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MB2-A

CARNÉ 202100081

TAREA PREPARATORIA 3.1

FECHA: 02 / 04 / 2022

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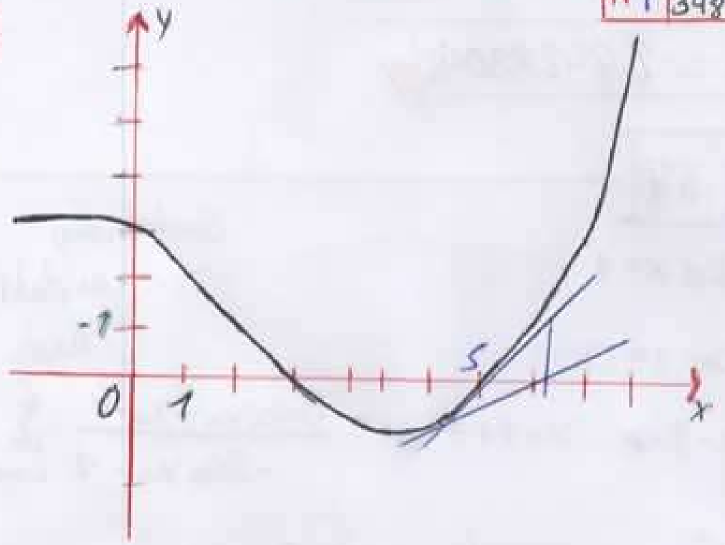
A1 Pág 348 #1

a)



La recta tangente en $x_1 = 6$ interseca en el eje x en $x = 7.3$. Por lo que $x_2 = 7.3$.

La recta tangente en $x = 7.3$ interseca en el eje x en $x = 6.8$. Por lo que $x_3 = 6.8$

b) $x_1 = 8$ sería una mejor primera aproximación por lo que la recta tangente en $x = 8$ se intersecta en el eje x más cerca de x que la primera aproximación $x_1 = 6$

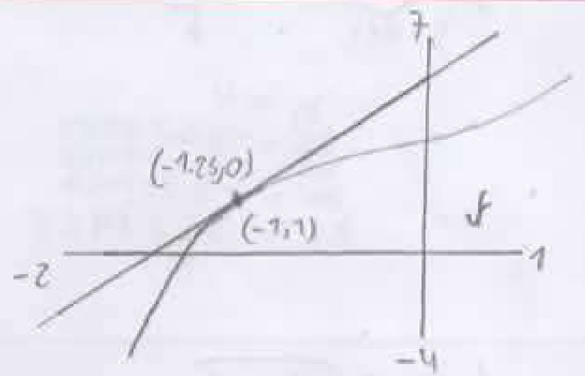
#2 Pág 349 #9

$x_1 = -1$

$f(x) = x^3 + x + 3$

$f'(x) = 3x^2 + 1$

Si $x_{n+1} = x_n - \dfrac{x_n^3 + x_n + 3}{3x_n^2 + 1}$



$x_1 = -1 \rightarrow x_2 = -1 - \dfrac{(-1)^3 + (-1) + 3}{3(-1)^2 + 1} = -1 - \dfrac{-1-1+3}{3+1} = -1 - \dfrac{1}{4} = -1.25$

El método de Newton sigue la recta tangente en $(-1, 1)$ hasta la intersección con el eje x en $(-1.25, 0)$, dando la segunda aproximación $x_2 = -1.25$

#3 Pág 349 #11

$\sqrt[4]{75}$

$x = \sqrt[4]{75} \rightarrow (x^4 = 75)$

$f(x) = x^4 - 75$

$f'(x) = 4x^3 \rightarrow$

$x_{n+1} = x_n - \dfrac{x_n^4 - 75}{4x_n^3} \rightarrow \sqrt[4]{81} = 3$ esta razonablemente cerca de $\sqrt[4]{75}$

$x_1 = 3$

$x_1 = 3 \rightarrow x_2 = 2.94 \rightarrow x_3 = 2.94283228 \rightarrow x_4 = 2.94283096 = x_5$

$$\boxed{\sqrt[4]{75} \approx 2.94283096}$$

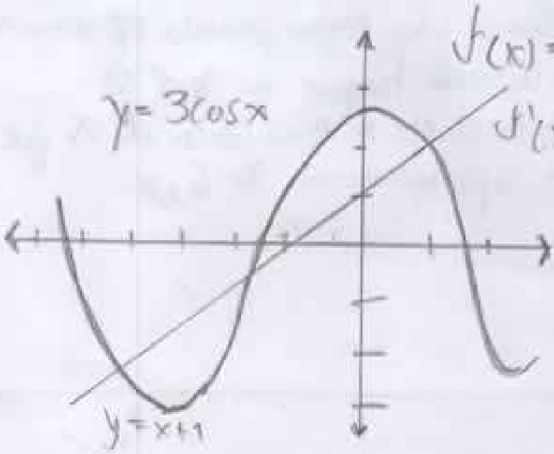
#4 Pág 349 #17

$3\cos x = x + 1$ Si $\rightarrow x = -4,\ x = -2$ o $x = 1$

$f(x) = 3\cos x - x - 1 = 0$

$f'(x) = -3\operatorname{sen} x - 1 \rightarrow x_{n+1} = x_n - \dfrac{3\cos x_n - x_n - 1}{-3\operatorname{sen} x_n - 1}$

Sustituimos con datos hasta 6 decimales



$x_1 = -2$

$x_2 = -1.856218$

$x_3 = -1.862356$

$x_4 = -1.862365 = x_5$

$x_1 = -1$

$x_2 = 0.892438$

$x_3 = 0.889473$

$x_4 = 0.889470 = x_5$

$x_1 = 4$

$x_2 = -3.652287$

$x_3 = -3.638060$

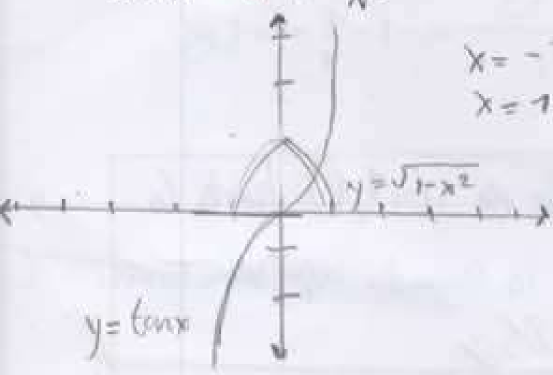
$x_4 = -3.637969$

$x_5 = -3.637958 = x_6$

$$\boxed{\begin{array}{r} -3.637958 \\ -1.862365 \\ 0.889470 \end{array}}$$

#5 Pág 349 #22

$\tan x = \sqrt{1 - x^2}$



$x = -1$

$x = 1$

$f(x) = \tan x - \sqrt{1-x^2} \rightarrow f(x) = \sec^2 x - \sqrt{1-x^2}$

$x_{n+1} = x_n - \dfrac{\tan x_n - \sqrt{1-x_n^2}}{\sec^2 x_n - \sqrt{1-x_n^2}}$

$x_1 = -0,545$

$x_2 = -0,54635$

$x_3 = -0,545351$

$x_1 = 0,5$

$x_2 = 0,55$

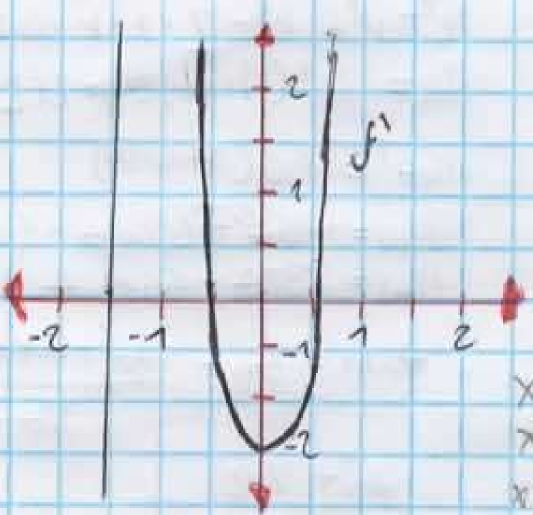
$x_3 = 0,545431$

$x_4 = 0,545351$

$$\boxed{\begin{array}{r} -0,545351 \\ 0,545351 \end{array}}$$

#6	Pág 349	#35

$f(x) = x^6 - x^4 + 3x^3 - 2x$



$f(x) = x^6 - x^4 + 3x^3 - 2x$

$f'(x) = 6x^5 - 4x^3 + 9x^2 - 2$

$f''(x) = 30x^4 - 12x^2 + 18x$

$x = -1.3, -0.4, 0.5$

$x_1 = -1.3$

$x_2 = x_1 - \frac{f'(x_1)}{f''(x_1)} = -1.293344$

$x_3 = -1.293227 = x_4$

$x_1 = -0.4$

$x_2 = -0.443755$

$x_3 = -0.441735$

$x_4 = -0.441731 = x_5$

$x_1 = 0.5$

$x_2 = 0.507937$

$x_3 = 0.507854 = x_4$

$x = -1.293227, -0.441731$ y 0.507854

$f(-1.293227) \approx -2.0212$

$f(0.507854) = -0.6721$

a) -1.293227
-0.441731
0.507854

b) $\rightarrow -2.0212$

#7	Pág 350	#38

$y = -\operatorname{sen} x$

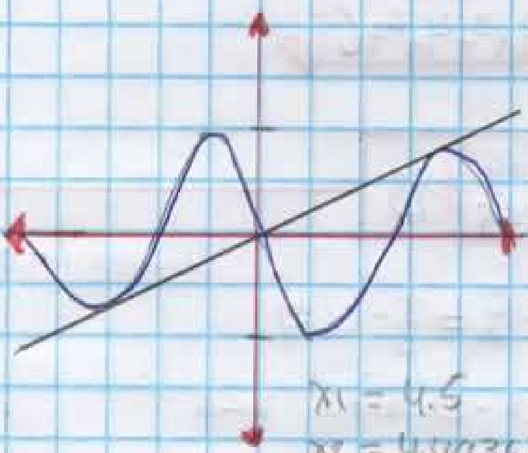
$f(x) = -\operatorname{sen} x$

$x = a \rightarrow f(a) = 1 - \cos a$

$f'(x) = -\cos x$

$y = \frac{-\operatorname{sen} a - 0}{a - 0}$

$f'(a) \rightarrow \frac{-\operatorname{sen} a}{a} = -\cos a \rightarrow \tan a = a$



$g(x) = \tan x - x \rightarrow g'(x) = \sec^2 x - 1$

$x_{n+1} = x_n - \frac{\tan x_n - x_n}{\sec^2 x_n - 1}$

$x_1 = 4.5$

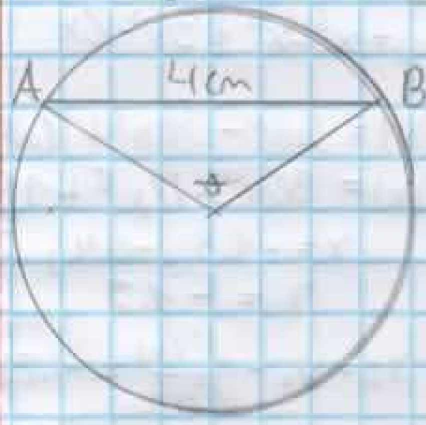
$x_2 = 4.493614$

$x_3 = 4.493410$

$x_4 = 4.493409 = x_5$

$\boxed{f'(x_5) = 0.217234}$

#8 Pág 350 #40



$5 = r\theta \qquad \rightarrow r = \frac{5}{\theta}$

$4^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos\theta$

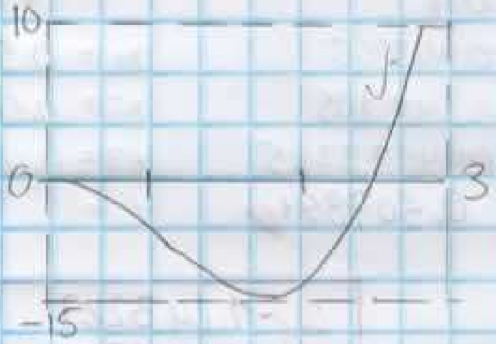
$16 = 2r^2(1-\cos\theta) = 2\left(\frac{5}{\theta}\right)^2(1-\cos\theta)$

$16\theta^2 = 50(1-\cos\theta)$

$f(\theta) = 16\theta^2 + 50\cos\theta - 50$

$f'(\theta) = 32\theta - 50\,\text{sen}\,\theta$

$$\theta_{n+1} = \theta_n - \frac{16\theta^2 + 50\cos\theta_n - 50}{32\theta - 50\,\text{sen}\,\theta}$$



$\theta_1 = 2.2$

$\theta_2 = 2.2662$

$\theta_3 = 2.2622 = \theta_4$

$\boxed{2.2622 \text{ radianes} = 130°}$

#9 Pág 355 #5

$f'(x) = x(12x+8)$

$\rightarrow f'(x) = x(12x+8) = 12x^2+8$

$f(x) = 12\frac{x^3}{3} + 8\frac{x^2}{2} + C = 4x^3+4x^2+C$

$\boxed{4x^3+4x^2+C}$

#10 Pág 355 #13

$f'(x) = \frac{1}{5} - \frac{2}{x}$

Dominio $(-\infty,0)\cup(0,\infty)$

$f'(x) = \frac{1}{5} - \frac{2}{x} = \frac{1}{5} - 2\left(\frac{1}{x}\right)$

$$f(x) = \begin{cases} \frac{1}{5}x - 2\ln|x| + C_1 & x<0 \\ \frac{1}{5}x - 2\ln|x| \; C_2 & x>0 \end{cases}$$

$$\boxed{f(x) = \begin{cases} \frac{1}{5}x - 2\ln|x| + C_1 & x<0 \\ \frac{1}{5}x - 2\ln|x| + C_2 & x>0 \end{cases}}$$

#11	Pág 355	#15

$$g(t) = \frac{1+t+t^2}{\sqrt{t}}$$

$$g(t) = \frac{1+t+t^2}{\sqrt{t}} = t^{-1/2} + t^{1/2} + t^{3/2}$$

$$g(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C$$

$$\boxed{g(t) = 2t^{1/2} + \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + C}$$

#12	Pág 355	#20

$$f(x) = 1 + 2\operatorname{sen} x + \frac{3}{\sqrt{x}}$$

$$f(x) = 1 + 2\operatorname{sen} x + \frac{3}{\sqrt{x}} = 1 + 2\operatorname{sen} x \; 3x^{-1/2}$$

$$f(x) = x - 2\cos x + 3\frac{x^{1/2}}{\frac{1}{2}} + C$$

$$= x - 2\cos x + 6\sqrt{x} + C$$

$$\boxed{f(x) = x - 2\cos x + 6\sqrt{x} + C}$$

#13	Pág 356	#26

$$f''(x) = x^6 - 4x^4 + x + 1$$

$$f''(x) = x^6 - 4x^4 + x + 1$$

$$f'(x) = \frac{1}{7}x^7 - \frac{4}{5}x^5 + \frac{1}{2}x^2 + x + C$$

$$f(x) = \frac{1}{56}x^8 - \frac{2}{15}x^6 + \frac{1}{6}x^3 + \frac{1}{2}x^2 + Cx + D$$

$$\boxed{f(x) = \frac{1}{56}x^8 - \frac{2}{15}x^6 + \frac{1}{6}x^3 + \frac{1}{2}x^2 + Cx + D}$$

#14 | Pág 366 | #37

$f'(t) = \sec t\,(\sec t + \tan t)\ ,\ -\frac{\pi}{2} < t < \frac{\pi}{2} \qquad f\left(\frac{\pi}{4}\right) = -1$

$f'(t) = \sec t\,(\sec t + \tan t) = \sec^2 t + \sec t \tan t\ ,\ -\frac{\pi}{2} < t < \frac{\pi}{2}$

$f(t) = \tan t + \sec t + C\ ;\ f\left(\frac{\pi}{4}\right) = 1 + \sqrt{2} + C$

$f\left(\frac{\pi}{4}\right) = -1 \rightarrow 1 + \sqrt{2} + C = -1$

$f(t) = \tan t + \sec t - 2 - \sqrt{2} \qquad C = -2 - \sqrt{2}$

$$\boxed{f(t) = \tan t + \sec t - 2 - \sqrt{2}}$$

#15 | Pág 366 | #41

$f''(\theta) = \operatorname{sen}\theta + \cos\theta\ ,\ f(0) = 3\ ,\ f'(0) = 4$

$f''(\theta) = \operatorname{sen}\theta + \cos\theta = f'(\theta) = -\cos\theta + \operatorname{sen}\theta + C \qquad f'(0) = -1 + C\ ;\ f'(0) = 4 \rightarrow C = 5$

$f'(\theta) = -\cos\theta + \operatorname{sen}\theta + 5 \rightarrow f(\theta) = -\operatorname{sen}\theta - \cos\theta + 5\theta + D$

$f(\theta) = -\operatorname{sen}\theta - \cos\theta + 5\theta + 4 \qquad f(0) = -1 + D\ ;\ f(0) = 3 \rightarrow D = 4$

$$\boxed{f(\theta) = -\operatorname{sen}\theta - \cos\theta + 5\theta + 4}$$

#16 | Pág 366 | #45

$f''(x) = e^x - 2\operatorname{sen} x \qquad f(0) = 2\ ,\ f(1) = 2$

$f\left(\frac{\pi}{2}\right) = e^{\pi/2} + 2 + \frac{\pi}{2}C + 2$

$f\left(\frac{\pi}{2}\right) = 0$

$f''(x) = e^x - 2\operatorname{sen} x \qquad f(0) = 1 + 0 + D \qquad e^{\pi/2} + 4 + \frac{\pi}{2}C = 0$

$f'(x) = e^x + 2\cos x + C \qquad f(0) = 3 \rightarrow D = 2 \qquad \frac{\pi}{2}C = -e^{\pi/2} - 4$

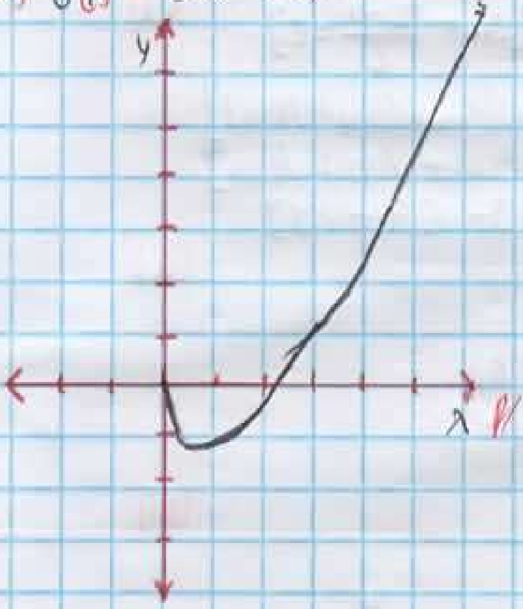
$f(x) = e^x + 2\operatorname{sen} x + Cx + D \qquad f(x) = e^x + 2\operatorname{sen} x + Cx + 2 \qquad C = -\frac{2}{\pi}\left(e^{\pi/2} + 4\right)$

$f(x) = e^x + 2\operatorname{sen} x + -\frac{2}{\pi}\left(e^{\pi/2} + 4\right)$

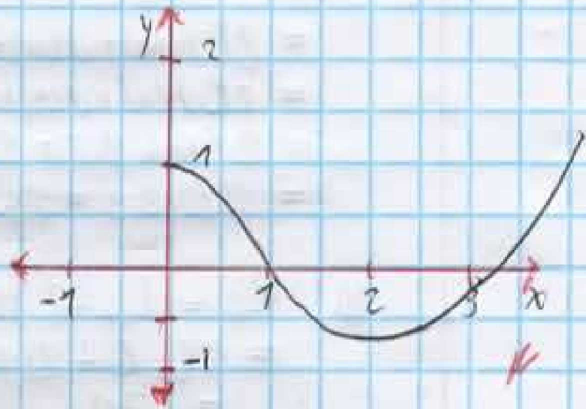
$$\boxed{f(x) = e^x + 2\operatorname{sen} x + -\frac{2}{\pi}\left(e^{\pi/2} + 4\right)}$$

#17	Pag 356	#66

a) $f(x) = 2x - 3\sqrt{x}$



b) $F(0) = 1$



c) $f(x) = 2x - 3\sqrt{x}$

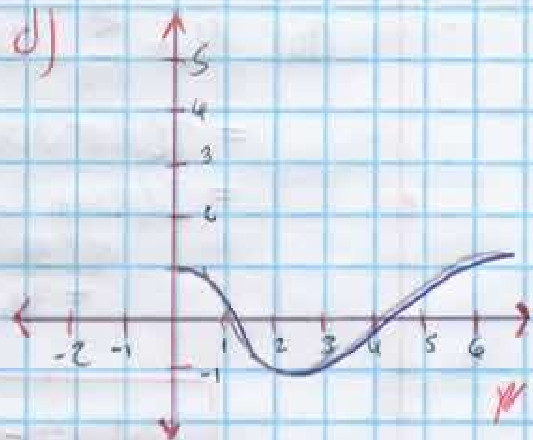
$F(x) = x^2 - 3\cdot\frac{2}{3}x^{3/2} + C$

$F(0) = C$ y $F(1) = 1 \to C = 1$

$F(x) = x^2 - 2x^{3/2} + 1$

$\boxed{F(x) = x^2 - 2x^{3/2} + 1}$

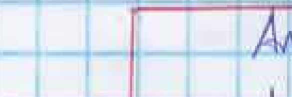
d)



#18	Pag 367	#79

$\to V = 80\,km/h$

$a = 7m/s$



$V_F^2 = V_i^2 + 2ad$

$V_0 = 80\,km/h = 22{,}22\,m/s$ $\quad d = ?$

$V_f = 0$

$a = 7m/s$

Remplazamos

$V_F^2 = V_0^2 + 2ad$

$0^2 = (22{,}22)^2 + 2(7)d$

$493.73 + 14d$

$-14d = 493.73$

$d = \dfrac{493.73}{35.27}$

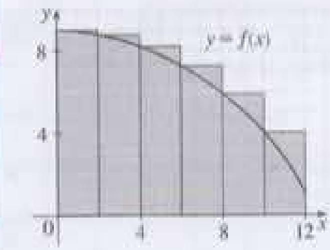
$d = 35.27$

Antes de detenerse

recorre $d = 35.27m$

a) (i) $L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x$ $\quad \left[\Delta x = \frac{12-0}{6} = 2\right]$

(i)



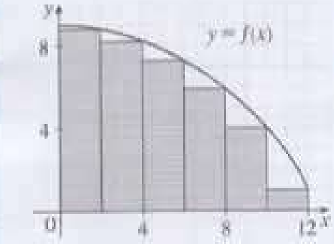
$= 2[f(x_0) + f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)]$

$= 2[f(0) + f(2) + f(4) + f(6) + f(8) + f(10)]$

$= 2[9 + 8.8 + 8.2 + 7.3 + 5.9 + 4.1]$

$= 2[43.3] = 86.6$

(ii)

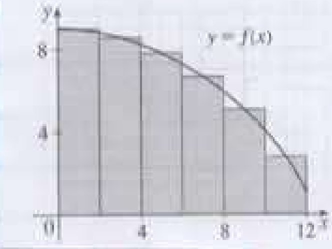


(ii) $R_6 = L_6 + 2f(12) - 2f(0)$

$= 86.6 + 2(1) - 2(9)$

$= 70.6$

(iii)



(iii) $M_6 = \sum_{i=1}^{6} f(\bar{x}_i)\Delta x$

$= 2[f(1) + f(3) + f(5) + f(7) + f(9) + f(11)]$

$= 2(8.9 + 8.5 + 7.8 + 6.6 + 5.1 + 2.8)$

$2(39.7)$

$= 79.4$

a) (i) = 86.6 $\quad$ Ver gráficas

(ii) = 70.6

(iii) = 79.4

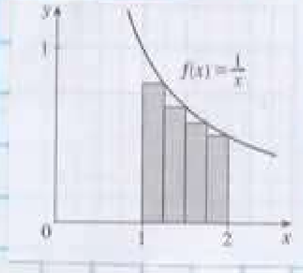
b) Ya que f está disminuyendo, obtenemos una sobre estimación mediante el uso de los puntos finales L_6

c) Ya que f está disminuyendo, obtenemos una subestimación mediante el uso de los puntos finales en R_6

d) M_6 da la mejor estimación, ya que el área de cada rectángulo parece estar más cerca del área verdadera que las sobreestimaciones y subestimaciones en L_6 y R_6

#20	Pag 375	#3

a) $f(x) = \frac{1}{x}$; $x = 1 \to x = 2$

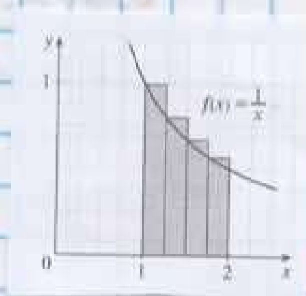


$$R_4 = \sum_{i=1}^{4} f(x_i)\Delta x \quad \left[\Delta x = \frac{2-1}{4} = \frac{1}{4}\right] = \left[\sum_{i=1}^{4} f(x_i)\right]\Delta x$$

$$= \left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right]\Delta x$$

$$= \left[\frac{4}{5} + \frac{2}{3} + \frac{4}{7} + \frac{1}{2}\right]\frac{1}{4} \approx 0.6345$$

b) $$L_4 = \sum_{i=1}^{4} f(x_{i-1})\Delta x = \left[\sum_{i=1}^{4} f(x_{i-1})\right]\Delta x$$



$$= \left[f(x_0) + f(x_1) + f(x_2) + f(x_3)\right]\Delta x$$

$$= \left[\frac{1}{1} + \frac{1}{5/4} + \frac{1}{6/4} + \frac{1}{7/4}\right]\frac{1}{4}$$

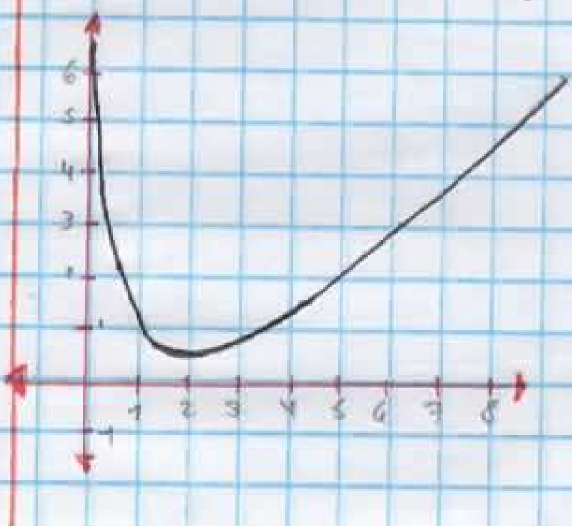
$$\left[1 + \frac{4}{5} + \frac{2}{3} + \frac{4}{7}\right]\frac{1}{4} \approx 0.7595$$

a) 0.6345 Se obtiene una subestimación mediante el uso de la aproximación correcta del punto final

b) 0.7595 es una sobreestimación

#21	Pag 375	#6

a) $f(x) = x - 2\ln x$ $\quad 1 \leq x \leq 5$

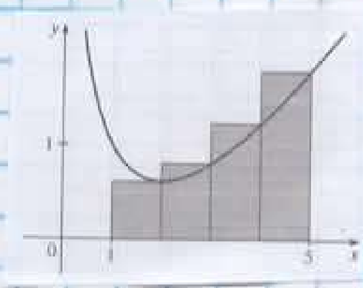


b) $f(x) = x - 2\ln x \to \Delta x = \frac{5-1}{4} = 1$

$$(i) \to R_4 = 1f(2) + 1f(3) + 1f(4) + 1f(5)$$

$$= (2 - 2\ln 2) + (3 - 2\ln 3) + (4 - 2\ln 4) + (5 - 2\ln 5)$$

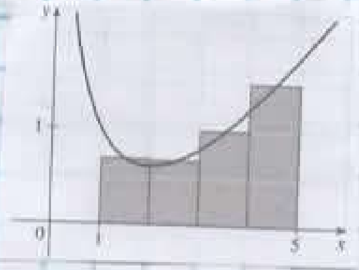
$$= 4.425$$



b) (ii) $M_4 = 1f(1.5) + 1f(2.5) + 1f(3.5) + 1f(4.5)$

$= (1.5 - 2\ln 1.5) + (2.5 - 2\ln 2.5) + (3.5 - 2\ln 3.5) + (4.5 - 2\ln 4.5)$

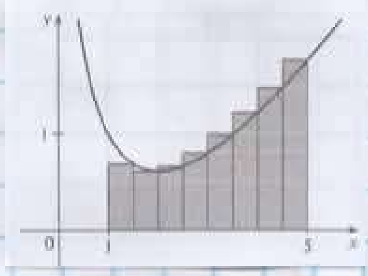
$= 3.843$



c) (i) $R_8 = \frac{1}{2}[f(1.5) + f(2) + f(3) + f(4) + f(5)]$

$= \frac{1}{2}[(1.5 - 2\ln 1.5) + (2 - 2\ln 2) + (3 - 2\ln 3) + (4 - 2\ln 4) + (5 - 2\ln 5)]$

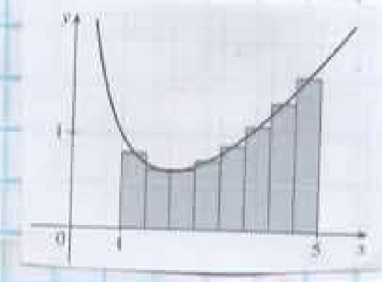
$= 4.134$



(ii) $M_8 = \frac{1}{2}[f(1.25) + f(1.75) + \cdots + f(4.75)]$

$= \frac{1}{2}[(1.25 - 2\ln 1.25) + (1.75 - 2\ln 1.75) + \cdots + (4.75 - 2\ln 4.75)]$

≈ 3.880



a) Ver gráfica

b)(i) 4.426 → Ver gráfica

(ii) 3.843 → Ver gráfica

c)(i) 4.134 → Ver gráfica

(ii) 3.889 → Ver gráfica

#22	Pag 376	#14

Tiempo (s)	Velocidad (m/s)
0	9.1
12	8.5
24	7.6
36	6.7
48	7.3
60	8.2

$d = v6$

a) $(9.1)(0)+(8.5)(12)+(7.6)(24)+(6.7)(36)+(7.3)(48)+(8.2)(60)$

$0 + 102 + 182.40 + 241.20 + 350.40 + 492$

1368m

b) $28.87(9.1+8.5+7.6+6.7+7.3+8.2)$

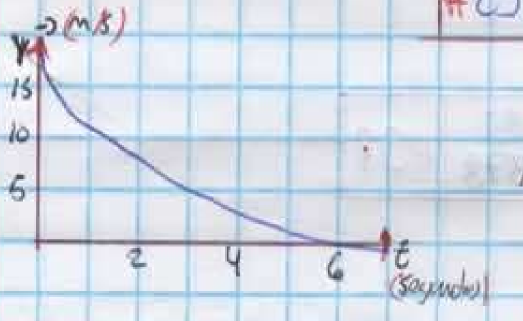
1368m

a) 1368 m

b) 1368m

c) La velocidad no aumenta ni disminuye en los intervalos dados por lo que las estimaciones en las partes a) y b) no son ni superiores o inferiores

#23	Pag 376	#17



$M_6 \to \Delta t = 1$

$M_6 = 1[v(0.5)+v(1.5)+v(2.5)+v(3.5)+v(4.5)+v(5.5)]$

$55+40+28+18+10+4$

155m

155m recorre a lo que aplican los frenos

#24	Pag 377	#21

$f(x) = \frac{2x}{x^2+1},\ 1 \le x \le 3$

$\Delta x = \frac{(3-1)}{n} = \frac{2}{n} \to x_i = 1 + i\Delta x = 1 + \frac{2i}{n}$

$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(1+\frac{2i}{n})}{(1+\frac{2i}{n})^2+1} \cdot \frac{2}{n}$

$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2(1+\frac{2i}{n})}{(1+\frac{2i}{n})^2+1} \cdot \frac{2}{n}}$

#25	Pag 377	#22

$f(x) = x^2 + \sqrt{1+2x}$, $4 \le x \le 7$

$x_i = 4 + i\Delta x = 4 + \frac{3i}{n}$

$\Delta x = \frac{(7-4)}{n} = \frac{3}{n}$

$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x =$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(4+\frac{3i}{n}\right)^2 + \sqrt{1+2\left(4+\frac{3i}{n}\right)}\right]\frac{3}{n}$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(4+\frac{3i}{n}\right)^2 + \sqrt{1+2\left(4+\frac{3i}{n}\right)}\right]\frac{3}{n}}$$

#26	Pag 377	#25

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan \frac{i\pi}{4n}$

Intervalo $\rightarrow$ $y = \tan x$ $\left[0, \frac{\pi}{4}\right]$

$\Delta x = \frac{\frac{\pi}{4} - 0}{n} = \frac{\pi}{4n}$

$x_i^* = x_i$

$x_i = a + \Delta x = \frac{i\pi}{4n}$

$y = \tan(x - k\pi) \rightarrow \left[k\pi, k\pi + \frac{\pi}{4}\right]$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{i\pi}{4}\right)\frac{\pi}{4n}$

$$\boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(\frac{i\pi}{4}\right)\frac{\pi}{4n}}$$

#27	Pag 378	#37

$y = \cos x$, $x = 0$, $x = b$; $0 \le b \le \frac{\pi}{2}$

$y = f(x) = \cos x$; $\Delta x = \frac{b-0}{n} = \frac{b}{n}$; $x_1 = 0 + i\Delta x = \frac{bi}{n}$

$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\frac{bi}{n}\right)\frac{b}{n}$

$= \lim_{n\to\infty} \left[\frac{b \operatorname{sen}\left(b\left(\frac{1}{2n}+1\right)\right)}{2n \operatorname{sen}\left(\frac{b}{2n}\right)} - \frac{b}{2n}\right] = \operatorname{sen} b$

$b = \frac{\pi}{2}$

$b = \frac{\pi}{2} \rightarrow A = \operatorname{sen}\frac{\pi}{2} = 1$

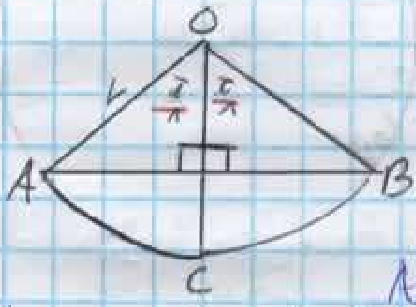
$$\boxed{A = \operatorname{sen}\frac{\pi}{2} = 1}$$

$A = \operatorname{sen}\frac{\pi}{2} = 1$

#28	Pág 378	#32

a) $A_n = \frac{1}{2} n r^2 \operatorname{sen}\left(\frac{2\pi}{n}\right)$

El diagrama nos muestra uno de n triángulos congruentes, ΔAOB con ángulo central $\frac{2\pi}{n}$. O es el centro del círculo y AB es uno de los lados del polígono. El radio OC se dibuja para dividir $\angle AOB$. Se deduce que OC interseca a AB en los ángulos rectos y biseca los de AB. Así ΔAOB dividiéndose en 2 triángulos rectángulos con patas de longitud $\frac{1}{2}(AB) = r \operatorname{sen}\left(\frac{\pi}{n}\right)$ y $r\cos\left(\frac{\pi}{n}\right)$.



ΔAOB con área de $2\frac{1}{2}\left[r\operatorname{sen}\left(\frac{\pi}{n}\right)\right]\left[r\cos\left(\frac{\pi}{n}\right)\right] = r^2 \operatorname{sen}\left(\frac{\pi}{n}\right)\cos\left(\frac{\pi}{n}\right) = \frac{1}{2} r^2 \operatorname{sen}\left(\frac{2\pi}{n}\right)$

$$A_n = n \text{ area}(\Delta AOB) = \frac{1}{2} n r^2 \operatorname{sen}\left(\frac{2\pi}{n}\right)$$

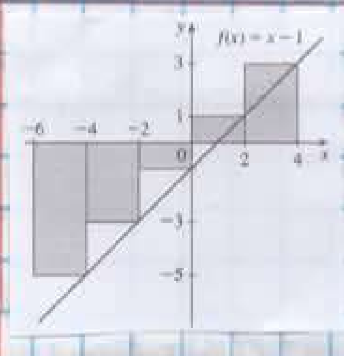
b) Usando la ecuación 3.3.2 $\lim_{\theta \to 0} \frac{\operatorname{sen}\theta}{\theta} = 1$

$\frac{2\pi}{n} \rightarrow \lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{1}{2} n r^2 \operatorname{sen}\left(\frac{2\pi}{n}\right)$

$$\lim_{n\to\infty} \frac{1}{2} n r^2 \frac{\operatorname{sen}\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}} = \lim_{n\to\infty} \frac{\operatorname{sen}\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}} \pi r^2 \rightarrow \theta = \frac{2\pi}{n}$$

$$n \to \infty,\ \theta \to 0;\ \lim_{n\to\infty} \frac{\operatorname{sen}\frac{2\pi}{n}}{\frac{2\pi}{n}} \pi r^2 = \lim_{\theta\to 0} \frac{\operatorname{sen}\theta}{\theta} \pi r^2 = (1)\pi r^2 = \boxed{\pi r^2}$$

#29	Pág 388	#1



$f(x) = x - 1;\ -6 \le x \le 4$

$\Delta x = \frac{b-a}{n} = \frac{4-(-6)}{5} = 2$

$$R_5 = \sum_{i=1}^{5} f(x_i)\,\Delta x$$

$$= (\Delta x)\left[f(x_1) + f(x_2) + f(x_3) + f(x_4) + f(x_5)\right]$$

$$= 2\left[f(-4) + f(-2) + f(0) + f(2) + f(4)\right]$$

$$= 2\left[-5 + (-3) + (-1) + 1 + 3\right]$$

$$= 2(-5)$$

$$= -10$$

-10

La Suma de Riemann representa la suma de las áreas de los dos rectángulos por encima del eje x menos la suma de las áreas del rectángulo debajo del x; su área neta respecto al eje x

#30	Pág 388	#5

a) $\int_0^{10} f(x)\,dx = R_5 = [f(2)+f(4)+f(6)+f(8)+f(10)]\Delta x$

$= [-1+0+(-2)+2+4](2)$

$= 3(2)$

$= 6$

b) $\int_0^{10} f(x)\,dx = L_5 = [f(0)+f(2)+f(4)+f(6)+f(8)]\Delta x$

$[3+(-1)+0+(-2)+2](2)$

$= 2(2)$

$= 4$

c) $\int_0^{10} f(x)\,dx = M_5 = [f(1)+f(3)+f(5)+f(7)+f(9)]\Delta x$

$[0+(-1)+(-1)+0+3](2)$

$= 1(2)$

$= 2$

Respuestas
$a = 6$
$b = 4$
$c = 2$

#31	Pág 388	#7

x	0	5	10	15	20	25
$f(x)$	-42	-37	-25	-6	15	36

$\int_0^{25} f(x)\,dx \le R_5$

$R_5 = \sum_{i=1}^{5} f(x_{i-1})\Delta x = 5[f(0)+f(5)+f(10)+f(15)+f(20)]$

$= 5[-42+(-5)+20+14+29]$

$= 5(16)$

$= 80$

$L_5 = \sum_{i=1}^{5} f(x_i)\Delta x = 5[f(5)+f(10)+f(15)+f(20)+f(25)]$

$5[-37+(-12)+(-6)+(-3)+33]$

$5(-25)$

$= -125$

Estimación Inferior $= -125$

Estimación Superior $= 80$

#32	Pág 389	#15

$\int_0^{\pi} \operatorname{sen} x \, dx \rightarrow n = 5, 10, 50, 100$

$\int_0^{\pi} \operatorname{sen} x \, dx \rightarrow$ $Y_1 = \operatorname{sen} x$, $X_{min} = 0$

$X_{max} = \pi$

$n = 5, 10, 50, 100$

n	R_n
5	1.933766
10	1.983524
50	1.999342
100	1.999836

#33	Pág 389	#17

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1 + x_i} \Delta x, \ [0, 1]$$

$$\boxed{\int_0^1 \frac{e^x}{1 + x_i} dx}$$

#34	Pág 389	#21

$\int_{-1}^{5} (1 + 3x) dx$

$\Delta x = \frac{5-1}{n} = \frac{4}{n}$

$\boxed{x_i = 3 + i\Delta x = 3 + \frac{4i}{n}}$

$$\int_{-1}^{5} (1 + 3x) dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} f\left(3 + \frac{4i}{n}\right) \frac{4}{n}$$

$$\lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^{n} \left[1 + 3\left(3 + \frac{4i}{n}\right)\right] = \lim_{n \to \infty} \frac{4}{n} \sum_{i=1}^{n} \left[10 + \frac{12i}{n}\right]$$

$$\lim_{n \to \infty} \frac{4}{n} \left(10 + \frac{12i}{n}\right) \sum_{i=1}^{n} i = \lim_{n \to \infty} \left(\frac{40}{n} + \frac{48i}{n^2}\right) \left[\frac{n(n+1)}{2}\right]$$

$$\lim_{n \to \infty} \left(\frac{40}{n} + \frac{48i}{n}\right) \left[\frac{n}{2}\right] = 42$$

$$\boxed{\int_{-1}^{5} (1 + 3x) dx = 42}$$

#35	Pag 389	#25

$$\int_0^1 (x^3 - 3x^2)\,dx$$

$$\boxed{\Delta x = \frac{1-0}{n} = \frac{1}{n} \qquad x_i = 0 + i\Delta x = \frac{i}{n}}$$

$$\int_0^1 (x^3-3x^2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\Delta x$$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n}\left[\frac{i^3}{n^3} - \frac{3i^2}{n^2}\right] = \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n^3}\sum_{i=1}^{n} i^3 - \frac{3}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$\lim_{n\to\infty}\left\{\frac{1}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{3}{n^3}\,\frac{n(n+1)(2n+1)}{6}\right\} = \lim_{n\to\infty}\left[\frac{1}{4}\,\frac{n+1}{n}\,\frac{n+1}{n} - \frac{1}{2}\,\frac{n+1}{n}\,\frac{2n+1}{n}\right]$$

$$\lim_{n\to\infty}\left[\frac{1}{4}\left(1+\frac{1}{n}\right)\left(1+\frac{1}{n}\right) - \frac{1}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right] = \frac{1}{4}(1)(1) - \frac{1}{2}(1)(1) = -\frac{3}{2}$$

$$\boxed{\int_0^1 (x^3 - 3x^2)\,dx = -\frac{3}{2}}$$

#36	Pag 389	#28

$$\int_a^b x^2\,dx = \frac{b^3 - a^3}{3} \to \int_a^b x^2\,dx = \lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n}\left[a + \frac{b-a}{n}i\right]^2$$

$$\lim_{n\to\infty} \frac{b-a}{n}\sum_{i=1}^{n}\left[a^2 + 2a\frac{b-a}{n}i + \frac{(b-a)^2}{n^2}i^2\right] = \lim_{n\to\infty}\left[\frac{(b-a)^3}{n^3}\sum_{i=1}^{n} i^2 + \frac{2a(b-a)^2}{n^2}\sum_{i=1}^{n} i + \frac{a^2(b-a)}{n}\sum_{i=1}^{n} 1\right]$$

$$\lim_{n\to\infty}\left[\frac{(b-a)^3}{n^3}\,\frac{n(n+1)(2n+1)}{6} + \frac{2a(b-a)^2}{n^2}\,\frac{n(n+1)}{2} + \frac{a^2(b-a)}{n}n\right]$$

$$\lim_{n\to\infty}\left[\frac{(b-a)^3}{6}\cdot 1\cdot\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + a(b-a)^2\cdot 1\cdot\left(1+\frac{1}{n}\right) + a^2(b-a)\right]$$

$$= \frac{(b-a)^3}{3} + a(b-a)^2 + a^2(b-a) = \frac{b^3 - 3ab^2 + 3a^2b - a^3}{3} + ab^2 - 2a^2b + a^3 + a^2b - a^3$$

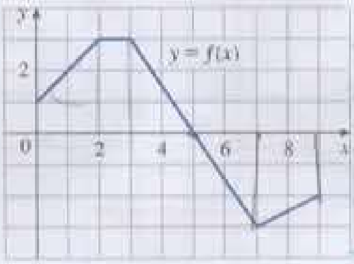
$$= \frac{b^3}{3} - \frac{a^3}{3} - ab^2 + a^2b + ab^2 - a^2b = \frac{b^3 - a^3}{3}$$

#37	Pág 384	#33

$A = \frac{1}{2}(b+B)h$

(a) $\int_0^2 f(x)\,dx$ (b) $\int_0^5 f(x)\,dx$

(c) $\int_5^7 f(x)\,dx$ (d) $\int_0^9 f(x)\,dx$



a) $\int_0^2 f(x)\,dx = \frac{1}{2}(1+3)2 = 4$

b) $\int_0^5 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^5 f(x)\,dx$

trapecio rectángulo triangulo

$= \frac{1}{2}(1+3)2 + 3(1) + \frac{1}{2}(2)(3)$

$4 + 3 + 3$

$= 10$

c) $\int_5^7 f(x)\,dx = -\frac{1}{2}(2)(3) = -3$

d) $\int_0^9 f(x)dx = \int_0^2 f(x)dx + \int_2^3 f(x) + \int_3^5 f(x) + \int_5^7 f(x) + \int_7^9$

$\frac{1}{2}(1+3)2 + 3(1) + \frac{1}{2}(2)(3) + -\frac{1}{2}(2)(3) + -\frac{1}{2}(3+2)2$

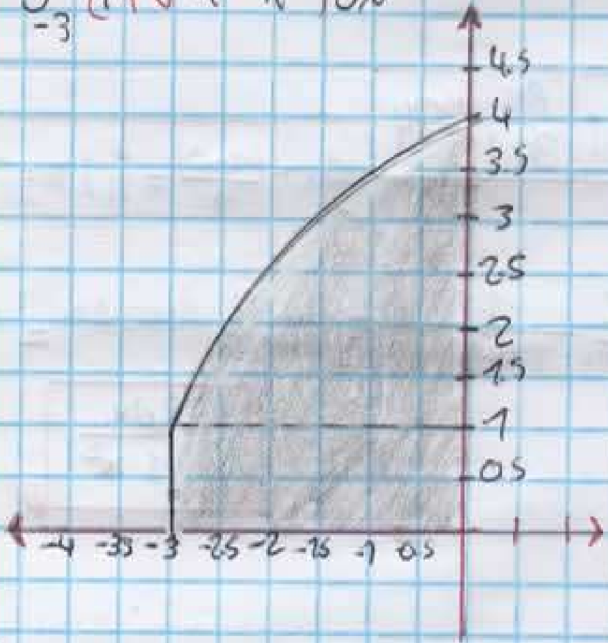
$4 + 3 + 3 - 3 - 5 = 2$

$$\boxed{\begin{array}{ll} a)\ \int_0^2 f(x)\,dx = 4 & b)\ \int_0^5 f(x)\,dx = 10 \\ c)\ \int_5^7 f(x)\,dx = -3 & d)\ \int_0^9 f(x)\,dx = 2 \end{array}}$$

#38	Pág 390	#37

$\int_{-3}^{0} (1+\sqrt{9-x^2})\,dx$

$f(x) = 1+\sqrt{9-x^2}$ $x=-3$ $x=0$



$\int_{-3}^{0} (1+\sqrt{9-x^2})\,dx$

$= \frac{1}{4}\pi(3)^2 + 1(3) = 3 + \frac{9}{4}\pi$

$$\boxed{\int_{-3}^{0} (1+\sqrt{9-x^2})\,dx = 3 + \frac{9}{4}\pi}$$