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202100081

Sección B

# Examen Final
## Matemática Intermedia 2

**#1 | Ej. 1**

$z = \sqrt{a^2 - x^2 - y^2}$

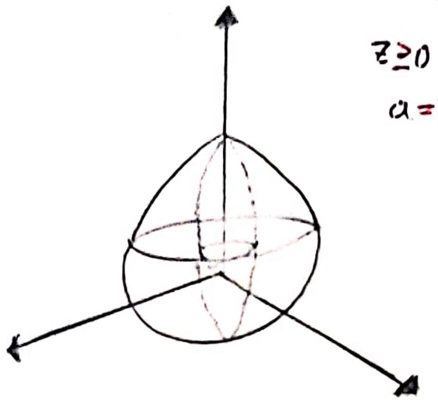
$\iint F \cdot n \, dS$

$z \geq 0$

$a = 2$

$F = x^3 i + y^3 j + z^3 k$

$z = \sqrt{a^2 - x^2 - y^2} \rightarrow z^2 = a^2 - x^2 - y^2 \rightarrow x^2 + y^2 + z^2 = a \rightarrow$ una esfera con centro $(0,0,0)$, radio $a = 2$



$\iint_{\partial S} F \cdot n \, dS = \iiint_S (\nabla \cdot F) \, dv; \quad \nabla \cdot F = \operatorname{div}(F)$

$\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right) \cdot (x^3, y^3, z^3) = \left(\frac{\partial x^3}{\partial x} + \frac{\partial y^3}{\partial y} + \frac{\partial z^3}{\partial z}\right)$

$3x^2 + 3y^2 + 3z^2 = 3(x^2 + y^2 + z^2)$

$3 \iiint_S (x^2 + y^2 + z^2) \, dv; \rightarrow$ coordenadas esféricas

Se observa que
$0 \leq \rho \leq 2$
$0 \leq \theta \leq 2\pi$ (Media esfera)
$0 \leq \varphi \leq \frac{\pi}{2}$

$dv = \rho^2 \operatorname{sen} \varphi \, d\rho \, d\varphi \, d\theta = \rightarrow \quad 3 \iiint \rho^2 \rho^2 \operatorname{sen} \varphi \, d\rho \, d\varphi \, d\theta;$ Aplicando los límites

$3 \int_0^{2\pi} \int_0^{\pi/2} \int_0^2 \rho^4 \operatorname{sen} \varphi \, d\rho \, d\varphi \, d\theta = 3 \int_0^{2\pi} \int_0^{\pi/2} \operatorname{sen} \varphi \cdot \frac{1}{5} \rho^5 \Big]_0^2 d\varphi \, d\theta = 3 \int_0^{2\pi} \int_0^{\pi/2} \frac{32}{5} \operatorname{sen} \varphi \, d\varphi \, d\theta$

$-\frac{96}{5} \int_0^{2\pi} \int_0^{\pi/2} -\operatorname{sen} \varphi \, d\varphi \, d\theta = -\frac{96}{5} \int_0^{2\pi} \int_0^{\pi/2} d(\cos\varphi) \, d\theta = -\frac{96}{5} \int_0^{2\pi} \cos\varphi \Big]_0^{\pi/2} d\theta = -\frac{96}{5} \int_0^{2\pi} (\cos\tfrac{\pi}{2}^{\,0} - \cos(0)^{\,1}) \, d\theta$

$-\frac{96}{5} \int_0^{2\pi} - d\theta = \frac{96}{5} \theta \Big]_0^{2\pi} = \frac{96}{5}(2\pi) = 120.64$

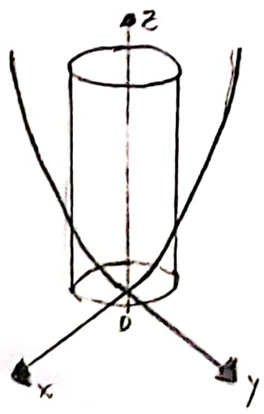
$\boxed{\iint F \cdot n \, dS = 120.64}$

**#2 | Ej. 2**

$a = 2$ Parábola $\rightarrow z = x^2 + y^2$; Cilindro $\rightarrow x^2 + y^2 = a^2$

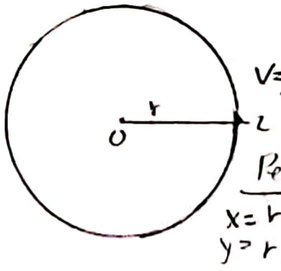
Pi = 3.1416

Plano xy - Volumen



Intersección entre ambos lados
$\left.\begin{array}{l} x^2 + y^2 = 4 \\ x^2 + y^2 = z \end{array}\right\} z = 4$ (Tope);

$0 \leq z \leq x^2 + y^2$
$0 \leq \theta \leq 2\pi$
$0 \leq r \leq 2$



$V = \int_0^{2\pi} \int_0^2 \int_0^{x^2+y^2} r \, dz \, dr \, d\theta$

Pero
$\left.\begin{array}{l} x = r\cos\theta \\ y = r\operatorname{sen}\theta \end{array}\right\} x^2 + y^2 = r^2$

$V = \int_0^{2\pi} \int_0^2 \int_0^{r^2} r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^2 r \int_0^{r^2} dz \, dr \, d\theta = \int_0^{2\pi} \int_0^2 r \cdot r^2 \, dr \, d\theta = \int_0^{2\pi} \int_0^2 r^3 \, dr \, d\theta$

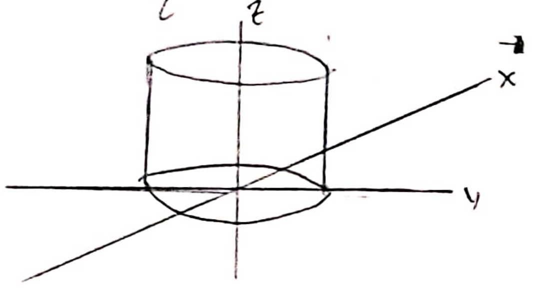
$\int_0^{2\pi} \frac{1}{4} r^4 \Big]_0^2 d\theta = \int_0^{2\pi} \frac{1}{4} 16 \, d\theta = 4 \int_0^{2\pi} d\theta = 4\theta \Big]_0^{2\pi} = 4 \cdot 2\pi = 8\pi \approx 25.13$

$\boxed{\text{Volumen } 8\pi \approx 25.13}$

**#3 | Ej 3**

$$\int_0^2\int_0^{\sqrt{36-9x^2}/2}\int_1^3 dz\,dy\,dx \rightarrow 1\le z\le 3,\; 0\le y\le \frac{\sqrt{36-9x^2}}{2};\; 0\le x\le 2$$

$$y=\frac{\sqrt{36-9x^2}}{2} = (2y)^2=(\sqrt{36-9x^2})^2 \rightarrow 4y^2=36-9x^2 \rightarrow 9x^2+4y^2=36 \rightarrow$$ Representa una elipse



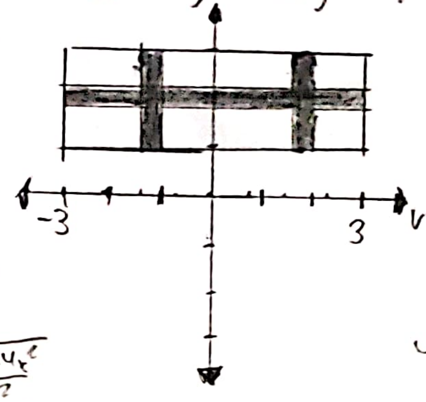

$\rightarrow$ Elipsoide

$9x^2+4y^2=36$

$9x^2=36-4y^2$

$3x=\sqrt{36-4y^2}$

$x=\frac{\sqrt{36-4y^2}}{3}$




$$\int_0^3\int_1^3\int_0^{\sqrt{36-4y^2}/3} dx\,dz\,dy$$

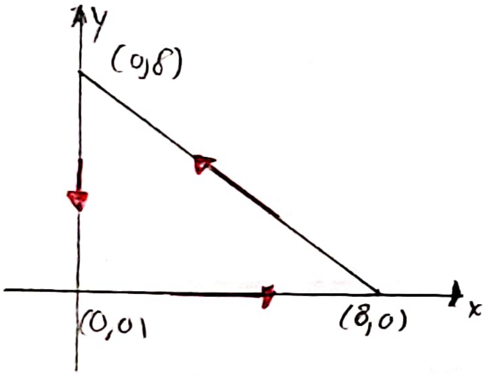
$$\int_0^2\int_0^{\sqrt{36-9x^2}/2}\int_1^3 dz\,dy\,dx \rightarrow \int_0^2\int_0^{\frac{\sqrt{36-9x^2}}{2}} z\Big|_1^3 = \int_0^2\int_0^{\frac{\sqrt{36-4x^2}}{2}} 2y\,dx$$

$$\int_0^2 2y\Big|_0^{\frac{\sqrt{36-9x^2}}{2}} = \int_0^2 3\sqrt{4-x^2} = 3\pi \approx 9.42$$

$$\boxed{3\pi \approx 9.42}$$

**#4 | Ej. 4**

$F(x,y)=(x^{3/2}-3y)\hat{i}+(6x+5y^{1/2})\hat{j}$ Triangulo $\rightarrow (0,0)\,(8,0),(0,8)$




Tramo horizontal; $y=0$; $\forall x\in\mathbb{R} \rightarrow \begin{matrix} x=t \\ y=0\end{matrix}$ $0\le t\le 8$

$$\left.\begin{matrix}\alpha(t)=\langle t,0\rangle \\ \alpha'(t)=\langle 1,0\rangle\end{matrix}\right\rangle \vec{F}(\alpha(t))=(t^{3/2},6t)$$

$$W_1=\int_1^8 \langle t^{3/2},6t\rangle\langle 1,0\rangle dt=\int_0^8 t^{3/2}dt$$

$$\frac{t^{5/2}}{\frac{5}{2}}\Big]_0^8 = \frac{2}{5}(8)^{5/2} \rightarrow W_1=72{,}41$$

$W_2 \Rightarrow$ Recta vertical

$\begin{matrix}x=0\\ y=t\end{matrix}$ $8\le t\le 0$; $F(\alpha,t)=\langle -3t,5t^{1/2}\rangle = W_2=\int_8^0 \langle -3t,5t^{1/2}\rangle\langle 0,1\rangle dt$

$$\alpha'(t)=\langle 0,1\rangle \rightarrow \int_8^0 5t^{1/2}dt=\frac{5t^{3/2}}{3/2}\Big]_8^0=\frac{10}{3}t^{3/2}\Big]_8^0=\frac{10}{3}(0-8^{3/2})=-75.43=W_2$$

$W_3 \rightarrow$ Recta lineal

Pendiente $\rightarrow m=\frac{8-0}{0-8}=\frac{8}{-8}=-1 \rightarrow y-0=-1(x-8) \rightarrow y=-x+8 \rightarrow y=8-x$, Si $x=t \rightarrow y=8-t$

$8\le x\le 0 \rightarrow$ Sentido contrario horario $\rightarrow$ $\begin{matrix}\alpha(t)=\langle t,8-t\rangle \\ \alpha'(t)=\langle 1,-1\rangle\end{matrix}$ $F(\alpha(t))=\langle t^{3/2}-3(8-t),6t+5(8-t)^{1/2}\rangle$

$\langle t^{3/2}-24+3t,\,6t+5(8-t)^{1/2}\rangle$

$$\int_8^0 \langle t^{3/2} - 24 + 3t - 6t \cdot 5(8-t)^{1/2} \rangle dt = \int_8^0 (t^{3/2} - 3t \cdot 24 - 5(8-t)^{1/2} dt =$$

$$\int_8^0 t^{3/2} dt - 3\int_8^0 t dt - 24\int_8^0 dt - 5\int_8^0 (8-t)^{1/2} dt = \frac{t^{5/2}}{5/2}\Big]_8^0 - 3\frac{1}{2}t^2\Big]_8^0 - 24t\Big]_8^0 - 5\frac{-(8-t)^{3/2}}{3/2}\Big]_8^0$$

$$\frac{2}{5}[0 - 8^{5/2}] - \frac{3}{2}(0-8^2] - 24[0-8] + \frac{10}{3}[8^{3/2} - 0] = -74{,}4077 + 96 + 192 + 75{,}4247$$

$\omega_3 = 291{,}02$ ; $\omega_T = \omega_1 + \omega_2 + \omega_3 = 287{,}997 = 288$ $\boxed{\omega = 288}$

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$\boxed{\#5 \mid Ej. 5}$

$\int_C \vec{F} \cdot dr$; $F(x,y) = \langle x^4y^2, 2/5 x^5 y + 1 \rangle$ ; $C: r(t) = \langle \sqrt{t}, 1 + t^3 \rangle$; $0 \le t \le 1$

a) $P_y(x,y)$

$P(x,y) = x^4y^2 \to \frac{\partial P(x,y)}{\partial y} = x^4 \cdot 2y = 2x^4y$

b) $Q_x(x,y)$

$Q(x,y) = 2/5 x^5 y + 1 \to \frac{\partial Q(x,y)}{\partial x} = \frac{2}{5} 5x^4y = 2x^4y$

c) Funcion potencial

$$\frac{\partial f(x,y)}{\partial x} = P(x,y) \to f(x,y) = \int^x x^4y^2 dx = y^2\int^x x^4 dx = y^2 \frac{1}{5}x^5 + h(y)$$

$$f(x,y) = \frac{1}{5}x^5y^2 + h(y) ; \frac{\partial f(x,y)}{\partial y} = Q(x,y) \to \frac{1}{5}x^5 \cdot 2y + h'(y) = \frac{2}{5}x^5y + 1$$

$$\frac{2}{5}x^5y + h'(y) = \frac{2}{5}x^5y + 1 \to h'(y) = 1 = \frac{dh(y)}{dy} \to dh(y) = dy \to h(y) = y + C$$

$$f(x,y) = \frac{1}{5}x^5y^2 + y + C$$

d) $\int_C F \cdot dr = \phi(x,y)\Big]_{\text{Inicial}}^{\text{Final}}$ Punto inicial $= \langle \sqrt{0}, 1+0 \rangle = \langle 0, 1 \rangle$

e) Punto final $= \langle \sqrt{1}, 1+1 \rangle = \langle 1, 2 \rangle$

f) Integral $\left(\frac{1}{5}x^5y^2 + y\right)\Big]_{(0,1)}^{(1,2)} = \left[\frac{1}{5}(1^5 \cdot 2^2) + 2\right] - \left(\frac{1}{5}(0^5 \cdot 1^2) + 1\right]$

$\frac{1}{5}(4 + 2 - (0+1] = \frac{4}{5} + 2 - 1 \to \oint_C \vec{F} \cdot d\vec{r} = \frac{9}{5} = 1.80$

a) Primera opcion
b) Primera opcion
c) Primera opcion
d) $\langle 0, 1 \rangle$
e) $\langle 1, 2 \rangle$
f) 1.80