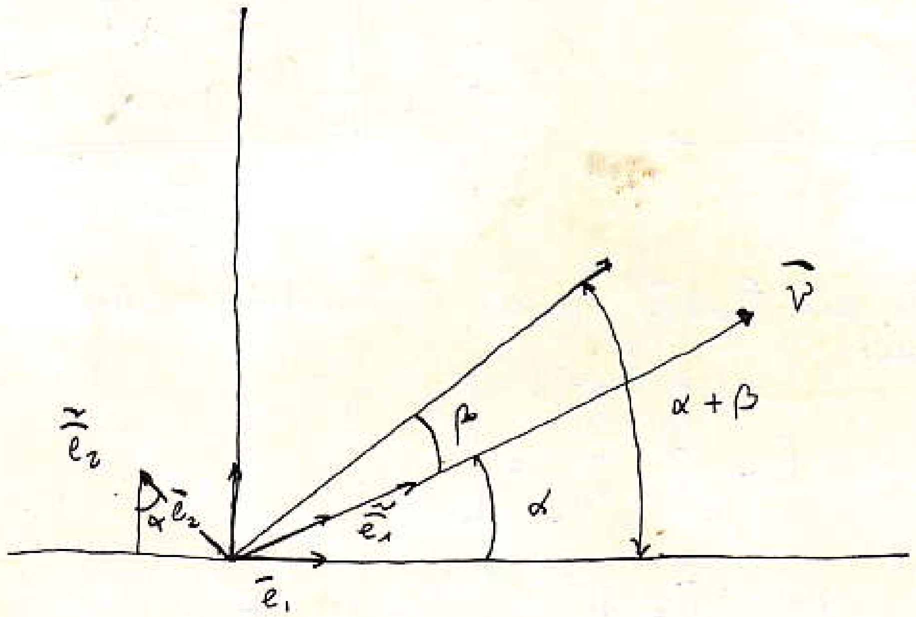
EXERCISES

find trigonometric expression for

$$\cos(\alpha+\beta) \text{ and } \sin(\alpha+\beta)$$



let's $\bar{v}$ be a vector in basis $\{\bar{e}_1, \bar{e}_2\}$ with components:

$$\bar{v} = \begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \end{bmatrix}$$

let's define new basis $\{\tilde{\bar{e}}_1, \tilde{\bar{e}}_2\}$ with components:

$$\tilde{\bar{e}}_1 = \cos\alpha\, \bar{e}_1 + \sin\alpha\, \bar{e}_2$$
$$\tilde{\bar{e}}_2 = -\sin\alpha\, \bar{e}_1 + \cos\alpha\, \bar{e}_2$$

then $\bar{v}$ can also be written as

$$\bar{v} = \cos\beta\, \tilde{\bar{e}}_1 + \sin\beta\, \tilde{\bar{e}}_1$$

Then the forward transformation F is given by:

$$F = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

and as $\bar{v}$ is an invariant vector then

$$\bar{v} = F\, \tilde{\bar{v}}$$

So

$$\begin{bmatrix} \cos(\alpha+\beta) \\ \sin(\alpha+\beta) \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$$

then, developping we obtain:

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$