3. VALORES PROPIOS COMPLEJOS:

$$\lambda_{1,2} = \alpha \pm i\beta$$

$$k_1 \, , k_2 \ldots k_n$$

$$x_G(t) = C_1(B_1 \cos(\beta t) - B_2 \sin(\beta t))e^{\alpha t} + C_2(B_2 \cos(\beta t) + B_1 \sin(\beta t))e^{\alpha t}$$

$$B_1 = \frac{1}{2}\left[k_1 + \overline{k_1}\right] \quad y \quad B_2 = \frac{i}{2}\left[k_1 - \overline{k_1}\right]$$

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Resolver El siguiente Sistema de Ecuaciones Diferenciales Homogéneo

$$X^{/} = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} x$$

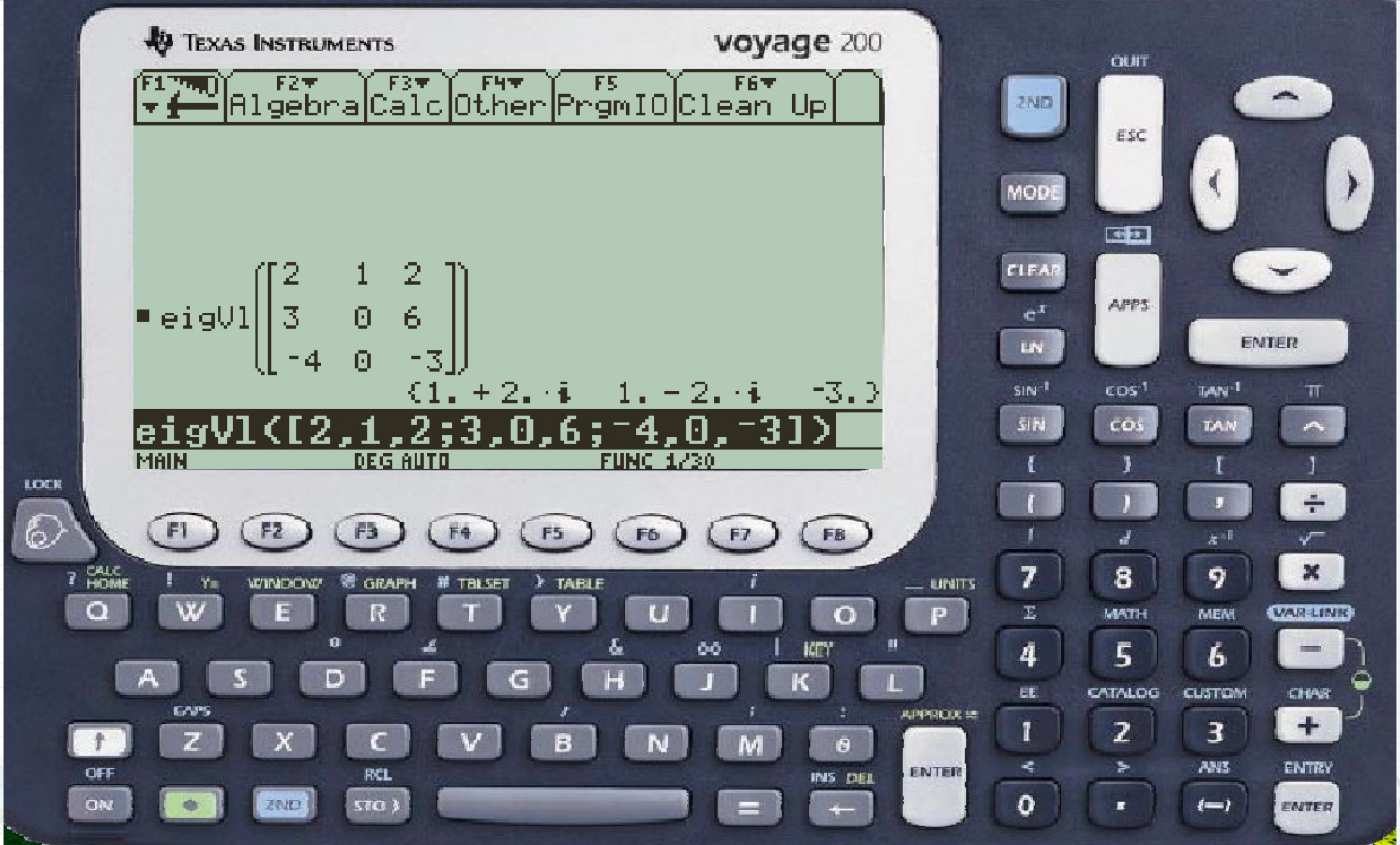
$$|a - \lambda I| = 0$$

$$\left| \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{vmatrix} = 0$$

$$-\lambda^3 - \lambda^2 + \lambda - 15 = 0$$

$$\lambda_1 = -3 \quad \lambda_{2,3} = 1 \pm 2i$$


TEXAS INSTRUMENTS
voyage 200
F1 F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
eigVl([2 1 2; 3 0 6; -4 0 -3])
{1. + 2. · i 1. - 2. · i -3.}
eigVl([2,1,2;3,0,6;-4,0,-3])
MAIN DEG AUTO FUNC 1/30
QUIT
2ND
ESC
MODE
CLEAR
APPS
ENTER
SIN
COS
TAN
MATH
MEM
CATALOG
CUSTOM
CHAR
ENTRY
ANS
ON
OFF
LOCK

Para $\lambda_1 = -3$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix} \begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2-(-3) & 1 & 2 \\ 3 & -(-3) & 6 \\ -4 & 0 & -3-(-3) \end{pmatrix} \begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 5 & 1 & 2 \\ 3 & 3 & 6 \\ -4 & 0 & 0 \end{pmatrix} \begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1/5 & 2/5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

$$k_2 + 2k_3 = 0$$

$$k_2 = -2k_3$$

$$k_1 = 0$$

$$k_1 + \left(\frac{1}{5}\right) k_2 + \left(\frac{2}{5}\right) k_3 = 0$$

$$k_1 + \left(\frac{1}{5}\right)(-2k_3) + \left(\frac{2}{5}\right) k_3 = 0$$

$$k_3 = 1$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \rightarrow K_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 K_1 e^{\lambda_1 t} = C_1 \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} e^{(-3)t}$$

Para: $\lambda_2 = 1 + 2i$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \qquad \begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2-(1+2\mathrm{i}) & 1 & 2 \\ 3 & -(1+2\mathrm{i}) & 6 \\ -4 & 0 & -3-(1+2\mathrm{i}) \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2\mathrm{i} & 1 & 2 \\ 3 & -1-2\mathrm{i} & 6 \\ -4 & 0 & -4-2\mathrm{i} \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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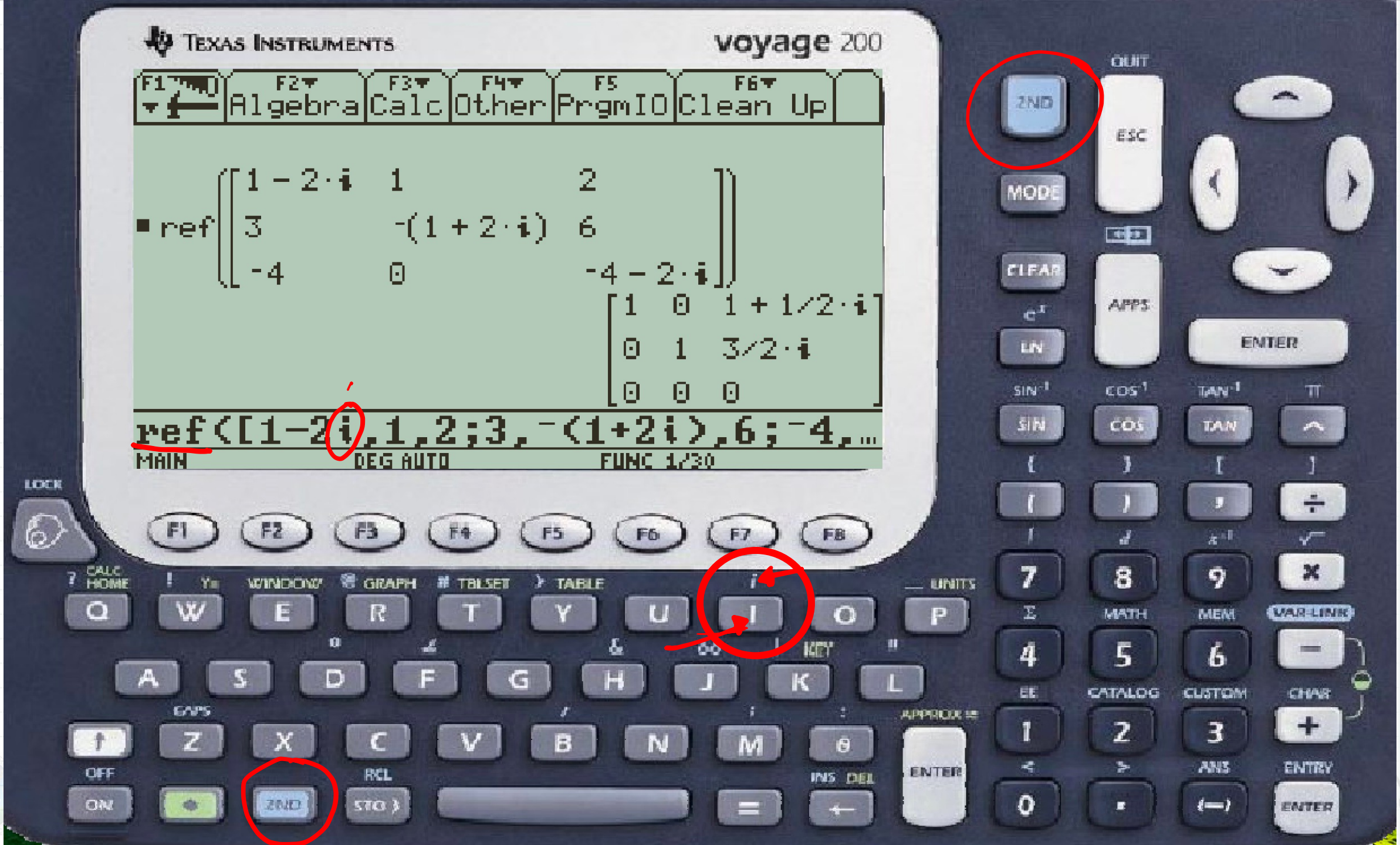
TEXAS INSTRUMENTS
voyage 200
F1 F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
ref([[1-2·i 1 2] [3 -(1+2·i) 6] [-4 0 -4-2·i]])
[[1 0 1+1/2·i] [0 1 3/2·i] [0 0 0]]
ref([1-2i,1,2;3,-(1+2i),6;-4,...
MAIN DEG AUTO FUNC 1/30
2ND
QUIT
ESC
MODE
CLEAR
APPS
ENTER
LN
SIN
COS
TAN
ENTER

Para: $\lambda_2 = 1 + 2i$

$$\vec{K}_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix}$$

$$\begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix} \cdot \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2-(1+2i) & 1 & 2 \\ 3 & -(1+2i) & 6 \\ -4 & 0 & -3-(1+2i) \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2i & 1 & 2 \\ 3 & -1-2i & 6 \\ -4 & 0 & -4-2i \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1+\frac{1}{2}i \\ 0 & 1 & \frac{3}{2}i \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_5 + \left(\frac{3}{2}i\right) k_6 = 0$$

$$k_4 + \left(1 + \frac{1}{2}i\right) k_6 = 0$$

$$k_5 = -\left(\frac{3}{2}i\right) k_6$$

$$k_4 = -\left(1 + \frac{1}{2}i\right) k_6$$

$$k_6 = 1$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \rightarrow K_2 = \begin{pmatrix} -1 - \frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} -1 - \frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix}$$

$$x_2(t) = C_2(B_1 \cos(\beta t) - B_2 \sin(\beta t))e^{\alpha t} + C_3(B_2 \cos(\beta t) + B_1 \sin(\beta t))e^{\alpha t}$$

$$B_1 = \frac{1}{2}[k_2 + \overline{k_2}] \quad y \quad B_2 = \frac{i}{2}[k_2 - \overline{k_2}]$$

$$B_1 = \frac{1}{2}\left[\begin{pmatrix} -1 - \frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix} + \begin{pmatrix} -1 + \frac{1}{2}i \\ \frac{3}{2}i \\ 1 \end{pmatrix}\right] = \frac{1}{2}\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$$B_2 = \frac{i}{2}\left[\begin{pmatrix} -1 - \frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix} - \begin{pmatrix} -1 + \frac{1}{2}i \\ \frac{3}{2}i \\ 1 \end{pmatrix}\right] = \frac{i}{2}\begin{pmatrix} -i \\ -3i \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{i^2}{2} \\ -\frac{3i^2}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 3/2 \\ 0 \end{pmatrix}$$

$$\vec{K}_2 = \begin{pmatrix} -1 - \frac{1}{2}i \\ 0 - \frac{3}{2}i \\ 1 + 0 \end{pmatrix} \qquad K_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{2}i \\ -\frac{3}{2}i \\ 0 \end{pmatrix}$$

$$K_2 = \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}}_{Re} + i\underbrace{\begin{pmatrix} -\frac{1}{2} \\ -\frac{3}{2} \\ 0 \end{pmatrix}}_{Im}$$

$$B_1 = Re[k_2] \quad y \quad B_2 = Im[k_2]$$

$$B_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \qquad B_2 = \begin{pmatrix} 1/2 \\ 3/2 \\ 0 \end{pmatrix}$$

$$\lambda_2 = \alpha + i\beta = \overset{1}{\tilde{\alpha}} + i\overset{2}{\tilde{\beta}}$$

$$x_2(t) = C_2(B_1\, cos(\beta t) - sin(\beta t))e^{\alpha t} + C_3(B_2\, cos(\beta t) + B_1\, sin(\beta t))e^{\alpha t}$$

$$x_2(t) = C_2\left(\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} cos(2t) - \begin{pmatrix} 1/2 \\ 3/2 \\ 0 \end{pmatrix} sin(2t)\right)e^t + C_3\left(\begin{pmatrix} 1/2 \\ 3/2 \\ 0 \end{pmatrix} cos(2t) + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} sin(2t)\right)e^t$$

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ECUACIONES DIFERENCIALES DE COEFICIENTES VARIABLES POR MEDIO DE SERIES DE POTENCIA

- $(x^2+1)\frac{d^2y}{dx^2} + \ln(x)\frac{dy}{dx} + e^x y = 0 \leftarrow$

(*) $y = \sum_{n=0}^{\infty} C_n x^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots$

$y' = \sum_{n=1}^{\infty} C_n n x^{n-1} = C_1 + 2C_2 x + 3C_3 x^2 + \cdots$

$y'' = \sum_{n=2}^{\infty} C_n n(n-1) x^{n-2}$

$y''' = \sum_{n=3}^{\infty} C_n n(n-1)(n-2) x^{n-3}$