Caso 2

Resolver El siguiente Sistema de Ecuaciones Diferenciales Homogéneo:

$$\boldsymbol{X'} = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}\boldsymbol{X}$$

$$|\boldsymbol{a} - \boldsymbol{\lambda I}| = 0 \qquad \left|\begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{vmatrix} = 0 \qquad -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

$$\lambda_1 = 1 \ , \qquad \lambda_{2,3} = 2$$

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$$\underline{\boldsymbol{Para\ \lambda_1 = 1:}}$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$$

$$\begin{pmatrix} 3-\lambda & -1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3-1 & -1 & -1 \\ 1 & 1-1 & -1 \\ 1 & -1 & 1-1 \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

$$k_2 - k_3 = 0$$

$$\boldsymbol{k_2 = k_3}$$

$$k_1 - \left(\frac{1}{2}\right)k_2 - \left(\frac{1}{2}\right)k_3 = 0$$

$$\boldsymbol{k_1 = k_3}$$

$$\boldsymbol{k_3 = 1}$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \rightarrow K_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 K_1 e^{\lambda_1 t} = C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{(1)\,t}$$

$$\underline{Para \quad \lambda_2 = 5}$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix}$$

$$\begin{pmatrix} 3-\lambda & -1 & -1 \\ 1 & 1-\lambda & -1 \\ 1 & -1 & 1-\lambda \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 3-2 & -1 & -1 \\ 1 & 1-2 & -1 \\ 1 & -1 & 1-2 \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_4 - k_5 - k_6 = 0$$

$$k_4 = k_5 + k_6$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \rightarrow K_2 = \begin{pmatrix} k_5 + k_6 \\ k_5 \\ k_6 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} k_5 \\ k_5 \\ 0 \end{pmatrix} + \begin{pmatrix} k_6 \\ 0 \\ k_6 \end{pmatrix}$$

$$k_5 \; y \; k_6 = 1$$

$$K_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$x_2(t) = C_2 K_2 e^{\lambda_2 t} = C_2 \left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right] e^{2t}$$

3. VALORES PROPIOS COMPLEJOS:

$$\lambda_{1,2} = \alpha \pm i\beta$$

$$k_1\ , k_2 \dots k_n$$

$$x_G(t) = C_1(B_1 \cos(\beta t) - B_2 \sin(\beta t))e^{\alpha t} + C_2(B_2 \cos(\beta t) + B_1 \sin(\beta t))e^{\alpha t}$$

$$B_1 = \frac{1}{2}\left[k_1 + \overline{k_1}\right] \quad y \quad B_2 = \frac{i}{2}\left[k_1 - \overline{k_1}\right]$$

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$$\boldsymbol{X}^{/} = \begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} & \mathbf{6} \\ \mathbf{-4} & \mathbf{0} & \mathbf{-3} \end{pmatrix} \boldsymbol{x}$$

$$|\boldsymbol{a} - \boldsymbol{\lambda I}| = 0$$

$$\left| \begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} & \mathbf{6} \\ \mathbf{-4} & \mathbf{0} & \mathbf{-3} \end{pmatrix} - \lambda \begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix} \right| = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{2 - \lambda} & \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{-\lambda} & \mathbf{6} \\ \mathbf{-4} & \mathbf{0} & \mathbf{-3 - \lambda} \end{vmatrix} = \mathbf{0}$$

$$-\boldsymbol{\lambda}^3 - \boldsymbol{\lambda}^2 + \boldsymbol{\lambda} - 15 = 0$$

$$\boldsymbol{\lambda}_1 = -3 \quad \boldsymbol{\lambda}_{2\,,3} = 1 \pm 2\boldsymbol{i}$$

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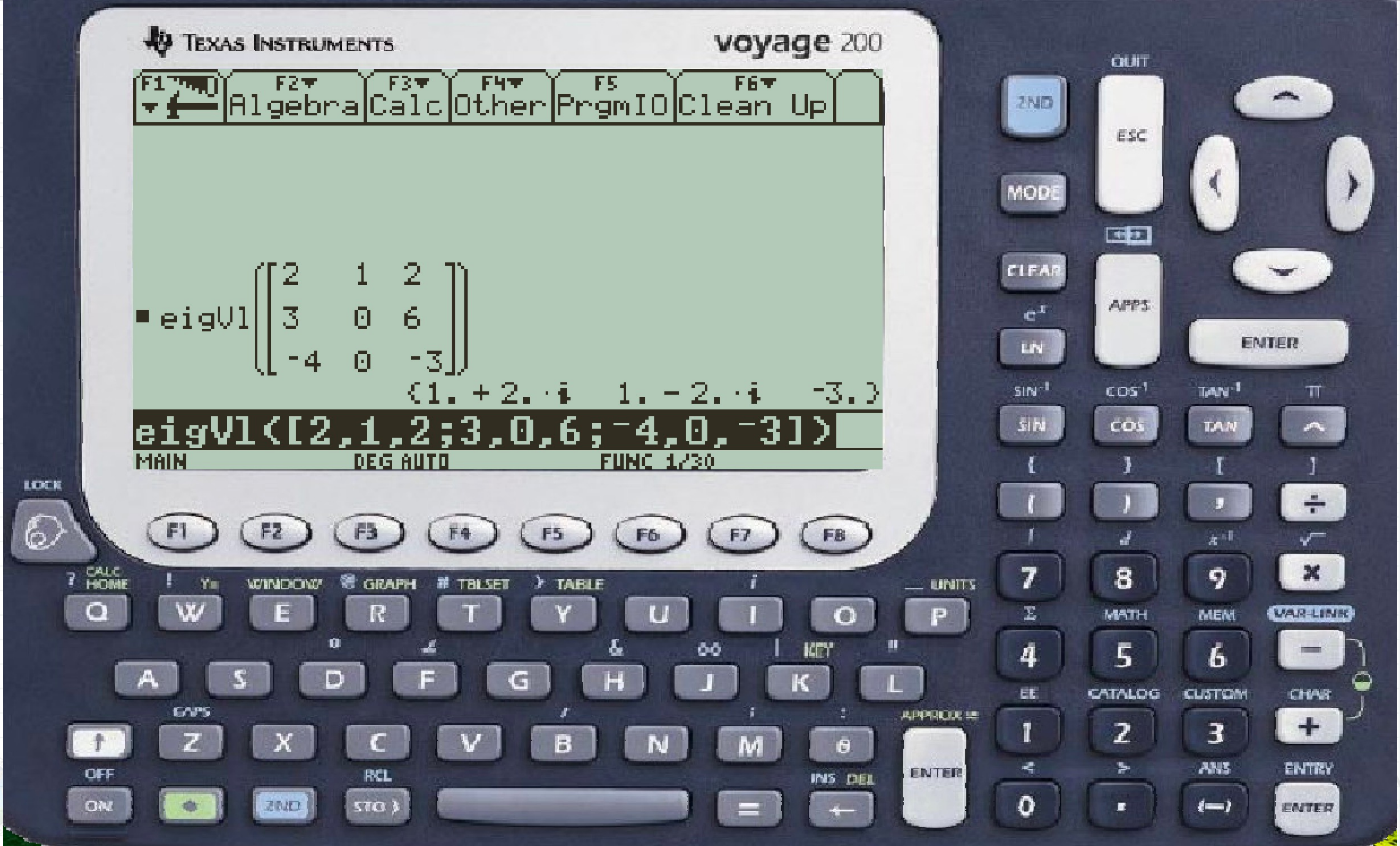
TEXAS INSTRUMENTS
voyage 200
F2▼ Algebra
F3▼ Calc
F4▼ Other
F5 PrgmIO
F6▼ Clean Up
eigVl([2 1 2; 3 0 6; -4 0 -3])
{1. + 2.·i 1. − 2.·i -3.}
eigVl([2,1,2;3,0,6;-4,0,-3])
MAIN
DEG AUTO
FUNC 1/30

$\underline{Para\ \lambda_1 = -3}$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \qquad \begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2-(-3) & 1 & 2 \\ 3 & -(-3) & 6 \\ -4 & 0 & -3-(-3) \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 5 & 1 & 2 \\ 3 & 3 & 6 \\ -4 & 0 & 0 \end{pmatrix}\begin{pmatrix} k1 \\ k2 \\ k3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1/5 & 2/5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

$$k_2 + 2k_3 = 0$$

$$k_2 = -2k_3$$

$$k_1 = 0$$

$$k_1 + \left(\frac{1}{5}\right)k_2 + \left(\frac{2}{5}\right)k_3 = 0$$

$$k_1 + \left(\frac{1}{5}\right)(-2k_3) + \left(\frac{2}{5}\right)k_3 = 0$$

$$k_3 = 1$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \rightarrow K_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1K_1e^{\lambda_1 t} = C_1\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}e^{(-3)t}$$

$\underline{\boldsymbol{Para}\text{:}\ \boldsymbol{\lambda_2} = 1 + 2\boldsymbol{i}}$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \qquad \begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2-(1+2\mathrm{i}) & 1 & 2 \\ 3 & -(1+2\mathrm{i}) & 6 \\ -4 & 0 & -3-(1+2\mathrm{i}) \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2\mathrm{i} & 1 & 2 \\ 3 & -1-2\mathrm{i} & 6 \\ -4 & 0 & -4-2\mathrm{i} \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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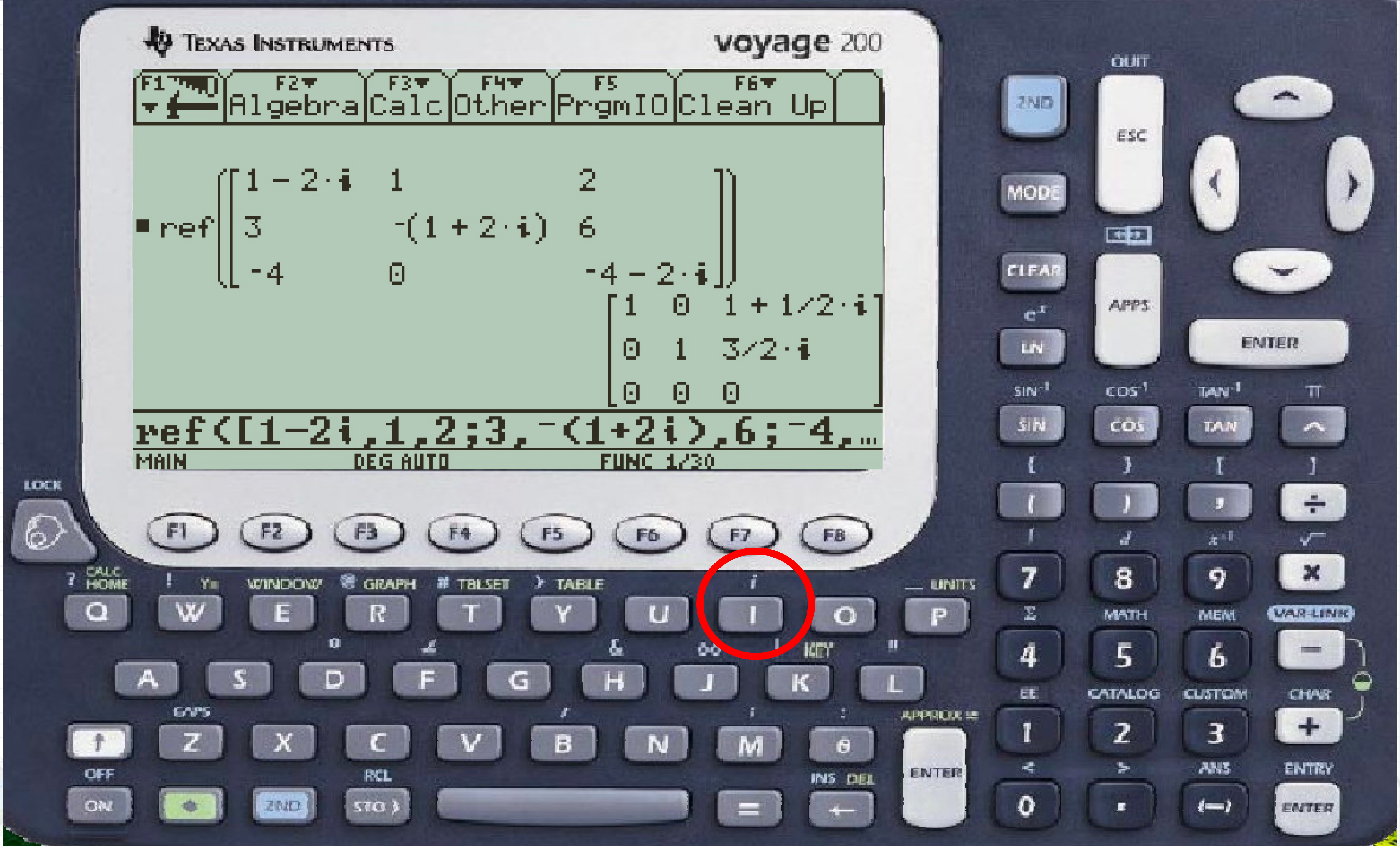
TEXAS INSTRUMENTS
voyage 200
F1 F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
■ ref([1 − 2·i 1 2; 3 −(1 + 2·i) 6; −4 0 −4 − 2·i])
[1 0 1 + 1/2·i; 0 1 3/2·i; 0 0 0]
ref([1−2i,1,2;3,−(1+2i),6;−4,…
MAIN DEG AUTO FUNC 1/30

$\underline{Para:\ \lambda_2 = 1 + 2i}$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \qquad \begin{pmatrix} 2-\lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3-\lambda \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2-(1+2i) & 1 & 2 \\ 3 & -(1+2i) & 6 \\ -4 & 0 & -3-(1+2i) \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-2i & 1 & 2 \\ 3 & -1-2i & 6 \\ -4 & 0 & -4-2i \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1 & 0 & 1+\frac{1}{2}i \\ 0 & 1 & \frac{3}{2}i \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_5 + \left(\frac{3}{2}i\right)k_6 = 0 \qquad k_4 + \left(1+\frac{1}{2}i\right)k_6 = 0$$

$$k_5 = -\left(\frac{3}{2}i\right)k_6 \qquad k_4 = -\left(1+\frac{1}{2}i\right)k_6$$

$$k_6 = 1$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \rightarrow K_2 = \begin{pmatrix} -1-\frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix}$$

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