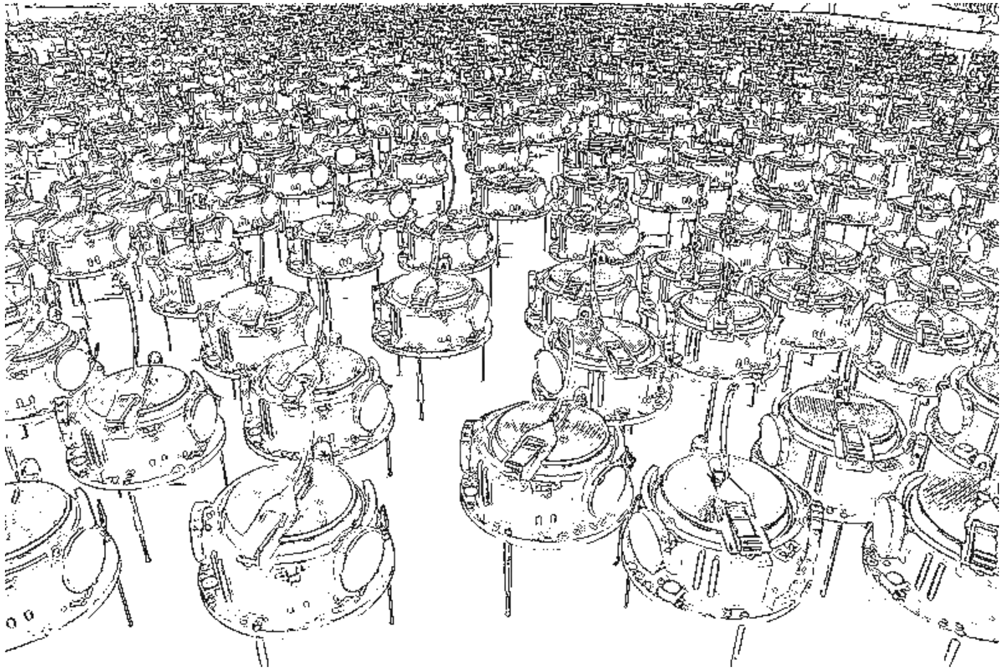
Intelligent Multi Agent Systems



Game Theory

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Agenda

- Introduction in Game Theory
- Some Example Games
- Solution Concepts:
  - Dominance
  - Pareto Optimality
  - Nash Equilibria
  - Correlated Equilibrium
  - MaxMin and MinMax strategies
  - Rationalizability

Game Theory

- In a multi-agent system, the decision of an agent may affect other agents.
- Typically, an agent will be uncertain about the actions of the other agents.
- Game theory is the study of multi-agent decision making under uncertainty.

It is based on two premises:

- The agents are rational.
- The agents reason strategically.

Class-room game

Lets play a little game:

- Everyone gets two cards (red and black)
- You will be matched with a random person

Rules of the game:

- Hold your card against the chest, reveal it simultanously
- Red card: you get 2 points
- Black card: your partner gets 3 points

Pay-Off Table:

Player II

Player I	black	red
black	3,3	0,5
red	5,0	2,2

Class-room game

Remixing partners...

Pay-Off Table, 2nd Round:

		Player II	
		black	**red**
Player I	**black**	8,8	0,10
	red	10,0	2,2

Class-room game

Remixing partners...

Pay-Off Table, 3rd Round, now we play 3 games against the same partner

Player II

Player I

	black	**red**
black	8,8	0,10
red	10,0	2,2

Class-room game

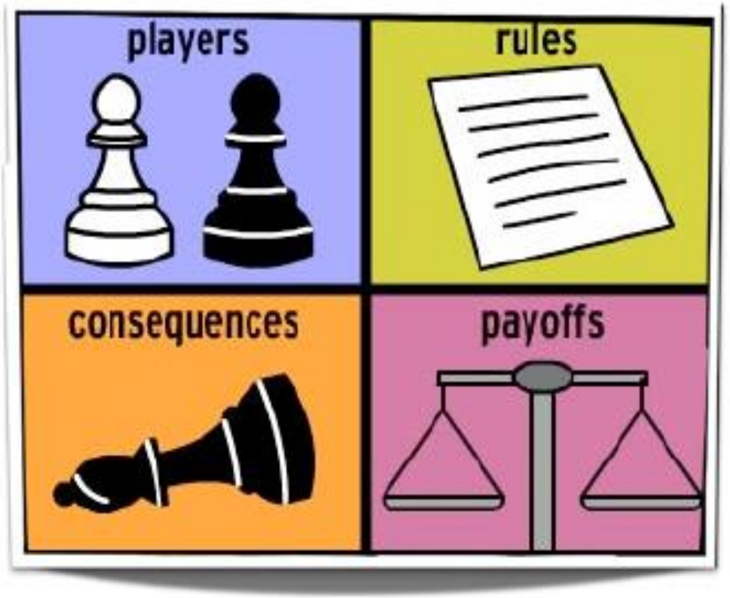
This game is actually called „prisoner‘s dilema“

This game extends to a variety of situations:

- ➡working on a joint project,
- ➡duopoly
- ➡arms race
- ➡use of a common property

Game Theory Premises

- ➡Simultaneous actions
- ➡No communication
- ➡Outcome depends on combination of actions
- ➡Utility (payoff) encapsulates everything about preferences over outcomes
- ➡(Typically) no repeated games



Strategic Games: Formal Definition

A strategic game is the simplest game-theoretic model of agent interactions.

- There are $n > 1$ agents in the world.
- Each agent i can choose an action a_i from his own action set A_i. The vector $(a_1, \ldots, a_n)$ of individual actions is called a joint action or an action profile, and is denoted by a or (a_i) .
- The profile (a_{-i}, a_i) indicates that agent i plays a_i.
- The game is played on a fixed world state s. The state consists of the n agents, their action sets A_i, and their payoffs.

Strategic Games: Formal Definition

- Each agent *i* has his own payoff function $u_i(a)$ that measures the goodness of the joint action a for the agent *i*.
- The state is fully observable to all agents.
- Common knowledge: all agents know
  - each other
  - the action sets of each other
  - the payoff functions of each other.
- Each agent chooses a single action. All agents choose their actions simultaneously and independently of each other.

2 Player Games

In the special case of two agents, a strategic game can be graphically represented by a payoff matrix:

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

- ➡ Original version of **prisoner‘s dilemma**
- ➡ A prisoner‘s dilema is any game with b < d < a < c

	C	D
C	a, a	b, c
D	c, b	d, d

Some more games

Stag-hunt game:

- ➡Players: 2 fishermen
- ➡Actions: fish, whale

	whale	fish
whale	2 / 2	1 / 0
fish	0 / 1	1 / 1

Also known as coordination game

Some more games

Chicken game:

- Players: 2 „brave“ car drivers
- Actions: straight, swerve

	swerve	straight
swerve	0 / 0	+1 / -1
straight	-1 / +1	-10 / -10

Some more games...

Matching Pennies:

➡Player 1: Match

➡Player 2: Dismatch

	head	tail
head	-1 +1	+1 -1
tail	+1 -1	-1 +1

Zero-sum game

➡Strictly competitive

„Optimal“ Action Selection

Which action will be chosen by each player?

➡ Theory of rational choice states that each player chooses the **best available action**

➡ Since this choice depends on the actions of the other player, each player must form a **belief** about the other players’ actions and preferences

➡ This belief is formed based on the **knowledge of the game and past experiences**

➡ BUT ! each play is considered in isolation (players do not know the selected actions of other players)

Which solution should we take

There are different solution concepts:

➡ Dominance

➡ Pareto Optimality

➡ Nash Equilibria

➡ MaxMin and MinMax strategies

Dominance

In any game, a player’s action *strictly dominates* another action if it is superior, no matter what the other player does

Steal strictly dominates Split

- If player 2 plays Split, then player 1 prefers Steal !
- If player 2 plays Steal, then player 1 also prefers Steal !

	Split	Steal
Split	3 3	7 0
Steal	0 7	1 1

Dominance

Definition:

In a strategic game player i's action a_i'' strictly dominates another action a_i' if

➡ $u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action

We say that a_i' is strictly dominated

Dominance

Definition:

In a strategic game player *i*'s action a_i'' weakly dominates another action a_i' if

- $u_i(a_i'',a_{-i}) >= u_i(a_i',a_{-i})$ for every list a_{-i} of the other player's action
- $u_i(a_i'',a_{-i}) > u_i(a_i',a_{-i})$ for some list a_{-i} of the other player's action

We say that a_i' is weakly dominated

Dominance

No matter what the column player does...

➡M weakly dominates T

➡B weakly dominates M

➡BUT: B strictly dominates T

	L	R
T	1	0
M	2	0
B	2	1

Dominance is not a real solution concept but can be used to eliminate actions.

Dominance

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

➡Dominance?

	swerve	straight
swerve	0 0	+1 -1
straight	-1 +1	-10 -10

➡Dominance?

Dominance

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

➡Neither *whale* nor *fish* strictly or weakly dominates the other player's action

ht}

	swerve	straight
swerve	0 0	+1 -1
straight	-1 +1	-10 -10

➡Dominance?

Dominance

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

➡Neither *whale* nor *fish* strictly or weakly dominates the other player’s action

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

➡Neither *swerve* nor *straight* strictly or weakly dominates the other player’s action

Identifying dominated strategies

However, actions can also be dominated by mixed strategies:

- M is not dominated by U or D
- But M is dominated by a mixed strategy that takes D and U with equal probability.

	L	C
U	$3, 1$	$0, 1$
M	$1, 1$	$1, 1$
D	$0, 1$	$4, 1$

Which solution should we take

There are different solution concepts:

➡Dominance

➡Pareto Optimality

➡Nash Equilibria

➡MaxMin and MinMax strategies

Pareto Optimality

"An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player. "

Pareto Optimal Solutions

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Which solution should we take

There are different solution concepts:

➡ Dominance

➡ Pareto Optimality

➡ Nash Equilibria

➡ MaxMin and MinMax strategies

Best Response and Nash Equilibrium

If you knew what everyone else was going to do, it would be easy to pick your own action

- Actions of other agents: a_{-i}

Best-response for player *i* given a_{-i}:

$$a_i^* \in B_i(a_{-i}) \text{ iff } \forall a_i \in A_i,\ u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i})$$

- a_i^* optimizes the utility of agent i given a_{-i}

Nash Equilibrium (pure strategy)

Idea: look for stable action profiles!

$a = (a_1, \ldots, a_n)$ is a ("pure strategy") Nash equilibrium iff a_i is the best response for all agents i, i.e, $\forall i,\ a_i \in BR(a_{-i})$

➡ $u_i(a^*) \geq u_i(a_i, a_{-i}^*)$ for every action a_i of player i

A NE corresponds to a stable "social norm": *if everyone follows it, no person will wish to deviate from this*

Example: The prisoner's dilemma

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Assume the profile :

(Quiet, Quiet) $u_1 \rightarrow 3 \quad u_2 \rightarrow 3$

(Fink, Quiet) $u_1 \rightarrow 7$

(Fink, Fink) $u_1 \rightarrow 1 \quad u_2 \rightarrow 1$

Example: The prisoner’s dilemma

	Quiet	Fink
Quiet	3 3	7 0
Fink	0 7	1 1

Assume the profile :

(Quiet, Quiet) $u_1 \to 3$ $u_2 \to 3$

(Fink, Quiet) $u_1 \to 7$

(Quiet, Fink) $u_2 \to 7$

(Fink, Fink) $u_1 \to 1$ $u_2 \to 1$

Example: The prisoner’s dilemma

The prisoner’s dilemma

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

Any deviation of this NE results in a worse outcome

➡ This NE is a strict Nash Equilibrium

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Example: The prisoner's dilemma

The prisoner's dilemma

	Quiet	Fink
Quiet	3, 3	0, 7
Fink	7, 0	1, 1

Any deviation of this NE results in a worse outcome

➡ This NE is a strict Nash Equilibrium

Comparison to Pareto Optimal Solutions

➡ Prisoner's dilemma paradoxon: Nash equilibrium is the only solution that is not Pareto Optimal!

(Fink, Fink) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

(Quiet, Fink) $u_1 \rightarrow 0$

(Fink, Quiet) $u_2 \rightarrow 0$

Examples: Chicken Game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(swerve, swerve) $u_1 \rightarrow 0 \quad u_2 \rightarrow 0$

(straight, straight) $u_1 \rightarrow -10 \quad u_2 \rightarrow -10$

Examples: Chicken Game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(swerve, swerve) $u_1 \rightarrow 0 \quad u_2 \rightarrow 0$

(straight, swerve) $u_1 \rightarrow +1$

(swerve, straight) $u_2 \rightarrow +1$

Both profiles are unstable!

(straight, straight) $u_1 \rightarrow -10 \quad u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$

(straight, swerve) $u_2 \rightarrow -1$

Examples: Chicken Game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

Examples: Chicken Game

Assume the profile :

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow 0$

(straight, straight) $u_2 \rightarrow -10$

Both profiles are stable!

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$

(swerve, swerve) $u_2 \rightarrow 0$

Examples: Chicken Game

Both profiles are strict NE!

	Swerve	Straight
Swerve	0 0	+1 -
Straight	- +1	-10 -10

Assume the profile :

(straight, swerve) $u_1 \rightarrow +1$ $u_2 \rightarrow -1$

(swerve, swerve) $u_1 \rightarrow 0$

(straight, straight) $u_2 \rightarrow -10$

(swerve, straight) $u_1 \rightarrow -1$ $u_2 \rightarrow +1$

(straight, straight) $u_1 \rightarrow -10$

(swerve, swerve) $u_2 \rightarrow 0$

Examples: Stag hunt

Assume the profile :

	whale	fish
whale	2, 2	0, 1
fish	1, 0	1, 1

(whale, whale) $u_1 \rightarrow 2$ $u_2 \rightarrow 2$

(fish, fish) $u_1 \rightarrow 1$ $u_2 \rightarrow 1$

Examples: Stag hunt

Both profiles are strict NE!

	whale	fish
whale	2 2	1 0
fish	0 1	1 1

Assume the profile :

(whale, whale) $u_1 \rightarrow 2 \quad u_2 \rightarrow 2$

(fish, whale) $u_1 \rightarrow 1$

(whale, fish) $u_2 \rightarrow 1$

(fish, fish) $u_1 \rightarrow 1 \quad u_2 \rightarrow 1$

(whale, fish) $u_1 \rightarrow 0$

(fish, whale) $u_2 \rightarrow 0$

Examples: Matching Pennies

Best response analysis

	head	tail
head	+1, -1	-1, +1
tail	-1, +1	+1, -1

	head	tail
head	+1, -1	-1, +1
tail	-1, +1	+1, -1

Examples: Matching Pennies

Best response analysis

	head	tail
head	+1, -1	-1
tail	-1, +1	+1, -1

Nash Equilibrium?

	head	tail
ead	+1, -1	-1, +1
tail	-1, +1	+1, -1

Mixed Strategies

It would be a pretty bad idea to play any deterministic strategy in matching pennies

Idea: confuse the opponent by playing randomly

- Define a strategy α_i for agent *i* as any probability distribution over the actions A_i.
- pure strategy a : only one action is played with positive probability
- mixed strategy α : more than one action is played with positive probability
  - $\alpha(a_k) \ldots$ probability of choosing action a_k
  - These actions are called the support of the mixed strategy
  - Aka. Lottery

von Neumann/Morgenstern Preferences

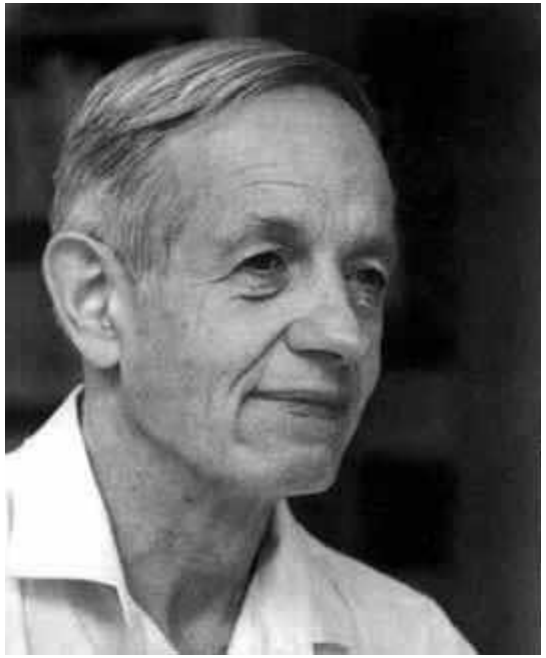
Preferences regarding distributions/lotteries α

➡Given utility function u compute expected utility

$$U(\alpha) = \sum_a \alpha(a)u(a)$$

Mixed Strategy Nash Equilibrium

Assume that (α_i', α_{-i}) is the **mixed** strategy profile in which every player j except i chooses its mixed strategy α_j as specified by α, whereas player i deviates to α_i'

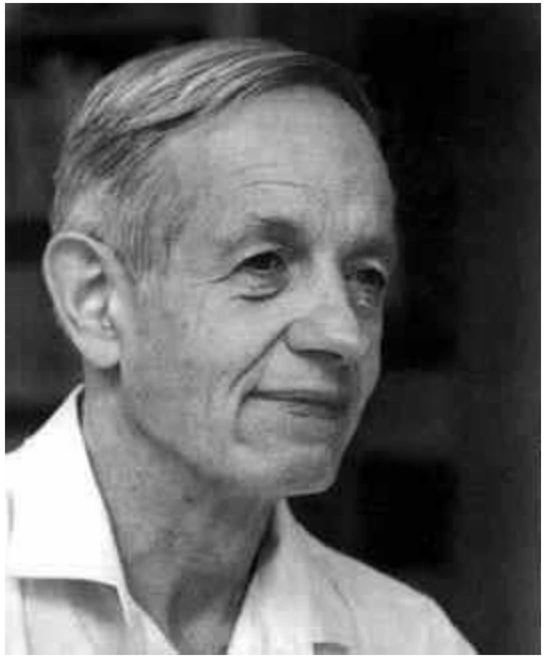


Definition :

The mixed strategy profile α^* in a strategic game is a **mixed strategy Nash Equilibrium** if for every player i and for every mixed strategy α_i of player i, the expected payoff to i in α^* is at least as large as the expected payoff to i in $(\alpha_i, \alpha_{-i}^*)$ according to a payoff function that represents player i's preferences over lotteries.

Mixed Strategy Nash Equilibrium

Definition :



Equivalently, for every player *i*, $U_i(\alpha^*) \geq U_i(\alpha_i, \alpha_{-i}^*)$ for every mixed strategy α_i of player *i*, where $U_i(\alpha)$ is the player's *i* expected payoff to the mixed strategy profile α

Every finite game has at least one (mixed) Nash equilibrium! [Nash, 1950]

Best-response

To find the mixed strategy NE, we can again make use of the notion of a Best-response.

- The mixed strategy profile α^* in a strategic game is a mixed strategy Nash Equilibrium if and only if α_i^* is in $B_i(\alpha_{-i}^*)$ for every player i

Definition :

- $B_i(\alpha_{-i})$ is the set of all player i's best mixed strategies when the list of the other players' mixed strategy is α_{-i}

Two Player Games

What is the set of best responses of player 1 to a mixed strategy of player 2?

	L (q)	R (1-q)
T (p)	pq	$p(1-q)$
B (1-p)	$(1-p)q$	$(1-p)(1-q)$

Two Player Games

What is the set of best responses of player 1 to a mixed strategy of player 2?

	L (q)	R (1-q)
T (p)	*pq*	*p(1-q)*
B (1-p)	*(1-p)q*	*(1-p) (1-q)*

$$U_1(\alpha) = p\,(qu_1(T,L) + (1-q)u_1(T,R)) + (1-p)\,(qu_1(B,L) + (1-q)u_1(B,R))$$

$$U_1(\alpha) = pU_1(T,\alpha_{-1}) + (1-p)U_1(B,\alpha_{-1})$$

➡ the expected payoff of player 1, given player 2's mixed strategy is a linear function of p

Two Player Games

The linearity implies 3 possible outcomes :

1. player 1's unique best response is the pure strategy T
   ➡(when $U_1(T, \alpha_{-1}) > U_1(B, \alpha_{-1})$)
2. player 1's unique best response is the pure strategy B
   ➡(when $U_1(T, \alpha_{-1}) < U_1(B, \alpha_{-1})$)
3. all player 1's mixed strategies are all best responses
   ➡(when $U_1(T, \alpha_{-1}) = U_1(B, \alpha_{-1})$)

Matching Pennies

player 1's expected payoff for the pure strategy *Head* (*p*) is

player 1's expected payoff for the pure strategy *Tail* (*1-p*) is

	head	tail
head	-1 +1	+1 -1
tail	+1 -1	-1 +1

Matching Pennies

player 1's expected payoff for the pure strategy *Head* (*p*) is

$$q \cdot 1 + (1-q) \cdot (-1) = 2q - 1$$

player 1's expected payoff for the pure strategy *Tail* (*1-p*) is

$$q \cdot (-1) + (1-q) \cdot 1 = 1 - 2q$$

	head	tail
head	-1 +1	+1 -1
tail	+1 -1	-1 +1

Matching Pennies

player 1’s expected payoff for the pure strategy *Head* (*p*) is

$$q \cdot 1 + (1 - q) \cdot (-1) = 2q - 1$$

player 1’s expected payoff for the pure strategy *Tail* (*1-p*) is

$$q \cdot (-1) + (1 - q) \cdot 1 = 1 - 2q$$

	head	tail
head	-1 +1	+1 -1
tail	+1 -1	-1 +1

1. 2q-1 < 1-2q when ***q < 1/2*** for any value of *p >0.0*

➡ Thus best response set is {Tail} or *p=0*

2. 2q-1 > 1-2q when ***q > 1/2*** for any value of *(1-p) > 0.0*

➡ Thus best response set is {Head} or *p=1*

3. 2q-1 = 1-2q when ***q = 1/2*** for any mixed strategy

➡ Thus best response set is the set of all mixed strategies

Matching Pennies

player 2's expected payoff for the pure strategy *H*

$$p \cdot 1 + (1-p) \cdot (-1) = 2p - 1$$

player 2's expected payoff for the pure strateg

$$p \cdot (-1) + (1-p) \cdot 1 = 1 - 2p$$

1. 2p-1 < 1-2p when ***p < 1/2*** for any value of *q >0.0*

➡Thus best response set is {Tail} or *q=0*

2. 2p-1 > 1-2p when ***p > 1/2*** for any value of *(1-q) > 0.0*

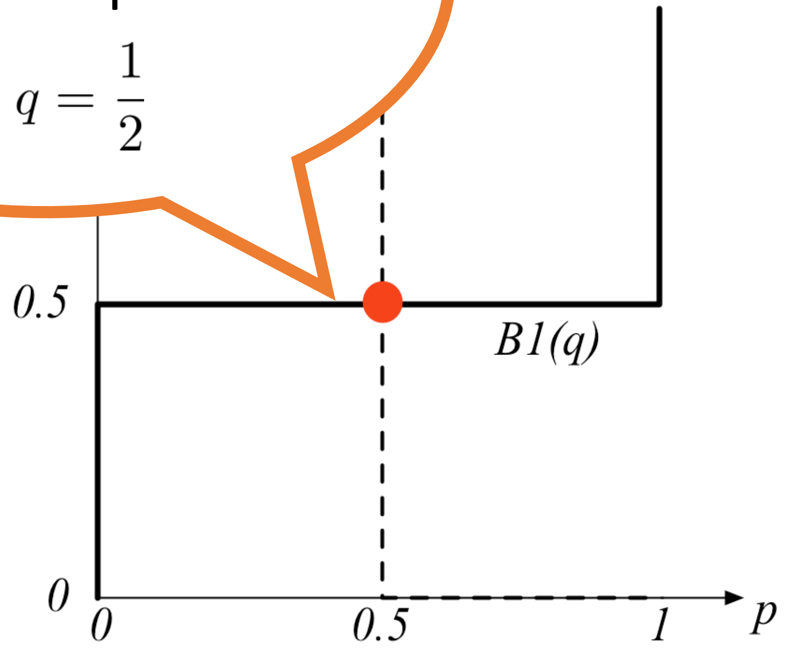
➡Thus best response set is {Head} or *q=1*

3. 2p-1 = 1-2p when ***p = 1/2*** for any mixed strategy

➡Thus best response set is the set of all mixed strategies

The only stable equilibrium point is

$$p = q = \frac{1}{2}$$



Interpreting mixed strategies

What does it mean to play a mixed strategy?

Different interpretations:

- Randomize to confuse your opponent
- Players randomize when they are uncertain about the other's action
- Mixed strategies are a concise description of what might happen in repeated play: count of pure strategies in the limit

Which solution should we take

There are different solution concepts:

➡Dominance

➡Pareto Optimality

➡Nash Equilibria

➡MaxMin and MinMax strategies

MaxMin Strategies

Player *i*'s maxmin strategy is a strategy that maximizes i's worst-case payoff, where all the other players *-i* play the strategies which cause the greatest harm to *i*.

Why would player *i* want to play a maxmin strategy?

- a conservative agent maximizing worst-case payoff
- a paranoid agent who believes everyone hates him

Definition (MaxMin):

- The maxmin strategy for player i is $\arg\max_{\alpha_i} \min_{\alpha_{-i}} u_i(\alpha_1, \alpha_2)$
  and the maxmin value for player i is $\max_{\alpha_i} \min_{\alpha_{-i}} u_i(\alpha_1, \alpha_2)$

MinMax Strategies

Player *i*'s minmax strategy against player *-i* in a 2-player game is a strategy that minimizes –*i*'s best-case payoff

Why would player *i* want to play a minmax strategy?

➡ to punish the other agent as much as possible (he really hates him)

Definition (Minmax, 2-player)

➡ In a two-player game, the minmax strategy for player *i* against player *-i* is $\arg\min_{\alpha_i} \max_{\alpha_{-i}} u_{-i}(\alpha_i, \alpha_{-i})$ and player *-i*'s minmax value is $\min_{\alpha_i} \max_{\alpha_{-i}} u_{-i}(\alpha_i, \alpha_{-i})$

MaxMin and MinMax in Zero Sum Games

Theorem (Minimax theorem (von Neumann, 1928))

In any finite, two-player, zero-sum game, in any Nash equilibrium each player receives a payoff that is equal to both his maxmin value and his minmax value.

- ➡ Each player's maxmin value is equal to his minmax value. By convention, the maxmin value for player 1 is called the value of the game.
- ➡ For both players, the set of maxmin strategies coincides with the set of minmax strategies.
- ➡ Any maxmin strategy profile (or, equivalently, minmax strategy profile) is a Nash equilibrium. Furthermore, these are all the Nash equilibria. Consequently, all Nash equilibria have the same payoff vector.

Summary

What we learned today:

- Definition of a game in normal form
- Rational Agents
- Solution Concepts: Pareto Optimality, Nash, ...
- Best Responses and Nash Equilibrium
- Mixed and Pure Nash Strategies

Correlated Equilibrium

Consider the following “traffic” game:

- What are the Nash equilibria?

	go	$wait$
go	$-100, -100$	$10, 0$
B	$0, 10$	$-10, -10$

What is the natural solution here?

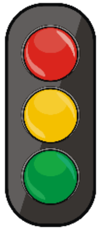
Correlated Equilibrium

Consider the following "traffic" game:

➡ What are the Nash equilibria?

	go	*wait*
go	$-100, -100$	$10, 0$
B	$0, 10$	$-10, -10$

What is the natural solution here?



➡ Traffic lights: a fair randomizing device that tells one of the agents to go and the other to wait.

Benefits:

➡ the negative payoff outcomes are completely avoided

➡ fairness is achieved

➡ the sum of social welfare might exceed that of any Nash equilibrium

Correlated Equilibrium

- Our example presumed that everyone perfectly observes the random event; not required.
- More generally, some random variable v with a commonly known distribution, and a private signal to each player about the outcome.
  - v can be used for coordination
  - signal doesn't determine the outcome or others' signals;
  - however, correlated
- For every Nash equilibrium there exists a corresponding correlated equilibrium.
- Not every correlated equilibrium is equivalent to a Nash equilibrium
  - Weaker notion than Nash

Correlated Equilibrium

We can view the problem now as finding a set of correlated strategies

➡The strategies of all agents can be expressed by a joint distribution *p(a)*

Some convenient notation:

$$\text{Expected Utility for action } a_i^j : \; U_i(a_i^j, p_{-i}) = \sum_{a_{-i} \in A_{-i}} p(a_{-i} | a_i^j) u_i(a_i^j, a_{-i})$$

$$\begin{aligned} \text{Expected Utility for agent } i : \; U_i(p) &= \sum_{a_i^j} p(a_i^j) U_i(a_i^j, p_{-i}) \\ &= \sum_{a_i^j} \sum_{a_{-i} \in A_{-i}} p(a_i^j) p(a_{-i} | a_i^j) u_i(a_i^j, a_{-i}) \\ &= \sum_{a \in A} p(a) u_i(a) \end{aligned}$$

Rationalizability

Rather than ask what is irrational, ask what is a best response to some beliefs about the opponent

- ➡ assumes opponent is rational
- ➡ assumes opponent knows that you and the others are rational

Rationalizable Strategy: Can we find a belief under which the strategy is rational?

Examples:

- ➡ is heads rational in matching pennies?
- ➡ is cooperate rational in prisoner's dilemma?

Will there always exist a rationalizable strategy?

- ➡ Yes, equilibrium strategies are always rationalizable.

Correlated Equilibrium

Definition (Correlated Equilibrium)

$$U_i(p) \geq U_i(a_i', p_{-i}) \qquad \forall i \in N,\ a_i' \in A_i$$

$$p(a) \geq 0 \qquad \forall a \in A$$

$$\sum_{a \in A} p(a) = 1$$

The expected utility for player i given the joint strategy profile p needs to be at least as high as the utility the player could get with another action a_i' (given profile p_{-i} of the other players)