Ecuación:

Es un enunciado que dice que dos expresiones matemáticas son iguales.

$$A = B$$

Ecuaciones aritméticas.

$$7 + 3 = 5 + 5$$
$$10 = 10$$

Ecuaciones algebraicas.

$a, b, c,$ x, y, z

Propiedades de la igualdad

1. $A = B \iff A + C = B + C$
2. $A = B \iff AC = BC$
3. $A = B \iff \frac{A}{C} = \frac{B}{C}$

Ecuación lineal

$$\boxed{ax + b = c}$$

donde a, b y c son números reales.

X = variable.

Ej. $2x + 3 = 7 - 6x$

$2x + 3 + 6x = 7 - 6x + 6x \rightarrow$ $2x + 6x = 7 - 3$

$8x + 3 = 7$ $\quad 8x = 4$

$8x + 3 - 3 = 7 - 3$ $\quad x = \frac{4}{8} = \frac{1}{2}$

$\left[8x = 4 \right] * \frac{1}{8} \rightarrow \frac{8}{8}x = \frac{4}{8}$

$$\boxed{x = \frac{1}{2}}$$

Ej. 2 Resolver $6(2y + 3) - 4(y - 5) = 0$

$12y + 18 - 4y + 20 = 0$ $\quad$ igualdad.

$8y + 38 = 0$ $\quad 8y = -38$ $\quad y = -\frac{38}{8} = -\frac{19}{4}$

$6(2y+3) - 4(y-5) = 0$ $\qquad y = -\frac{19}{4}$

$6\left(2\left(-\frac{19}{4}\right)+3\right) - 4\left(-\frac{19}{4} - 5\right) = 0$

$6\left(-\frac{19}{2}+3\right) - 4\left(-\frac{39}{4}\right) = 0$

$6\left(\frac{-13}{2}\right) - 4\left(\frac{-39}{4}\right) = 0 \rightarrow 3(-13) + 39 = 0$

$-39+39 = 0 \rightarrow 0 = 0$

Ej. $\frac{3x+1}{6x-2} = \frac{2x+5}{4x-13}$

$(3x+1)(4x-13) = (2x+5)(6x-2)$

$12x^2 - 39x + 4x - 13 = 12x^2 - 4x + 30x - 10$

$-39x + 4x + 4x - 30x = -10 + 13$

$-69x + 0x = 3$

$-61x = 3 \rightarrow \boxed{x = -\frac{3}{61}}$

Ej. $2x - \frac{x}{2} + \frac{x+1}{4} = 6x$

$2x - \frac{x}{2} + \frac{x}{4} + \frac{1}{4} = 6x$

$2x - \frac{x}{2} + \frac{x}{4} - 6x = -\frac{1}{4}$

$x\left(2 - \frac{1}{2} + \frac{1}{4} - 6\right) = -\frac{1}{4}$ $\qquad x = \frac{-1}{-17}$

$x\left(\frac{8-2+1-24}{4}\right) = -\frac{1}{4} \rightarrow x(-17) = -1$ $\qquad x = \frac{1}{17}$

Ecuaciones cuadráticas.

$ax^2 + bx + c = 0$ $\qquad a, b$ y c son ctes.

1. factorización 2. completación al cuadrado
3. fórmula cuadrática.

1. factorización.

$\rightarrow ax^2 + bx + c = 0$

$a_1x \times c_1 \;\Big|\; a_1c_2 +$

$a_2x \times c_2 \;\Big|\; \underline{a_2c_1}$

b

$(a_1x + c_1)(a_2x + c_2) = 0$

$a_1x + c_1 = 0 \rightarrow x = -\frac{c_1}{a_1}$

$a_2x + c_2 = 0 \rightarrow x = -\frac{c_1}{a_2}$

Ej. Resolver $x^2 + 8x + 12 = 0$

$x \times 6 \;\Big|\; 2x$

$x \times 2 \;\Big|\; \underline{6x}$

$8x$

$(x+6)(x+2) = 0 \rightarrow x+6=0 \rightarrow x = -6$

$x+2=0 \rightarrow x = -2$

Ej. $15x^2 - 14 = 29x$

$15x^2 - 29x - 14 = 0$

$5x \times +2 \;\Big|\; -35x$

$3x \times -7 \;\Big|\; \underline{6x}$

$-29x$

$(5x+2)(3x-7) = 0 \rightarrow 5x+2=0 \rightarrow 5x = -2 \rightarrow x = -\frac{2}{5}$

$\rightarrow 3x-7=0 \rightarrow x = \frac{7}{3}$

2. Completación al cuadrado.

$ax^2 + bx + c = 0$

$a = 1 \quad x^2 + bx = -c$

$x = -\frac{b}{2} \pm \sqrt{-c + \frac{b^2}{4}}$

$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$

$\left(x + \frac{b}{2}\right)^2 = \pm\sqrt{-c + \frac{b^2}{4}}$

Ej. Resolver $x^2 - 10x + 20 = 0$

$a = 1$

$x^2 - 10x = -20$

$x^2 - 10x + 25$

$\rightarrow x^2 - 10x + \left(\frac{10}{2}\right)^2 = -20 + \left(\frac{10}{2}\right)^2$

$(x-5)^2$

$x^2 - 10x + 25$

$(x-5)^2 = -20 + 25$

$\sqrt{(x-5)^2} = \pm\sqrt{5}$

$x - 5 = \pm\sqrt{5}$

$x = 5 \pm \sqrt{5} \rightarrow x_1 = 5 + \sqrt{5}$

$x_2 = 5 - \sqrt{5}$ ✗

Ej. 2 $4x^2 - 12x - 11 = 0$

$4x^2 - 12x = 11$

$4(x^2 - 3x) = 11$

$4(x^2 - 3x + \frac{9}{4}) = 11 + \frac{9}{4}$

$4(x - \frac{3}{2})^2 = 20$

$x - \frac{3}{2} = \pm\sqrt{5} \rightarrow x_1 = \frac{3}{2} + \sqrt{5}$

$x_1 = \frac{3}{2} - \sqrt{5}$ ✗

$4(x^2 - 3x + \frac{9}{4}) = 20$

$x^2 - 3x + \frac{9}{4} = 5$

$\sqrt{(x - \frac{3}{2})^2} = \sqrt{5}$

$\rightarrow \sqrt{(x - \frac{3}{2})^2} = \sqrt{5}$

$\frac{1}{4} \cdot 4 = 9$

Ej. $30x + 9 = -25x^2$

$25x^2 + 30x + 9 = 0 \rightarrow 25x^2 + 30x = -9$

$25x^2 + 30x$

$25(x^2 + \frac{30}{25}x) = -9 \rightarrow 25(x^2 + \frac{6}{5}x) = -9$

$b = \frac{6}{5}$

$(\frac{b}{2})^2 \rightarrow (\frac{6}{10})^2$

$(\frac{3}{5})^2$

$25(x^2 + \frac{6}{5}x + \frac{9}{25}) = -9 + 9$

$25(x + \frac{3}{5})^2 = 0 \rightarrow \sqrt{(x + \frac{3}{5})^2} = \sqrt{0}$

$x + \frac{3}{5} = 0$

$x_1 = -\frac{3}{5}$ $x_2 = -\frac{3}{5}$

3. Formula cuadratica

$$ax^2+bx+c=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Ej. Resolver.

$$\left[\frac{5}{w^2}-\frac{10}{w}+2=0\right]*w^2$$

$$5-10w+2w^2=0$$

$2w^2-10w+5=0 \quad a=2 \quad b=-10 \quad c=5$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(2)(5)}}{2(2)} \qquad \sqrt{15\cdot 4}$$

$$x=\frac{10\pm\sqrt{100-40}}{4}\rightarrow\frac{10\pm\sqrt{60}}{4}=\frac{10\pm 2\sqrt{15}}{4}$$

$$x_1=\frac{5}{2}+\frac{\sqrt{15}}{2}=\frac{5+\sqrt{15}}{2} \qquad 4.44$$

$$x_2=\frac{5}{2}-\frac{\sqrt{15}}{2}=\frac{5-\sqrt{15}}{2} \qquad 0.56$$

Ej. Resolver: $\frac{3x}{x^2+9}=-2$ $\quad 3x=-2x^2-18$

$2x^2+3x+18\rightarrow c$

$3x=-2(x^2+9)=-2x^2-18$

$(0=-2x^2-3x-18)*-1\rightarrow 0=2x^2+3x+18$

$a=-2 \quad b=-3 \quad c=-18$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(18)}}{2(2)}$$

$\sqrt{-135}\rightarrow$ Error

$$x=\frac{3\pm\sqrt{9-144}}{4}=\frac{3}{4}\pm\frac{\sqrt{-135}}{4}\rightarrow\frac{3}{4}\pm\frac{\sqrt{135}}{4}i$$

$\sqrt{11(-11}$

$\rightarrow x^2+1=0$

$\sqrt{x^2}=\sqrt{-1}\rightarrow x=\sqrt{-1}=\pm i$

Propiedades de i

$$i = \sqrt{-1} \qquad i^2 = -1$$

Numero complejo → $A = \alpha + \beta i$ (Parte real; Parte imaginaria.)

α y β son numeros reales.

Conjugado de un numero complejo

$$\bar{A} = \alpha - \beta i$$

Operaciones con numeros complejos

$A = a + bi \qquad B = c + di$

Suma → $A + B = (a+bi) + (c+di) = (a+c) + (b+d)i$

Resta → $A - B = (a+bi) - (c+di) = (a-c) + (b-d)i$

Producto → $A \cdot B = (a+bi)(c+di) =$

$ac + adi + bci + bdi^2$

-1

$(ac - bd) + (ad+bc)i$

Cociente: $\frac{A}{B} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di} =$

$\frac{ac - adi + bci - bdi^2}{c^2 - cdi + cdi - d^2 i^2} = \frac{(ac+bd) + (bc-ad)i}{c^2 + d^2}$

-1

Ej. Calcular $A+B$, $B-C$, AB, $\frac{B}{C}$

$A = (2+5i) \qquad B = (4-3i) \qquad C = (3-2i)$

$A+B = (2+5i) + (4-3i) = (2+4) + (5-3)i = 6+2i$

$-3+2i$

$B-C = (4-3i) - (3-2i) = (4-3) + (-3-2)i$

$= 1 - 5i \quad -1$

$AB = (2+5i)(4-3i) = 8 - 6i + 20i - 15i^2$

$23 + 14i$

$$\frac{B}{C} = \frac{4-3i}{3-2i} * \frac{3+2i}{3+2i} = \frac{12+8i-9i-6i^2}{9+4}$$ -1

$$\frac{B}{C} = \frac{18-i}{13} = \frac{18}{13} - \frac{i}{13}$$

otros Tipos de Ecuaciones.

1. Ecuaciones que se resuelven por factorizacion
2. Ecuaciones con radicales $\sqrt{}$
3. Ecuaciones que se resuelven por medio de una sustitucion. x^6, x^4, $x^{1/2}$, $x^{-1/4}$, $x^{1/3}$...
4. Valor absoluto. $|\ |$

Ej. Resolver $9x^3 - 18x^2 - 4x + 8 = 0$

$$9x^2(x-2) - 4(x-2) = 0$$

$$(9x^2 - 4)(x-2) = 0$$

$9x^2 - 4 = 0 \rightarrow 9x^2 = 4$

$\sqrt{x^2} = \sqrt{\frac{4}{9}} =$ → $x_1 = \frac{2}{3}$ / → $x_2 = -\frac{2}{3}$ /

$x - 2 = 0 \rightarrow x = 2$ /

Ej. $15x^5 - 20x^4 = 6x^3 - 9x^2$

$15x^5 - 20x^4 - 6x^3 + 9x^2 = 0$

$5x^4(3x-4) - 2x^2(3x-4) = 0$

$(5x^4 - 2x^2)(3x-4) = 0$

$5x^4 - 2x^2 = 0$ → $x^2 = 0$ → $x_1 = 0$, $x_2 = 0$

$x^2(5x^2 - 2) = 0$

$5x^2 - 2 = 0$ $\sqrt{x^2} = \sqrt{\frac{2}{5}}$

$x_3 = +\sqrt{\frac{2}{5}}$ $x_4 = -\sqrt{\frac{2}{5}}$

$3x - 4 = 0 \rightarrow x_5 = \frac{4}{3}$

Ej. Resolver.

$$\frac{x}{2x+7} - \frac{x+1}{x+3} = 1$$

$$\frac{x(x+3) - (x+1)(2x+7)}{(2x+7)(x+3)} = 1$$

$$\frac{x^2+3x-(2x^2+7x+2x+7)}{2x^2+6x+7x+21} = 1$$

$$-x^2-6x-7 = 1(2x^2+13x+21)$$

$$0 = 3x^2+19x+28$$

$$x = \frac{-19 \pm \sqrt{19^2-4(3)(28)}}{2(3)}$$

$x_1 = -7/3$

$x_2 = -4$

$$3x \quad 7 \quad 12x$$
$$x \quad 4 \quad 7x$$
$$19x$$

$$(3x+7)(x+4) = 0$$

$$x = -\frac{7}{3}$$

$$x = -4$$

Ej. Resolver $\left(\sqrt{2x-9}\right)^2 = \left(\frac{1}{2}\right)^2$

$(\sqrt{a})^2 = a \qquad \sqrt{a^2} = a$

$$2x-9 = \frac{1}{4} \rightarrow 2x = \frac{1}{4}+9 = \frac{37}{4}$$

$$x = \frac{37}{8}$$

Ej. 2 Resolver. $\sqrt{3x+1} = 2+\sqrt{x+1}$

$$\left(\sqrt{3x+1} - \sqrt{x+1}\right)^2 = 1^2$$

$$\left(\sqrt{3x+1}-\sqrt{x+1}\right)^2=2^2$$

$$\left(\sqrt{3x+1}\right)^2-2\sqrt{3x+1}\cdot\sqrt{x+1}+\left(\sqrt{x+1}\right)^2=4$$

$$3x+1-2\sqrt{3x+1}\cdot\sqrt{x+1}+x+1=4$$

$4(3x+1)(x+1)$ $\quad 4x+2$

$$\rightarrow\left(-2\sqrt{3x+1}\cdot\sqrt{x+1}\right)^2=(-4x+2)^2$$

$$\left(-2\sqrt{(3x+1)\cdot(x+1)}\right)^2=(-4x+2)^2$$

$$4(3x+1)(x+1)=16x^2-16x+4$$

$$4(3x^2+4x+1)=16x^2-16x+4$$

$$12x^2+16x+4=16x^2-16x+4$$

$$4x^2-32x=0 \qquad x(-4x+32)=0$$

$$x=0$$

$$-4x+32=0 \rightarrow -4x=-32 \rightarrow x=\frac{-32}{-4}=8$$

$x=0$ ✓

$$\sqrt{1}-\sqrt{1}=2 \rightarrow 0=2$$

no es solución.

$x=8$ ✓ $\quad \sqrt{24+1}-\sqrt{8+1}=2$

Si es $\quad 5-3=2 \rightarrow \boxed{2=2}$

solución

Ej. Resolver $\left|\sqrt{7-2x}-\sqrt{5+x}\right|^2=\left|\sqrt{4+3x}\right|^2$

$$\left(\sqrt{7-2x}\right)^2=\left(\sqrt{4+3x}+\sqrt{5+x}\right)^2$$

$$7-2x=4+3x+2\sqrt{(4+3x)(5+x)}+5+x$$

$$\begin{cases}-2-6x=2\sqrt{(4+3x)(5+x)} & \times -\frac{1}{2}\\ 1+3x=-\sqrt{(4+3x)(5+x)}\end{cases}$$

$$|1+3x|^2 = \left(-\sqrt{(4+3x)(5+x)}\right)^2$$

$$1+6x+9x^2 = (4+3x)(5+x)$$

$$1+6x+9x^2 = 20+19x+3x^2$$

$$6x^2-13x-19=0$$

$$\begin{matrix} 6x & -19 \\ x & 1 \end{matrix} \Big\} = \begin{matrix} 6x \\ -19x \\ \hline -13x \end{matrix}$$

$$(6x-19)(x+1)=0 \longrightarrow x_1 = \frac{19}{6}$$

$x=-1$ si es sol.

$$\sqrt{7-2(-1)} - \sqrt{5+(-1)} = \sqrt{4+3(-1)} \qquad x=-1$$

$$\sqrt{9}-\sqrt{4}=\sqrt{1}$$

$$3-2=1 \longrightarrow 1=1$$

Ej. Resolver $36x^{-4} - 13x^{-2} + 1 = 0$

$u = x^{-2}$

$u^2 = x^{-4}$

$$36u^2 - 13u + 1 = 0$$

$$\begin{matrix} 9u & -1 \\ 4u & -1 \end{matrix} \Big\} \begin{matrix} -9u \\ -4u \\ \hline -13u \end{matrix}$$

$$(9u-1)(4u-1)=0 \longrightarrow \begin{cases} 9u-1=0 \rightarrow u=\frac{1}{9} \\ 4u-1=0 \rightarrow u=\frac{1}{4} \end{cases}$$

$(u) = (x^{-2})^{-1/2}$

$u^{-1/2} = x$

$\sqrt{9} = \sqrt{x^2}$

$x=3$

$u=\frac{1}{9} \longrightarrow x = \left(\frac{1}{a}\right)^{-1/2} = \frac{1}{(1/9)^{1/2}} = \left(\frac{1}{\frac{1}{3}}\right) = 3$ //

$u=\frac{1}{4} \longrightarrow x = \left(\frac{1}{4}\right)^{-1/2} = \frac{1}{(\frac{1}{4})^{1/2}} = \left(\frac{1}{\frac{1}{2}}\right) = 2$ //

$9x^{-2}=1 \rightarrow \frac{9}{x^2}=1$

Ej. Resolver

$$x^{1/2} - 3x^{1/3} = 3x^{1/6} - 9$$

$$x^{1/2} - 3x^{1/3} - 3x^{1/6} + 9 = 0$$

$u = x^{1/6}$

$u^2 = (x^{1/6})^2 = x^{1/3}$

$u^3 = (x^{1/6})^3 = x^{1/2}$

$$u^3 - 3u^2 - 3u + 9 = 0$$

$$u^2(u-3) - 3(u-3) = 0$$

$$(u^2 - 3)(u-3) = 0$$

$u^2 - 3 = 0 \rightarrow u^2 = 3 \rightarrow u = \pm\sqrt{3}$

$u - 3 = 0 \rightarrow u = 3$

$(u)^6 = (x^{1/6})^6 \rightarrow x = u^6$

$u = \sqrt{3} \rightarrow x = (\sqrt{3})^6 \quad (3^{1/2})^6 = 3^3 = 27$

$u = 3 \quad x = 3^6 = 729$

Ej. Resolver

$$\left(x + \frac{2}{x}\right)^{3/2} + \left(x + \frac{2}{x}\right)^{1/2} = 12\left(x + \frac{2}{x}\right)^{-1/2}$$

$u = x + \frac{2}{x}$

$u^{3/2} + u^{1/2} - 12u^{-1/2} = 0$

$u^{-1/2}(u^2 + u - 12) = 0$

$(u^{-1/2})^2 = (0)^2 \rightarrow u = 0$

$u^2 + u - 12 = 0$

$u - 3 \mid uu$

$u + 4 \mid -3u$

u

$x - 2 \mid -2x$

$x - 1 \mid -x$

$-3x$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

$(u-3)(u+4) = 0$

$u = 3 \quad u = -4$

$u = 0 \left[0 = x + \frac{2}{x}\right] x$

$0 = x^2 + 2$

$u = 3$

$\left[3 = x + \frac{2}{x}\right] x$

$3x = x^2 + 2$

$x^2 - 3x + 2 = 0$

$$U = x + \frac{2}{x}$$

$$U \cdot x = \left[-4 = x + \frac{2}{x} \right] \cdot x$$

$$-4x = x^2 + 2 \qquad x^2 + 4x + 2 = 0$$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(2)}}{2(1)}$$

$$x = -2 \pm \frac{\sqrt{8}}{2} = -2 \pm \frac{\sqrt{4 \cdot 2}}{2} = -2 \pm \frac{2\sqrt{2}}{2}$$

$$x_3 = -2 + \sqrt{2}$$

$$x_4 = -2 - \sqrt{2}$$

Valor absoluto

$$\rightarrow |a| = b \rightarrow \begin{cases} a = b \\ -a = b \end{cases}$$

$a = b$

$a = -b$

Ej. $|x - 5| = 3$

$x - 5 = 3 \rightarrow x = 8$

$x - 5 = -3 \rightarrow x = 2$

Ej. $|3x + 2| + |3 - x| = 7$

$|a| = b \rightarrow a = b$ y $a = -b$

$$|3x + 2| = \begin{cases} 3x + 2 \\ -(3x + 2) \end{cases} \qquad |3 - x| = \begin{cases} 3 - x \\ -|3 - x| \end{cases}$$

Ej. $|3x+2| + |3-x| = 7$

$|a| = b \rightarrow a = b$ y $a = -b$

$|3x+2| = \begin{matrix} 3x+2 \\ -(3x+2) \end{matrix}$ $\quad |3-x| = \begin{matrix} 3-x \\ -|3-x| \end{matrix}$

$3x+2 + 3-x = 7 \rightarrow 2x = 2$ $\boxed{x = 1}$

$-(3x+2) + 3-x = 7$

$3x+2 -(3-x| = 7$

$-(3x+2) - (3-x| = 7$

$-3x-2+3-x = 7$

$-4x = 6$ $\boxed{x = -\frac{3}{2}}$

$3x+2-3+x = 7$

$4x = 8$ $\boxed{x = 2}$

$-3x-2-3+x = 7$

$-2x = 12$ $\boxed{x = -6}$.