x\to\infty} \frac{mx^a}{nx^b} = \begin{cases} 0, & a < b \\ \frac{m}{n}, & a = b \\ \infty, & a > b \end{cases}$$

Límites de funciones generales

Si $\lim_{x\to c} f(x) = F$ y $\lim_{x\to c} g(x) = G$ entonces

$$\lim_{x\to c}[f(x) \pm g(x)] = F \pm G$$

$$\lim_{x\to c}[a \cdot f(x)] = a \cdot F$$

$$\lim_{x\to c}[f(x)g(x)] = F \cdot G$$

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{F}{G} \qquad \text{si } G \neq 0$$

$$\lim_{x\to c} f(x)^n = F^n \qquad \text{si } n \text{ es entero positivo}$$

$$\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{F} \qquad \text{si } n \text{ es entero positivo,}$$

y si n es par, entonces $F > 0$

Composición de funciones

Si $f(x)$ es continua $\lim_{x\to c} g(x) = G$ entonces

$$\lim_{x\to c} f(g(x)) = f\left(\lim_{x\to c} g(x)\right) = f(G)$$

Límites y Derivadas

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

$$\lim_{h\to 0} \sqrt[n]{\frac{f(x+h)}{f(x)}} = \exp\left(\frac{f'(x)}{f(x)}\right)$$

$$\lim_{h\to 0} \sqrt[n]{\frac{f(x+h\cdot x))}{f(x)}} = \exp\left(\frac{xf'(x)}{f(x)}\right)$$

Regla de L'Hopital

si $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = 0$ o

si $\lim_{x\to c} f(x) = \lim_{x\to c} g(x) = \pm\infty$ entonces

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

Aplicaciones de L'Hopital

$$\lim_{x\to c} f(x)^{g(x)} = \lim_{x\to c} \exp[g(x) \cdot \ln(f(x))] =$$

$$\lim_{x\to c} \exp\left(\frac{\ln(f(x))}{1/g(x)}\right) = \exp\left(\lim_{x\to c} \frac{\ln(f(x))}{1/g(x)}\right)$$

luego aplicar L'Hopital

Transformaciones de otras formas indeterminadas a $\left[\frac{0}{0}\right]$, para aplicar L'Hopital

∞/∞	$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{1/g(x)}{1/f(x)}$
$0 \cdot \infty$	$\lim_{x\to c} f(x)g(x) = \lim_{x\to c} \frac{f(x)}{1/g(x)}$
$\infty - \infty$	$\lim_{x\to c}(f(x) - g(x)) = \lim_{x\to c} \frac{1/g(x) - 1/f(x)}{1/(f(x)g(x))}$
0^0	$\lim_{x\to c} f(x)^{g(x)} = \exp\left(\lim_{x\to c} \frac{g(x)}{1/\ln f(x)}\right)$
1^∞	$\lim_{x\to c} f(x)^{g(x)} = \exp\left(\lim_{x\to c} \frac{\ln f(x)}{1/g(x)}\right)$
∞^0	$\lim_{x\to c} f(x)^{g(x)} = \exp\left(\lim_{x\to c} \frac{g(x)}{1/\ln f(x)}\right)$

Teorema de Sandwich

Si $f(x) \leq g(x) \leq h(x)$ para todo x en un intervalo abierto que contiene a, excepto posiblmemente en a, y

$$\lim_{x\to c} f(X) = \lim_{x\to c} h(x) = L, \qquad \text{entonces}$$

$$\lim_{x\to c} g(X) = L$$

Infinitésimos Equivalente

Estas funciones de la forma $\lim_{x\to c} f(x) = 0$ son infinitésimos equivalentes cuando $x \to c$. Si $\lim_{x\to c} \frac{f(x)}{g(x)}$ tiene la forma $\left[\frac{0}{0}\right]$ entonces son intercambiables:

$$x \sim \sin(x)$$

$$x \sim \arcsin(x)$$

$$x \sim \sinh(x)$$

$$x \sim \tan(x)$$

$$x \sim \arctan(x)$$

$$x \sim \ln(1+x)$$

$$1 - \cos x \sim \frac{x^2}{2}$$

$$\cosh(x) - 1 \sim \frac{x^2}{2}$$

$$a^x - 1 \sim x\ln(a)$$

$$e^x - 1 \sim x$$

$$(1+x)^a - 1 \sim ax$$

$$\log_a(1+x) \sim \frac{x}{\ln a}$$

Funciones Trigonométricas

$$\lim_{x\to 0}\frac{\sin(x)}{x}=1$$

$$\lim_{x\to 0}\frac{\sin(ax)}{ax}=1 \qquad \text{para } a\neq 0$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x}=0$$

$$\lim_{x\to 0}\frac{1-\cos(x)}{x^2}=\frac{1}{2}$$

$$\lim_{x\to n^{\pm}}\tan\left(\pi x+\frac{\pi}{2}\right)=\mp\infty \qquad \text{para todo entero } n$$

$$\lim_{x\to 0}\frac{\sin(ax)}{x}=a$$

$$\lim_{x\to 0}\frac{\sin(ax)}{bx}=\frac{a}{b} \qquad \text{para } b\neq 0$$

Límites Especiales Notables

$$\lim_{x\to 0^+}x^x=1$$

$$\lim_{x\to 0}(1+x)^{1/x}=e$$

$$\lim_{x\to +\infty}\left(1+\frac{1}{x}\right)^x=e$$

$$\lim_{n\to\infty}\frac{n}{\sqrt[n]{n!}}=e$$

$$\lim_{x\to +\infty}\left(1-\frac{1}{x}\right)^x=\frac{1}{e}$$

$$\lim_{x\to +\infty}\left(1+\frac{k}{x}\right)^{mx}=e^{mk}$$

$$\lim_{x\to +\infty}\left(\frac{x}{x+k}\right)^x=\frac{1}{e^k}$$

$$\lim_{x\to 0}\frac{a^x-1}{x}=\ln a=log_e(a)$$

$$\lim_{x\to 0}\frac{c^{ax}-1}{bx}=\frac{a}{b}\ln c$$

$$\lim_{x\to 0}\frac{\sin(x)}{x}=1$$

$$\lim_{x\to 0}\frac{\tan(x)}{x}=1$$

$$\lim_{x\to 0}\frac{\cos(x)-1}{x}=0$$

$$\lim_{x\to 0}\frac{(1+x)^n-1}{x}=n$$

$$\lim_{x\to 0}\frac{x^n-a^n}{x-a}=0$$

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

$$\lim_{x\to 0}\frac{e^{ax}-1}{bx}=\frac{a}{b}$$

$$\lim_{x\to 0}\left(1+a\left(e^{-x}-1\right)\right)^{-1/x}=e^a$$

Logaritmos y exponentes

$$\lim_{x\to\infty}xe^{-x}=0$$

$$\lim_{x\to 1}\frac{\ln(x)}{x-1}=1$$

$$\lim_{x\to 0}\frac{\ln(x+1)}{x}=1$$

$$\lim_{x\to 0}\frac{\ln(1+ax)}{bx}=\frac{a}{b}$$

$$\lim_{x\to 0}\frac{\log_c(1+ax)}{bx}=\frac{a}{b\ln c}$$

$$\lim_{x\to 0}\frac{-\ln\left(1+a\cdot\left(e^{-x}-1\right)\right)}{x}=a$$

Ejemplos de Técnicas

Factorar y Cancelar

$$\begin{aligned}\lim_{x\to 2}\frac{x^2+4x-12}{x^2-2x}&=\lim_{x\to 2}\frac{(x-2)(x+6)}{x(x-2)}\\&=\lim_{x\to 2}\frac{x+6}{x}=\frac{8}{2}=4\end{aligned}$$

Racionalizar numerador/denominador

$$\begin{aligned}\lim_{x\to 9}\frac{3-\sqrt{x}}{x^2-81}&=\lim_{x\to 9}\frac{3-\sqrt{x}}{x^2-81}\cdot\frac{3+\sqrt{x}}{3+\sqrt{x}}\\&=\lim_{x\to 9}\frac{9-x}{(x^2-81)\left(3+\sqrt{x}\right)}\\&=\lim_{x\to 9}\frac{-1}{(x+9)(3+\sqrt{x})}\\&=\frac{-1}{(18)(6)}=-\frac{1}{108}\end{aligned}$$

Combinar expresiones racionales

$$\begin{aligned}\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{x+h}-\frac{1}{x}\right)&=\lim_{h\to 0}\frac{1}{h}\left(\frac{x-(x+h)}{x(x+h)}\right)\\&=\lim_{h\to 0}\frac{1}{h}\left(\frac{-h}{x(x+h)}\right)\\&=\lim_{h\to 0}\frac{-1}{x(x+h)}=-\frac{1}{x^2}\end{aligned}$$

Polinomios al Infinito

$$\begin{aligned}\lim_{x\to\infty}\frac{3x^2-4}{5x-2x^2}&=\lim_{x\to\infty}\frac{x^2\left(3-\frac{4}{x^2}\right)}{x^2\left(\frac{5}{x}-2\right)}\\&=\lim_{x\to\infty}\frac{3-\frac{4}{x^2}}{\frac{5}{x}-2}=-\frac{3}{2}\end{aligned}$$

Equivalencia de Infinitésimos

$$\lim_{x\to0}\frac{1}{x^3}\left[\left(\frac{2+\cos x}{3}\right)^x-1\right]$$
$$=\lim_{x\to0}\frac{1}{x^3}\left[\exp\left(x\ln\frac{2+\cos x}{3}\right)-1\right] \qquad \leftarrow y^x=\exp(x\ln y)$$
$$=\lim_{x\to0}\frac{1}{x^3}\left[x\ln\frac{2+\cos x}{3}\right] \qquad \leftarrow e^x-1\sim x$$
$$=\lim_{x\to0}\frac{1}{x^2}\ln\left(\frac{(3-1)+\cos x}{3}\right)$$
$$=\lim_{x\to0}\frac{1}{x^2}\ln\left(\left(\frac{3}{3}\right)+\frac{-1+\cos x}{3}\right)$$
$$=\lim_{x\to0}\frac{1}{x^2}\ln\left(1+\frac{\cos(x)-1}{3}\right)$$
$$=\lim_{x\to0}\frac{\cos(x)-1}{3x^2} \qquad \leftarrow x\sim\ln(1+x)$$
$$=\lim_{x\to0}\frac{-(1-\cos(x))}{3x^2}$$
$$=\lim_{x\to0}\frac{-x^2/2}{3x^2} \qquad \leftarrow 1-\cos x\sim\frac{x^2}{2}$$
$$=-\frac{1}{6}$$

Equivalencia de Infinitésimos

$$\lim_{x\to1}\frac{\sin(x-1)}{x^4-1}$$
$$=\lim_{t\to0}\frac{\sin t}{(t+1)^4-1} \qquad \leftarrow t=x-1,\ x\to1\Rightarrow t\to0$$
$$=\lim_{t\to0}\frac{\sin t}{(t^4+4t^3+6t^2+4t+1)-1}$$
$$=\lim_{t\to0}\frac{\sin t}{t^4+4t^3+6t^2+4t}$$
$$=\lim_{t\to0}\frac{t}{t\,(t^3+4t^2+6t+4)} \qquad \leftarrow \sin t\sim t$$
$$=\lim_{t\to0}\frac{1}{(t^3+4t^2+6t+4)}=\frac{1}{4}$$

Equivalencia de Infinitésimos

$$\lim_{x\to e}\frac{\ln(\ln x)}{x-e}=\lim_{x\to e}\frac{\ln(\ln x+1-1)}{x-e}$$
$$=\lim_{x\to e}\frac{\ln[1+(\ln x-1)]}{x-e}$$
$$=\lim_{x\to e}\frac{\ln x-1}{x-e} \qquad \leftarrow ln(1+x)\sim x$$
$$=\lim_{x\to e}\frac{\ln x-\ln e}{x-e} \qquad \leftarrow 1=\ln(e)$$
$$=\lim_{x\to e}\frac{\ln\left(\frac{x}{e}\right)}{x-e}=\lim_{x\to e}\frac{\ln\left[1+\left(\frac{x}{e}-1\right)\right]}{x-e}$$
$$=\lim_{x\to e}\frac{\frac{x}{e}-1}{x-e} \qquad \leftarrow ln(1+x)\sim x$$
$$=\lim_{x\to e}\frac{\frac{x-e}{e}}{x-e}=\lim_{x\to e}\left(\frac{1}{e}\right)\frac{x-e}{x-e}=\frac{1}{e}$$

1. Fórmulas de Derivadas

1.1. Formas básicas y propiedades de las derivadas

1. $\frac{d}{dx}c = 0$

2. $\frac{d}{dx}x = 1$

3. $\frac{d}{dx}(u + v - w) = \frac{d}{dx}u + \frac{d}{dx}v - \frac{d}{dx}w$

4. $\frac{d}{dx}(u \cdot v) = u' \cdot v + v' \cdot u$

5. $\frac{d}{dx}[c \cdot u] = c \cdot \frac{d}{dx}u$

6. $\frac{d}{dx}u^n = n \cdot u^{n-1} \cdot \frac{d}{dx}u$

7. $\frac{d}{dx}\sqrt{u} = \frac{\frac{d}{dx}u}{2 \cdot \sqrt{u}}$

8. $\frac{d}{dx}\frac{u}{v} = \frac{u' \cdot v - v' \cdot u}{v^2}$

9. $\frac{d}{dx}\frac{u}{c} = \frac{1}{c} \cdot \frac{d}{dx}u$

10. $\frac{d}{dx}\frac{c}{u} = \frac{-c \cdot \frac{d}{dx}u}{u^2}$

1.2. Fórmulas de derivadas trigonométricas

11. $\frac{d}{dx}\sin u = \cos u \cdot \frac{d}{dx}u$

12. $\frac{d}{dx}\cos u = -\sin u \cdot \frac{d}{dx}u$

13. $\frac{d}{dx}\tan u = \sec^2 u \cdot \frac{d}{dx}u$

14. $\frac{d}{dx}\cot u = -\csc^2 u \cdot \frac{d}{dx}u$

15. $\frac{d}{dx}\sec u = \sec u \cdot \tan u \cdot \frac{d}{dx}u$

16. $\frac{d}{dx}\csc u = -\csc u \cdot \cot u \cdot \frac{d}{dx}u$

1.3. Fórmulas de derivadas trigonométricas inversas

17. $\frac{d}{dx}\sin^{-1} u = \frac{\frac{d}{dx}u}{\sqrt{1 - u^2}}$

$$\left[-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right]$$

18. $\frac{d}{dx}\cos^{-1} u = -\frac{\frac{d}{dx}u}{\sqrt{1 - u^2}}$

$$\left[0 < \cos^{-1} u < \pi\right]$$

19. $\frac{d}{dx}\tan^{-1} u = \frac{\frac{d}{dx}u}{1 + u^2}$

$$\left[-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right]$$

20. $\frac{d}{dx}\cot^{-1} u = -\frac{\frac{d}{dx}u}{1+u^2}$
$[0 < \cot^{-1} u < \pi]$

21. $\frac{d}{dx}\sec^{-1} u = \frac{\frac{d}{dx}u}{|u| \cdot \sqrt{u^2-1}}$

22. $\frac{d}{dx}\csc^{-1} u = -\frac{\frac{d}{dx}u}{|u| \cdot \sqrt{u^2-1}}$

1.4. Fórmulas de derivadas exponenciales y logarítmicas

23. $\frac{d}{dx}\ln u = \frac{\frac{d}{dx}u}{u} = \frac{d}{dx}\log_e u$

24. $\frac{d}{dx}e^u = e^u \cdot \frac{d}{dx}u$

25. $\frac{d}{dx}\log_a u = \frac{\frac{d}{dx}u}{u \cdot \ln a} = \frac{\log_a e}{u} \cdot \frac{d}{dx}u$

26. $\frac{d}{dx}u^v = \frac{d}{dx}e^{v \cdot \ln u} = e^{v \cdot \ln u}\frac{d}{dx}[v \cdot \ln u] = v \cdot u^{v-1}\frac{du}{dx} +$
$u^v \cdot \ln u \frac{dv}{dx}$

1.5. Fórmulas de derivadas hiperbólicas

27. $\frac{d}{dx}\sinh u = \cosh u \cdot \frac{d}{dx}u$

28. $\frac{d}{dx}\cosh u = \sinh u \cdot \frac{d}{dx}u$

29. $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \cdot \frac{d}{dx}u$

30. $\frac{d}{dx}\coth u = -\operatorname{csch}^2 u \cdot \frac{d}{dx}u$

31. $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \cdot \tanh u \cdot \frac{d}{dx}u$

32. $\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \cdot \coth u \cdot \frac{d}{dx}u$

1.6. Fórmulas de derivadas hiperbólicas inversas

33. $\frac{d}{dx}\sinh^{-1} u = \frac{1}{\sqrt{u^2+1}} \cdot \frac{d}{dx}u$

34. $\frac{d}{dx}\cosh^{-1} u = \frac{1}{\sqrt{u^2-1}} \cdot \frac{d}{dx}u$

35. $\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2} \cdot \frac{d}{dx}u$

36. $\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2} \cdot \frac{d}{dx}u$

37. $\frac{d}{dx}\operatorname{sech}^{-1} u = \frac{-1}{u \cdot \sqrt{1-u^2}} \cdot \frac{d}{dx}u$

38. $\frac{d}{dx}\operatorname{csch}^{-1} u = \frac{-1}{|u| \cdot \sqrt{1+u^2}} \cdot \frac{d}{dx}u$

1.7. Representacin de las derivadas de orden superior

39. *Segunda derivada*
$\frac{d^2y}{dx^2} = f''(x) = y'' = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

40. Tercera derivada
$\frac{d^3y}{dx^3} = f'''(x) = y''' = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)$

41. *N-ésima derivada*
$\frac{d^ny}{dx^n} = f^n(x) = y^n = \frac{d}{dx}\left(\frac{d^{n-1}y}{dx^{n-1}}\right)$