Método del operador Anulador:

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1}y}{dx^{n-1}} + \dots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

D

$$a_n D^n y + a_{n-1} D^{n-1} y + \dots + a_2 D^2 y + a_1 D y + a_0 y = g(x)$$

$$y = y_c + y_p$$

operador que anule a la función $g(x)$

$$g(x) = x^3 - 2x^2 + 3x + 1$$

$$D(g(x)) = 3x^2 - 4x + 3$$

$$D^2 g(x) = 6x - 4$$

$$D^3 g(x) = 6 \qquad \rightarrow D^4[x^3 - 2x^2 + 3x + 1] = 0$$

$$D^4 g(x) = 0$$

1. El operador diferencial D^n anula a cada una de las funciones.

$$1, \ x, \ x^2, \ x^3, \dots \ x^{n-1}$$

$$x^{4-1} \rightarrow n = 4$$

$$D^4[g(x)] = 0$$

2. El operador diferencial $(D-\alpha)^n$ anula a cada una de las funciones.

$$e^{\alpha x}, \ xe^{\alpha x}, \ x^2 e^{\alpha x}, \dots \ x^{n-1} e^{\alpha x}.$$

$$g(x) = x^2 e^{-3x} \quad \rightarrow \alpha = -3 \qquad (D+3)^3[x^2 e^{-3x}] = 0$$

$$n = 3$$

3. El operador diferencial $[D^2 - 2\alpha D + (\alpha^2 + \beta^2)]^n$ anula a cada una de las funciones.

- $e^{\alpha x}\cos\beta x,\quad xe^{\alpha x}\cos\beta x,\quad x^2e^{\alpha x}\cos\beta x, \ldots\; x^{n-1}e^{\alpha x}\cos\beta x$
- $e^{\alpha x}\operatorname{sen}\beta x,\quad xe^{\alpha x}\operatorname{sen}\beta x,\quad x^2e^{\alpha x}\operatorname{sen}\beta x, \ldots\; x^{n-1}e^{\alpha x}\operatorname{sen}\beta x$

$$g(x) = 5xe^{2x}\cos 3x + 9e^{-5x}\operatorname{sen} 4x$$

$n=2 \quad \alpha = 2 \quad \beta = 3 \qquad\qquad \alpha = -5 \quad \beta = 4$

$$[D^2 - 2(2)D + (4+9)]^2 \qquad\qquad [D^2 - 2(-5)D + (25+16)]$$

$$(D^2 - 4D + 13)^2 \qquad\qquad D^2 + 10D + 41$$

$$(D^2 - 4D + 13)^2(D^2 + 10D + 4)[g(x)] = 0$$

4. Cuando $\alpha = 0 \quad n = 1$

$$(D^2 + \beta^2)\begin{cases}\cos\beta x \\ \operatorname{sen}\beta x\end{cases} = 0$$

$x^1\cos 2x \to \beta = 2$

$n = 2 \quad \to (D^2 + 4)^2$

Ej. Determine un operador que anule a la función $g(x)$.

$$g(x) = 3x^2e^{6x} + 2e^{6x} - e^{2x}\cos x + x^4$$

$n=3 \quad \alpha = 6 \qquad \alpha = 6 \qquad \alpha = 2 \quad \beta = 1 \qquad n = 5$

$(D-6)^3 \to (D-6)(D-6)^2 \to (D-6) \qquad (D^2 - 2(2)D + (4+1)) \qquad D^5$

$$\underbrace{D^5(D-6)^3(D^2 - 4D + 5)}[g(x)] = 0$$

Ej. Resolver $y'' + 4y = 4\cos x + 3\operatorname{sen} x - 8$

y_c $\qquad y = y_c + y_p$

$y'' + 4y = 0 \rightarrow m^2 + 4 = 0 \rightarrow \sqrt{m^2} = \sqrt{-4}$

$m = \pm 2i$

$y_c = C_1 \cos 2x + C_2 \operatorname{sen} 2x$ $\qquad m = \pm 2i$

y_p $\qquad g(x) = 4\cos x + 3\operatorname{sen} x - 8$

$(D^2+1) \qquad (D^2+1) \qquad D$

$D(D^2+1)[4\cos x + 3\operatorname{sen} x - 8] = 0$

$D(D^2+1) = 0 \longrightarrow D = 0$

$D^2 + 1 = 0 \rightarrow D = \pm i$

$y_p = A + B\cos x + C\operatorname{sen} x$

$y' = -B\operatorname{sen} x + C\cos x$

$y'' = -B\cos x - C\operatorname{sen} x$

$-B\cos x - C\operatorname{sen} x + 4(A + B\cos x + C\operatorname{sen} x) = 4\cos x + 3\operatorname{sen} x - 8$

$\cos x(-B + 4B) = 4\cos x \rightarrow B = \frac{4}{3}$

$\operatorname{sen} x(-C + 4C) = 3 \rightarrow C = 1$

$4A = -8 \qquad A = -2$

$y = C_1\cos 2x + C_2 \operatorname{sen} 2x - 2 + \frac{4}{3}\cos x + \operatorname{sen} x$

Ej. Resolver.

$y'' + 2y' + y = x^2 e^{-x}$

$y = y_c + y_p$

y_c $\quad n^2 + 2n + 1 = 0$

$$\begin{array}{cc|c} n & 1 & n \\ n & 1 & n \\ & & \overline{2n} \end{array} \rightarrow (n+1)(n+1) = 0 \rightarrow \begin{array}{l} n = -1 \\ n = -1 \end{array}$$

$$y_c = C_1 e^{-x} + C_2 x e^{-x}$$

y_p

$$g(x) = x^2 e^{-x}$$

$$n = 3 \qquad \alpha = -1$$

$$(D+1)^3 [x^2 e^{-x}] = 0$$

$$(D+1)^3 = 0 \rightarrow \begin{array}{l} D+1 = 0 \\ D+1 = 0 \\ D+1 = 0 \end{array} \rightarrow \begin{array}{l} D = -1 \\ D = -1 \\ D = -1 \end{array}$$

$$y_p = Ax^2 e^{-x} + Bx^3 e^{-x} + Cx^4 e^{-x}$$

$$y' = -Ax^2 e^{-x} + 2Axe^{-x} - Bx^3 e^{-x} + 3Bx^2 e^{-x} - Cx^4 e^{-x} + 4Cx^3 e^{-x}$$

$$y'' = Ax^2 e^{-x} - 2Axe^{-x} - 2Axe^{-x} + 2Ae^{-x} + Bx^3 e^{-x} - 3Bx^2 e^{-x} - 3Bx^2 e^{-x} + 6Bxe^{-x}$$
$$+ Cx^4 e^{-x} - 4Cx^3 e^{-x} - 4Cx^3 e^{-x} + 12Cx^2 e^{-x}$$

$$Ax^2 e^{-x} - 4Axe^{-x} + 2Ae^{-x} + Bx^3 e^{-x} - 6Bx^2 e^{-x} + 6Bxe^{-x} + Cx^4 e^{-x} - 8Cx^3 e^{-x} + 12Cx^2 e^{-x}$$

$$-2Ax^2 e^{-x} + 4Axe^{-x} \qquad -2Bx^3 e^{-x} + 6Bx^2 e^{-x} \qquad -2Cx^4 e^{-x} + 8Cx^3 e^{-x}$$

$$Ax^2 e^{-x} \qquad Bx^3 e^{-x} \qquad Cx^4 e^{-x} \qquad +$$

$$2Ae^{-x} + 6Bxe^{-x} + 12Cx^2 e^{-x} = x^2 e^{-x}$$

$$2Ae^{-x} = 0 \rightarrow A = 0$$

$$6Bxe^{-x} = 0 \rightarrow B = 0$$

$$12C = 1 \rightarrow C = \frac{1}{12}$$

$$y_p = \frac{1}{12} x^4 e^{-x}$$

Sol

$$y = y_c + y_p \rightarrow \quad y = C_1 e^{-x} + C_2 x e^{-x} + \frac{1}{12} x^4 e^{-x}$$

$\operatorname{tg} x$, $e^x \csc x$, $\dfrac{e^x}{e^x + 1}$

Variación de parámetros

$$y = y_c + y_p$$

$$a_2(x) \frac{d^2y}{dx^2} + a_1(x) \frac{dy}{dx} + a_0(x) y = g(x)$$

$$a y'' + b y' + c y = 0$$

y_c

m_1 y m_2

$$y_c = C_1 y_1 + C_2 y_2$$

y_p

$$y = U_1 y_1 + U_2 y_2$$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$$

$$U_1 = -\int \frac{y_2\, g(x)}{W} dx$$

$$U_2 = \int \frac{y_1\, g(x)}{W} dx$$

Ej. Resolver $y'' + y = \tan x$

$$y = y_c + y_p$$

y_c

$$m^2 + 1 = 0 \rightarrow \quad m = \pm i$$

$$y = C_1 \underbrace{\cos x}_{y_1} + C_2 \underbrace{\operatorname{sen} x}_{y_2}$$

$$W = \begin{vmatrix} \cos x & \operatorname{sen} x \\ -\operatorname{sen} x & \cos x \end{vmatrix} = \underbrace{\cos^2 x + \operatorname{sen}^2 x}_{1}$$

y_p

$$y = U_1 y_1 + U_2 y_2$$

$$W = 1$$

$$U_1 = -\int \frac{y_2\, g(x)}{W}\, dx = -\int \frac{\operatorname{sen} x \cdot \tan x}{1}\, dx$$

$$U_1 = -\int \operatorname{sen} x \left(\frac{\operatorname{sen} x}{\cos x}\right) dx = -\int \frac{\operatorname{sen}^2 x}{\cos x}\, dx$$

$$U_1 = -\int \frac{1-\cos^2 x}{\cos x}\, dx = -\int (\sec x - \cos x)\, dx$$

$$U_1 = -\left[\ln|\tan x + \sec x| - \operatorname{sen} x\right]$$

$$U_1 = \operatorname{sen} x - \ln|\tan x + \sec x|$$

$$U_2 = \int \frac{y_1\, g(x)}{W}\, dx = \int \frac{\cos x \cdot \tan x}{1}\, dx$$

$$U_2 = \int \frac{\cancel{\cos x}\left(\frac{\operatorname{sen} x}{\cancel{\cos x}}\right)}{1}\, dx = \int \operatorname{sen} x\, dx = -\cos x$$

$$y = \cos x(\operatorname{sen} x - \ln|\tan x + \sec x|) - \operatorname{sen} x \cos x$$

$$y_c = \cancel{\cos x \operatorname{sen} x} - \cos x \ln|\tan x + \sec x| - \cancel{\operatorname{sen} x \cos x}$$

$$y_c = -\cos x \ln|\tan x + \sec x|$$

$$y = y_c + y_p$$

$$y = C_1 \cos x + C_2 \operatorname{sen} x - \cos x \ln|\tan x + \sec x|$$

Ej. Resolver $\quad y'' - 2y' + y = \dfrac{e^x}{1+x^2}$

$$y = y_c + y_p$$

$\underline{y_c}$ $\quad m^2 - 2m + 1 = 0$

$$\begin{array}{ll|l} m & -1 & -m \\ m & -1 & -m \\ \hline & & -2m \end{array}$$

$$(m-1)(m-1) = 0$$

$$m = 1$$

$$m = 1$$

$$y_c = c_1 \underset{y_1}{e^x} + c_2 \underset{y_2}{x e^x}$$

$\underline{y_p}$

$$y = u_1 y_1 + u_2 y_2$$

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{vmatrix} = \cancel{xe^{2x}} + e^{2x} - \cancel{xe^{2x}}$$

$$W = e^{2x}$$

$$u_1 = -\int \frac{y_2 g(x)}{W} dx$$

$$u_1 = -\int \frac{xe^x \cdot \frac{e^x}{1+x^2}}{e^{2x}} dx = -\int \frac{\frac{x\cancel{e^{2x}}}{1+x^2}}{\cancel{e^{2x}}} dx = -\int \frac{x}{1+x^2} dx$$

$$u = 1+x^2 \qquad du = 2x\,dx \qquad \frac{du}{2} = x\,dx$$

$$-\frac{1}{2}\int \frac{du}{u} = -\frac{1}{2} \ln u = -\frac{1}{2} \ln(1+x^2)$$

$$u_2 = \int \frac{y_1 g(x)}{W} dx = \int \frac{e^x \cdot \frac{e^x}{1+x^2}}{e^{2x}} dx = \int \frac{\frac{\cancel{e^{2x}}}{1+x^2}}{\cancel{e^{2x}}} dx$$

$$u_2 = \int \frac{dx}{1+x^2} = tg^{-1} x$$

$$y_c = e^x\left(-\frac{1}{2}\ln(1+x^2)\right) + xe^x tg^{-1} x$$

$\underline{Sol}$

$$y = c_1 e^x + c_2 x e^x - \frac{1}{2} e^x \ln(1+x^2) + x e^x tg^{-1} x$$