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## Homework 5: Seafloor Subsidence due to cooling

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```
close all
clear all
clc
```

### Part 1 Conductive heat flow

#### Step 1: A model

Imagine an infinitely long and wide solid plate. The plate has thickness  $d$ . The temperature at the top of the plate is  $T_1$  and the temperature at the bottom of the plate is  $T_2$ . Draw a diagram of this plate and label these parameters.

```
figure
rectangle('Position',[1 3 8 3]);
axis ([0 10 0 10]);
axis off;
title('Diagram of a Solid Plate');

dThick = '\leftarrow d';
tempT1 = 'T_{1}';
tempT2 = 'T_{2}';

text(1, 4, dThick);
```

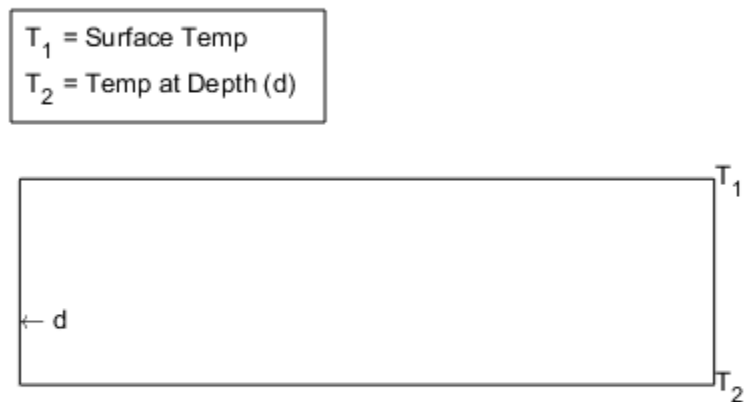
```

text(9, 6, tempT1);
text(9, 3, tempT2);

dim = [.2 .5 .3 .3];
str = {'T_{1} = Surface Temp', 'T_{2} = Temp at Depth (d)'};
annotation('textbox',dim,'String',str,'FitBoxToText','on');

```

### Diagram of a Solid Plate



## Step 2: Heat flow

The rate of heat flow per unit area ( $Q$ ) *down* through the plate is

$$Q = -k * (T_2 - T_1 / d)$$

Why does heat flow *down* in this equation?

Heat flows down because of the negative nature of  $k$ .  $Q$  is a *rate*. At the surface a greater *rate* occurs because of a greater temperature gradient than at depth where the gradient is less (everything is hotter).

## Step 3: Thermal conductivities

Thermal Conductivity Values (in  $W/m \cdot C$ )

◆ Silver: 429

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◆ Magnesium: 156

◆ Glass: 1.05

◆ Rock: 2-7

◆ Wood (oak): 0.17

## Step 4: The heat transport equation

Substitute our values into heat transport equation:

$$Q = -k * ((T(z + \delta z) - t(z)) / (Z + \delta z))$$

Write the derivative of the right hand side of the equation:

$$-k(\delta T / \delta z)$$

Heat flow equation using derivative

$$Q = -k(\delta T / \delta z)$$

%Write the same thing using the gradient operator?  
%  
%

## Step 5: The conservation equation

Insert the calculated Q (above) into the conservation equation

$$c_p \rho (\delta T / \delta t) = A - (\delta / \delta z) (-k(\delta T / \delta z))$$

Assuming that internal heat generation is zero we get:

$$c_p \rho (\delta T / \delta t) = -k(\delta / \delta z) (\delta T / \delta z)$$

By dividing  $c_p$  and  $\rho$  through we get  $\kappa = -K / c_p * \rho$  and finally:

$$(\delta T / \delta t) = k(\delta^2 T / \delta z^2)$$

What is the value of K in this case?

$$\kappa = -K / c_p * \rho$$

## Part 2 : Oceanic lithosphere cooling

Step 1: Setup the model domain and compute

---

```

depthOcean = linspace(0 , 100, 100);
timeInt = linspace(0 , 100, 100);
constantK = 31.56; %km^2/Ma
morT = 640;
%
T = zeros(length(depthOcean),length(timeInt));

tic
for it = 1 : length(timeInt);
    for iz = 1 : length(depthOcean);
        T(iz,it) = morT * erf(depthOcean(iz)/
(2*(sqrt(constantK*timeInt(it)))));
    end
end
toc
%

figure
imagesc(timeInt, depthOcean, T); axis ij;
c=colorbar;
colormap(jet)
xlabel('Time [Ma]')
ylabel ('Ocean depth [km]')
c.Label.String = ('Temperature^{o}C')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Figure out mesh grid%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% meshT = meshgrid(timeInt, depthOcean);

Elapsed time is 0.007337 seconds.

c =

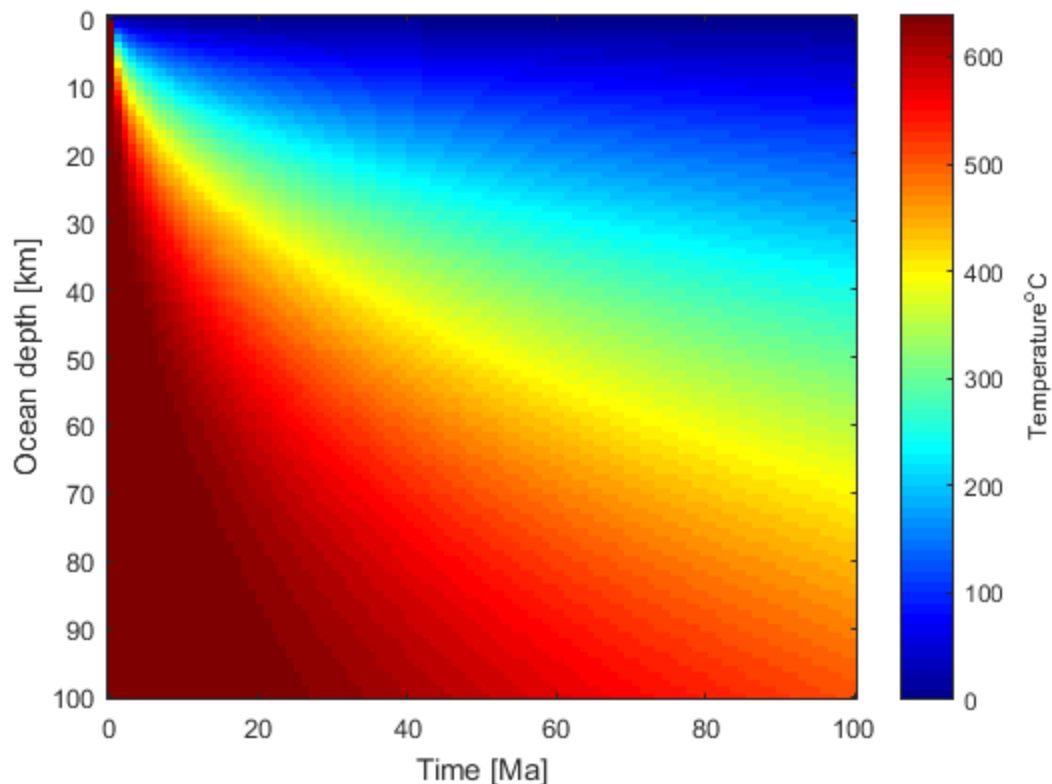
ColorBar (Temperature^{o}C) with properties:

    Location: 'eastoutside'
      Limits: [0 640]
    FontSize: 9
   Position: [0.8222 0.1095 0.0476 0.8167]
      Units: 'normalized'

Use GET to show all properties

```

---



## Step 2: Analyze the model output

Does your model make sense given the boundary conditions used to derive the solution? \bullet Yes the model makes sense given the boundary condition supplied. At  $T_{\{0\}}$  the original temp at the mid-ocean ridge is 640, as temperature decreases with depth, as we move further from the MOR our depth value increases and temperature decreases, as it does using this model.

What controls the rate at which the temperature decays? (List all things you can think of.) \bullet The rate of temperature decay is controlled by the composition/heat conductivity of the rock, the temperature at the ocean floor, and the original heat of the material controlling the temperature gradient between the rock being cooled and the ocean temperature.

How could we convert this model from age to distance from ridge axis? \bullet We could convert from age to distance from ridge axis by determining the relationship between age and distance (using velocity and time) and substitute the appropriate equation in for age related to distance. What would be a more appropriate boundary condition at  $T(z=0)$  given what we know about the oceans? \bullet A more appropriate boundary condition  $T(z=0)$  would be a pixel by pixel elevation for what  $z$  actually is over space. Is 640°C an appropriate value for the temperature at a mid-ocean ridge? Why or why not? \bullet 640°C might be an appropriate temperature for certain spots along the mid-ocean ridge however this value can vary by up to 250°C. Using one value for the entire length of the MOR simplifies the problem.

## Part 3: Plate velocity and the depths of oceans

Step 1: Load and plot sea-floor depth

---

```
load('spreadingData.mat');

fields(Bath)

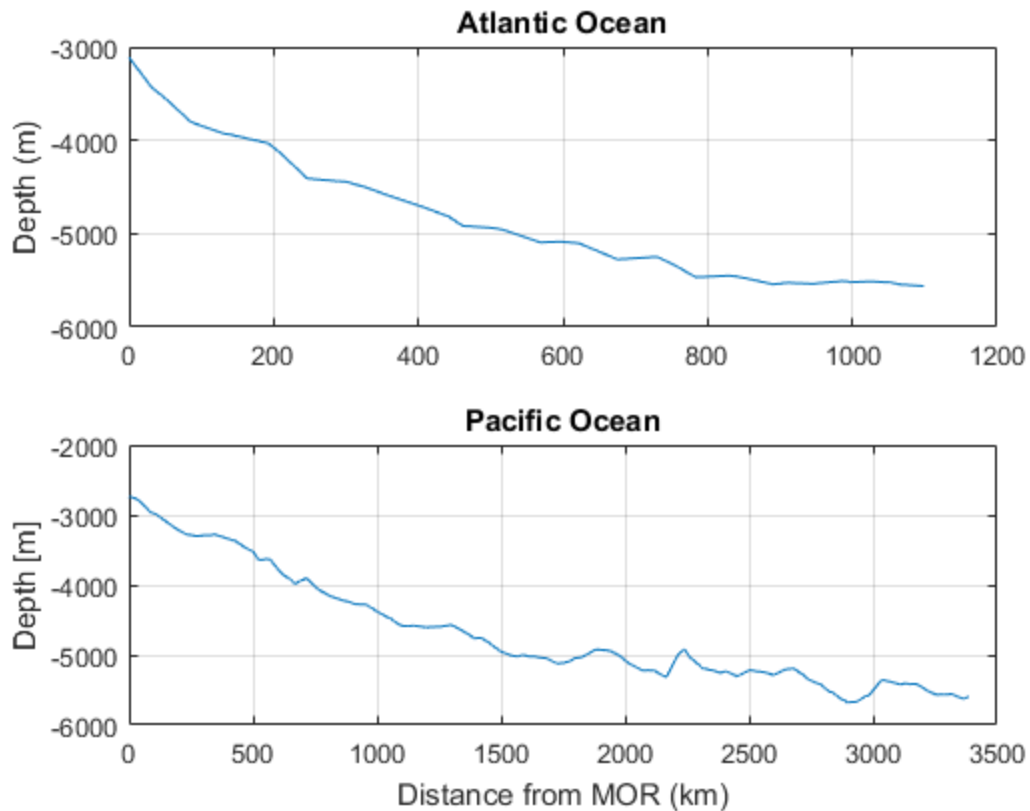
subplot (2,1,1);

plot (Bath.atlanticx,Bath.atlanticz);
ylabel ('Depth (m)')

title ('Atlantic Ocean')
grid on
hold on
subplot (2,1,2);
plot (Bath.pacificx,Bath.pacificz);
xlabel ('Distance from MOR (km)')
ylabel('Depth [m]')
title ('Pacific Ocean')
grid on

ans =

    'atlanticz'
    'atlanticx'
    'pacificx'
    'pacificz'
```



## Step 2: A half-space model

```

depthPredictPac = zeros(length(Bath.pacificx),1); % Create an empty
vector for predicted Pacific depths
Velocity = 45; %km/Ma
for ix = 1 : length(Bath.pacificx); % Run a loop to populate
depthPredictPac using the depth half-space model
    depthPredictPac(ix) = -(2.65 + .345 *(Bath.pacificx(ix)/
Velocity)^(1/2)); % run Bath.pacific through the loop
end

depthPredictAtl = zeros(length(Bath.atlanticx),1); % Create an empty
vector for predicted Pacific depths
VelocityAtl = 20
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%
% Fix Atlantic%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for ix = 1 : length(Bath.atlanticx); % Run a loop to populate
depthPredictAtl using the depth half-space model
    depthPredictAtl(ix) = -(3.12 + .345 *(Bath.atlanticx(ix)/
VelocityAtl)^(1/2)); % run Bath.atlanticx through the loop
end

figure(3)

```

---

```

subplot (2,1,1);

plot (Bath.atlanticx,Bath.atlanticz,'b');
ylabel ('Depth (m)')

title ('Atlantic Ocean')
grid on
hold on
plot(Bath.atlanticx,(depthPredictAtl*1000), 'r')

subplot (2,1,2);
plot (Bath.pacificx,Bath.pacificz,'b');
hold on
xlabel ('Distance from MOR (km)')
ylabel ('Depth (m)')
title ('Pacific Ocean')
grid on
plot(Bath.pacificx,(depthPredictPac*1000), 'r')

legend on
legend('Actual','Predicted','Location','Best')

% What does 2.65 represent in the equation above?
% 2.65 represents the death of the MOR in km's. I adjusted the value
% to
% better fit the actual depths in the Atlantic Ocean.
% What are the velocity rates that best fit each ocean?
%
% Pacific Ocean = 45 km/Ma
% Atlantic Ocean = 20 km/Ma
%
% Convert to cm/year and compare to literature

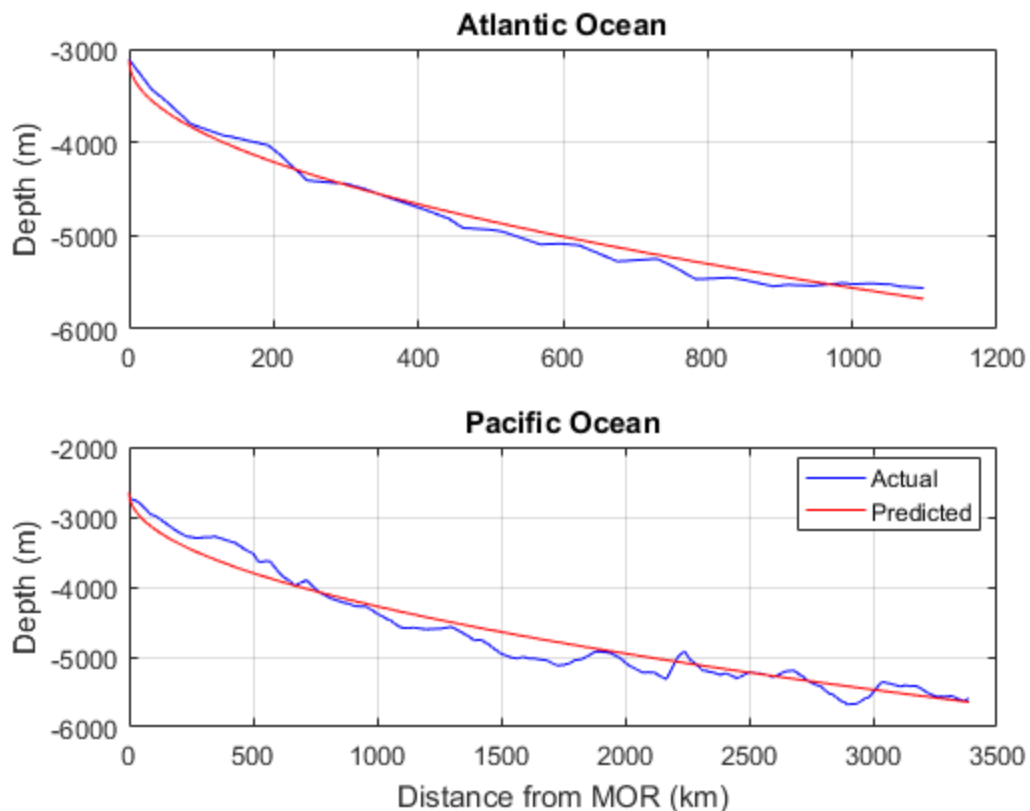
velPac = Velocity*1000/1000000; % = 0.045 cm/yr

velAtl = VelocityAtl*1000/1000000; % = 0.02 cm/yr
% Measured velocities
% The velocities vary throughout both ocean but in general the PACific
% is
% spreading from 40-100 mm/yr while the atlantic is spreading slower
% from
% 10-20 mm/yr (NOAA, 2008) . Our model shows a similar trend.

VelocityAtl =

```





## Part 4: Global oceanic plate ages

Step 1: Load topo data and plot seafloor depths

```
load('topo.mat')

load('coastlines')

[platelat, platelon] = importPlates('All_boundaries.txt');
Undefined function 'importPlates' for input arguments of type 'char'.

Error in GEOS397_HW5_Roehner (line 255)
[platelat, platelon] = importPlates('All_boundaries.txt');
```

## Step 2: Kill the topography and convert to km

```
topo (topo>0) = 0; % kill the topography
topo = topo/1000; % convert depth to km

g = figure;
hold on
g.InvertHardcopy = 'off'; %Figure background color set to black when
printing
```

---

```

g.Color = 'k'; %Set figure window background color to black
g.Position = [100 100 1000 500]; %The location and size of figure's
    drawable area set to this extent
g.PaperPositionMode = 'auto'; %Printed figure matches the displayed
    figure size
%
ax = axesm('Mollweid', 'Frame', 'on', 'Grid', 'on'); %Defined the
    projection, turned frame and grid on
setm(ax, 'MLabelLocation', 60); %Set meridian label locations to every
    60 degrees
setm(ax, 'PLabelLocation', 30); %Set parallel label locations to every
    30 degrees
mlabel('MLabelParallel', 0); %Sets the location of the parallel labels
plabel('PLabelMeridian', -25); %Sets the location of the meridian
    labels
axis('off'); %Turns off axis labeling, tickmarks and background
setm(ax, 'FontColor', [0.9 0.9 0.9]); %Sets font color to nearly white
setm(ax, 'GColor', [0.9 0.9 0.9]);
setm(ax, 'fontweight', 'bold', 'fontsize', 16);
%
plotm(coastlat, coastlon);
LAT = topolatlim(1):topolatlim(2);
LON = topolonlim(1):topolonlim(2);
%
idx180 = find(LON > 180, 1); % find first index of longitude array
    that is larger than 180 deg
LON(idx180:end) = LON(idx180:end) - 360; % subtract 360 degrees to
    make lon interval [-180 180] instead of [0 360]
%

[lon, lat] = meshgrid(LON, LAT); % compute the lat/lon of every grid
    point in topo
pcolorm(lat, lon, topo); % plot the matrix of elevations on the map
plotm(platelat, platelon)

%
c = colorbar;
colormap((jet(20))); % colorbar with jet scale.
c.Color = [1 1 1]; % colorbar text white

c.Label.String = ('Ocean depth [km]'); %colormap title

```

## Step 3: Compute seafloor age

$$d = 2.65 + 0.345t^{1/2} \quad \text{Solve for } t$$

$$t = ((d - 2.65)/0.345)^2$$

## Step 4: Plot the oceanic lithosphere age map

```
ageSeafloor= zeros(180,360);
```

---

```

depth = -topo;

for ix = 1:numel(depth);

    ageSeafloor(ix) = ((depth(ix) - 2.65)/(0.345))^2;

    if ageSeafloor(ix) == ((0- 2.65)/(0.345))^2;
        ageSeafloor(ix) = -10;
    end
end

l = figure;
hold on
l.InvertHardcopy = 'off'; %Figure background color set to black when
    printing
l.Color = 'k'; %Set figure window background color to black
l.Position = [100 100 1000 500]; %The location and size of figure's
    drawable area set to this extent
l.PaperPositionMode = 'auto'; %Printed figure matches the displayed
    figure size
%
ax = axesm('Mollweid', 'Frame', 'on', 'Grid', 'on'); %Defined the
    projection, turned frame and grid on
setm(ax, 'MLabelLocation', 60); %Set meridian label locations to every
    60 degrees
setm(ax, 'PLabelLocation', 30); %Set parallel label locations to every
    30 degrees
mlabel('MLabelParallel', 0); %Sets the location of the parallel labels
plabel('PLabelMeridian', -25); %Sets the location of the meridian
    labels
axis('off'); %Turns off axis labeling, tickmarks and background
setm(ax, 'FontColor', [0.9 0.9 0.9]); %Sets font color to nearly white
setm(ax, 'GColor', [0.9 0.9 0.9]);
setm(ax, 'fontweight', 'bold', 'fontsize', 16);
%
plotm(coastlat, coastlon);
LAT = topolatlim(1):topolatlim(2);
LON = toponlim(1):toponlim(2);
%
idx180 = find(LON > 180, 1); % find first index of longitude array
    that is larger than 180 deg
LON(idx180:end) = LON(idx180:end) - 360; % subtract 360 degrees to
    make lon interval [-180 180] instead of [0 360]
%

[lon, lat] = meshgrid(LON, LAT); % compute the lat/lon of every grid
    point in topo
pcolorm(lat, lon, ageSeafloor); % plot the matrix of elevations on the
    map
plotm(platelat, platelon, 'b')

```

---

---

```
cmap = flipud( jet(20) ); % create a flipped jet colormap
cmap = [0.5 0.5 0.5 ; cmap]; %
cmap(end,:) = []; % (1 pt.) your comment here
colormap(cmap); % (1 pt.) your comment here
cmap = colorbar;

cmap.Color = ([1 1 1]);
cmap.Label.String = ('Age [Ma]');
```

## Step 5: Discussion

```
% • Does your map of ocean ages make sense given the plate boundaries?
% • What is the oldest age in your map?
% • Where does this oldest age occur and does this make sense
  geologically?
% • Where do the youngest ages occur? Does this conform to your
  knowledge of oceanic lithosphere
% generation?
% • Are there any assumptions that have gone into this model that
  might not be accurate?
```

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