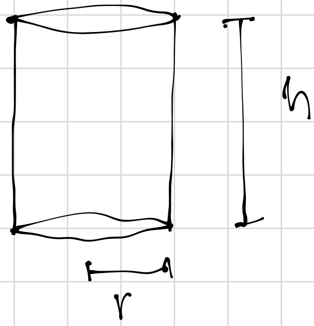
Ej. El radio r y la altura h de un cilindro circular recto se miden con errores maximos posibles de 4 por ciento y el 2 por ciento, respectivamente, ¿cual es el error relativo que se puede cometer al medir el volumen.

error relativo



radio → 4%

altura → 2%

$$\%E.R. = \frac{V}{dV} * 100$$

$$V = \pi r^2 h$$

$$dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh = V_r dr + V_h dh$$

$$\frac{\Delta r}{r} = \frac{4}{100}$$

$$dV = 2\pi r h\, dr + \pi r^2 dh$$

$$\frac{\Delta h}{h} = \frac{2}{100}$$

$$E.R. = \frac{dV}{V} = \frac{2\pi r h dr + \pi r^2 dh}{\pi r^2 h}$$

$$E.R. = \frac{2\cancel{\pi}\cancel{r}\cancel{h}\, dr}{\cancel{\pi} r^{\cancel{2}} \cancel{h}} + \frac{\cancel{\pi}\cancel{r^2}\, dh}{\cancel{\pi}\cancel{r^2} h}$$

$$E.R. = 2\frac{dr}{r} + \frac{dh}{h}$$

$$\frac{\Delta r}{r} \approx \frac{dr}{r} = \frac{4}{100} \qquad \frac{\Delta h}{h} \approx \frac{dh}{h} = \frac{2}{100}$$

$$E.R. = 2\left(\frac{4}{\cancel{100}_{50}}\right) + \left(\frac{2}{100}\right) = \frac{4}{50} + \frac{1}{50} = \frac{5}{50} = \frac{1}{10}$$

$$\%E.R. = \frac{1}{10} * 100 = 10\%$$

Regla de la cadena caso I

↳ funciones compuestas $\quad r \quad (\ )^n$

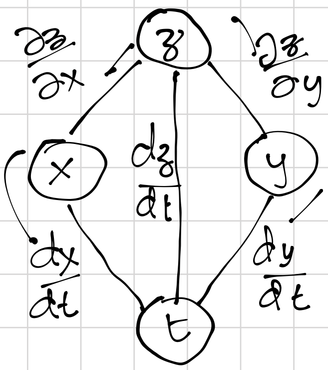
$$y = f(x) \qquad x = g(t) \rightarrow y = f(g(t))$$

$$\frac{dy}{dt} = \frac{dy}{\cancel{dx}} \cdot \frac{dx}{dt} = \frac{dy}{dt}$$

$$z = f(x,y) \qquad x = g(t) \qquad y = h(t)$$

Suponga que $z = f(x,y)$ es una función derivable de x e y donde $x = g(t)$ y $y = h(t)$ son funciones derivables de t, entonces z es una función derivable de t y

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$



$$\frac{dz}{dt}$$

Ej. Use la regla de la cadena para determinar dz/dt

$$z = f(x,y) = \frac{x-y}{x+2y} \qquad x = e^{\pi t} \qquad y = e^{-\pi t}$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = \frac{(x+2y)(1) - (x-y)(1)}{(x+2y)^2} = \frac{\cancel{x}+2y-\cancel{x}+y}{(x+2y)^2} = \frac{3y}{(x+2y)^2}$$

$$\frac{dx}{dt} = \pi e^{\pi t} \qquad \frac{dy}{dt} = -\pi e^{-\pi t}$$

$$\frac{\partial f}{\partial y} = \frac{(x+2y)(-1) - (x-y)(2)}{(x+2y)^2} = \frac{-x - \cancel{2y} - 2x + 2y}{(x+2y)} = \frac{-3x}{(x+2y)^2}$$

$$\frac{dz}{dt} = \frac{3y}{(x+2y)^2}(\pi e^{\pi t}) - \frac{3x}{(x+2y)^2}(-\pi e^{-\pi t})$$

$$\frac{dz}{dt} = \frac{3e^{-\pi t}\,\pi e^{\pi t}}{(e^{\pi t}+2e^{-\pi t})^2} - \frac{3e^{\pi t}}{(e^{\pi t}+2e^{-\pi t})^2}(-\pi e^{-\pi t})$$

$$\frac{dz}{dt} = \frac{3\pi + 3\pi}{(e^{\pi t}+2e^{-\pi t})^2} = \frac{6\pi}{(e^{\pi t}+2e^{-\pi t})^2}$$

Ej. Use regla de la cadena para determinar dz/dt

$$z = f(x,y) = \sqrt{1+xy} = (1+xy)^{1/2} \qquad x = \tan t \qquad y = tg^{-1} t$$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(1+xy)^{-1/2}\cdot y \qquad \frac{dx}{dt} = \sec^2 t$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(1+xy)^{-1/2}x \qquad \frac{dy}{dt} = \frac{1}{1+t^2}$$

$$\frac{dz}{dt} = \left(\frac{y}{2\sqrt{1+xy}}\right)(\sec^2 t) + \left(\frac{x}{2\sqrt{1+xy}}\right)\left(\frac{1}{1+t^2}\right)$$

$$\frac{dz}{dt} = \left(\frac{\tan^{-1} t \sec^2 t}{2\sqrt{1+\tan t\cdot \tan^{-1} t}}\right) + \left(\frac{\tan t}{2(1+t^2)\sqrt{1+\tan t\cdot tg^{-1} t}}\right)$$

$\tan tg^{-1}(t)$ $\qquad \tan t \cdot tg^{-1} t$

$tg^{-1} tg(t)$

Ej. sea $p(t) = f(g(t), h(t))$ donde f es derivable,
$g(2) = 4$ $\quad g'(2) = -3$ $\quad h(2) = 5$, $\quad h'(2) = 6$, $\quad f_x(4,5) = 2$
$f_y(4,5) = 8$
Determine $p'(2)$.

$x = g(t) \qquad y = h(t)$

$$p = f(x, y)$$

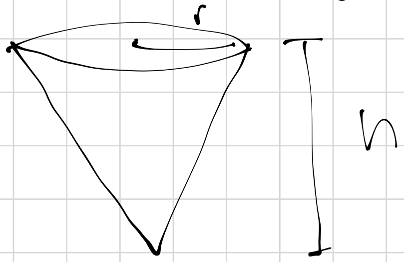
$$p'(t)\Big|_{t=2} = \frac{\partial p}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial p}{\partial y} \cdot \frac{dy}{dt}$$

$$\qquad f_x \qquad g'(t) \qquad f_y \qquad h'(t)$$

$$p'(2) = (2)(-3) + (8)(6) = -6 + 48$$

$$p'(2) = 42$$

Ej. El radio de un cono circular recto aumenta a razón de 4.6 cm/s, mientras su altura disminuye a razón de 6.5 cm/s. ¿A qué razón cambia el volumen del cono cuando el radio es de 300 cm y la altura es de 350 cm?



$$\frac{dr}{dt} = 4.6 \text{ cm/s}$$

$$\frac{dh}{dt} = -6.5 \text{ cm/s}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dV}{dt} = ?$$

$$\frac{dV}{dt} = \frac{\partial V}{\partial r}\frac{dr}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt}$$

$$\frac{dV}{dt} = \left(\frac{2}{3}\pi r h\right)\left(\frac{dr}{dt}\right) + \left(\frac{1}{3}\pi r^2\right)\left(\frac{dh}{dt}\right)$$

$$\frac{dV}{dt} = \left(\frac{2}{3}\pi(300)(350)\right)(4.6) + \left(\frac{1}{3}\pi(300)^2\right)(-6.5)$$

$$\frac{dV}{dt} = 398{,}983.2 \text{ cm}^3/\text{s}.$$

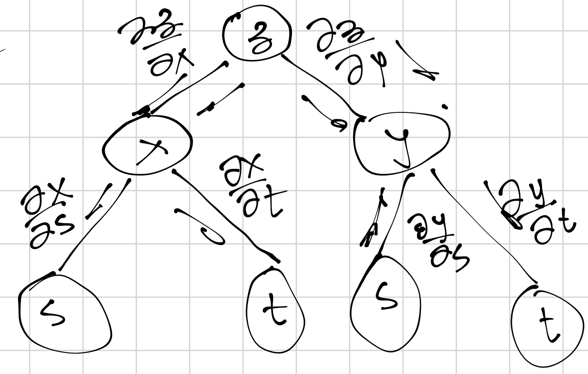
Caso II

$z = f(x,y) \qquad x = u(s,t) \qquad y = v(s,t)$

Suponga que $z = f(x,y)$ es una función derivable de x e y donde $x = g(s,t)$ $y = h(s,t)$ son funciones derivables de s y t, entonces

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$



Ej. Calcular $\dfrac{\partial z}{\partial s}$, $\dfrac{\partial z}{\partial t}$ $\qquad x^{1/2}$

$$z = \sqrt{x}\, e^{xy} \qquad x = 1+st \qquad y = s^2 - t^2$$

$$\frac{\partial z}{\partial x} = \sqrt{x}\cdot y\, e^{xy} + \frac{1}{2}x^{-1/2} e^{xy}$$

$$\frac{\partial x}{\partial s} = t \qquad \frac{\partial z}{\partial y} = \sqrt{x}\cdot x e^{xy} = x^{3/2} e^{xy}$$

$$\frac{\partial y}{\partial s} = 2s$$

$$\frac{\partial z}{\partial s} = \left(\sqrt{x}\, y e^{xy} + \frac{1}{2}x^{-1/2} e^{xy}\right)(t) + \left(x^{3/2} e^{xy}\right)(2s)$$

$$\frac{\partial z}{\partial s} = \left[\sqrt{1+st}\cdot(s^2-t^2)e^{(1+st)(s^2-t^2)} + \frac{1}{2}(1+st)^{-1/2}e^{(1+st)(s^2-t^2)}\right]t$$
$$+ 2s(1+st)^{3/2} e^{(1+st)(s^2-t^2)}$$

$$\frac{\partial x}{\partial t} = s \qquad \frac{\partial y}{\partial t} = -2t$$

$$\frac{\partial z}{\partial t} = \left(\sqrt{x}\, y e^{xy} + \frac{1}{2}x^{-1/2} e^{xy}\right)(s) + \left(x^{3/2} e^{xy}\right)(-2t)$$

$$\frac{\partial z}{\partial t} = \left[\sqrt{1+st}\cdot(s^2-t^2)\,e^{(1+st)(s^2-t^2)} + \frac{1}{2}(1+st)\,e^{-1/2\,(1+st)(s^2-t^2)}\right] s \; +$$

$$(1+st))^{3/2}\, e^{(1+st)(s^2-t^2)}\,(-2t) \quad //$$

Ej.. Dados

$$w = xy + yz + zx$$

$$x = r\cos\theta \qquad y = r\,\text{sen}\,\theta \qquad z = r\theta$$

$$\frac{\partial w}{\partial r}, \quad \frac{\partial w}{\partial \theta} \qquad \text{cuando } r=2 \quad \theta = \frac{\pi}{2}$$

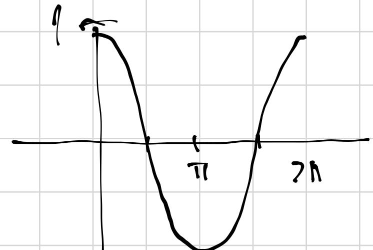
$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial x} = (y+z) \qquad \frac{\partial w}{\partial y} = (x+z) \qquad \frac{\partial w}{\partial z} = (y+x)$$

$$\frac{\partial x}{\partial r} = \cos\theta \qquad \frac{\partial y}{\partial r} = \text{sen}\,\theta \qquad \frac{\partial z}{\partial r} = \theta$$

$$\frac{\partial w}{\partial r} = (y+z)(\cos\theta) + (x+z)(\text{sen}\,\theta) + (y+x)(\theta)$$

$$\frac{\partial w}{\partial r} = (r\,\text{sen}\,\theta + r\theta)\cos\theta + (r\cos\theta + r\theta)\,\text{sen}\,\theta + (r\,\text{sen}\,\theta + r\cos\theta)\theta$$



$$\frac{\partial w}{\partial r} = \left(2\,\text{sen}\,\tfrac{\pi}{2} + 2\left(\tfrac{\pi}{2}\right)\right)\cos\tfrac{\pi}{2} + \left(2\cos\tfrac{\pi}{2} + 2\left(\tfrac{\pi}{2}\right)\right)\text{sen}\,\tfrac{\pi}{2} + \left(2\,\text{sen}\,\tfrac{\pi}{2} + 2\cos\tfrac{\pi}{2}\right)\tfrac{\pi}{2}$$

$$\frac{\partial w}{\partial r} = (2+\pi)\,0 + (\pi)(1) + (2)\left(\tfrac{\pi}{2}\right) = 2\pi \quad //$$

$$\frac{\partial w}{\partial \theta} = ?$$

$$\frac{\partial x}{\partial \theta} = -r\,\text{sen}\,\theta \qquad \frac{\partial y}{\partial \theta} = r\cos\theta \qquad \frac{\partial z}{\partial \theta} = r$$

$$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta} + \frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial \theta}$$

$$\frac{\partial x}{\partial \theta} = -2\,\text{sen}\,\frac{\pi}{2} = -2 \qquad \frac{\partial y}{\partial \theta} = 2\cos\frac{\pi}{2} = 0 \qquad \frac{\partial z}{\partial \theta} = 2$$

$$\frac{\partial w}{\partial \theta} = (2+\pi)(-2) + (\pi)(0) + 2(2) = -4 - 2\pi + 4 = -2\pi$$