Calcular $\mathcal{L}\{\ \}$ de La Ecuacion Diferencial :

2) $y'' - 2y' + 5y = 1 + t\,,\quad y(0) = 0\,,\ y'(0) = 4$

$$\mathcal{L}\{y''\} - 2\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{1+t\}$$

$n=2 \qquad n=1$

$$s^2Y(s) - s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{4} - 2[sY(s) - \underbrace{y(0)}_{0}] + 5Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s)[s^2 - 2s + 5] = \frac{1}{s} + \frac{1}{s^2} + 4$$

$$s^2 - 2s + 1 - 1 + 5 \qquad \frac{s + 1 + 4s^2}{s^2}$$

$$(s-1)^2 + 4$$

$$Y(s) = \frac{4s^2 + s + 1}{s^2 \cdot [(s-1)^2 + 4]} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{(s-1)^2 + 4}$$

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Calcular $\mathcal{L}\{\ \}$ de La Ecuacion Diferencial :

2) $y'' - 2y' + 5y = 1 + t\,,\quad y(0) = 0\,, y'(0) = 4$

$$Y(s) = \frac{7/25}{s} + \frac{1/5}{s^2} - \frac{(7/25)s}{(s-1)^2+4} + \frac{109/25}{(s-1)^2+4}$$

$$- \frac{(7/25)(s-1+1)}{(s-1)^2+4} + \frac{109/25}{(s-1)^2+4}$$

$$-\frac{(7/25)(s-1)}{(s-1)^2+4} - \frac{7/25}{(s-1)^2+4} + \frac{109/25}{(s-1)^2+4}$$

$$Y(s) = \frac{7/25}{s} + \frac{1/5}{s^2} - \frac{(7/25)(s-1)}{(s-1)^2+4} + \frac{102/25}{(s-1)^2+4} \quad \Big|\ \mathcal{L}^{-1}\{\ \}_{s-1\to s}$$

$$y(t) = \frac{7}{25} + \frac{1}{5}t - \frac{7}{25}e^{t}\cos(2t) + \frac{102}{25\cdot 2}e^{t}\,\mathrm{SEN}(2t)$$

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Calcular $\mathcal{L}\{\ \}$ de La Ecuacion Diferencial :

2) $y'' + 2y' + y = 0$, $\quad y(1) = 2$, $y'(0) = 2$ $\qquad |\mathcal{L}\{\ \}$

$n=2 \qquad n=1$

$$s^2 Y(s) - s\,\cancel{y(0)}^{C_1} - \underbrace{y'(0)}_{2} + 2[sY(s) - \cancel{y(0)}^{C_2}] + Y(s) = 0$$

$$Y(s)\underbrace{[s^2+2s+1]}_{(s+1)^2} = sC_1 + 2 + 2C_2$$

$$Y(s) = \frac{sC_1}{(s+1)^2} + \frac{2}{(s+1)^2} + \frac{2C_2}{(s+1)^2}$$

$$Y(s) = \frac{(s+1-1)C_1}{(s+1)^2} + \frac{2}{(s+1)^2} + \frac{2C_2}{(s+1)^2}$$

$$Y(s) = \frac{(s+1)C_1}{(s+1)^2} - \frac{C_1}{(s+1)^2} + \frac{2}{(s+1)^2} + \frac{2C_2}{(s+1)^2}$$

$$Y(s) = \frac{C_1}{s+1} - \frac{C_1}{(s+1)^2} + \frac{2}{(s+1)^2} + \frac{2C_2}{(s+1)^2}$$

$$y(t) = C_1 e^{-t} - C_1 e^{-t} t + 2e^{-t} t + 2C_2 e^{-t} t$$

$y(1) = 2$

$$2 = C_1 e^{-1} - C_1 e^{-1} + 2e^{-1} + 2C_2 e^{-1}$$

$$2 = 2e^{-1} + 2C_2 e^{-1} \quad \Big| * \frac{1}{2}$$

$$\frac{1 - e^{-1}}{e^{-1}} = C_2$$

$$\boxed{C_2 = e^{1} - 1}$$

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Calcular $\mathcal{L}\{\ \}$ de La Ecuacion Diferencial :

2) $y'' + 2y' + y = 0\,,\quad y(1) = 2\,, y'(0) = 2$

$$y'(t) = -c_1 e^{-t} - c_1\left[-e^{-t}t + e^{-t}\right] + 2\left[-e^{-t}t + e^{-t}\right] + 2c_2\left[-e^{-t}t + e^{-t}\right]$$

$$y'(0) = 2 \rightarrow 2 = -c_1 - c_1 + 2 + 2c_2$$

$$2 = -2c_1 + 2 + 2c_2$$

$$c_1 = c_2 \rightarrow c_1 = e^1 - 1$$

$$y(t) = (e^1-1)e^{-t} - (e^1-1)e^{-t}t + 2e^{-t}t + 2(e^1-1)e^{-t}t$$

$$y(t) = (e^1-1)e^{-t}t + (e^1-1)e^{-t} + 2e^{-t}t$$

$$y(t) = e^1e^{-t}t - e^{-t}t + e^1e^{-t} - e^{-t} + 2e^{-t}t = e^1e^{-t}t + e^{-t}t + e^1e^{-t} - e^{-t}$$

$$y(t) = e^{-t}t(e^1+1) + e^{-t}(e^1-1)$$

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