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Project 2.

1. Let X be exponential with parameter $\lambda = 2$, density $f(x) = \lambda e^{-\lambda x}$, $EX = \frac{1}{\lambda}$, $VarX = \frac{1}{\lambda^2}$.

 Plot the graph of f(x) on [0,5].

Illustrate the Central Limit Theorem (CLT) for X as follows

(a) Simulate $X = X_1 + ... + X_{10}$ n = 100 times, use the (practical) range $0 \leq X \leq 20$ and bin size = 1

Graph: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [0, 20]

Label the three graphs appropriately

(b) Simulate $X = X_1 + ... + X_{30}$ n = 1000 times, use the (practical) range $0 \leq X \leq 30$ and bin size = 1

Graph: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [0, 30]

Label the three graphs appropriately

(c) Simulate $X = X_1 + ... + X_{100}$ n = 10000 times, use the (practical) range $30 \leq X \leq 70$ and bin size = 1

Graph: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [30, 70]

Label the three graphs appropriately

(c2) Do (c) with standardization and compare the frequencies with Standard Normal N(0,1) density

$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

Illustrate the Law of Large Numbers (LLN) for X exponential with parameter $\lambda = \frac{1}{3}$, $EX = \frac{1}{\lambda} = 3$ as follows:

(d) Graph : all averages $\{X_1, \frac{X_1 + X_2}{2}, ..., \frac{X_1 + ... + X_{10000}}{10000}\}$, first 50 averages, averages from 5000 to 10000

(e) Use "Do loop" to simulate 3 different samples of $\frac{X_1 + ... + X_n}{n}$ for n = 1000000. Take the average of the three runs

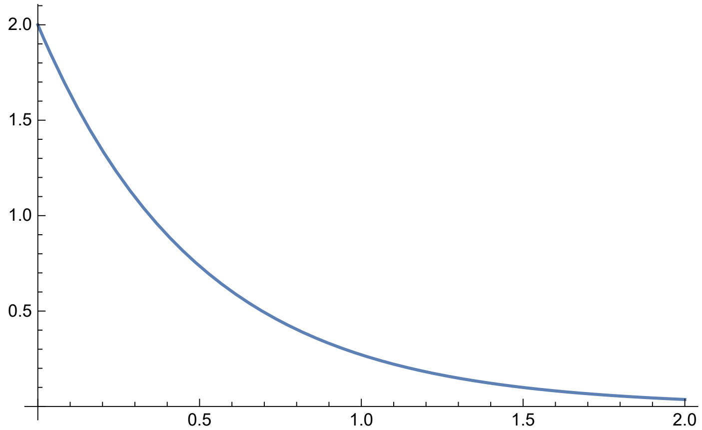
to get a better approximation to EX

(f) OPTIONAL (extra credit). If "your computer is fast enough" try (e) for $n = 10^7$ or 10^8 or even 10^9.

In[3]:=
```
<< BarCharts`
Plot[2 * Exp[-2 * x], {x, 0, 2}, PlotRange -> All]
```

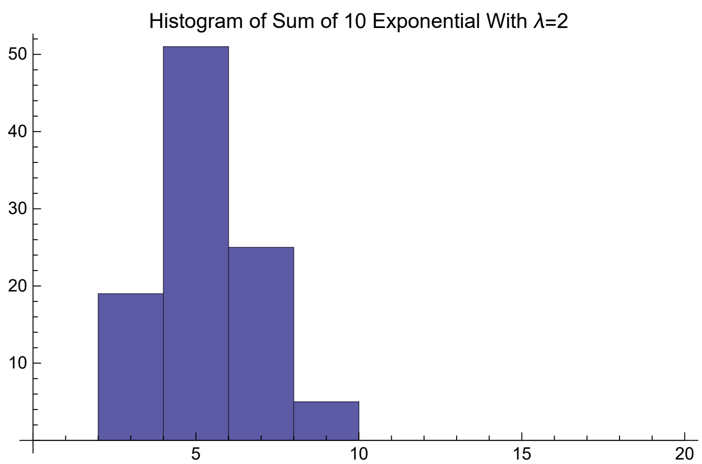
General: BarCharts` is now obsolete. The legacy version being loaded may conflict with current functionality. See the Compatibility Guide for updating information.

Out[4]=

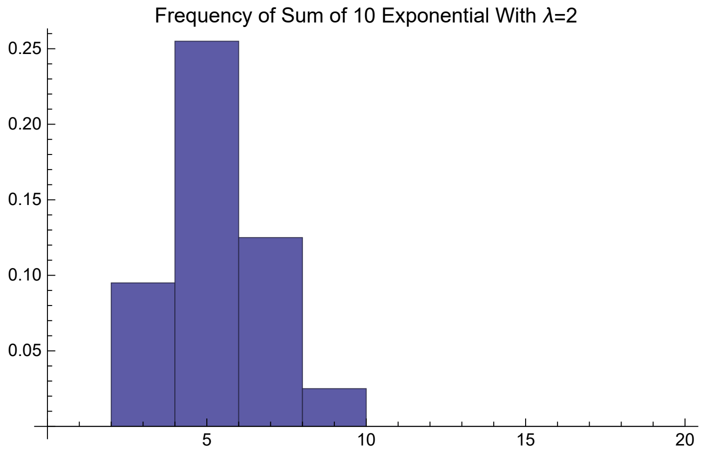


In[ ]:=
```
X := Sum[Random[ExponentialDistribution[2]], {10}]
Sample100 = Table[X, 100];
bcount = BinCounts[Sample100, {0, 20, 2}];
GeneralizedBarChart[Table[{1 + (i - 1) * 2, bcount[[i]], 2}, {i, 1, 10}],
 PlotLabel → "Histogram of Sum of 10 Exponential With λ=2"]
GeneralizedBarChart[Table[{1 + (i - 1) * 2, bcount[[i]] / (2 * 100), 2}, {i, 1, 10}],
 PlotLabel → "Frequency of Sum of 10 Exponential With λ=2"]
Show[GeneralizedBarChart[Table[{1 + (i - 1) * 2, bcount[[i]] / (2 * 100), 2}, {i, 1, 10}],
  PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / √(5 * Pi) * Exp[- (x - 5)^2 / 5], {x, 0, 10}, PlotRange → All]]
```

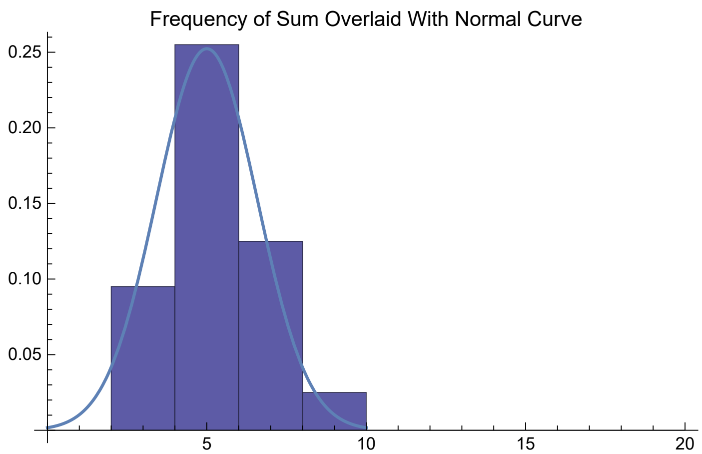
Out[ ]=



Out[ ]=



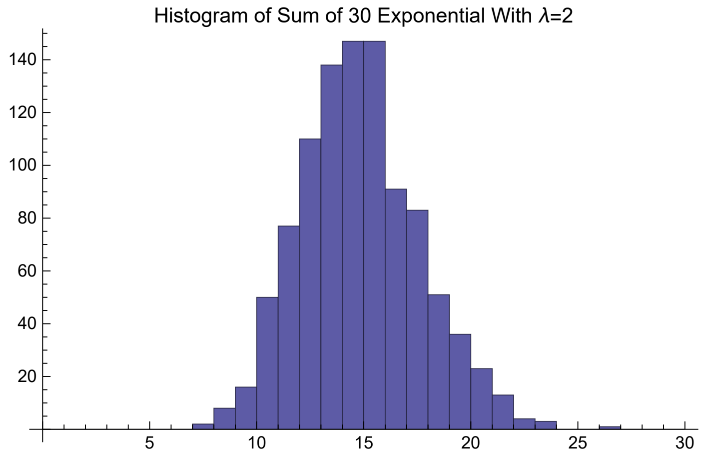
Out[ ]=



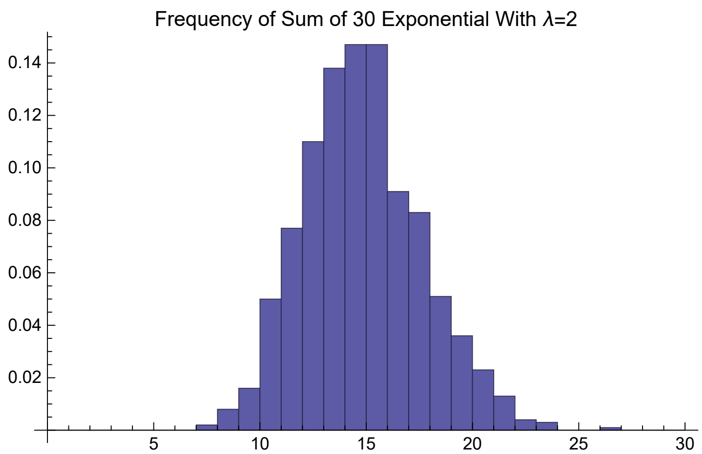
In[ ]:=

```
X := Sum[Random[ExponentialDistribution[2]], {30}]
Sample100 = Table[X, 1000];
bcount = BinCounts[Sample100, {0, 30, 1}];
GeneralizedBarChart[Table[{.5 + (i - 1), bcount[[i]], 1}, {i, 1, 30}],
 PlotLabel → "Histogram of Sum of 30 Exponential With λ=2"]
GeneralizedBarChart[Table[{.5 + (i - 1), bcount[[i]] / (1 * 1000), 1}, {i, 1, 30}],
 PlotLabel → "Frequency of Sum of 30 Exponential With λ=2"]
Show[GeneralizedBarChart[Table[{.5 + (i - 1), bcount[[i]] / (1 * 1000), 1}, {i, 1, 30}],
  PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / √(15 * Pi) * Exp[- (x - 15)^2 / 15], {x, 0, 30}, PlotRange → All]]
```

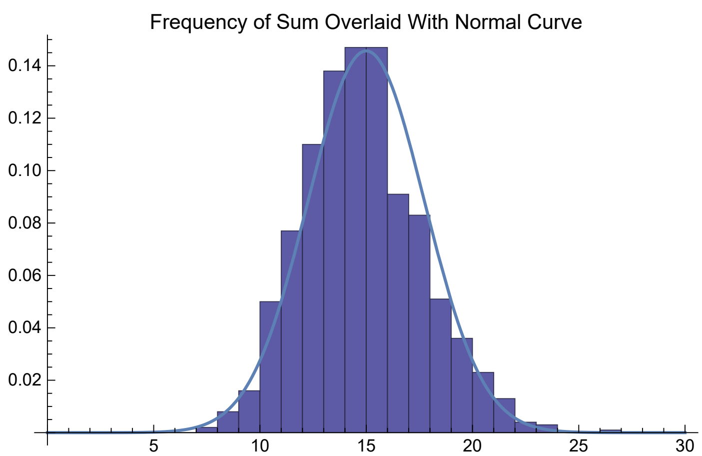
Out[ ]=


Histogram of Sum of 30 Exponential With λ=2
140
120
100
80
60
40
20
5
10
15
20
25
30

Out[ ]=


Frequency of Sum of 30 Exponential With λ=2
0.14
0.12
0.10
0.08
0.06
0.04
0.02
5
10
15
20
25
30

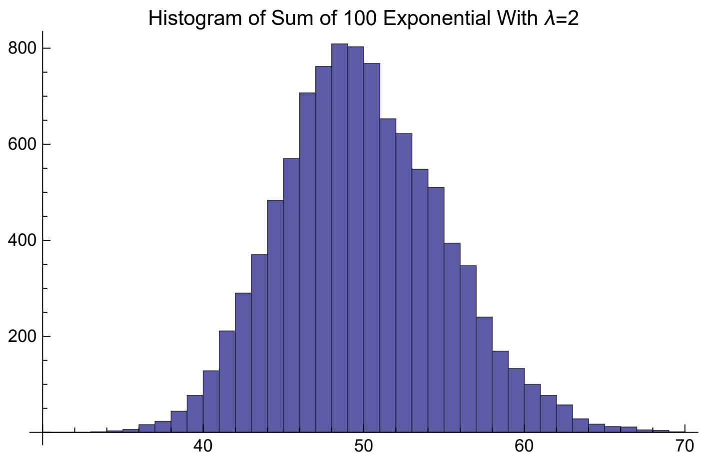
Out[ ]=


Frequency of Sum Overlaid With Normal Curve
0.14
0.12
0.10
0.08
0.06
0.04
0.02
5
10
15
20
25
30

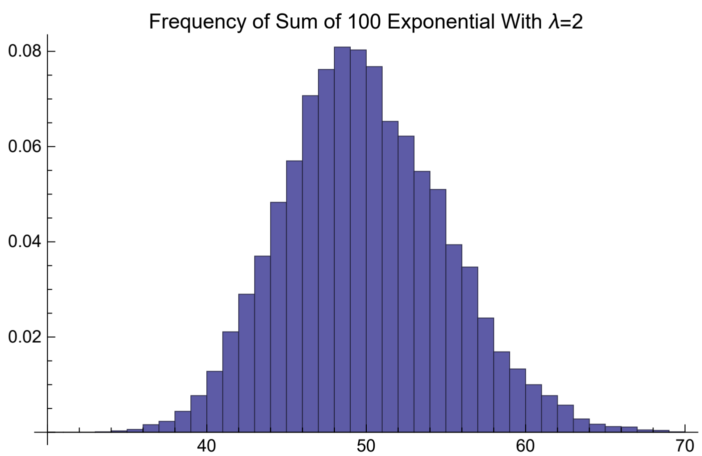
In[ ]:=

```
X := Sum[Random[ExponentialDistribution[2]], {100}]
Sample100 = Table[X, 10000];
bcount = BinCounts[Sample100, {30, 70, 1}];
GeneralizedBarChart[Table[{30.5 + (i - 1), bcount[[i]], 1}, {i, 1, 40}],
 PlotLabel → "Histogram of Sum of 100 Exponential With λ=2"]
GeneralizedBarChart[Table[{30.5 + (i - 1), bcount[[i]] / (1 * 10000), 1}, {i, 1, 40}],
 PlotLabel → "Frequency of Sum of 100 Exponential With λ=2"]
Show[GeneralizedBarChart[Table[{30.5 + (i - 1), bcount[[i]] / (1 * 10000), 1}, {i, 1, 40}],
  PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / √(50 * Pi) * Exp[- (x - 50)^2 / 50], {x, 30, 70}, PlotRange → All]]
Show[GeneralizedBarChart[Table[{-20.5 + (i - 1), bcount[[i]] / (1 * 10000), 1}, {i, 1, 40}],
  PlotLabel → "Frequency of Sum Overlaid With Standard Normal"],
 Plot[1 / √(2 * Pi) * Exp[-x^2 / 2], {x, -20, 20}, PlotRange → All]]
```

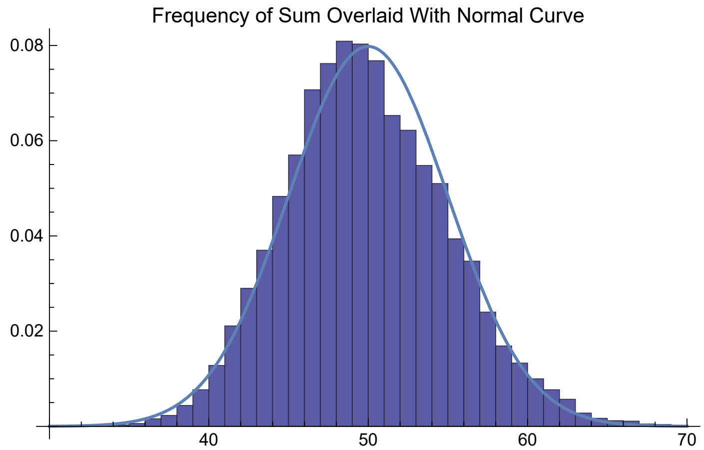
Out[ ]=



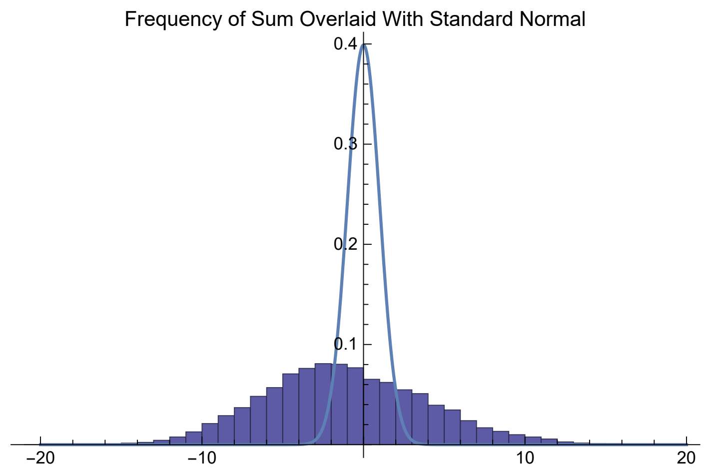
Out[ ]=



Out[ ]=



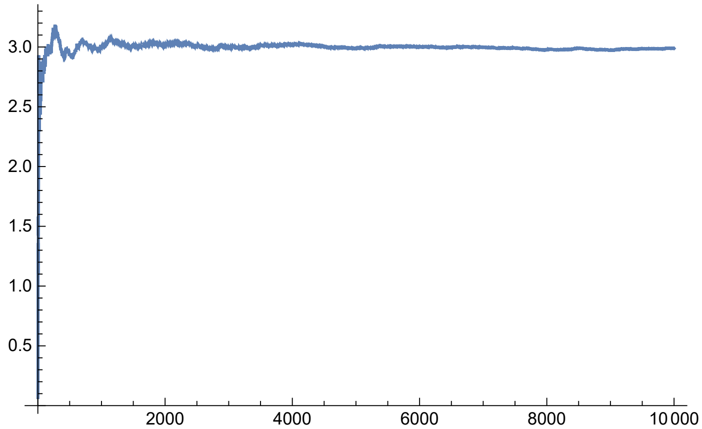
Out[ ]=



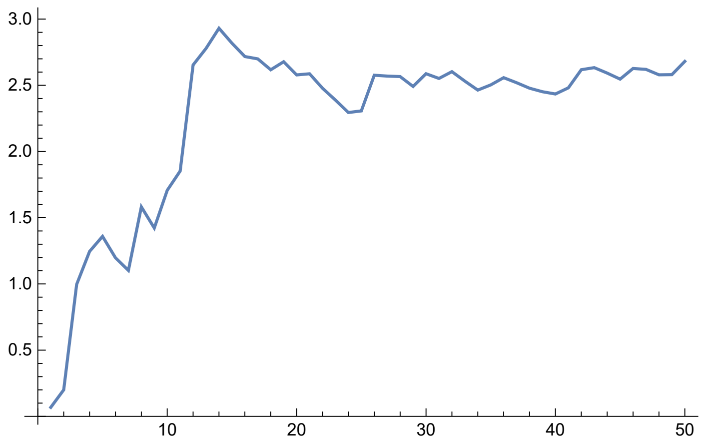
In[ ]:=

```
data = Table[Random[ExponentialDistribution[1 / 3]], 10000];
sumdata = Accumulate[data];
average = Table[sumdata[[i]] / i, {i, 1, 10000}];
ListPlot[average, Joined → True, PlotRange → All]
ListPlot[Drop[average, {51, 10000}], Joined → True, PlotRange → All]
ListPlot[Drop[average, {1, 5000}], Joined → True, PlotRange → All]
s = 0; n = 1000000; sum = 0;
For[i = 1, i < 4, i++, Do[r = Random[ExponentialDistribution[1 / 3]];
   s = s + r, {i, 1, n}];
 sum = sum + s;
 s = 0]
Print[sum / n / 3 // N]
```

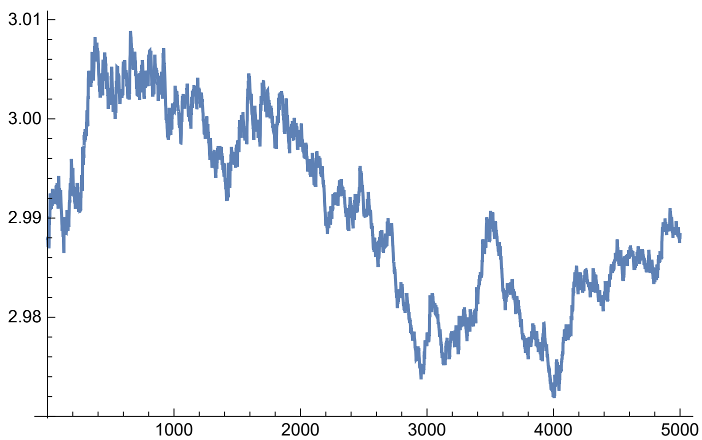
Out[∘]=



Out[∘]=



Out[∘]=



`2.99799`

2. Let X be Poisson with $\lambda = 2$, density $f(x) = \frac{\lambda^x}{x!} e^{-\lambda x}$, x = 0,1,2,..., EX = λ, VarX = λ.

 Plot the graph of f(x) on {0,1,...,15}.

Illustrate the Central Limit Theorem (CLT) for X as follows:

(a) Simulate $X = X_1 + \ldots + X_{10}$ n = 100 times, use the (practical) range $0 \le X \le 40$ and bin size = 1

Graph: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [0, 40]

Label the three graphs appropriately

(b) Simulate $X = X_1 + ... + X_{30}$ n = 1000 times, use the (practical) range $30 \le X \le 90$ and bin size = 1

Display: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [30, 90]

Label the three graphs appropriately

(c) Simulate $X = X_1 + ... + X_{100}$ n = 10000 times, use the (practical) range $150 \le X \le 250$ and bin size = 1

Graph: Histogram of bin counts, Frequencies, and combined Frequencies with the Normal curve from CLT on [150, 250]

Label the three graphs appropriately

Illustrate the Law of Large Numbers (LLN) for X Poisson with parameter $\lambda = 5$, $EX = \lambda = 5$ as follows:

(d) Graph : $\{X_1, \frac{X_1 + X_2}{2}, ..., \frac{X_1 + ... + X_{10000}}{10000}\}$, first 50 averages, averages from 5000 to 10000

(e) Use "Do loop" to simulate 3 different samples of $\frac{X_1 + ... + X_n}{n}$ for n = 1000000. Take the average of the three runs

to get a better approximation to EX

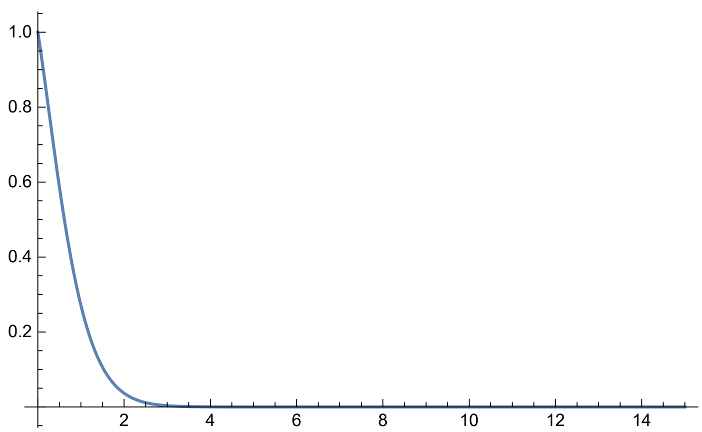
(f) OPTIONAL (extra credit). If "your computer is fast enough" try (e) for $n = 10^7$ or 10^8 or even 10^9.

(* Do loop *) s = 0; n = 1000000; Do[r = random distribution of choice ; s = s + r, {i, 1, n}]; Print[s/n//N]

In[ ]:=

```
Plot[2^x / x! * Exp[-2 * x], {x, 0, 15}, PlotRange -> All]
```

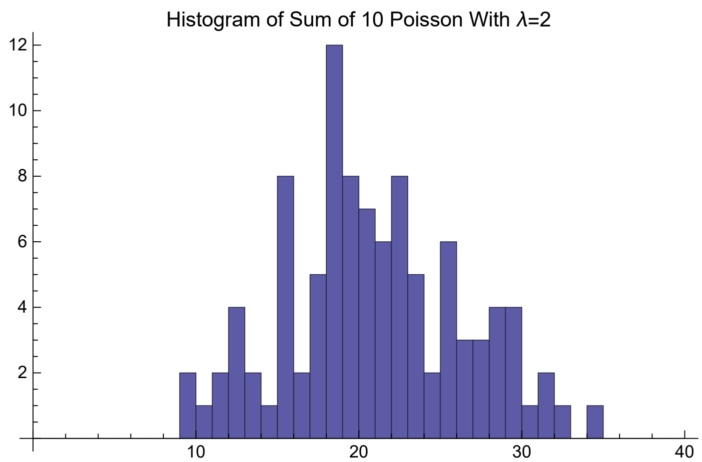
Out[ ]=



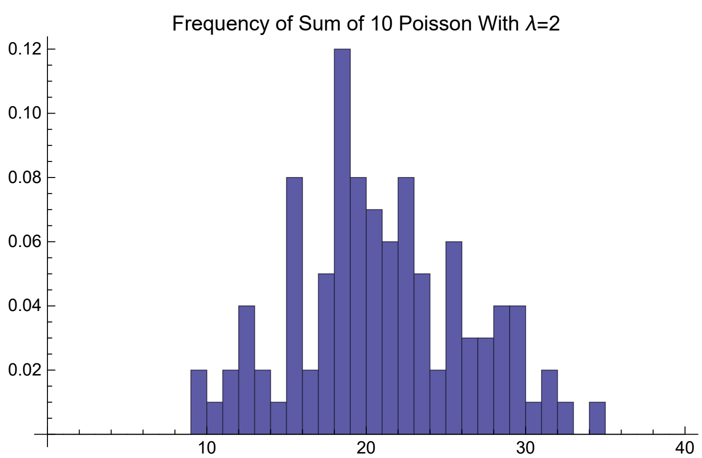
In[ ]:=

```
X := Sum[Random[PoissonDistribution[2]], {10}]
Sample100 = Table[X, 100];
bcount = BinCounts[Sample100, {0, 40, 1}];
GeneralizedBarChart[Table[{.5 + (i - 1) * 1, bcount[[i]], 1}, {i, 1, 40}],
 PlotLabel → "Histogram of Sum of 10 Poisson With λ=2"]
GeneralizedBarChart[Table[{.5 + (i - 1) * 1, bcount[[i]] / (1 * 100), 1}, {i, 1, 40}],
 PlotLabel → "Frequency of Sum of 10 Poisson With λ=2"]
Show[GeneralizedBarChart[Table[{.5 + (i - 1) * 1, bcount[[i]] / (1 * 100), 1}, {i, 1, 40}],
  PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / Sqrt[40 * Pi] * Exp[- (x - 20)^2 / 40], {x, 0, 40}, PlotRange → All]]
```

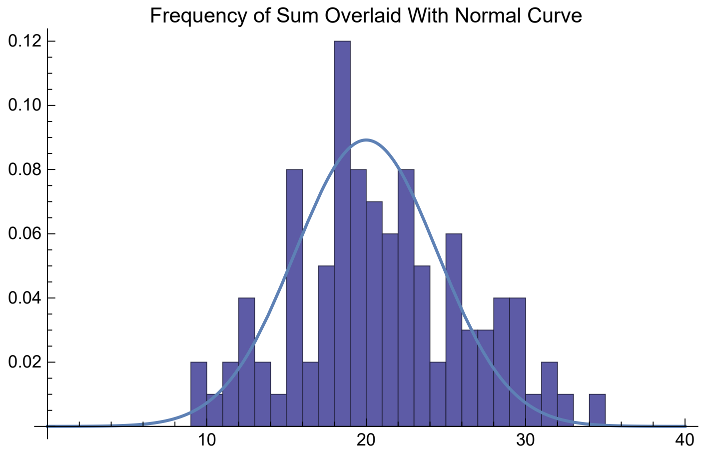
Out[ ]=



Out[ ]=



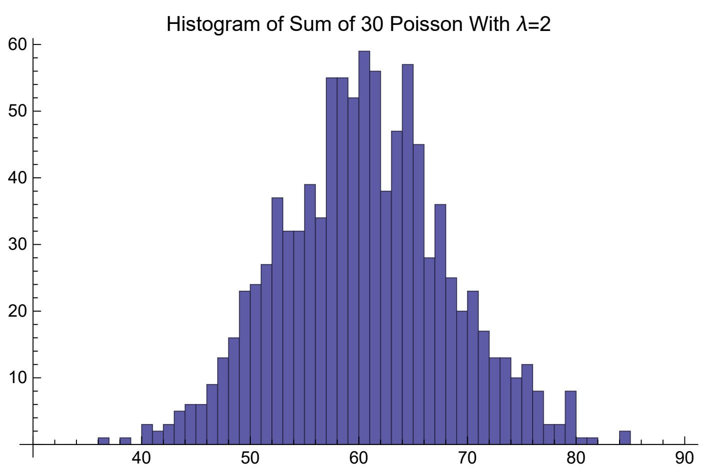
Out[ ]=



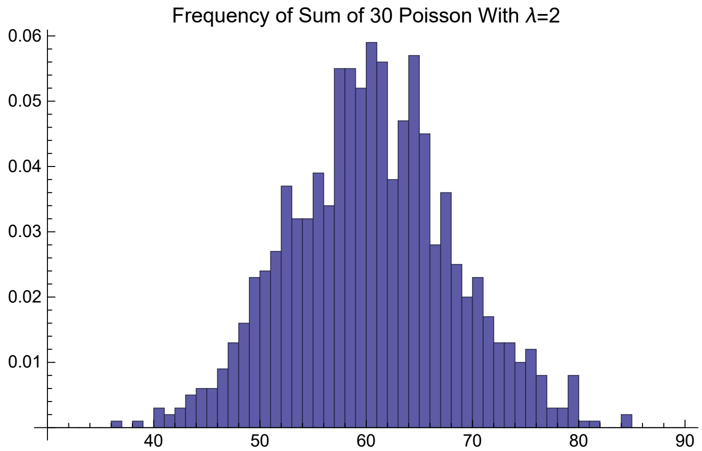
In[ ]:=

```
X := Sum[Random[PoissonDistribution[2]], {30}]
Sample100 = Table[X, 1000];
bcount = BinCounts[Sample100, {30, 90, 1}];
GeneralizedBarChart[Table[{30.5 + (i - 1) * 1, bcount[[i]], 1}, {i, 1, 60}],
 PlotLabel → "Histogram of Sum of 30 Poisson With λ=2"]
GeneralizedBarChart[Table[{30.5 + (i - 1) * 1, bcount[[i]] / (1 * 1000), 1}, {i, 1, 60}],
 PlotLabel → "Frequency of Sum of 30 Poisson With λ=2"]
Show[GeneralizedBarChart[Table[{30.5 + (i - 1) * 1, bcount[[i]] / (1 * 1000), 1}, {i, 1, 60}],
  PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / Sqrt[120 * Pi] * Exp[-(x - 60)^2 / 120], {x, 30, 90}, PlotRange → All]]
```

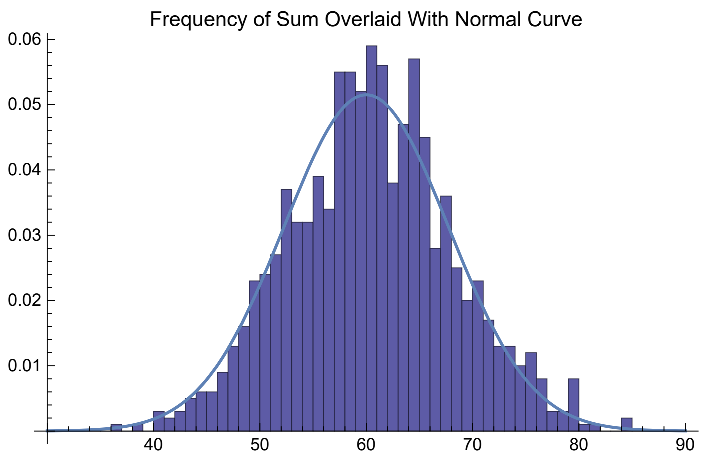
Out[ ]=



Out[ ]=



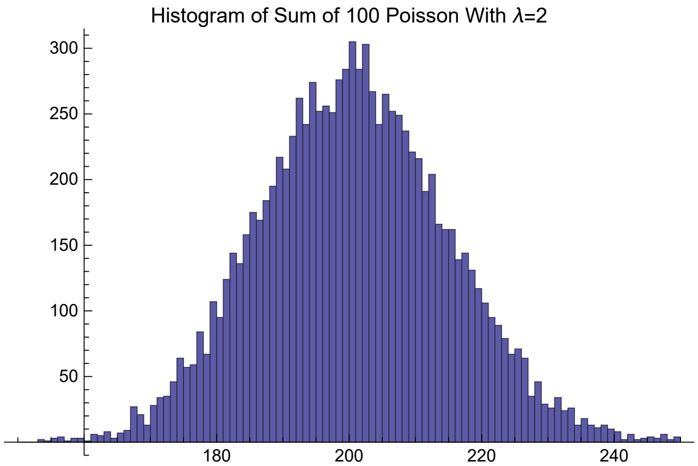
Out[ ]=



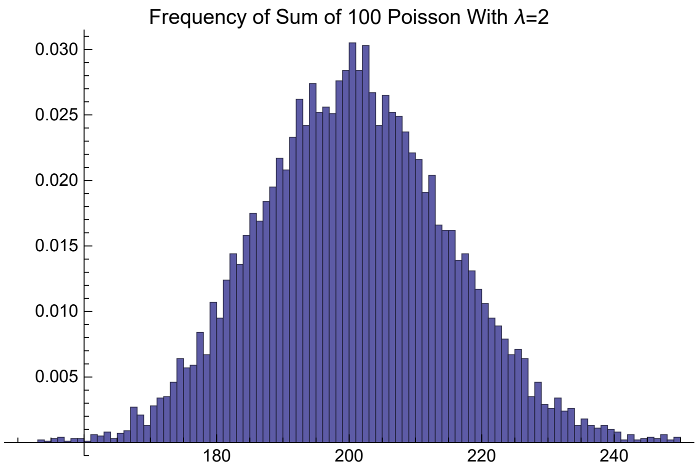
In[ ]:=

```
X := Sum[Random[PoissonDistribution[2]], {100}]
Sample100 = Table[X, 10000];
bcount = BinCounts[Sample100, {150, 250, 1}];
GeneralizedBarChart[Table[{150.5 + (i - 1) * 1, bcount[[i]], 1}, {i, 1, 100}],
 PlotLabel → "Histogram of Sum of 100 Poisson With λ=2"]
GeneralizedBarChart[Table[{150.5 + (i - 1) * 1, bcount[[i]] / (1 * 10000), 1}, {i, 1, 100}],
 PlotLabel → "Frequency of Sum of 100 Poisson With λ=2"]
Show[GeneralizedBarChart[Table[{150.5 + (i - 1) * 1, bcount[[i]] / (1 * 10000), 1},
    {i, 1, 100}], PlotLabel → "Frequency of Sum Overlaid With Normal Curve"],
 Plot[1 / Sqrt[400 * Pi] * Exp[- (x - 200)^2 / 400], {x, 150, 250}, PlotRange → All]]
```

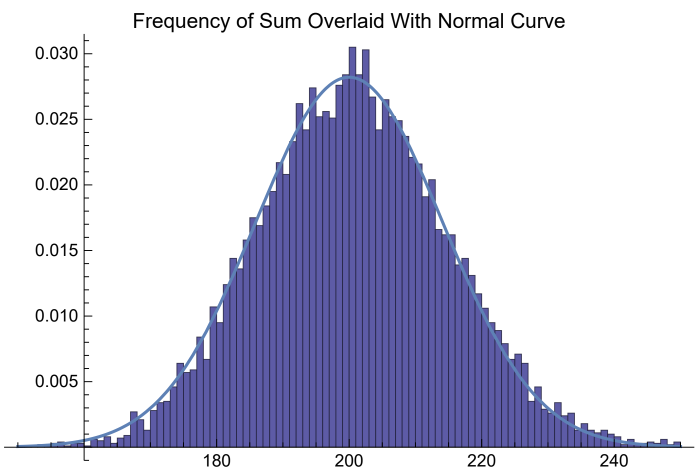
Out[ ]=


Histogram of Sum of 100 Poisson With λ=2
300
250
200
150
100
50
180
200
220
240

Out[ ]=


Frequency of Sum of 100 Poisson With λ=2
0.030
0.025
0.020
0.015
0.010
0.005
180
200
220
240

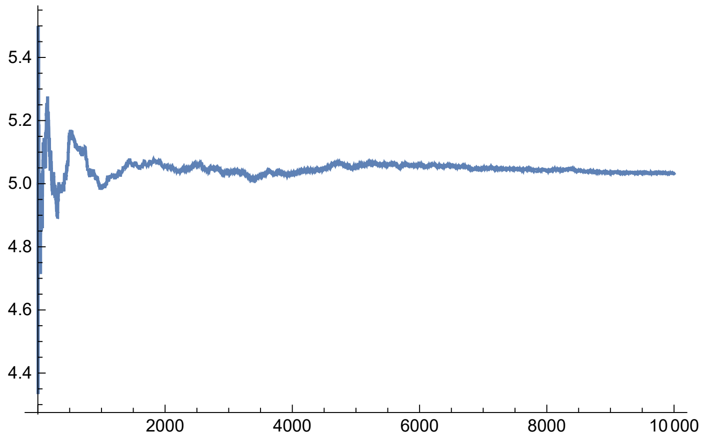
Out[ ]=


Frequency of Sum Overlaid With Normal Curve
0.030
0.025
0.020
0.015
0.010
0.005
180
200
220
240

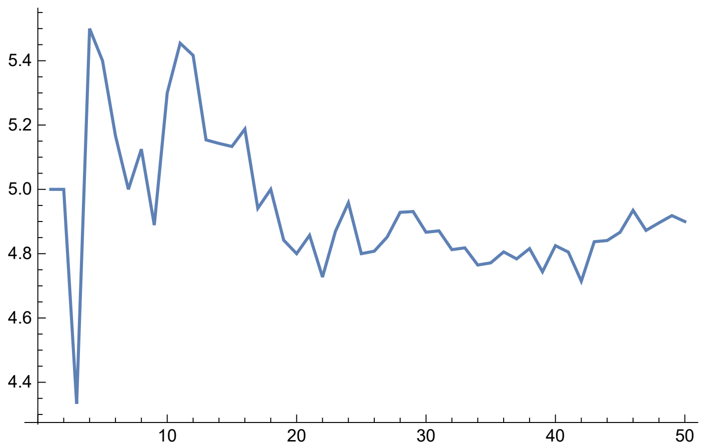
In[ ]:=

```
data = Table[Random[PoissonDistribution[5]], 10 000];
sumdata = Accumulate[data];
average = Table[sumdata[[i]] / i, {i, 1, 10 000}];
ListPlot[average, Joined → True, PlotRange → All]
ListPlot[Drop[average, {51, 10 000}], Joined → True, PlotRange → All]
ListPlot[Drop[average, {1, 5000}], Joined → True, PlotRange → All]
s = 0; n = 1 000 000; sum = 0;
For[i = 1, i < 4, i++, Do[r = Random[PoissonDistribution[5]];
   s = s + r, {i, 1, n}];
  sum = sum + s;
  s = 0]
Print[sum / n / 3 // N]
```

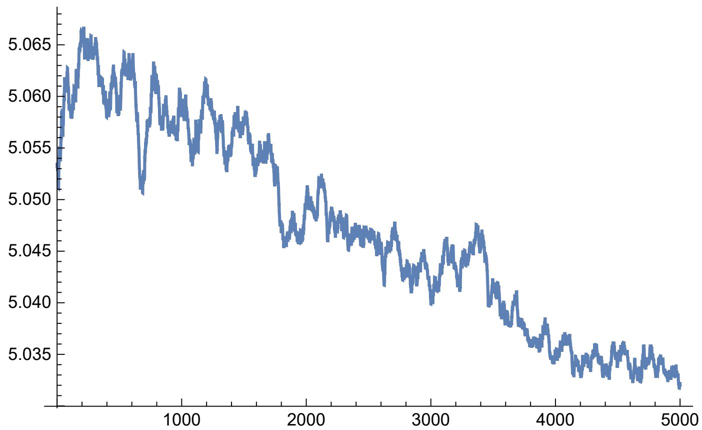
Out[ ]=



Out[ ]=



Out[•]=



4.99952