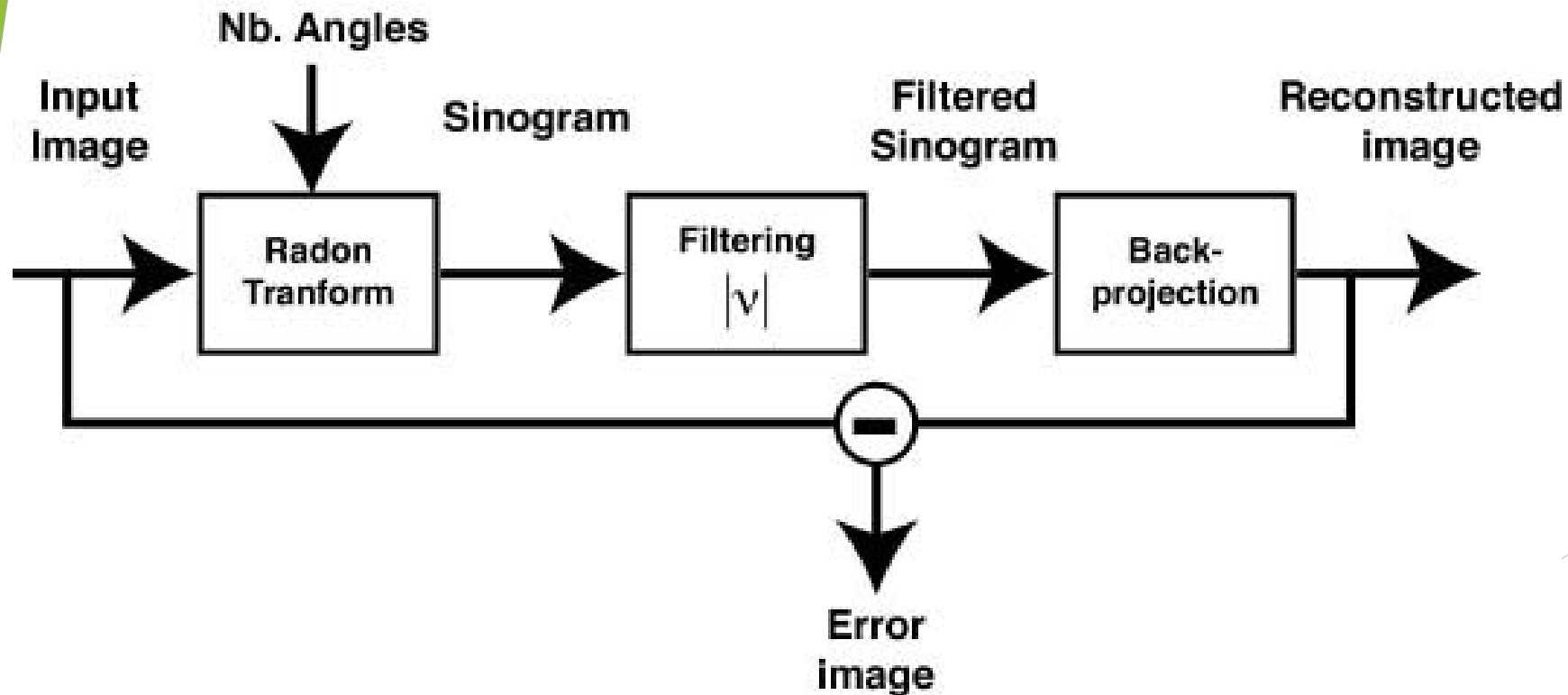# Mathematics Behind Medical Image Reconstruction
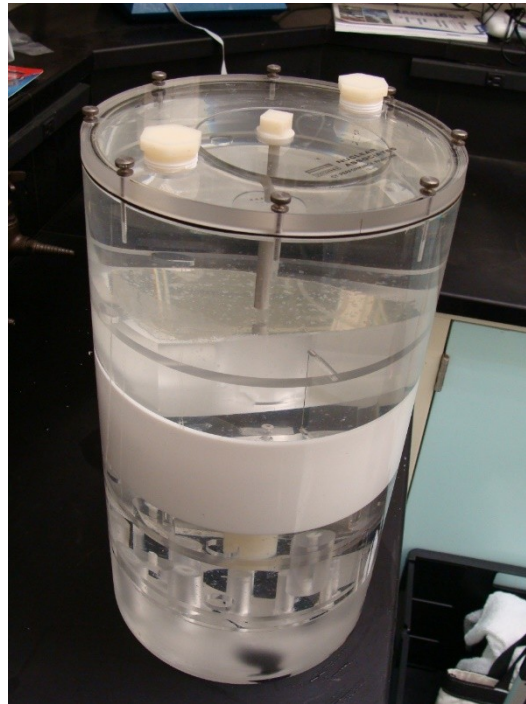
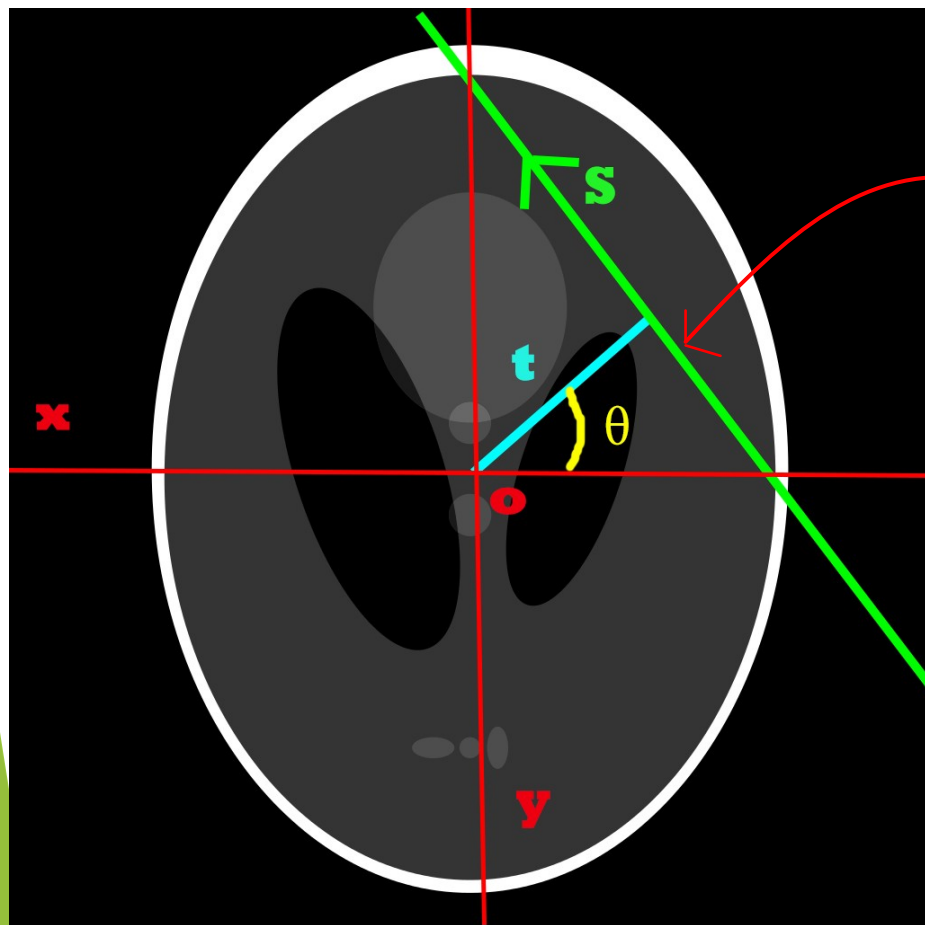Javier A. Salazar II

Phantoms

Shepp-Logan phantom

Real world phantom used for CT performance
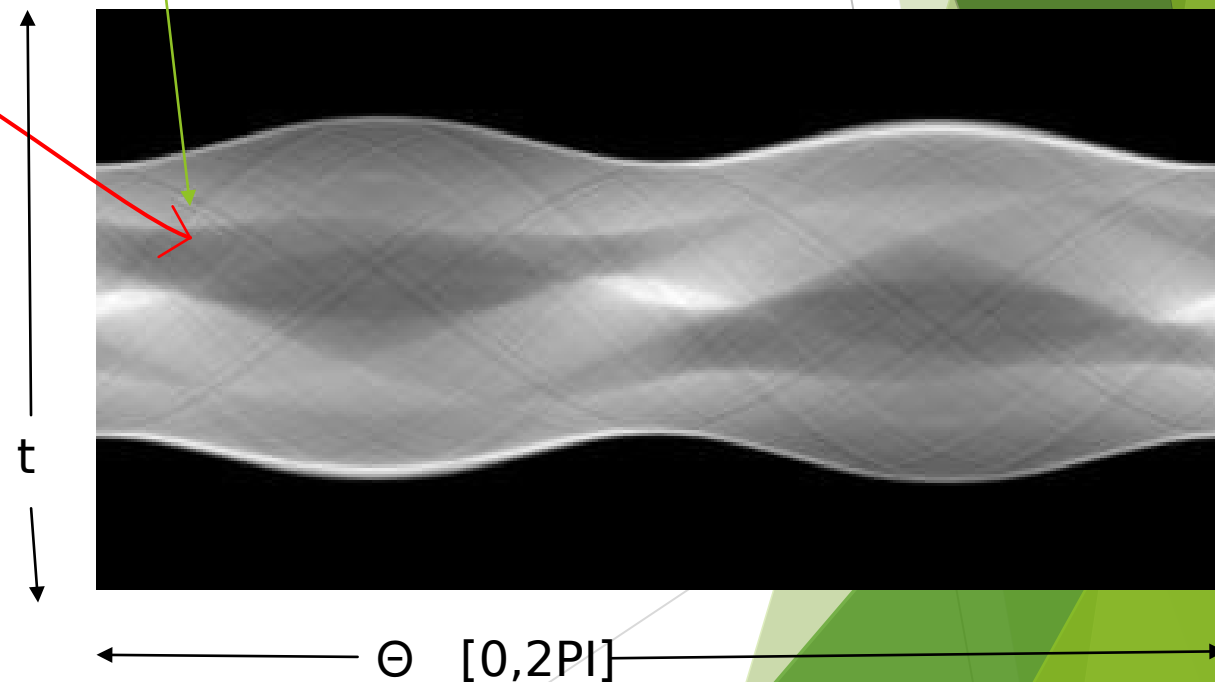
XCAT phantom used for 3D scans

Radon Transform

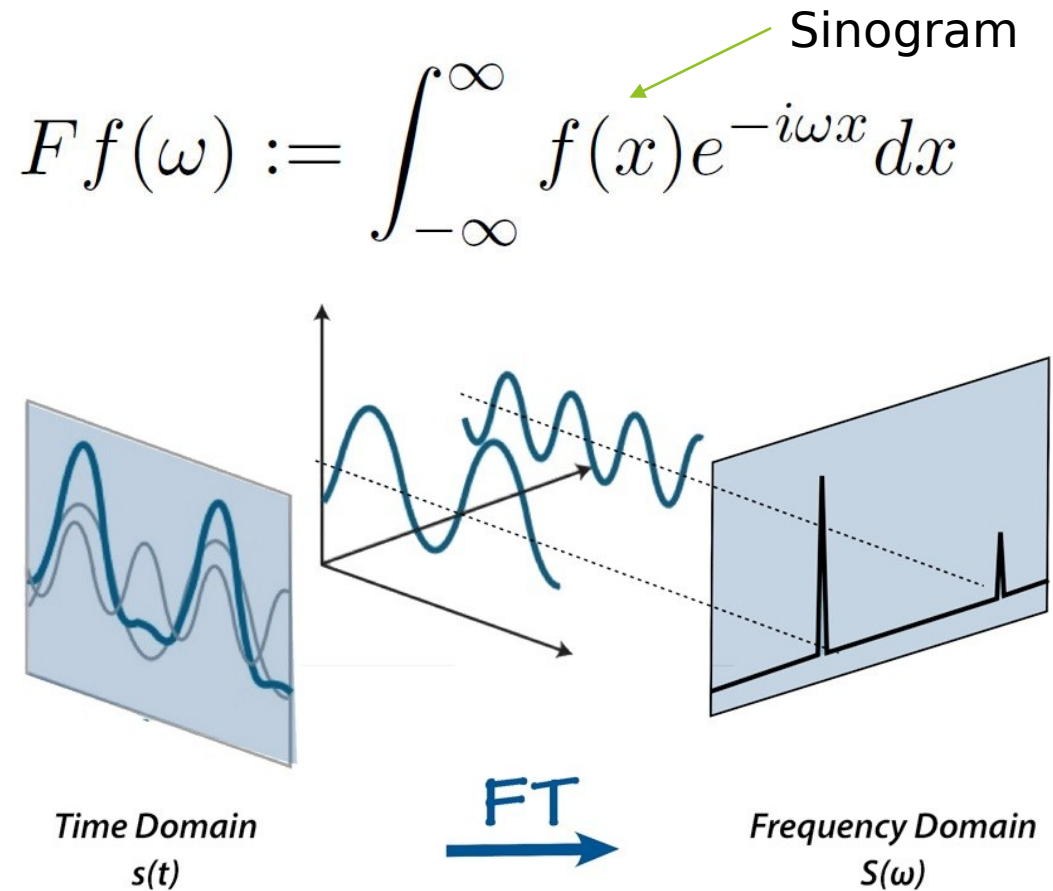$$Rf(t,\theta) := \int_{s=-\infty}^{s=\infty} f((t\cos(\theta) - s\sin(\theta)), t\sin(\theta) + s\cos(\theta))ds$$
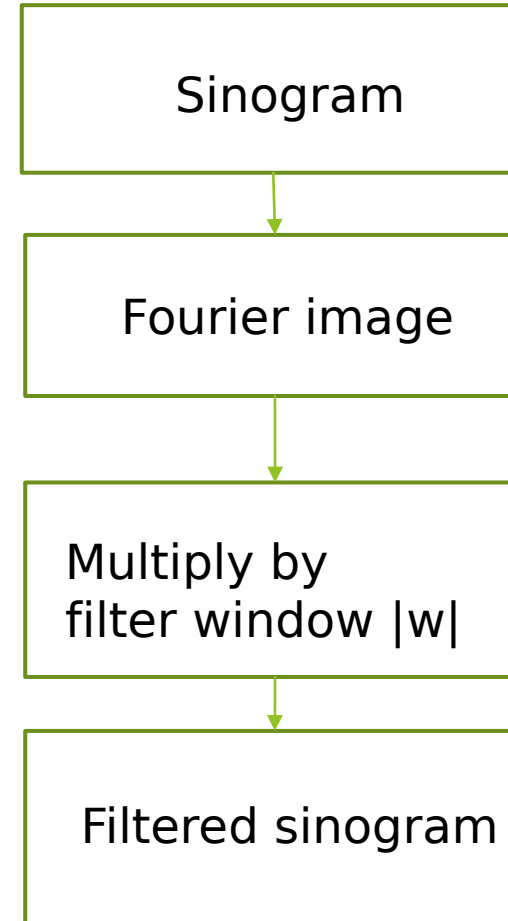

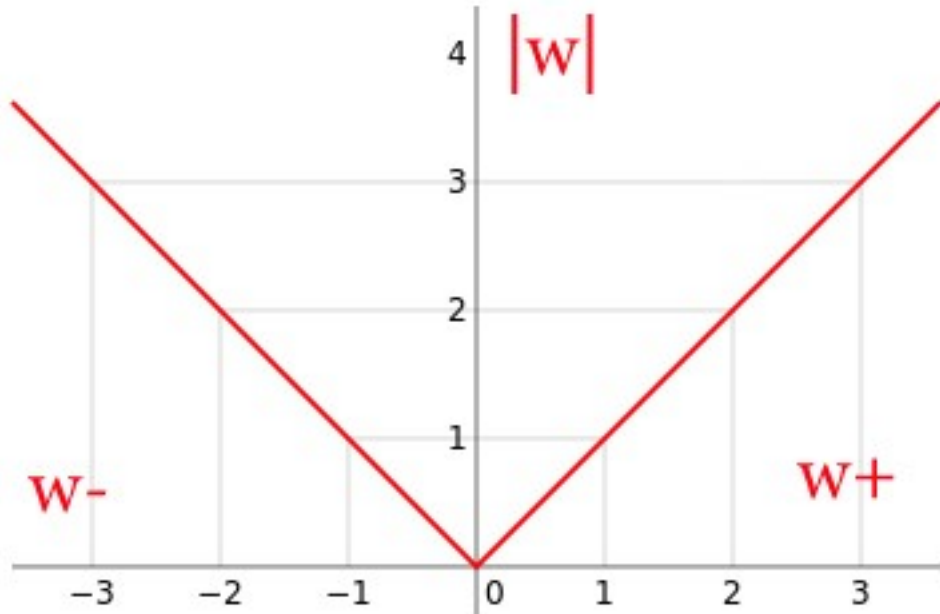One line = One point

Sinogram

Θ   [0,2PI]

Fourier Transform

Sinogram

$$Ff(\omega) := \int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$$

Time Domain
s(t)

FT

Frequency Domain
S(ω)

$$F^{-1}g(x) := \frac{1}{2\pi}\int_{\omega=-\infty}^{\infty} g(\omega)e^{i\omega x}d\omega$$

Filtering

Sinogram

↓

Fourier image

↓

Multiply by
filter window |w|

↓

Filtered sinogram

Filter Design
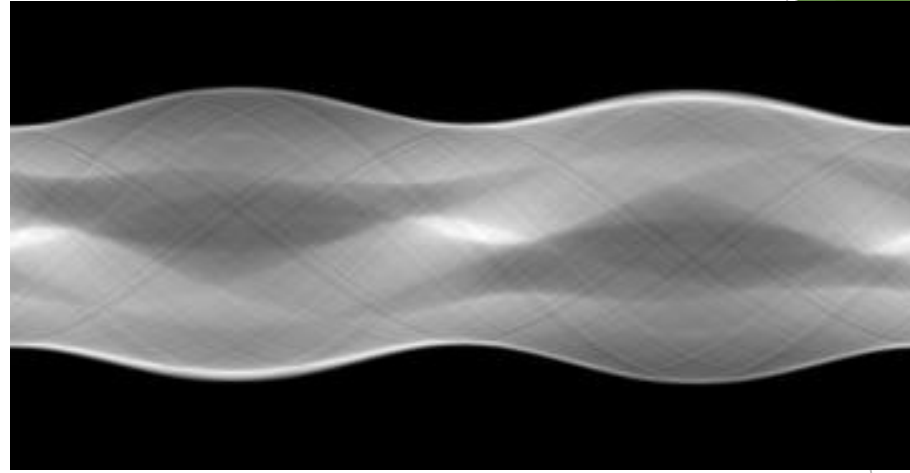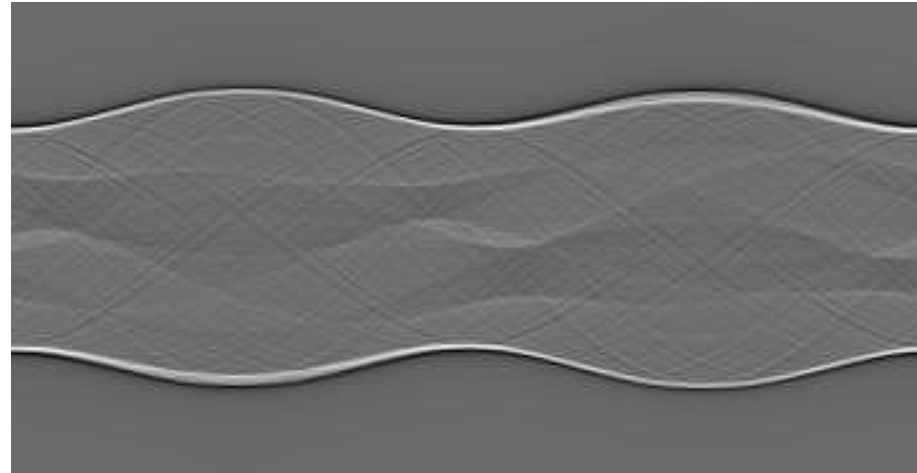

High-pass filter

High frequencies pass and are amplified while low frequencies are de-emphasized
Remove high frequencies ☐ blur effect

Remove low frequencies ☐ sharpen effect
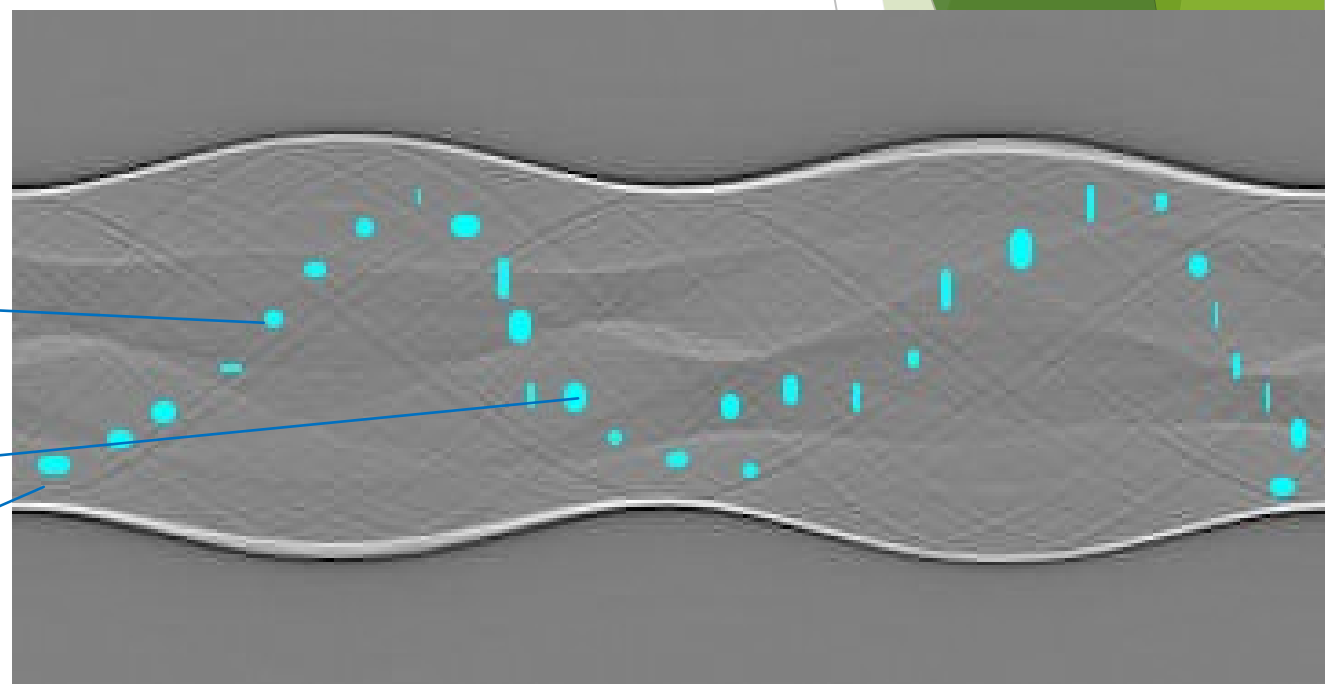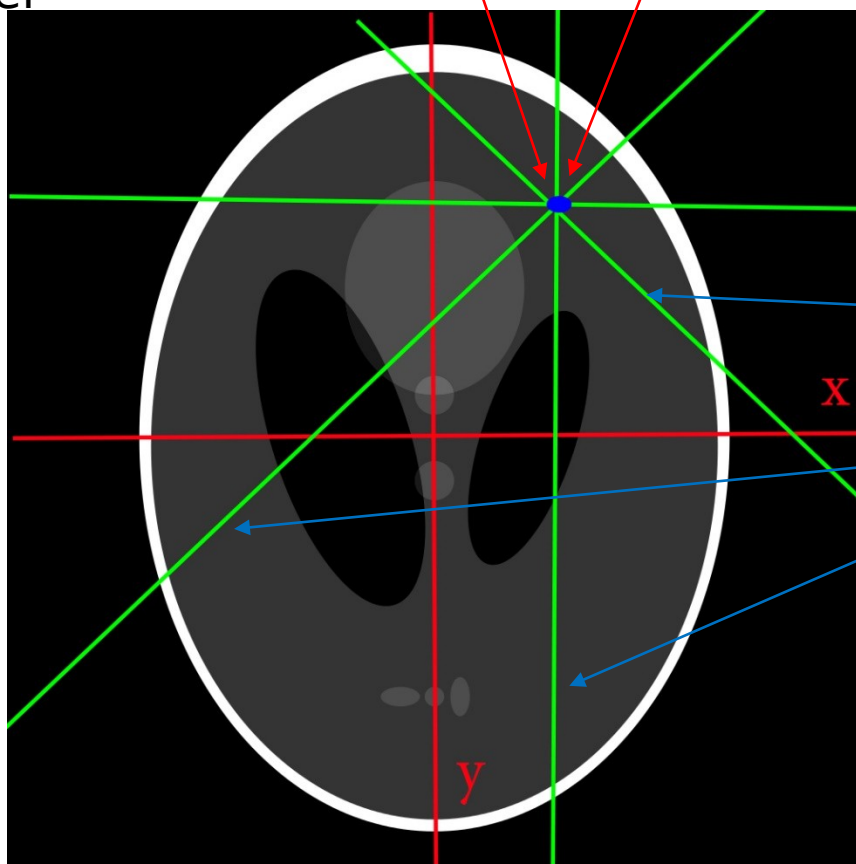
Unfiltered sinogram

Filtered sinogram

Back Projection

$$Bh(x,y) := \frac{1}{\pi} \int_{\theta=0}^{\theta=\pi} h(x\cos(\theta) + y\sin(\theta)), \theta)d\theta$$
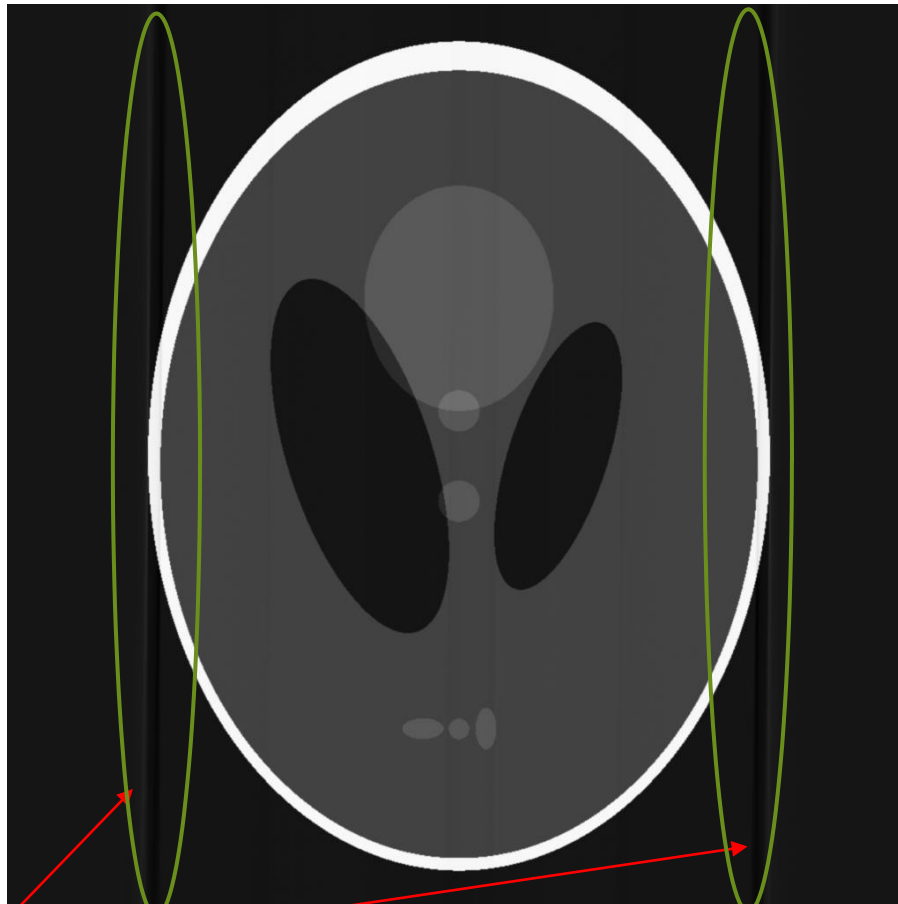
Sinogram

Reconstructed pixel

Final Results

Back projected image without using filter |w| Back projected image using filter |w|

Noise artifact residues

Summary