Mathematics

Introduction

- Mathematics is the abstract study of topics encompassing quantity, structure, space, change, and other properties. Mathematicians seek out patterns and formulate new conjectures. Mathematicians resolve the truth or falsity of conjectures by mathematical proof.
- It has become customary to view mathematical research as establishing truth by rigorous deduction from appropriately chosen axioms and definitions.
- There is not even consensus on whether mathematics is an art or a science. A great many professional mathematicians take no interest in a definition of mathematics, or consider it undefinable.

Equations

- An equation, in a mathematical context, is generally understood to mean a mathematical statement that asserts the equality of two expressions. For example:

$$x + 3 = 5$$

- Asserts that x+3 is equal to 5. The = symbol was invented by Robert Recorde (1510-1558), who considered that nothing could be more equal than parallel straight lines with the same length.
- Equations often express relationships between given quantities, the knowns, and quantities yet to be determined, the unknowns. By convention, unknowns are denoted by letters at the end of the alphabet, x, y, z, w, …, while knowns are denoted by letters at the beginning, a, b, c, d, … .

Equations

TYPES OF EQUATIONS

- An **algebraic equation or polynomial equation** is an equation in which a polynomial is set equal to another polynomial. These equations are further classified by their degree:
  - A linear equation has degree one
  - Quadratic equation has degree two
  - Cubic equation has degree three
  - Quartic equation has degree four
- A **Diophantine equation** is an equation where the unknowns are required to be integers.

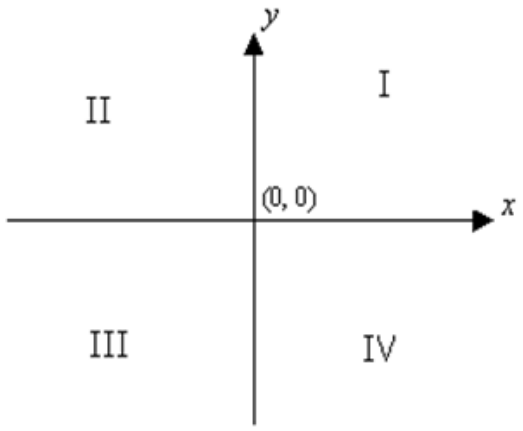
Equations

- An **indeterminate equation** has an infinite set of solutions, which only give one variable in terms of the others.
- A **transcendental equation** is an equation involving a transcendental function of one of its variables.
- A **functional equation** is an equation in which the unknowns are functions rather than simple quantities.
- A **differential equation** is an equation involving derivatives.
- An **integral equation** is an equation involving integrals.
- A **parametric equation** includes variables which are all functions of one or more common variables (called parameters).

Functions and Graphs

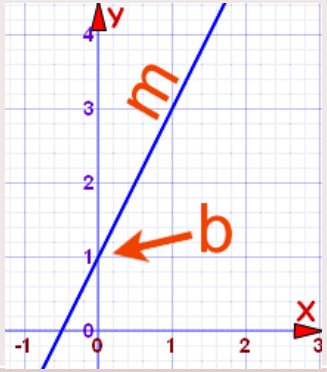
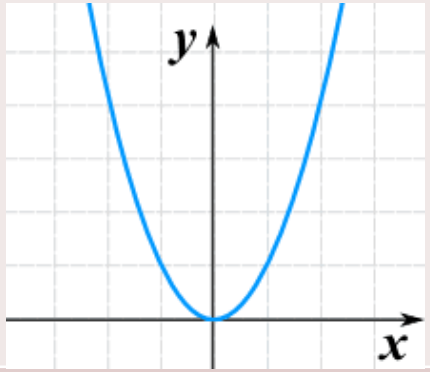
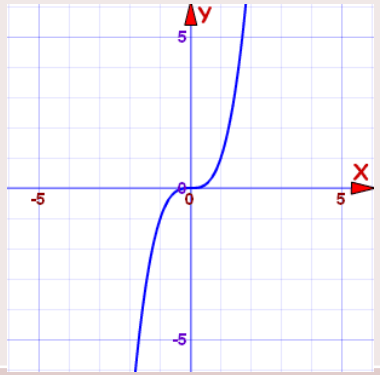
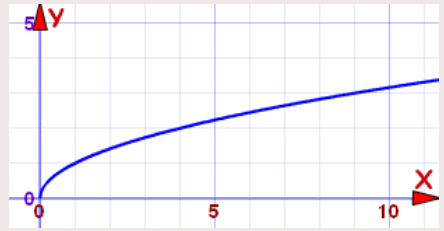
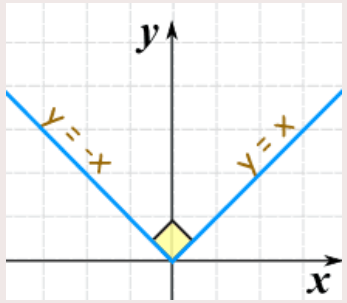
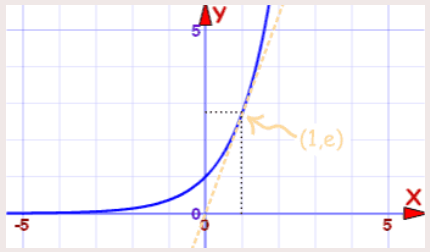
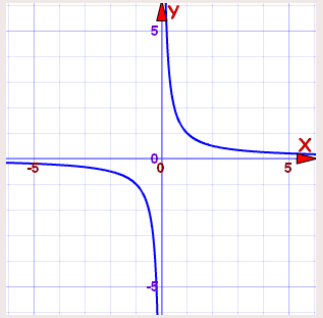
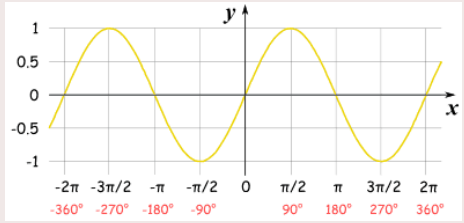
- The graph of a function f is the set of all points in the plane of the form (x, f(x)). We could also define the graph of f to be the graph of the equation y = f(x). So, the graph of a function is a special case of the graph of an equation.
- A good way of presenting a function is by graphical representation. Graphs give us a visual picture of the function. The most common way to graph a function is to use the **rectangular coordinate system**. This consists of:
  - The x-axis;
  - The y-axis;
  - The origin (0,0); and
  - The four quadrants, normally labelled I, II, III and IV.

Functions and Graphs



- Normally, the values of the independent variable (generally the x-values) are placed on the horizontal axis, while the values of the dependent variable (generally the y-values) are placed on the vertical axis.
  - The x-value, called the abscissa, is the perpendicular distance of P from the y-axis.
  - The y-value, called the ordinate, is the perpendicular distance of P from the x-axis.
  - The values of x and y together, written as (x, y) are called the coordinates of the point

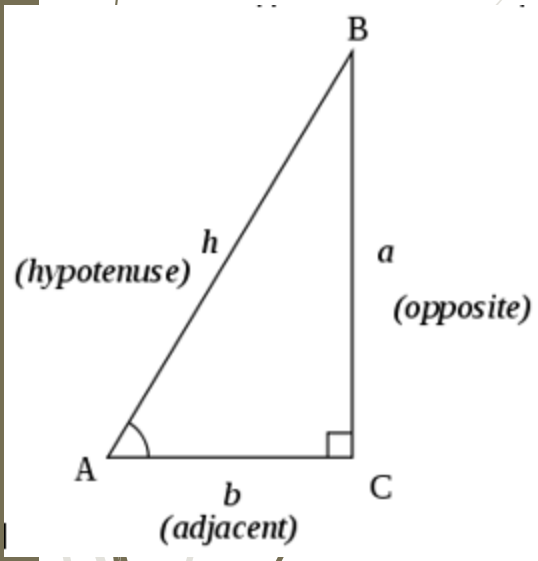
Commonly used functions

FUNCTIONS			
Linear	**Square**	**Cube**	**Square Root**
			
Absolute value function	**Exponential**	**Reciprocal**	**Sine**
			

Trigonometric Functions

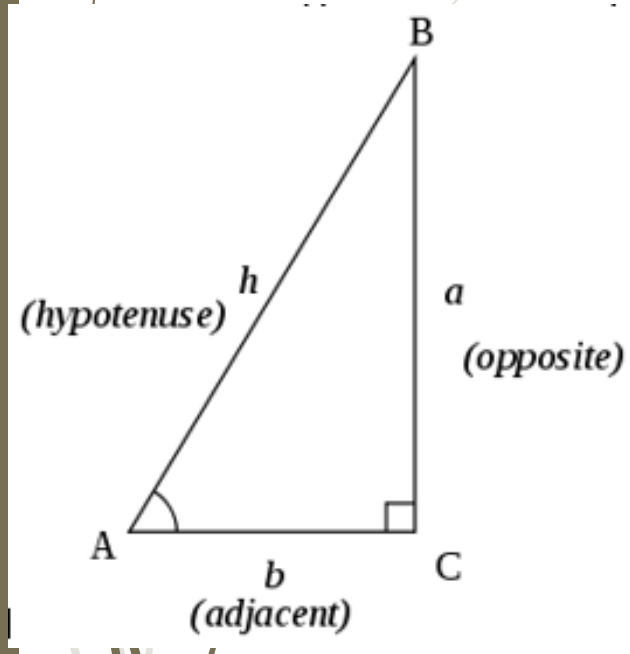
- They are used to relate the angles of a triangle to the lengths of the sides of a triangle. Trigonometric functions are important in the study of triangles and modeling periodic phenomena, among many other applications.
- Trigonometric functions are used, for instance, in navigation, engineering, and physics. A common use in elementary physics is resolving a vector into Cartesian coordinates. The sine and cosine functions are also commonly used to model periodic function phenomena such as sound and **light waves, the position and velocity of harmonic oscillators, sunlight intensity and day length, and average temperature** variations through the year.

Trigonometric Functions



- To define the trigonometric functions for the angle A, start with any right triangle that contains the angle A.
- The hypotenuse is the side opposite the right angle, in this case side h. The hypotenuse is always the longest side of a right-angled triangle.
- The opposite side is the side opposite to the angle we are interested in (angle A), in this case side a.
- The adjacent side is the side having both the angles of interest (angle A and right-angle C), in this case side b.

Trigonometric Functions



Function	Abbreviation	Description
Sine	Sin	opposite / hypotenuse
Cosine	Cos	adjacent / hypotenuse
Tangent	tan (or tg)	opposite / adjacent
Cotangent	cot (or cotan or cotg or ctg or ctn)	adjacent / opposite
Secant	Sec	hypotenuse / adjacent
Cosecant	csc (or cosec)	hypotenuse / opposite