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Tercer Examen Parcial
MA3Q

1) El método de convergencia acelerada se aplica a una sucesión por medio de: $P_0=0.2$ y $P_2=0.6$ para obtener $P=0.65$ $P_1=?$

$P_2^{(1)} =$ Steffensen $g(x)=\sqrt[3]{6x}$; $P_0=1$

$$\frac{x_{n+1}-P}{x_n-P} = \frac{x_{n+2}-P}{x_{n+1}-P} \rightarrow \frac{P_1-P}{P_0-P} = \frac{P_2-P}{P_1-P}$$

$(P_1-P)^2=(0.6-0.65)(0.2-0.65) \rightarrow (P_1-P)^2 = 0.0225 \rightarrow P_1-P=-0.15$

$P_1=P\pm0.15 \rightarrow P_1=0.65-0.15$

$P_1=0.50$

$\boxed{P_1=0.50}$

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2) El valor de $P_2^{(2)}$ por steffensen y la sucesión $g(x)=\sqrt[3]{6x}$; $p_0=1$

$g(x)=\sqrt[3]{6x}$; $P_0=1$; $x_1=f(1)=\sqrt[3]{6}$; $P_2=f(\sqrt[3]{6})=\sqrt[3]{6\sqrt[3]{6}}$ $= f(1)=1.817121$

$f(2)=2.217405$

$$x_3=x_0-\frac{(x_1-x_0)}{x_2-2x_1+x_0} = 1-\frac{(1.817121-1)^2}{2.217405-2(1.817121)+1}$$

$x_3=2.601793 \rightarrow f(2.601793)=2465963$

$\boxed{2.465963 \text{ opción e}}$

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3) Si se utiliza un polinomio de Lagrange de grado 3 para aproximar $f(1.2)$; $(1.1, 1.21), (1.3, 1.69), (1.4, 1.96), (1.7, 2.89)$ el valor de L_2 es:?

Lagrange

$L_2=-0.555556$

$$P(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_1)(x_0-x_3)}f(x_0)$$

$$P_1(x)=\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}f(x_1)$$

$$P_2(x)=\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}f(x_2)=\frac{(1.2-1.1)(1.2-1.3)(1.2-1.7)}{(1.4-1.1)(1.4-1.3)(1.4-1.7)}\cdot(1.96)=-0.555556x$$

$\boxed{\begin{array}{c}0.555556\\ \text{Opción b}\end{array}}$

4) El coeficiente x^2 en el polinomio de Lagrange de segundo grado que se construye con la función $f(x)=\sqrt{1+x}$; $x_0=0$; $x_1=1$; $x_2=3$

$$P(x)=\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0)+\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1)+\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2)$$

$$\frac{(x-1)(x-3)}{\underbrace{(0-1)(0-3)}_{3}}f(0)+\frac{(x-0)(x-3)}{\underbrace{(1-0)(1-3)}_{-2}}f(1)+\frac{(x-0)(x-1)}{\underbrace{(3-0)(3-1)}_{6}}f(3)$$

$$\frac{(x-1)(x-3)}{3}f(0)+\frac{(x-0)(x-3)}{-2}f(1)+\frac{(x-0)(x-1)}{6}f(3)$$

$f(x)=\sqrt{1+x} \rightarrow f(0)=\sqrt{1}=1$; $f(1)=\sqrt{2}$; $f(3)=\sqrt{4}=2$

$$\frac{x^2-4x+3}{3}(1)+\frac{x^2-3x}{-2}(\sqrt{2})+\frac{x^2-x}{6}(2)$$

$$\frac{x^2-4x+3}{3}+\frac{3x-x^2}{2}\sqrt{2}+\frac{x^2-x}{6}2$$

$$\frac{1}{3}x^2-\frac{4}{3}x+1+\frac{3\sqrt{2}}{2}x-\frac{\sqrt{2}}{2}x^2+\frac{1}{3}x^2-\frac{1}{3}x=-0,04044x^2$$

$$\boxed{\begin{array}{c}-0,0404407\\ \text{opción C}\end{array}}$$

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5) La aproximación a $f(4,5)$ por medio de un polinomio de Lagrange y los puntos $(1,1)$; $(2,0{,}5)$; $(4,0{,}25)$; $(5,0.2)$

$$P(x)=\frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}f(x_0)+\frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}f(x_1)+\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}f(x_2)$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}f(x_3);\quad x=4,5$$

$$f(4,5)\ \frac{(4,5-2)(4,5-4)(4,5-5)(1)}{(1-2)(1-4)(1-5)}+\frac{(4,5-1)(4,5-4)(4,5-5)(0,5)}{(2-1)(2-4)(2-5)}+\frac{(4,5-1)(4,5-2)(4,5-5)(0,25)}{(4-1)(4-2)(4-5)}+$$

$$\frac{(4,5-1)(4,5-2)(4,5-4)(0,2)}{(5-1)(5-2)(5-4)}=\frac{-0,625}{-12}+\frac{-0,4375}{6}+\frac{-1,09375}{-6}+\frac{0,875}{12}$$

$f(4,5)=0,234375$

$$\boxed{\begin{array}{c}f(4,5)=0,234375\\ \text{opción C}\end{array}}$$