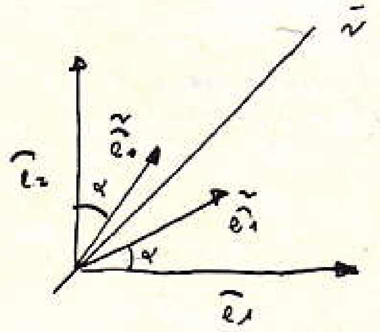

I want $\tilde{e}_1, \tilde{e}_2$ such as $\vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is a bissectrice

That means $(0, \hat{e}_1, \tilde{e}_1) = (0, \tilde{e}_2, \hat{e}_2) = \alpha$

Then we can write:

$$\tilde{e}_1 = \cos\alpha\, \hat{e}_1 + \sin\alpha\, \hat{e}_2$$
$$\tilde{e}_2 = \sin\alpha\, \hat{e}_1 + \cos\alpha\, \hat{e}_2$$

Hence we have our F transformation:

$$F = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Then let's apply our transformation rules for $\vec{v}^i$ and α_i:

$$\hat{\vec{v}}^i = B\,\vec{v}^i \quad \text{we have to find } B$$

now F is <u>not orthonormal</u> so we have to compute F^{-1}

$$|F| = \cos^2\alpha - \sin^2\alpha \quad \Rightarrow \quad F^{-1} = |F|^{-1} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = B$$

$$\hat{\vec{v}}^i = \begin{bmatrix} \cos\alpha - \sin\alpha \\ \cos\alpha - \sin\alpha \end{bmatrix} \frac{1}{\cos^2\alpha - \sin^2\alpha} = \begin{bmatrix} \dfrac{1}{\cos\alpha + \sin\alpha} \\ \dfrac{1}{\cos\alpha + \sin\alpha} \end{bmatrix}$$

for $\hat{\alpha}_i$ we apply the transformation rule, so we have

$$\hat{\alpha}_i = F\,\alpha_i$$

$$\hat{\alpha}_i = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\hat{\alpha}_i = \begin{bmatrix} \cos\alpha + \sin\alpha \,, & \sin\alpha + \cos\alpha \end{bmatrix}$$

So we can see that as F is not an orthonormal Transformation Then $\hat{\alpha}_i \not\equiv \hat{\vec{v}}^i$

So far we have visited two kinds of tensors: Vectors and Covectors. Vectors are <u>contravariant</u> since their component transform using BACKWARDS transformation, whereas covectors are covariant since their component transforms using FORWARD transformation.
Additionaly, vectors are noted $\vec{v}^i$ refering to a <u>column vector</u> (with upper-indice) and covectors are noted α_i refering to a row <u>vector</u> (with lower-indice)

In the next chapter we will see a new kind of Tensors: Linear Maps.