Ej: Resuelva

N

$$(2y - \frac{1}{x} + \cos 3x)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 - 3y\,\text{sen}\,3x = 0$$

$Mdy + Ndy = 0$ $\qquad$ $Mdy + Ndx = 0$

$$\int(2y - \frac{1}{x} + \cos 3x)dy + \int(\frac{y}{x^2} - 4x^3 - 3y\,\text{sen}\,3x)dx = \int 0$$

$-x^{-1}$

$$\boxed{\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}}$$

yx^{-2}

$$\frac{\partial M}{\partial x} = \frac{1}{x^2} - 3\,\text{sen}\,3x$$

$$\frac{\partial N}{\partial y} = \frac{1}{x^2} - 3\,\text{sen}\,3x$$

no es exacta. $\qquad$ si es exacta.

$y^2 - \frac{y}{x} + y\cos 3x$ $\qquad$ $-\frac{y}{x} - x^4 + y\cos 3x$

Sol. $y^2 - \frac{y}{x} + y\cos x - x^4 = C$

$Mdx + Ndy = 0$ $\qquad$ $Mdy + Ndx = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\qquad$ $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

Ej. Resolver

$$(x - y^3 + y^2\,\text{sen}\,x)dx = (3xy^2 + 2y\cos x)dy$$

$$\int(\underbrace{x - y^3 + y^2\,\text{sen}\,x}_{M})dx - \int(\underbrace{3xy^2 + 2y\cos x}_{N})dy = \int 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = -3y^2 + 2y\,\text{sen}\,x$$

$$\frac{\partial N}{\partial x} = -3y^2 + 2y\,\text{sen}\,x$$

si es exacta

$\frac{1}{2}x^2 - xy^3 - y^2\cos x \qquad -xy^3 + y^2\cos x \qquad = c$

$\underline{\text{Sol}} \quad \frac{1}{2}x^2 - xy^3 - y^2\cos x = c$

Ej. Resolver

$$\left(\frac{1}{t} + \frac{1}{t^2} - \frac{y}{t^2+y^2}\right)dt + \left(ye^y + \frac{t}{t^2+y^2}\right)dy = 0$$

$\underbrace{\qquad}_{M} \qquad \underbrace{\qquad}_{N}$

$(t^2+y^2-2y^2)$

$$\frac{\partial M}{\partial y} = -\left[\frac{(t^2+y^2)(1) - y(2y)}{(t^2+y^2)^2}\right] = \frac{y^2-t^2}{(t^2+y^2)^2}$$

$$\frac{\partial N}{\partial t} = \frac{(t^2+y^2)(1) - t(2t)}{(t^2+y^2)^2} = \frac{y^2-t^2}{(t^2+y^2)^2}$$

Si es exacta.

$\ln t - \frac{1}{t} - tg^{-1}\frac{t}{y} \qquad ye^y - e^y + tg^{-1}\frac{y}{t} = c$

$-y\int\frac{1}{t^2+y^2}dt = -\frac{y}{y}tg^{-1}\frac{t}{y}$

$\int ye^y dy \quad \sim \quad u = y \quad du = dy$

$\int dv = \int e^y dy \quad v = e^y$

$\int\frac{t}{t^2+y^2}dy = t\int\frac{dy}{t^2+y^2} = \frac{\cancel{t}}{\cancel{t}}tg^{-1}\frac{y}{t}$

$\underline{\text{Sol}} \quad \ln t - \frac{1}{t} - tg^{-1}\frac{t}{y} + ye^y - e^y + tg^{-1}\frac{y}{t} = c$

Ej. Determine $M(x,y)$ de modo que la ecuación diferencial sea exacta.

$$M(x,y)dx + \underbrace{\left(xe^{xy} + 2xy + \frac{1}{x}\right)}_{N}dy = 0$$

x^{-1}

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial N}{\partial x} = xye^{xy} + 2y - x^{-2} + e^{xy}$$

$$\frac{\partial M}{\partial y} = xye^{xy} + 2y - x^{-2}$$

$$\int \partial M = \int (xye^{xy} + e^{xy} + 2y - x^{-2})\, dy$$

$$x\int ye^{xy}\, dy \qquad u = y \qquad du = dy$$

$$\int dv = \int e^{xy} dy \qquad v = \frac{1}{x} e^{xy}$$

$$\frac{y}{x} e^{xy} - \frac{1}{x}\int e^{xy} dy = \frac{y}{x} e^{xy} - \frac{1}{x^2} e^{xy}$$

$$M(x,y) = \frac{y}{x} e^{xy} - \frac{1}{x^2} e^{xy} + \frac{1}{x} e^{xy} + y^2 - \frac{y}{x^2}$$

$$\left(\frac{y}{x} e^{xy} - \frac{1}{x^2} e^{xy} + \frac{1}{x} e^{xy} + y^2 - \frac{y}{x^2}\right) dx + \left(xe^{xy} + 2xy + \frac{1}{x}\right) dy = 0$$

Ecuaciones Reducibles a exactas

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \rightarrow \text{no es exacta}$$

Factor de integración.

$$\mu(x) = e^{\int \frac{M_y - N_x}{N} dx}$$

$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy}$$

se multiplica el factor de integración por toda la ecuación y se vuelve a alizar si es exacta.

$$\mu(x)\, M(x,y)\,dx + \mu(x)\, N(x,y)\,dy = 0$$

$$\mu(y)\, M(x,y)\,dx + \mu(y)\, N(x,y)\,dy = 0$$

Ej. Resolver

$$\underbrace{6xy}_{M}\,dx + (\underbrace{4y+9x^2}_{N})\,dy = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 6x \\ \frac{\partial N}{\partial x} &= 18x \end{aligned}\right\} \text{ no es exacta}$$

$$\mu(x) = e^{\int \frac{6x-18x}{4y+9x^2}\,dx} = e^{\int \frac{-12x}{4y+9x^2}\,dx}$$

$$\mu(y) = e^{\int \frac{18x-6x}{6xy}\,dy} = e^{\int \frac{12\cancel{x}}{6\cancel{x}y}\,dy} = e^{\int \frac{2}{y}\,dy}$$

$$\mu(y) = e^{2\int \frac{dy}{y}} = e^{2\ln y} = e^{\ln y^2} = y^2$$

$$y^2\left[6xy\,dx + (4y+9x^2)\,dy\right] = 0$$

$$\int \underbrace{6xy^3}_{M}\,dx + \int \underbrace{4y^3+9x^2y^2}_{N}\,dy = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 18xy^2 \\ \frac{\partial N}{\partial x} &= 18xy^2 \end{aligned}\right\} \text{ si es exacta}$$

$3x^2y^3$ $\qquad$ $y^4 + 3x^2y^3 = C$

Sol $\quad 3x^2y^3 + y^4 = c$

Ej. Resolver

$$(\underbrace{y^2 + xy^3}_{M})dx + (\underbrace{5y^2 - xy + y^3 \operatorname{sen} y}_{N})dy = 0$$

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= 2y + 3xy^2 \\ \frac{\partial N}{\partial x} &= -y\end{aligned}\right\} \text{ no es exacta}$$

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}dx} = e^{\int \frac{2y + 3xy^2 - (-y)}{5y^2 - xy + y^3 \operatorname{sen} y}dx}$$

$$\mu(y) = e^{\int \frac{N_x - M_y}{M}dy} = e^{\int \frac{-y - (2y + 3xy^2)}{y^2 + xy^3}dy}$$

$$\mu(y) = e^{\int \frac{-3y - 3xy^2}{y^2 + xy^3}dy} = e^{\int \frac{-3y\cancel{(1+xy)}}{y^2\cancel{(1+xy)}}dy}$$

$$\mu(y) = e^{\int \frac{-3}{y}dy} = e^{-3\int \frac{dy}{y}} = e^{-3\ln y} = e^{\ln y^{-3}}$$

$$\mu(y) = y^{-3} = \frac{1}{y^3}$$

$$\frac{1}{y^3}(y^2 + xy^3)dx + \frac{1}{y^3}(5y^2 - xy + y^3 \operatorname{sen} y)dy = 0$$

$$\int\left(\frac{1}{y} + x\right)dx + \int\left(\frac{5}{y} - \frac{x}{y^2} + \operatorname{sen} y\right)dy = \int 0$$

$$\left.\begin{aligned}\frac{\partial M}{\partial y} &= -\frac{1}{y^2} \\ \frac{\partial N}{\partial x} &= -\frac{1}{y^2}\end{aligned}\right\} \text{ si es exacta}$$

$$-xy^{-2} = \frac{x}{y}$$

$$\frac{x}{y} + \frac{1}{2}x^2 \qquad 5\ln y + \frac{x}{y} - \cos y \qquad = C$$

Sol.

$$\frac{x}{y} + \frac{1}{2}x^2 + 5\ln y - \cos y = C$$

Ej. Resolver

$$\underbrace{x}_{M}\,dx + (\underbrace{x^2y + 4y}_{N})\,dy = 0 \qquad y(4) = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 0 \\ \frac{\partial N}{\partial x} &= 2xy \end{aligned}\right\}$$ no es exacta.

$$\mu(x) = e^{\int \frac{M_y - N_x}{N} dx} = e^{\int \frac{0 - 2xy}{x^2y + 4y} dx}$$

$$\mu(y) = e^{\int \frac{N_x - M_y}{M} dy} = e^{\int \frac{2\cancel{x}y - 0}{\cancel{x}} dy} = e^{\int 2y\,dy}$$

$$\mu(y) = e^{y^2}$$

$$e^{y^2}x\,dx + e^{y^2}(x^2y + 4y)\,dy = 0$$

$$\int \underbrace{xe^{y^2}}_{M}dx + \int (\underbrace{x^2ye^{y^2} + 4ye^{y^2}}_{N})\,dy = 0$$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 2xye^{y^2} \\ \frac{\partial N}{\partial x} &= 2xye^{y^2} \end{aligned}\right\}$$ si es exacta.

$$\frac{1}{2}x^2e^{y^2} \qquad \frac{1}{2}x^2e^{y^2} + 2e^{y^2} = C$$

$x^2 \int y e^{y^2} dy$ $\qquad$ $u = y^2$

$du = 2y\, dy$ $\qquad$ $\frac{du}{2} = y\,dy$

$\frac{x^2}{2} \int e^u du = \frac{1}{2} x^2 e^u$

$\int 4y e^{y^2} dy$ $\qquad$ $u = y^2$

$du = 2y\, dy$

$\frac{4}{2} \int e^u du = 2e^u$

Sol $\quad \frac{1}{2} x^2 e^{y^2} + 2e^{y^2} = C \rightarrow$ sol general.

$y(0) = 4$

$\frac{1}{2} (0)^2 e^{4^2} + 2e^{4^2} = C$

$C = 2e^{16}$

Sol $\quad \frac{1}{2} x^2 e^{y^2} + 2e^{y^2} = 2e^{16} \rightarrow$ Sol. particular.