Application of evolutionary multiobjective algorithms for informed decision making in complex problems of the industrial-logistics field.

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Abstract

Two problems of interest from the field of industrial-logistics based on the single objective problems proposed in Inditex Zara Data Challenge [1], have been extended as multiobjective problems and are solved through the use of three state-of-the-art evolutionary multi-objective algorithms: NSGAII, SMS-EMOA and SPEA2. They are the following:

Problem 1 Blocks: A set of blocks is given, in which each one has a value (forecast of sales) and a weight (number of products). A subset of those blocks must be chosen so that sales are maximized while choosing exactly n products contained in the blocks chosen. The multiobjective formulation implies maximizing sales worth while simultaneously minimizing number of blocks.

Problem 2 Stores: A set of n stores will be opened in a country. The coordinates that maximizes sales must be found while maintaining a minimun distance between both new and stablished stores. The multiobjective formulation implies maximizing sales while simultaneously maximizing sum of distances between cities.

For each algorithm, a set of nine parameters varying population size and mutation rates have been tested and analysed, using hypervolume indicator convergence and final values. Moreover, global results among methods were also compared and analysed. Both, with statistical significance methods including Friedman tests and Schaffer post-hoc analysis. Results succeed in obtaining non-dominated solutions in both problems, and allow to extract useful conclusions about parameter setting and algorithm selection.

Keywords: *MOEA, NSGAII, SPEA2, SMS-EMOA, logistics, industrial.*

1. Introduction

This is a research article written as summary of the Final Master Thesis entitled "Aplicación de algoritmos evolutivos de optimización multi-objetivo para la toma de decisiones informadas en problemas complejos del ámbito logístico industrial" [2] submitted to fulfil the requirements for completion of the master's degree "Master Universitario en Sistemas Inteligentes y Aplicaciones Numéricas en Ingeniería" at the Instituto Universitario de Sistemas Inteligentes y Aplicaciones Numéricas en Ingeniería (SIANI) [3], of the Universidad de Las Palmas de Gran Canaria [4], which has been tutorized by Dr. David Greiner.

Logistics and industrial problems often involve mixed variable types such as integer and floating point, along with highly non-linear constrains and non-convex functions. Moreover, it is common to have functions with many variables, even hundreds in some cases, as the ones presented in this study. Classical optimization algorithms have trouble dealing with such problems, specially when there are complex objective functions and the optimization involves a tradeoff between the objectives. In this context, multi-objective evolutionary algorithms (MOEAs) come into play, they can handle such problems and constraints and deal elegantly with mixed variable types. Moreover, the use of MOEAs produce a set of solutions, which is an advantage since the decision maker will have an array of posilibilities from which to choose and experiment based on different criteria.

The algorithms for the study have been choosen so that at least two different MOEAs paradigms are used, both the indicator based and the Pareto based. An introduction to such paradigms and MOEAs in general can be found at [5] of Michael T. M. Emmerich and Andre H. Deutz.

1.1. Pareto based MOEAs

Algorithms based on this paradigm aim to achieve two goals, find solutions as close as possible to the Pareto front through the employment of the Pareto dominance criteria and, achieve a uniform distribution of solutions along the best set of solutions which form the best front found. The algorithms NSGAII proposed by K. Deb et al. in [6] and SPEA2 by Zitzler and Thiele in [7] will be used from this paradigm in the present study. For instance, the NSGAII algorithm uses a two-ranking scheme, the first one uses the Pareto dominance criteria and the second one, classifies the population based on the contribution of an individual to the population diversity.

1.2. Indicator based MOEAs

Indicator based algorithms use a metric to measure the quality of the population. For example, the hypervolume metric [8]. The algorithm SMS-EMOA [9] proposed by *Emmerich et al.* will be used in this study from this paradigm.

2. Methodology

The methodology followed by this paper is inspired in [10]. Nine parameter configurations are experimentally tested to obtain the best one for each algorithm in the range studied. Then, the best configurations of each algorithm are compared through statistical tests in order to find the best performing algorithm for each problem. The testing methodology used is as the outlayed in [11], wich is specially useful for comparing multiple algorithms over different sets of data and is primarily based on Friedman's tests. All the input data used can be found at the repository [12] along with some additional useful resources for the reproducibility of the experiment.

One thousand eighty (1080) optimizations were run, some of which took more than one day to finish. The large number of executions was necessary to achieve both statistical significance and a wide range of parameter configurations. Both mutation rate and population size were varied. The mutation rates were calculated by adding and substrating 50% from the recommended value, calculated as suggested in [13] as the inverse of the number of genes in the chromosome of an individual. On the other hand, populations sizes of 50, 100 and 150 were used. The cartesian product of mutation rates and populations sizes gives nine parameter configurations for each algorithm. The number of executions is calculated as the multiplication of the number of algorithms, problems, mutations rates, population sizes and executions:

$$3 \cdot 2 \cdot 3 \cdot 3 \cdot 20 = 1080 \quad optimizations$$

One million fifty thousand (1050000) functions evaluations were used for each execution. In every case the uniform binary crossover and uniform mutation operators were used. Moreover, the chromosomes of the individuals in both cases were represented by a binary string, in which a one indicates that the element referenced by that index must be included in the set of solutions. This binary encoding is specially useful in the problems at hand, since it allows to eliminate one restriction from both of them, because every element can only be included once. By employing binary encoding and each bit representing one element, the possibility of the same element being included more than once in a solution is eliminated, and thus both the mathematical and computational complexity of the problems reduced.

In order to obtain the best configuration and algorithm for each problem in the range of parameters studied, statistical tests are used, specifically, Friedman test. The null hypothesis is that all the parameter configurations produce equal results, so when it is rejected, empirical evidence is found and post-hoc test (particularly here, Schaffer approach is used) are performed to establish which parameters/methods have statistical significant differences among them.

3. Problems definition

3.1. Blocks

The first objective is to maximize the sales of the sets of blocks choosen. The function is calculated as the sum of the sales of all the products contained in the blocks. The second objective is to minimize the number of blocks. On the other hand, a restriction of equality must be met, since exactly *M* products must be choosen. The problem is framed as a knapsack problem [14], in which the value of an object is the sales of the block, and the weigth, its number of products.

$$\sum_{i=1}^{N} E_{B_i} x_i = M = 50$$

$$x_i \in \{0,1\}; \quad 0 < E_{B_i} < 11; \quad N > 0; \quad V_i >= 0$$

F_1, F_2**:** Objective functions.

V: Amount of sales for the block.

E_B**:** Number of products in a block.

x_i**:** Value of the binary digit in the position i within the chromosome of an individual

N: Number of blocks to consider in the study.

M: Number of total products included in the choosen blocks.

I: Binary string of N digits. Chromosome of an individual.

i: Index that points to an specific block.

3.2. Stores

It is desired to find the set of near optimal coordinates in order to place new stores, according to the sales criteria which must be maximized while keeping a minimun distance between stores. Data of online sales was available for the country in which the stores needed to be opened. Such data contained postal codes and worth of sales for more than eigth thousand coordinates. In order to build an objective function, the sales in a radius of 10 Km were assigned to each location, so every postal code had an estimate value of total sales if a new store were placed in it. For the second objective, the total distance between stores is calculated as the sum of the distances between all stores.

There is one main restriction, the stores must be separated by at least 20 Km between each other, both within new and old ones, that is, there must not be a pair of stores that have a distance smaller than 20 Km. The Haversine distance is used for measurements, since all the stores are located whithin a radius of 1000 Km, the error incurred due to the deviation of the earth from a perfect sphere is acceptable.

$$maxF_2 = \sum_{i=1}^{N} x_i \sum_{j=i+1}^{N} Haversine(i,j)$$

$$Haversine(i,j) =$$

$$2r \cdot \sqrt{sin^2 \tfrac{\phi_i - \phi_j}{2} + cos(\phi_i)cos(\phi_j)sin^2 \tfrac{\lambda_i - \lambda_j}{2}}$$

$$Haversine(i,j) > D = 20 \quad \forall \quad i \neq j$$

$$\sum_{i=1}^{N} I_i = M = 15$$

$$x_i \in \{0,1\}; \quad N = 350; \quad V_i > 0$$

F_1, F_2**:** Objective functions.

V: Calculated value of sales.

N: Number of locations to consider in the study.

M: Number of new locations to choose.

D: Minimun distance between locations.

ϕ**:** Latitude.

λ**:** Longitude.

r: Sphere's radius (Earth's radius).

I: Binary string of N digits. Chromosome of an individual.

x_i**:** Value of bit *i* in the chromosome of an individual.

i,j: Indices of specific bits inside chromosome.

The objective function that measures the distance is calculated as the sum of all the distances between stores.

$$F_2 = \sum_{i=1}^{N} x_i \sum_{j=i+1}^{N} Haversine(i,j)$$

The number of distances is thus:

$$Distances = (N-1) + (N-2) + ... + 1 =$$

$$= 14 + 13 + 12 + ... + 1 = \frac{(1+14)14}{2} = 105$$

This result comes from an arithmetic sequence given by the number of distances calculated, which is a function of the number of elements to be choosen for the final set of solutions.

4. Results

Detailed comparisons and statistical analysis of parameters for each of the three methods are available in the full Masther Thesis document. Due to space constraints, here only the final comparison among methods is shown, in addition to the accumulated final non-dominated solutions. For each of the three tested evolutionary multi-objective algorithms, the best ordered parameter combination in the Friedman test has been selected for the following final comparisons.

The best parameter configuration was found for each algorithm and problem, then, the algorithms are compared. Moreover, the best set of solutions obtained (best front) is calculated by taking the non-dominated solutions from all the runs of all the algorithms and configurations for each problem. In the problems at hand, a lower value in the Friedman test ranking means better results.

4.1. Blocks problem

Algorithm	Ranking
SPEA2-150pz15mr	**1.60000**
NSGAII-150pz15mr	1.85000
SMS-EMOA-150pz15mr	2.54999

Figure 1. Blocks, Friedman test ranking of the best parameter configuration for each algorithm.

The calculated p-value of the Friedman test is 0.00783. Due to the magnitude of such value the null hypothesis can be rejected and thus, the results produced by the algorithms are not equal. The Friedman ranking (see 4.1) sugests that the best algorithm is the SPEA2, followed by the NSGAII and then the SMS-EMOA. Statistical differences were found between the pairs (SPEA2,SMS-EMOA) and (NSGAII,SMS-EMOA) according to the adjusted Shaffer p-values obtained (see 4.1), suchs results indicate the worst algorithm is the SMS-EMOA, since the other two are better positioned in the ranking and are staticaly different from SMS-EMOA. On the other hand, due the magnitude of the adjusted p-value for the pairwise comparison of (SPEA2,NSGAII), it is not possible to reject the null hypothesis for that particular case. However, since SPEA2 is the best one in the Friedman ranking, the suggested algorithm for the problem is the SPEA2 with a population size of 150 and a multiplier of the mutation rate of 1.5.

Hypothesis	p_{Shaf} adjusted
SPEA2-150pz15mr vs .SMS-EMOA-150pz15mr	0.0079893
NSGAII-150pz15mr vs .SMS-EMOA-150pz15mr	0.0268566
NSGAII-150pz15mr vs .SPEA2-150pz15mr	0.4291953

Figure 2. Blocks, pairwise comparison for each algorithm with its best parameter configuration.

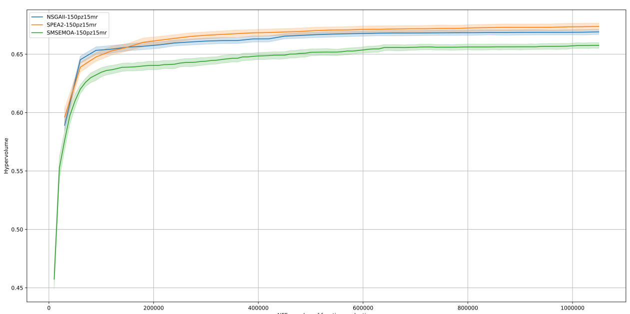

Figure 3. Bloques, hipervolumen normalizado en función del número de evaluaciones de funciones objetivo para las mejores configuraciones de parámetros.

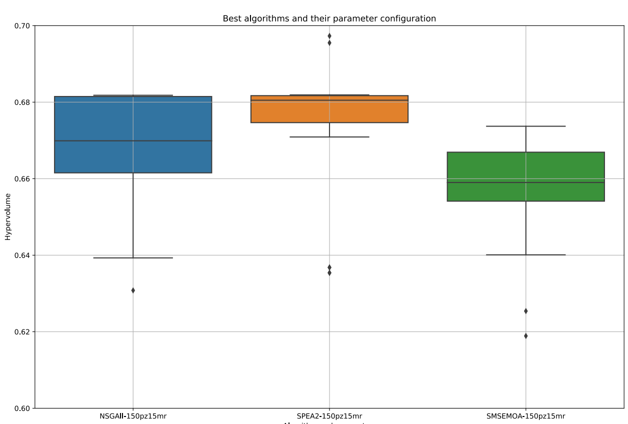

Figure 4. Blocks, hypervolume as a box plot with the best parameter configurations for each algorithm.

The best front front obtained is similar to an exponential curve of an integer variable. The specific values of each solution can be seen at the table 4.1. The results imply that in

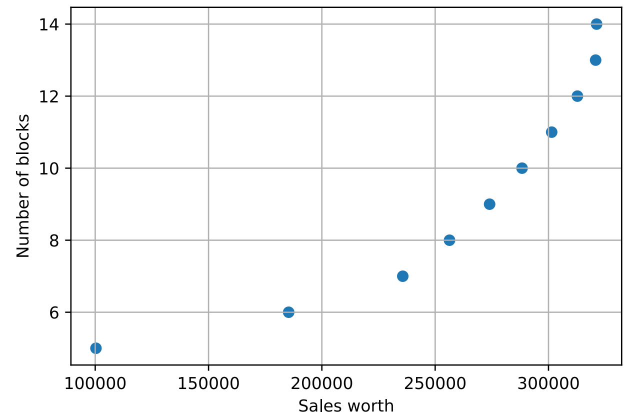

Figure 5. Blocks, best front of solutions found.

order to enhance one objective the other must be sacrified in an exponential manner and vice-versa, which is common in multi-objective optimization problems. In this problem, the values of the objective functions differ drastically between minimun and maximun values, moreover, they present large differences in their magnitudes. The individual with greatest sales has 220% $(321236/100319 = 3.202)$ more sales and 180% $(14/5 = 2.8)$ more blocks than the one with the lowest sales. The value of the sales diminishes at a greater rate when the number of blocks is lower than nine than when it is higher than nine. Notice how for the individual with the lowest sales the value of sales would be incremented by 84.7% $(185355/100319 = 1.847)$ if only one more block was added, whereas for the individual with greatest sales a substraction of one block means only a worsening of 0.1% $(321236/320805 = 1.847)$, this facts showcase again the exponential nature of the set of solutions obtained for the front.

4.2. Stores problem

The calculated p-value for the Friedman Test is 0.38674. Due to the magnitude of such value, it is not possible to reject the null hypothesis and conclude that there are statistically significant differences between the best parameter configurations of each algorithm. However, the Friedman ranking obtained (see 4.2) sugests that the best algorithm is the SPEA2, followed by the NSGAII and the SMS-EMOA. On the other hand, the best parameter configuration found for each algorithm was the same, with a population size of 150 and a a mutation rate multiplier of 1.5, such values were indeed found with statistical significance (see 4.2), so for the coordinates optimization problem this parameter configuration would be the recommended one along with the SPEA2 algorithm.

In order to analyze specific solutions, the number of calculated distances (105) will be used. The total distance is divided by 105 in order to obtain the mean distance between

Ventas	Num. Bloques	Ventas norm.	Num. B. Norm.	Identificador de los bloques
100319	5	0.832	0.0	1143 726 1121 2223 2574
185355	6	0.549	0.1	2443 2128 812 1143 726 1121
235705	7	0.381	0.2	2443 1412 2128 812 1580 358 1143
256325	8	0.312	0.3	2443 1412 2128 812 1580 358 2037 1546
274017	9	0.253	0.4	2443 1412 2128 812 1580 358 2037 2321 1850
288333	10	0.206	0.5	2443 1412 2128 812 1580 358 2037 861 1850 96
301410	11	0.162	0.6	2443 1412 2128 812 358 2037 2321 861 1850 2306 96
312773	12	0.124	0.7	2443 1412 2128 358 2037 1546 2321 861 1850 2306 96 2335
320805	13	0.0969	0.8	2443 1412 2128 812 2037 2321 861 1850 2254 2306 96 2335 2359
321236	14	0.096	0.9	2443 1412 2128 2037 2321 861 1850 2254 2306 96 2335 2503 2359 1679

Figure 6. Bloques, mejor frente de soluciones no-dominadas: Tabla con algunos individuos selectos.

Algorithm	Ranking
SPEA2-150pz15mr	**1.7500**
NSGAII-150pz15mr	2.1000
SMS-EMOA-50pz15mr	2.1500

Figure 7. Stores, pairwise comparison for each algorithm with its best parameter configuration.

stores. If one looks at the front found (see 11) there is a tradeoff between the value of the sales and the distances of the stores. The extreme points in the front have the minimun and maximun found values for both sales and distances. Notice how the value of the best sales represents an increment of 72% ($1416948/820510 = 1.7269$) with respect to the worst, whereas the worst distance represents and increment of 64% ($42956/26194 = 1.6399$) with respect to the best. On the other hand, for individuals that are in the middle of the front the sales increases rapidly without worsening too much the distances. For instance, an individual with approximate sales of 1.2e6 and a distance of 4e3 is located approximately in the middle of the front, such as the one (1238562,39420), this element has increased sales of 51% ($1238562/820510 = 1.5095$) and worsened distance of 9% ($42956/39420 = 1.0897$) with respect to the worst individual in sales. From the best set of solutions, it is possible to see how extreme variations are produced in objective functions near the extreme of the front, whereas in middle the variations are smaller. There are also high rates of change for the distances, for example, the individual (1319947,37458) differs only 7% ($1416948/1319947 = 1.0734$) in sales with respect to the best one, whereas the difference in distance is 43% ($42956/37458 = 1.43$). This analysis showcases the nature of the front, in which for some parts the rate of change of the values of the objective functions is small (center of the front), whereas in other zones the rate of change is high (near the extremes).

Hypothesis	p_{Shaf} adjusted
SPEA2-150pz15mr vs .SMS-EMOA-50pz15mr	0.617709
NSGAII-150pz15mr vs .SPEA2-150pz15mr	0.617605
NSGAII-150pz15mr vs .SMS-EMOA-50pz15mr	0.874367

Figure 8. Stores, Friedman test ranking of the best parameter configuration for each algorithm.

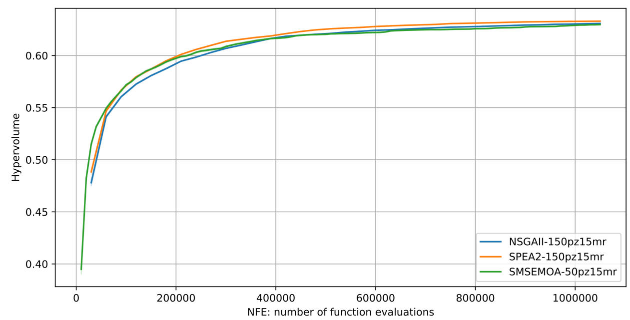

Figure 9. Tiendas, hipervolumen normalizado en función del número de evaluaciones de funciones objetivo para las mejores configuraciones de parámetros.

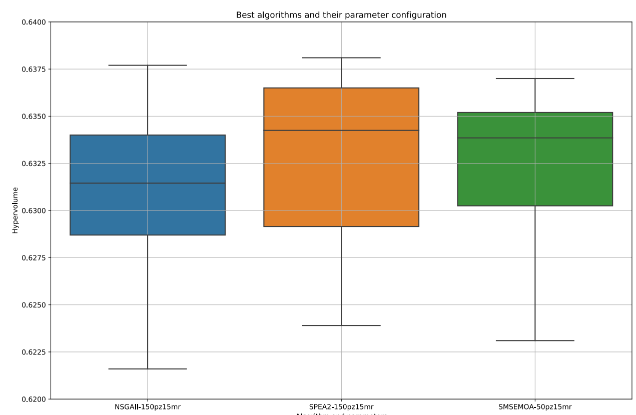

Figure 10. Stores, hypervolume as a box plot with the best parameter configurations for each algorithm.

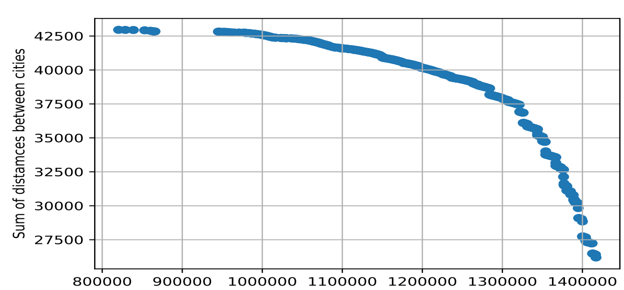

Figure 11. Stores, best front of solutions found.

5. Conclusions and perspectives

Two complex multi-objective problems of mixed variable type and highly non-linear constraints were solved through three different state-of-the-art MOEAs, and two different paradigms, Pareto and indicator based. In each problem the best parameter configuration was found for the range studied for each algorithm and then, the best algorithm is choosen based on statistical significance tests. Finally, some specific solutions were analysed in the context of each problem.

In almost every configuration of both problems, statistical evidence was found that indicates that higher mutation rates and population sizes lead to better solutions in the range of parameters studied. The facts seem to indicate that for the problems studied the balance between exploration and exploitation is inclined towards exploration. In future works it would be reasonable to increase the higher end of the parameters studied to search for better solutions.

Even though statistical test could not find a best algorithm for the stores problem, the solutions of each algorithm converge towards a similar set of postal codes, which adds confidence to the final election of locations, because very similar solutions were found with three different algorithms with different paradigms.

The normalization of the objetives is fundamental in the optimization process, specially when dealing with different orders of magnitude, because one of the functions might dominate the other if normalization is not applied.

Given the results obtained, some future lines of work arise. It would be beneficial to study different mutation operators and perform more executions per configuration to search for statistical differences whithin the algorithms with new parameters. Other algorithms and paradigms could also be included from the decomposition paradigm such as MOEA/D [15] or NSGAIII [16]. Additionally, it could be useful to vary the objetive function that quantifies minimun distances in the locations problem to search for new geographical distribution of stores. Overall, the present study solved two complex optimizations problems of highly non-linear constraints with state-of-the-art MOEAs and found uniformly distributed solutions along the front, so business decisions could be made from a wide range of posibilities by transforming single-objective problems into multi-objective ones.

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