TRANSFORMADA INVERSA DE DERIVADA DE TRANSFORMADA

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Únicamente Para funciones $\ln(S)$, $\tan^{-1}(S)$ y $\cot^{-1}(S)$

$$\mathcal{L}^{-1}\{F(S)\} = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{dS}\{F(S)\}\right\}$$

$$\mathcal{L}\{t\,f(t)\} = (-1)^1 \frac{d}{ds}[\mathcal{L}\{f(t)\}] = -\frac{d}{ds}[F(s)] \quad \Big| \mathcal{L}^{-1}\{\}$$

$$t\,f(t) = -\mathcal{L}^{-1}\left\{\frac{d}{ds}[F(s)]\right\} \Rightarrow f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}[F(s)]\right\}$$

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Calcular la Tranformada Inversa $\mathcal{L}^{-1}\{F(S)\}$ de :

1) $\boldsymbol{F(s)} = \ln\left(\frac{s}{s+1}\right) = \ln(s) - \ln(s+1)$

$$\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}\left[\ln(s) - \ln(s+1)\right]\right\}$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$f(t) = -\frac{1}{t}\left[1 - e^{-t}\right]$$

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Calcular la Tranformada Inversa $\mathcal{L}^{-1}\{F(S)\}$ de :

2) $F(s) = \cot^{-1}\left(\frac{1}{s}\right)$

$$\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{d}{ds}\left[\cot^{-1}\left(\frac{1}{s}\right)\right]\right\}$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{-1}{1+\left(\frac{1}{s}\right)^2}\cdot\left(\frac{-1}{s^2}\right)\right\}$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{\left(\frac{s^2+1}{\cancel{s^2}}\right)}\cdot\left(\frac{1}{\cancel{s^2}}\right)\right\} = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$f(t) = -\frac{\text{SEN}(t)}{t}$$

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TEOREMA DE CONVOLUCION

$$f(\cancel{t})^{\tau} * g(\cancel{t})^{\beta}$$

$$\mathcal{L}\{f(t) * g(t)\} = \left(\int_0^\infty \underline{e}^{-s\tau} f(\tau)\, d\tau\right)\left(\int_0^\infty \underline{e}^{-s\beta} g(\beta)\, d\beta\right)$$

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty \int_0^\infty e^{-s(\tau+\beta)} \underline{f(\tau)} g(\beta)\, \underline{d\tau}\, d\beta$$

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty f(\tau)\, d\tau \int_0^\infty e^{-s(\tau+\beta)} g(\beta)\, d\beta.$$

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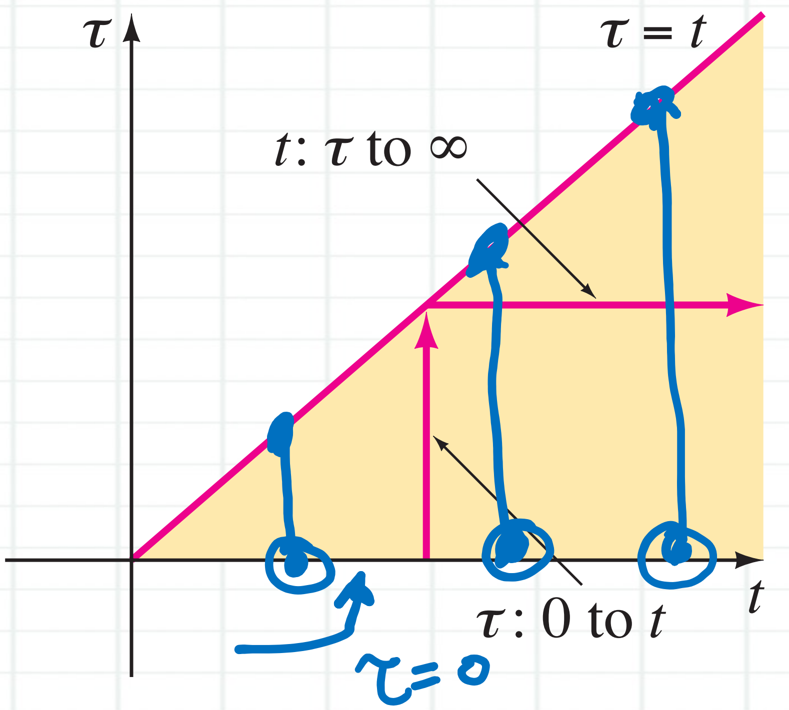
$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty f(\tau)\, d\tau \int_0^\infty e^{-s\underbrace{(\tau+\beta)}_{t}} g(\underbrace{\beta}_{t-\tau})\, \underbrace{d\beta}_{dt}.$$

hacemos $t = \tau + \beta$, $dt = d\beta$.

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty f(\tau)\, d\tau \int e^{-st} g(t - \tau)\, dt.$$

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty e^{-st}\, dt \int f(\tau) g(t - \tau)\, d\tau$$

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty e^{-st} \left\{ \int_0^t f(\tau)\, g(t - \tau)\, d\tau \right\} dt$$



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$$\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\left\{\int_0^t f(\tau) \cdot g(t-\tau)d\tau\right\}$$

$$= \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\left\{\int_0^t f(\tau) \cdot g(t-\tau)d\tau\right\}$$

$$\mathcal{L}\{f(t) * g(t)\} = \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\}$$

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