# ECUACIONES INTEGRALES Ó INTEGRODIFERENCIALES

$$f(t) = g(t) + \int_0^t f(\tau) \cdot h(t - \tau)\, d\tau \, .$$


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Use la Transformada de Laplace para Resolver la Ecuacion integral o Integrodiferencial :

$$y' = 1 - sin(t) - \int_0^t y(\tau)d\tau \ , \qquad y(0) = 0$$

$$\mathcal{L}\{y'\} = \mathcal{L}\{1 - \text{sen}(t)\} - \mathcal{L}\{\int_0^t \underbrace{y(\tau)}_{\tau\to t}d\tau\}$$

$t-\tau$ $\quad$ $\tau \to t$

$$\mathcal{L}\{1 * y(t)\} = \mathcal{L}\{1\} \cdot \mathcal{L}\{y(t)\}$$

$$sY(s) - \cancel{y(0)}^{\,0} = \frac{1}{s} - \frac{1}{s^2+1} - \frac{Y(s)}{s}$$

$$sY(s) + \frac{Y(s)}{s} = \frac{s^2+1-s}{s(s^2+1)}$$

$$Y(s)\left[\frac{s^2+1}{\cancel{s}}\right] = \frac{s^2-s+1}{\cancel{s}(s^2+1)}$$

$$Y(s) = \frac{s^2-s+1}{(s^2+1)^2}$$

$$Y(s) = \frac{1}{s^2+1} - \frac{s}{(s^2+1)^2}$$


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$$\frac{s}{(s^2+1)^2} \rightarrow \left(\frac{s}{s^2+1}\right)\cdot\left(\frac{1}{s^2+1}\right)$$

$$\int_0^t \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\}_{t\to t-\theta} \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\}_{t\to\theta} d\theta$$

$$\int_0^t \cos(t-\theta)\,\text{sen}(\theta)\,d\theta$$

$$\int_0^t \left[\cos(t)\cos(\theta)+\text{sen}(t)\,\text{sen}(\theta)\right]\text{sen}(\theta)\,d\theta$$

$$\cos(t)\int_0^t \cos(\theta)\,\text{sen}(\theta)\,d\theta + \text{sen}(t)\int_0^t \text{sen}^2(\theta)\,d\theta$$

$$\left(\frac{1-\cos(2\theta)}{2}\right)$$

$u=\text{sen}(\theta)$

$du=\cos(\theta)\,d\theta$

$$\cos(t)\left[\frac{\text{sen}^2(\theta)}{2}\Big|_0^t\right]+\text{sen}(t)\left[\frac{1}{2}\theta-\frac{1}{4}\text{sen}(2\theta)\Big|_0^t\right]$$

$$\frac{1}{2}\cos(t)\,\text{sen}^2(t)+\frac{1}{2}t\,\text{sen}(t)$$

$$-\frac{1}{4}\text{sen}(t)\,\text{sen}(2t)$$

$$\cancel{\frac{1}{2}\cos(t)\,\text{sen}^2(t)}+\frac{1}{2}t\,\text{sen}(t)$$

$$\cancel{-\frac{1}{4}\text{sen}(t)\left[2\,\text{sen}(t)\right.}\left.\cos(t)\right]$$

$$Y(s)=\frac{1}{s^2+1}-\frac{s}{(s^2+1)^2} \quad \Big|\ \mathcal{L}^{-1}\{\}$$

$$y(t)=\text{sen}(t)-\frac{1}{2}t\,\text{sen}(t)$$


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Use la Transformada de Laplace para Resolver la Ecuacion integral o Integrodiferencial :

$$f(t) + 2\int_0^t f(\tau)\cdot cos(t-\tau)d\tau = 4e^{-t} + sin(t)$$

$$\mathcal{L}\{f(t)\} + 2\mathcal{L}\left\{\int_0^t f(\tau)\cdot \cos(t-\tau)\,d\tau\right\} = \mathcal{L}\{4e^{-t} + \operatorname{sen}(t)\}$$

$\tau \to t \quad t-\tau \to t$

$$\mathcal{L}\{f(t) * \cos(t)\}$$

$$F(s) + \frac{2F(s)\cdot s}{s^2+1} = \frac{4}{s+1} + \frac{1}{s^2+1}$$

$$F(s)\left[\frac{s^2+1+2s}{\cancel{s^2+1}}\right] = \frac{4(s^2+1)+s+1}{(s+1)\cancel{(s^2+1)}}$$

$$F(s) = \frac{4s^2+s+5}{(s+1)(s^2+2s+1)}$$

$$F(s) = \frac{4s^2+s+5}{(s+1)^3}$$


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$$F(s) = \frac{4}{s+1} - \frac{7}{(s+1)^2} + \frac{8}{(s+1)^3} \quad \Big| \mathcal{L}^{-1}\{\;\}_{s+1 \to s}$$

$$f(t) = e^{-t}(4) - 7e^{-t}t + \frac{8}{2!}e^{-t}t^2$$


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