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Primer Examen Parcial
Intermedia 2

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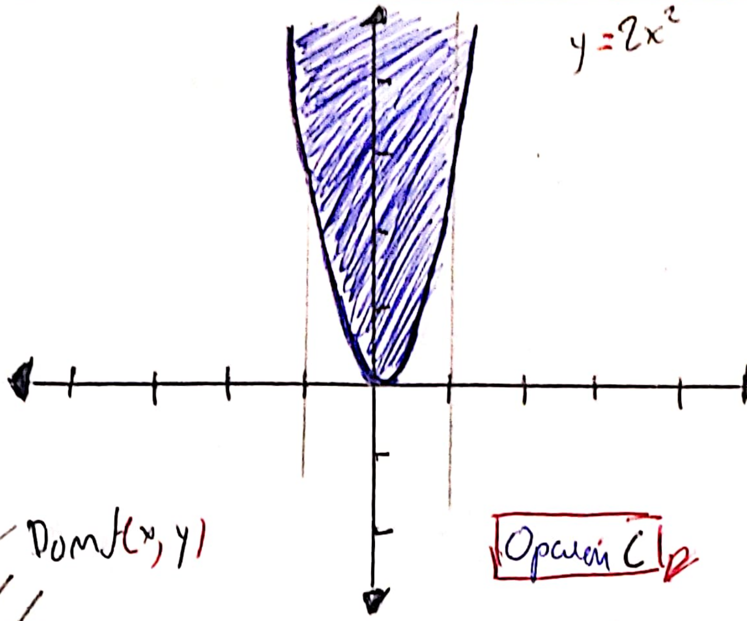
Sección B

#1 $f(x,y) = \sqrt{y-2x^2} \rightarrow \text{Dom} f(x,y)$

1.1 $y-2x^2 \geq 0 \rightarrow$ como $x^2 > 0$ $\forall x \in \mathbb{R} \rightarrow y \geq 2x^2$

Si $y = 2x^2$ es una función cuadrática

$.P_1(0,4)$
$4 \geq 2(0)^2$
$.P_2(8,0)$
$0 \geq 2(8)^2$ Falso



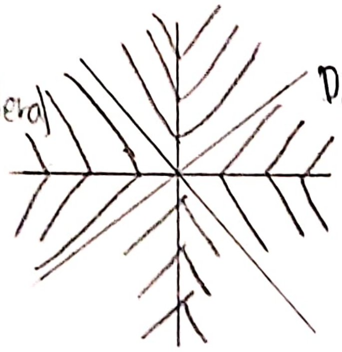
$\boxed{\text{Opción C}}$

1.2 $f(x,y) = \ln(x^2-y^2-1)$

$K \geq 0$

$x^2-y^2-1=0$

$x^2-y^2=1$ (Hipérbola equilatera)



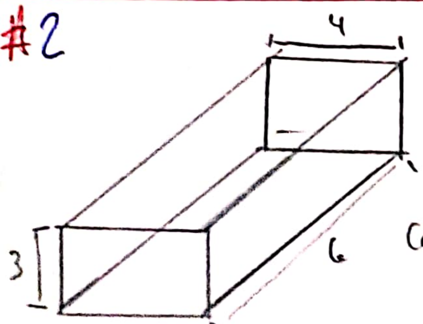
Dom $f(x,y)$

Respuestas

1.1 → Opción C

1.2 → HIPERBOLAS

#2



6 pies → Largo
4 pies → Ancho
3 pies → Profundidad

Cobre hoja → $\frac{1}{4}$ pulgada espesor

Cantidad Aproximada de Cobre = ?

$A \times h$; x = ancho, y = Largo, z = altura

Sueldo$(x,y,z) = 2xz + 2yz + xy$

$dS(x_0) = \left(\frac{dS}{dx}, \frac{dS}{dy}, \frac{dS}{dz}\right)\Big]_{x_0}$

$dS(x_0) = (2z+y, 2z+x, 2x+2y)\Big]_{(4,6,3)}$

$(6+6, 6+4, 8+12) = (12,10,20)$ pies

Área de las cuatro laterales $= L \times h$

Área del fondo $= L \times A$

$S(x_0) = S(x_0) + dS(x_0)(x-x_0)$

$S(x) = 84\text{ pies}^2 + (12,10,20)\begin{pmatrix} x-4 \\ y-6 \\ z-3 \end{pmatrix} \rightarrow e^{1/4} \rightarrow$ pulgada a pie $= 0,02083$ pies → Se suma en cada lado del área en el punto.

$x = (4,02083, 6,02083, 3,02083) \rightarrow S(x) = 84 + (12,10,20)\begin{pmatrix} 4,02083-4 \\ 6,02083-6 \\ 3,02083-3 \end{pmatrix} \rightarrow 84 + (12,10,20)\begin{pmatrix} 0,2083 \\ 0,2083 \\ 0,2083 \end{pmatrix}$

$= 84 + 12(0,2083) + 10(0,0283) + 20(0,0283) \rightarrow 84,88$ pies

$A\,\text{total} = 84,87486\ \text{pie}^2 \rightarrow$ $\boxed{A = 84,88\ \text{pie}^2}$

#3 $z=\sqrt{16-x^2-y^2}$ ← curva ; plano → $x+y=4$

$\vec{u}= x+y=4 \to y=4-x$

$z=\sqrt{16-x^2-(4-x)^2} = \sqrt{16-x^2-(16-8x-x^2)} = \sqrt{16-x^2-16+8x-x^2} = \sqrt{8x-2x^2}$

Sea → $x=t$; $y=4-t$; $z=\sqrt{8t-2t^2}$

$f(x,y,z) = f(t) = \langle t, 4-t, \sqrt{8t-2t^2} \rangle$

$$\boxed{r(t)= ti+(4-t)j+\sqrt{8t-2t^2}k}$$

#4 $t\to 0 \left(\frac{t-1}{t+2}i + \frac{\sqrt{t+4}-2}{t}j + \frac{t}{\operatorname{Sen} t}k\right)$

a) $\lim_{t\to 0}\left\langle \frac{t-1}{t+2}, \frac{\sqrt{t+4}-2}{t}, \frac{t}{\operatorname{Sen} t}\right\rangle \to \lim_{t\to 0} \frac{t-1}{t+2} = \frac{0-1}{0+2} = \frac{-1}{2} = \frac{-1}{2} \approx -0.5$

$\lim_{t\to 0}\frac{\sqrt{t-4}-2}{t} = \frac{0}{0} \to \lim_{t\to 0}\frac{\sqrt{t+4}-2}{t} \cdot \frac{(\sqrt{t+4}-2)}{(\sqrt{t+4}-2)} = \lim_{t\to 0}\frac{(\sqrt{t+4})^2(2)^2}{t(\sqrt{t+4}+2)} = \lim_{t\to 0}\frac{\cancel{t+4-4}}{t(\sqrt{t+4}-2}$

$\lim_{t\to 0}\frac{1}{\sqrt{t+4}-2} = \frac{1}{\sqrt{0+4}+2} = \frac{1}{\sqrt{4}+2} = \frac{1}{2+2} = \frac{1}{4} \approx 0.25$

$\lim_{t\to 0}\frac{t}{\operatorname{Sen} t} = \frac{1}{\cancel{\lim_{t\to 0}\frac{\operatorname{Sen} t}{t}}} = \frac{1}{1} = 1 \to \lim_{t\to 0} f(t) = \langle -0.50, 0.25, 1.00\rangle$

b) $K=\frac{2}{13}$ $a=?$ $r(t)=2\cos t i + 2\operatorname{Sen} t j + atk \to r(t) = \langle 2\cos t, 2\operatorname{Sen} t, at\rangle$

$K=\frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3} \to r'(t)=\langle -2\operatorname{Sen} t, 2\cos t, a\rangle$; $r''(t)=\langle -2\cos t, -2\operatorname{Sen} t, 0\rangle$

$r'(t)\times r''(t) = \begin{vmatrix} i & j & k \\ -2\operatorname{Sen} t & 2\cos t & a \\ -2\cos t & -2\operatorname{Sen} t & 0\end{vmatrix} = \begin{matrix} i(0+2a\operatorname{Sen} t)-j(0+2a\cos t) \\ +k(4\operatorname{Sen}^2 t+4\cos^2 t) \to (2a\operatorname{Sen} t, -2a\cos t, 4)\end{matrix}$

$\|r'(t)\| = \sqrt{[2a\operatorname{Sen} t]^2+[-2a\cos t]^2+(4)^2} = \sqrt{4a^2\operatorname{Sen}^2 t+4a^2\cos^2 t+16} = \sqrt{4a^2+16}$;

$\|r'(t)\| = \sqrt{(2\operatorname{Sen} t]^2+[2\cos t]^2+(a)^2} = \sqrt{4\operatorname{Sen}^2 t+4\cos^2 t+a^2} = \sqrt{4+a^2}$

$$K=\frac{\sqrt{4a^2+16}}{\sqrt{(4+a^2)^3}}=\frac{2}{13}\rightarrow 13\sqrt{4a^2+16}=2\sqrt{(4+a^2)^3}$$

$$169(4a^2+16)=4(4+a^2)^3\rightarrow 676a^2+2704=4(64+48a^2+12a^4+a^6)$$

$$676a^2+2704=256+192a^2+48a^4+4a^6\rightarrow 4a^6+48a^4+192a^2+256-676a^2-2704=0$$

$$4a^6+48a^4-484a^2-2448=0$$

$$\text{Sea } w=a^2\rightarrow 4w^3+48w^2-484w-2448=0 \quad \text{Hallamos } w$$

$$w=9\rightarrow a^2=9\rightarrow a=3$$

c) $L_c;\ 0\leq t\leq 2\pi$

$$L=\int_a^b \|r'(t)\|dt;\ \|r'(t)\|=\sqrt{4+a^2}\rightarrow\int_0^{2\pi}\sqrt{4+a^2}\,dt;\quad \text{con } a=3$$

$$\sqrt{4+a^2}\int_0^{2\pi}dt=\sqrt{4+a^2}\,t\Big]_0^{2\pi}=2\pi\sqrt{4+a^2}\rightarrow \text{Si } a=3$$

$$L_c=2\pi\sqrt{4+9}\rightarrow L_c=2\pi\sqrt{13}\approx 22{,}65$$

Respuestas

a) $\lim_{t\to 0}\langle -0.50, 0.25, 1.00\rangle$

b) $a=3$

c) $0\leq t\leq 2\pi\rightarrow$ Longitud de la curva 22.65

#5 $t\to 0\left(\frac{t-1}{t+2}\hat{i}+\frac{\sqrt{t+4}-2}{t}\hat{j}+\frac{t}{\operatorname{Sen}t}\hat{k}\right)\rightarrow$

a) $\lim_{t\to 0}\left\langle\frac{t-1}{t+2},\frac{\sqrt{t+4}-2}{t},\frac{t}{\operatorname{Sen}t}\right\rangle$

$$\cdot\lim_{t\to 0}\frac{t-1}{t-2}=\frac{0-1}{0-2}=\frac{-1}{2}=-\frac{1}{2}=-0.50$$

$$\cdot\lim_{t\to 0}\frac{\sqrt{t+4}-2}{t}\rightarrow\frac{0}{0}\rightarrow\lim_{t\to 0}\frac{\sqrt{t+4}-2}{t}\cdot\frac{(\sqrt{t+4}-2)}{(\sqrt{t+4}-2)},\ \lim_{t\to 0}\frac{(\sqrt{t+4})^2(2)^2}{t(\sqrt{t+4}+2)}=\lim_{t\to 0}\frac{t+4-4}{t(\sqrt{t+4}+2)}$$

$$\lim_{t\to 0}\frac{1}{\sqrt{t+4}+2}=\frac{1}{\sqrt{0+4}+2}=\frac{1}{\sqrt{4}+2}=\frac{1}{2+2}=\frac{1}{4}=0.25$$

$$\cdot\lim_{t\to 0}\frac{t}{\operatorname{Sen}t}=\frac{1}{\lim_{t\to 0}\frac{\operatorname{Sen}t}{t}}=\frac{1}{1}=1\rightarrow\lim_{t\to 0}r(t)=\langle -0.50, 0.25, 1.00\rangle$$

b) $R(t) = (t^2-3t)i + \sqrt{8-2t}\, j + \frac{1-t}{\sqrt{t}} k$ ← Dominio

$\cdot\ t^2-3t$; $\in \mathbb{R}$

$\cdot\ \sqrt{8-2t} \rightarrow 8-2t \geq 0 \rightarrow 2t-8 \leq 0 \rightarrow 2t \leq 8 \rightarrow t \leq 4$

$t \in (-\infty, 4]$

$\frac{\sqrt{1-t}}{\sqrt{t}} = \left.\begin{array}{l}\sqrt{t} \neq 0 \\ t \geq 0\end{array}\right\} t > 0$; $t \in (0, +\infty)$

$\text{Dom } R(t) = \text{Dom } R_1(t) \cap \text{Dom } R_2(t) \cap \text{Dom } R_3(t) = (0, 4]$

Respuestas

a) $\lim_{t \to 0} \langle -0.50, 0.25, 1 \rangle$

b) Dominio $(0, 4]$