

Calcular la Transformada de Laplace $\mathcal{L}\{f(t)\}$ de :

$$4) f(t) = \cos^2(t) = \frac{1 + \cos(2t)}{2} = \frac{1}{2} + \frac{1}{2} \cdot \cos(2t)$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\left\{\frac{1}{2}\right\} + \frac{1}{2} \mathcal{L}\{\cos(2t)\}$$

$\cos(kt); k=2$

$$F(s) = \frac{1/2}{s} + \frac{1}{2} \left[\frac{s}{s^2 + 4} \right]$$

Calcular la Transformada de Laplace $\mathcal{L}\{f(t)\}$ de :

5) $f(t) = \sin(t) \cos(t)$;

$\text{SEN}(2x) = 2 \text{SEN}(x) \cos(x)$

$\frac{\text{SEN}(2t)}{2} = \text{SEN}(t) \cos(t)$

$\mathcal{L}\{f(t)\} = \frac{1}{2} \mathcal{L}\{\text{SEN}(2t)\}$
 $\text{SEN}(kt) ; k=2$

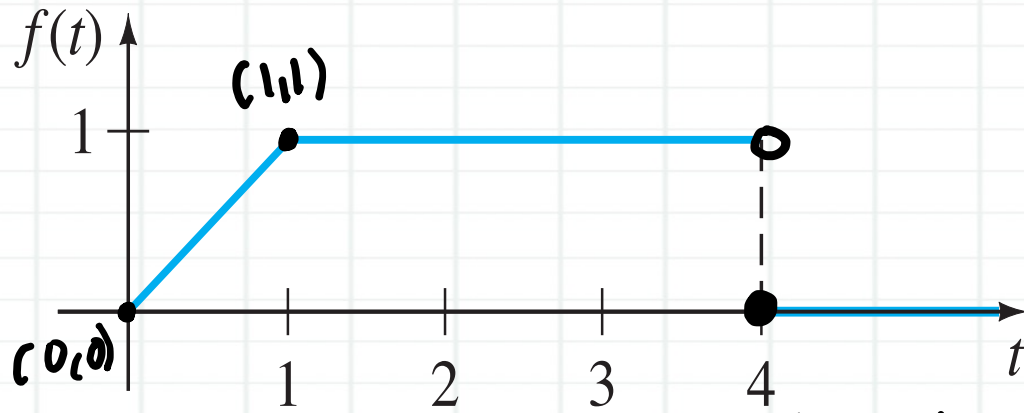
$F(s) = \frac{1}{2} \left[\frac{2}{s^2 + 4} \right] = \frac{1}{s^2 + 4}$



TRANSFORMADA DE LAPLACE DE FUNCIONES POR TRAMOS



Calcular la Transformada de Laplace $\mathcal{L}\{f(t)\}$ de la función por tramos:



⊛ $f_1(t) = t$, $f_2(t) = 1$, $f_3(t) = 0$

$$f(t) = \begin{cases} t & \text{si } 0 \leq t \leq 1 \\ 1 & \text{si } 1 \leq t < 4 \\ 0 & \text{si } t \geq 4 \end{cases}$$

$$\mathcal{L}\{f(t)\} = \lim_{b \rightarrow \infty} \left[\int_0^1 e^{-st}(t) dt + \int_1^4 e^{-st}(1) dt + \int_4^b e^{-st}(0) dt \right]$$

Handwritten integration table for $\int t e^{-st} dt$:

d	∫
t	e^{-st}
$\frac{1}{s}$	$-\frac{1}{s} e^{-st}$
$-\frac{1}{s^2}$	$\frac{1}{s^2} e^{-st}$

$$F(s) = \left(-\frac{t}{s} e^{-st} - \frac{1}{s^2} e^{-st} \Big|_0^1 \right) + \left(-\frac{1}{s} e^{-st} \Big|_1^4 \right)$$

$$F(s) = \frac{-1}{s} e^{-s} - \frac{1}{s^2} e^{-s} + \frac{1}{s^2} - \frac{1}{s} e^{-4s} + \frac{1}{s} e^{-s}$$

$$F(s) = -\frac{1}{s^2} e^{-s} + \frac{1}{s^2} - \frac{1}{s} e^{-4s}$$

TRANSFORMADA INVERSA DE LAPLACE ($\mathcal{L}^{-1} \{F(s)\}$)



$$\text{a)} \quad 1 = \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\}$$

$$\text{b)} \quad t^n = \mathcal{L}^{-1} \left\{ \frac{n!}{s^{n+1}} \right\}, \quad n = 1, 2, 3, \dots$$

$$\text{c)} \quad e^{at} = \mathcal{L}^{-1} \left\{ \frac{1}{s - a} \right\}$$

$$\text{d)} \quad \sin kt = \mathcal{L}^{-1} \left\{ \frac{k}{s^2 + k^2} \right\}$$

$$\text{e)} \quad \cos kt = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + k^2} \right\}$$

$$\text{f)} \quad \sinh kt = \mathcal{L}^{-1} \left\{ \frac{k}{s^2 - k^2} \right\}$$

$$\text{g)} \quad \cosh kt = \mathcal{L}^{-1} \left\{ \frac{s}{s^2 - k^2} \right\}$$

Calcular la Transformada Inversa $\mathcal{L}^{-1}\{F(s)\}$ de :

$$1) F(s) = \frac{2}{s} \quad ; \quad \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{s}\right\} ; \quad \frac{2}{s}$$

$$f(t) = 2$$

