Comparación de modelos y selección de variables

Contents

1 Introducción

```r
d = read.csv("datos/Hitters.csv")
d = d[,-1]
str(d)
```

```
## 'data.frame':    322 obs. of  20 variables:
##  $ AtBat    : int  293 315 479 496 321 594 185 298 323 401 ...
##  $ Hits     : int  66 81 130 141 87 169 37 73 81 92 ...
##  $ HmRun    : int  1 7 18 20 10 4 1 0 6 17 ...
##  $ Runs     : int  30 24 66 65 39 74 23 24 26 49 ...
##  $ RBI      : int  29 38 72 78 42 51 8 24 32 66 ...
##  $ Walks    : int  14 39 76 37 30 35 21 7 8 65 ...
##  $ Years    : int  1 14 3 11 2 11 2 3 2 13 ...
##  $ CAtBat   : int  293 3449 1624 5628 396 4408 214 509 341 5206 ...
##  $ CHits    : int  66 835 457 1575 101 1133 42 108 86 1332 ...
##  $ CHmRun   : int  1 69 63 225 12 19 1 0 6 253 ...
##  $ CRuns    : int  30 321 224 828 48 501 30 41 32 784 ...
##  $ CRBI     : int  29 414 266 838 46 336 9 37 34 890 ...
##  $ CWalks   : int  14 375 263 354 33 194 24 12 8 866 ...
##  $ League   : chr  "A" "N" "A" "N" ...
##  $ Division : chr  "E" "W" "W" "E" ...
##  $ PutOuts  : int  446 632 880 200 805 282 76 121 143 0 ...
##  $ Assists  : int  33 43 82 11 40 421 127 283 290 0 ...
##  $ Errors   : int  20 10 14 3 4 25 7 9 19 0 ...
##  $ Salary   : num  NA 475 480 500 91.5 750 70 100 75 1100 ...
##  $ NewLeague: chr  "A" "N" "A" "N" ...
```

Comprobamos si hay missing obsevations (NA) en el salario:

```r
sum(is.na(d$Salary))
```

```
## [1] 59
```

Eliminamos estos datos:

```r
d = na.omit(d)
```

2 Comparación de modelos

Se pueden utilizar las siguientes métricas para comparar modelos:

- R-cuadrado
- Residual Sum of Squares, RSS: sum((observed - predicted)ˆ2).
- Mean Squared Error, MSE = mean((observed - predicted)ˆ2). Cuanto menor sea el MSE, mejor.
- Residual Standard Error, RSE = sum((observed - predicted)ˆ2)/(n-k-1).
- Mean Absolute Error, MAE = mean(abs(observed - predicted)).

El problema con estas métricas es que dependen del número de regresores considerados. Por tanto se pueden utilizar para **comparar modelos con el mismo número de regresores**. Otras métricas que no tienen este problema son:

- Akaike Information criteria:

$$AIC = \frac{1}{n\hat{\sigma}^2}(RSS + 2k\hat{\sigma}^2)$$

- Estadístico Cp de Mallows:

$$Cp = \frac{1}{n}(RSS + 2k\hat{\sigma}^2)$$

Cp y AIC son proporcionales, $C_p = AIC * \hat{\sigma}^2$.

- Bayesian Information Criteria:

$$BIC = \frac{1}{n}(RSS + \log(n)k\hat{\sigma}^2)$$

- R-cuadrado ajustado:

$$R^2 - ajustado = 1 - \frac{RSS/(n-k-1)}{TSS/(n-1)}$$

donde:

- k: número de regresores.
- $\hat{\sigma}^2$: estimación del error del modelo, la varianza residual.
- RSS: Residual sum of squares

$$RSS = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

- TSS: Total Sum of Squares

$$TSS = \sum_{i=1}^{n}(y_i - \bar{y}_i)^2$$

Por último, se puede utilizar el método del subconjunto de validación y el de validación cruzada para comparar modelos, sobre todo **desde un punto de vista predictivo**.

3 Métodos de construcción de modelos a partir de un conjunto de variables

3.1 Selección de variables significativas

Algoritmo:

1. Mp es el modelo con todos los regresores.
2. Para k = p, . . . , 1
   a. Se estima el modelo con k regresores, Mk.
   b. Se elimina la variable con mayor pvalor de los contrastes individuales.
3. Elegir el modelo con el mayor número de regresores significativos.

Importante: solo se puede eliminar un regresor no significativo en cada iteración.

```r
m_1 = lm(Salary ~ ., data = d)
summary(m_1)
```

```
##
## Call:
## lm(formula = Salary ~ ., data = d)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -907.62 -178.35  -31.11  139.09 1877.04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 163.10359   90.77854   1.797 0.073622 .
## AtBat        -1.97987    0.63398  -3.123 0.002008 **
## Hits          7.50077    2.37753   3.155 0.001808 **
## HmRun         4.33088    6.20145   0.698 0.485616
## Runs         -2.37621    2.98076  -0.797 0.426122
## RBI          -1.04496    2.60088  -0.402 0.688204
## Walks         6.23129    1.82850   3.408 0.000766 ***
## Years        -3.48905   12.41219  -0.281 0.778874
## CAtBat       -0.17134    0.13524  -1.267 0.206380
## CHits         0.13399    0.67455   0.199 0.842713
## CHmRun       -0.17286    1.61724  -0.107 0.914967
## CRuns         1.45430    0.75046   1.938 0.053795 .
## CRBI          0.80771    0.69262   1.166 0.244691
## CWalks       -0.81157    0.32808  -2.474 0.014057 *
## LeagueN      62.59942   79.26140   0.790 0.430424
## DivisionW  -116.84925   40.36695  -2.895 0.004141 **
## PutOuts       0.28189    0.07744   3.640 0.000333 ***
## Assists       0.37107    0.22120   1.678 0.094723 .
## Errors       -3.36076    4.39163  -0.765 0.444857
## NewLeagueN  -24.76233   79.00263  -0.313 0.754218
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 315.6 on 243 degrees of freedom
## Multiple R-squared:  0.5461, Adjusted R-squared:  0.5106
## F-statistic: 15.39 on 19 and 243 DF,  p-value: < 2.2e-16
```

```r
m_2 = lm(Salary ~ . - CHmRun, data = d)
summary(m_2)
```

```
##
## Call:
## lm(formula = Salary ~ . - CHmRun, data = d)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -914.51 -175.66  -31.72  137.49 1876.79
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 163.08380   90.59426   1.800 0.073071 .
## AtBat        -1.97939    0.63268  -3.129 0.001970 **
## Hits          7.44499    2.31485   3.216 0.001475 **
## HmRun         4.03304    5.52892   0.729 0.466429
## Runs         -2.27127    2.80872  -0.809 0.419504
## RBI          -0.96237    2.47840  -0.388 0.698131
## Walks         6.20550    1.80884   3.431 0.000707 ***
## Years        -3.42721   12.37355  -0.277 0.782031
## CAtBat       -0.17461    0.13146  -1.328 0.185336
## CHits         0.18359    0.48861   0.376 0.707440
## CRuns         1.40160    0.56455   2.483 0.013714 *
## CRBI          0.73870    0.25026   2.952 0.003468 **
## CWalks       -0.80172    0.31424  -2.551 0.011343 *
## LeagueN      63.12305   78.94944   0.800 0.424756
## DivisionW  -116.85917   40.28499  -2.901 0.004062 **
## PutOuts       0.28224    0.07721   3.655 0.000315 ***
## Assists       0.37319    0.21986   1.697 0.090902 .
## Errors       -3.38913    4.37472  -0.775 0.439262
## NewLeagueN  -25.31356   78.67426  -0.322 0.747916
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 314.9 on 244 degrees of freedom
## Multiple R-squared:  0.5461, Adjusted R-squared:  0.5126
## F-statistic: 16.31 on 18 and 244 DF,  p-value: < 2.2e-16
```

```r
m_3 = lm(Salary ~ . - CHmRun - Years, data = d)
summary(m_3)
```

```
##
## Call:
## lm(formula = Salary ~ . - CHmRun - Years, data = d)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -912.02 -180.92  -34.89  138.05 1881.51
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 148.43333   73.41097   2.022 0.044268 *
## AtBat        -1.95091    0.62309  -3.131 0.001953 **
## Hits           7.39141    2.30241   3.210 0.001503 **
## HmRun          4.08280    5.51558   0.740 0.459869
## Runs          -2.23967    2.80111  -0.800 0.424737
## RBI           -0.99402    2.47109  -0.402 0.687845
## Walks          6.19706    1.80517   3.433 0.000701 ***
## CAtBat        -0.19133    0.11657  -1.641 0.102010
## CHits          0.20673    0.48050   0.430 0.667398
## CRuns          1.42497    0.55716   2.558 0.011144 *
## CRBI           0.74147    0.24958   2.971 0.003265 **
## CWalks        -0.80376    0.31356  -2.563 0.010966 *
## LeagueN       64.19282   78.70619   0.816 0.415521
## DivisionW   -116.06176   40.10620  -2.894 0.004148 **
## PutOuts        0.28303    0.07702   3.675 0.000292 ***
## Assists        0.37732    0.21894   1.723 0.086083 .
## Errors        -3.31999    4.35935  -0.762 0.447044
## NewLeagueN   -24.88922   78.51099  -0.317 0.751502
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 314.3 on 245 degrees of freedom
## Multiple R-squared:  0.546,  Adjusted R-squared:  0.5144
## F-statistic: 17.33 on 17 and 245 DF,  p-value: < 2.2e-16
```

Y así sucesivamente. Este método de suele utilizar cuando utilizamos el modelo para explicar relaciones entre variables, por lo que estamos interesados en variables significativas. Cuando el objetivo es predecir se utilizan los métodos que se indican a continuación.

3.2 Best subset selection

Algoritmo:

Para k = 1, 2, . . . , p:

- Estimar todos los modelos de k regresores (hay $\binom{p}{k}$ modelos posibles).
- Elegir el que tenga menor RSS o mayor R^2. Este será el modelo M_k.

```r
library(leaps)
m2 = regsubsets(Salary ~ ., data = d)
summary(m2)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = d)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 8
## Selection Algorithm: exhaustive
##          AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 ) " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 ) " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 ) "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 ) " "   "*"  " "   " "  " " "*"   " "   "*"    "*"   "*"    " "   " "
## 8  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   " "    " "   "*"    "*"   " "
##          CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 ) " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 ) " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 ) " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 ) "*"    " "     "*"       "*"     " "     " "    " "
```

El resultado son los mejores 8 modelos (por defecto):

- la primera línea es el mejor modelo (en términos de R^2) de una variable. La variable seleccionada es la que aparece con un asterisco, **CRBI**.
- la segunda linea es el mejor modelo (en términos de R^2) de dos variables, **Hits** y **CRBI**.
- y así sucesivamente.

Podemos seleccionar el numero de modelos que nos devuelve con *nvmax*:

```r
m_best = regsubsets(Salary ~ ., data = d, nvmax = 19)
summary(m_best)
```

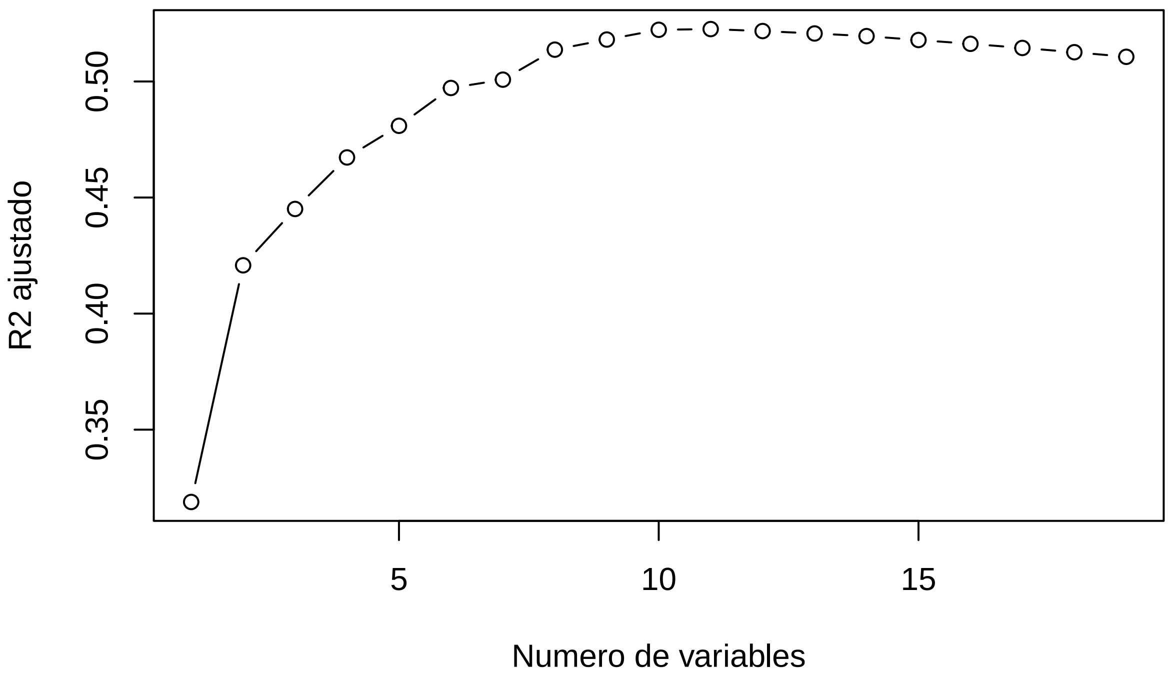
```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = d, nvmax = 19)
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: exhaustive
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  " "   "*"  " "   " "  " " "*"   " "   "*"    "*"   "*"    " "   " "
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   "*"    "*"   " "
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

Podemos trabajar con R2 o con las otras métricas:

```
m_best_summary = summary(m_best)
names(m_best_summary)
```

```
## [1] "which"  "rsq"    "rss"    "adjr2"  "cp"     "bic"    "outmat" "obj"
```

```r
plot(m_best_summary$adjr2, xlab = "Numero de variables", ylab = "R2 ajustado", type = "b")
```



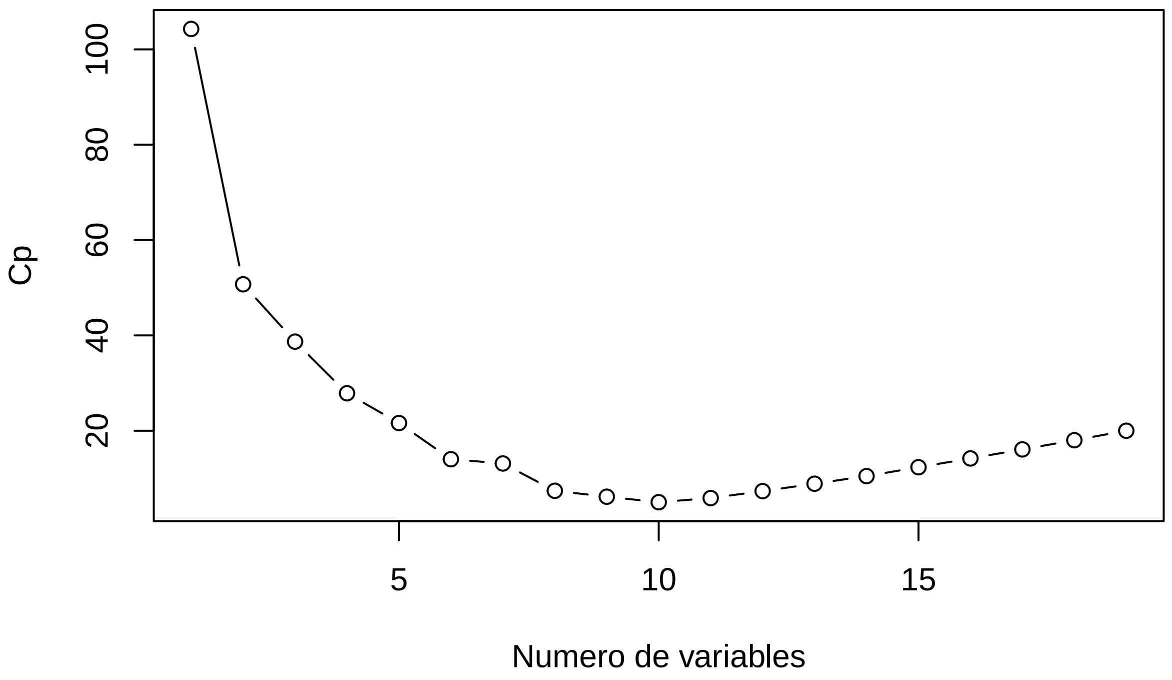
Buscamos el máximo:

```r
which.max(m_best_summary$adjr2)
```

```
## [1] 11
```

Si utilizamos el criterio del Cp (que es equivalente al AIC):

```r
plot(m_best_summary$cp, xlab = "Numero de variables", ylab = "Cp", type = "b")
```



```r
which.min(m_best_summary$cp)
```

```
## [1] 10
```

Justificación: el RSS disminuye con el número de regresores k. Por eso penalizamos incluyendo un término que contiene a k.

Los coeficinetes estimados con ese modelo son:

```r
coef(m_best,10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns 
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798    1.4082490 
##         CRBI       CWalks    DivisionW      PutOuts      Assists 
##    0.7743122   -0.8308264 -112.3800575    0.2973726    0.2831680 
```

3.3 Método Forward-Stepwise

Algoritmo:

1. M0 es el modelo sin regresores.
2. Para k = 0, ..., (p-1)
   a. A partir del modelo con k regresores, M_k, estimar todos los modelos posibles con (k+1) regresores.
   b. Elegir el que tenga menor RSS o mayor R^2. Este será el modelo M_{k+1}.
3. Elegir el mejor modelo de M0, ..., M_p utilizando validación cruzada, Cp, AIC, BIC, R2-ajustado.

```r
m_fwd = regsubsets(Salary ~ ., data = d, nvmax = 19, method = "forward")
summary(m_fwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = d, nvmax = 19, method = "forward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: forward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    " "   "*" 
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*" 
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 4  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 5  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    " "   "*"
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 7  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    " "   "*"
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   "*"
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*"
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*"
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*"
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*"
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*"
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```r
m_fwd_summary = summary(m_fwd)
which.min(m_fwd_summary$cp)
```

```
## [1] 10
```

```r
coef(m_fwd,10)
```

```
##   (Intercept)          AtBat           Hits          Walks         CAtBat          CRuns
##   162.5354420     -2.1686501      6.9180175      5.7732246     -0.1300798      1.4082490
##          CRBI         CWalks      DivisionW        PutOuts        Assists
##     0.7743122     -0.8308264   -112.3800575      0.2973726      0.2831680
```

3.4 Método Backward-Stepwise

Algoritmo:

1. Mp es el modelo con todos los regresores.
2. Para k = p, . . . , 1

    a. A partir del modelo con k regresores, M_k, estimar todos los modelos posibles con (k-1) regresores.
    b. Elegir el que tenga menor RSS o mayor R^2. Este será el modelo M_{k-1}.
3. Elegir el mejor modelo de M0, …, M_p utilizando validación cruzada, Cp, AIC, BIC, R2 ajustado.

```r
m_bwd = regsubsets(Salary ~ ., data = d, nvmax = 19, method = "backward")
summary(m_bwd)
```

```
## Subset selection object
## Call: regsubsets.formula(Salary ~ ., data = d, nvmax = 19, method = "backward")
## 19 Variables  (and intercept)
##            Forced in Forced out
## AtBat          FALSE      FALSE
## Hits           FALSE      FALSE
## HmRun          FALSE      FALSE
## Runs           FALSE      FALSE
## RBI            FALSE      FALSE
## Walks          FALSE      FALSE
## Years          FALSE      FALSE
## CAtBat         FALSE      FALSE
## CHits          FALSE      FALSE
## CHmRun         FALSE      FALSE
## CRuns          FALSE      FALSE
## CRBI           FALSE      FALSE
## CWalks         FALSE      FALSE
## LeagueN        FALSE      FALSE
## DivisionW      FALSE      FALSE
## PutOuts        FALSE      FALSE
## Assists        FALSE      FALSE
## Errors         FALSE      FALSE
## NewLeagueN     FALSE      FALSE
## 1 subsets of each size up to 19
## Selection Algorithm: backward
##           AtBat Hits HmRun Runs RBI Walks Years CAtBat CHits CHmRun CRuns CRBI
## 1  ( 1 )  " "   " "  " "   " "  " " " "   " "   " "    " "   " "    "*"   " " 
## 2  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " " 
## 3  ( 1 )  " "   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " " 
## 4  ( 1 )  "*"   "*"  " "   " "  " " " "   " "   " "    " "   " "    "*"   " " 
## 5  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " " 
## 6  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " " 
## 7  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   " " 
## 8  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   " "    " "   " "    "*"   "*" 
## 9  ( 1 )  "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 10  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 11  ( 1 ) "*"   "*"  " "   " "  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 12  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 13  ( 1 ) "*"   "*"  " "   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 14  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    " "   " "    "*"   "*" 
## 15  ( 1 ) "*"   "*"  "*"   "*"  " " "*"   " "   "*"    "*"   " "    "*"   "*" 
## 16  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*" 
## 17  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   " "   "*"    "*"   " "    "*"   "*" 
## 18  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   " "    "*"   "*" 
## 19  ( 1 ) "*"   "*"  "*"   "*"  "*" "*"   "*"   "*"    "*"   "*"    "*"   "*" 
##           CWalks LeagueN DivisionW PutOuts Assists Errors NewLeagueN
## 1  ( 1 )  " "    " "     " "       " "     " "     " "    " "       
## 2  ( 1 )  " "    " "     " "       " "     " "     " "    " "       
## 3  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 4  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 5  ( 1 )  " "    " "     " "       "*"     " "     " "    " "
## 6  ( 1 )  " "    " "     "*"       "*"     " "     " "    " "
## 7  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 8  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 9  ( 1 )  "*"    " "     "*"       "*"     " "     " "    " "
## 10  ( 1 ) "*"    " "     "*"       "*"     "*"     " "    " "
## 11  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 12  ( 1 ) "*"    "*"     "*"       "*"     "*"     " "    " "
## 13  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 14  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 15  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 16  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    " "
## 17  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 18  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
## 19  ( 1 ) "*"    "*"     "*"       "*"     "*"     "*"    "*"
```

```
m_bwd_summary = summary(m_bwd)
which.min(m_bwd_summary$cp)
```

```
## [1] 10
```

```
coef(m_bwd,10)
```

```
##  (Intercept)        AtBat         Hits        Walks       CAtBat        CRuns
##  162.5354420   -2.1686501    6.9180175    5.7732246   -0.1300798    1.4082490
##         CRBI       CWalks    DivisionW      PutOuts      Assists
##    0.7743122   -0.8308264 -112.3800575    0.2973726    0.2831680
```

Es el mismo que antes.

3.5 Eligiendo el mejor modelo utilizando subconjuntos de validación

```
set.seed(1)
n = nrow(d)
n_train = round(0.6*n)
n_test = n - n_train

pos = 1:n
pos_train = sample(pos,n_train,replace = F) # muestreo sin reemplazamiento
pos_test = pos[-pos_train]

# dividimos los datos en training set y validation set
datos_train = d[pos_train,]
datos_test = d[pos_test,]
```

Estimamos todos los modelos posibles con los datos de entrenamiento

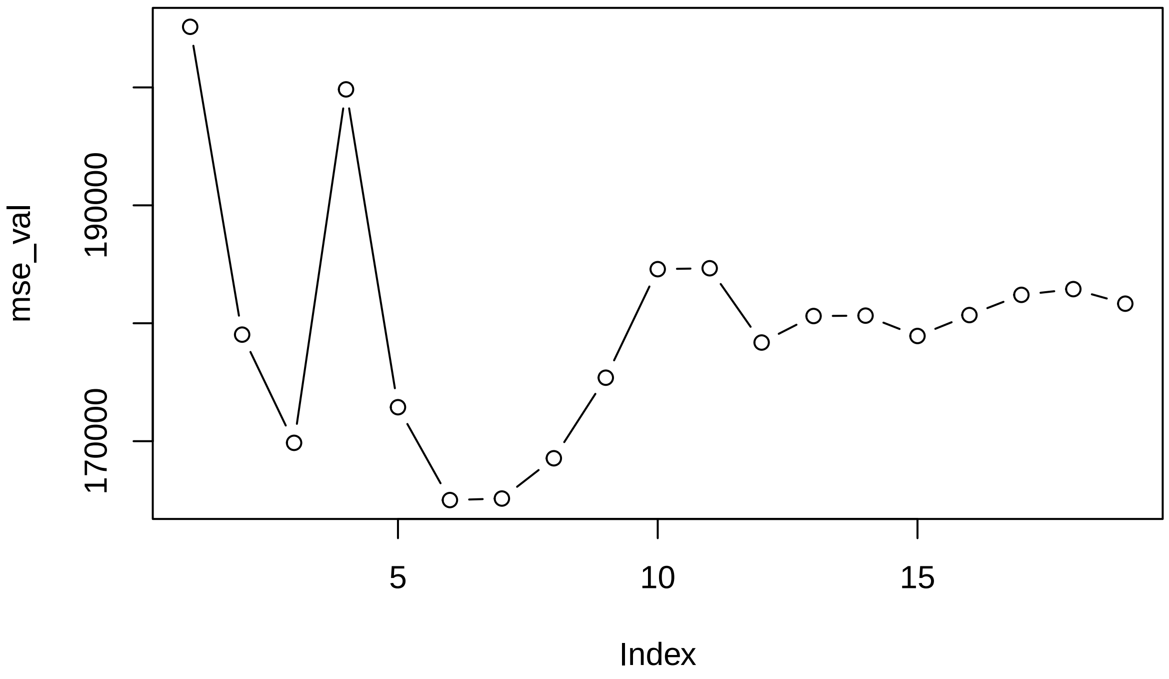
```
m_val = regsubsets(Salary ~ ., data = datos_train, nvmax = 19)
```

Vamos calcular el error de prediccion (MSE) de este modelo en los datos test. Como no existe la funcion predecir a partir de modelos estimados con *regsubsets()*, se ha programado en (descargar):

```
source("funciones/regsubsets_predict.R")
predict.regsubsets
```

```
## function (objeto_regsubsets, newdata, id)
## {
##     formu = as.formula(objeto_regsubsets$call[[2]])
##     X_mat = model.matrix(formu, newdata)
##     coefi = coef(objeto_regsubsets, id)
##     X_col = names(coefi)
##     pred = X_mat[, X_col] %*% coefi
##     return(pred)
## }
```

```
source("funciones/MSE.R")
#
mse_val = rep(0,19)
for (i in 1:19){
  pred_i = predict(m_val,datos_test,i)
  mse_val[i] = MSE(datos_test$Salary,pred_i)
}
plot(mse_val, type = "b")
```



```
which.min(mse_val)
```

```
## [1] 6
```

Para calcular los coeficientes del modelo de regresión finales, es preferible hacerlo con todos los datos:

```
m_val_final = regsubsets(Salary ~ ., data = d, nvmax = 19)
coef(m_val_final,6)
```

```
##  (Intercept)        AtBat         Hits        Walks         CRBI    DivisionW
##   91.5117981   -1.8685892    7.6043976    3.6976468    0.6430169 -122.9515338
##      PutOuts
##    0.2643076
```

3.6 Eligiendo el mejor modelo utilizando Cross-Validation

Función para obtener las posiciones de train y de test:

```r
source("funciones/validacion_cruzada.R")
```

Datos de los folds:

```r
num_folds = 10
set.seed(1)
pos = validacion_cruz_pos(nrow(d),num_folds)
```

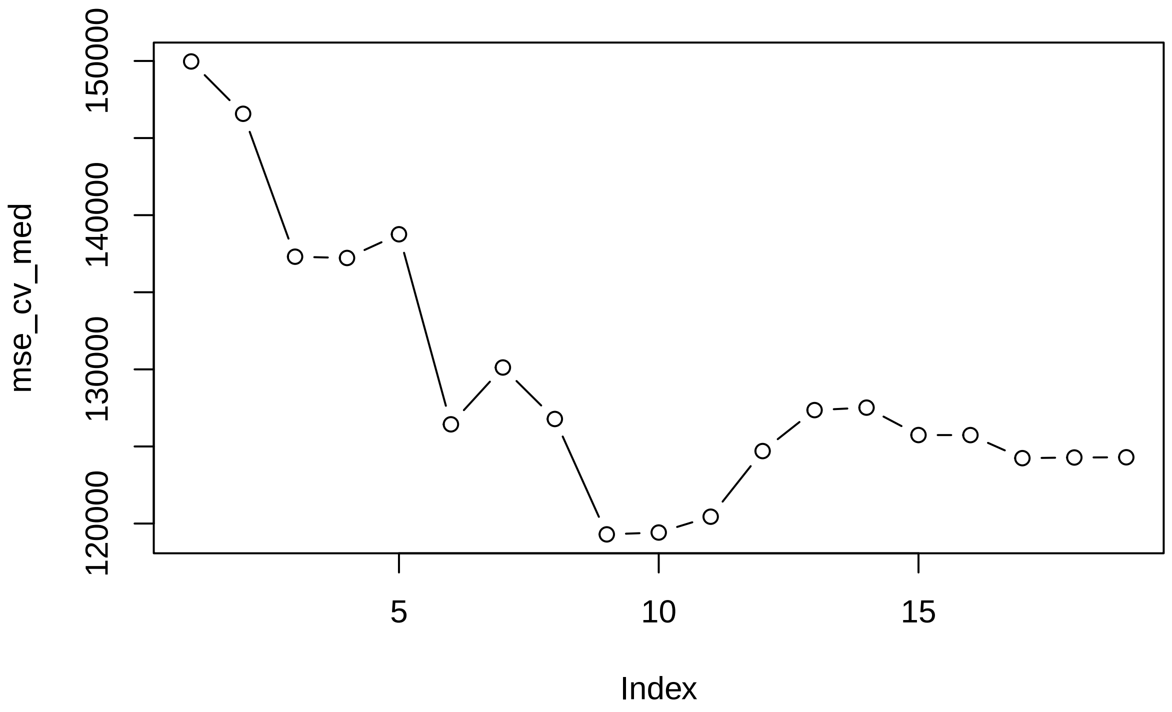
Calculamos el error cometido en cada fold por cada modelo:

```r
mse_cv = matrix(0, nrow = num_folds, ncol = 19)
for (i in 1:num_folds){
  # datos de training y de validation de cada fold
  datos_train = d[pos$train[[i]],]
  datos_test = d[pos$test[[i]],]

  m_cv = regsubsets(Salary ~ .,data = datos_train, nvmax = 19)

  for (j in 1:19){
    pred = predict(m_cv,newdata = datos_test, id = j)
    mse_cv[i,j] = MSE(datos_test$Salary,pred)
  }
}
```

```r
mse_cv_med = apply(mse_cv, 2, mean)
plot(mse_cv_med, type = "b")
```



El que tiene menor error es el de 9 variables. Lo aplicamos a todos los datos:

```r
m_cv_final = regsubsets(Salary ~ ., data = d, nvmax = 19)
coef(m_cv_final,9)
```

```
##   (Intercept)         AtBat          Hits         Walks        CAtBat
##  146.24960033   -1.93676754    6.65672102    5.55204413   -0.09953904
##         CRuns          CRBI        CWalks     DivisionW       PutOuts
##    1.25067124    0.66176849   -0.77798498 -115.34950146    0.27773062
```