

DERIVADA DE TRANSFORMADA



$$\begin{aligned} \blacksquare \frac{d}{ds} [p\{f(t)\}] &= \int_0^{\infty} \frac{d}{ds} [\bar{e}^{st}] f(t) dt = \int_0^{\infty} -\bar{e}^{st} \underbrace{t f(t)} dt \\ -\frac{d}{ds} [p\{f(t)\}] &= \int_0^{\infty} \bar{e}^{st} t f(t) dt \end{aligned}$$

$$\blacksquare -\frac{d}{ds^2} [p\{f(t)\}] = \int_0^{\infty} \frac{d}{ds} [\bar{e}^{st}] \cdot t f(t) dt = -\int_0^{\infty} \bar{e}^{st} t^2 f(t) dt$$

$$\frac{d}{ds^2} [p\{f(t)\}] = \int_0^{\infty} \bar{e}^{st} t^2 f(t) dt$$

$$\blacksquare \frac{d}{ds^3} [p\{f(t)\}] = \int_0^{\infty} \frac{d}{ds} [\bar{e}^{st}] t^2 f(t) dt = -\int_0^{\infty} \bar{e}^{st} t^3 f(t) dt$$

$$-\frac{d}{ds^3} [p\{f(t)\}] = \int_0^{\infty} \bar{e}^{st} t^3 f(t) dt$$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d}{ds^n} [\mathcal{L}\{f(t)\}]$$



Calcular $\mathcal{L}\{f(t)\}$ de :

1) $f(t) = t^2 \cdot e^t$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{t^2\} \quad s \rightarrow s-1$$

$$F(s) = \frac{2}{s^3} \quad | \quad s \rightarrow s-1$$

$$F(s) = \frac{2}{(s-1)^3} \quad \text{✓}$$

$$\mathcal{L}\{f(t)\} = (-1)^2 \frac{d}{ds^2} [\mathcal{L}\{e^t\}]$$

$$F(s) = \frac{d}{ds^2} \left[\frac{1}{s-1} \right]$$

$$F(s) = \frac{d}{ds} \left[\frac{-1}{(s-1)^2} \right]$$

$$F(s) = \frac{2}{(s-1)^3} \quad \text{✓}$$

Calcular $\mathcal{L}\{f(t)\}$ de :

2) $f(t) = t \cdot \cos(2t)$

$$\mathcal{L}\{f(t)\} = (-1)' \frac{d}{ds} [\mathcal{L}\{\cos(2t)\}]$$

$$F(s) = - \frac{d}{ds} \left[\frac{s}{s^2 + 4} \right]$$

$$F(s) = - \left[\frac{(1)(s^2 + 4) - s(2s)}{(s^2 + 4)^2} \right]$$

$$F(s) = - \left[\frac{4 - s^2}{(s^2 + 4)^2} \right] = \frac{s^2 - 4}{(s^2 + 4)^2}$$



Calcular $\mathcal{L}\{f(t)\}$ de :

3) $f(t) = t \cdot e^{3t} \cdot \sin(2t)$

$$\mathcal{L}\{f(t)\} = (-1)' \frac{d}{ds} [\mathcal{L}\{\sin(2t)\}] \Big|_{s \rightarrow s-3}$$

$$F(s) = -\frac{d}{ds} \left[\frac{2}{s^2+4} \right] \Big|_{s \rightarrow s-3}$$

$$F(s) = - \left[\frac{(0)(s^2+4) - 2(2s)}{(s^2+4)^2} \right] \Big|_{s \rightarrow s-3}$$

$$F(s) = \frac{4s}{(s^2+4)^2} \Big|_{s \rightarrow s-3} = \frac{4(s-3)}{((s-3)^2+4)^2}$$

