COMPUTER VISION II - Hom

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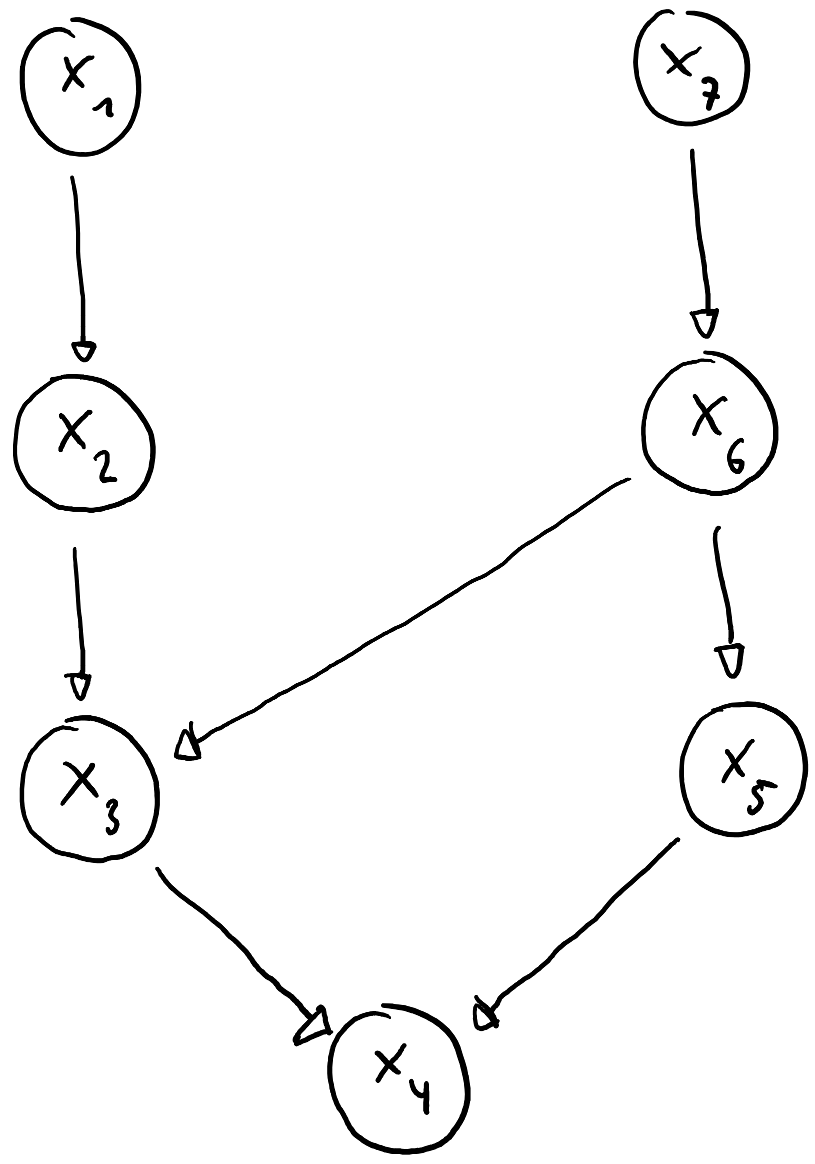
PROBLEM 1

1. Graphical models help to formalize and visualize the structure of probabilistic models. Especially, they enable to figure out the conditional independence of variables and to set up a simplified factorization of the probabilistic model (slides 8, 34).

2.



Markov blanket: $\{x_7, x_3, x_5, x_2\}$

3.

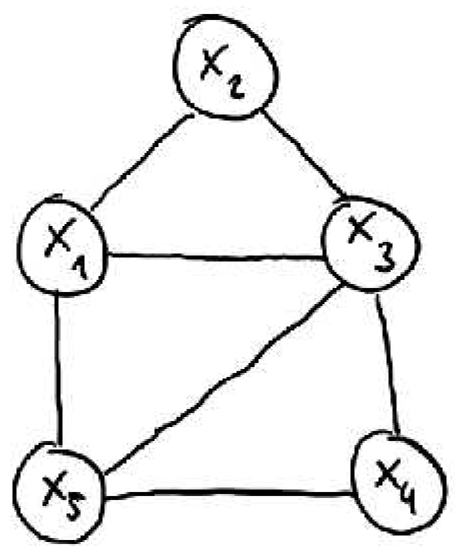
a)

$$p(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) =$$
$$p(x_1) \cdot p(x_4 | x_1) \cdot p(x_8 | x_4, x_5) \cdot$$
$$p(x_5 | x_1, x_4) \cdot p(x_6 | x_3, x_5) \cdot$$
$$p(x_3) \cdot p(x_9 | x_6, x_7) \cdot p(x_7)$$

b)

$$p(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) =$$
$$\frac{1}{Z} \cdot f_1(x_1, x_4, x_5) \cdot f_2(x_4, x_5, x_8) \cdot$$
$$f_3(x_5, x_6) \cdot f_4(x_3, x_6) \cdot$$
$$f_5(x_6, x_7, x_9)$$

4.



is x_3, x_5, x_4 not a clique?????

5.

x_1	x_2	x_3	$Z \cdot p(x_1, x_2, x_3)$	$p(x_1, x_2, x_3)$
0	0	0	$1 \cdot 1 \cdot 1 = 1$	0.034
0	0	1	$1 \cdot 0.2 \cdot 1 = 0.2$	0.007
0	1	0	$0.2 \cdot 0.5 \cdot 1 = 0.1$	0.003
0	1	1	$0.2 \cdot 3 \cdot 0.5 = 0.3$	0.010
1	0	0	$0.5 \cdot 1 \cdot 0.2 = 0.1$	0.003
1	0	1	$0.5 \cdot 0.2 \cdot 3 = 0.3$	0.010
1	1	0	$3 \cdot 0.5 \cdot 0.2 = 0.3$	0.010
1	1	1	$3 \cdot 3 \cdot 3 = 27$	0.922
			$\Sigma = Z = 29.3$	

6.

Markov Random Fields (left)

$p(x_1, x_2, x_3, x_4) =$

$$\frac{1}{Z} \cdot f_1(x_1, x_2) \cdot f_2(x_1, x_3) \cdot f_3(x_2, x_4) \cdot f_4(x_3, x_4)$$

$x_1 \perp\!\!\!\perp x_4 \mid x_2, x_3$:

use Markov blanket here?

$$p(x_1, x_4 \mid x_2, x_3) = \frac{p(x_1, x_2, x_3, x_4)}{p(x_2, x_3)}$$

slide 47

$$= \frac{p(x_1, x_2, x_3, x_4)}{\sum_{x_2} \sum_{x_3} p(x_1, x_2, x_3, x_4)}$$

$$= \frac{\frac{1}{Z} \cdot f_1(x_1, x_2) \cdot f_2(x_1, x_3) \cdot f_3(x_2, x_4) \cdot f_4(x_3, x_4)}{\sum_{x_2} \sum_{x_3} \frac{1}{Z} \cdot f_1(x_1, x_2) \cdot f_2(x_1, x_3) \cdot f_3(x_2, x_4) \cdot f_4(x}$$

$=$ ______

$\cancel{x_2 \perp\!\!\!\perp x_3 \mid x_1, x_4}$

Bayesian Network (right):

$p(x_1, x_2, x_3, x_4) =$

$$p(x_1) \cdot p(x_2|x_1) \cdot p(x_3|x_1) \cdot p(x_4|x_2, x_3)$$

$x_1 \perp\!\!\!\perp x_4 \mid x_2, x_3$:

$$p(x_1, x_4 | x_2, x_3) = \frac{p(x_1, x_2, x_3, x_4)}{p(x_2, x_3)} = \frac{"}{\sum_{x_1} p(x_1)\, p(x_2|x_1)}$$

$p(a,b) = p(b) \cdot p(a|b)$

$$= \frac{p(x_1) \cdot p(x_2|x_1) \cdot p(x_3|x_1) \cdot p(x_4|x_2, x_3)}{p(x_3) \cdot p(x_2 | x_3)}$$

oder $p(x_2) \cdot p(x_3|x_2)$

$$\cancel{=} \frac{p(x_1) \cdot p(x_2|x_1) \cdot p(x_3|x_1) \cdot \frac{p(x_2, x_3, x_4)}{p(x_2, x_3)}}{p(x_2, x_3)}$$

$x_2 \perp\!\!\!\perp x_3 \mid x_1$:

$$p(x_2, x_3 | x_1) = \frac{p(x_1, x_2, x_3)}{p(x_1)}$$

$$= \frac{\sum_{x_4} p(x_1, x_2, x_3, x_4)}{p(x_1)}$$

$$= \frac{p(x_1)\, p(x_2|x_1)\, p(x_3|x_1) \overbrace{\sum_{x_4} p(x_4|x_2, x_3)}^{1}}{p(x_1)}$$

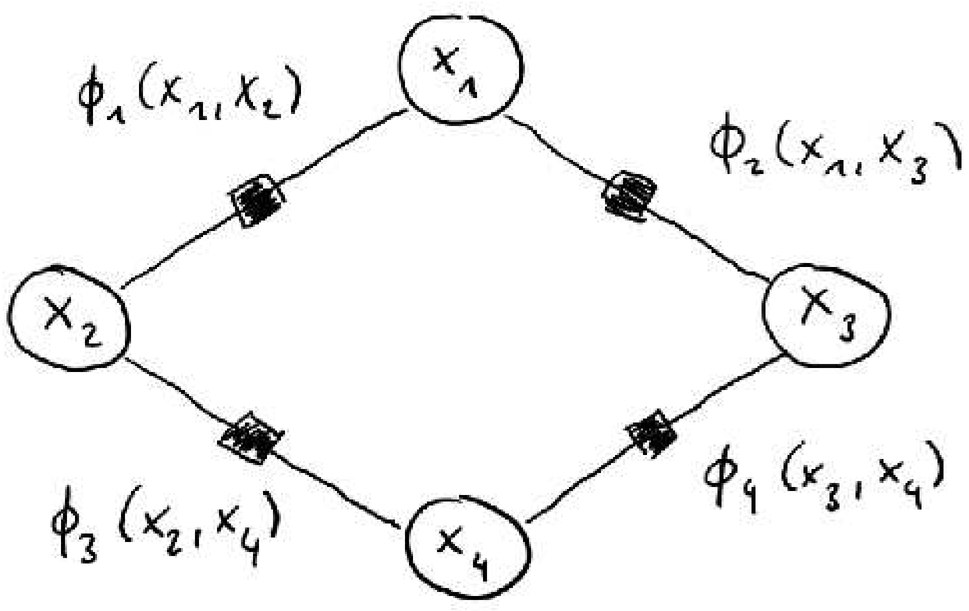
$$= \frac{p(x_1)\, p(x_2|x_1)\, p(x_3|x_1)}{p(x_1)}$$

$$= p(x_2|x_1)\, p(x_3|x_1)$$

$\square$

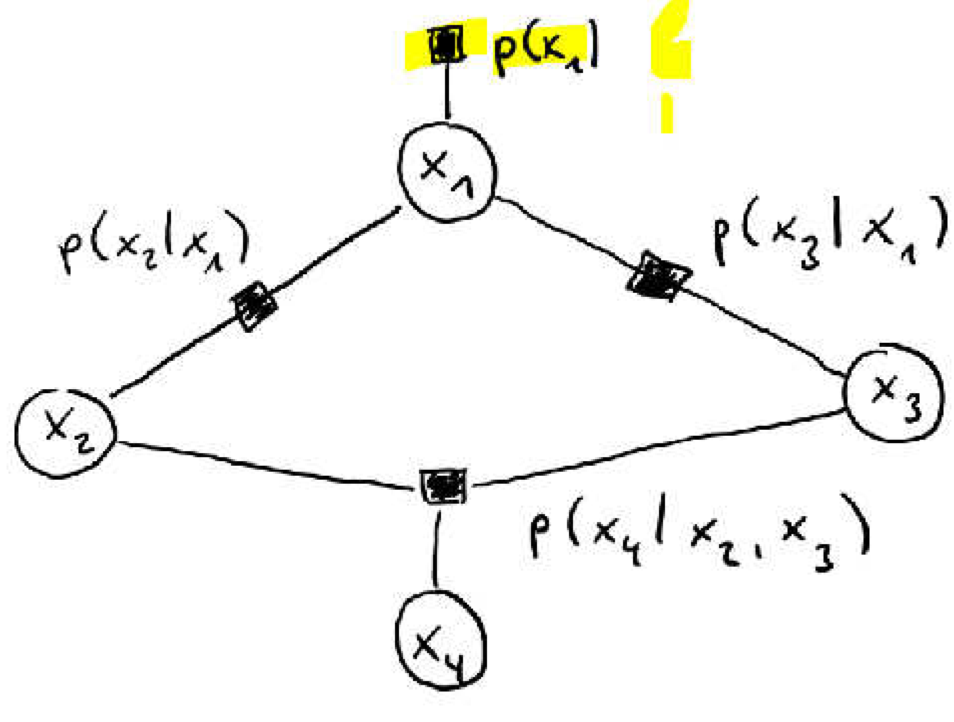
7. <u>left:</u>

$$p(x_1, x_2, x_3, x_4) = \frac{1}{Z} \cdot \phi_1(x_1, x_2) \cdot \phi_2(x_1, x_3) \cdot \phi_3(x_2, x_4) \cdot \phi_4(x_3, x_4)$$



<u>right:</u>

$$p(x_1, x_2, x_3, x_4) = p(x_1) \cdot p(x_2 | x_1) \cdot p(x_3 | x_1) \cdot p(x_4 | x_2, x_3)$$

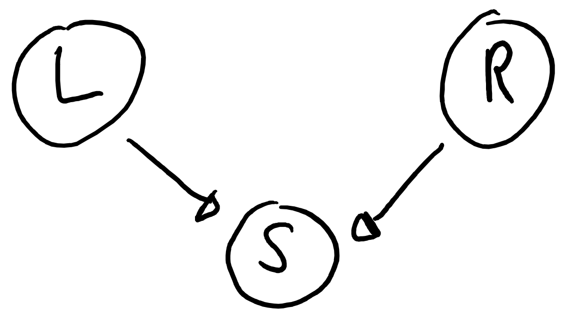


8. Markov blanket of ω_2 (left): $\{\omega_1, x_2, \omega_5, \omega_6\}$

8. Markov blanket of w_2 (left): $\{w_1, x_2, w_5, w_4\}$

9. Markov blanket of w_8 (right): $\{w_5, w_7, w_9, w_{11}, x_8\}$

PROBLEM 2



$$p(S,L,R) = p(L) \cdot p(R) \cdot p(S \mid L,R)$$

1.

$$p(L=1 \mid S=0) = \frac{p(L=1, S=0)}{p(S=0)}$$

$$= \frac{\sum_R p(S=0, L=1, R)}{\sum_L \sum_R p(S=0, L, R)}$$

$$= \frac{p(L=1) \cdot \sum_R p(R) \cdot p(S=0 \mid L=1, R)}{\sum_L \sum_R p(L) \cdot p(R) \cdot p(S=0 \mid L, R)}$$

$$= \frac{p(L=1) \cdot \sum_R p(R) \cdot (1 - p(S=1 \mid L=1, R))}{\sum_L \sum_R p(L) \cdot p(R) \cdot (1 - p(S=1 \mid L, R))}$$

$$= \frac{0.1 \cdot (0.2 \cdot 0.9 + 0.8 \cdot 0.7)}{0.1 \cdot 0.2 \cdot 0.9 + 0.1 \cdot 0.8 \cdot 0.7 + 0.9 \cdot 0.2 \cdot 0.8 + 0.9 \cdot 0.8 \cdot 0.2}$$

$$= \frac{0.074}{0.362} = 0.2044$$

2.

$$p(L=1 \mid S=0, R=1) = \frac{p(S=0, L=1, R=1)}{p(S=0, R=1)}$$

$$= \frac{p(L=1) \cdot p(R=1) \cdot p(S=0 \mid R=1, L=1)}{\sum_L p(S=0, L, R=1)}$$

$$= \frac{0.1 \cdot 0.2 \cdot (1 - p(S=1 \mid R=1, L=1))}{}$$

$$= \frac{0.1 \cdot 0.2 \cdot (1 - p(S=1 \mid R=1, L=1))}{\sum_L p(L) \cdot p(R=1) \cdot p(S=0 \mid R=1, L)}$$

$$= \frac{0.1 \cdot 0.2 \cdot 0.9}{0.1 \cdot 0.2 \cdot \underbrace{0.9}_{(1-p(S=1 \mid R=1, L=1))} + 0.9 \cdot 0.2 \cdot \underbrace{0.8}_{(1-p(S=1 \mid R=1, L=0))}}$$

$$= 0.\overline{1}$$

3. Even though L and R do not have a relation (no arrow between them) their probabilities can – paradoxically enough – change when information is available about the other random variable.
I.e. it is possible to state that the probability for $L=1$ decreases when we know that certain events of the other variable ($R=1$) occur. We assume that the state of S is primarily accounted for by R rather than L, even though it still may be due to L. However, by knowing about R the assumed influence of L is "explained away", i.e. gets less weight.

PROBLEM 3 Only my notes

gradient of student-t:

$$\frac{\partial f}{\partial d} = \frac{2d}{2\sigma^2} \cdot (-\alpha) \cdot \left(1 + \frac{1}{2\sigma^2} d^2\right)^{-\alpha-1}$$

$$= -\frac{d\alpha}{\sigma^2} \cdot \text{student-t (with } \alpha+1)$$

use slide 19

$$\frac{\partial}{\partial d_{k,l}} f(d_{k,l}) = \frac{\frac{\partial}{\partial d_{k,l}} f(d_{k,l})}{f(d_{k,l})}$$

$$= \frac{-\frac{d\alpha}{\sigma^2} \cdot \left(1 + \frac{1}{2\sigma^2} d^2\right)^{-\alpha} : \left(1 + \frac{1}{2\sigma^2} d^2\right)^1}{\left(1 + \frac{d^2}{2\sigma^2}\right)^{-\alpha}}$$

$$= -\frac{d\alpha}{\sigma^2 \cdot \left(1 + \frac{1}{2\sigma^2} d^2\right)} = -\frac{d\alpha}{\sigma^2 \cdot \text{student (with } \alpha = 1)}$$

$$\log f(d) = -\alpha \cdot \log\left(1 + \frac{d^2}{2\sigma^2}\right)$$

$$\frac{\partial \log f(d)}{\partial d} = -\alpha \cdot \frac{d}{\sigma^2} \cdot \frac{1}{1 + \frac{d^2}{2\sigma^2}} = -\frac{\alpha d}{\sigma^2 \cdot \left(1 + \frac{d^2}{2\sigma^2}\right)}$$

$$= -\frac{\alpha d}{\sigma^2 + \frac{d^2}{\sigma^2}} \quad \alpha \quad =\checkmark$$

$$\log p(I_0, I_1 \mid d) = \sum_{i,j} \log \text{student-t}(I_0 - I_1 ; \sigma, \alpha)$$

$$= \sum_{i,j} \log \left(1 + \frac{(\overbrace{I^0_{i,j} - I^1_{i,j-d}}^{\text{diff}})^2}{2\sigma^2}\right)^{-\alpha}$$

$$= \sum_{i,j} -\alpha \cdot \log\left(1 + \frac{\text{diff}^2}{2\sigma^2}\right)$$

$$\frac{\partial \log p(I_0, I_1 \mid d)}{\partial d} = \sum_{i,j} -\alpha \cdot \frac{\text{diff}}{\sigma^2} \cdot \frac{1}{1 + \frac{\text{diff}^2}{2\sigma^2}}$$

$$= \ldots = \sum_{i,j} -\frac{\alpha \cdot \text{diff}}{\sigma^2 + \frac{\text{diff}^2}{\sigma^2}}$$