Regla de la cadena Si g es derivable en x y f es derivable en $g(x)$, entonces la función compuesta $F = f \circ g$ definida mediante $F(x) = f(g(x))$ es derivable en x, y F' está dada por el producto

$$F'(x) = f'(g(x)) \cdot g'(x)$$

En la notación de Leibniz, si $y = f(u)$ y $u = g(x)$ son funciones derivables, entonces

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$$

Obtenga la derivada

$$y = \cos\left(\sqrt{\operatorname{sen}(\tan \pi x)}\right)$$

$f = \cos$

$g = \sqrt{\quad}$

$$y' = \underbrace{-\operatorname{SEN}\left(\sqrt{\operatorname{SEN}(\tan \pi x)}\right)}_{f'} \cdot \underbrace{\left(\sqrt{\operatorname{SEN}(\tan \pi x)}\right)'}_{g'}$$

$f = \sqrt{\quad}$

$g = \operatorname{SEN}$

$$y' = -\operatorname{SEN}\left(\sqrt{\operatorname{SEN}(\tan \pi x)}\right) \cdot \frac{1}{2\sqrt{\operatorname{SEN}(\tan \pi x)}} \cdot \left(\operatorname{SEN}(\tan \pi x)\right)'$$

$f = \operatorname{SEN}$

$g = \tan$

$$y' = \frac{-\operatorname{SEN}\left(\sqrt{\operatorname{SEN}(\tan \pi x)}\right)}{2\sqrt{\operatorname{SEN}(\tan \pi x)}} \cdot \cos(\tan \pi x) \cdot \underbrace{(\tan \pi x)'}$$

$f = \tan$

$g = \pi x$

$$y' = \frac{-\operatorname{SEN}\left(\sqrt{\operatorname{SEN}(\tan \pi x)}\right)}{2\sqrt{\operatorname{SEN}(\tan \pi x)}} \cos(\tan \pi x) \cdot \sec^2(\pi x) \cdot \pi$$

Se da una tabla de valores de f, g, f' y g'.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

(a) Si $h(x) = f(g(x))$, encuentre $h'(1)$.

(b) Si $H(x) = g(f(x))$, determine $H'(1)$.

a) $h(x) = f(g(x))$, $h'(1) = ?$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(1) = f'(g(1)) \cdot g'(1) = \underbrace{f'(2)} \cdot 6 = 5 \cdot 6 = 30$

b) $H(x) = g(f(x))$, $H'(1) = ?$

$H'(x) = g'(f(x)) \cdot f'(x)$

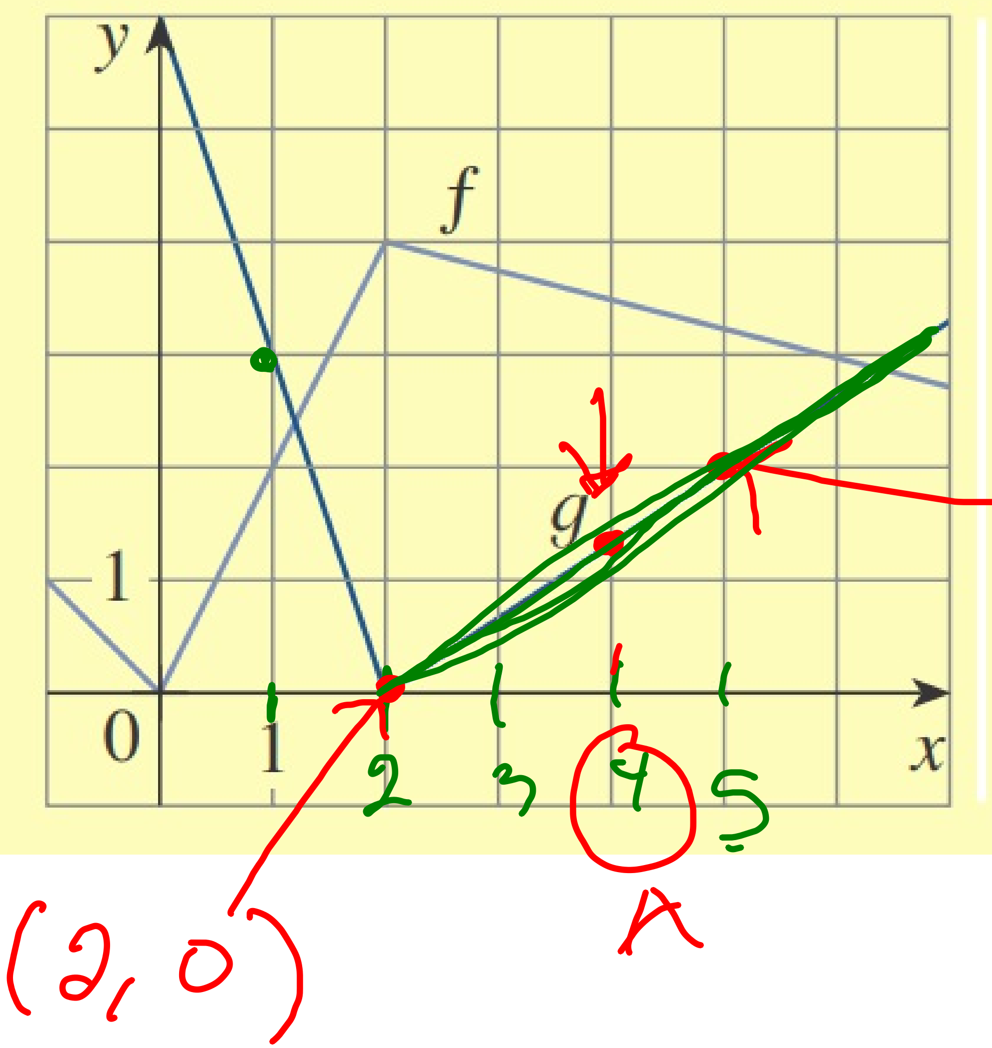
$H'(1) = g'(\underbrace{f(1)}_{3}) \cdot f'(1) = g'(3) \cdot \underbrace{f'(1)} = 9 \cdot 4 = 36$

Sean f y g las funciones cuyas gráficas se muestran; sea $u(x) = f(g(x))$, $v(x) = g(f(x))$ y $w(x) = g(g(x))$. Encuentre, si existe, cada derivada. Si no existe, explique por qué.

(a) $u'(1)$ (b) $v'(1)$ (c) $w'(1)$

EJEMPLO

$$g'(4) = \frac{2}{3}$$



$$m = \frac{2-0}{5-2} = \frac{2}{3}$$

Si $F(x) = f(3f(4f(x)))$, donde $f(0) = 0$ y $f'(0) = 2$, encuentre $F'(0)$.

$$F'(0) = ?$$

$$F(x) = f(\underbrace{3f(4f(x))}_{g})$$

$$F'(x) = f'(3f(4f(x)))(\underbrace{3f(4f(x))}_{g(x)})'$$

$$F'(x) = f'(3f(4f(x)))\,3\,(f'(4f(x))\cdot(4f(x))')$$

$$F'(x) = f'(3f(4f(x)))\,3\,f'(4f(x))\cdot 4f'(x)$$

$$F'(x) = 12\,f'(3f(4f(x)))\,f'(4f(x))\,f'(x)$$

$$F'(0) = 12\,f'(3f(4f(0)))\,f'(4f(0))\,f'(0)$$

$$F'(0) = 12\,f'(3\overset{0}{f(0)})\,f'(0)\cdot 2$$

$$F'(0) = 12\,f'(0)\cdot 2\cdot 2$$

$$F'(0) = 12\cdot 2\cdot 2\cdot 2 = 96$$

Derivación Implícita

$y = f(x)$

$y = \sqrt{x}$

$y = \cos x$

$H = \operatorname{sen} t$

$\rightarrow \frac{d}{dt} H = H'$

$y = f(x)$ EXPLÍCITA

$f(x, y) = 0$ IMPLÍCITA

Encuentre dy/dx por derivación implícita.

$$x^3 - xy^2 + y^3 = 1 \qquad \frac{dy}{dx} = y'$$

$$(x^3)' - \left(y^2 \cdot x' + x\,(y^2)'\right) + (y^3)' = (1)'$$

$$3x^2 - \left(y^2 \cdot 1 + x\,(2y\,y')\right) + 3y^2 \cdot y' = 0$$

$$(y^2)' = \left(\left[f(x)\right]^2\right)' = 2f(x)\cdot f'(x) = 2y\cdot y'$$

$$y = f(x)$$

$$3x^2 - y^2 - 2xy\,y' + 3y^2 y' = 0$$

$$-2xy\,y' + 3y^2 y' = -3x^2 + y^2$$

$$y'(-2xy + 3y^2) = -3x^2 + y^2$$

$$\boxed{y' = \frac{-3x^2 + y^2}{-2xy + 3y^2} = \frac{dy}{dx}}$$

Encuentre dy/dx por derivación implícita.

$$xy = \sqrt{x^2 + y^2}$$

IMPLÍCITA

f g

$$y \cdot 1 + x y' = \frac{1}{2\sqrt{x^2+y^2}} \cdot (x^2 + y^2)'$$

$$y + x y' = \frac{1}{2\sqrt{x^2+y^2}} (2x + 2yy')$$

TAREA DESPEJAR y'

Encuentre dy/dx por derivación implícita.

$$1 + x = \operatorname{sen}(xy^2)$$

$f \quad g$

$0 + 1 = \cos(xy^2) \cdot (x \cdot y^2)'$

$1 = \cos(xy^2)(y^2 \cdot 1 + x \cdot 2yy')$

$y' = ?$

Si $g(x) + x\,\mathrm{sen}(g(x)) = x^2$, determine $g'(0)$.

$y = g(x)$

$y + x\,\mathrm{sen}\,y = x^2, \quad y'(0)$

$g(x) + \overset{\downarrow f}{x} \cdot \mathrm{sen}\overset{\downarrow g}{(g(x))} = x^2$

$g'(x) + \mathrm{sen}(g(x)) \cdot 1 + x(\cos(g(x))g'(x)) = 2x$

$g'(x) + \mathrm{sen}(g(x)) + x\cos(g(x))\,g'(x) = 2x$

$$g'(x) = \frac{2x - \mathrm{sen}(g(x))}{1 + x\cos(g(x))}$$

$$g'(0) = \frac{2(0) - \mathrm{sen}(g(0))}{1 + x\cos(g(0))}$$

$$(y^n)' = n\,y^{n-1} \cdot y'$$

$$y' = f'(x) \qquad \underbrace{\left[\left(f(x)\right)\right]^n}_{} = n\left[f(x)\right]^{n-1} \cdot f'(x)$$

$$n\,y^{n-1} \cdot y'$$