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Recommended Citation

O'Dwyer, Aidan: A summary of PI and PID controller tuning rules for processes with time delay. Part 2: PID controller tuning rules. Proceedings of PID '00: IFAC Workshop on Digital Control, pp. 242-247, Terrassa, Spain, April 4-7, 2000.

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A SUMMARY OF PI AND PID CONTROLLER TUNING RULES FOR PROCESSES WITH TIME DELAY. PART 2: PID CONTROLLER TUNING RULES.

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Abstract: The ability of proportional integral (PI) and proportional integral derivative (PID) controllers to compensate many practical industrial processes has led to their wide acceptance in industrial applications. The requirement to choose either two or three controller parameters is perhaps most easily done using tuning rules. A summary of tuning rules for the PID control of single input, single output (SISO) processes with time delay is provided in this paper. Copyright ©2000 IFAC

Keywords: PID controllers, rules, time delay.

1. INTRODUCTION

This paper summarises some of the most directly applicable tuning rules for PID controllers that have been developed to compensate SISO processes with time delay, modeled in either first order lag plus delay (FOLPD) form or integral plus delay (IPD) form. It is a companion paper to that of O'Dwyer (2000a) and the two papers have similar structure. A comprehensive summary of PID controller tuning rules for processes with time delay is available from the author (O'Dwyer, 2000b).

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s)$$
 (1)

with K_c = proportional gain, T_i = integral time and T_d = derivative time. Many tuning rules have been defined for this PID structure. Tuning rules have also been defined for a range of alternative PID controller structures. One example of such structure is the 'classical' form of the PID controller:

$$G_{c}(s) = K_{c} \left(1 + \frac{1}{T_{i}s} \right) \left(\frac{1 + T_{d}s}{1 + T_{d}s/N} \right)$$
 (2)

Tuning rules for these and other such PID controller structures are explicitly indicated; in all cases, numerical data is quoted to a maximum of two places of decimals. Most authors recommend application of the tuning rules for a range of model time delay to time constant $(\tau_{\rm m}/T_{\rm m})$ between 0.1 and 1.0; this data, together with other relevant comments, is provided by O'Dwyer (2000b). Results from the analytical calculation of robustness criteria associated with a number of tuning rules, for a range of $\tau_{\rm m}/T_{\rm m}$ values, are presented in Section 4. A list of symbols and abbreviations used in the paper is provided in the appendix.

2. PID TUNING RULES – $\frac{K_m e^{-s\tau_m}}{1 + sT_m}$ MODEL

Rule	K _c	T_{i}	T_d		
<u>Ideal controller</u> – $G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s)$					
Process reaction					
Ziegler and Nichols (1942)	$\frac{aT_{m}}{K_{m}\tau_{m}}$ $a = [1.2,2]$	$2 au_{ m m}$	$0.5 au_{ m m}$		

Rule	K _c	T_{i}	T_{d}
Astrom and Hagglund (1995)	$\frac{0.94T_{m}}{K_{m}\tau_{m}}$	$2 au_{ m m}$	0.5T _m
Chien <i>et</i> al. (1952) –regulator – 0% o.s.	$\frac{0.95T_{_{m}}}{K_{_{m}}\tau_{_{m}}}$	$2.38 \tau_{m}$	$0.42 au_{ m m}$
Chien <i>et</i> al. (1952) – regulator – 20% o.s.	$\frac{1.2T_{_{m}}}{K_{_{m}}\tau_{_{m}}}$	$2 au_{ m m}$	0.42τ _m
Chien <i>et</i> al. (1952) – servo – 0% o.s.	$\frac{0.6T_{_{m}}}{K_{_{m}}\tau_{_{m}}}$	T_{m}	0.5t _m
Chien et al. (1952) - servo - 20% o.s.	$\frac{0.95T_{m}}{K_{m}\tau_{m}}$	1.36T _m	0.47t _m
Cohen and Coon (1953)	¹ K _c ⁽¹⁾	$T_i^{(1)}$	T _d (1)
	Reg	gulator	
Murrill (1967) – min. IAE	$\frac{1.44}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.92}$	$\frac{T_{\rm m}}{0.88} \bigg(\frac{T_{\rm m}}{\tau_{\rm m}}\bigg)^{0.75}$	$0.48T_m \left(\frac{\tau_m}{T_m}\right)^{1.14}$
Murrill (1967) – min. ISE	$\frac{1.50}{K_{m}} \bigg(\frac{T_{m}}{\tau_{m}}\bigg)^{0.95}$	$\frac{T_m}{1.10} \bigg(\frac{T_m}{\tau_m}\bigg)^{0.77}$	$0.56T_m \bigg(\frac{\tau_m}{T_m}\bigg)^{1.01}$
Zhuang and Atherton ²	$\frac{1.47}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.97}$	$\frac{T_{\rm m}}{1.12} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.75}$	$0.55T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.95}$
(1993) – min. ISE	$\frac{1.52}{K_m} \bigg(\frac{T_m}{\tau_m}\bigg)^{0.74}$	$\frac{T_{\rm m}}{1.13} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.64}$	$0.55T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.85}$
Murrill (1967) – min. ITAE	$\frac{1.36}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.95}$	$\frac{T_{\scriptscriptstyle m}}{0.84} \bigg(\frac{T_{\scriptscriptstyle m}}{\tau_{\scriptscriptstyle m}}\bigg)^{0.74}$	$0.38T_m \bigg(\frac{\tau_m}{T_m}\bigg)^{1.00}$
Zhuang and Atherton ²	$\frac{1.47}{K_m} \bigg(\frac{T_m}{\tau_m}\bigg)^{0.97}$	$\frac{T_{\rm m}}{0.94} \bigg(\frac{T_{\rm m}}{\tau_{\rm m}}\bigg)^{0.73}$	$0.44T_m \left(\frac{\tau_m}{T_m}\right)^{0.94}$
(1993) – min. ISTSE	$\frac{1.52}{K_m} \bigg(\frac{T_m}{\tau_m}\bigg)^{0.73}$	$\frac{T_{\scriptscriptstyle m}}{0.96} \bigg(\frac{T_{\scriptscriptstyle m}}{\tau_{\scriptscriptstyle m}}\bigg)^{0.60}$	$0.44 T_m \left(\frac{\tau_m}{T_m}\right)^{0.85}$

$$^{1} \ K_{c}^{(1)} = \frac{1}{K_{m}} \left(1.35 \frac{T_{m}}{\tau_{m}} + 0.25 \right), T_{i}^{(1)} = T_{m} \left(\frac{2.5 \frac{\tau_{m}}{T_{m}} + 0.46 \left(\frac{\tau_{m}}{T_{m}} \right)^{2}}{1 + 0.61 \frac{\tau_{m}}{T_{m}}} \right)^{2}$$

 $T_d^{(1)} = 0.37 \tau_m / (1 + 0.2 [\tau_m / T_m])$

 2 For $0.1 \! \leq \! \frac{\tau_{_m}}{T_{_m}} \! \leq \! 1$ and $1.1 \! \leq \! \frac{\tau_{_m}}{T_{_m}} \! \leq \! 2$, respectively

Rule	17	T	Т
	K _c	T _i	T _d
Zhuang and Atherton ²	$\frac{1.53}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.96}$	$\frac{T_{m}}{0.97} \bigg(\frac{T_{m}}{\tau_{m}}\bigg)^{0.75}$	$0.41T_m \left(\frac{\tau_m}{T_m}\right)^{0.93}$
(1993) – min. ISTES	$\frac{1.59}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.71}$	$\frac{T_{m}}{0.96} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.60}$	$0.41T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.85}$
	S	ervo	L
Rovira et	$\frac{1.09}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.87}$	$T_{\rm m}$	$(\tau)^{0.91}$
al. (1969)	$\frac{1.05}{K_m} \left(\frac{r_m}{\tau_m} \right)$	$\frac{T_{\rm m}}{0.74 - 0.13 \frac{\tau_{\rm m}}{T_{\rm m}}}$	$0.35T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.57}$
– min.	m · m·	T _m	, m
IAE			
Zhuang	$\frac{1.05}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.90}$	$T_{\rm m}$	$0.49T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{0.89}$
and Atherton ²	$\overline{K_{m}}(\overline{\tau_{m}})$	$\frac{1.20 - 0.37 \frac{\tau_{\rm m}}{T_{\rm m}}}{1.20 - 0.37 \frac{\tau_{\rm m}}{T_{\rm m}}}$	$0.49 I_{\rm m} \left(\overline{T_{\rm m}} \right)$
(1993) –	$1.15 (T_m)^{0.57}$	$T_{\rm m}$	$(\tau_m)^{0.71}$
min. ISE	$\frac{1.15}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.57}$	$\frac{T_{\rm m}}{1.05 - 0.22 \frac{\tau_{\rm m}}{T_{\rm m}}}$	$0.49T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{3.77}$
Rovira et	$\frac{0.97}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.85}$	$T_{\rm m}$	(τ) 0.93
al. (1969)	$\frac{0.57}{K_{m}} \left(\frac{r_{m}}{\tau_{m}} \right)$	$\frac{T_{\rm m}}{0.80 - 0.15 \frac{\tau_{\rm m}}{T}}$	$0.31T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.93}$
– min.	m < m	$T_{\rm m}$	\ m/
ITAE			
Zhuang	$\frac{1.04}{K} \left(\frac{T_{\rm m}}{\tau}\right)^{0.90}$	T _m	$0.39T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{0.91}$
and	$\overline{K_{m}}(\overline{\tau_{m}})$	$\frac{T_{m}}{0.99 - 0.24 \frac{\tau_{m}}{T_{m}}}$	$0.391_{\rm m} \left(\overline{T_{\rm m}} \right)$
Atherton ²	0.50	m	0.04
(1969) –	$\frac{1.14}{K_{\rm m}} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.58}$	$\frac{T_m}{0.92-0.17\frac{\tau_m}{T_m}}$	$0.38T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{0.84}$
min. ISTSE	$K_{m} \setminus \tau_{m}$	$0.92 - 0.17 \frac{\tau_{\rm m}}{T}$	$T_{\rm m}$
13131		1 m	
Zhuang	0.97 (т.)	T _m	(τ) 0.89
and	$\frac{0.97}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.90}$	$\frac{1}{0.98-0.25\frac{\tau_{\rm m}}{T}}$	$0.32T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{0.89}$
Atherton ²	ιτ _m (τ _m)	$0.98 - 0.23 {T_{\rm m}}$	(I _m)
(1969) –	106 (T) 0.58	$T_{\rm m}$	(τ) 0.83
min.	$\frac{1.06}{K_{\rm m}} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.38}$	$\frac{T_{\rm m}}{0.89 - 0.17 \frac{\tau_{\rm m}}{T_{\rm m}}}$	$0.32T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{3.32}$
ISTES	m · m·	I _m	· m·
		synthesis	
Smith and		T	0.5
Corripio	$T_{\rm m}$	T_{m}	$0.5\tau_{\mathrm{m}}$
(1985) –	$K_{_m}\tau_{_m}$		
regulator Smith and			
Corripio	5T	т	0.5τ _m
(1985) –	$\frac{5T_{\rm m}}{cV}$	$T_{\rm m}$	U.Jt _m
servo	$6K_{m}\tau_{m}$		
Smith and			
Corripio	$T_{\rm m}$	$T_{\rm m}$	$0.5\tau_{\mathrm{m}}$
(1985) –	$\frac{1}{2K_{m}\tau_{m}}$	III	""
servo –	ZIX _m v _m		
5% o.s.			
Abbas			$T_m \tau_m$
(1997)	$K_c^{(2) 3}$	$T_{\rm m} + 0.5\tau_{\rm m}$	$2T_m + \tau_m$

$$^{3} \ K_{c}^{\ (2)} = \frac{0.18 + 0.35 \bigg(\frac{\tau_{m}}{T_{m}}\bigg)^{-1.00}}{K_{m}(0.53 - 0.36V^{0.71})} \ ,$$

$$0 \leq V \leq 0.2 \ , \ V = overshoot$$

ı		t	 			
Rule	\mathbf{K}_{c}	T_{i}	T_d			
	Re	obust				
Fruehauf	5T _m					
et al.	$\frac{\overline{9\tau_{\rm m}K_{\rm m}}}{2\tau_{\rm m}K_{\rm m}}$	$5\tau_{ m m}$	$\leq 0.5\tau_{\mathrm{m}}$			
(1993)						
	$T_{\rm m}$	т	$\leq 0.5\tau_{\rm m}$			
	$2\tau_m K_m$	$T_{\rm m}$	≥ 0.5 t _m			
	Ultim	ate cycle				
Zhuang		$0.05T_{\rm u}$				
and	$0.51K_u$	$(3.30K_{m}K_{u} + 1)$	$0.13T_{\rm u}$			
Atherton		servo				
(1993) –	4	SCIVO				
min.	$\mathbf{K}_{c}^{(3)}$	T _i ⁽³⁾	0.14T _u			
ISTSE	\mathbf{K}_{c}		O.I T I _u			
	G1 : 1	regulator				
		controller –				
($G(s) = K \int_{1-s}^{1}$	$-\frac{1}{T_{i}s}\Bigg)\left(\frac{1+T_{d}}{1+T_{d}s}\right)$	<u>s_</u>)			
`	$S_c(s) - K_c(1)$	$T_i s / (1 + T_d s)$	'N)			
	Proces	s reaction				
Hang et al.	0.83T _m	1.5t _m	$0.25\tau_{\mathrm{m}}$,			
(1993)		1.5 v _m	N = 10			
()	$K_{\scriptscriptstyle m}\tau_{\scriptscriptstyle m}$		N = 10			
Witt and	aT	$ au_{\mathrm{m}}$	$ au_{ m m}$			
Waggoner	$\frac{aT_{m}}{K_{m}\tau_{m}}$,	· m	N = [10,20]			
(1990)			11 - [10,20]			
	a = [0.6,1]	<u> </u> gulator				
Vove and			(> 0.90			
Kaya and Scheib	$\frac{0.98}{K_{\rm m}} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.76}$	$\frac{T_{\rm m}}{0.91} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{1.05}$	$0.60T_m \left(\frac{\tau_m}{T_m}\right)^{0.90}$			
(1988) –	$K_{m} (\tau_{m})$	$0.91 (\tau_{\rm m})$				
min. IAE			N = 10			
Kaya and	112 (T) 0.90	T. (T.) 0.95	(7)0.88			
Scheib	$\frac{1.12}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.90}$	$\frac{T_{\rm m}}{0.80} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.95}$	$0.55T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.88}$			
(1988) –	κ _m (ι _m)	0.80 (t _m)				
min. IAE			N = 10			
Kaya and	$\frac{0.78}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{1.06}$	т (т)	(τ)1.04			
Scheib	$\frac{K_m}{K_m} \left(\frac{\tau_m}{\tau_m} \right)$	$\frac{T_{\rm m}}{1.14} \left(\frac{T_{\rm m}}{\tau_{\rm m}}\right)^{0.71}$	$0.57T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{1.04}$			
(1988) –		· · · · · · · · · · · · · · · · · · ·	N = 10			
min. ITAE			11 = 10			
Servo						
Kaya and	$\frac{0.65}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{1.04}$	$T_{\rm m}$	$0.51T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{1.08}$			
Scheib	$K_{m} (\overline{\tau_{m}})$	$0.99 + 0.10 \frac{\tau_{\rm m}}{T_{\rm m}}$	$\overline{T_m}$			
(1988) –		1 _m	N = 10			
min. IAE	, . 102		086			
Kaya and	$\frac{0.72}{K_{}} \left(\frac{T_{\rm m}}{\tau_{}} \right)^{1.03}$		$0.55T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{0.86}$			
Scheib (1988) –	$K_{m} \setminus \tau_{m}$	$\frac{T_{\rm m}}{1.13 - 0.18 \frac{\tau_{\rm m}}{T_{\rm m}}}$	m(T _m)			
min. ISE		* m	N = 10			
Kaya and	112 (7) 0.80	Т	(\ \1.01			
Scheib	$\frac{1.13}{K_{m}} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.80}$		$0.43T_{\rm m} \left(\frac{\tau_{\rm m}}{T}\right)^{1.01}$			
(1988) –	$K_{m} \setminus \tau_{m}$	$\frac{T_{\rm m}}{1.00 + 0.03 \frac{\tau_{\rm m}}{T_{\rm m}}}$	$\langle T_{\rm m} \rangle$			
(1700)	(1700) - m					
		_				

$$\begin{split} \overline{K_c^{(3)}} &= \frac{4.43 K_m K_u - 0.97}{5.12 K_m K_u + 1.73} \, K_u \,, \\ T_i^{(3)} &= \frac{1.75 K_m K_u - 0.61}{3.78 K_m K_u + 1.39} \, T_u \end{split}$$

min. ITAE					N = 10		
Rule	K _c		T_{i}		$T_{\rm d}$		
Direct synthesis							
Tsang and	1				0.5	т	
Rad			$T_{\rm m}$		$0.5\tau_{\rm m}$,		
(1995)	$K_{m}\tau_{m}$					N = 5	
Tsang et		Γ _m	Т	m	$0.25\tau_{\mathrm{m}}$,		
al. (1993)	K _m	τ_{m}			N = 2.5		
	a	ξ	a	ξ	aξ		
	1.68	0.0	0.86	0.4	0.54	0.8	
	1.38	0.1	0.75	0.5	0.50	0.9	
	1.16	0.2	0.67	0.6	0.46	1.0	
	0.99	0.3 R (0.60 obust	0.7			
	1 (T	1	0.5	τ _m ,	
Chien	$\frac{1}{K_{m}} \left(\frac{1}{\lambda} \right)$	$\frac{I_{\rm m}}{+0.5\tau}$	•	m			
(1988)					N = 10		
(/		$_{\rm m}$, $T_{\rm m}$]	0.5	,	_		
	$\frac{1}{K_m} \left(\frac{1}{\lambda} \right)$	$0.5\tau_{\rm m}$	0.5	$\tau_{_{ m m}}$	T_{m} ,		
					N =	: 10	
	$\lambda = [\tau$						
	1 .		ate cyc				
Shinskey	0.95T _m		1.43	$3\tau_{ m m}$	$0.52\tau_{\mathrm{m}}$		
(1988)	0.95T _m	$/K_{\rm m}\tau_{\rm m}$	1.17	$'\tau_{ m m}$	0.48		
min. IAE	$1.14T_{\rm m}$	$/K_{m}\tau_{m}$	1.03	$8\tau_{ m m}$	0.40	$0\tau_{\mathrm{m}}$	
– regulator	1.39T _m	$/K_{\rm m}\tau_{\rm m}$	0.77	$7\tau_{ m m}$	0.35	$5\tau_{ m m}$	
– varying							
$\tau_{\rm m}/T_{\rm m}$		1		11			
	,	dustrial				`	
U(s) =	$= K_c \left(1 - \frac{1}{c} \right)$	$+\frac{1}{T_i s}$	R(s) -	$\frac{1 + T_d}{1 + T_d s_d}$	$\frac{1}{N}$ Y(s)		
		Reg	ulator				
Kaya and Scheib $\frac{0.91}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.79}$				Γ.,) 1.00	0.54T _m	τ) 0.78	
Scheib	\overline{K}_{m}	$\left(\frac{m}{r_{\rm m}}\right)$	1.01	<u>"</u>	0.54T _m	$\frac{\overline{T_{m}}}{T_{m}}$	
(1988) –					N = 10		
min. IAE		0.00					
Kaya and	1.11	$\Gamma_{\underline{m}}$ 0.90	$\frac{T_{m}}{0.93} \left(\frac{T_{m}}{\tau_{m}}\right)^{0.88}$		$0.57T_{m} \left(\frac{\tau_{m}}{T_{m}}\right)^{0.91}$		
Scheib	K _m \ 1	t _m)	$\overline{0.93} \left(\overline{\tau_{\rm m}} \right)$		$O_{m} \left(\overline{T_{m}} \right)$		
(1988) – min. ISE					N = 10		
Kaya and	071(D 0.89	Tr. (1	0.99	(0.97	
Scheib	$\frac{0.71}{K_{m}}$	1 _m	$\frac{T_{\rm m}}{1.03} \left(\frac{T_{\rm m}}{\tau} \right)$	<u>m</u>	$0.60T_{\rm m}$	$\frac{\tau_{\rm m}}{T}$	
(1988) –	II. m	m /	1.03 ()	'm /		*m/	
min. ITAE					N =	: 10	
		S	ervo				
Kaya and	0.82 (T_m	T	m		τ_m	
Scheib	$\frac{0.82}{K_m}$	<u>π</u>	1.09 – 0	$0.22 \frac{\tau_{\rm m}}{T_{\rm m}}$	$0.44T_{\rm m}$	$\left(\frac{\overline{T_{m}}}{T_{m}}\right)$	
(1988) –				T_{m}	N =	: 10	
min. IAE		0.04				0.50	
Kaya and	$\frac{1.14}{K_{m}} \left(\frac{7}{1} \right)$	$\left(\frac{\Gamma_{\rm m}}{\Gamma_{\rm m}}\right)^{0.94}$	T	$\frac{m}{0.35 \frac{\tau_m}{T_m}}$	035T	$\left(\frac{\tau_{\rm m}}{\tau_{\rm m}}\right)^{0.78}$	
Scheib	K _m \	r _m /	0.99 – 0	$0.35\frac{\tau_{\rm m}}{T}$	$0.35T_{\rm m} \left(\frac{\tau_{\rm m}}{T_{\rm m}}\right)^{0.78}$		
(1988) –				1 _m	N =	: 10	
min. ISE	- /		т		,	1.11	
Kaya and	0.83	$\Gamma_{\rm m}$		<u>π</u>	$0.44T_{\rm m}$	$ \tau_{\rm m} $	
-	17	_ '			0rm1	T !	
Scheib (1988) –	$\frac{0.83}{K_{\rm m}} \left(\frac{1}{1} \right)$	τ _m /	1.00 + 0	$\frac{1}{0.01 \frac{\tau_{\rm m}}{T_{\rm m}}}$	N =	· m·	

3. PID TUNING RULES – $\frac{K_m e^{-s\tau_m}}{s}$ MODEL

Rule	K _c	T_{i}	T_d			
Ideal	<u>Ideal controller</u> $G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s)$					
	Process reaction					
Ford (1953)	$\frac{1.48}{K_m\tau_m}$	$2\tau_{_{m}}$	$0.37\tau_{\mathrm{m}}$			
Astrom and Hagglund (1995)	$\frac{0.94}{K_{\rm m}\tau_{\rm m}}$	$2 au_{ m m}$	0.5τ _m			
	Direct	synthesis				
Cluett and	$\frac{0.96}{\mathrm{K_m}\tau_{\mathrm{m}}}$	$3.04\tau_{\rm m}$	$0.39\tau_{\mathrm{m}}$			
Wang (1997) – designed	$\frac{0.62}{K_{m}\tau_{m}}$	5.26τ _m	0.26t _m			
closed loop time constant	$\frac{0.47}{K_{\rm m}\tau_{\rm m}}$	$7.23\tau_{\mathrm{m}}$	$0.21 au_{ m m}$			
equals $\tau_{\rm m}$ to $6\tau_{\rm m}$,	$\frac{0.38}{\mathrm{K_{m}}\tau_{\mathrm{m}}}$	9.19τ _m	$0.17 au_{_{ m m}}$			
respectively	$\frac{0.31}{\mathrm{K_{m}}\tau_{\mathrm{m}}}$	11.16τ _m	$0.15\tau_{m}$			
	$\frac{0.27}{K_{m}\tau_{m}}$	$13.14\tau_{\rm m}$	$0.13\tau_{\mathrm{m}}$			
	Classical	controller -				
$G_{c}(s) = K_{c} \left(1 + \frac{1}{T_{i}s} \right) \left(\frac{1 + T_{d}s}{1 + T_{d}s/N} \right)$						
Regulator						
Shinskey (1996) –	0.93	$1.57\tau_{m}$	$0.56\tau_{\rm m}$			
min. IAE	$K_{m}\tau_{m}$					
	0.93	1.60τ _m	0.58τ _m ,			
Shinskey (1994) –	$K_{m}\tau_{m}$		N = 10			
(1994) – min. IAE	0.93	1.48t _m	$0.63\tau_{\mathrm{m}}$,			
	$K_{m}\tau_{m}$		N = 20			

4. SIMULATION RESULTS

Space considerations dictate that only representative simulation results may be provided. In these results, approximate gain margin and phase margin are analytically calculated, using the method outlined by Ho, *et al.* (1996), for processes compensated using an appropriately tuned PID controller. The MATLAB package has been used in the simulations. The same tuning rules are used in Figures 1 and 2; similarly, the same tuning rules are used in Figures 3 and 4, and in Figures 5 and 6.

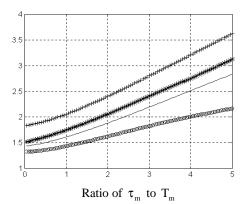


Figure 1: Gain margin

- = Ziegler-Nichols (1942)
- + = Astrom-Hagglund (1995)
- o = Cohen-Coon (1953)
- * = Chien et al. (1952) reg 20% o.s.

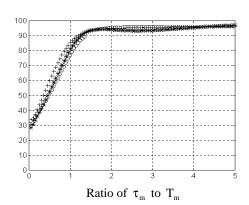


Figure 2: Phase margin

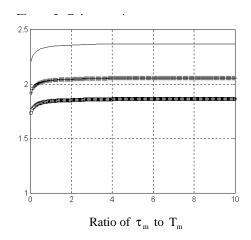
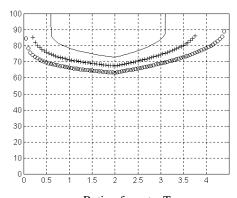
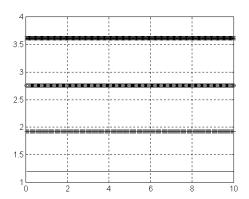


Figure 3: Gain margin

- = Abbas (1997) 0% o.s.
- + = Abbas (1997) 10% o.s.
- o = Abbas (1997) 20% o.s.



 $\begin{array}{c} \text{Ratio of } \tau_{\scriptscriptstyle m} \text{ to } T_{\scriptscriptstyle m} \\ \text{Ratio of } \tau_{\scriptscriptstyle m} \text{ to } T_{\scriptscriptstyle m} \\ \text{Figure 4: Phase margin} \end{array}$



 $\begin{array}{c} \text{Ratio of } \tau_{\scriptscriptstyle m} \text{ to } T_{\scriptscriptstyle m} \\ \text{Ratio of } \tau_{\scriptscriptstyle m} \text{ to } T_{\scriptscriptstyle m} \\ \text{Figure 5: Gain margin} \end{array}$

- = Tsang et al. (1993) - ξ = 0.1 + = Tsang et al. (1993) - ξ = 0.4 o = Tsang et al. (1993) - ξ = 0.7 * = Tsang et al. (1993) - ξ = 1.0

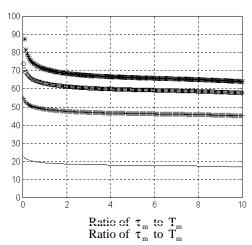


Figure 6: Phase margin

These simulations reveal the following:

- (1) Typically, the analytical calculation of the phase margin is real (and positive) in a restricted range of ratios of τ_m/T_m ; the range allowed is very limited for many tuning rules. Typically, the gain margin is real and positive over a much wider range.
- (2) The process reaction curve tuning rule of Cohen and Coon (1953) gives rise to a smaller gain margin (and approximately equal phase margin) to that of Ziegler and Nichols (1942), indicating that the closed loop response associated with the application of the former tuning rule may be expected to be more oscillatory. This is compatible with application experience.
- (3) Both the gain and phase margins are larger for the tuning rule of Abbas (1997), when the design criteria is to achieve 0% overshoot in the closed loop response, compared to when the design criterion is to achieve 20% overshoot. This is as expected.
- (4) The tuning method of Tsang *et al.* (1993) gives a constant gain margin and an almost constant phase margin. The nature of this tuning rule has interesting similarities to the tuning rules that give rise to constant gain and phase margins when a PI controller is used (O'Dwyer, 2000a). It is also clear that the tuning rules may be used at ratios of $\tau_{\rm m}/T_{\rm m}$ outside the normally recommended range of 0.1 to 1.0.
- (5) If the data in Figures 1 and 2 is compared with the corresponding data (O'Dwyer, 2000a), it is clear that the gain margin of the PID controller is significantly lower than that of the corresponding PI controller, when the Ziegler and Nichols (1942) tuning rules are used. The phase margin is also mostly higher for the PI controller. This indicates that the PID controller should offer a faster response (to a step input in servo mode, for example). Similar comments apply for many other tuning rules. A fuller panorama of simulation results show that stability tends to be assured when a PI controller tuning rule is used. Thus, a cautious design approach is to use a PI controller, with an appropriate tuning rule, particularly at larger ratios of time delay to time constant.

5. CONCLUSIONS

A large number of PID controller tuning rules have been defined in the literature to compensate SISO processes with time delays. The paper has presented a flavour of the variety of tuning rules defined. Some results associated with the analytical calculation of the gain margin and phase margin of compensated delayed systems, as the ratio of time delay to time constant varies, have also been presented. Future work will concentrate on further analytical evaluation of the robustness of delayed processes compensated using tuning rule based PID controllers.

- Astrom, K.J. and Hagglund, T. (1995). *PID Controllers: Theory, Design and Tuning*, page 139, Instrument Society of America, Research Triangle Park, North Carolina, 2nd Edition.
- Chien, K.-L., Hrones, J.A. and Reswick, J.B. (1952).
 On the automatic control of generalised passive systems. *Transactions of the ASME*, **February**, pp. 175-185.
- Chien, I.-L. (1988). IMC-PID controller design an extension. Proceedings of the IFAC Adaptive Control of Chemical Processes Conference, pp. 147-152, Copenhagen, Denmark.
- Cohen, G.H. and Coon, G.A. (1953). Theoretical considerations of retarded control. *Transactions of* the ASME, May, pp. 827-834.
- Cluett, W.R. and Wang, L. (1997). New tuning rules for PID control. *Pulp and Paper Canada*, **3**, pp. 52-55.
- Ford, R.L. (1953). The determination of the optimum process-controller settings and their confirmation by means of an electronic simulator. *Proceedings of the IEE, Part 2*, **101**, April, pp. 141-155 and pp. 173-177.
- Fruehauf, P.S., Chien, I.-L. and Lauritsen, M.D. (1993). Simplified IMC-PID tuning rules. Proceedings of the ISA/93 Advances in Instrumentation and Control Conference, McCormick Place, Chicago, Illinois, pp. 1745-1766.
- Hang, C.C., Lee, T.H. and Ho, W.K. (1993). *Adaptive Control*, page 76, Instrument Society of America, Research Triangle Park, North Carolina.
- Ho, W.K., Gan, O.P., Tay, E.B. and Ang, E.L. (1996). Performance and gain and phase margins of well-known PID tuning formulas. *IEEE Transactions on Control Systems Technology*, 4, pp. 473-477.
- Kaya, A. and Scheib, T.J. (1988). Tuning of PID controls of different structures. Control Engineering, July, pp. 62-65.
- Murrill, P.W. (1967). *Automatic control of processes*. International Textbook Co.
- O'Dwyer, A. (2000a). A summary of PI and PID controller tuning rules for processes with time delay: Part 1: PI controller tuning rules. *Proceedings of IFAC Workshop on Digital Control*, Terrassa, Spain.
- O'Dwyer, A. (2000b). PI and PID controllers for time delay processes: a summary. *Technical Report AOD/00/01*, Dublin Institute of Technology, http://www.docsee.kst.ie/aodweb/.
- Rovira, A.A., Murrill, P.W. and Smith, C.L. (1969). Tuning controllers for setpoint changes. *Instruments and Control Systems*, 42, **December**, pp. 67-69.
- Shinskey, F.G. (1988). Process Control Systems Application, Design and Tuning. McGraw-Hill Inc., New York, 3rd Edition.

- Shinskey, F.G. (1994). Feedback controllers for the process industries. McGraw-Hill Inc., New York.
- Shinskey, F.G. (1996). Process Control Systems -Application, Design and Tuning. McGraw-Hill Inc., New York, 4th Edition.
- Smith, C.A. and Corripio, A.B. (1985). Principles and practice of automatic process control, John Wiley and Sons, New York.
- Tsang, K.M., Rad, A.B. and To, F.W. (1993). Online tuning of PID controllers using delayed state variable filters, *Proceedings of the IEEE Region 10 Conference on Computer, Communication, Control and Power Engineering*, **4**, pp. 415-419.
- Tsang, K.M. and Rad, A. B. (1995). A new approach to auto-tuning of PID controllers. *International Journal of Systems Science*, **26**, pp. 639-658.
- Witt, S.D. and Waggoner, R.C. (1990). Tuning parameters for non-PID three-mode controllers, *Hydrocarbon Processing*, **June**, pp. 74-78.
- Zhuang, M. and Atherton, D.P. (1993). Automatic tuning of optimum PID controllers. *IEE Proceedings, Part D*, **140**, pp. 216-224.
- Ziegler, J.G. and Nichols, N.B. (1942). Optimum settings for automatic controllers. *Transactions of* the ASME, November, pp. 759-768.

APPENDIX: LIST OF SYMBOLS AND ABBREVIATIONS USED

- $G_c(s) = PID$ controller transfer function
- IAE = integral of absolute error, ISE = integral of squared error
- ISTES = integral of squared time multiplied by error, all to be squared
- ISTSE = integral of squared time multiplied by squared error
- ITAE = integral of time multiplied by absolute error
- K_c = Proportional gain of the controller, K_m = Gain of the process model
- N = Indication of the amount of filtering on the derivative term
- o.s. = overshoot
- R(s) = Desired variable
- T_d = Derivative time of the controller, T_i = Integral time of the controller
- $T_{\!\!\!\mbox{\scriptsize m}}$ = Time constant of the process model, $T_{\!\!\!\mbox{\scriptsize u}}$ = Ultimate time
- U(s) = manipulated variable, Y(s) = controlled variable
- λ = Parameter that determines robustness of compensated system.
- ξ = damping factor of the compensated system
- τ_m = time delay of the process model

Tn