Project 7 Javier Salazar 1001144647.

Survival of Family Names. One quarter of married couples in certain society have no children. The other three quarters have exactly three children, each equally likely to be a boy or a girl. Using the Branching Processes model

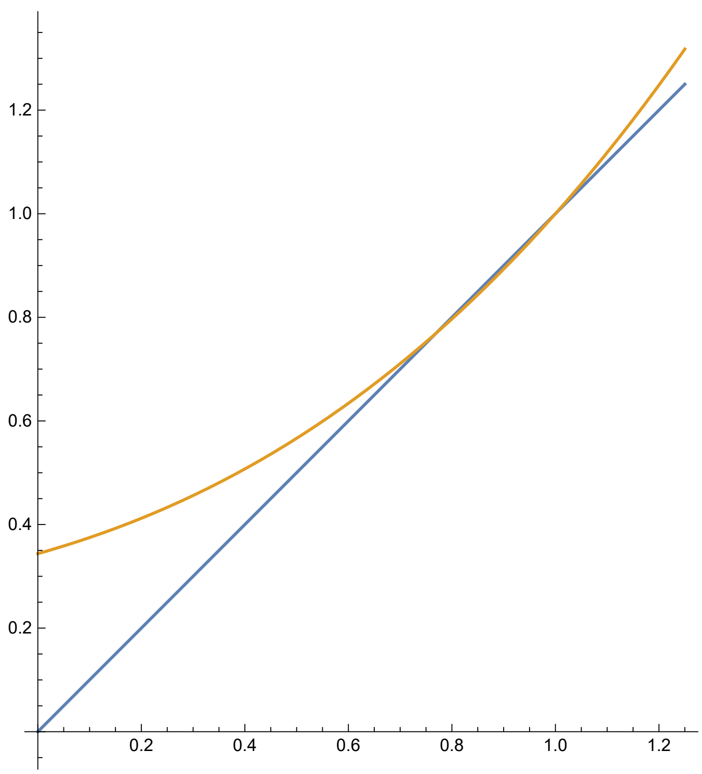
(a) Find the PGF (*Probability Generating Function*) φ(s)

In[ ]:=
```
Clear[s]; Solve[s == 11/32 + s 9/32 + s^2 9/32 + s^3 3/32, s];
```

(b) Graph φ(s) together with the function g(s) = s.

```
Plot[{s, 11/32 + s 9/32 + s^2 9/32 + s^3 3/32}, {s, 0, 1.25}, AspectRatio → Automatic]
```

Out[ ]=



(c) Find the expected size of the male population in the 7-th generation.

In[ ]:=
```
Print[
 "The expected size of the male population in the 7-th generation is ", (9/8)^7 // N]
```

```
The expected size of the male population in the 7-th generation is 2.2807
```

(d) What is the probability that the male line of descent of a particular husband will die out

(i) in the 3-rd generation

```
In[]:= π1 = 11/32 // N;
π2 = 11/32 + π1 9/32 + π1^2 9/32 + π1^3 3/32 // N;
π3 = 11/32 + π2 9/32 + π2^2 9/32 + π2^3 3/32 // N;
Print["The probability that the male descent of a
   particular husband will die out in the 3-rd generation is ", π3 - π2]
```

```
The probability that the male descent of a
   particular husband will die out in the 3-rd generation is 0.0748915
```

(ii) by the 7-th generation

```
In[]:= π1 = 11/32 // N;
π2 = 11/32 + π1 9/32 + π1^2 9/32 + π1^3 3/32 // N;
π3 = 11/32 + π2 9/32 + π2^2 9/32 + π2^3 3/32 // N;
π4 = 11/32 + π3 9/32 + π3^2 9/32 + π3^3 3/32 // N;
π5 = 11/32 + π4 9/32 + π4^2 9/32 + π4^3 3/32 // N;
π6 = 11/32 + π5 9/32 + π5^2 9/32 + π5^3 3/32 // N;
π7 = 11/32 + π6 9/32 + π6^2 9/32 + π6^3 3/32 // N;
Print["The probability that the male descent of a
   particular husband will die out by the 7-th generation is ", π7]
```

```
The probability that the male descent of a
   particular husband will die out by the 7-th generation is 0.678367
```

(iii) eventually

```
In[]:= extinct = 11/32;
For[i = 2, i ≤ 100000, i++, extinct = (11/32 + extinct 9/32 + extinct^2 9/32 + extinct^3 3/32) // N]
Print["The probability that the male descent
   of a particular husband will die out eventually is ", extinct]
```

```
The probability that the male descent of a particular husband will die out eventually is 0.768875
```

(e) Simulate n = 100000 samples of the first 7 male generations, compute the frequency of extinction by

the 7-th generation and the average size of the 7-th generation of males. Compare with exact values.

Hint. Use total probability formula to determine the probabilities of a father having k boys P_k, k = 0, 1, 2, 3.

```
In[•]:= Plot[Piecewise[{{0, 0 ≤ x ≤ 11/32}, {1, 11/32 < x ≤ 20/32},
            {2, 20/32 < x ≤ 29/32}, {3, 29/32 < x ≤ 1}}], {x, 0, 1}];

In[•]:= X := RandomReal[]
        g[x_] := Piecewise[{{0, 0 ≤ x ≤ 11/32},
           {1, 11/32 < x ≤ 20/32}, {2, 20/32 < x ≤ 29/32}, {3, 29/32 < x ≤ 1}}]
        n = 100000; NumGenerations = 7; size = 0; count1 = 0;
        Do[ x = 1;
           Do[
              If[x > 0, a = Table[g[X], {x}]; x = Apply[Plus, a]],
              {NumGenerations}];
           If[x == 0, count1 = count1 + 1, size = size + x ],
           {n}];
        Print["frequency(extinction by 7-th generation) = ", count1/n // N];
        Print["π7 = ", 0.678367];
        Print["average(size of 7-th generation) = ", (size/n) // N];
        Print["E X7 = ", (9/8)^7 // N];

        frequency(extinction by 7-th generation) = 0.67647

        π7 = 0.678367

        average(size of 7-th generation) = 2.29617

        E X7 = 2.2807
```

2. **Survival of Family Names(B).** Married couple in certain society have no children, one child or 2 children with equal probabilities.

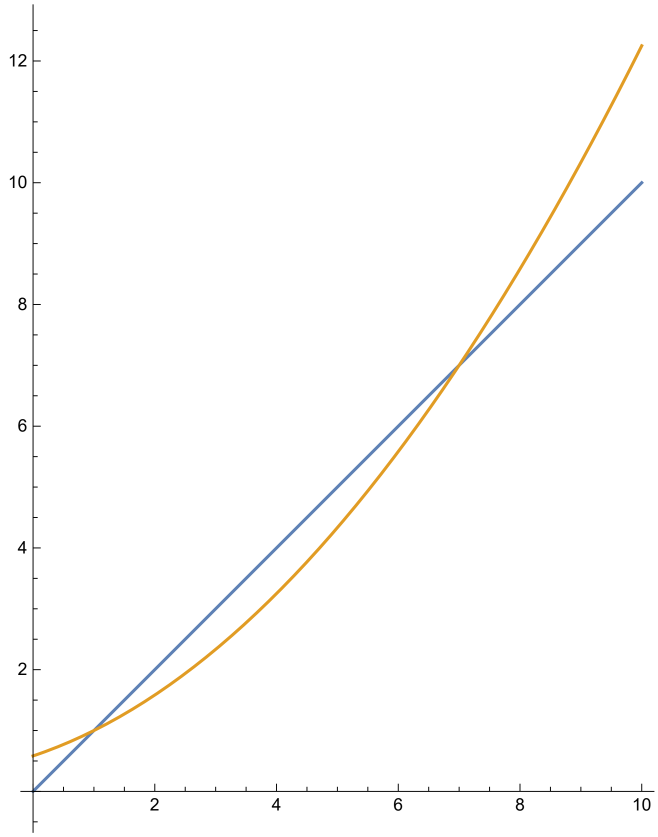
Each child is equally likely to be a boy or a girl.

(a) Find the PGF (*Probability Generating Function*) φ(s)

```
Clear[s]; Solve[s == 7/12 + s 4/12 + + s^2 1/12, s];
```

(b) Graph φ(s) together with the function g(s) = s.

In[∘]:=
```
Plot[{s, 7/12 + s 4/12 + + s^2 1/12}, {s, 0, 10}, AspectRatio → Automatic]
```

Out[∘]=


(c) Find the expected size of the male population in the 7-th generation.

In[∘]:=
```
Print[
 "The expected size of the male population in the 7-th generation is ", (1/2)^7 // N]
```

```
The expected size of the male population in the 7-th generation is 0.0078125
```

(d) What is the probability that the male line of descent of a particular husband will die out

(*i*) in the 3-rd generation

In[∘]:=
```
π1 = 7/12 // N;
π2 = 7/12 + π1 4/12 + π1^2 1/12 // N;
π3 = 7/12 + π2 4/12 + π2^2 1/12 // N;
Print["The probability that the male descent of a
   particular husband will die out in the 3-rd generation is ", π3 - π2]
```

```
The probability that the male descent of a
   particular husband will die out in the 3-rd generation is 0.100065
```

(*ii*) by the n-th generation for n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

In[∘]:=
```
π1 = 7/12 // N
π2 = 7/12 + π1 4/12 + π1^2 1/12 // N
π3 = 7/12 + π2 4/12 + π2^2 1/12 // N
π4 = 7/12 + π3 4/12 + π3^2 1/12 // N
π5 = 7/12 + π4 4/12 + π4^2 1/12 // N
π6 = 7/12 + π5 4/12 + π5^2 1/12 // N
π7 = 7/12 + π6 4/12 + π6^2 1/12 // N
π8 = 7/12 + π7 4/12 + π7^2 1/12 // N
π9 = 7/12 + π8 4/12 + π8^2 1/12 // N
π10 = 7/12 + π9 4/12 + π9^2 1/12 // N
```

Out[∘]= 0.583333

Out[∘]= 0.806134

Out[∘]= 0.906199

Out[∘]= 0.953833

Out[∘]= 0.977094

Out[∘]= 0.988591

Out[∘]= 0.994306

Out[∘]= 0.997156

Out[∘]= 0.998579

Out[∘]= 0.999289

(*iii*) eventually

In[∘]:=
```
extinct = 7/12;
For[i = 2, i ≤ 100000, i++, extinct = (7/12 + extinct 4/12 + extinct^2 1/12) // N]
Print["The probability that the male descent
   of a particular husband will die out eventually is ", extinct]
```

```
The probability that the male descent of a particular husband will die out eventually is 1.
```

(e) Simulate n = 100000 samples of the first 7 male generations, compute the frequency of extinction by the 7-th generation and the average size of the 7-th generation of males. Compare with exact values.

```
In[∘]:= X := RandomReal[]
g[x_] := Piecewise[{{0, 0 ≤ x ≤ 7/12}, {1, 7/12 < x ≤ 11/12}, {2, 11/12 < x ≤ 1}}]
n = 100000; NumGenerations = 7; size = 0; count1 = 0;
Do[ x = 1;
   Do[
      If[x > 0, a = Table[g[X], {x}]; x = Apply[Plus, a]],
      {NumGenerations}];
   If[x == 0, count1 = count1 + 1, size = size + x ],
   {n}];
Print["frequency(extinction by 7-th generation) = ", count1/n // N];
Print["π7 = ", 0.99431];
Print["average(size of 7-th generation) = ", (size/n) // N];
Print["E X7 = ", (1/2)^7 // N];
```

```
frequency(extinction by 7-th generation) = 0.99445

π7 = 0.99431

average(size of 7-th generation) = 0.00757

E X7 = 0.0078125
```

3. **Survival of Family Tree.** Assume that the number of children of a given family in certain region (along will all decedents of future
generations) follows binomial distribution b(5,1/2).

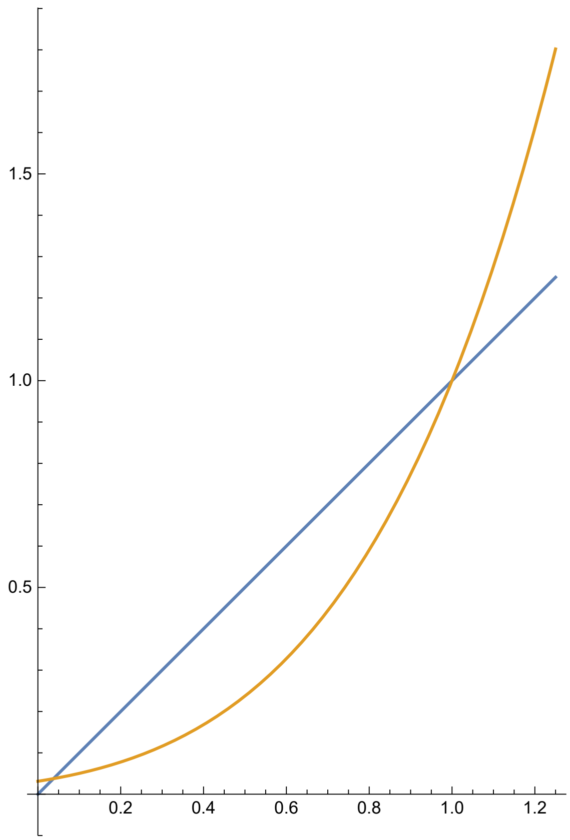
(a) Find the PGF (*Probability Generating Function*) φ(s).

```
In[∘]:= Clear[s];
Solve[s == 0.03125 + s * 0.15625 + s^2 0.3125 + s^3 0.3125 + s^4 0.15625 + s^5 0.03125, s];
```

(b) Graph φ(s) together with the function g(s) = s.

In[ ]:=
```
Plot[{s, 0.03125 + s * 0.15625 + s^2 0.3125 + s^3 0.3125 + s^4 0.15625 + s^5 0.03125},
  {s, 0, 1.25}, AspectRatio → Automatic]
```

Out[ ]=



(c) Find the expected size of population in the 10-th generation.

In[ ]:=
```
Print["The expected size of the population in the 7-th generation is ", (2.5)^10 // N]
```

```
The expected size of the population in the 7-th generation is 9536.74
```

(d) What is the probability that the family dies out

(i) in the 10-rd generation

```
In[•]:= π1 = 0.03125;
π2 = 0.03125 + π1 * 0.15625 + π1^2 * 0.3125 + π1^3 * 0.3125 + π1^4 * 0.15625 + π1^5 * 0.03125;
π3 = 0.03125 + π2 * 0.15625 + π2^2 * 0.3125 + π2^3 * 0.3125 + π2^4 * 0.15625 + π2^5 * 0.03125;
π4 = 0.03125 + π3 * 0.15625 + π3^2 * 0.3125 + π3^3 * 0.3125 + π3^4 * 0.15625 + π3^5 * 0.03125;
π5 = 0.03125 + π4 * 0.15625 + π4^2 * 0.3125 + π4^3 * 0.3125 + π4^4 * 0.15625 + π4^5 * 0.03125;
π6 = 0.03125 + π5 * 0.15625 + π5^2 * 0.3125 + π5^3 * 0.3125 + π5^4 * 0.15625 + π5^5 * 0.03125;
π7 = 0.03125 + π6 * 0.15625 + π6^2 * 0.3125 + π6^3 * 0.3125 + π6^4 * 0.15625 + π6^5 * 0.03125;
π8 = 0.03125 + π7 * 0.15625 + π7^2 * 0.3125 + π7^3 * 0.3125 + π7^4 * 0.15625 + π7^5 * 0.03125;
π9 = 0.03125 + π8 * 0.15625 + π8^2 * 0.3125 + π8^3 * 0.3125 + π8^4 * 0.15625 + π8^5 * 0.03125;
π10 = 0.03125 + π9 * 0.15625 + π9^2 * 0.3125 + π9^3 * 0.3125 + π9^4 * 0.15625 + π9^5 * 0.03125;
Print[
 "The probability that the family will die out in the 10-th generation is ", π10 - π9]
```

```
The probability that the family will die out in the 10-th generation is 5.90803×10^-9
```

(*ii*) by the 10-th generation

```
In[•]:= Print["The probability that the family will die out by the 10-th generation is ", π10]
```

```
The probability that the family will die out by the 10-th generation is 0.0375801
```

(*iii*) eventually (*Hint.* Use NRoots[.] or Solve [.])

```
In[•]:= extinct = 0.03125;
For[i = 2, i ≤ 100000, i++,
 extinct = (0.03125 + extinct * 0.15625 + extinct^2 * 0.3125 + extinct^3 * 0.3125 +
     extinct^4 * 0.15625 + extinct^5 * 0.03125) // N]
Print["The probability that the family will die out eventually is ", extinct]
```

```
The probability that the family will die out eventually is 0.0375801
```

(e) Simulate n = 100000 samples of the first 10 generations, compute the frequency of extinction by the 10-th generation and the average size and standard deviation of the 10-th generation of males . Compare with exact values.

In[ ]:=
```
X := RandomReal[]
h[x_] := Piecewise[{{0, 0 ≤ x ≤ 0.03125}, {1, 0.03125 < x ≤ 0.1875}, {2, 0.1875 < x ≤ 0.5},
    {3, 0.5 < x ≤ 0.8125}, {4, 0.8125 < x ≤ 0.96875}, {5, 0.96875 < x ≤ 1}}]
n = 100000; NumGenerations = 10; size = 0; count1 = 0;
Do[ x = 1;
   Do[
      If[x > 0, a = Table[h[X], {x}]; x = Apply[Plus, a]],
      {NumGenerations}];
   If[x == 0, count1 = count1 + 1, size = size + x ],
   {n}];
Print["frequency(extinction by 10-th generation) = ", count1/n // N];
Print["π10 = ", 0.0375801];
Print["average(size of 10-th generation) = ", (size/n) // N];
Print["E X10 = ", (2.5)^10 // N];
```

frequency(extinction by 10-th generation) = 0.03746

π_{10} = 0.0375801

average(size of 10-th generation) = 9533.54

E X_{10} = 9536.74