

# Extensiones del modelo lineal: modelos aditivos

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## 1 Modelo

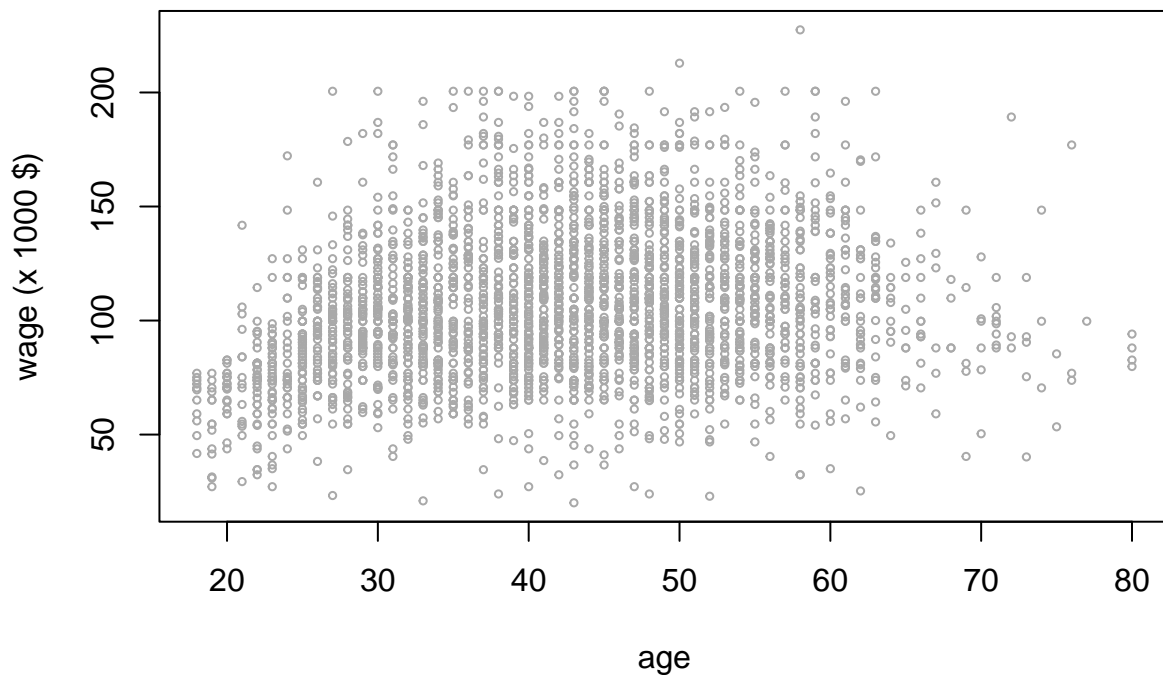
- Los extensiones del modelo lineal estudiados hasta ahora utilizan solo un regresor.
- Los Modelos Aditivos constituyen la manera natural de extender el modelo:
- Datos:  $\{y_i, x_{1i}, x_{2i}, \dots, x_{pi}\}$ ,  $i = 1, \dots, n$

$$y_i = \beta_0 + f(x_{1i}) + f(x_{2i}) + \dots + f(x_{pi}) + u_i$$

donde cada  $f(x_i)$  puede ser un polinomio, un spline, un término  $\beta_k x_{ki}$ , una interacción,...

## 2 Ejemplo

```
d = read.csv("datos/Wage.csv")
d = d[d$wage<250,]
plot(d$age,d$wage, cex = 0.5, col = "darkgrey", ylab = "wage (x 1000 $)", xlab = "age")
```



Queremos trabajar con el modelo:

$$wage \sim \beta_0 + f_1(year) + f_2(age) + f_3(education) + \epsilon$$

- Cuando las  $f_i$  están dadas en términos de **funciones base**, se puede estimar el modelo utilizando mínimos cuadrados:

```
m1 = lm(wage ~ poly(age, degree = 3) + education, data = d)
summary(m1)
```

```
##
## Call:
## lm(formula = wage ~ poly(age, degree = 3) + education, data = d)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-100.235	-16.770	-0.634	16.194	90.930

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	85.371	1.622	52.621	< 2e-16 ***
poly(age, degree = 3)1	307.473	26.666	11.530	< 2e-16 ***
poly(age, degree = 3)2	-346.911	26.641	-13.022	< 2e-16 ***
poly(age, degree = 3)3	92.928	26.569	3.498	0.000476 ***
education2. HS Grad	9.971	1.832	5.442	5.71e-08 ***
education3. Some College	21.329	1.930	11.049	< 2e-16 ***
education4. College Grad	33.318	1.925	17.311	< 2e-16 ***
education5. Advanced Degree	48.243	2.125	22.705	< 2e-16 ***

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.53 on 2913 degrees of freedom
## Multiple R-squared:  0.3119, Adjusted R-squared:  0.3102
## F-statistic: 188.6 on 7 and 2913 DF,  p-value: < 2.2e-16
```

```
m2 = lm(wage ~ year + poly(age, degree = 3) + education, data = d)
summary(m2)
```

```
##
## Call:
## lm(formula = wage ~ year + poly(age, degree = 3) + education,
##     data = d)
##
## Residuals:
```

	Min	1Q	Median	3Q	Max
	-103.84	-16.77	-0.81	15.76	92.14

```
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-2175.3992	485.2418	-4.483	7.64e-06 ***
year	1.1271	0.2419	4.659	3.32e-06 ***
poly(age, degree = 3)1	302.7188	26.5913	11.384	< 2e-16 ***
poly(age, degree = 3)2	-350.2735	26.5566	-13.190	< 2e-16 ***
poly(age, degree = 3)3	96.7958	26.4880	3.654	0.000262 ***
education2. HS Grad	9.9366	1.8257	5.443	5.69e-08 ***
education3. Some College	21.3851	1.9237	11.117	< 2e-16 ***
education4. College Grad	33.2497	1.9179	17.336	< 2e-16 ***

```
## education5. Advanced Degree    48.1041    2.1174  22.718  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.43 on 2912 degrees of freedom
## Multiple R-squared:  0.317, Adjusted R-squared:  0.3151
## F-statistic: 168.9 on 8 and 2912 DF,  p-value: < 2.2e-16

library(splines)
m3 = lm(wage ~ ns(year, df = 4) + poly(age, degree = 3) + education, data = d)
summary(m3)

##
## Call:
## lm(formula = wage ~ ns(year, df = 4) + poly(age, degree = 3) +
##     education, data = d)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -103.784  -16.784   -0.805   15.869   92.258
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)      81.415      1.909  42.648 < 2e-16 ***
## ns(year, df = 4)1       7.482      2.642   2.832 0.004663 **
## ns(year, df = 4)2       3.338      2.254   1.481 0.138761
## ns(year, df = 4)3       8.609      3.204   2.686 0.007262 **
## ns(year, df = 4)4       6.178      1.826   3.383 0.000726 ***
## poly(age, degree = 3)1  303.571     26.597  11.414 < 2e-16 ***
## poly(age, degree = 3)2 -349.741     26.587 -13.155 < 2e-16 ***
## poly(age, degree = 3)3   97.189     26.499   3.668 0.000249 ***
## education2. HS Grad      9.858      1.827   5.397 7.33e-08 ***
## education3. Some College 21.240      1.926  11.030 < 2e-16 ***
## education4. College Grad 33.223      1.919  17.310 < 2e-16 ***
## education5. Advanced Degree 48.033      2.118  22.680 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 26.43 on 2909 degrees of freedom
## Multiple R-squared:  0.3177, Adjusted R-squared:  0.3152
## F-statistic: 123.2 on 11 and 2909 DF,  p-value: < 2.2e-16
```

- Comparamos los tres modelos con el contraste de la F:

```
anova(m1,m2)

## Analysis of Variance Table
##
## Model 1: wage ~ poly(age, degree = 3) + education
## Model 2: wage ~ year + poly(age, degree = 3) + education
##   Res.Df    RSS Df Sum of Sq    F    Pr(>F)
## 1     2913 2049905
## 2     2912 2034737   1     15168 21.707 3.319e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Los modelos no son equivalentes, y como en m2 ha salido significativo el coeficiente de year, nos quedamos con m2.

```
anova(m2,m3)
```

```
## Analysis of Variance Table
```

```
##
```

```
## Model 1: wage ~ year + poly(age, degree = 3) + education
```

```
## Model 2: wage ~ ns(year, df = 4) + poly(age, degree = 3) + education
```

```
##   Res.Df      RSS Df Sum of Sq    F Pr(>F)
```

```
## 1    2912 2034737
```

```
## 2    2909 2032438  3      2299.3 1.097 0.3491
```

Los modelos son equivalentes, nos quedamos con m2 ya que tiene menos parámetros.