

Caso 2

Resolver El siguiente Sistema de Ecuaciones Diferenciales Homogéneo:

$$X' = \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} X$$

$$|a - \lambda I| = 0 \quad \left| \begin{pmatrix} 3 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 3 - \lambda & -1 & -1 \\ 1 & 1 - \lambda & -1 \\ 1 & -1 & 1 - \lambda \end{vmatrix} = 0 \quad -\lambda^3 + 5\lambda^2 - 8\lambda + 4 = 0$$

$$\lambda_1 = 1, \quad \lambda_{2,3} = 2$$



Para $\lambda_1 = 1$:

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$$

$$\begin{pmatrix} 3 - \lambda & -1 & -1 \\ 1 & 1 - \lambda & -1 \\ 1 & -1 & 1 - \lambda \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 - 1 & -1 & -1 \\ 1 & 1 - 1 & -1 \\ 1 & -1 & 1 - 1 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_2 - k_3 = 0$$

$$k_2 = k_3$$

$$\begin{pmatrix} 1 & -1/2 & -1/2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0$$

$$k_1 - \left(\frac{1}{2}\right)k_2 - \left(\frac{1}{2}\right)k_3 = 0$$

$$k_1 = k_3$$

$$k_3 = 1$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \rightarrow K_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 K_1 e^{\lambda_1 t} = C_1 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} e^{(1) t}$$



Para $\lambda_2 = 5$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix}$$

$$\begin{pmatrix} 3 - \lambda & -1 & -1 \\ 1 & 1 - \lambda & -1 \\ 1 & -1 & 1 - \lambda \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 3 - 2 & -1 & -1 \\ 1 & 1 - 2 & -1 \\ 1 & -1 & 1 - 2 \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_4 - k_5 - k_6 = 0$$

$$k_4 = k_5 + k_6$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \rightarrow K_2 = \begin{pmatrix} k_5 + k_6 \\ k_5 \\ k_6 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} k_5 \\ k_5 \\ 0 \end{pmatrix} + \begin{pmatrix} k_6 \\ 0 \\ k_6 \end{pmatrix}$$

$$k_5 \text{ y } k_6 = 1$$

$$K_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$x_2(t) = C_2 K_2 e^{\lambda_2 t} = C_2 \left[\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right] e^{2t}$$

3. VALORES PROPIOS COMPLEJOS:

$$\lambda_{1,2} = \alpha \pm i\beta$$

$$k_1, k_2 \dots k_n$$

$$x_G(t) = C_1(B_1 \cos(\beta t) - B_2 \sin(\beta t))e^{\alpha t} + C_2(B_2 \cos(\beta t) + B_1 \sin(\beta t))e^{\alpha t}$$

$$B_1 = \frac{1}{2} [k_1 + \overline{k_1}] \quad y \quad B_2 = \frac{i}{2} [k_1 - \overline{k_1}]$$



Resolver El siguiente Sistema de Ecuaciones Diferenciales Homogéneo

$$X' = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} x$$

$$|a - \lambda I| = 0$$

$$\left| \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2 - \lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3 - \lambda \end{vmatrix} = 0$$

$$-\lambda^3 - \lambda^2 + \lambda - 15 = 0$$

$$\lambda_1 = -3 \quad \lambda_{2,3} = 1 \pm 2i$$





TEXAS INSTRUMENTS

voyage 200

F1 F2 F3 F4 F5 F6
Algebra Calc Other PrgmIO Clean Up

$$\text{eigV1} \begin{bmatrix} 2 & 1 & 2 \\ 3 & 0 & 6 \\ -4 & 0 & -3 \end{bmatrix}$$

(1. + 2.·i 1. - 2.·i -3.)

eigV1([2,1,2;3,0,6;-4,0,-3])

MAIN

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Para $\lambda_1 = -3$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \quad \begin{pmatrix} 2 - \lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3 - \lambda \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 - (-3) & 1 & 2 \\ 3 & -(-3) & 6 \\ -4 & 0 & -3 - (-3) \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 5 & 1 & 2 \\ 3 & 3 & 6 \\ -4 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1/5 & 2/5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = 0 \quad \begin{matrix} k_2 + 2k_3 = 0 \\ k_2 = -2k_3 \\ k_1 = 0 \end{matrix}$$

$$\begin{matrix} k_1 + \left(\frac{1}{5}\right)k_2 + \left(\frac{2}{5}\right)k_3 = 0 \\ k_1 + \left(\frac{1}{5}\right)(-2k_3) + \left(\frac{2}{5}\right)k_3 = 0 \end{matrix}$$

$$k_3 = 1$$

$$K_1 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} \rightarrow K_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$$x_1(t) = C_1 K_1 e^{\lambda_1 t} = C_1 \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} e^{(-3)t}$$



Para: $\lambda_2 = 1 + 2i$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix}$$

$$\begin{pmatrix} 2 - \lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3 - \lambda \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \mathbf{0}$$

$$\begin{pmatrix} 2 - (1 + 2i) & 1 & 2 \\ 3 & -(1 + 2i) & 6 \\ -4 & 0 & -3 - (1 + 2i) \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 - 2i & 1 & 2 \\ 3 & -1 - 2i & 6 \\ -4 & 0 & -4 - 2i \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$





TEXAS INSTRUMENTS

voyage 200

F1 F2 F3 F4 F5 F6
Algebra Calc Other PrgmIO Clean Up

$$\text{ref} \begin{bmatrix} 1-2i & 1 & 2 \\ 3 & -(1+2i) & 6 \\ -4 & 0 & -4-2i \end{bmatrix}$$
$$\begin{bmatrix} 1 & 0 & 1+1/2i \\ 0 & 1 & 3/2i \\ 0 & 0 & 0 \end{bmatrix}$$

ref([1-2i,1,2;3,-(1+2i),6;-4,...

MAIN DEG AUTO FUNC 1/30

LOCK



F1 F2 F3 F4 F5 F6 F7 F8

7 CALC HOME 1 Y= WINDOW/ GRAPH # TBLSET > TABLE
Q W E R T Y U I O P
A S D F G H J K L
Z X C V B N M
OFF ON RCL STO ENTER



QUIT
2ND ESC
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CLEAR
c⁻¹ APPS
LN ENTER
SIN⁻¹ COS⁻¹ TAN⁻¹ TT
SIN COS TAN
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4 5 6 -
EE CATALOG CUSTOM CHAS
1 2 3 +
< > ANS ENTRY
0 . (−) ENTER



Para: $\lambda_2 = 1 + 2i$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix}$$

$$\begin{pmatrix} 2 - \lambda & 1 & 2 \\ 3 & -\lambda & 6 \\ -4 & 0 & -3 - \lambda \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = 0$$

$$\begin{pmatrix} 2 - (1 + 2i) & 1 & 2 \\ 3 & -(1 + 2i) & 6 \\ -4 & 0 & -3 - (1 + 2i) \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 - 2i & 1 & 2 \\ 3 & -1 - 2i & 6 \\ -4 & 0 & -4 - 2i \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 + \frac{1}{2}i \\ 0 & 1 & \frac{3}{2}i \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$k_5 + \left(\frac{3}{2}i\right)k_6 = 0$$

$$k_4 + \left(1 + \frac{1}{2}i\right)k_6 = 0$$

$$k_5 = -\left(\frac{3}{2}i\right)k_6$$

$$k_4 = -\left(1 + \frac{1}{2}i\right)k_6$$

$$k_6 = 1$$

$$K_2 = \begin{pmatrix} k_4 \\ k_5 \\ k_6 \end{pmatrix} \rightarrow K_2 = \begin{pmatrix} -1 - \frac{1}{2}i \\ -\frac{3}{2}i \\ 1 \end{pmatrix}$$

