Javier Andres Monjes Solorzano
20210081 - 3026467407 01

# Segundo Examen Parcial

M. Intermedia 3
Seccion B

#1 1

$$T(x,y,z)=\frac{80}{1+x^2+2y^2+3z^2} \;; P.(2,-2,1)$$

$$\nabla T=\frac{\partial T}{\partial x}\hat{i}+\frac{\partial T}{\partial y}\hat{j}+\frac{\partial T}{\partial z}\hat{k} \rightarrow \nabla T=\left(\frac{\partial T}{\partial x},\frac{\partial T}{\partial y},\frac{\partial T}{\partial z}\right)$$

a) $$\frac{\partial T}{\partial x}=\frac{(1+x^2+2y^2+3z^2)(80)'-80(1+x^2+y^2+3z^2)'}{1+x^2+2y^2+3z^2}=\frac{80}{(1+x^2+2y^2+3z^2)}\Bigg]_{(2,2,1)}=\frac{-80(2)(-2}{(1+(-2)^2+2(-2)^2+3(1^2)^2}$$

$$\frac{\partial T}{\partial x}=\frac{320}{256}=\frac{5}{4}$$

$$\frac{\partial T}{\partial y}=\frac{-80(4y)}{(1+x^2+2y^2+3z)^2}\Bigg]_{(2,-2,1)} \rightarrow \frac{80(4)(-2)}{256}=\frac{5}{2}$$

$$\frac{\partial T}{\partial z}=\frac{-80(6z)}{(1+x^2+2y^2+3z)^2}\Bigg]_{(2,-2,1)} \rightarrow \frac{80(6)(1)}{256}=-\frac{15}{8}$$

$$\nabla T=\left(\frac{5}{4}\hat{i},\frac{5}{2}\hat{j},-\frac{15}{8}\right)$$

$$\|\nabla T\|=\sqrt{\left(\frac{5}{4}\right)^2+\left(\frac{5}{2}\right)^2+\left(\frac{-15}{8}\right)^2}=\sqrt{\frac{25}{16}+\frac{25}{4}+\frac{225}{64}}=\sqrt{\frac{725}{64}}=\frac{\sqrt{25}\sqrt{29}}{\sqrt{64}}=\frac{5\sqrt{29}}{8}$$

$$\bar{U}_{\nabla}=\frac{\nabla T}{\|\nabla T\|}=\frac{\frac{5}{4},\frac{5}{2},\frac{-15}{8}}{\frac{5\sqrt{29}}{8}}=\left(\frac{\frac{5}{4}}{\frac{5\sqrt{29}}{8}},\frac{\frac{5}{2}}{\frac{5\sqrt{29}}{8}},\frac{\frac{-15}{8}}{\frac{5\sqrt{29}}{8}}\right)=\left(\frac{40}{20\sqrt{29}},\frac{40}{10\sqrt{29}},-\frac{120}{40\sqrt{29}}\right)$$

$$\bar{U}_{\nabla}=\left\langle\frac{2}{\sqrt{29}},\frac{4}{\sqrt{29}},\frac{-3}{\sqrt{29}}\right\rangle$$

b) $$\bar{U}_{min}=\bar{U}_{max}=\left(\frac{-2}{\sqrt{29}},\frac{-4}{\sqrt{29}},\frac{3}{\sqrt{29}}\right)$$

Tasa maxima $\rightarrow \frac{5\sqrt{25}}{8}=3{,}37$

Tasa minima $=-\|\nabla T\| \rightarrow$ Tasa minima $=-3{,}37$

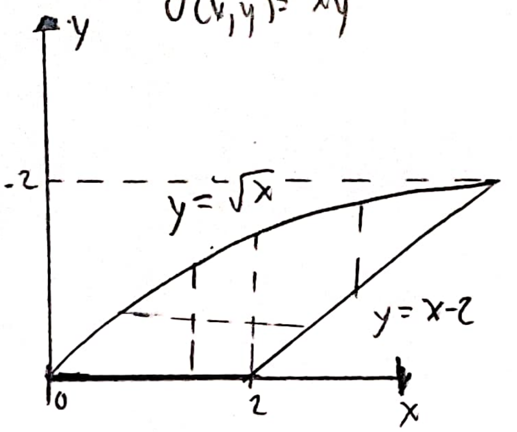
Respuestas

a) $\left\langle\frac{2}{\sqrt{29}},\frac{4}{\sqrt{29}},\frac{-3}{\sqrt{29}}\right\rangle ; \left\langle\frac{5}{4},\frac{5}{2},\frac{-15}{8}\right\rangle$

b) Tasa minima 3.37

| # 2 | 2 |
|---|---|

$f(x,y) = xy$




$$\left.\begin{array}{l} y=\sqrt{x} \\ y = x-2 \end{array}\right\} \quad \sqrt{x} = x-2 \to (\sqrt{x})^2 = (x-2)$$

$$x = x^2-4x+4 \to x^2-4x-4+4=0$$

$$x^2-5x+4=0 \to x = \frac{5 \pm \sqrt{25+4(1)(4)}}{2}$$

$$\frac{5\pm\sqrt{25-16}}{2} = \frac{5\pm\sqrt{9}}{2} \quad \frac{5\pm 3}{2} \to \begin{array}{l} x_1 = 4 \\ x_2 = 1 \end{array}$$

$y_1 = \sqrt{4} \pm 2$

$y_2 = \sqrt{1} = \pm 1$

a) $\iint f(x,y)\,da = A \quad \begin{array}{l} 0 \le x \le 2 \\ 0 \le y \le \sqrt{x} \end{array} \; y \; \begin{array}{l} 2 \le x \le 4 \\ x-2 \le y \le \sqrt{x} \end{array}$

$$\int_0^2 \int_0^{\sqrt{x}} xy\,dy\,dx + \int_2^4 \int_{x-2}^{\sqrt{x}} xy\,dy\,dx \quad \to f(x,y)\ xy$$

$$\iint f(x,y)\,da \to \begin{array}{l} 0 \le y \le 2 \\ y^2 \le x \le y+2 \end{array} \to \int_0^2 \int_{y^2}^{y+2} xy\,dx\,dy \to \text{opcion } 1$$

b) opcion 2

c) $$\int_0^2 \int_{y^2}^{y+2} xy\,dx\,dy = \int_0^2 y \left(\left[\int_{y^2}^{y+2} x\,dx\right]\right) dy = \int_0^2 y \cdot \frac{1}{2}x^2 \Big)_{y^2}^{y+2} dx$$

$$\int_0^2 y \frac{1}{2}\left[(y+2)^2 - (y^2)^2\right] dy = \frac{1}{2}\int_0^2 y\left[y^2+4y+4-y^4\right] dy$$

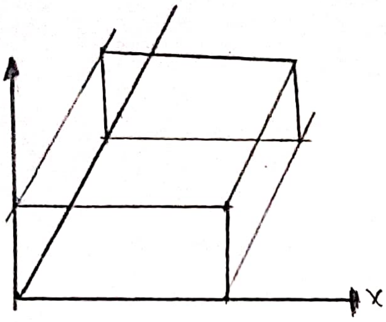
$$\frac{1}{2}\int_0^2 \left[y^3+4y^2+4y-y^5\right] dy = \frac{1}{2}\int_0^2 y^3\,dy + \frac{1}{2}4\int_0^2 y^2\,dy + \frac{1}{2}4\int_0^2 y\,dy - \frac{1}{2}\int_0^2 y^5\,dy$$

$$\frac{1}{2}\frac{1}{4}y^4\Big)_0^2 + 2\frac{1}{3}y^3\Big]_0^2 + 2\frac{1}{2}y^2\Big)_0^2 - \frac{1}{2}\frac{1}{6}y^6\Big)_0^2 = \frac{1}{8}(2^4) + \frac{2}{3}(2^3) + 1(2^2) - \frac{1}{12}(2^6) = 2 + \frac{16}{3} + 4 - \frac{16}{3}$$

$$= 6$$

Respuestas

| |
|---|
| a) opcion 1 $\int_0^2 \int_{y^2}^{y+2} xy\,dx\,dy$ |
| b) opcion 2 $\to \int_0^2 \int_0^{\sqrt{x}} xy\,dy\,dx + \int_2^4 \int_{x-2}^{\sqrt{x}} xy\,dy\,dx$ |
| c) opcion 4 $\to$ 6 |



| # 3 | 4 |
|---|---|

a) Arista $= x+x+x+x+y+y+y+y+z+z+z+z$

$4x+4y+4z=68 \rightarrow x+y+z=17$

$V=xyz \xrightarrow{\text{Sustituido } a} x+y+z=12$

$V(x,y,z)=xyz$ $\qquad V(x,y)=xy(17-x-y)$

$17xy-x^2y-xy^2$

$f(x,y,z)=x+y+z=17-x-y \rightarrow$ $\qquad V(x,y)=17xy-x^2y-xy^2 \rightarrow$ opción 2

b) $D(a,b)=f_{xx}(a,b)\,;\,f_{yy}(a,b)-[f_{xy}(a,b)]^2$

$V(x,y)=17xy-x^2y-xy^2$

$\left.\begin{array}{l} f_x=17y-2xy-y^2 \\ f_y=17x-x^2-2xy \end{array}\right\}$ extremos $\quad \begin{array}{l} f_x=0 \rightarrow 17y=2xy+y^2 \\ f_y=0 \rightarrow 17x=2xy+x^2 \end{array} \rightarrow \left\{\begin{array}{l} y^2+2xy=17y \\ 17x=2xy+x^2 \end{array}\right.$

$17x-x^2=2xy$

$y=\frac{17x-x^2}{2x} \rightarrow y=\frac{17-x}{2}$

$\left(\frac{17-x}{2}\right)^2+2\left(\frac{17-x}{2}\right)x=17\left(\frac{17-x}{2}\right) \rightarrow \frac{(17-x)^2}{4}+x(17-x)=\frac{17}{2}(17-x)$

$\frac{289-34x+x^2}{4}+(17x-x^2)=\frac{354-17x}{2}$

$\frac{289-34x+x^2+68x-4x^2}{4}=\frac{578-34x}{4}$

$289-34x+x^2+68x-4x^2-578+34x= \rightarrow -3x^2+68x-289=0$

$3x^2-68x+289=0 \rightarrow x=\frac{68\pm\sqrt{68^2-4(3)(289)}}{6}=\frac{68\pm\sqrt{1156}}{6}=\frac{68\pm 34}{6}$

$x_1 \rightarrow 17 \rightarrow y_1=0$

$x_2 \rightarrow 17/3 \rightarrow y_2=17/3$

$f_{xx}=\frac{\partial}{\partial x}(17y-2xy-y^2)=-2x$ $\qquad f_{xy}=\frac{\partial}{\partial x}(17x-x^2-2xy)=17-2x-2y$

$f_{yy}=\frac{\partial}{\partial y}(17x-x^2-2xy)=-2y$ $\qquad f_{yx}=\frac{\partial}{\partial y}(17y-2xy-y^2)=17-2x-2y$

| $P(x,y)$ | $f_{xx}$ | $f_{yy}$ | $f_{xy}$ | D |
|---|---|---|---|---|
| 17,0 | 0 | 34 | -17 | -289 ← Punto silla |
| 17/3, 17/3 | -34/3 | -34/3 | -17/3 | 289/3 ← Máximo |

$D\left(\frac{17}{3},\frac{17}{3}\right)=\frac{289}{3}=96.33$

$x$ $17/3$

$y=17/3 \rightarrow z=17-\frac{17}{3}-\frac{17}{3}=17/3$

$x=y=z=17/3 \rightarrow 5.67$

Respuestas → a) $17xy\ yx^2\ xy^2$ $\quad$ c) 5.67 todas

b) 96.33