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Project 6.

1. **Classical Gambler's Ruin.**

(a) Simulate n =10,000 samples and compute the sample average frequency to win K, sample average of the game duration
for K = 40, i = 5, p = $\frac{18}{38}$ and compare with exact values of P_i and E T. Animate a sample for K = 20, i = 2.

In[∘]:=
```
p = 18/38;
Y := If[Random[] ≤ p, 1, -1]
n = 10000; win = 0; K = 40; i = 5; TotalTime = 0;
Do[m = i; T = 0;
  While[0 < m < K, T = T + 1; If[0 < m < K, m = m + Y]];
  If[m == K, win = win + 1]]; TotalTime = TotalTime + T,
 {n}]
Print["frequency(win K) = ", win/n // N]
Print["Pi = ", (1 - ((1-p)/p)^i)/(1 - ((1-p)/p)^K) // N]
Print["average(T) = ", TotalTime/n // N]
Print["E T = ", i/(1 - 2 p) - K/(1 - 2 p) (1 - ((1-p)/p)^i)/(1 - ((1-p)/p)^K) // N]
```

frequency(win K) = 0.0123

P_i = 0.0104045

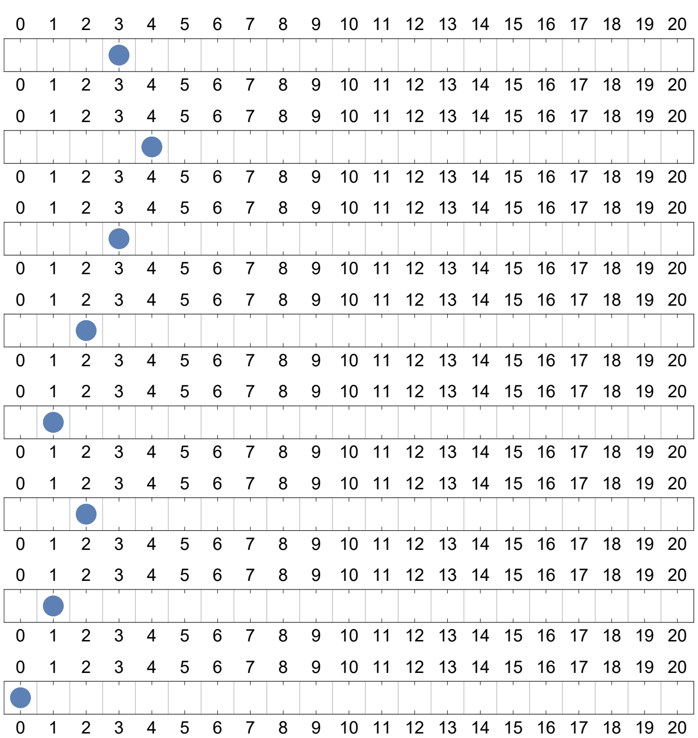
average(T) = 87.297

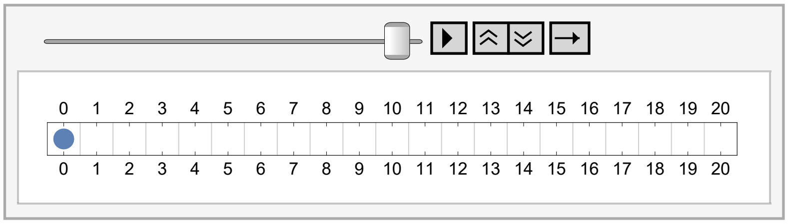
E T = 87.0926

In[•]:=
```
Clear[A];
X := 2 RandomInteger[] - 1;
i = 2;  K = 20;  m = i; Num = 0; j = 0;

A[0] = ListPlot[{{m - .5, 1 - .5}}, PlotStyle -> PointSize[.03],
    Frame → Automatic, AspectRatio → Automatic, Axes → None,
    FrameTicks → {{{-.5, 0}, {0.5, 1}, {1.5, 2}, {2.5, 3}, {3.5, 4}, {4.5, 5}, {5.5, 6},
        {6.5, 7}, {7.5, 8}, {8.5, 9}, {9.5, 10}, {10.5, 11}, {11.5, 12}, {12.5, 13}, {13.5, 14},
        {14.5, 15}, {15.5, 16}, {16.5, 17}, {17.5, 18}, {18.5, 19}, {19.5, 20}}, None},
    GridLines → {{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
      {0, 1}}, PlotRange → {{-1, 20}, {0, 1}}];

While[0 < m < K,
    If[0 < m < K, m = m + X];
   Print[j = j + 1;
    A[j] = ListPlot[{{m - .5, 1 - .5}}, PlotStyle -> PointSize[.03],
      Frame → Automatic, AspectRatio → Automatic, Axes → None, FrameTicks →
       {{{-.5, 0}, {0.5, 1}, {1.5, 2}, {2.5, 3}, {3.5, 4}, {4.5, 5}, {5.5, 6}, {6.5, 7},
         {7.5, 8}, {8.5, 9}, {9.5, 10}, {10.5, 11}, {11.5, 12}, {12.5, 13}, {13.5, 14},
         {14.5, 15}, {15.5, 16}, {16.5, 17}, {17.5, 18}, {18.5, 19}, {19.5, 20}}, None},
      GridLines → {{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20},
        {0, 1}}, PlotRange → {{-1, 20}, {0, 1}}]];
     ];
Num = j;
ListAnimate[Table [A[j], {j, 0, Num}], AnimationRate → 1]
```



Out[ ]=


(b) Consider a slightly unfavorable game with p = .49 and K =100. For what initial capital i does the game last

longest on average ? Find the corresponding maximum duration of the game. What is the probability of winning

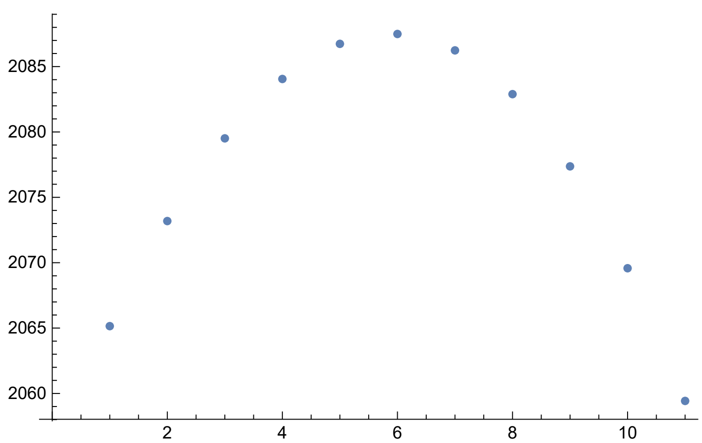
100 in this case?

In[ ]:=

$$\text{MeanGameDuration}[K\_, p\_, i\_] := \frac{i}{1-2p} - \frac{K}{1-2p}\,\frac{1-\left(\frac{1-p}{p}\right)^{i}}{1-\left(\frac{1-p}{p}\right)^{K}}$$

$$\text{ProbabilityWin}[K\_, p\_, i\_] := \frac{1-\left(\frac{1-p}{p}\right)^{i}}{1-\left(\frac{1-p}{p}\right)^{K}}$$

```
ListPlot[Table[MeanGameDuration[100, 0.49, i], {i, 60, 70}]]
Table[MeanGameDuration[100, 0.49, i], {i, 60, 70}]
MeanGameDuration[100, 0.49, 65] // N
ProbabilityWin[100, 0.49, 65] // N
```

Out[ ]=


Out[ ]= {2065.15, 2073.19, 2079.51, 2084.05, 2086.74, 2087.49, 2086.24, 2082.89, 2077.37, 2069.58, 2059.43}

Out[ ]= 2087.49

Out[ ]= 0.232501

2. **Bold Play Strategy.** Let K= 2M and p be the probability of winning a single game. Consider a gambler that follows the

the so called *bold play strategy* defined as follows:

bet i = the current fortune (to win i and have the fortune 2 i or lose i and become bankrupt with 0), whenever $0 < i \leq M$

or bet K - i (to win K - i and reach the goal of K or lose K - i to end up with 2 i - K), whenever M < i < 2M.

(a) Let K = 20. Find the Markov transition matrix P as a function of p, q (p + q =1).

In[ ]:=
```
Clear[p, q];
```

$$P = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 \\
q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & p \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix};$$

(b) Re-label the states $\{0 \to 19, 1 \to 0, 2 \to 1, \ldots 19 \to 18, 20 \to 20\}$ to obtain a decomposition $P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$.

In[ ]:=
```
Pnew = RotateLeft[P, 1];
For[i = 1, i < 22, i++, Pnew[[i]] = RotateLeft[Pnew[[i]], 1]];
Pnew // MatrixForm;
```

In[ ]:=

$$Q = \begin{pmatrix}
0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & q & 0
\end{pmatrix};$$

$$R = \begin{pmatrix}
0 & q \\
0 & q \\
0 & q \\
0 & q \\
0 & q \\
0 & q \\
0 & q \\
0 & q \\
0 & q \\
p & q \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0 \\
p & 0
\end{pmatrix};$$

$$Id = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix};$$

$$\Theta = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix};$$

(c) Find the matrix U of probabilities to win K or become bankrupt as a function of p and q.

In[]:=
```
W = Inverse[IdentityMatrix[19] - Q];
U = W.R; U // MatrixForm
```

Out[]//MatrixForm=

$$\begin{pmatrix}
-\frac{p^5}{-1+p^2 q^2}-\frac{p^5 q}{-1+p^2 q^2} & q+p\,q-\frac{p^2 q}{-1+p^2 q^2}-\frac{p^3 q}{-1+p^2 q^2} \\
-\frac{p^4}{-1+p^2 q^2}-\frac{p^4 q}{-1+p^2 q^2} & q-\frac{p\,q}{-1+p^2 q^2}-\frac{p^2 q}{-1+p^2 q^2} \\
-\frac{p^3}{-1+p^2 q^2}-\frac{p^5 q}{-1+p^2 q^2} & q+p\,q-\frac{p^2 q^2}{-1+p^2 q^2}-\frac{p^3 q^2}{-1+p^2 q^2} \\
-\frac{p^3}{-1+p^2 q^2}-\frac{p^3 q}{-1+p^2 q^2} & \frac{q}{1-p^2 q^2}-\frac{p\,q}{-1+p^2 q^2} \\
p^2 & q+p\,q \\
-\frac{p^2}{-1+p^2 q^2}-\frac{p^4 q}{-1+p^2 q^2} & q-\frac{p\,q^2}{-1+p^2 q^2}-\frac{p^2 q^2}{-1+p^2 q^2} \\
p^2-\frac{p^3 q}{-1+p^2 q^2}-\frac{p^3 q^2}{-1+p^2 q^2} & q-\frac{p\,q^2}{-1+p^2 q^2}-\frac{p^2 q^4}{-1+p^2 q^2} \\
-\frac{p^2}{-1+p^2 q^2}-\frac{p^2 q}{-1+p^2 q^2} & -\frac{q}{-1+p^2 q^2}-\frac{p\,q^3}{-1+p^2 q^2} \\
p^2-\frac{p^2 q}{-1+p^2 q^2}-\frac{p^2 q^2}{-1+p^2 q^2} & q-\frac{p\,q^4}{-1+p^2 q^2}-\frac{p^2 q^4}{-1+p^2 q^2} \\
p & q \\
p-\frac{p^4 q}{-1+p^2 q^2}-\frac{p^4 q^2}{-1+p^2 q^2} & q^2-\frac{p\,q^2}{-1+p^2 q^2}-\frac{p^2 q^2}{-1+p^2 q^2} \\
\frac{p}{1-p^2 q^2}-\frac{p^3 q}{-1+p^2 q^2} & -\frac{q^2}{-1+p^2 q^2}-\frac{p\,q^2}{-1+p^2 q^2} \\
p-\frac{p^2 q}{-1+p^2 q^2}-\frac{p^4 q^2}{-1+p^2 q^2} & q^2-\frac{p\,q^3}{-1+p^2 q^2}-\frac{p^2 q^3}{-1+p^2 q^2} \\
p-\frac{p^2 q}{-1+p^2 q^2}-\frac{p^2 q^2}{-1+p^2 q^2} & -\frac{q^2}{-1+p^2 q^2}-\frac{p\,q^4}{-1+p^2 q^2} \\
p+p\,q & q^2 \\
\frac{p}{1-p^2 q^2}-\frac{p\,q}{-1+p^2 q^2} & -\frac{q^3}{-1+p^2 q^2}-\frac{p\,q^3}{-1+p^2 q^2} \\
p+p\,q-\frac{p^2 q^2}{-1+p^2 q^2}-\frac{p^2 q^3}{-1+p^2 q^2} & -\frac{q^3}{-1+p^2 q^2}-\frac{p\,q^5}{-1+p^2 q^2} \\
p-\frac{p\,q}{-1+p^2 q^2}-\frac{p\,q^2}{-1+p^2 q^2} & -\frac{q^4}{-1+p^2 q^2}-\frac{p\,q^4}{-1+p^2 q^2} \\
p+p\,q-\frac{p\,q^2}{-1+p^2 q^2}-\frac{p\,q^3}{-1+p^2 q^2} & -\frac{q^5}{-1+p^2 q^2}-\frac{p\,q^5}{-1+p^2 q^2}
\end{pmatrix}$$

(d) Find the matrix W.1 of the expected time of the game as a function of p and q.

In[]:=
```
DtoA = Table[Apply[Plus, Table[W[[i, j]], {j, 1, 7}]], {i, 1, 7}];
sum = Total[DtoA]
```

Out[]=
$$6+2\,p+\frac{1}{1-p^2 q^2}-\frac{p}{-1+p^2 q^2}-\frac{p^2}{-1+p^2 q^2}-\frac{p\,q}{-1+p^2 q^2}-\frac{p^2 q}{-1+p^2 q^2}-\frac{p^2 q^3}{-1+p^2 q^2}$$

(e) Compare probabilities of winning \$20 and game duration for a classical play versus bold play for all initial capital values $i = 1, 2, \ldots, 19$ for $p = \frac{18}{38}$, $p = \frac{1}{2}$

Hint. Use Print[Table[{i, Preg[[i]], Pbold[[i]], MeanGameDuration[20,18/38,i], Dbold[[i]]}, {i, 1, 19}]// TableForm]

In[∘]:=
```
p = 18/38;
q = 1 - p;
Pbold = U[[All, 1]];
Print[Table[{i, ProbabilityWin[20, 18/38, i] // N,
    Pbold[[i]] // N, MeanGameDuration[20, 18/38, i] // N,
    Apply[Plus, Table[W[[i, j]], {j, 1, 7}]] // N}, {i, 1, 19}] // TableForm]
```

1	0.0153781	0.0388112	13.1563	1.71293
2	0.032465	0.0819348	25.6633	1.50508
3	0.0514503	0.126711	37.4489	1.5996
4	0.0725452	0.172974	48.4328	1.06627
5	0.0959839	0.224377	58.5261	1.
6	0.122027	0.2675	67.6298	1.26583
7	0.150964	0.315415	75.6338	1.03488
8	0.183115	0.365166	82.4161	0.13991
9	0.21884	0.41657	87.8409	0.0736371
10	0.258533	0.473684	91.7573	0.
11	0.302637	0.516808	93.9978	0.792146
12	0.351642	0.564723	94.376	0.561197
13	0.406091	0.614474	92.6852	0.666226
14	0.466591	0.665877	88.6955	0.0736371
15	0.533812	0.722992	82.1513	0.
16	0.608503	0.770907	72.7688	0.295367
17	0.691493	0.824146	60.2328	0.0387564
18	0.783703	0.879425	44.1927	0.155456
19	0.88616	0.936539	24.2593	0.081819

In[∘]:=
```
p = .5
q = 1 - p;
Pbold = U[[All, 1]];
Print[Table[{i, i/20 // N, Pbold[[i]] // N, i * (20 - i) // N,
    Apply[Plus, Table[W[[i, j]], {j, 1, 7}]] // N}, {i, 1, 19}] // TableForm]
```

Out[∘]= 0.5

1	0.05	0.05	19.	1.76667
2	0.1	0.1	36.	1.53333
3	0.15	0.15	51.	1.63333
4	0.2	0.2	64.	1.06667
5	0.25	0.25	75.	1.
6	0.3	0.3	84.	1.26667
7	0.35	0.35	91.	1.03333
8	0.4	0.4	96.	0.133333
9	0.45	0.45	99.	0.0666667
10	0.5	0.5	100.	0.
11	0.55	0.55	99.	0.766667
12	0.6	0.6	96.	0.533333
13	0.65	0.65	91.	0.633333
14	0.7	0.7	84.	0.0666667
15	0.75	0.75	75.	0.
16	0.8	0.8	64.	0.266667
17	0.85	0.85	51.	0.0333333
18	0.9	0.9	36.	0.133333
19	0.95	0.95	19.	0.0666667

3. **Mystery Escape.** An 11 story mystery building has 10 doors on each floor. Each door is labelled UP or DOWN

and leads to a staircase to a different floor. Namely, 11-th floor has 10 doors labelled DOWN, 10-th floor has 9 doors
labelled DOWN and one labelled UP, . . . , second floor has 1 door labelled DOWN and 9 labelled UP and the first floor
has 10 doors labelled UP. In addition, all staircases are separated and invisible to each other. Visitors on the 11-th
floor are asked to use stairs to get to the first floor with an idea of using the minimum number of doors. Since the
visitors are unfamiliar with the building, they choose at random the doors labelled DOWN .

Assign states {0, 1, . . . , 10} to floors 1-11 and consider the Markov chain X_n with 0 being an absorbing state.

(a) Re-label the states {0→10, 1→0, 2→1, . . . , 10→9} to obtain a decomposition $P = \begin{pmatrix} Q & R \\ O & I \end{pmatrix}$.

(b) How many doors to walk through will it take on average before reaching the 1-st floor.
In other words, if T = number of doors used to reach the ground floor, find $E[T \mid X_0 = 10]$.

(c) Find the answer to (a) and (b) in the case the doors were unlabeled.

In[ ]:=

$$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 0 & 0 & 0 & 0 & 0 \\ 1/7 & 1/7 & 1/7 & 1/7 & 1/7 & 1/7 & 1/7 & 0 & 0 & 0 & 0 \\ 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 1/8 & 0 & 0 & 0 \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 0 & 0 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 \end{pmatrix};$$

```
Pnew = RotateLeft[P, 1];
For[i = 1, i < 12, i++, Pnew[[i]] = RotateLeft[Pnew[[i]], 1]];
Pnew // MatrixForm;
```

$$Q = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & \frac{1}{7} & 0 & 0 & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & 0 & 0 & 0 \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & 0 & 0 \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 \end{pmatrix};$$

$$R = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{5} \\ \frac{1}{6} \\ \frac{1}{7} \\ \frac{1}{8} \\ \frac{1}{9} \\ \frac{1}{10} \end{pmatrix};$$

```
Id = ( 1 );
Θ = ( 0 0 0 0 0 0 0 0 0 0 );
W = Inverse[IdentityMatrix[10] - Q];
W // MatrixForm;
DtoA = Table[Apply[Plus, Table[W[[i, j]], {j, 1, 10}]], {i, 1, 10}];
Print["The average number of doors used to reach the ground floor is ", 7381 / 2520 // N]
```

The average number of doors used to reach the ground floor is 2.92897

In[ ]:= $$P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 & 1/10 \\ 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 1/10 & 0 \end{pmatrix};$$

```
Pnew = RotateLeft[P, 1];
For[i = 1, i < 12, i++, Pnew[[i]] = RotateLeft[Pnew[[i]], 1]];
Pnew // MatrixForm
```

$$Q = \begin{pmatrix} 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} \\ \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 \end{pmatrix};$$

$$R = \begin{pmatrix} \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \\ \frac{1}{10} \end{pmatrix};$$

$$Id = \begin{pmatrix} 1 \end{pmatrix};$$

$$\Theta = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix};$$

```
W = Inverse[IdentityMatrix[10] - Q];
W // MatrixForm;
```

```
DtoA = Table[Apply[Plus, Table[W[[i, j]], {j, 1, 10}]], {i, 1, 10}];
Print["The average number of doors used to reach the ground floor is ", 10]
```

Out[ ]//MatrixForm=

$$\begin{pmatrix}
0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} & \frac{1}{10} \\
\frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & \frac{1}{10} & 0 & \frac{1}{10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

```
The average number of doors used to reach the ground floor is 10
```