

Image Compression Using Singular Value Decomposition

Gowri.P¹, Senbaga Priya.K², Hari Prasath.R.K³, Pavithra.S⁴

¹Assistant Professor, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore-641008, TamilNadu, India.

^{2,3,4}PG Scholar, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore-641008, TamilNadu, India.

Abstract - In this modern era, everyone is fond of taking photos and selfies. But, they look for their storage space running out and the speed of their device getting reduced. In that case, we have many algorithm based on image compression techniques. Here we are to discuss how Linear algebra plays its role in the compression of images. Basically, we will be discussing how SVD is extensively used in image compression process resulting in saving computer's memory. The main technique used in this paper is SVD algorithm and is implemented using MATLAB. Using Singular value decomposition, JPEG file is compressed and the compression ratio is obtained. We apply linear algebra (SVD) on this matrix and get a reduced matrix out of the original matrix and the image that corresponds to the reduced matrix requires much lesser storage space compared to the original image.

KEYWORDS - Image compression, singular value decomposition, techniques involved, JPEG image-compression ratio, image coding

I. INTRODUCTION

With rapid demand of storing large amount of data, the space to store this data (in the form of files) in the hard drives of the computer systems or onto the servers of large companies is getting less compared to the amount of data that is to be stored. As a result of it, various compression techniques are in demand which can help to reduce the size of data files.

Now-a-days, images play a vital role in capturing a high resolution picture with better quality. Image compression has an impact in the field of digital image processing, which deals with a technique to reduce storage required for transmitting it. High resolution images require larger bandwidth and storage for transmission. Thus compressed images results in the reduction of storage space and increased transmission rate. A typical image can be reduced to 80% of its original image. It has its own application in various fields like medical imaging, galleries, telecommunication, security etc.,

This paper focuses on Singular Value Decomposition which involves factorizing matrices into a series of linear approximations that expose the underlying structure of matrices. SVD is a dominant idea that algebra possess. SVD decomposes any $m \times n$ rectangular matrix into three elements such that $A = U \cdot S \cdot V^T$

II. SINGULAR VALUE DECOMPOSITION

Singular value decomposition is a matrix factorization where any arbitrary matrix A is factored into $A = USV^T$. Where the Eigen vectors of AA^T and $A^T A$ is the columns of U and V , V^T is the conjugate transpose of V and S is the diagonal matrix of a non-zero Eigen values of both AA^T and $A^T A$ of a rank r . Here, A is a $m \times n$ matrix, U is a $m \times m$ matrix, V is a $n \times n$ matrix and S is a $m \times n$ matrix.

SVD performs better than DCT (Discrete cosine transform) when the images have high standard deviation (i.e., higher pixel quality). Singular value decomposition is linear in data transformation that is used for compressing images and the image matrix is represented as the product of these matrices $A = USV^T$. It expresses image data in terms of number of Eigen vectors depending upon the dimension of an image and the mainly Psycho-visual redundancies in an image are used for compression. Image can be compressed without affecting the quality of the image and thus reduces the storage space.

The Most used technique is JPEG and SVD gives high energy compression and optimal energy compaction. In SVD the image is decomposed into three elements by the Eigen values of the image.

III. IMAGE COMPRESSION USING SVD

Image compression is minimizing the size in bytes of a graphic file without degrading the quality of the image to an unacceptable level. This reduction in file size allows more images to be stored in a harddisk or memory space. This also reduces the time required for images sent over the Internet or downloaded from Web pages. An image file can be compressed in many ways and one such way to compress a JPEG image file is through Singular Value Decomposition. This JPEG method is commonly used for photographs, while GIF method is used for line art and other images in which geometric shapes are relatively simple.

A $m \times n$ pixels image can be represented by $m \times n$ matrix representation. Suppose we have 9 megapixel gray-scale images, which is 3000×3000 pixels (a 3000×3000 matrix). For each pixel, we have some level of black and white colour, given by some integer between 0 and 255. 0 represents the black colour and 255 represent the white colour. Each of the integers (and hence each pixel) require approximately 1 byte to store, resulting in an approximately 8.6 Mb image. A colour image consists of three components, a red, a green, and a blue (RGB). Each of these is represented by a matrix; hence storing colour images requires three times the space (25.8 Mb).

IV. APPLICATIONS OF SVD

1. SVDs application in world of image and data compression are very useful and resource saving.
2. SVD allows us to arrange the portions of a matrix in order of importance. The most important singular values will produce the important unit eigen vectors.
3. We can eliminate large portions of our matrix without losing quality.
4. Therefore, an optimum value of k must be chosen, with an acceptable error, which conveys most of the information contained in the original image, and has an acceptable file size too.
5. Image compression alone can experience redundancies which can be overcome by singular value decomposition (SVD) of the image matrix. Image compression and coding techniques explore three types of redundancies namely...
 - Coding redundancy,
 - Inter-pixel redundancy,
 - Psycho-visual redundancy.

V. PROBLEMS ON SVD

Consider a matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ from an image displayed as pixel values and compute the singular value decomposition.

SOLUTION:

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

Subtracting λI from A

$$|A - \lambda I| = 0$$

$$\left| \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$(\lambda - 6)(\lambda - 1) = 0$$

$$\lambda = 6; \lambda = 1$$

Therefore our eigenvalues are 6 and 1. We construct the matrix S^2 by placing the eigenvalues along the main diagonal in decreasing order.

$$S^2 = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, taking the square root of matrix S2 gives,

$$S = \begin{bmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{bmatrix}$$

Now we need to find the eigenvectors of A which are the columns of V. First we will show where $\lambda = 6$;

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$-x + 4y = 0 \rightarrow 1$$

$$x - 4y = 0 \rightarrow 2$$

on solving we get,

$$x = 4y; x = 4; y = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Since V has an orthonormal basis, \vec{v}_1 needs to be of length one. We divide \vec{v}_1 by its magnitude to accomplish this. Thus,

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 0.2 \end{pmatrix}$$

Similarly for $\lambda = 1$

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$4x + 4y = 0 \rightarrow 3$$

$$x + y = 0 \rightarrow 4$$

on solving we get,

$$x = -y; x = -1; y = 1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Since V has an orthonormal basis, \vec{v}_2 needs to be of length one. We divide \vec{v}_2 by its magnitude to accomplish this. Thus,

$$\vec{v}_2 = \begin{pmatrix} -0.7 \\ 0.7 \end{pmatrix}$$

Now we need to construct the augmented orthogonal matrix V ,

$$V = (\vec{v_1} \quad \vec{v_2})$$

$$V = \begin{pmatrix} 1 & -0.7 \\ 0.2 & 0.7 \end{pmatrix}$$

Now we need to find the eigenvectors for A .Finding the eigenvectors using the eigen values previously found.
First we show where $\lambda = 6$;

$$\left(\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$x = -5; y = 0$$

$$\vec{u_1} = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$$

$$\left(\begin{pmatrix} 6 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$\begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$x = 0; y = 5$$

$$\vec{u_2} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

$$U = (\vec{u_1} \quad \vec{u_2})$$

$$U = \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix}$$

$$A = U * S * V^T$$

$$= \begin{pmatrix} -5 & 0 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} \sqrt{6} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.2 \\ -0.7 & 0.7 \end{pmatrix}$$

$$A = \begin{pmatrix} -12 & -2.4 \\ -3.5 & 3.5 \end{pmatrix}$$

VI. LAB VIEW IMPLEMENTATION USING SVD



Fig- 1. Colour and grayscale image

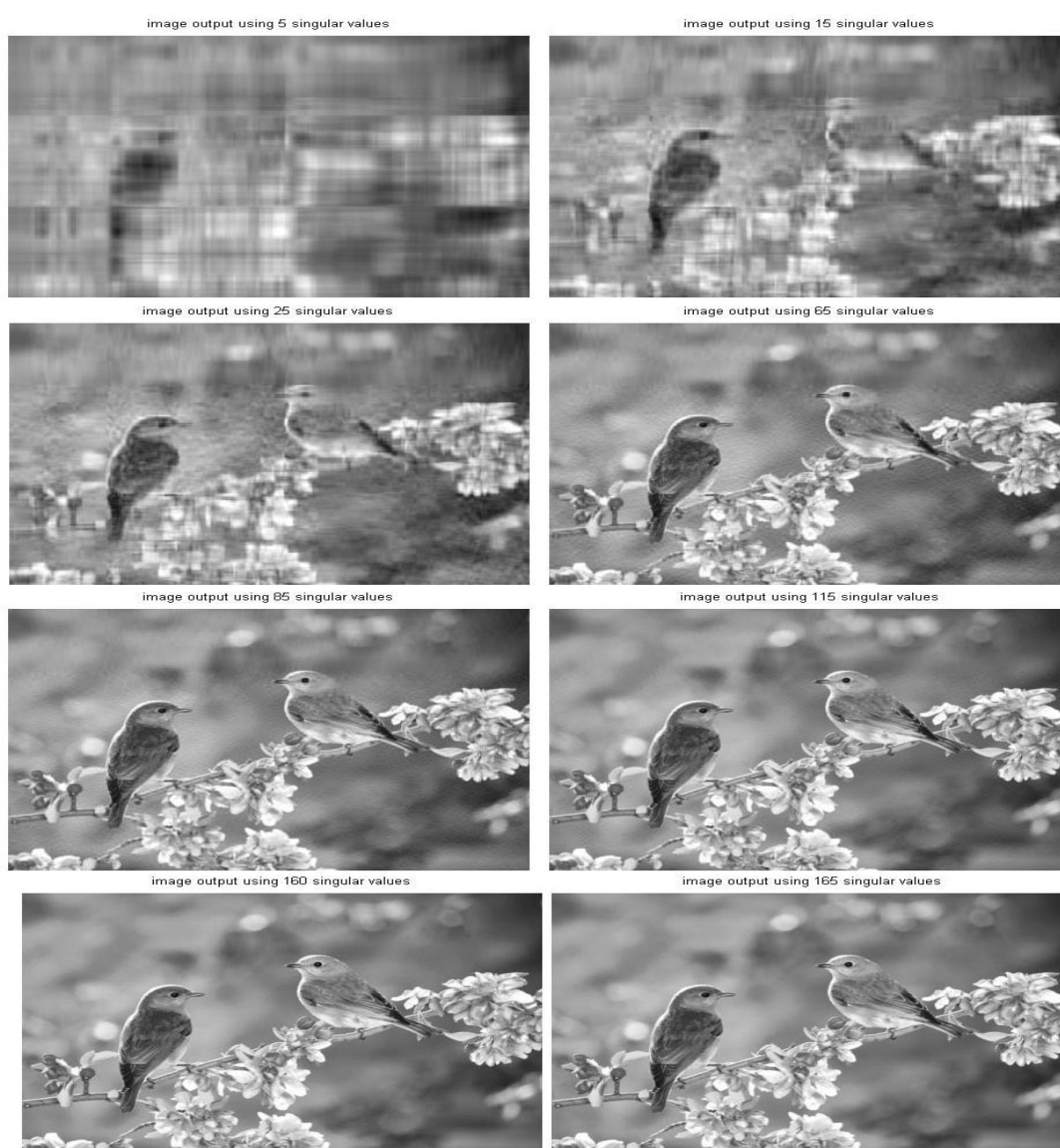


Fig-2 . Images formed in MATLAB

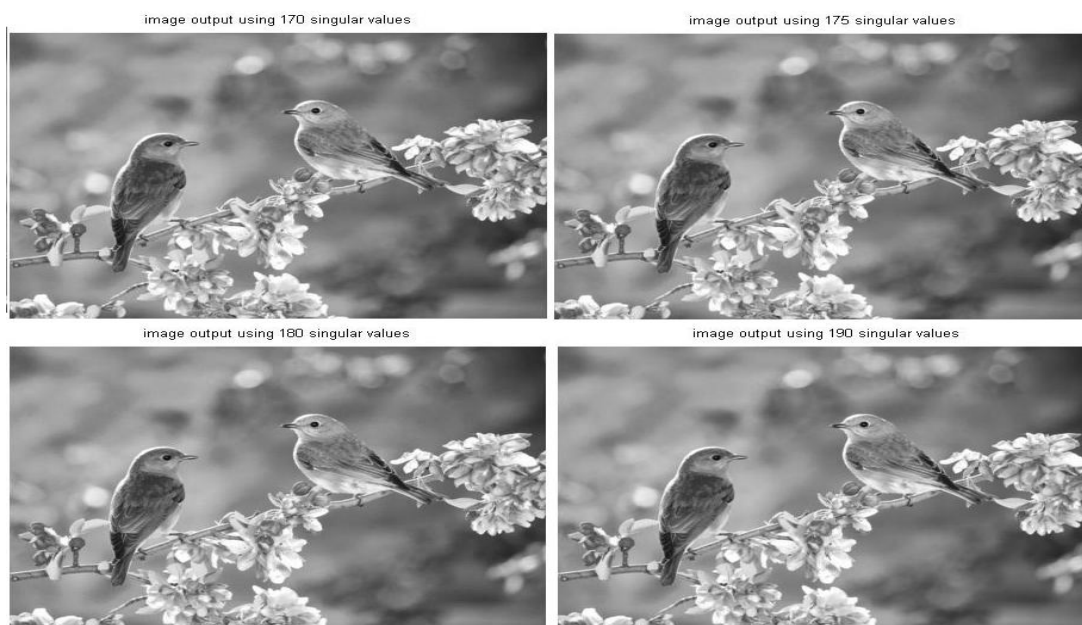


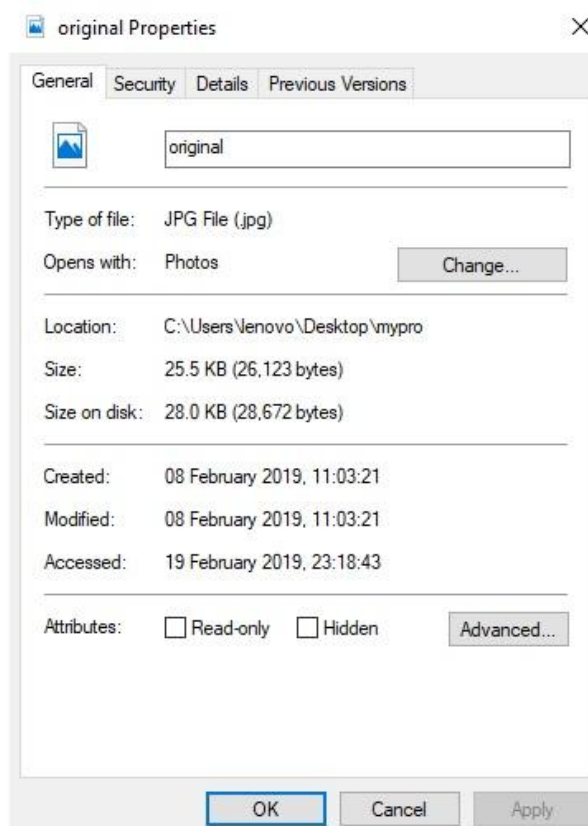
Fig-3 . Images formed in MATLAB

VII. RESULTS AND DISCUSSION



Original image

Fig- 4. File size of an original image



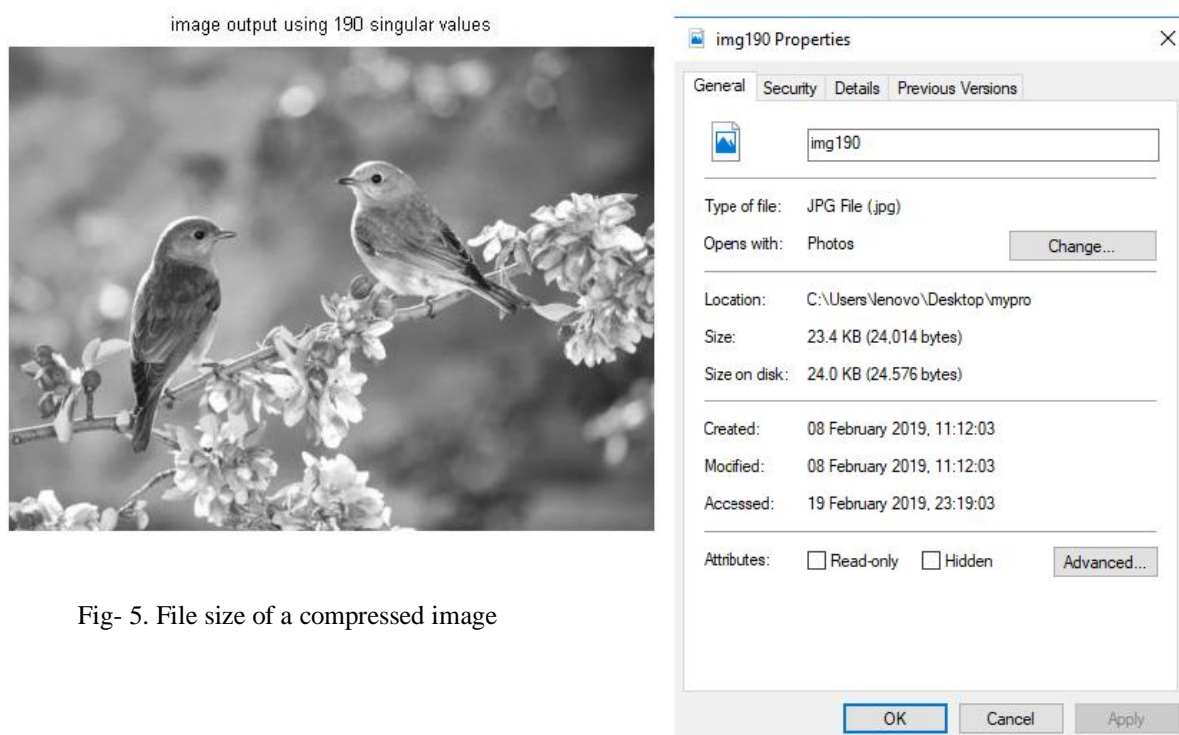


Fig- 5. File size of a compressed image

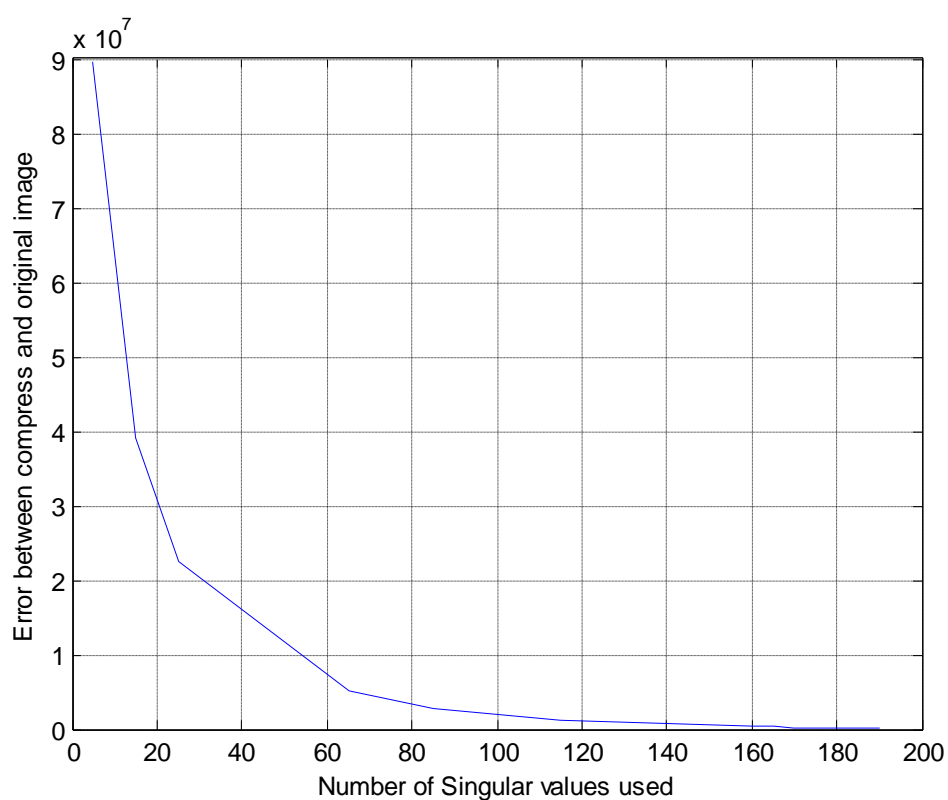


Fig-6. Graphical representation using MATLAB

VIII. CONCLUSION

Using SVD, we can save lot of memory which can be used for other purposes. From personal computers to servers of large companies, we can save memory using SVD. This paper establishes a lab view framework for image compression using MATLAB software. The compression ratio varies with respect to the number of singular values considered. The size of the original image is 25.5KB where, the size of the compressed image is 23.4KB which is comparatively less compared to the original image.

Thus, it is observed that SVD gives good compression results compared to other compression techniques. The degree of compression varies with the value of k (No. of singular values). Through this image compression using Singular value decomposition, we can reduce the size of the image without the loss of clarity and can save memory.

REFERENCES

- [1] K. Mounika, D. Sri Navya Lakshmi, K. Alekya; "SVD Based Image Compression", International Journal of Engineering Research and General Science Volume 3, Issue 2, March-April, 2015: ISSN 2091-2730
- [2] H.S. Prasantha , H.L. Shashidhara , K.N. Balasubramanya Murthy; "Image compression using SVD", International conference on computational intelligence and multimedia applications (ICCIMA-2007) volume:3
- [3] B.Anusha, Y.Soujanya, E.Vyshnavi, T.Harish, S.Teja; "Image Compression Using SVD ON Labview With Vision Module", International Journal of Computational Intelligence Research, ISSN 0973-1873 Volume 14, Number 1 (2018), pp. 59-68
- [4] Henri Bruno Razafindradina, Paul Auguste Randriamitantsoa, Nicolas Raft Razafindrakoto; "Image Compression with SVD", A New Quality Metric Based On Energy Ratio; IJCSN International Journal of Computer Science and Network, Volume 5, Issue 6, December 2016
- [5] Brady Mathews, "Image Compression using Singular Value Decomposition (SVD)", 12 December 2014, The University of Utah
- [6] Sunny Verma, Jakkam Phanindra Krishna; "Image Compression and Linear Algebra", November 15, 2013
- [7] Miss Samruddhi Kahu Ms. Reena Rahate; "Image Compression using Singular Value Decomposition", International Journal of Advancements in Research & Technology, Volume 2, Issue 8, August-2013 244
- [8] https://en.wikipedia.org/wiki/Image_compression
- [9] https://www.researchgate.net/publication/316682406_Tutorial_review_on_existing_image_compression_techniques
- [10] https://en.wikipedia.org/wiki/Singular_value_decomposition