SISTEMAS DE ECUACIONES DIFERENCIALES

Dichos sistemas de ecuaciones diferenciales se resolverán por la regla de Cramer, que dice lo siguiente:

$$\begin{aligned} ax + by &= k_1 \\ cx + dy &= k_2 \end{aligned}$$

Entonces la solución queda escrita como:

$$x = \frac{\begin{vmatrix} k_1 & b \\ k_2 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} \quad , \quad y = \frac{\begin{vmatrix} a & k_1 \\ c & k_2 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

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Use la Transformada de Laplace para Resolver el siguiente Sistemas de Ecuaciones Diferenciales:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + \frac{dy}{dt} = 0$$

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 4\frac{dx}{dt} = 0$$

$x(0) = 1, \quad x'(0) = 0,$

$y(0) = -1, \quad y'(0) = 5$

- $s^2X(s) - s\cancel{x(0)}^{1} - \cancel{x'(0)}^{0} + sX(s) - \cancel{x(0)}^{1} + sY(s) - \cancel{y(0)}^{-1} = 0$

$s^2Y(s) - s\cancel{y(0)}_{-1} - \cancel{y'(0)}_{5} + sY(s) - \cancel{y(0)}_{-1} - 4[sX(s) - \cancel{x(0)}_{1}] = 0$

- $X(s)[s^2+s] + sY(s) = s$
$-4sX(s) + Y(s)[s^2+s] = -s$

; $ax + by = k_1$
$cx + dy = k_2$

$$X(s) = \frac{\begin{vmatrix} k_1 & b \\ k_2 & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{\begin{vmatrix} s & s \\ -s & s^2+s \end{vmatrix}}{\begin{vmatrix} s^2+s & s \\ -4s & s^2+s \end{vmatrix}} = \frac{s^2(s+2)}{s^2(s^2+2s+5)} = \frac{s+2}{s^2+2s+5}$$

det([s, s ;-s, s^2+s]) ; det([s^2+s, s; -4s, s^2+s])

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$$X(s)=\frac{s+2}{s^2+2s+5}=\frac{s+2}{s^2+2s+1-1+5}=\frac{s+2}{(s+1)^2+4}=\frac{(s+1-1)+2}{(s+1)^2+4}$$

$$X(s)=\frac{s+1}{(s+1)^2+4}+\frac{1}{(s+1)^2+4}\quad \Big|\, \mathcal{L}^{-1}\{\ \}_{s+1\to s}$$

$$x(t)=e^{-t}\cos(2t)+\frac{e^{-t}}{2}\operatorname{sen}(2t)$$

$$Y(s)=\frac{\begin{vmatrix} a & k_1 \\ c & k_2 \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}=\frac{\begin{vmatrix} s^2+s & s \\ -4s & -s \end{vmatrix}}{\begin{vmatrix} s^2+s & s \\ -4s & s^2+s \end{vmatrix}}=\frac{-s^2(s-3)}{s^2(s^2+2s+5)}=\frac{3-s}{s^2+2s+5}$$

$$Y(s)=\frac{3-(s+1-1)}{(s+1)^2+4}=\frac{4}{(s+1)^2+4}-\frac{(s+1)}{(s+1)^2+4}\quad \Big|\, \mathcal{L}^{-1}\{\ \}_{s+1\to s}$$

$$y(t)=2e^{-t}\operatorname{sen}(2t)-e^{-t}\cos(2t)$$

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$$\frac{dx}{dt} = 4x - 2y + 2\mathcal{U}(t-1)$$

$$\frac{dy}{dt} = 3x - y + \mathcal{U}(t-1)$$

$$x(0) = 0, \quad y(0) = \frac{1}{2}$$

$$sX(s) - \cancel{x(0)}^{\,0} = 4X(s) - 2Y(s) + \frac{2}{s}e^{-s}$$

$$sY(s) - \cancel{y(0)}^{\,1/2} = 3X(s) - Y(s) + \frac{1}{s}e^{-s}$$

$$X(s)[s-4] + 2Y(s) = \frac{2}{s}e^{-s}$$

$$-3X(s) + Y(s)[s+1] = \frac{1}{s}e^{-s} + \frac{1}{2}$$

$$X(s) = \frac{\begin{vmatrix} \frac{2}{s}e^{-s} & 2 \\ \frac{1}{s}e^{-s} + \frac{1}{2} & s+1 \end{vmatrix}}{\begin{vmatrix} s-4 & 2 \\ -3 & s+1 \end{vmatrix}} = \frac{-(e^{s}-2)e^{-s}}{s^2-3s+2} = \frac{-1+2e^{-s}}{(s-2)(s-1)}$$

$$X(s) = \frac{-1}{(s-2)(s-1)} + e^{-s}\left[\frac{2}{(s-2)(s-1)}\right] = \frac{1}{s-1} - \frac{1}{s-2} + e^{-s}\left[\frac{2}{s-2} - \frac{2}{s-1}\right] \Big| \mathcal{L}^{-1}\{\}$$

$$x(t) = e^{t} - e^{2t} + \mathcal{U}(t-1)\left[2e^{2(t-1)} - 2e^{(t-1)}\right]$$

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$$Y(s)=\frac{\begin{vmatrix} s-4 & \frac{2}{s}e^{-s} \\ -3 & \frac{1}{s}e^{-s}+\frac{1}{2} \end{vmatrix}}{\begin{vmatrix} s-4 & 2 \\ -3 & s+1 \end{vmatrix}}=\frac{\left(\frac{\frac{1}{2}\left(s(s-4)e^{s}+2(s+2)\right)e^{-s}}{s}\right)}{(s-2)(s-1)}$$

$$Y(s)=\frac{\frac{1}{2}(s-4)}{(s-2)(s-1)}+e^{-s}\left[\frac{s+2}{s(s-2)(s-1)}\right]$$

$$Y(s)=\frac{3/2}{s-1}-\frac{1}{s-2}+e^{-s}\left[-\frac{3}{s-1}+\frac{2}{s-2}+\frac{1}{s}\right] \quad \Big| \mathcal{L}^{-1}\{\}$$

$$y(t)=\frac{3}{2}e^{t}-e^{2t}+u(t-1)\left[-3e^{(t-1)}+2e^{2(t-1)}+1\right]$$

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