Project 9 - Javier Salazar 1001144647.

Use the Metropolis-Hastings Algorithm to simulate samples from distributions 1- 3 below for sample sizes of 40,00 and 100,000. Plot together simulated probability density histogram and $\pi(x)$ in all cases.

1. $\pi(k) = \frac{27720}{83711} \frac{1}{k+1}, \quad k = 0, 1, \ldots, 10$

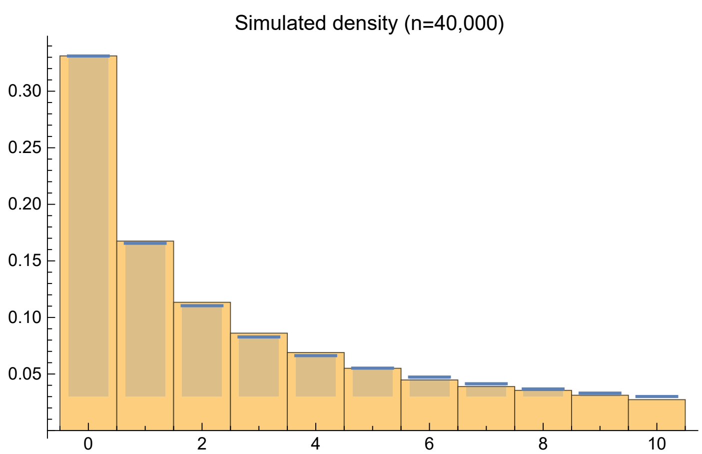
Hint. Use symmetric random walk on {0,1, . . . , 10} with sticky boundary at {0} and {10} from Lab5_Notebook.

In[◦]:=
```
Clear[simu1, X, Y];
X[0] = 5; Print["start = ", X[0]]; n = 40000;
Do[Which[0 < X[k] < 10, Y = X[k] + 2 RandomInteger[] - 1; m = Min[1, (X[k] + 1)/(Y + 1)],
   X[k] == 0, Y = X[k] + RandomInteger[]; m = Min[1, (X[k] + 1)/(Y + 1)],
   X[k] == 10, Y = X[k] + RandomInteger[] - 1; m = Min[1, (X[k] + 1)/(Y + 1)]];
  {U = RandomReal[], If[U ≤ m, X[k + 1] = Y, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}];
Print["sample size = ", Length[simu1]];
Show[Histogram[simu1, {-0.5, 10.5, 1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=40,000) "],
 DiscretePlot[27720/83711 1/(k + 1), {k, 0, 10}, ExtentSize → .7, PlotRange → All]]
```

start = 5

sample size = 40000

Out[◦]=

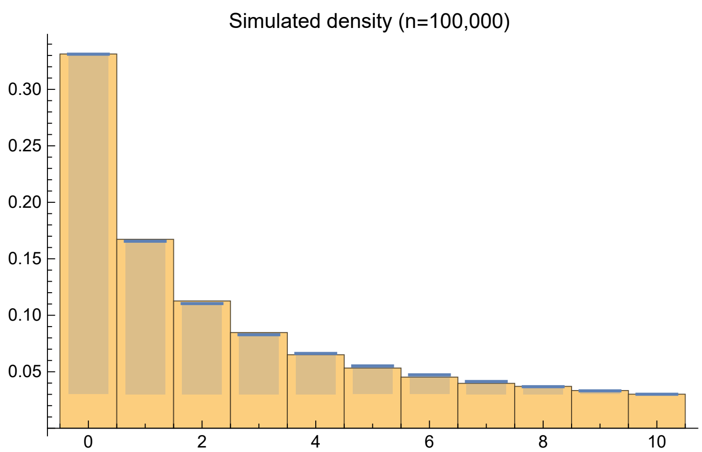


In[ ]:=
```
Clear[simu1, X, Y];
X[0] = 5; Print["start = ", X[0]]; n = 100000;
Do[Which[0 < X[k] < 10, Y = X[k] + 2 RandomInteger[] - 1; m = Min[1, (X[k] + 1)/(Y + 1)],
   X[k] == 0, Y = X[k] + RandomInteger[]; m = Min[1, (X[k] + 1)/(Y + 1)],
   X[k] == 10, Y = X[k] + RandomInteger[] - 1; m = Min[1, (X[k] + 1)/(Y + 1)]];
  {U = RandomReal[], If[U ≤ m, X[k + 1] = Y, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}];
Print["sample size = ", Length[simu1]];
 Show[Histogram[simu1, {-0.5, 10.5, 1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=100,000) "],
 DiscretePlot[27720/83711 1/(k + 1), {k, 0, 10}, ExtentSize → .7, PlotRange → All]]
```

start = 5

sample size = 100000

Out[ ]=



2. $\pi(x) = \cos(x)/2$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

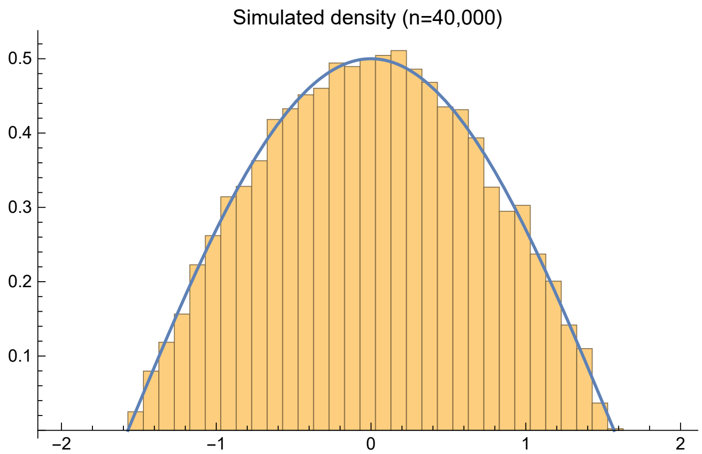
In[ ]:=
```
X[0] = 0; Print["start = ", X[0]]; n = 40000; a = 1; Print["a = ", a];
Do[{Y = RandomReal[{X[k] - a, X[k] + a}], U = RandomReal[], m = Min[1, Cos[Y] / Cos[X[k]]],
   If[U ≤ m, X[k + 1] = Y, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}]; Print["sample size = ", Length[simu1]];
Show[Histogram[simu1, {-Pi / 2 - .5, Pi / 2 + .5, .1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=40,000)"],
 Plot[Cos[x] / 2, {x, -Pi / 2, Pi / 2}, PlotRange → All]]
```

```
start = 0

a = 1

sample size = 40000
```

Out[ ]=



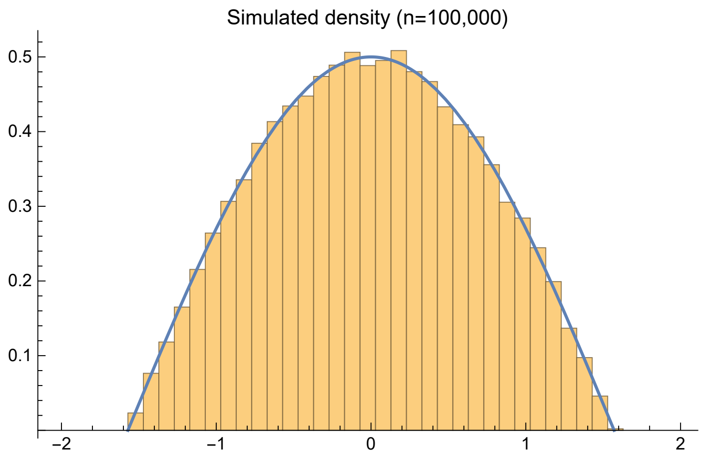
In[ ]:=
```
X[0] = 0; Print["start = ", X[0]]; n = 100000; a = 1; Print["a = ", a];
Do[{Y = RandomReal[{X[k] - a, X[k] + a}], U = RandomReal[], m = Min[1, Cos[Y] / Cos[X[k]]],
   If[U ≤ m, X[k + 1] = Y, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}]; Print["sample size = ", Length[simu1]];
Show[Histogram[simu1, {-Pi / 2 - .5, Pi / 2 + .5, .1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=100,000)"],
 Plot[Cos[x] / 2, {x, -Pi / 2, Pi / 2}, PlotRange → All]]
```

```
start = 0

a = 1

sample size = 100000
```

Out[ ]=



3. $\pi(x) = \frac{1}{3} f(x) + \frac{2}{3} g(x)$, $0 \le x \le 3$, is a convex mixture of densities

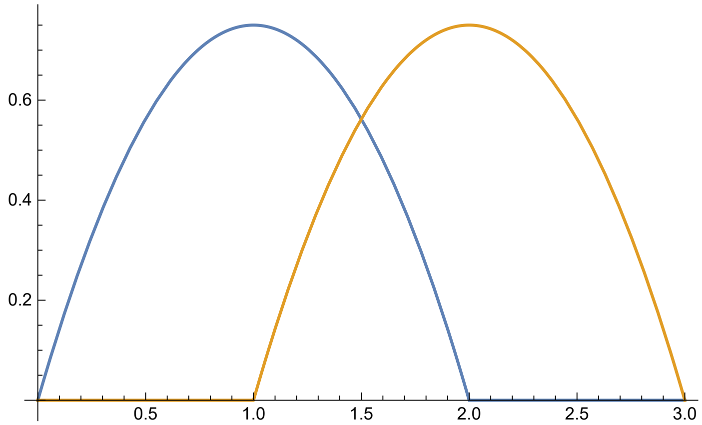
$f(x) = \frac{3}{4} x (2 - x)$ on $[0, 2]$ and $g(x) = \frac{3}{4} (x - 1)(3 - x)$ on $[1, 3]$

In[ ]:=
```
f[x_] := x (2 - x) 3/4 /; x ≤ 2 ∧ x ≥ 0
f[x_] := 0 /; (x > 2 || x < 0)
```

In[ ]:=
```
g[x_] := (x - 1) (3 - x) 3/4 /; x ≤ 3 ∧ x ≥ 1
g[x_] := 0 /; (x > 3 || x < 1)
```

In[ ]:=
```
Plot[{f[x], g[x]}, {x, 0, 3}]
```

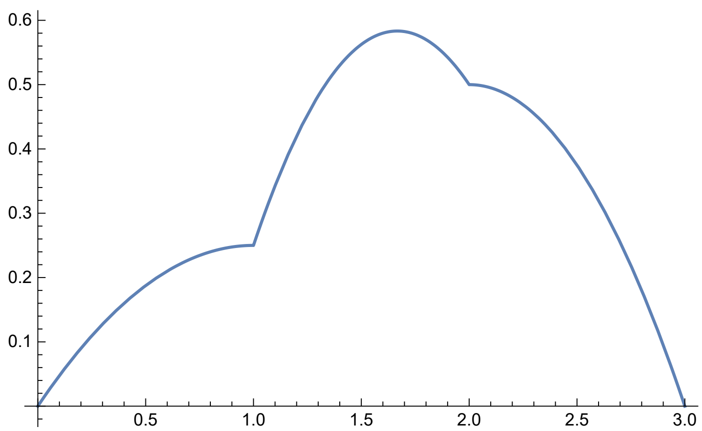
Out[ ]=



In[ ]:= $h[x\_] := \frac{1}{3} f[x] + \frac{2}{3} g[x]$ (* $\pi(x) = h(x)$ *)

In[ ]:=
```
Plot[h[x], {x, 0, 3}]
```

Out[ ]=



In[ ]:=
```
X[0] = 1.5; Print["start = ", X[0]]; n = 40000;
Do[{Y = Random[NormalDistribution[X[k], 1]], U = RandomReal[], m = Min[1, h[Y]/h[X[k]]],
   If[U ≤ m, X[k + 1] = Y;
    Accepted = Accepted + 1, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}]; Print["sample size = ", Length[simu1]];
Show[Histogram[simu1, {0 - .05, 3 + .05, .1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=40,000) vs. h[x]"],
 Plot[h[x], {x, 0, 3}, PlotRange → All]]
```

start = 1.5

$RecursionLimit: Recursion depth of 1024 exceeded during evaluation of 1 + Accepted.

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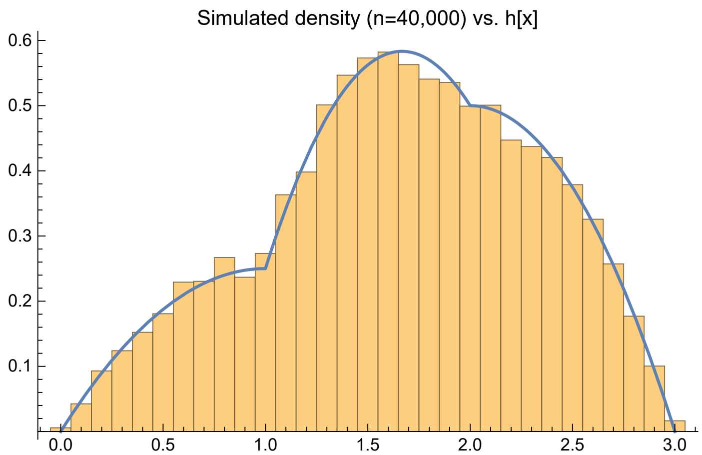
General: Further output of $RecursionLimit::reclim2 will be suppressed during this calculation.

sample size = 40000

Out[ ]=



In[ ]:=
```
X[0] = 1.5; Print["start = ", X[0]]; n = 100000;
Do[{Y = Random[NormalDistribution[X[k], 1]], U = RandomReal[], m = Min[1, h[Y]/h[X[k]]],
   If[U ≤ m, X[k + 1] = Y;
    Accepted = Accepted + 1, X[k + 1] = X[k]]}, {k, 0, n - 1}];
simu1 = Table[X[k], {k, 0, n - 1}]; Print["sample size = ", Length[simu1]];
Show[Histogram[simu1, {0 - .05, 3 + .05, .1}, "ProbabilityDensity",
  PlotRange → All, PlotLabel → "Simulated density (n=100,000) vs. h[x]"],
 Plot[h[x], {x, 0, 3}, PlotRange → All]]
```

```
start = 1.5
```

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General: Further output of $RecursionLimit::reclim2 will be suppressed during this calculation.

```
sample size = 100000
```

Out[ ]=