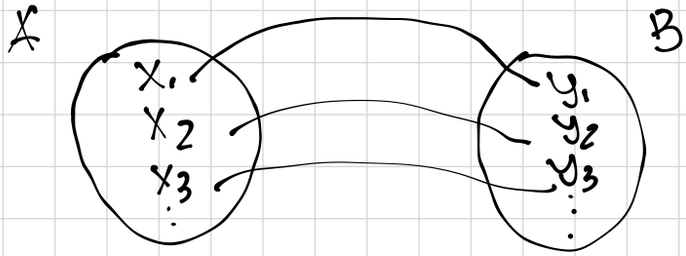
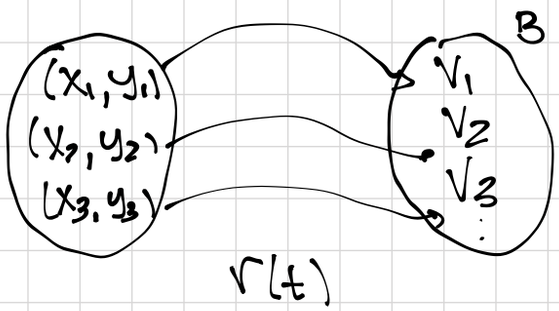
Funciones Vectoriales:

función → regla de correspondencia.



Dominio $y = f(x)$ Rango Parametro

F.V. A



$$x = x(t) = f(t)$$
$$y = y(t) = g(t)$$
$$z = z(t) = h(t)$$

$\mathbb{R}^2$ → Espacio bidimensional

$$r(t) = f(t)i + g(t)j$$

$\mathbb{R}^3$ → Espacio tridimensional

$$r(t) = f(t)i + g(t)j + h(t)k$$

Ej. Determine el dominio

$$r(t) = \ln(t+1)i + \frac{t}{\sqrt{9-t^2}}j + 2^t k$$

Para i $\quad f(t) = \ln(t+1)$

$t+1 > 0 \rightarrow t > -1 \rightarrow (-1, \infty)$

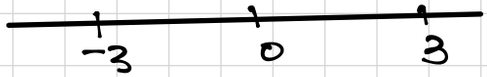
Para j $\quad g(t) = \frac{t}{\sqrt{9-t^2}}$

$9 - t^2 > 0$

$(3-t)(3+t) > 0$

$3 - t = 0 \rightarrow t = 3$

$3 + t = 0 \rightarrow t = -3$



$(-\infty, -3)$ $(-3, 3)$ $(3, \infty)$

	$(-\infty, -3)$	$(-3, 3)$	$(3, \infty)$
$(3-t)$	+	+	−
$(3+t)$	−	+	+
	−	+	−

R// $(-3, 3)$

Para k $h(t) = 2^t$

Reales.

Dominio →

$(-1, 3]$

$\{ t / t \in (-1, 3] \}$

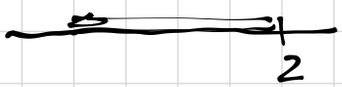
Ej. Determine el dominio

$$r(t) = \sqrt{2 - t}\, i + \frac{e^t - 1}{t} j + \ln(t+1) k$$

Para i $f(t) = \sqrt{2 - t}$

$2 - t \geq 0 \rightarrow -t \geq -2$ $\times -1$

$t \leq 2$



$(-\infty, 2]$

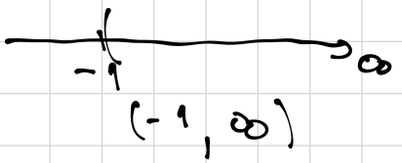
Para j $g(t) = \frac{e^t - 1}{t}$

$t \neq 0$

Para k $\quad h(t) = \ln(t+1)$

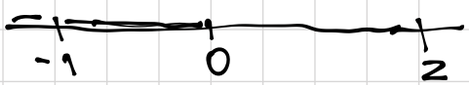
$t+1 > 0$

$t > -1$



$(-1, \infty)$

Dominio



$(-1, 0) \quad (0, 2]$

$\{t \mid t \in (-1,0) \cup (0,2]\}$

límites de funciones vectoriales

si $\quad r(t) = f(t)\,i + g(t)\,j + h(t)\,k$

$$\lim_{t \to a} r(t) = \lim_{t \to a} f(t)\,i + \lim_{t \to a} g(t)\,j + \lim_{t \to a} h(t)$$

siempre y cuando los límites de las funciones componentes existan.

forma indeterminada $\quad \frac{0}{0}, \frac{\infty}{\infty}$

⇓

L'Hôpital

Ej. Calcular

$$\lim_{t \to 1} \left(\frac{t^2-1}{t-1}\,i + \sqrt{t+5}\,j + \frac{\operatorname{sen} \pi(t)}{\ln t}\,k \right)$$

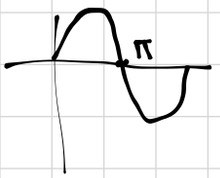
i $\quad \lim_{t \to 1} \frac{t^2-1}{t-1} \to \lim_{t \to 1} \frac{1^2-1}{1-1} = \frac{0}{0} \to$ F. I.

$$\lim_{t \to 1} \frac{(t-1)(t+1)}{t-1} = \lim_{t \to 1} t+1$$

$$\lim_{t\to 1} \; 1+1 = 2$$

j: $\lim_{t\to 1} \sqrt{t+5}$

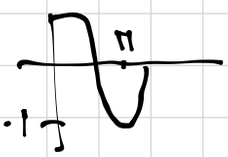
$$\lim_{t\to 1} \sqrt{1+5} = \sqrt{6}$$



k: $\lim_{t\to 1} \dfrac{\operatorname{sen} \pi t}{\ln t}$

$$\lim_{t\to 1} \frac{\operatorname{sen} \pi(1)}{\ln(1)} = \frac{0}{0} \to \text{F. I.}$$

$$\lim_{t\to 1} \frac{\pi \cos \pi t}{\frac{1}{t}}$$



$$\lim_{t\to 1} \frac{\pi \cos \pi(1)}{1/1} = \frac{\pi(-1)}{1} = -\pi$$

$$\lim_{t\to 1} r(t) = 2i + \sqrt{6}j - \pi k$$

Ej. Determine el límite

$$\lim_{t\to\infty} \left\langle \underbrace{\frac{e^{2t}}{2e^{2t}+t}}_{i}, \underbrace{\frac{e^{-t}}{2e^{-t}+5}}_{j}, \underbrace{\tan^{-1} t}_{k} \right\rangle$$

i: $\lim_{t\to\infty} \dfrac{e^{2t}}{2e^{2t}+t}$

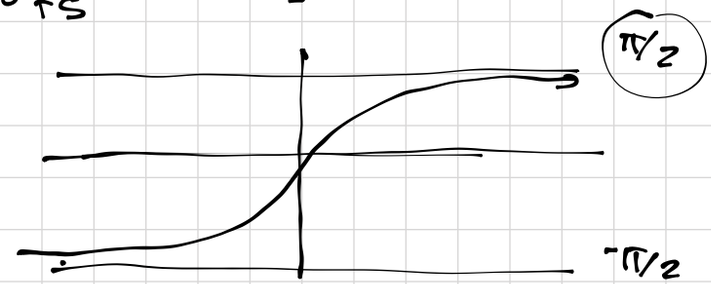
$$\lim_{t\to\infty} \frac{2e^{2t}}{4e^{2t}+1} = \lim_{t\to\infty} \frac{2e^{2(\infty)}}{4e^{2(\infty)}+1} = \frac{\infty}{\infty}$$

$$\lim_{t\to\infty} \frac{4e^{2t}}{8e^{2t}} = \lim_{t\to\infty} \frac{4}{8} = \frac{1}{2}$$

j: $\lim_{t\to\infty} \frac{e^t}{2e^t+5}$

$\lim_{t\to\infty} \frac{e^{-\infty}}{2e^{\infty}+5} = \frac{0}{0+5} = \frac{0}{5} = 0$

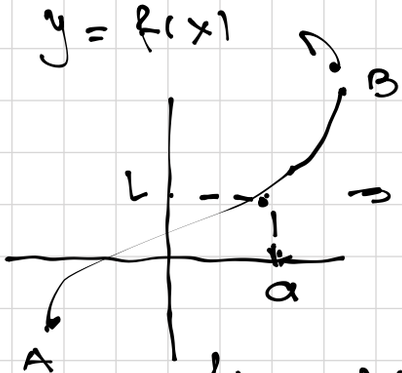
k: $\lim_{t\to\infty} \tan^{-1} t$



$\lim_{t\to\infty} \tan^{-1}\infty = \frac{\pi}{2}$

Sol $\lim_{t\to\infty} r(t) = \frac{1}{2}i + 0j + \frac{\pi}{2}k$

Continuidad



→ función es continua

$\lim_{x\to a} f(x) = f(a) = L$

↳ se puede determinar la grafica de una función.

función vectorial.

$\lim_{t\to a} r(t) = r(a)$ → función vectorial es continua

⇓

se puede obtener la grafica de la función vectorial

⇓

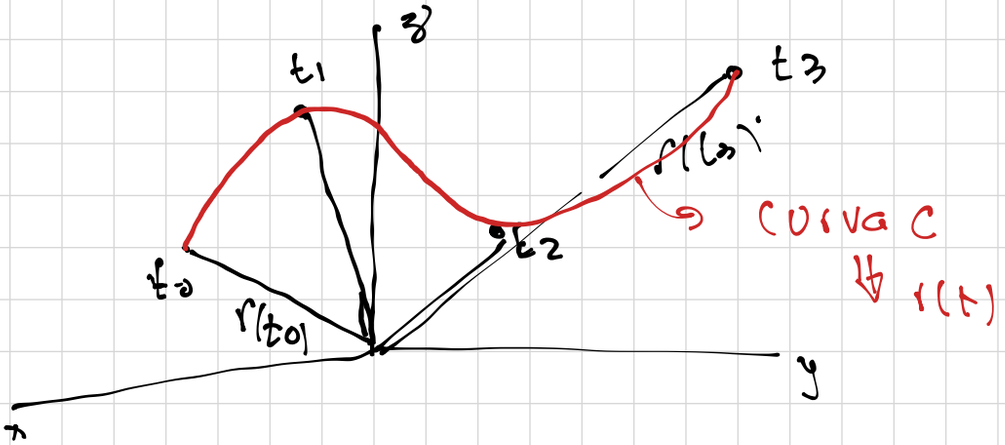
Curva en el espacio

Curva en el espacio

$r(t) = f(t)\,i + g(t)\,j + h(t)\,k \qquad a \le t \le b$

$x = f(t) \qquad y = g(t) \qquad z = h(t)$

Ecuaciones paramétricas de la curva C.



t	x	y	z
t_0	x_0	y_0	z_0
t_1	x_1	y_1	z_1
t_2	x_2	y_2	z_2
t_3	x_3	y_3	z_3

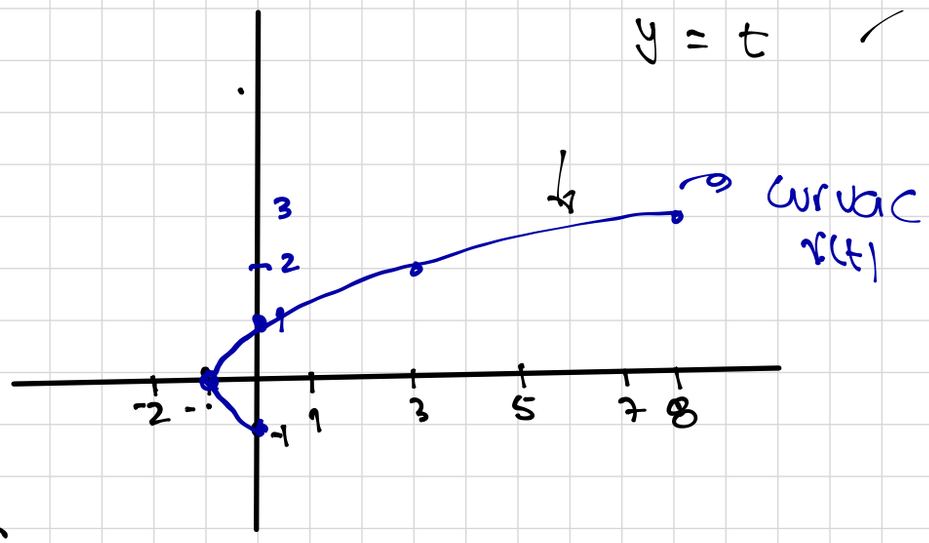
Ej. trace la curva de la función

$r(t) = (t^2 - 1)\,i + t\,j$

t	x	y
-1	0	-1
0	-1	0
1	0	1
2	3	2
3	8	3

$x = t^2 - 1$

$y = t$



$x = t^2 - 1 \qquad y = t$

$\sqrt{x+1} = \sqrt{t^2}$

$t = \sqrt{x+1} \qquad y = t$

$t = t$

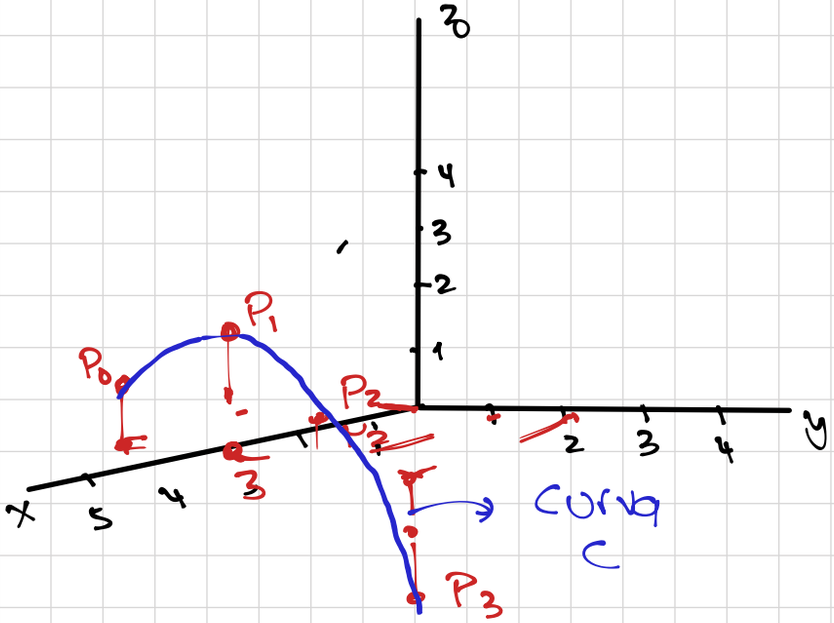
$(\sqrt{x+1})^2 = (y)^2 \rightarrow \boxed{x+1 = y^2}$

Ej. trace la grafica

$r(t) = 3i + tj + (2 - t^2)k$

$x = 3 \qquad y = t \qquad z = 2 - t^2$

t	x	y	z	
-1	3	-1	1	P_0
0	3	0	2	P_1
1	3	1	1	P_2
2	3	2	-2	P_3



Ej. trace la grafica.

$r(t) = t\cos t\, i + t \operatorname{sen} t\, j + t\, k$

$(x)^2 = (t\cos t)^2 \qquad (y)^2 = (t \operatorname{sen} t) \qquad (z)^2 = (t)^2$

$x^2 = t^2\cos^2 t \qquad y^2 = t^2 \operatorname{sen}^2 t \qquad z^2 = t^2$

$x^2 = t^2\cos^2 t +$

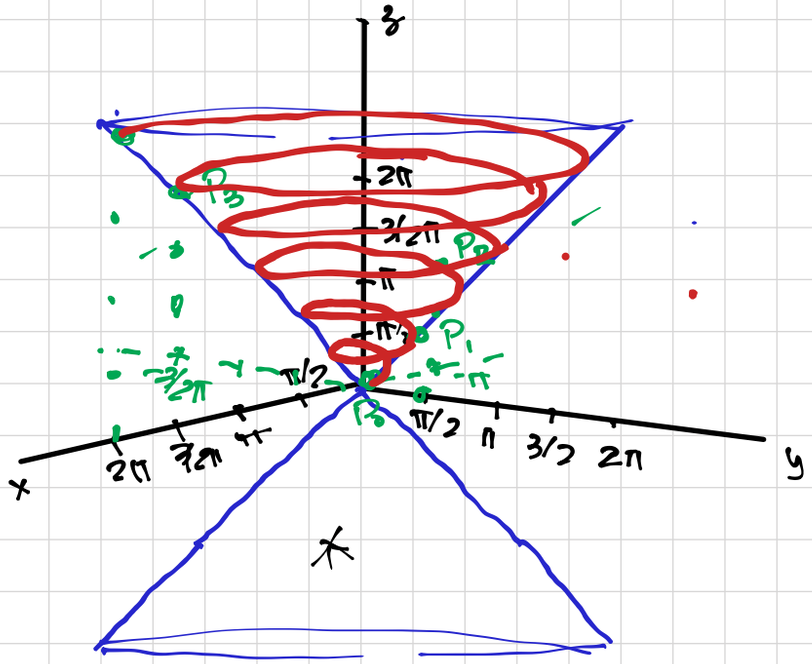
$y^2 = t^2 \operatorname{sen}^2 t$

$x^2 + y^2 = t^2\cos^2 t + t^2 \operatorname{sen}^2 t = t^2(\cos^2 t + \operatorname{sen}^2 t)$

$x^2 + y^2 = t^2$

$x^2 + y^2 = z^2$ → cono.

$x = t\cos t$
$y = t\sen t$
$z = t$



t	x	y	z	
0	0	0	0	P_0
π/2	0	π/2	π/2	P_1
π	−π	0	π	P_2
3/2π	0	−3/2π	3/2π	P_3
2π	2π	0	2π	P_4

$r(t) = \cos t\, i + \sen t\, j + t\, k$