UNIVERSIDAD DE SAN CARLOS DE GUATEMALA
FACULTAD DE INGENIERÍA
ESCUELA DE CIENCIAS
DEPARTAMENTO DE MATEMÁTICA
MATEMÁTICA INTERMEDIA 2

MI2B

TAREA No. _#3__

DESCRIPCIÓN DE CALIFICACIÓN		CALIFICACIÓN
Presentación		
Ejercicios resueltos		
Ejercicio calificado 1	#	
Ejercicio calificado 2	#	
CALIFICACIÓN TOTAL		

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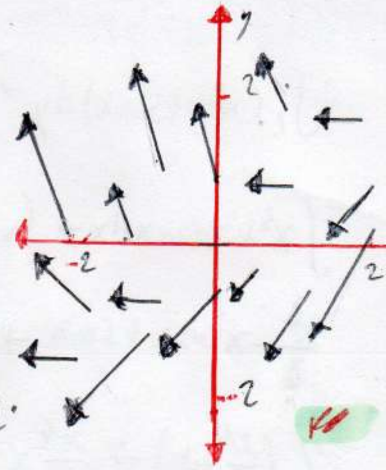
Fecha: 30 / 12 / 2022

16.1 EJERCICIOS

| # 1 | P. 1073 | Ej. 3 |

1-10 Trace el campo vectorial **F** dibujando un diagrama como la figura 5 o la figura 9.

3. $\mathbf{F}(x, y) = -\frac{1}{2}\mathbf{i} + (y - x)\mathbf{j}$



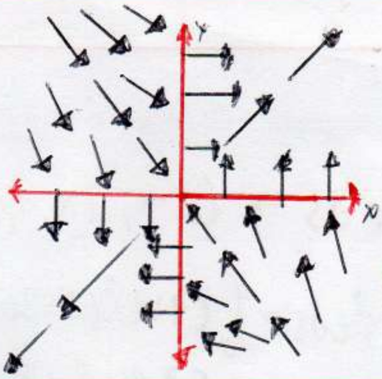
$F(x,y) = -\frac{1}{2}\hat{\imath} + (y-x)\hat{\jmath}$

Longitud del vector $\to -\frac{1}{2}\hat{\imath} + (y-x)\hat{\jmath} \to \sqrt{\frac{1}{4} + (y-x)^2}$

Los vectores a lo largo de la línea $y = x$ son horizontales con longitud $\frac{1}{2}$.

5. $\mathbf{F}(x, y) = \dfrac{y\,\mathbf{i} + x\,\mathbf{j}}{\sqrt{x^2 + y^2}}$

| # 2 | P. 1073 | Ej. 5 |



$F(x,y) = \dfrac{y\hat{\imath} + x\hat{\jmath}}{\sqrt{x^2+y^2}}$

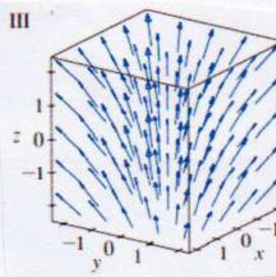
La longitud del vector $\dfrac{y\hat{\imath} + x\hat{\jmath}}{\sqrt{x^2+y^2}}$ es 1

15-18 Asocie los campos vectoriales **F** en $\mathbb{R}^3$ con los diagramas rotulados I-IV. Dé razones de sus decisiones.

| # 3 | P. 1074 | Ej. 17 |

17. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\,\mathbf{k}$

III



$F(x,y,z) = x\hat{\imath} + y\hat{\jmath} + 3k \to$ corresponde al gráfico III

La proyección de cada vector en el plano x-y es $x\hat{\imath} + y\hat{\jmath}$, que apunta lejos del origen, y los vectores apuntan generalmente hacia arriba porque sus componentes z son todos 3.

16.2 EJERCICIOS

| # 4 | P. 1084 | Ej. 3 |

1-16 Evalúe la integral de línea, donde C es la curva dada.

3. $\int_C xy^4\,ds$, C es la mitad derecha del círculo $x^2 + y^2 = 16$

Paramedrica en C

$\int_C xy^4\,ds$; $C \Rightarrow x = 4\cos t$

$y = 4\operatorname{sen} t$; $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$$\int_C xy^4\,ds \to \int_{-\pi/2}^{\pi/2} (4\cos t)(4\operatorname{sen} t)^4 \sqrt{(-4\operatorname{sen} t)^2 + (4\cos t)^2}\,dt = \int_{-\pi/2}^{\pi/2} 4^5 \cos t \operatorname{sen}^4 t \sqrt{16(\operatorname{sen}^2 t - \cos^2 t)}\,dt$$

$$4^5 \int_{-\pi/2}^{\pi/2} (\operatorname{sen}^4 t \cos t)(4)\,dt = (4)^6 \left[\frac{1}{5}\operatorname{sen}^5 t\right]_{-\pi/2}^{\pi/2} = 4^6 \cdot \frac{2}{5} = 1638.40 \to \boxed{\int_C xy^4\,ds = 1638.40}$$

5 | P. 1084 | Ej. 5

5. $\int_C (x^2y + \operatorname{sen} x)\, dy$,
C es el arco de la parábola $y = x^2$ de $(0, 0)$ a (π, π^2)

$y=x^2$; $(0,0)$ (π,π^2)

parametro → x ; $C \to x = x$; $y = x^2$; $0 \le x \le \pi$

$$\int_C (x^2y+\operatorname{sen}x)\,dy \to \int_0^\pi [x^2(x^2)+\operatorname{sen}x]\,2x\,dx = 2\int_0^\pi (x^4+\operatorname{sen}x)\,dx \to \int_0^\pi x^5 + x\operatorname{sen}x\,dx$$

$$\int x^5 + x\operatorname{sen}x\,dx \to \int x^5\,dx + \int x\operatorname{sen}x\,dx \to \text{usando: } \int x^n dx = \frac{x^{n+1}}{n+1},\ n \ne -1 \to \frac{x^6}{6} + \int x\operatorname{sen}x\,dx$$

$$\frac{x^6}{6} - x\cos x + \operatorname{sen}x \to \left(\frac{x^6}{6} - x\cos x + \operatorname{sen}x\right)\Big]_0^\pi \to \frac{\pi^6}{6} - \pi\cos\pi + \operatorname{sen}\pi - \left(\frac{0^6}{6} - 0\cos 0 + \operatorname{sen}0\right) = \frac{\pi^6}{6} + \pi$$

$$2\left(\frac{\pi^6}{6}+\pi\right) \to \frac{\pi^6}{3} + 2\pi$$

$$\boxed{\int_C (x^2y+\operatorname{sen}x)\,dy = \frac{\pi^6}{3} + 2\pi}$$

10. $\int_C y^2z\, ds$,
C es el segmento de recta de $(3, 1, 2)$ a $(1, 2, 5)$

6 | P. 1084 | Ej. 10

$(3,1,2)$; $(1,2,5)$ $\quad 0 \le t \le 1$

$$\int_C y^2z\,ds \to C \to ;\ (3,1,2)\ (1,2,5)\ ;\ C;\ x = 2-2t\ ;\ y = 1+t\ ;\ z = 2+3t.$$

$$\int_C y^2z\,ds = \int_0^1 (1+t)^2(2+3t)\sqrt{(-2)^2+1^2+3^2}\,dt = \sqrt{14}\int_0^1 (3t^3+8t^2+7t+2)\,dt$$

$$= \sqrt{14}\left[\frac{3}{4}t^4 + \frac{8}{3}t^3 + \frac{7}{2}t^2 + 2t\right]_0^1 = \sqrt{14}\left(\frac{3}{4}+\frac{8}{3}+\frac{7}{2}+2\right) = \frac{107}{12}\sqrt{14}$$

$$\boxed{\int_C y^2z\,ds = \frac{107}{12}\sqrt{14} \approx 33.36}$$

12. $\int_C xyz^2\, ds$,
C es el segmento de recta de $(-1, 5, 0)$ a $(1, 6, 4)$

7 | P. 1084 | Ej. 12

$C \to (-1,5,0)\ (1,6,4)$

$$\int_C xyz^2\,ds \to \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{dz}{dt}\right)^2} = \sqrt{1^2+(-2\operatorname{sen}2t)^2+(2\cos 2t)^2} = \sqrt{1+4(\operatorname{sen}^2 2t+\cos^2 2t)} = \sqrt{5}$$

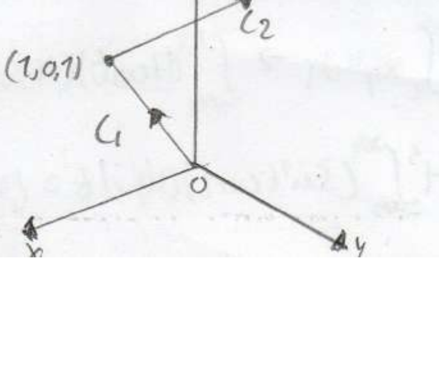
$$\int_C (x^2+y^2+z^2)\,ds = \int_0^{2\pi} (t^2+\cos^2 2t+\operatorname{sen}^2 2t)\sqrt{5}\,dt = \sqrt{5}\int_0^{2\pi}(t^2+1)\,dt = \sqrt{5}\left[\frac{1}{3}t^3+t\right]_0^{2\pi}$$

$$\sqrt{5}\left[\frac{1}{3}(8\pi^3)+2\pi\right] = \sqrt{5}\left(\frac{8}{3}\pi^3+2\pi\right)$$

$$\boxed{\int_C xyz^2\,ds = \sqrt{5}\left(\frac{8}{3}\pi^3+2\pi\right)}$$

16. $\int_C (y + z)\, dx + (x + z)\, dy + (x + y)\, dz$,
C consta de segmentos de recta de $(0, 0, 0)$ a $(1, 0, 1)$ y de $(1, 0, 1)$ a $(0, 1, 2)$

8 | P. 1084 | Ej. 16



En → C_1: $x = t \Rightarrow dx = dt$; $y = 0 \to dy = 0\,dt$, $z = t$
$dz = dt$; $0 \le t \le 1$

En → C_2; $x = 1-t \Rightarrow dx = -dt$, $y = t$
$dy = dt$; $z = 1+t \Rightarrow dz = dt$; $0 \le t \le 1$

$$\int_C (y+z)dx + (x+z)dy + (x+y)dz$$

$$\int_{C_1} (y+z)dx + (x+z)dy + (x+y)dz + \int_{C_2} (y+z)dx + (x+z)dy + (x+y)dz$$

$$\int_0^1 (0+t)dt + (t+t)0dt + (t+0)dt + \int_0^1 (t+1+t)(-dt) + (1-t+1+t)dt + (1-t+t)dt$$

$$\int_0^1 2t\,dt + \int_0^1 (-2t+2)dt = [t^2]_0^1 + [-t^2+2t]_0^1 = 1+1 = 2$$

$$\boxed{\int_C (y+z)dx + (x+z)dy + (x+y)dz = 2}$$

# 9	G. 1085	P. 76

19-22 Evalúe la integral de línea $\int_C \mathbf{F} \cdot d\mathbf{r}$, donde C está dada por la función vectorial $\mathbf{r}(t)$.

21. $\mathbf{F}(x, y, z) = \operatorname{sen} x\, \mathbf{i} + \cos y\, \mathbf{j} + xz\, \mathbf{k}$,
$\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\,\mathbf{k}$, $0 \leqslant t \leqslant 1$

$F(x,y,z) = \operatorname{sen} x\,\hat{i} + \cos y\,\hat{j} + xz\,k$

$r(t) = t^3\hat{i} - t^2\hat{j} + t\,k \;;\; 0 \leq t \leq 1$

$$\int_C F \cdot dr = \int_0^1 (\operatorname{sen} t^3, \cos(-t^2), t^4)(3t^2, -2t, 1)\,dt \to \int_0^1 (3t^2 \operatorname{sen} t^3 - 2t\cos t^2 + t^4)\,dt$$

$$\left[-\cos t^3 - \operatorname{sen} t^2 + \frac{1}{5}t^5\right]_0^1 = \frac{6}{5} - \cos 1 - \operatorname{sen} 1$$

$$\boxed{\int_C F\,dr = \frac{6}{5} - \cos 1 - \operatorname{sen} 1}$$

# 10	P. 1086	G. 39

9. Encuentre el trabajo realizado por el campo de fuerzas

$$\mathbf{F}(x, y) = x\,\mathbf{i} + (y+2)\,\mathbf{j}$$

para mover un objeto a lo largo de un arco del cicloide

$$\mathbf{r}(t) = (t - \operatorname{sen} t)\,\mathbf{i} + (1 - \cos t)\,\mathbf{j} \qquad 0 \leqslant t \leqslant 2\pi$$

$F(x,y) = x\hat{i} + (y+2)\hat{j}$ $\qquad 0 \leq t \leq 2\pi$

$r(t) = (t - \operatorname{sen} t)\hat{i} + (1-\cos t)\hat{j}$

$$W = \int_C F \cdot dr = \int_0^{2\pi} \langle t - \operatorname{sen} t, 3 - \cos t\rangle \langle 1-\cos t, \operatorname{sen} t\rangle\,dt$$

$$\int_0^{2\pi} (t - t\cos t - \operatorname{sen} t + \operatorname{sen} t\cos t + 3\operatorname{sen} t - \operatorname{sen} t\cos t)\,dt \to \int_0^{2\pi} (t - t\cos t + 2\operatorname{sen} t)\,dt$$

$$= \int_0^{2\pi} t\,dt - \int_0^{2\pi} t\cos(t)\,dt + \int_0^{2\pi} 2\operatorname{sen}(t)\,dt \to \int_0^{2\pi} t\,dt \to \left[\frac{t^{1+1}}{1+1}\right]_0^{2\pi} = \left.\frac{t^2}{2}\right]_0^{2\pi} \to \lim_{t\to 0}\left(\frac{t^2}{2}\right) = 0$$

$$\lim_{t\to 2\pi}\left(\frac{t^2}{2}\right) = 2\pi^2$$

$$= 2\pi^2 - 0 \to 2\pi^2; \to \int_0^{2\pi} t\cos t\,dt = 0 \to \left[t\operatorname{sen} t - \int \operatorname{sen} t\,dt\right]_0^{2\pi} \to \int \operatorname{sen} t\,dt = 0 \to \int_0^{2\pi} t\cos t\,dt$$

$\int uv' = uv - \int u'v$

$u = t$

$v' = \cos t$

$$\to u' = \frac{d}{dt}t = 1;\; v = \int \cos t\,dt = \operatorname{sen} t \to \left[t\operatorname{sen} t - \int 1\operatorname{sen} t\,dt\right]_0^{2\pi} \to \left[t\operatorname{sen} t - \int \operatorname{sen} t\,dt\right]_0^{2\pi}$$

$$\int \cos t\,dt \to \operatorname{sen} t + C$$

$$\int \operatorname{sen} t\,dt = -\cos t \to [t\operatorname{sen} t + \cos t]_0^{2\pi} \to \begin{cases} \lim_{t\to 0}(t\operatorname{sen} t + \cos t) = 1 \\ \lim_{t\to 2\pi}(t\operatorname{sen} t + \cos t) = 1 \end{cases} = 1 - 1 = 0$$

$$\int_0^{2\pi} 2\operatorname{sen} t\,dt \to 2[-\cos t]_0^{2\pi} \to 0 \;;\; 2\cdot 0 \;;\; 2\pi^2 - 0 + 0 \to 2\pi^2$$

$$\boxed{\int_C F\,dr = 2\pi^2}$$

16.3 EJERCICIOS

-10 Determine si **F** es un campo vectorial conservativo o no. Si o es, halle una función f tal que $\mathbf{F} = \nabla f$.

# 11	P. 1094	G. 7

$\mathbf{F}(x, y) = (ye^x + \operatorname{sen} y)\,\mathbf{i} + (e^x + x\cos y)\,\mathbf{j}$

$F(x,y) = (ye^x + \operatorname{Sen} y)i + (e^x + x\cos y)j$

$\dfrac{\partial(ye^x+\operatorname{Sen}y)}{\partial y} = e^x + \cos y \Rightarrow \dfrac{\partial(e^x+\cos y)}{\partial x}$

$F \to \mathbb{R}^2 \quad f \to \nabla f = F \;;\; f_x(x,y) = ye^x + \operatorname{Sen} y$

$f(x,y) = ye^x + x\operatorname{Sen}y + g(y)$

$f_y(x,y) = e^x + x\cos y + g'(y)$

$f_y(x,y) = e^x + x\cos y + g'(y) \to f_y(x,y) = e^x + x\cos y \;;\; g(y) = K \;;\; f(x,y) = ye^x + x\operatorname{Sen}y + K$

$\boxed{f(x,y) = ye^x + x\operatorname{Sen}y + K}$

4. $\mathbf{F}(x, y) = x^2\,\mathbf{i} + y^2\,\mathbf{j}$,
C es el arco de la parábola $y = 2x^2$ de $(-1, 2)$ a $(2, 8)$

# 12	P. 1094	G. 14

$F(x,y) = x^2 i + y^2 j \to F(x,y) = x^2 e^{xy} \to f(x,y) = xe^{xy} + g(x) \to f_x(x,y) = xye^{xy} + e^{xy} + g'(x)$

a) $f_x(x,y) = (1+xy)e^{xy} \to g'(x) = 0 \to g(x) = K \;;\; K = 0$ $\qquad (1+xy)e^{xy} + g'(x)$

$f(x,y) = xe^{xy}$

b) $C \to r(0) = (1,0) \;;\; r\left(\frac{\pi}{2}\right) = (0,2) \to \int_C F\cdot dr = f(0,2) - f(1,0) = 0 - e^0 = -1$

Respuestas

a) $f(x,y) = xe^{xy}$

b) -1

7. $\mathbf{F}(x, y, z) = yze^{xz}\,\mathbf{i} + e^{xz}\,\mathbf{j} + xye^{xz}\,\mathbf{k}$,
C: $\mathbf{r}(t) = (t^2 + 1)\,\mathbf{i} + (t^2 - 1)\,\mathbf{j} + (t^2 - 2t)\,\mathbf{k}$,
$0 \leq t \leq 2$

# 13	P. 1095	G. 17

a) $f_x(x,y,z) = yze^{xz} \to f(x,y,z) = ye^{xz} + g(y,z) \;;\; f_y(x,y,z) = e^{xz} + g_y(y,z)$

$f_y(x,y,z) = e^{xz} \to f_z(x,y,z) = xye^{xz} \;;\; h'(z) = 0 \to h(z) = K \;;\; f(x,y,z) = ye^{xz}$ $\qquad K = 0$

b) $r(0) = \langle 1,-1,0\rangle \;;\; r(2) = \langle 5,3,0\rangle \;;\; \int_C F\,dr = f(5,3,0) - f(1,-1,0) = 3e^0 + e^0 = 4$

a) $f(x,y,z) = ye^{xz}$

b) 4

#14 | P. 1095 | Ej. 24

23-24 Halle el trabajo realizado por el campo de fuerzas **F** para mover un objeto de P a Q.

24. $\mathbf{F}(x, y) = (2x + y)\,\mathbf{i} + x\,\mathbf{j}$; $P(1, 1)$, $Q(4, 3)$

$\rightarrow F(x,y) = (2x+y)\hat{i} + x\hat{j}$; $P(1,1)$; $Q(4,3)$

$W = \int_C F\,dr$; $\frac{\partial(2x+y)}{\partial y} = 1 = \frac{\partial(x)}{\partial x}$; $\nabla f = F\,ln$; $f_x(x,y) = 2x+y \rightarrow f(x,y) = x^2 + xy + g(y)$

$f_y(x,y) = x + g'(y)$; $f_y(x,y) = x$; $g'(y) = 0 \rightarrow g(y) = K$. $K = 0 \rightarrow f(x,y) = x^2 + xy$

$W = \int_C F\,dr = f(4,3) - f(1,1) = (16+12) - (1+1) = 26$

$\boxed{W = \int_C F\,dr = 26}$

16.4 EJERCICIOS

#15 | P. 1101 | Ej. 2

1-4 Evalúe la integral de línea por dos métodos: (a) directamente y (b) usando el teorema de Green.

2. $\oint_C y\,dx - x\,dy$,
C es el círculo con centro en el origen y radio 4

a) $C \rightarrow x = 4\cos t$; $y = 4\operatorname{sen} t$; $0 \le t \le 2\pi$

$dx = -4\operatorname{sen} t\,dt$; $dy = 4\cos t\,dt$

a) $\oint_C y\,dx - x\,dy = \int_0^{2\pi} [(4\operatorname{sen} t)(-4\operatorname{sen} t) - (4\cos t)(4\cos t)]\,dt = -16\int_0^{2\pi} (\operatorname{sen}^2 t + \cos^2 t)\,dt$

$-16\int_0^{2\pi} 1\,dt = -16(2\pi) = -32\pi$

b) $\oint_C y\,dx - x\,dy = \iint_D \left[\frac{\partial}{\partial x}(-x) - \frac{\partial}{\partial y}(y)\right] dA = \iint_D (-1-1)\,dA = -2\iint_D dA \rightarrow -2(\text{área de } D)$

$= -2\pi(4)^2 = -32\pi \approx -100.53$

$\boxed{a)\ -32\pi \qquad b)\ -32\pi}$

#16 | P. 1102 | Ej. 6

La región D está encerrada por C

5-10 Use el teorema de Green para evaluar la integral de línea a lo largo de la curva con orientación positiva dada.

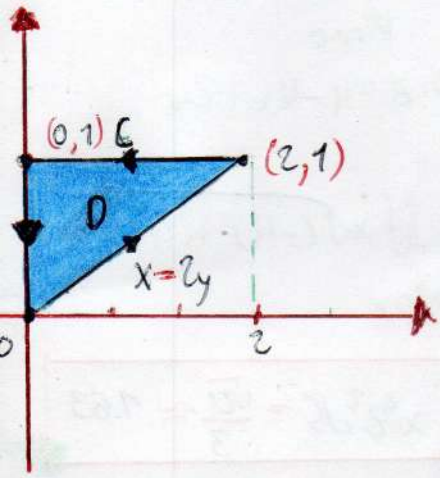
6. $\int_C (x^2 + y^2)\,dx + (x^2 - y^2)\,dy$,
C es el triángulo con vértices $(0, 0)$, $(2, 1)$ y $(0, 1)$

$\{(x,y) \mid 0 \le y \le 1 ;\ 0 \le x \le 2y\}$

$\int_C (x^2+y^2)\,dx + (x^2-y^2)\,dy$

$\iint_D \left[\frac{\partial}{\partial x}(x^2-y^2) - \frac{\partial}{\partial y}(x^2+y^2)\right] dA \rightarrow \int_0^1 \int_0^{2y} (2x - 2y)\,dx\,dy$

$\int_0^1 [x^2 - 2xy]_{x=0}^{x=2y}\,dy \rightarrow \int_0^1 (4y^2 - 4y^2)\,dy = \int_0^1 0\,dy = 0$



$\boxed{\int_C (x^2+y^2)\,dx + (x^2-y^2)\,dy = 0}$

8. $\int_C \cos y\,dx + x^2 \operatorname{sen} y\,dy$,
C es el rectángulo con vértices (0, 0), (5, 0), (5, 2) y (0, 2)

#17 P. 1102 Ej. 8

$\int_C \cos y\,dx + x^2 \operatorname{sen} y\,dy$; (0,0) (5,0) (5,2) (0,2)

$$\int_C y^4 dx + 2xy^3 dy = \iint_D \left[\frac{\partial}{\partial x}(2xy^3) - \frac{\partial}{\partial y}(y^4)\right] dA = \iint_D (2y^3 - 4y^3)\,dA$$

$$= -2\iint_D y^3\,dA = 0$$

$$\boxed{-2\iint_D y^3 dA = 0}$$

#18 P. 1102 Ej. 13

11-14 Use el teorema de Green para evaluar $\int_C \mathbf{F}\cdot d\mathbf{r}$. (Verifique la orientación de la curva antes de aplicar el teorema.)

13. $\mathbf{F}(x, y) = \langle y - \cos y, x \operatorname{sen} y\rangle$,
C es el círculo $(x-3)^2 + (y+4)^2 = 4$ orientado en el sentido de las manecillas del reloj

$\int_C F\cdot dr$; $F(x,y) = \langle y - \cos y, x \operatorname{sen} y\rangle$
C ; $(x-3)^2 + (y+4)^2 = 4$ sentido del reloj

$F(x,y) = \langle y - \cos y, x \operatorname{sen} y\rangle$ La región D esta encerrada por C, es el disco con un radio 2 centrado en (3,-4); se atraviesa en el sentido de las agujas del reloj, por lo que $-C$ da la orientación positiva.

$$\int_C F\,dr = -\int_{-C} (y-\cos y)dx + (x\operatorname{sen} y)dy = -\iint_D \left[\frac{\partial}{\partial x}(x\operatorname{sen} y) - \frac{\partial}{\partial y}(y - \cos y)\right] dA$$

$$-\iint_D (\operatorname{sen} y - 1 - \operatorname{sen} y)\,dA = \iint_D dA = \text{área de } D = \pi(2)^2 = 4\pi$$

$$\boxed{\iint_D dA = 4\pi}$$

16.7 EJERCICIOS

5-20 Evalúe la integral de superficie.

#19 P. 1133 Ej. 11

. $\iint_S x^2 z^2\,dS$,
S es la parte del cono $z^2 = x^2 + y^2$ que está entre los planos $z = 1$ y $z = 3$

$\iint_S x^2 z^2 dS \to z^2 = x^2 + y^2$; $z = 1$; $z = 3 \to$

plano

$(1,0,0)(0,-2,0)(0,0,4) \to 4x - 2y + z = 4 \to z = 4 - 4x + 2y$

Sobre $\to D \to \{(x,y) \mid 0 \le x \le 1, 2x-2 \le y \le 0\}$; $\iint_S x^2 z^2 dS \to \iint x\sqrt{(-4)^2 + (2)^2 + 1}\,dA$

$$\sqrt{21}\int_0^1\int_{2x-2}^0 x\,dy\,dx = \sqrt{21}\int_0^1 [xy]_{y=2x-2}^{y=0} dx \to \sqrt{21}\int_0 (-2x^2 + 2x)dx$$

$$\sqrt{21}\left[-\frac{2}{3}x^3 + x^2\right]_0^1 = \sqrt{21}\left(-\frac{2}{3} + 1\right) = \frac{\sqrt{21}}{3} \approx 1.53$$

$$\boxed{\iint_S x^2 z^2 dS = \frac{\sqrt{21}}{3} \simeq 1.53}$$

4. $\iint_S y^2z^2\,dS$;
S es la parte del cono $y = \sqrt{x^2 + z^2}$ dado por $0 \le y \le 5$

#20 | P. 1133 | Ej 14

$r(x,z) = x\hat{i} + \sqrt{x^2+z^2}\,\hat{j} + zk$; $x^2+z^2 \le 25$

$r_x \times r_z = \left(\hat{i} + \frac{x}{\sqrt{x^2+z^2}}\hat{j}\right)\left(\frac{z}{\sqrt{x^2+z^2}}\hat{j} + k\right) = \frac{x}{\sqrt{x^2+z^2}}\hat{i} - \hat{j} + \frac{z}{\sqrt{x^2+z^2}}k$

$|r_x \times r_z| = \sqrt{\frac{x^2}{x^2+z^2} + 1 + \frac{z^2}{x^2+z^2}} = \sqrt{\frac{x^2+z^2}{x^2+z^2} + 1} = \sqrt{2}$

$\iint_S y^2z^2\,dS = \iint_{x^2+z^2\le 25} (x^2+z^2)z^2\sqrt{2}\,dA = \sqrt{2}\int_0^{2\pi}\int_0^5 r^2(r\sin\theta)^2 r\,dr\,d\theta$

$\sqrt{2}\int_0^{2\pi}\sin^2\theta\,d\theta\int_0^5 r^5\,dr = \sqrt{2}\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}\left[\frac{1}{6}r^6\right]_0^5$

$\sqrt{2}(\pi)\,\frac{1}{6}(15{,}625 - 0) = \frac{15.625\sqrt{2}}{6}\pi$

$\boxed{\frac{15.625\sqrt{2}}{6}\pi}$

21-32 Evalúe la integral de superficie $\iint_S \mathbf{F}\cdot d\mathbf{S}$ para el campo vectorial dado $\mathbf{F}$ y la superficie orientada S. En otras palabras, determine el flujo de $\mathbf{F}$ por S. Para superficies cerradas, use la orientación positiva (hacia fuera).

24. $\mathbf{F}(x, y, z) = xz\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$,
S es el hemisferio $x^2 + y^2 + z^2 = 25$, $y \ge 0$ orientado en la dirección del eje y positivo

#21 | P. 1133 | Ej. 24

$F(x,y,z) = xz\hat{i} + x\hat{j} + yk$

$y(x,y) = \sqrt{xz^2 + y^2} \to \{(x,y) \mid 1 \le x^2+y^2 \le 25\}$

$\iint_S F\,dS = -\iint_D \left[-(-x)\left(\frac{x}{\sqrt{x^2+y^2}}\right) - (-y)\left(\frac{y}{\sqrt{x^2+y^2}}\right) + y\right]dA$

$= -\iint_D \left[\frac{x^2+y^2}{\sqrt{x^2+y^2}} + \left(\sqrt{x^2+y^2}\right)^3\right]dA = -\int_0^{2\pi}\int_1^3\left(\frac{r^2}{r} + r^3\right)r\,dr\,d\theta \Rightarrow \int_0^{2\pi}d\theta\int_1^2(r^2+r^4)\,dr = -[\theta]_0^{2\pi}\left[\frac{1}{3}r^3 + \frac{1}{5}r^5\right]_1^3$

$2\pi\left(9 + \frac{243}{5} - \frac{1}{3} - \frac{1}{5}\right) = -\frac{1712}{15}\pi \approx -358.56$

$\boxed{\iint_S F\,dS = -\frac{1712}{15}\pi \approx -358.56}$

27. $\mathbf{F}(x, y, z) = y\,\mathbf{j} - z\,\mathbf{k}$,
S consta del paraboloide $y = x^2 + z^2$, $0 \le y \le 1$ y el disco $x^2 + z^2 \le 1$, $y = 1$

#22 | P. 1133 | Ej. 27

$\iint_S F\,dS = \iint_D F(r(u,v))\,(r_u \times r_v)\,dA$

$r_u = \frac{\partial x}{\partial u}\hat{i} + \frac{\partial y}{\partial u}\hat{j} + \frac{\partial z}{\partial u}k$

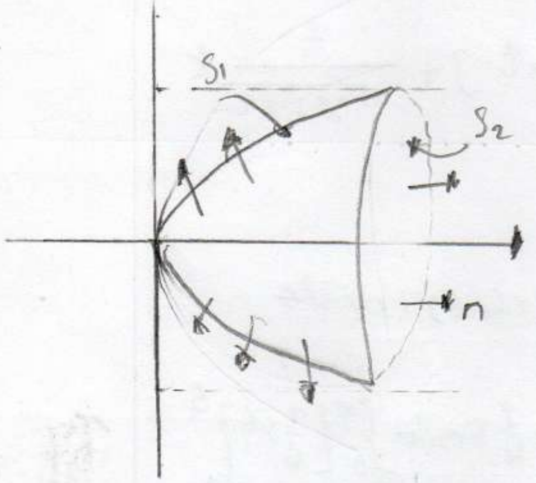
$r_v = \frac{\partial x}{\partial v}\hat{i} + \frac{\partial y}{\partial v}\hat{j} + \frac{\partial z}{\partial v}k$

$S_1 = y = x^2+z^2$

$S_2 = x^2+z^2 \le 1,\ y = 1$

$$\iint_S F\,dS = \iint_{S_1} F\,dS_1 + \iint_{S_2} F\,dS_2 \rightarrow \quad r(u,v) = \langle u, u^2+v^2, v\rangle$$

$$F(r(u,v)) = \langle 0, u^2+v^2, -v\rangle$$



$$x = u \rightarrow \frac{\partial x}{\partial u} = 1 \,;\ \frac{\partial x}{\partial v} = 0$$

$$y = u^2+v^2 \rightarrow \frac{\partial y}{\partial u} = 2u \,;\ \frac{\partial y}{\partial v} = 2v$$

$$z = v \rightarrow \frac{\partial z}{\partial u} = 0 \,;\ \frac{\partial z}{\partial v} = 1$$

$$r_u = \langle 1, 2u, 0\rangle \,;\ r_v = \langle 0, 2v, 1\rangle$$

$$r_u \times r_v = \begin{bmatrix} i & j & k \\ 1 & 2u & 0 \\ 0 & 2v & 1 \end{bmatrix} \rightarrow i\begin{bmatrix} 2u & 0 \\ 2v & 1 \end{bmatrix} - j\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + k\begin{bmatrix} 1 & 2u \\ 0 & 2v \end{bmatrix} \rightarrow 2ui - 1j + 2vk$$

$$= \langle 2u, -1, 2v\rangle$$

$$F(r(u,v))(r_u \times r_v) = \langle 0, u^2+v^2, -v\rangle\langle 2u, -1, 2v\rangle$$

$$-(u^2+v^2) - 2v^2$$

$$\iint_{S_1} F\,dS = \iint_D F(r(u,v))(r_u \times r_v)\,dA \rightarrow \iint_{0\le u^2+v^2\le 1} (-(u^2+v^2) - 2v^2)\,du\,dv$$

$$\int_0^{2\pi}\int_0^1 (-r^2 - 2r^2 sen^2\theta)\, r\,dr\,d\theta \rightarrow u = r\,sen\,\theta;\ v = r\cos\theta \rightarrow \int_0^{2\pi}\int_0^1 (r^3 + 2r^3 sen^2\theta)\,dr\,d\theta$$

$$-\int_0^{2\pi}\left[\frac{r^4}{4} + \frac{2r^4}{4} sen^2\theta\right]_0^1 d\theta \rightarrow -\int_0^{2\pi}\left(\frac{1}{4} + \frac{sen^2\theta}{2}\right)d\theta \rightarrow -\int_0^{2\pi}\frac{1}{4}\,d\theta - \frac{1}{2}\int_0^{2\pi} sen^2\theta\,d\theta$$

$$-\frac{2\pi}{4} - \frac{1}{2}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta \rightarrow -\frac{\pi}{2} - \frac{1}{2}\left[\int_0^{2\pi}\frac{d\theta}{2} - \int_0^{2\pi}\frac{\cos 2\theta}{2}\,d\theta\right] = -\frac{\pi}{2} - \frac{2\pi}{4} + 0$$

$$= -\pi \quad \rightarrow \iint_{S_1} F\,dS_1 = -\pi \quad ; \quad r(u,v) = \langle u, 1, v\rangle;\ u^2+v^2 \le 1 \rightarrow r_u = \langle 1,0,0\rangle$$

$$r_v = \langle 0,0,1\rangle$$

$$r_u \times r_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \rightarrow = j \rightarrow = \langle 0,1,0\rangle \,;\ F(r(u,v)) = (0,1,0)$$

$$F(r(u,v))(r_u \times r_v) = \langle 0, 1, -v\rangle\langle 0,1,0\rangle = 1 \rightarrow \iint_{S_2} F\,dS_2 = \iint_D F(r(u,v))(r_u \times r_v)\,dA$$

$$= \iint_{u^2+v^2\le 1} 1\,dA \,;\ \text{Área de } D \rightarrow \pi \,;\ \iint_{S_1} F\,dS_2 = \pi \rightarrow \iint_S F\,dS = -\pi + \pi = 0$$

$$\boxed{\iint_S F\,dS = 0}$$

16.9 EJERCICIOS

A 23 | P. 1045 | Ej. 2

1-4 Verifique que el teorema de la divergencia es cierto para el campo vectorial **F** en la región *E*.

2. $\mathbf{F}(x, y, z) = y^2z^3\,\mathbf{i} + 2yz\,\mathbf{j} + 4z^2\,\mathbf{k}$,
E es el sólido encerrado por el paraboloide $z = x^2 + y^2$ y el plano $z = 9$

$S \to z = g(x,y)$

$F(x,y,z) = y^2z^3 i + 2yz j + 4z^2 K$

$\iint_S F\,dS = \iint_D P\left(\frac{\partial g}{\partial x}\right) + Q\left(\frac{\partial g}{\partial y}\right) - R\,dA$

$F = Pi + Qj + RK$; $\iint_{S_1} F\,dS$; $S_1 = z = x^2+y^2$

$\iint_{S_1} F\,dS = \iint_D P\left(\frac{\partial y}{\partial x}\right) + Q\left(\frac{\partial y}{\partial y}\right) - R\,dA$; $F(x,y,z) = y^2z^2 i + 2yz j + 4z^2 K$

$\iint_D (y^2z^3)(2x) + (2yz)(2y) - 4z^2\,dA = \iint_D 2xy^2z^3 + 4y^2z - 4z^2\,dA$; $z = x^2+y^2$

$\iint_D 2xy^2(x^2+y^2)^3 + 4y^2(x^2+y^2) - 4(x^2+y^2)^2\,dA$. $S_1 = x^2+y^2 = 9$; $z = 0$

$\int_0^{2\pi}\int_0^2 \left(2r^3\sin^2\theta\cos\theta(r^2)^3 + 4r^2\sin^2\theta(r^2) - 4(r^2)^2\right) r\,dr\,d\theta \to \int_0^{2\pi}\int_0^3 2r^{10}\sin^2\theta\cos\theta + 4r^5\sin\theta - 4r^5\,dr\,d\theta$

$\int_0^{2\pi}\left[\frac{2r^{11}\sin^2\theta\cos\theta}{11} + \frac{2r^6\sin^2\theta}{3} - \frac{2r^6}{3}\right]_0^3 d\theta$; $\int_0^{2\pi} \frac{2\cdot 3^{11}\sin^2\theta\cos\theta}{11} + 486\sin^2\theta - 486\,d\theta$

$= \frac{354294}{11}\int_0^{2\pi} \sin^2\theta\cos\theta\,d\theta - 486\int_0^{2\pi}\cos^2\theta\,d\theta$; $\sin\theta = u$; $\cos\theta\,d\theta = du$; $\int_0^{2\pi} \to \int_{\sin 0}^{\sin 2\pi} = \int_0^0$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$. $\iint_{S2} F\,dS \to \iint_{S2} F\,dS = \iint_D -P\left(\frac{\partial z}{\partial x}\right) - Q\left(\frac{\partial z}{\partial y}\right) + R\,dA$; $F(x,y,z) = y^2z^2 i + 2yz j + 4z^2 k$

$\iint_D -(y^2z^2)(0) - (2yz)(0) + 4z^2\,dA \to \iint_D 4z^2\,dA$; $z = 9$ $\to \iint_D 4\cdot 9^2\,dA = 324\iint_D dA = 324\cdot 9\pi$

2916π

$\iint_S F\,dS = \iint_{S_1} F\,dS + \iint_{S_2} F\,dS = -486\pi + 2916\pi = 2430\pi$; $\iiint_E \operatorname{div} F\,dV$

$F(x,y,z) = y^2z^2 i + 2yz j + 4z^2 K$

$\to \{(r,\theta,z) \in E \mid r^2 \le z \le 9,\ 0 \le r \le 3,\ 0 \le \theta \le 2\pi\}$

$\operatorname{div} F = \frac{\partial(y^2z^2)}{\partial x} + \frac{\partial(2yz)}{\partial y} + \frac{\partial(4z^2)}{\partial z} = 0 + 2z + 8z = 10z$

$\iiint_E \operatorname{div} F\,dV = \int_0^{2\pi}\int_0^3\int_{r^2}^9 10z\,r\,dz\,dr\,d\theta \to \int_0^{2\pi}\int_0^3 \left[5z^2 r\right]_{r^2}^9 dr\,d\theta$

$\int_0^{2\pi}\int_0^3 405r - 5r^5\,dr\,d\theta \to \int_0^{2\pi}\left[\frac{405}{2} - \frac{5r^6}{6}\right]_0^3 d\theta + \int_0^{2\pi} \frac{405\cdot 3^2}{2} - \frac{5\cdot 3^6}{6}\,d\theta$

$$\int_0^{2\pi} 1215\,d\theta \rightarrow 1215 \cdot 2\pi = 2430\pi \;;\; \iint_S F\cdot dS = \iiint_E \operatorname{div} F\,dV = 2430\pi$$

$$\iint_S F\cdot dS = \iiint_E \operatorname{div} F\,dV = 2430\pi$$

$$\boxed{\iint_S F\cdot dS = \iiint_E \operatorname{div} F\,dV = 2430\pi}$$

5-15 Use el teorema de la divergencia para calcular la integral de superficie $\iint_S \mathbf{F}\cdot d\mathbf{S}$; es decir, calcule el flujo de $\mathbf{F}$ a través de S.

7. $\mathbf{F}(x,y,z) = 3xy^2\,\mathbf{i} + xe^z\,\mathbf{j} + z^3\,\mathbf{k}$,
S es la superficie del sólido acotado por el cilindro $y^2+z^2=1$ y los planos $x=-1$ y $x=2$

#24	P.1145	Ej. 7

$y = r\cos\theta \;;\; z = r\operatorname{sen}\theta \;;\; x = x$

$$F = 3y^2 + 0 + 3z^2 \;;\; \iint_S F\,dS = \iiint_E (3y^2+3z^2)\,dV$$

$$\int_0^{2\pi}\int_0^1\int_{-1}^2 (3r^2\cos^2\theta + 3r^2\operatorname{sen}^2\theta)\,r\,dx\,dr\,d\theta \rightarrow 3\int_0^{2\pi} d\theta \int_0^1 r^3\,dr \int_{-1}^2 dx$$

$$3\,[\theta]_0^{2\pi}\left[\tfrac{1}{4}r^4\right]_0^1 [x]_{-1}^2 = 3(2\pi)\left(\tfrac{1}{4}\right)(3) = \frac{9\pi}{2}$$

$$\boxed{\iint_S F\cdot dS = \frac{9\pi}{2}}$$

$F(x,y,z) = z\,i + y\,j + zx\,k$

#25	P.1146	Ej. 10

S es la superficie del tetraedo encerrado por los planos de coordenadas y el plano:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$$

Vertices del Tetraedro $\rightarrow (0,0,0),(a,0,0),(0,b,0),(0,0,c)$

$$E = \left\{(x,y,z) \mid 0\le x\le a \;;\; 0\le y\le b\left(1-\frac{x}{a}\right) \;;\; 0\le z\le c\left(1-\frac{x}{a}-\frac{y}{b}\right)\right\} \rightarrow \text{tenemos } F = 0+1+x = x+1$$

$$\iint_S F\,dS = \iiint_E (x+1)\,dV = \int_0^a\int_0^{b(1-\frac{x}{a})}\int_0^{c(1-\frac{x}{a}-\frac{y}{b})} (x+1)\,dz\,dy\,dx$$

$$= \int_0^a\int_0^{b(1-\frac{x}{a})} (x+1)\left[c\left(1-\frac{x}{a}-\frac{y}{b}\right)\right]dy\,dx = c\int_0^a (x+1)\left[\left(1-\frac{x}{a}\right)y - \frac{1}{2b}y^2\right]_{y=0}^{y=b(1-\frac{x}{a})} dx$$

$$= c\int_0^a (x+1)\left[\left(1-\frac{x}{a}\right)\cdot b\left(1-\frac{x}{a}\right) - \frac{1}{2b}\cdot b^2\left(1-\frac{x}{a}\right)^2\right]dx = \frac{1}{2}bc\int_b^a (x+1)\left(1-\frac{x}{a}\right)^2 dx$$

$$= \frac{1}{2}bc\int_0^a \left(\frac{1}{a^2}x^3 + \frac{1}{a^2}x^2 - \frac{2}{a}x^2 + x - \frac{2}{a}x + 1\right)dx \rightarrow \frac{1}{2}bc\left[\frac{1}{4a^2}x^4 + \frac{1}{3a^2}x^3 - \frac{2}{3a}x^3 + \frac{1}{2}x^2 - \frac{1}{a}x^2 + x\right]_0^a$$

$$= \frac{1}{2}bc\left(\frac{1}{4}a^2 + \frac{1}{3}a - \frac{2}{3}a^2 + \frac{1}{2}a^2 - a + a\right) = \frac{1}{2}bc\left(\frac{1}{12}a^2 + \frac{1}{3}a\right) = \frac{1}{24}abc(a+4)$$

$$\boxed{\iint_S F\cdot dS = \frac{1}{24}abc(a+4)}$$