Ecuaciones Homogeneas

Función homogenea.

$$f(x,y) \rightarrow f(tx, ty) = t^n f(x,y) \rightarrow \text{F.H.}$$

$$f(x,y) = x^4y^2 + x^3y^3 + x^6$$

$$f(tx,ty) = (tx)^4(ty)^2 + (tx)^3(ty)^3 + (tx)^6$$

$$= t^4x^4\overset{t^6}{t^2}y^2 + t^3x^3\overset{t^6}{t^3}y^3 + t^6x^6$$

$$= t^6\underline{(x^4y^2 + x^3y^3 + x^6)} \rightarrow \text{grado } 6.$$

Ec. Diferencial.

$$M(x,y)dx + N(x,y)dy = 0$$

$$M(tx,ty) = t^n M(x,y)$$

$$N(tx,ty) = t^n N(x,y)$$

$n = n$ Ec. diferencial homogenea.

Sustitución $\boxed{y = vx}$ $\boxed{dy = vdx + xdv} \rightarrow$

se sustituye en la Ec. dif. de tal manera que la ecuación este expresado en terminos de v y de x. el resultado de la ecuación se resuelve por variables separables.
se regresa a la sustitución de $\boxed{y = vx}$ para expresar la solución en terminos de x e y.

$$x = vy \qquad dx = vdy + ydv$$

Ej. Resolver

$$(\overset{t^2}{y^2} + \overset{t^2}{xy})dx - \overset{t^2}{x^2}dy = 0$$

$$y = vx \qquad dy = vdx + xdv$$

$$((vx)^2 + x(vx))dx - x^2(vdx + xdv) = 0$$

$$(x^2v^2 + x^2v)dx - (x^2vdx + x^3dv) = 0$$

$$(x^2v^2 + \cancel{x^2v} - \cancel{x^2v})\,dx - x^3\,dv = 0$$

$$\left[\cancel{x^2}\cancel{v^2}\,dx - \cancel{x^3}\,dv = 0\right] * \frac{1}{\cancel{v^2}\cdot\cancel{x^3}}$$

$$\int \frac{dx}{x} - \int \frac{dv}{v^2} = 0 \qquad \int v^{-2}\,dv = \frac{v^{-1}}{-1}$$

$$\ln x + \frac{1}{v} = C \qquad y = vx \qquad v = \frac{y}{x}$$

Sol $\quad \ln x + \left(\frac{1}{\frac{y}{x}}\right) = C$

$$\ln x + \frac{x}{y} = C$$

Ej. Resolver $\quad x\dfrac{dy}{dx} = y + \sqrt{x^2 - y^2} \qquad x > 0$

$$M\,dx + N\,dy = 0$$

$$x\,dy = (y + \sqrt{x^2 - y^2})\,dx$$

$$x\,dy - (y + \sqrt{x^2 - y^2})\,dx = 0$$

$$M(x,y) = x \rightarrow M(tx, ty) = t^1 x \qquad \alpha = 1$$

$$N(x,y) = -(y + \sqrt{x^2 - y^2}) \rightarrow N(tx, ty) = -(ty + \sqrt{t^2x^2 - t^2y^2})$$

$$= -(ty + \sqrt{t^2(x^2 - y^2)}) = -(ty + t\sqrt{x^2 - y^2})$$

$$= -t^1(y + \sqrt{x^2 - y^2}) \qquad \alpha = 1$$

Si es homogenea.

$$y = vx \qquad x = uy \qquad dx = u\,dy + y\,du$$

$$uy\,dy - (y + \sqrt{u^2y^2 - y^2})(u\,dy + y\,du) = 0$$

$$\cancel{uy\,dy} - (\cancel{uy\,dy} + y^2\,du - u\sqrt{u^2y^2 - y^2}\,dy + y\sqrt{u^2y^2 - y^2}\,du) = 0$$

$$dy\left(\cancel{uy}-\cancel{uy}-u\sqrt{y^2(u^2-1)}\right) \quad -\left[y^2+y\sqrt{y^2(u^2-1)}\right]du$$

$$-uy\sqrt{u^2-1}\,dy-\left(y^2+y^2\sqrt{u^2-1}\right)du$$

$$\left[-u\,y\sqrt{u^2-1}\,dy-y^2\left(1+\sqrt{u^2-1}\right)du\right]*\frac{1}{u\sqrt{u^2-1}\,y^2}$$

$$-\int\frac{dy}{y}-\int\frac{\left(1+\sqrt{u^2-1}\right)}{u\sqrt{u^2-1}}du=\int 0$$

$$-\int\frac{dy}{y}-\left[\int\frac{du}{u\sqrt{u^2-1}}+\int\frac{du}{u}\right]=0$$

$$-\ln y+\ln u$$

$$\int\frac{du}{u\sqrt{u^2-1}}=tg^{-1}\sqrt{u^2-1}$$

Sol $$-\ln y+\ln u+tg^{-1}\sqrt{u^2-1}=C$$

$$x=uy \qquad u=\frac{x}{y}$$

$$-\ln y+\ln\frac{x}{y}+tg^{-1}\sqrt{\frac{x^2}{y^2}-1}=C$$

Ej. $(x+ye^{y/x})dx-xe^{y/x}dy=0 \qquad y(1)=0$

$\alpha=1 \qquad (tx)e^{ty/tx} \qquad$ Si es

$\alpha=1 \qquad$ Homogenea.

$$y=vx \qquad dy=vdx+xdv$$

$$(x+vxe^{vx/x})dx-xe^{vx/x}(vdx+xdv)=0$$

$$(x+vxe^{v})dx-xve^{v}dx-x^2e^{v}dv=0$$

$$(x+\cancel{vxe^{v}}-\cancel{vxe^{v}})dx-x^2e^{v}dv=0$$

$$\left[x\,dx - x^2 e^v dv = 0 \right] * \frac{1}{x^2}$$

$$\frac{x}{x^2} dx - e^v dv = 0$$

$$\int \frac{dx}{x} - \int e^v dv = \int 0$$

$$\ln x - e^v = c$$

$$y = vx \qquad v = \frac{y}{x}$$

$$\ln x - e^{y/x} = c$$

$$y(1) = 0$$

$$\cancel{\ln 1} - \cancel{e}^{0/1} = c \quad \Rightarrow c = -1$$

sol

$$\ln x - e^{y/x} = -1$$

Ecuación $\frac{dy}{dx} = f(ax + by + c) \rightarrow$ función lineal.

$$U = ax + by + c$$

$$\frac{dU}{dx} = a + b\frac{dy}{dx} \quad \rightarrow \quad \frac{dy}{dx} = \frac{\frac{dU}{dx} - a}{b}$$

$$\frac{\frac{dU}{dx} - a}{b} = f(U)$$

Ej. Resolver $\frac{dy}{dx} = \frac{1 - x - y}{x + y}$

$$\frac{dy}{dx} = \frac{1 - (x + y)}{x + y}$$

$$U = x + y$$

$$\frac{dv}{dx} = 1 + \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\frac{dv}{dx} - 1 = \frac{1 - v}{v} = \frac{1}{v} - 1$$

$$\frac{dv}{dx} \cancel{-1} = \frac{1}{v} \cancel{-1}$$

$$\frac{dv}{dx} = \frac{1}{v}$$

$$\int v\,dv = \int dx \longrightarrow \frac{1}{2} v^2 = x + C$$

$$\frac{1}{2}(x+y)^2 = x + C$$

Ej. Resolver $\frac{dy}{dx} = 1 + e^{y - x + 5}$

$$v = y - x + 5$$

$$\frac{dv}{dx} = \frac{dy}{dx} - 1 \longrightarrow \frac{dy}{dx} = \frac{dv}{dx} + 1$$

$$\frac{dv}{dx} \cancel{+1} = \cancel{1} + e^{v}$$

$$\frac{dv}{dx} = e^{v} \longrightarrow \frac{dv}{e^{v}} = dx$$

$$\int e^{-v}\,dv = \int dx$$

$$-e^{-v} = x + C \longrightarrow -e^{-(y-x+5)} = x + C$$

Ej. Resolver $\frac{dy}{dx} = \frac{3x+2y}{3x+2y+2}$ $\quad y(-1) = -1$

$$u = 3x+2y$$

$$\frac{du}{dx} = 3 + 2\frac{dy}{dx} \rightarrow \frac{dy}{dx} = \frac{\frac{du}{dx} - 3}{2}$$

$$\frac{\frac{du}{dx} - 3}{2} = \frac{u}{u+2}$$

$$\frac{du}{dx} = \frac{2u}{u+2} + 3 = 2u + 3(u+2) = \frac{2u+3u+6}{u+2}$$

$$\frac{du}{dx} = \frac{5u+6}{u+2}$$

$$\int \frac{u+2}{5u+6}\,du = \int dx$$

$$\begin{array}{r|l} & 1/5 \\ \hline 5u+6 & u+2 \\ & -u - \frac{6}{5} \\ \hline & 4/5 \end{array}$$

$$2 - \frac{6}{5} = \frac{4}{5}$$

$$\int \frac{1}{5} + \frac{4/5}{5u+6}\,du = \int dx$$

$$v = 5u+6 \qquad dv = 5\,du \qquad \frac{dv}{5} = du$$

$$\frac{1}{5}u + \frac{4}{5}\left(\frac{1}{5}\right)\ln v = x + C$$

$$\searrow (5u+6$$

$$\frac{1}{5}(3x+2y) + \frac{4}{25}\ln|(5(3x+2y)+6)| = x + C$$

$$y(-1) = -1$$

$$\frac{1}{5}\left(3(-1)+2(-1)\right)+\frac{4}{25}\ln\left[5(-3-2)+6\right]=-1+C$$

$$\cancel{\frac{1}{5}(-5)}+\frac{4}{25}\ln|-19|=\cancel{-1}+C$$

$$C=\frac{4}{25}\ln 19$$

Sol

$$\frac{1}{5}(3x+2y)+\frac{4}{25}\ln\left(5(3x+2y)+6\right)=x+\frac{4}{25}\ln 19$$