Ley de enfriamiento de Newton

$$\frac{dT}{dt} \propto T - T_a \qquad T_a = \text{tenperatura ambiente.}$$

$$\frac{dT}{dt} = k(T - T_a)$$

$$\int \frac{dT}{T-T_a} = \int k\,dt \rightarrow e^{\ln(T-T_a)} = e^{kt+C} = e^{kt}\cdot \cancel{e}^{C}$$

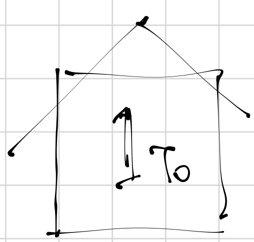
$$T - T_a = Ce^{kt}$$

$$T = T_a + Ce^{kt} \longrightarrow T(0) = T_0$$

$$T_0 = T_a + C\cancel{e^{k(0)}} \rightarrow C = T_0 - T_a$$

$$\boxed{T = T_a + (T_0 - T_a)e^{kt}}$$

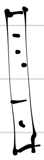
Ej. Un termometro se lleva de una habitación hasta el ambiente exterior, donde la temperatura del aire es de 5°F. Despues de 1 minu el termometro



$\rightarrow T_a = 5°F$

$T = 55°F$

$t = 1min$



$T = 30°F$

$t = 5min$

t	T
0	T_0
1	55
5	30°F

$$T = T_a + Ce^{kt}$$

$t = 0 \quad T = T_0$

$$T_0 = 5 + C\cancel{e^{k(0)}} \rightarrow \boxed{T_0 = 5 + C} \quad (1)$$

$t = 1min$

$T = 55$

$$55 = 5 + Ce^{k(1)} \rightarrow 50 = Ce^{k} \rightarrow \boxed{C = \frac{50}{e^{k}}} \quad (2)$$

$t = 5\,min$
$T = 30$

$30 = 5 + Ce^{5k} \rightarrow 25 = Ce^{5k}$

$\boxed{C = \frac{25}{e^{5k}}}$ (3).

(2) = (3)

$\frac{50}{e^{k}} = \frac{25}{e^{5k}}$

$\frac{e^{5k}}{e^{k}} = \frac{25}{50} = \frac{1}{2} \rightarrow e^{5k} \cdot e^{-k} = \frac{1}{2}$

$\ln e^{4k} = \ln \frac{1}{2} \rightarrow 4k = \ln 1/2$

$k = \frac{\ln 1/2}{4} = -0.17329$

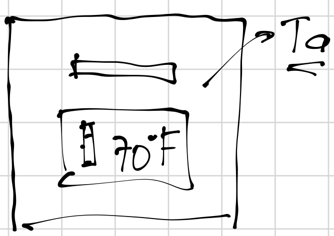
(2)

$C = \frac{50}{e^{-0.17329}} = 59.46$

(1) $T_0 = 5 + C \rightarrow T_0 = 5 + 59.46$

$\boxed{T_0 = 64.46\,°F}$

Ej. Un termometro que indica 70°F se coloca en un horno precalentado a una temperatura constante. A traves de una ventana de vidrio en la puerta del horno un observador registra que el termometro lee 110°F despues de 1/2 minuto, y 145°F despues de 1 minuto. ¿Cual es la temperatura del horno.



t	T °F
0	70
1/2	110
1	145

$\frac{1}{2}\,min - T = 110°F$

$1\,min \rightarrow T = 145°F$

$T = T_a + Ce^{kt}$

$t = 0 \quad T = 70$

$70 = T_a + Ce^{k(0)} \rightarrow \boxed{70 = T_a + C}$ (1)

$t = 1/2\,min \quad T = 110°F$

$110 = T_a + Ce^{k(1/2)}$ (2)

$t = 1\,\text{min} \quad T = 145°F$

$$145 = T_a + C e^{k(1)} \quad (3)$$

(1) $C = 70 - T_a$

$$110 = T_a + (70 - T_a) e^{1/2 k}$$

$$110 = T_a + 70 e^{1/2 k} - T_a e^{1/2 k}$$

$$110 - 70 e^{1/2 k} = T_a(1 - e^{1/2 k}) \rightarrow T_a = \frac{110 - 70 e^{1/2 k}}{(1 - e^{1/2 k})}$$

(2) $145 = T_a + (70 - T_a) e^{k}$

$$145 = T_a + 70 e^{k} - T_a e^{k}$$

$$145 - 70 e^{k} = T_a(1 - e^{k}) \rightarrow T_a = \frac{145 - 70 e^{k}}{1 - e^{k}}$$

(1) = (2) $\quad T_a = T_a$

$$\frac{110 - 70 e^{1/2 k}}{1 - e^{1/2 k}} = \frac{145 - 70 e^{k}}{1 - e^{k}}$$

$$(110 - 70 e^{1/2 k})(1 - e^{k}) = (145 - 70 e^{k})(1 - e^{1/2 k})$$

$$\cancel{110} - 110 e^{k} - 70 e^{1/2 k} + \cancel{70 e^{3/2 k}} = \cancel{145} - 145 e^{1/2 k} - \cancel{70 e^{k}} + \cancel{70 e^{3/2 k}}$$

$$-35 - 40 e^{k} + 75 e^{1/2 k} = 0 \quad) \times -\frac{1}{5}$$

$$8 e^{k} - 15 e^{1/2} + 7 = 0 \quad \rightarrow \quad u = e^{1/2 k}$$

$$u^2 = e^{k}$$

$$\begin{array}{ll|l} 8e^{1/2k} & -7 & -8e^{1/2k} \\ e^{1/2k} & -1 & -7e^{1/2k} \\ & & -15 e^{1/2k} \end{array}$$

$$(8 e^{1/2 k} - 7)(e^{1/2 k} - 1) = 0$$

$$(8 e^{1/2 k} - 7) = 0 \rightarrow 8 e^{1/2 k} = 7 \quad \ln e^{1/2 k} = \ln \frac{7}{8}$$

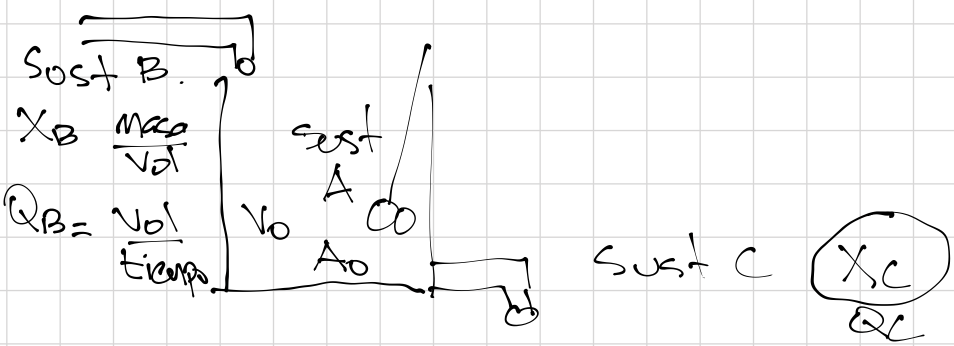
$$\frac{1}{2} k = \ln 7/8$$

$$k = -0.26706$$

$$T_a = \frac{145 - 70e^{k}}{1 - e^{k}}$$

$$T_a = \frac{145 - 70e^{-0.26706}}{1 - e^{-0.26706}} = 390$$

Mezclas



$$\frac{dA}{dt} = \left[\text{Cantidad de sust que entra} - \text{Cantidad de sust. que sale}\right]$$

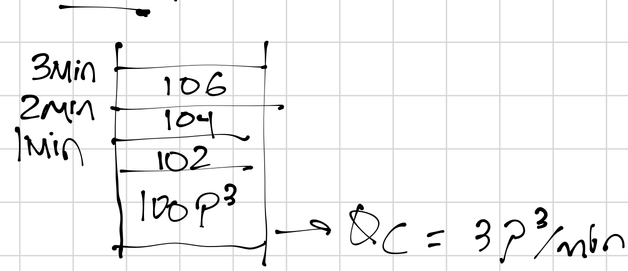
$$\frac{dA}{dt} = R_1 - R_2 \qquad R_1 = X_B Q_B$$

$$R_2 = X_C Q_C$$

$$\frac{dA}{dt} = X_B Q_B - X_C Q_C \rightarrow \text{Ec. lineal.}$$

Caso 1 $\quad Q_B = Q_C$ $\qquad Q_B = 5p^3/min$

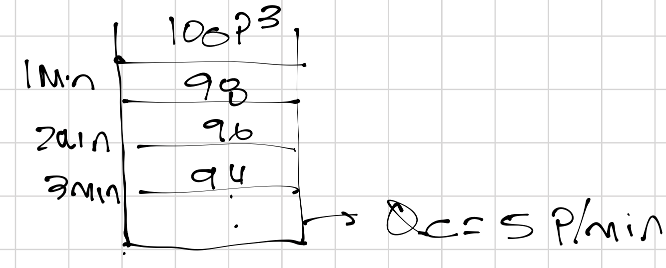
$$X_C = \frac{A}{V_0}$$



Caso 2 $\quad Q_B > Q_C$

$$X_C = \frac{A}{V_0 + (Q_B - Q_C)t}$$

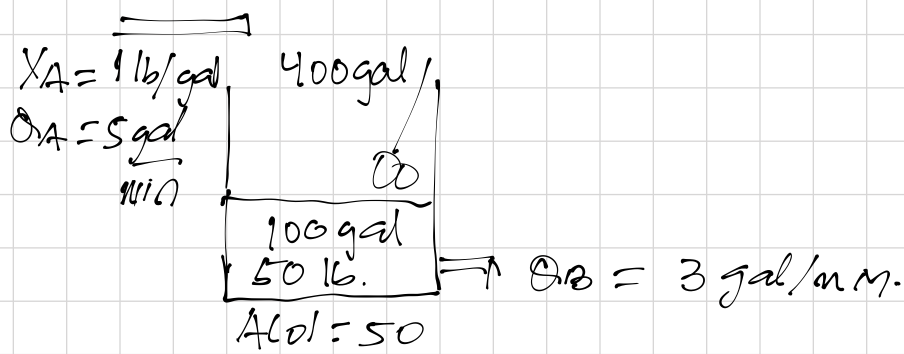
$Q_B = 3p^3/min$



Caso 3 $\quad Q_B < Q_C$

$$x_C = \frac{A}{V_0 + (Q_B - Q_C)t}$$

Ej. Inicialmente, un tanque de 400 gal contiene 100 gal de Salmuera con 50 lb de sal. Salmuera con 1 lb de sal por galón entra al tanque a razón de 5 gal/min y la mezcla total de salmuera del recipiente sale a razón de 3 gal/min. ¿Cuánta sal contendrá el tanque cuando esté completamente lleno de salmuera?



$$x_B = \frac{A}{V_0}$$

$$V = V_0 + (Q_A - Q_B)t = 100 + (5-3)t$$

$$V = 100 + 2t \qquad V = 400\,gal$$

$$400 = 100 + 2t \rightarrow 300 = 2t$$

$$t = \frac{300}{2} = 150 \text{ minutos.}$$

$$x_B = \frac{A}{V_0 + (Q_A - Q_B)t} = \frac{A}{100 + 2t}$$

$$\frac{dA}{dt} = R_1 - R_2 = x_A Q_A - x_B Q_B$$

$$\frac{dA}{dt} = (1\,lb/gal)(5\,gal/min) - (3\,gal/min)\left(\frac{A}{100+2t}\right)\frac{lb}{gal}$$

$$\frac{dA}{dt} = 5 - \frac{3A}{100+2t}$$

$$\frac{dA}{dt} + \frac{3A}{\underset{2(50+t)}{100+2t}} = 5$$

$$\frac{dA}{dt} + \frac{3}{2}\,\frac{A}{50+t} = 5$$

$$F.I. = e^{\int P(t)\,dt} = e^{\frac{3}{2}\int \frac{dt}{50+t}} = e^{\frac{3}{2}\ln(50+t)}$$

$$F.I. = e^{\ln(50+t)^{3/2}}$$

$$F.I. = (50+t)^{3/2}$$

$$\frac{d}{dt}\left[(50+t)^{3/2} A\right] = 5(50+t)^{3/2}$$

$u = 50+t$

$du = dt$

$$\int d\left[(50+t)^{3/2} A\right] = \int 5(50+t)^{3/2}\,dt$$

$5\int u^{3/2}\,du$

$5\, \frac{u^{3/2+1}}{\frac{3}{2}+1}$

$\frac{5}{2}$

$$(50+t)^{3/2} A = \cancel{5}\, \frac{(50+t)^{5/2}}{\frac{5}{2}} + C$$

$$(50+t)^{3/2} A = 2(50+t)^{5/2} + C$$

$$A = \frac{2(50+t)^{5/2} + C}{(50+t)^{3/2}}$$

$$A = 2(50+t)^{5/2} \cdot (50+t)^{-3/2} + C(50+t)^{-3/2}$$

$$A = 2(50+t) + C(50+t)^{-3/2}$$

$A(0) = 50$ lb.

$$50 = 2(50+0) + C(50+0)^{-3/2}$$

$$-50 = C(50)^{-3/2} \rightarrow C = \frac{-50}{50^{-3/2}}$$

$$C = -50 \cdot 50^{3/2} = -50^{5/2}$$

$$\boxed{A(t) = 2(50+t) - 50^{5/2}(50+t)^{-3/2}}$$

$$A(150) = 2(50+150) - 50^{5/2}(50+150)^{-3/2}$$

$$A(150) = 500 - \qquad = 393.75 \text{ lbs.}$$