SEGUNDO TEOREMA DE TRASLACION , TRASLACION EN EL EJE "t" (Escalón Unitario $u(t-a)$)

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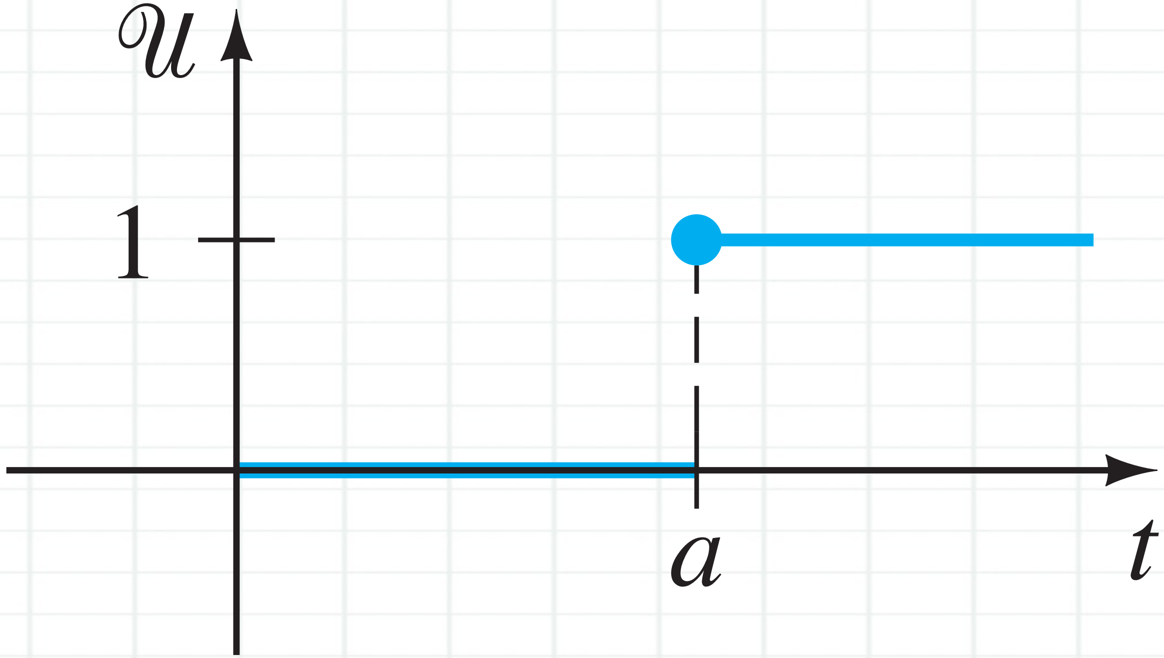
La función escalón unitario $\mathscr{U}(t - a)$ se define como

$$\mathscr{U}(t - a) = \begin{cases} 0, & 0 \leq t < a \\ 1, & t \geq a. \end{cases}$$



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Dibujar $f(t) = (t+1)\,u(t-3)$:

$t=0 \rightarrow f(0)=(0+1)\cancel{u(t-3)}^{0} = 0$

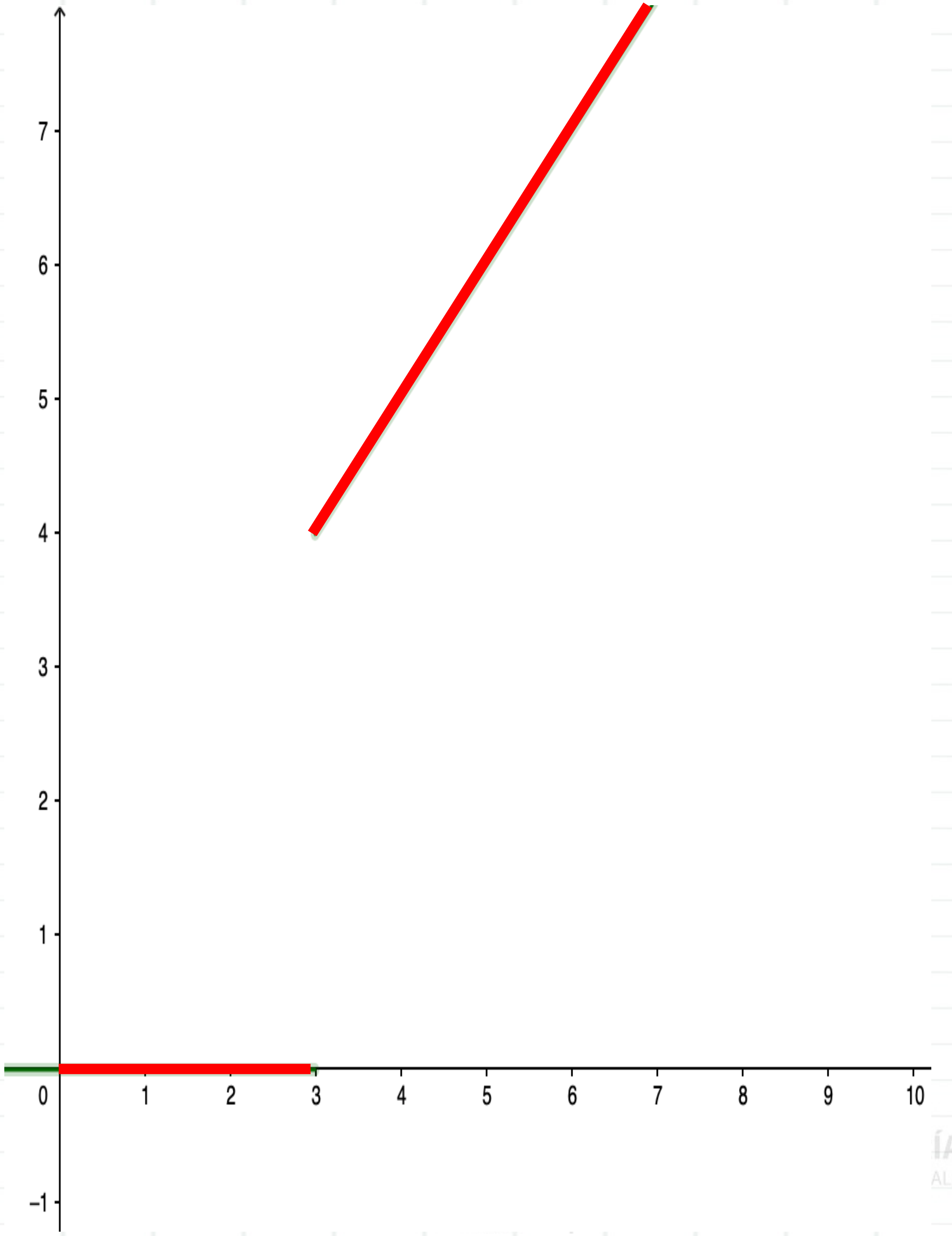
$t=1 \rightarrow f(1)=(1+1)\cancel{u(t-3)}^{0} = 0$

$t=2 \rightarrow f(2)=(2+1)\cancel{u(t-3)}^{0} = 0$

$t=3 \rightarrow f(3)=(3+1)\cancel{u(t-3)}^{1} = 4$

$t=4 \rightarrow f(4)=(4+1)\cancel{u(t-3)}^{1} = 5$

$* \; t=1 \rightarrow f(1)=(1+1)\,u(1-3)$



$$\mathcal{L}\{\mathbb{C}u(t-a)\}$$

$$\mathcal{L}\{\mathbb{C}u(t-a)\} = \int_0^a \underbrace{e^{-st}}\,\mathbb{C}\,\cancelto{0}{u(t-a)}\,dt + \int_a^\infty \underbrace{e^{-st}}\cdot\mathbb{C}\,\cancelto{1}{u(t-a)}\,dt$$

$$\mathcal{L}\{\mathbb{C}u(t-a)\} = \int_a^\infty e^{-st}\,\mathbb{C}\,dt$$

$$\mathcal{L}\{\mathbb{C}u(t-a)\} = -\frac{\mathbb{C}}{s}e^{-st}\Big|_a^\infty$$

$$\boldsymbol{\mathcal{L}\{\mathbb{C}u(t-a)\} = \frac{\mathbb{C}}{s}e^{-at}}$$

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$$\mathscr{L}\{f(t-a)\,\mathscr{U}(t-a)\} = \int_0^a e^{-st} f(t-a)\cdot \underbrace{\overset{0}{\cancel{\mathscr{U}(t-a)}}}_{\substack{\text{cero para}\\ 0\le t<a}}\, dt + \int_a^\infty e^{-st} f(t-a)\cdot \underbrace{\overset{1}{\cancel{\mathscr{U}(t-a)}}}_{\substack{\text{uno para}\\ t\ge a}}\, dt$$

$$\mathscr{L}\{f(t-a)\,\mathscr{U}(t-a)\} = \int_a^\infty e^{-st} f(t-a)\, dt.$$

Ahora si hacemos $v = t-a$, $dv = dt$

$$\mathscr{L}\{f(t-a)\,\mathscr{U}(t-a)\} = \int_0^\infty e^{-s(v+a)}\cdot f(v)\, dv$$

$$\mathscr{L}\{f(t-a)\,\mathscr{U}(t-a)\} = e^{-as}\int_0^\infty e^{-sv} f(v)\, dv$$

$$\mathscr{L}\{f(t-a)\,\mathscr{U}(t-a)\} = e^{-as}\mathscr{L}\{f(t)\}.$$

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Calcular $\mathcal{L}\{f(t)\}$ de :

1) $\boldsymbol{f(t) = (t+2)\,u(t-3)}$ $\quad | \; t \to t-3$

$f(t) = ((t-3+3)+2)\,u(t-3)$

$f(t) = ((t-3)+5)\,u(t-3) \quad | \; t-3 \to t$

$f(t) = (t+5)\,u(t-3) \quad | \mathcal{L}\{\}$

$\mathcal{L}\{f(t)\} = e^{-3s}\,\mathcal{L}\{t+5\}$

$F(s) = e^{-3s}\left[\frac{1}{s^2} + \frac{5}{s}\right]$

Calcular $\mathcal{L}\{f(t)\}$ de :

2) $\boldsymbol{f(t) = e^{2-t}\,u(t-4)}$ $\quad | \; t \to t-4$

$$f(t) = e^{2-(t-4+4)}\,u(t-4)$$

$$f(t) = e^{-2-(t-4)}\,u(t-4) \quad | \; t-4 \to t$$

$$f(t) = e^{-2-t}\,u(t-4) = e^{-2}\cdot e^{-t}\,u(t-4) \quad | \; \mathcal{L}\{\}$$

$$\mathcal{L}\{f(t)\} = e^{-4s}\,\mathcal{L}\{e^{-2}\cdot e^{-t}\}$$

$$F(s) = e^{-4s}\left[\frac{e^{-2}}{s+1}\right]$$

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