Ej Resolver

$$[x^2 y' + xy = 1] * \frac{1}{x^2}$$

$$y' + \frac{\cancel{x}}{x^2} y = \frac{1}{x^2} \rightarrow \frac{dy}{dx} + P(x)y = Q(x)$$

$$y' + \frac{1}{x} y = \frac{1}{x^2} \quad — \quad P(x) = \frac{1}{x}$$

$$Q(x) = \frac{1}{x^2}$$

$$F.I. = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$

$$x y' + \frac{1}{\cancel{x}}(x) y = \frac{\cancel{x}}{x^2} = \frac{1}{x}$$

$$\frac{d}{dx}(xy) = xy' + (1)y$$

$$\frac{d}{dx}(xy) = \frac{1}{x}$$

$$\int d(xy) = \int \frac{1}{x} dx$$

$$xy = \ln x + C \rightarrow y = \frac{\ln x}{x} + \frac{C}{x} = \frac{\ln x + C}{x}$$

Ej. Resolver

$$\left[(1+x)\frac{dy}{dx} - xy = x + x^2\right] * \frac{1}{1+x}$$

$$\frac{dy}{dx} - \frac{x}{1+x} y = \frac{x+x^2}{1+x} = \frac{x(\cancel{1+x})}{\cancel{1+x}}$$

$$P(x) = -\frac{x}{1+x} \qquad Q(x) = x$$

$$F.I. = e^{\int P(x)dx} = e^{(-)\int \frac{x}{1+x}dx}$$

$$x+1 \overline{\smash{)}\,x\,} \quad 1$$
$$\underline{-x-1}$$
$$-1$$

$$\rightarrow \quad F.I. = e^{-\int 1 - \frac{1}{x+1}\,dx}$$

$$F.I. = e^{-[x + \ln(x+1)]} = e^{-x} \cdot e^{+\ln(x+1)}$$

$$F.I. = e^{-x}\, e^{\ln(x+1)^{+1}} = e^{-x}(x+1)^{1} = e^{-x}(x+1)$$

$$\frac{d}{dx}\left(e^{-x}(x+1)\,y\right) = x e^{-x}(x+1) = x^2 e^{-x} + x e^{-x}$$

$$\int d\left(e^{-x}(x+1)\,y\right) = \int (x^2 e^{-x} + x e^{-x})\,dx$$

$$\int x^2 e^{-x}\,dx$$

$$u = x^2 \qquad du = 2x\,dx$$
$$\int dv = \int e^{-x}dx \qquad v = -e^{-x}$$
$$-x^2 e^{-x} + 2\int x e^{-x}\,dx$$

$$u = x \qquad du = dx$$
$$\int dv = \int e^{-x}dx \qquad v = -e^{-x}$$
$$-xe^{-x} + \int e^{-x}dx$$
$$-xe^{-x} - e^{-x}$$

$$e^{-x}(x+1)\,y = -x^2 e^{-x} + 2(-xe^{-x} - e^{-x}) + (-xe^{-x} - e^{-x}) + C$$
$$e^{-x}(x+1)\,y = -x^2 e^{-x} - 3xe^{-x} - 3e^{-x} + C$$
$$y = \frac{-x^2 e^{-x} - 3xe^{-x} - 3e^{-x} + C}{e^{-x}(x+1)}$$

Ej. Resolver

$$(x+2)^2 \frac{dy}{dx} = 5 - 8y - 4xy$$

$$(x+2)^2 \frac{dy}{dx} = 5 - 4y(2+x)$$

$$\left[(x+2)^2 \frac{dy}{dx} + 4y(2+x) = 5 \right] * \frac{1}{(x+2)^2}$$

$$\frac{dy}{dx} + \frac{4y}{(x+2)} = \frac{5}{(x+2)^2}$$

$u = x+2$
$du = dx$

$$P(x) = \frac{4}{x+2} \qquad Q(x) = \frac{5}{(x+2)^2}$$

$$e^{4\int \frac{du}{u}} \quad e^{4\ln u} \quad e^{\ln u^4}$$

$$F.I. = e^{\int P(x)dx} = e^{4\int \frac{dx}{x+2}} = e^{4\ln(x+2)}$$

$$F.I. = e^{\ln(x+2)^4} = (x+2)^4$$

$$\frac{d}{dx}(\underbrace{(x+2)^4 y}_{F.I. \cdot y}) = \frac{5}{(x+2)^2}(x+2)^4 = 5(x+2)^2$$

$u = x+2$
$du = dx$
$5\int u^2 du$

$$\int d(x+2)^4 y = \int 5(x+2)^2 dx$$

$$(x+2)^4 y = \frac{5}{3}(x+2)^3 + C$$

$$y = \frac{5}{3}\frac{(x+2)^3}{(x+2)^4} + \frac{C}{(x+2)^4}$$

Sol

$$y = \frac{5}{3}\frac{1}{(x+2)} + \frac{C}{(x+2)^4}$$

$$y = \frac{5}{3}(x+2)^{-1} + C(x+2)^{-4}$$

Ej. Resolver

$$y' + \tan x \, y = \cos^2 x \qquad y(0) = -1$$

$$P(x) = \tan x \qquad Q(x) = \cos^2 x$$

$$F.I. = e^{\int P(x)dx} = e^{\int \tan x \, dx} = e^{\int \frac{\operatorname{sen} x}{\cos x} dx}$$

$u = \cos x \qquad du = -\operatorname{sen} x \, dx$

$$F.I. = e^{-\int \frac{du}{u}} = e^{-\ln u} = e^{\ln u^{-1}} = u^{-1}$$

$$F.I. = (\cos x)^{-1} = \frac{1}{\cos x}$$

$$\frac{d}{dx}\left(\frac{1}{\cos x}\, y\right) = \frac{\cos^{\cancel{2}} x}{\cancel{\cos x}}$$

$$\int d\left(\frac{y}{\cos x}\right) = \int \cos x \, dx$$

$$\frac{y}{\cos x} = \operatorname{sen} x + C$$

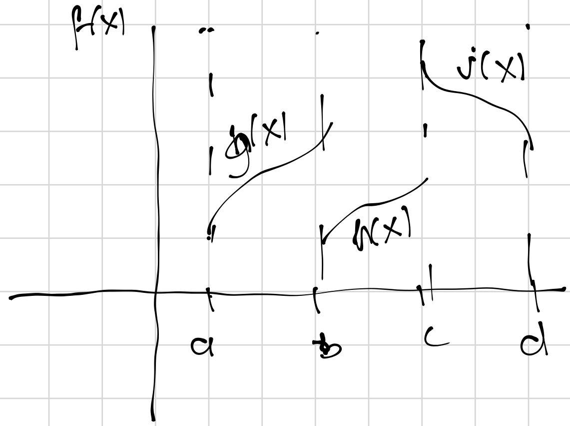
$$y = \operatorname{sen} x \cos x + C \cos x$$

$$y(0) = -1$$

$$-1 = \operatorname{sen} 0 \cos 0 + C \cos 0 \rightarrow C = -1$$

Sol $y = \operatorname{sen} x \cos x - \cos x$

Ecuaciones lineales con coeficientes discontinuos



$$f(x) = \begin{cases} g(x) & a \le x < b \\ h(x) & b \le x < c \\ j(x) & c \le x < d \end{cases}$$

$$\frac{dy}{dx} + P(x)y = f(x) \qquad f(x) = \begin{cases} g(x) & a \le x < b \\ h(x) & b \le x < c \end{cases}$$

$$a \le x < b \rightarrow \frac{dy}{dx} + P(x)y = g(x)$$

$$b \le x < c \rightarrow \frac{dy}{dx} + P(x)y = h(x)$$

Ej. Resolver

$$(1+x^2)\frac{dy}{dx} + 2xy = f(x) \qquad y(0)=0$$

$$f(x) = \begin{cases} x & 0 \le x < 1 \\ -x & x \ge 1 \end{cases}$$

$0 \le x < 1 \rightarrow \left[(1+x^2)\frac{dy}{dx} + 2xy = x \right] * \frac{1}{(1+x^2)}$

$$\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{x}{1+x^2}$$

$$p(x) = \frac{2x}{1+x^2} \qquad u = 1+x^2$$

$$du = 2x\,dx$$

$$F.I. = e^{\int p(x)dx} = e^{\int \frac{2x}{1+x^2}dx}$$

$$F.I. = e^{\int \frac{du}{u}} = e^{\ln u} = \cancel{e}^{\cancel{\ln}(1+x^2)}$$

$$F.I. = (1+x^2)$$

$$\frac{d}{dx}\left[(1+x^2)y\right] = \frac{x}{\cancel{1+x^2}}\cancel{(1+x^2)} = x$$

$$\cancel{\int d}\left[(1+x^2)y\right] = \int x\,dx$$

$$(1+x^2)y = \frac{1}{2}x^2 + C_1$$

$$y = \frac{1}{2}\frac{x^2}{1+x^2} + \frac{C_1}{1+x^2}$$

$y(0)=0$

$$0 = \frac{1}{2}\cancel{\left(\frac{0^2}{1+0^2}\right)} + \frac{C_1}{1+0^2} \rightarrow \boxed{C_1 = 0}$$

$$y = \frac{1}{2}\frac{x^2}{1+x^2}$$

$\int p(x)$

$x \ge 1 \quad \left[(1+x^2)\frac{dy}{dx} + 2xy = -x \right] * \frac{1}{1+x^2}$

$$\frac{dy}{dx} + \frac{2x}{1+x^2}\,y = \frac{-x}{1+x^2}$$

$$F.I. = e^{\int \frac{2x}{1+x^2}dx} = e^{\int \frac{du}{u}} = \cancel{e}^{\cancel{\ln} u} = (1+x^2)$$

$$\frac{d}{dx}\left((1+x^2)\,y\right) = \frac{-x}{\cancel{1+x^2}}\,(\cancel{1+x^2}) = -x$$

$$\int d\left((1+x^2)\,y\right) = \int -x\,dx$$

$$(1+x^2)\,y = -\frac{1}{2}x^2 + C_2$$

$$y = -\frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{C_2}{1+x^2}$$

$$y = \begin{cases} \frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{C_1}{1+x^2} & 0 \le x < 1 \\ -\frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{C_2}{1+x^2} & x \ge 1 \end{cases}$$

$$y = \begin{cases} \frac{1}{2}\,\frac{x^2}{1+x^2} & 0 \le x < 1 \\ -\frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{C_2}{1+x^2} & x \ge 1 \end{cases}$$

$$\lim_{x\to 1^-} y = \lim_{x\to 1^+} y$$

$$\lim_{x\to 1^-} \frac{1}{2}\,\frac{x^2}{1+x^2} = \lim_{x\to 1^+} -\frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{C_2}{1+x^2}$$

$$\frac{1}{2}\,\frac{(1)^2}{1+1^2} = -\frac{1}{2}\,\frac{(1)^2}{1+1^2} + \frac{C_2}{1+1^2}$$

$$\frac{1}{4} = -\frac{1}{4} + \frac{C_2}{2} \rightarrow \frac{1}{\cancel{2}} = \frac{C_2}{\cancel{2}} \rightarrow C_2 = 1$$

Sol

$$y = \begin{cases} \frac{1}{2}\,\frac{x^2}{1+x^2} & 0 \le x < 1 \\ -\frac{1}{2}\,\frac{x^2}{1+x^2} + \frac{1}{1+x^2} & x \ge 1 \end{cases}$$

Ej. Resolver $\frac{dy}{dx} + P(x)\, y = 4x \qquad y(0) = 3$

$$P(x) = \begin{cases} 2 & 0 \le x \le 1 \quad y_1 \\ -\frac{2}{x} & x > 1 \quad y_2 \end{cases}$$

$0 \le x < 1$

$$\frac{dy}{dx} + 2y = 4x$$

$$F.I. = e^{\int 2 dx} = e^{2x}$$

$\int x e^{2x} dx$

$u = x \quad du = dx$

$dv = e^{2x} dx \quad v = \frac{1}{2} e^{2x}$

$$\frac{d}{dx}(e^{2x} y) = 4x e^{2x}$$

$$\int d(e^{2x} y) = \int 4x e^{2x} dx$$

$$e^{2x} y = 4\left(\frac{1}{2} x e^{2x} - \frac{1}{2}\int e^{2x} dx\right)$$

$$e^{2x} y = 2x e^{2x} - e^{2x} + C_1$$

$$y = \frac{2x \cancel{e^{2x}}}{\cancel{e^{2x}}} - \frac{\cancel{e^{2x}}}{\cancel{e^{2x}}} + \frac{C_1}{e^{2x}}$$

$$y = 2x - 1 + C_1 e^{-2x}$$

$y(0) = 3$

$3 = 2(0) - 1 + C_1 \cancel{e^{-2(0)}}^{1}$

$\boxed{4 = C_1}$

$\rightarrow y = 2x - 1 + 4e^{-2x}$

$x \ge 1 \qquad \frac{dy}{dx} + \left[-\frac{2}{x}\right] y = 4x$

$$F.I. = e^{-2\int \frac{dx}{x}} = e^{-2 \ln x} = \cancel{e}^{\cancel{\ln} x^{-2}} = x^{-2}$$

$$\frac{d}{dx}(x^{-2} y) = \frac{4\cancel{x}}{\cancel{x^{-2}}} = \frac{4}{x}$$

$$\int d(x^{-2}y) = \int \frac{4}{x}\,dx$$

$$x^{-2}y = 4\ln x + C_2$$

$$y = \frac{4\ln x}{x^{-2}} + \frac{C_2}{x^{-2}}$$

$$y = 4x^2\ln x + C_2x^2$$

$$\lim_{x\to 1^-} 2x-1+4e^{-2x} = \lim_{x\to 1^+} 4x^2\ln x + C_2x^2$$

$$(2)(1) - 1 + 4e^{-2(1)} = \cancel{4(1)^2\ln 1} + C_2(1)^2$$

$$C_2 = 1 + 4e^{-2}$$

Sol

$$y = \begin{cases} 2x - 1 + 4e^{-2x} & 0 \le x < 1 \\ 4x^2\ln x + (1+4e^{-2})x^2 & x \ge 1 \end{cases}$$