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Primer Examen Parcial
Matematica Intermedia 3

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Seccion B

1

Cuando el azúcar se disuelve en el agua, la cantidad A que permanece sin disolverse después de t minutos satisface la ecuación diferencial $\frac{dA}{dt} = -KA$ para $(K>0)$. Si 25% del azúcar se disuelve después de 1 minuto.

a) Determinar la tasa de decaimiento exponencial de la disolución del azúcar.

b) ¿Cuánto tiempo toma para que la mitad del azúcar se disuelva?

a) $\frac{dA}{dt} = -KA$; A = Cantidad sin disolver

después de 1 minuto se disuelve el 25% → queda el 75%

$$\frac{dA}{A} = -Kdt \rightarrow \int_{A_0}^{A} \frac{1}{A}\, dA = -K\int_0^t dt \rightarrow Ln[A]_{A_0}^{A} = -Kt\Big]_0^t$$

$$Ln[A] - Ln[A_0] = -Kt \rightarrow Ln\left(\frac{A}{A_0}\right) = -Kt \text{ ; ahora: } A = 0.75A_0$$

$$Ln\left(\frac{0.75A_0}{A_0}\right) = -Kt \rightarrow K = \frac{Ln(0.75)}{-1min} = \rightarrow K = 0.28768$$

b) ahora si $A = 0,5$

$$Ln\left(\frac{A}{A_0}\right) = -Kt \rightarrow ln\left(\frac{0,5A_0}{A_0}\right) = -0,28768^{-1} \cdot t$$

$$t = \frac{-ln(0,5)}{0,28768} \rightarrow t = 2,4 \text{ minutos}$$

Respuestas

A) 0.28768
b) 2.4 minutos

#2

$$3xy^2\frac{dy}{dx} = 3x^4 + y^3$$

a) Tipo de ecuación diferencial

4) Bernoulli //

b) Factor Integración

$\mu = e^{\int P(x)dx}$, $P(x) = -\frac{1}{3x} \rightarrow e^{\int -\frac{1}{3x}dx} = e^{-1/3\int \frac{1}{x}dx} = e^{-\frac{1}{3}\ln x} = e^{\ln x^{-\frac{1}{3}}} = x^{-1/3} = \frac{1}{x^{1/3}}$ (5) Ninguna

c) $y'y^2 - \frac{1}{3x}y^2y = 3x^3y^{-2}y^2 \rightarrow y^2y' - \frac{1}{3x}y^3 = 3x^3$

Sea $w = y^3 \rightarrow dw = 3y^2 dy \rightarrow 3[y^2y' - \frac{1}{3x}y^3] = 3[3x^3]$

$3y^2y' - \frac{1}{x}y^3 = 9x^3 \rightarrow dw - \frac{1}{x}w = 9x^3$; $P(x) = \frac{-1}{x}$

$\mu e^{\int P(x)dx} = e^{-\int \frac{1}{x}dx} = e^{-\ln(x)} = e^{\ln x^{-1}} = x^{-1}$

$w \cdot x^{-1} = \int 9x^3 \cdot x^{-1}dx = \int 9x^2 dx = \cancel{x}\frac{1}{3}x^3 + C = 3x^3 + C$

$w = \frac{3x^3+C}{x^{-1}} \rightarrow w = 3x^4 + xC = y^3 \rightarrow y^3 = 3x^3 + Cx \rightarrow y(x) = (3x^3 + Cx)^{1/3}$ // (5) Ninguna

Respuestas

a) Bernoulli

b) 5 (Ninguna) $\rightarrow \frac{1}{x^{1/3}}$

c) 5 (Ninguna) $\rightarrow y(x) = (3x^3 + Cx)^{1/3}$

#3

$$(xy - x^2 - 1)dx + (x^2+1)dy = 0$$

A) Tipo; $P(x,y)$ $xy - x^2 - 1$ $\rightarrow \frac{\partial P}{\partial y} = x$; $\frac{\partial P}{\partial x} = 2x \rightarrow$ No exacta (Reducible exacta)

$P(x,y) = x^2+1$

$$\frac{\frac{\partial P}{\partial y} - \frac{\partial P}{\partial x}}{a} = g(x) \rightarrow g(x) = \frac{x - 2x}{x^2+1} = \frac{-x}{x^2+1} = g(x);$$

b) $y = e^{\int g(x)dx}$; $\int g(x)dx = \int \frac{-x}{x^2-1} dx$; $w = x^2+1$, $dw = 2x dx$

$-\int \frac{x}{x^2+1} dx = -\frac{1}{2}\int \frac{2x dx}{x^2+1} = -\frac{1}{2}\int \frac{d(w)}{w} = -\frac{1}{2} \ln(x)$

$-\frac{1}{2}\ln(x^2+1) = \ln(x^2+1)^{-1/2} \rightarrow y = e^{\ln(x^2+1)^{-1/2}} \rightarrow y = (x^2+1)^{-1/2}$

$y = \frac{1}{(x^2+1)^{1/2}} = \frac{1}{\sqrt{x^2+1}}$ //

c) Solución

$(x^2+1)^{-1/2}(xy-x^2-1)dx + (x^2+1)^{-1/2}(x^2+1)dy = 0$

$(x^2+1)^{-1/2}(xy-x^2-1)dx\,(x^2+1)^{1/2}dy = 0$

$P(x,y) = (x^2+1)^{-1/2}(xy-x^2-1)$

$P(x,y) = (x^2+1)^{1/2}dy$

$\frac{du}{dx} = P \rightarrow \frac{\partial u}{dy} = P \rightarrow \frac{du}{dy} = P \rightarrow U = \int P(x,y)dy = \int (x^2+1)^{1/2} dy +$

$u = (x^2+1)^{1/2} y + h(x)$

$\frac{\partial u}{dx} = P \rightarrow y = \frac{1}{2}(x^2+1)^{-1/2}(2x) + h'(x) = xy(x^2+1)^{-1/2} + h'(x) = (x^2+1)^{-1/2}(xy-x^2-1)$

$h'(x) = (x^2+1)^{-1/2}(xy) - (x^2+1)^{-1/2}(x^2+1) - xy(x^2+1)^{-1/2}$

$h'(x) = -(x^2+1)^{1/2} = \frac{dh(x)}{dx} \rightarrow h(x) = -\int \sqrt{x^2+1}\, dx$

$h(x) = \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\ln|x+\sqrt{1+x^2}|$; Por tanto

$$\boxed{z(x,y) = \sqrt{x^2+1} - y + \frac{1}{2}x\sqrt{x^2+1} + \frac{1}{2}\ln|x+\sqrt{1+x^2}| + C}$$

Respuestas

a) 5 Reducible exacta
b) Ninguna de las anteriores 5
c) 5 Ninguna de las anteriores

#4

a) $\left(\frac{d^4y}{dx^4}\right) + \left(\frac{dy}{dx}\right)^2 - \frac{2}{x}y = -\frac{2}{3}xy^{5/2}$

$y^{iv} + (y')^2 - \frac{2}{x}y = \frac{-2}{3}xy^{5/2} \rightarrow$ Orden 4 No lineal E.D.O

Respuestas

a) E.D.O. Orden 4 No lineal ✓
b) E.D.O. Orden 1 No lineal ✓

b) $\left(\frac{dy}{dx}\right)^3 - \frac{x}{x^2+1}y = xy^{-1}$

$xy^{-1} \rightarrow (y')^3 = \frac{x}{x^2+1}y = xy^{-1}$; Orden 1 no lineal E.D.O

#5

$(x^{-1/2}y^{1/2} + \frac{x}{x^2+y})dx + p(x,y)dy = 0$

$\frac{d}{dy}\frac{\partial}{\partial y}\left(x^{-1/2}y^{1/2} + \frac{x}{x^2+y}\right] = x^{-1/2}\left(\frac{1}{2}\right)y^{-1/2} + \left[\frac{\cancel{(x^2+y)(x)'}^{0} - (x)(x^2+y)'}{(x^2+y^2)^2}\right]$

$\frac{1}{2}x^{-1/2}y^{-1/2} - \frac{x}{(x^2+y)^2} = \frac{dQ}{dx}$; siendo $Q = P(x,y)$

$\frac{d\,P(x,y)}{dx} = \frac{1}{2}x^{-1/2}y^{-1/2} - \frac{x}{x^2+y^2} \rightarrow P(x,y) = \int^x \frac{1}{2}x^{-1/2}y^{-1/2}dx - \int^x \frac{x}{(x^2+y)^2}dx$

$\frac{1}{2}y^{-1/2}\int x^{-1/2}dx - \frac{1}{2}\int\frac{2x\,dx}{(x^2+y)^2}$; $w = x^2+y$, $dw = 2x\,dx$

$/\frac{1}{2}y^{-1/2}\frac{x^{1/2}}{\frac{1}{2}} - \frac{1}{2}\frac{(x^2+y)^{-1}}{-1} = x^{1/2}y^{-1/2} + \frac{1}{2(x^2+y)} = P(x,y)$

b) $(x^{-1/2}y^{1/2} + \frac{x}{x^2+y})dx + (x^{1/2}y^{-1/2} + \frac{1}{2(x^2+y)})dy = 0$ $\quad P(x,y) = x^{1/2}y^{-1/2} + \frac{1}{2}(x^2+y)dy = 0$

$\frac{du}{dx} = x^{-1/2}y^{1/2} + \frac{x}{x^2+y} \rightarrow u(x,y) = \int^x x^{-1/2}y^{1/2}dx + \int^x \frac{x}{x^2+y}dx$

$u(x,y) = y^{1/2}\cdot\frac{x^{1/2}}{\frac{1}{2}} + \frac{1}{2}\ln|x^2+y| + h(y)$

$u(x,y) = \frac{1}{2}x^{1/2}y^{1/2} + \frac{1}{2}h(x^2+y) + h(y)$

$$\frac{du}{dy} = P \to \frac{\partial}{\partial y} = \frac{1}{2} x^{1/2} \cdot \frac{1}{2} y^{-1/2} + \frac{1}{2} \frac{1}{x^2+y} + h'(y) =$$

$$\frac{1}{4} x^{1/2} y^{-1/2} + \frac{1}{2(x^2+y)} + h'(y) = x^{1/2} y^{-1/2} + \frac{1}{2(x^2+y)}$$

$$h'(y) = \frac{3}{4} x^{1/2} y^{-1/2} = \frac{dh(y)}{dy} \to h(y) = \frac{3}{4} x^{1/2} \int y^{-1/2} dy =$$

$$\frac{3}{4} x^{1/2} \frac{y^{1/2}}{\frac{1}{2}} = \frac{3}{4} \cdot 2 x^{1/2} y^{1/2} = \frac{3}{2} x^{1/2} y^{1/2} + C$$

$$u(x,y) = 2x^{1/2} y^{1/2} + \frac{1}{2} \ln(x^2+y) = C$$

Rezultat

a) $P(x,y) = x^{1/2} y^{-1/2} + \frac{1}{2}(x^2+y)dy = 0$ Opcija A

b) $2x^{1/2} y^{1/2} + \frac{1}{2} \ln(x^2+y) = C$ Opcija C