

$$\int_a^b f(x) dx = \int_{-2}^2 f(x) dx + \int_2^5 f(x) dx - \int_{-2}^1 f(x) dx$$

$$\int_{-2}^2 f(x) dx + \int_2^5 f(x) dx - \int_{-2}^1 f(x) dx = \int_{-2}^5 f(x) dx + \int_{-1}^2 f(x) dx$$

$$\int_{-1}^5 f(x) dx$$

$$\int_{-1}^5 f(x) dx$$

$$2 \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq 2\sqrt{2}$$

$$-1 \leq x \leq 1; 0 \leq x^2 \leq 1$$

$$1 \leq 1+x^2 \leq 2; 1 \leq \sqrt{1+x^2} \leq \sqrt{2}$$

$$1[1-(-1)] \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq \sqrt{2}[1-(-1)]$$

$$2 \leq \int_{-1}^1 \sqrt{1+x^2} dx \leq 2\sqrt{2}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{1+(\frac{i}{n})^2}$$

$$\Delta x = \frac{(1-0)}{n} = \frac{1}{n}; x_i = 0 + i\Delta x = \frac{i}{n}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{1+(\frac{i}{n})^2} = \lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{1+(\frac{i}{n})^2} \cdot \frac{1}{n} = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \int_0^1 f(x) dx = \int_0^1 \frac{1}{1+x^2} dx$$

$$\int_0^1 \frac{dx}{1+x^2}$$

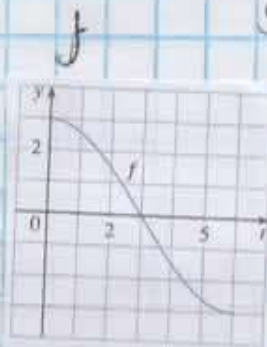
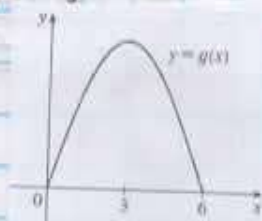
$$\int_0^1 \frac{dx}{1+x^2}$$

#42 Pág 399 #4

$$g(x) = \int_0^x f(t) dt$$

a) $g(x) = \int_0^x f(t) dt \rightarrow g(0) = 0$
 $g(6) = 0$

e) $y = g(x)$



b) $g(1) = 2.8$

$g(2) = 4.4$

$g(3) = 5.7$

$g(4) = g(3) - g(2) = 0.8$

$g(5) = 2.8$

c) En el intervalo de $(0, 3)$

d) g tiene un máximo valor en $x = 3$

f) El gráfico de g es una parábola en el gráfico g obteniendo un gráfico que se parece a f

#43 Pág 399 #7

$$g(x) = \int_0^x \sqrt{t+t^3} dt$$

$$g(x) = \int_0^x \sqrt{t+t^3} dt \rightarrow g'(x) = f(x) = \sqrt{x+x^3}$$

$$g'(x) = f(x) = \sqrt{x+x^3}$$

#44 Pág 399 #11

$$F(x) = \int_x^0 \sqrt{1+\sec t} dt$$

$$F(x) = \int_x^0 \sqrt{1+\sec t} dt = -\int_0^x \sqrt{1+\sec t} dt \rightarrow F'(x) = -\frac{d}{dx} \int_0^x \sqrt{1+\sec t} dt = -\sqrt{1+\sec x}$$

$$F'(x) = -\sqrt{1+\sec x}$$

#45 Pág 400 #18

$$y = \int_{1-3x}^1 \frac{u^3}{1+u^2} du$$

$$y' = \frac{d}{dx} \left(\int_{1-3x}^1 \frac{u^3}{1+u^2} du \right) \rightarrow y' = \frac{d}{dx} \int_1^{1-3x} \frac{u^3}{1+u^2} du \rightarrow$$

$$y' = \frac{d}{dx} \int_1^{1-3x} -\frac{u^3}{1+u^2} du \rightarrow y' = \frac{d}{dx} \int_1^u -\frac{u^3}{1+u^2} du \rightarrow y' = \frac{d}{du} \int_1^u -\frac{u^3}{1+u^2} du \left(\frac{du}{dx} \right)$$

$$y' = -\frac{u^3}{1+u^2} \cdot \frac{d}{dx} u \rightarrow y' = -\frac{(1-3x)^3}{1(1-3x)^2} \cdot \frac{d}{dx} (1-3x) \rightarrow y' = -\frac{(1-3x)^3}{1(1-3x)^2} (-3)$$

$$y' = \frac{3-27x+81x^2-81x^3}{2-6x+9x^2}$$

$$y' = \frac{3-27x+81x^2-81x^3}{2-6x+9x^2}$$

#46 P49 400 #19

$$\int_1^3 (x^2+2x-4) dx$$

$$\int_1^3 (x^2+2x-4) dx = \left[\frac{1}{3}x^3 + x^2 - 4x \right]_1^3 = (9+2-12) \cdot \left(\frac{1}{3} + 1 - 4 \right) = 6 + \frac{8}{3} = \frac{26}{3}$$

#47 P49 400 #27

$$\int_0^2 x(2+x^5) dx$$

$$\int_0^2 x(2+x^5) dx \rightarrow \int_0^2 x(2+x^5) dx = \int_0^2 2x + x^6 dx = \int_0^2 2x dx + \int_0^2 x^6 dx$$

$$x^2 + \frac{x^7}{7} = \left[x^2 + \frac{x^7}{7} \right]_0^2 = 2^2 + \frac{2^7}{7} - (0^2 + \frac{0^7}{7}) = \frac{156}{7}$$

$$\int_0^2 x(2+x^5) dx = \frac{156}{7} \approx 22.29$$

#48 P49 400 #35

$$\int_1^2 \frac{v^3+3v^6}{v^4} dv$$

$$\int_1^2 \frac{v^3+3v^6}{v^4} dv = \int_1^2 \left(\frac{1}{v} + 3v^2 \right) dv = \left[\ln|v| + v^3 \right]_1^2 = (\ln 2 + 8) - (\ln 1 + 1) = \ln 2 + 7$$

$$\int_1^2 \frac{v^3+3v^6}{v^4} dv = \ln 2 + 7 \approx 7.69315$$

#49 Pág 400 #37

$$\int_0^1 (xe + e^x) dx$$

$$\int_0^1 (xe + e^x) dx = \left[\frac{x^2 + 1}{e + 1} + e^x \right]_0^1 = \left(\frac{1}{e + 1} + e \right) - (0 + 1) = \frac{1}{e + 1} + e - 1$$

$$\int_0^1 (xe + e^x) dx = \frac{1}{e + 1} + e - 1 \approx 1.98722$$

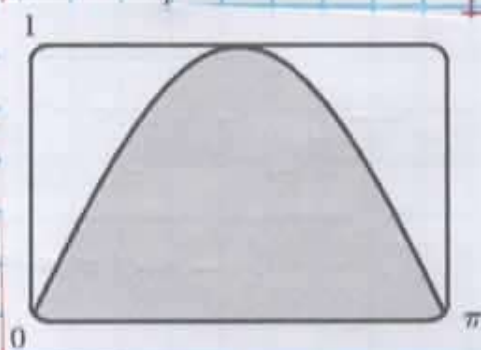
#50 Pág 400 #40

$$\int_1^3 \frac{y^3 - 2y^2 - y}{y^2} dy$$

$$\int_1^3 \frac{y^3 - 2y^2 - y}{y^2} dy = \int_1^3 \left(y - 2 - \frac{1}{y} \right) dy = \left[\frac{1}{2} y^2 - 2y - \ln|y| \right]_1^3 = \left(\frac{9}{2} - 6 - \ln 3 \right) - \left(\frac{1}{2} - 2 - 0 \right) = \ln 3$$

$$\int_1^3 \frac{y^3 - 2y^2 - y}{y^2} dy = -\ln 3 \approx -1.09861$$

#51 Pág 400 #51



$$y = \sin x; 0 \leq x \leq \frac{\pi}{2}$$

$$\int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = (-\cos \pi) - (-\cos 0) = -(-1) + 1 = 2$$

El área es de 2

#52 Pág 400 #60

$$g(x) = \int_{1-2x}^{1+2x} t \sin t dt$$

$$g(x) = \int_{1-2x}^{1+2x} t \sin t dt = \int_{1-2x}^0 t \sin t dt + \int_0^{1+2x} t \sin t dt = -\int_0^{1-2x} t \sin t dt + \int_0^{1+2x} t \sin t dt$$

$$g'(x) = -(1-2x)\sin(1-2x) \cdot \frac{d}{dx}(1-2x) + (1+2x)\sin(1+2x) \cdot \frac{d}{dx}(1+2x) \\ = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$$

$$g'(x) = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x)$$

#53 Pág 400 #63

$$y = \int_{\cos x}^{\sin x} \ln(1+2v) dv$$

$$y = \int_{\cos x}^{\sin x} \ln(1+2v) dv = \int_{\cos x}^0 \ln(1+2v) dv + \int_0^{\sin x} \ln(1+2v) dv = -\int_0^{\cos x} \ln(1+2v) dv + \int_0^{\sin x} \ln(1+2v) dv$$

$$y' = -\ln(1+2\cos x) \frac{d}{dx} \cos x + \ln(1+2\sin x) \frac{d}{dx} \sin x = \sin x \ln(1+2\cos x) + \cos x \ln(1+2\sin x)$$

$$y' = \sin x \ln(1+2\cos x) + \cos x \ln(1+2\sin x)$$

#54 Pág 401 #75

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right)$$

[0,1]

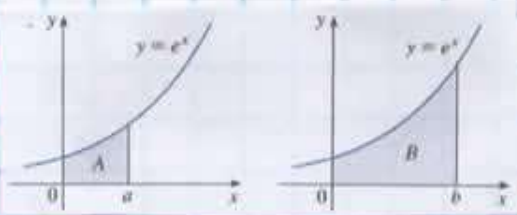
$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right) = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^4} + \frac{i}{n} \right) \frac{1}{n} = \lim_{n \rightarrow \infty} \frac{1+0}{n} \sum_{i=1}^n \left[\left(\frac{i}{n} \right)^4 + \frac{i}{n} \right] = \int_0^1 (x^4 + x) dx$$

$$= \left[\frac{1}{5} x^5 + \frac{1}{2} x^2 \right]_0^1 = \left(\frac{1}{5} + \frac{1}{2} \right) - 0 = \frac{7}{10} \approx 0.7$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i^4}{n^5} + \frac{i}{n^2} \right) = \frac{7}{10} \approx 0.7$$

#55 Pág 401 #84

B = 3A



$$\int_0^b e^x dx = 3 \int_0^a e^x dx \rightarrow [e^x]_0^b = 3[e^x]_0^a$$

$$e^b - 1 = 3(e^a - 1) \rightarrow e^b = 3e^a - 2$$

$$b = \ln(3e^a - 2)$$

$$b = \ln(3e^a - 2)$$

#56 Pág 409 #8

$$\int (u^6 - 2u^5 - u^3 + \frac{2}{7}) du$$

$$\int (u^6 - 2u^5 - u^3 + \frac{2}{7}) du \rightarrow \frac{1}{7} u^7 - 2 \frac{1}{6} u^6 - \frac{1}{4} u^4 + \frac{2}{7} u + C = \frac{1}{7} u^7 - \frac{1}{3} u^6 - \frac{1}{4} u^4 + \frac{2}{7} u + C$$

$$\boxed{\frac{1}{7} u^7 - \frac{1}{3} u^6 - \frac{1}{4} u^4 + \frac{2}{7} u + C}$$

#57 Pág 409 #18

$$\int \frac{\sin 2x}{\sin x} dx$$

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx = \int 2 \cos x dx = 2 \sin x + C$$

$$\boxed{\int \frac{\sin 2x}{\sin x} dx = 2 \sin x + C}$$

#58 Pág 409 #29

$$\int_1^4 \left(\frac{4+6u}{\sqrt{u}} \right) du$$

$$\int_1^4 \left(\frac{4+6u}{\sqrt{u}} \right) du = \int_1^4 \left(\frac{4}{\sqrt{u}} + \frac{6u}{\sqrt{u}} \right) du = \int_1^4 (4u^{-1/2} + 6u^{1/2}) du = \left[8u^{1/2} + 4u^{3/2} \right]_1^4$$

$$(16+32) - (8+4) = 36$$

$$\boxed{\int_1^4 \left(\frac{4+6u}{\sqrt{u}} \right) du = 36}$$

#59 Pág 409 #37

$$\int_0^{2\pi} \frac{1+\cos^2 \theta}{\cos^2 \theta} d\theta \rightarrow \int_0^{2\pi} \frac{1+\cos^2 \theta}{\cos^2 \theta} d\theta = \int_0^{2\pi} \left(\frac{1}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \right) d\theta = \int_0^{2\pi} (\sec^2 \theta + 1) d\theta$$

$$[\tan \theta + \theta]_0^{2\pi} = \left(\tan \frac{\pi}{4} + \frac{\pi}{4} \right) - (0+0) = 1 + \frac{\pi}{4}$$

$$\boxed{\int_0^{2\pi} \frac{1+\cos^2 \theta}{\cos^2 \theta} d\theta = 1 + \frac{\pi}{4} \approx 1.7854}$$

#60 Page #43
409

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{t^4-1} dt \quad \int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{t^4-1} dt = \int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{(t^2+1)(t^2-1)} dt$$

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1} dt = [\arctan t]_0^{\frac{1}{\sqrt{3}}} = \arctan\left(\frac{1}{\sqrt{3}}\right) - \arctan(0) = \frac{\pi}{6} - 0 = \frac{\pi}{6}$$

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{t^4-1} dt = \frac{\pi}{6}$$

#61 Page #44
409

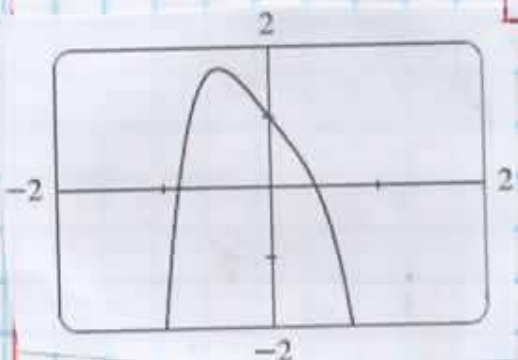
$$\int_0^2 |2x-1| dx \rightarrow |2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 \geq 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} = \begin{cases} 2x-1 & \text{if } x \geq \frac{1}{2} \\ 1-2x & \text{if } x < \frac{1}{2} \end{cases}$$

$$\int_0^2 |2x-1| dx = \int_0^{\frac{1}{2}} (1-2x) dx + \int_{\frac{1}{2}}^2 (2x-1) dx = \left[x - x^2 \right]_0^{\frac{1}{2}} + \left[x^2 - x \right]_{\frac{1}{2}}^2$$

$$= \left(\frac{1}{2} - \frac{1}{4} \right) - 0 + (4 - 2) - \left(\frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4} + 2 - \left(-\frac{1}{4} \right) = \frac{5}{2}$$

$$\int_0^2 |2x-1| dx = \frac{5}{2} = 2.5$$

#62 Page #47
409



$$y = 1 - 2x - 5x^4$$

$$x = a \approx -0.86$$

$$x = b \approx 0.42$$

$$\int_a^b (1 - 2x - 5x^4) dx = \left[x - x^2 - x^5 \right]_a^b$$

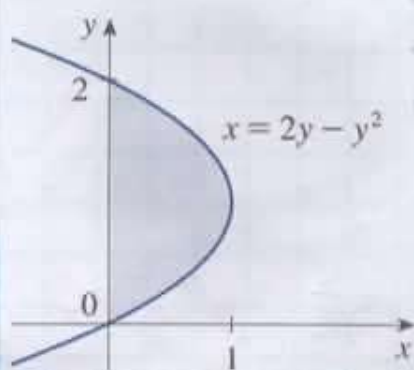
$$(b - b^2 - b^5) - (a - a^2 - a^5) = 1.36$$

$$1.36$$

$$x = 2y - y^2$$

#63 Pág 409 #49

$$\int_0^2 (2y - y^2) dy ; x = 2y - y^2 \rightarrow y=0, y=2$$



$$A = \int_0^2 (2y - y^2) dy$$

$$\left[y^2 - \frac{1}{3} y^3 \right]_0^2 = \left(4 - \frac{8}{3} \right) - 0 = \frac{4}{3}$$

$$A = \int_0^2 (2y - y^2) dy = \frac{4}{3} \approx 1.33$$

$$r(t) = 200 - 4t$$

$$0 \leq t \leq 50$$

10 minutos

$$\int_0^{10} r(t) dt = \int_0^{10} (200 - 4t) dt = \left[200t - 2t^2 \right]_0^{10} = (2000 - 200) - 0 = 1800$$

1800 litros en los primeros 10 minutos

$$\int x \sqrt{1-x^2} dx$$

#65 Pág 418 #7

$$u = 1 - x^2 ; du = -2x dx ; x dx = -\frac{1}{2} du$$

$$\int x \sqrt{1-x^2} dx = \int \sqrt{u} \left(-\frac{1}{2} du \right) = -\frac{1}{2} \left(\frac{2}{3} u^{3/2} \right) + C = -\frac{1}{3} (1-x^2)^{3/2} + C$$

$$-\frac{1}{3} (1-x^2)^{3/2} + C$$

$$\int \frac{a + bx^3}{\sqrt{3ax + bx^3}} dx \rightarrow \int \frac{a + bx^3}{\sqrt{3ax + bx^3}} dx = \int \frac{\frac{1}{3} du}{\frac{u}{\sqrt{u}}} = \frac{1}{3} \int u^{-1/2} du$$

$$\frac{1}{3} 2u^{1/2} + C = \frac{2}{3} \sqrt{3ax + bx^3} + C$$

$$\frac{2}{3} \sqrt{3ax + bx^3} + C$$

#67 Page 419 #25

$$u = 1 + e^x; du = e^x dx$$

$$\int e^x \sqrt{1 + e^x} dx$$

$$\int e^x \sqrt{1 + e^x} dx = \int \sqrt{u} du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (1 + e^x)^{3/2} + C$$

$$u = \sqrt{1 + e^x}; u^2 = 1 + e^x; 2u du = e^x dx$$

$$\int e^x \sqrt{1 + e^x} dx = \int u \cdot 2u du = \frac{2}{3} u^3 + C = \frac{2}{3} (1 + e^x)^{3/2} + C$$

$$\boxed{\int e^x \sqrt{1 + e^x} dx = \frac{2}{3} (1 + e^x)^{3/2} + C}$$

#68 Page 419 #29

$$u = 5^t; du = 5^t \ln 5 dt$$

$$5^t dt = \frac{1}{\ln 5} du$$

$$\int 5^t \sin(5^t) dt$$

$$\int 5^t \sin(5^t) dt = \int \sin u \left(\frac{1}{\ln 5} du \right) = -\frac{1}{\ln 5} \cos u + C = -\frac{1}{\ln 5} \cos(5^t) + C$$

$$\boxed{-\frac{1}{\ln 5} \cos(5^t) + C}$$

#69 Page 419 #38

$$u = 1 + \tan t; du = \sec^2 t dt$$

$$\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}}$$

$$\int \frac{dt}{\cos^2 t \sqrt{1 + \tan t}} = \int \frac{\sec^2 t dt}{\sqrt{1 + \tan t}} = \int \frac{du}{\sqrt{u}} = \int u^{-1/2} du = \frac{u^{1/2}}{1/2} + C = 2\sqrt{1 + \tan t} + C$$

$$\boxed{2\sqrt{1 + \tan t} + C}$$

#70 Page 479 #44

$$u = x^2; du = 2x dx$$

$$\int \frac{x}{1 + x^4} dx \rightarrow \int \frac{x}{1 + x^4} dx \rightarrow \int \frac{\frac{1}{2} du}{1 + u^2} = \frac{1}{2} \tan^{-1} u + C = \frac{1}{2} \tan^{-1}(x^2) + C$$

$$\boxed{\int \frac{x}{1 + x^4} dx = \frac{1}{2} \tan^{-1}(x^2) + C}$$

$$\int x(2x+5)^8 dx$$

#71 Pág 419 #47

$$u = 2x+5; du = 2dx \\ x = \frac{1}{2}(u-5)$$

$$\int x(2x+5)^8 dx = \int \frac{1}{2}(u-5)u^8 \left(\frac{1}{2}du\right) = \frac{1}{4} \int (u^9 - 5u^8) du$$

$$= \frac{1}{4} \left(\frac{1}{10} u^{10} - \frac{5}{9} u^9 \right) + C = \frac{1}{40} (2x+5)^{10} - \frac{5}{36} (2x+5)^9 + C$$

$$\boxed{\int x(2x+5)^8 dx = \frac{1}{40} (2x+5)^{10} - \frac{5}{36} (2x+5)^9 + C}$$

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

#72 Pág 419 #63

$$u = 1+2x; du = 2dx \\ x=0; u=1; x=13; u=27$$

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = \int_1^{27} u^{-2/3} \left(\frac{1}{2}du\right) = \left[\frac{1}{2} \cdot 3u^{1/3} \right]_1^{27} = \frac{3}{2} (3-1) = 3$$

$$\boxed{\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}} = 3}$$

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx$$

#73 Pág 419 #66

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x dx = 0$$

$\rightarrow f(x) = x^4 \sin x$ es una función impar

$$\int_0^1 \frac{e^z + 1}{e^z + z} dz$$

#74 Pág 419 #71

$$\int_0^1 \frac{e^z + 1}{e^z + z} dz = \int_1^{e+1} \frac{1}{u} du$$

$$u = e^z + z; du = (e^z + 1) dz \\ z=0; u=1; z=1; u=e+1$$

$$[\ln |u|]_1^{e+1} = \ln |e+1| - \ln |1| = \ln(e+1)$$

$$\boxed{\int_0^1 \frac{e^z + 1}{e^z + z} dz = \ln(e+1) \approx 1.31326}$$

#75

Pag

419

#77

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} dx$$

Semi-circle
radius = 2 $I_1 = 0 \rightarrow$ Teorema 7(b)

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} dx = I_1 + I_2 = \int_{-2}^2 x\sqrt{4-x^2} dx + \int_{-2}^2 3\sqrt{4-x^2} dx$$

$$f(x) = x\sqrt{4-x^2} \quad x_2 = -2; x = 2 \rightarrow I_2 \rightarrow I = 0 + 3 \frac{1}{2} (x \cdot 2^2) = 6x$$

$$\int_{-2}^2 (x+3)\sqrt{4-x^2} dx \rightarrow I = 0 + 3 \frac{1}{2} (x \cdot 2^2) = 6x$$

#76

Pag

420

#87

$$\int_0^4 f(x) dx = 10 \quad ; \quad \text{encontre} \int_0^2 f(2x) dx$$

$$u = 2x \quad \int_0^2 f(2x) dx = \int_0^4 f(u) \left(\frac{1}{2} du\right) = \frac{1}{2} \int_0^4 f(u) du = \frac{1}{2} (10) = 5$$

$$\int_0^4 f(x) dx = 10 \rightarrow \int_0^2 f(2x) dx = 5$$