Intelligent Multi Agent Systems

TECHNISCHE UNIVERSITÄT DARMSTADT

Summer Semester 2016, Homework 3 (26 points)

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Due date: Wednesday 20th July, 2016, 23:59 CEST (that's 10:59 AM on the following day in Pago Pago!)

Problem 3.1 Theoretical Questions [0 Points]

a) **POMDP** **[4 Points]**

1) What is a POMDP?

POMDP stands for Partial Observable Markov Decision Proces. In such a Process the real state can not be obtained by the agent but an observation and it's confidence. [c.f. 08-PoMDPs p. 4]

2) How is it defined? (formal)

A POMDP is defined by:

1. *its state space* $s \in S$
2. *its action space* $a \in A$
3. *its observation space* $o \in O$
4. *its transition dynamics* $\Pr(s'|s,a) = P^a_{s's}$
5. *its observation probabilities* $O(o|s) = O_{os}$
6. *its reward function* $r(s,a) = R^a_s$
7. *its initial state probabilities* $\mu_0(s)$
8. *the history* $h_t = (o_1, a_1, ..., a_{t-1}, o_t)$, *that contains all past states and actions*

[from 08-PoMDPs p. 5]

3) How is it different from a common MDP?

The ***true state*** *is hidden. Thus the agents have to work with observations. For that, all the history of the actions and observations of all the agents have to be saved. (To make assumptions on the real state and thus the expected reward.)*

4) Given an example of a POMDP

A typical example for this problem is the tiger-door problem. The agent can choose to open one of two doors, behind one of them is a reward and one the other contains punishment. However the agent can only observe the the true state with a probability distribution, through listening. [c.f. 08-PoMDPs p. 7]

b) **State Definitions** **[3 Points]**

1) Name the different state definitions, introduced in the lecture.

1. *Information state*

2. *Belief state*

2) How do they differ from one another?

The information state contains the whole history of actions and observations. The belief state describes a probability distribution over states, given the past observations.

3) What are their advantages/disadvantages?

The information state allows for explicit evaluation for each time step. But is grows exponentially in the amount of steps (T). The belief state is a possibly much shorter description of the true state. For discrete states: In order to compute the Bellman equation we need to discretize the belief state. [c.f. from 08-PoMDPs pp. 10 ff.]

c) **Alpha Vectors** **[3 Points]**

1) What are alpha vectors? 2) How do they relate to POMDPs?

One alpha vectors represent the reward from the value function over the state space for one action. The set of alpha vectors that lie on the upper convex hull represent the best possible actions under the current belief state. [c.f. from 08-PoMDPs pp. 24-26]

3) Which property allows us to use them?

The reward function of the Bellman equation for the value Iteration is linear in the belief state b ($b : p(b,a) = \sum_s b(s)\, r(s,a)$). The max-pooling over all actions implies a piecewise linearity in b for $V^*(b)$ (convexity).

d) **Belief-State Value Iteration** **[4 Points]**

1) Describe Belief-State Value Iteration

We iterate over the most probable action-observations transitions to obtain the maximal reward in the previous step. This results in the state, we belief to have obtained for the chosen action.

2) How are alpha vectors used?

An alpha vector is used to compute the reward function of an belief state for a policy tree. [c.f. 08-PoMDPs p.28]

3) What are the main update steps?

There are three update steps:

1. *Get the* $\max_\alpha(s)$ *for the action space and the possible thereafter observations* o'
2. *Averaging over the observations* o' *and adding reward leads to the optimal Q values* Q^*_{t-1}
3. *Consider the best actions* ($\max_\alpha b$) *to obtain* V^*_{t-1}

[c.f. 08-PoMDPs pp. 29-30]

e) **Point-Based Value Iteration** **[4 Points]**

1) Describe Point-Based Value Iteration

Same as the Belief-State Value Iteration, but only rely on a small set of belief states. Updates the belief state in polynomial time.

2) How are alpha vectors used?

Each alpha vectors represent a reward "function" for the state space.

3) What are the main update steps?

1. *"Compute alpha vectors for actions and observations"*
2. *For a subset of belief states, "average over observations, add reward".*
3. *"Find best action for each belief point."*

[c.f. 08-PoMDPs p. 34]

f) **Bayes Adaptive MDPs** **[4 Points]**

1) What is the main concept of BAMDPs? When are they useful?

"A Bayes Adaptive MDP defines a POMDP with the MDP model as unobserved state". We put a posterior in the state space. The agent is aware and can define any uncertainty in its partial observed model. [c.f. 08-PoMDPs p. 76]

2) Why is a Dirichlet distribution a good prior for discrete systems?

"[T]he Dirichlet distribution is the conjugate prior of the categorical distribution and multinomial distribution." [c.f. https://en.wikipedia.org/wiki/Dirichlet_distribution] Which fits to our case if the set of states S and set of actions A are discrete (nominal).

3) When does a BAMDP become a BAPOMDP?

BAMDPs become BAPOMDPs, when we can measure the true states s and s', but only an observation model $O_{os} = p(o|s)$. [c.f. 08-PoMDPs p. 79]

4) Can we use Belief-State Value Iteration on BAPOMDPs? Is it a good idea?

The belief state easily becomes huge, as we have to consider the transition history. This makes the Belief-State Value Iteration on BAPOMDPs feasible, only for small problems. [c.f. 08-PoMDPs pp. 79-80]

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g) **Decentralized POMDPs** **[4 Points]**

1) How do they differ from common to POMDPs??

POMDPs consider problems, or more specific the view on the problem in the perspective of a single agent. Decentralized POMDPs, consider partial observable from a point of view with multiple agents. [c.f. 09-DecPoMDPs p. 1]

2) How is a Dec-POMDP defined? (formal)

A Dec-POMDP is defined by:

1. *it's state space* $s \in S$
2. *an action space* A_i *for each agent* i
3. *an observation space* O_i *for each agent* i
4. *the state-action transition dynamics* $P(s'|s,a) = P^a_{s's}$
5. *an observation model per agent* $i:\ O_i(o|s) = O^i_{os}$
6. *a shared reward function for all agents* I*:* $r(s,a) = R^a_s$
7. *the initial state probabilities* $\mu_0(s)$.

[c.f. 09-DecPoMDPs p. 4]

3) Can we use the same algorithms as for common POMDPs? Why?

We can not use the PoMDP algorithm directly. The state representation of each agent now not only consists of it's on observation history, but every possible other agents observation history as well. A local policy is map from an observation history $o^i = \left(o^i_1, ..., o^i_t\right)$ *to actions. Since each agent can only observe the other actions with some certainty, each agent* i *have to save a set of possible local policies* Π_i *for each other agent* $\notin i$*. The set of polices for each other agent is denoted as joint policy* $\langle \pi_1, ..., \pi_n \rangle$*. [c.f. 09-DecPoMDPs p. 7 and 9] In order to solve a Dec-POMDP we can now perform an exhaustive search over the piecewise product of combinations of the set of joint policies to find the optimal set of local policies. [c.f. 09-DecPoMDPs pp.12 ff.]*

4) Are there other ways to solve Dec-POMDPs? Explain one conceptually

The joint policy may be represented as a set of trees. The root and the nodes (as well as the leafs) are the actions, which are connected via the observations (edges). We can evaluate the values of each tree. [c.f. 09-DecPoMDPs pp. 7 and 12] Since there will be many trees that do not contribute to the optimal solutions, we can prune (exclude) these trees. [c.f. 09-DecPoMDPs p. 16] "We can prune a tree q_i *if there is a distribution* $x\left(q_i^{-1}\right)$ *of other trees that has a higher value for all system states and trees of the other agent." [09-DecPoMDPs p. 18]*

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4.2 a)

It depends on the case, but most probably Jolly. By the text description we deduce Roberto did not talk at the start, so that means the (B,W,W) state does not occur. Thus it is Filipes turn and knowing Roberto kept quiet he discards this state and if he sees ~~Filipe~~ Jolly with a white foot he can be sure his is black. If he does not speak then Jolly can be sure his foot is black. Thus the game is always solved in maximum 3 steps. By the wording in the text ("all three were quiet") we deduce Jolly saved them with his foot being black.

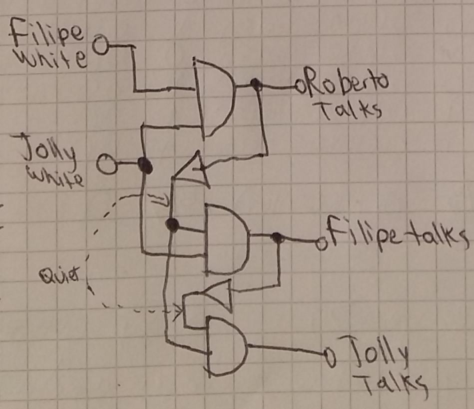
Given a uniform distribution of the seven possible states the propabilities of each winning are:

Roberto with Black Foot = 1/7

Filipe with Black Foot = 2/7

Jolly with Black Foot = 4/7

A Very simple way for these ruffians to solve this problem Is to plug their observations in the WildWest™ Circuit to solve their conundrum! As cheap as 49.99 € per piece!



Truth tables

F	J	RT
0	0	0
0	1	0
1	0	0
1	1	1

$\overline{RT}$	J	FT
0	0	0
0	1	0
1	0	0
1	1	1

$\overline{RT}$	$\overline{FT}$	JT
0	0	0
0	1	0
1	0	0
1	1	1

R	F	J	RT	FT	JT
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	0	0
1	0	0	0	0	1
1	0	1	0	1	0
1	1	0	0	0	1

Note: Truth table approach takes 3 hours less than the probabilistic approach.

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Basically it means that if roberto sees 2 white foot he knows his is black, thus he speaks. by observing Roberto kept quiet filipe knows both him and Jolly cannot have white, thus if Jolly has white then filipe knows his is black and speaks. By observing Filipe kept quiet (and therefore also roberto) Jolly can be certain his color is black, because if his wasn't black either man would have spoken out. The game will never last 4 turns.

Now for the IMAS formulation!

- $S = \{ \underset{s_0}{B,B,B}; \underset{s_1}{B,B,W}; \underset{s_2}{B,W,B}; \underset{s_3}{B,W,W}; \underset{s_4}{W,B,B}; \underset{s_5}{W,B,W}; \underset{s_6}{W,W,B} \}$

  So 7 possible states. (Considering the game is over once a solution is found or a false guess leads to an ending.)

- a {stay quiet, talk} can either guess right or wrong

  same for all 3 agents

alternative
- a {stay quiet, say black, say white}

not needed

- O_R { Filipe's foot, Jolly's foot, Jolly kept quiet }

  O_F { Jolly's foot, Roberto kept quiet }

  O_J { filipe kept quiet }

Note, If the previous kept quiet that assumes the one before also did, thus no use putting extra observations

- $P(s'|s,a) = 1$. The states don't change what we change is our belief of a state being the true underlying state and we win when we are certain that state is the true state

Roberto only

- $O(FF = W | s_0) = 0$
  $O(FF = W | s_1) = 0$
  $O(FF = W | s_2) = 1$
  $O(FF = W | s_3) = 1$
  $O(FF = W | s_4) = 0$
  $O(FF = W | s_5) = 0$
  $O(FF = W | s_6) = 1$

Filipe & Roberto

$O(JF = W | s_0) = 0$
$O(JF = W | s_1) = 1$
$O(JF = W | s_2) = 0$
$O(JF = W | s_3) = 1$
$O(JF = W | s_4) = 0$
$O(JF = W | s_5) = 1$
$O(JF = W | s_6) = 0$

Filipe

$O(RKQ = Yes | s_0) = 1$
$O(RKQ = Yes | s_1) = 1$
$O(RKQ = Yes | s_2) = 1$
$O(RKQ = Yes | s_3) = 0$
$O(RKQ = Yes | s_4) = 1$
$O(RKQ = Yes | s_5) = 1$
$O(RKQ = Yes | s_6) = 1$

Jolly

$O(FKQ = Yes | s_0) = 1$
$O(FKQ = Yes | s_1) = 0$
$O(FKQ = Yes | s_2) = 1$
$O(FKQ = Yes | s_3) = 1$
$O(FKQ = Yes | s_4) = 1$
$O(FKQ = Yes | s_5) = 0$
$O(FKQ = Yes | s_6) = 1$

given uniform dist. of states

$\rightarrow O(FF = W) \approx 3/7$ $\quad O(JF = W) = 3/7$ $\quad O(RKQ = Yes) = 6/7$ $\quad O(FKQ = Yes) = 5/7$

! Do note while S is supposed to be 7, in reality Filipe can only have 6 possible states where he gets to have a choice and Jolly only gets 4 states

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It kind of fits into the Dec POMDP definition because there are multiple agents in a partially observable world where every agent has their actions and their observations. Every agent (Jolly, Filipe and Roberto) has to decide by themselves without communicating directly (decentralized). It does defer slightly because while they share a common goal (stay alive) they also get to keep the foot if they answer (weighted though). They also indirectly know what the others observed through assumptions based on their actions.

History (given longest case) Ex. So

$$h_t = (\underbrace{FF = B, JF = B}_{O_R \leftarrow JKQ \text{ doesn't matter}};\ a_R = \text{Keep Quiet};\ \underbrace{JF = B, RKQ = \text{Yes}}_{O_F};\ a_F\ \text{Keep Quiet};\ \underbrace{FKQ = \text{Yes}}_{O_J};\ a_J = \text{Say Black})$$

b) • $O_i^{x|V}$ represents the visual (hence V) observations. It contains for Roberto the colour of Filipes and Jollys foot and for Filipe the color of Jallys foot. Jolly gets no visual cues.

• $O_i^{x|A}$ shockingly represents the Auditive observations (yes it was quite clear). For Filipe it contains wether Roberto kept Quiet or not (if he talked the game is over though) and for Jolly wether Filipe kept Quiet (Roberto's is implied anyways)

• The visual changes depending on the agent (x) and the auditive depending on the time (i)

• It shows the probabilities of an agent x observing O observations based on the states the agent its in, basically shows that each state will have corresponding observations and given these probabilities an agent can infer the state by their observations

• Suppose $O_{x|V} = FF \,\&\, JF$ $O_{x|A} = JKQ$ then we have ~~[illegible]~~

$O_{x\mid V}$	JKQ
FF	0....
JF	0....

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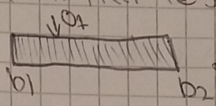
- because the belief state between states is correlated, that is there is a true underlying state (just one) and we believe with different certainty which is the most probable. If we start to believe in one more we believe in another less. this allows nice representations ex 

. The belief thus depends on the observations

- $b_1^R(s_0 \mid BB) = 0.5$ $\quad b_1^R(s_0 \mid BW) = 0$ $\quad b_1^R(s_0 \mid WB) = 0$ $\quad b_1^R(s_0 \mid WW) = 0$
  $(s_1 \mid BB) = 0$ $\quad (s_1 \mid BW) = 0.5$ $\quad (s_1 \mid WB) = 0$ $\quad (s_1 \mid WW) = 0$
  $(s_2 \mid BB) = 0$ $\quad (s_2 \mid BW) = 0$ $\quad (s_2 \mid WB) = 0.5$ $\quad (s_2 \mid WW) = 0$
  $(s_3 \mid BB) = 0$ $\quad (s_3 \mid BW) = 0$ $\quad (s_3 \mid WB) = 0$ $\quad (s_3 \mid WW) = 1$
  $(s_4 \mid BB) = 0.5$ $\quad (s_4 \mid BW) = 0$ $\quad (s_4 \mid WB) = 0$ $\quad (s_4 \mid WW) = 0$
  $(s_5 \mid BB) = 0$ $\quad (s_5 \mid BW) = 0.5$ $\quad (s_5 \mid WB) = 0$ $\quad (s_5 \mid WW) = 0$
  $(s_6 \mid BB) = 0$ $\quad (s_6 \mid BW) = 0$ $\quad (s_6 \mid WB) = 0.5$ $\quad (s_6 \mid WW) = 0$

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b_1^R	BB	BW	WB	WW
s_0	0.5	0	0	0
s_1	0	0.5	0	0
s_2	0	0	0.5	0
s_3	0	0	0	1
s_4	0.5	0	0	0
s_5	0	0.5	0	0
s_6	0	0	0.5	0

- It is a Greedy policy given that the (−) reward for choosing wrong (death) is way bigger than the reward for picking the lucky foot to ensure it is only taken when the certainty is 100% Ex $r(\text{wrong choice}) = -100000$
  $r(\text{right choice}) = 1$
  $r(\text{quiet}) = 0$

$\pi^R(BB) =$ Keep Quiet
$\pi^R(BW) =$ Keep Quiet
$\pi^R(WB) =$ Keep Quiet
$\pi^R(WW) =$ Say black

0.5 is not enough certainty! we want 100%) and $\emptyset$ is the only state with that certainty

- Given It says they all kept quiet he chose to keep quiet (duh) he saw that either (or both) of them had a black foot so he could not be 100% of the colour of his foot and decided to shut up. had he observed two whites he would have said his was black and won

- π given the observation tries to estimate π given the state. as we can only guess with uncertainty at the true state π_O is what we use. If we knew our state for certain we'd use π_S.

- If our belief of $(S \mid O) = 1$ then we know with 100% security our state and then $\pi(a \mid s) = \pi(a \mid b)$. If not we use the amount of belief we have
  $\pi(a \mid s) = \pi(a \mid b)\, b(S \mid O)$
  so $\pi(a \mid s) =$

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- $\pi^R(S_0)$ = ~~Keep Quiet~~ Say Black
  $\pi^R(S_1)$ = ~~Keep Quiet~~ Say Black
  $\pi^R(S_2)$ = ~~Keep Quiet~~ Say Black
  $\pi^R(S_3)$ = Say Black
  $\pi^R(S_4)$ = ~~Keep Quiet~~ Say White
  $\pi^R(S_5)$ = ~~Keep Quiet~~ Say white
  $\pi^R(S_6)$ = ~~Keep Quiet~~ Say white

If we know the state we can choose with 100% certainty the right action

$\pi(S_0 \mid KQ) = 0$	$\pi(S_0 \mid \text{Say white}) = -10000$	$\pi(\text{Say Black} \mid S_0) = 1$
$\pi(S_1 \mid KQ) = 0$	$\pi(S_1 \mid \text{Say white}) = -10000$	$\pi(\text{Say Black} \mid S_1) = 1$
$\pi(S_2 \mid KQ) = 0$	$\pi(S_2 \mid \text{Say white}) = -10000$	$\pi(\text{Say Black} \mid S_2) = 1$
$\pi(S_3 \mid KQ) = 0$	$\pi(S_3 \mid \text{Say white}) = -10000$	$\pi(\text{Say Black} \mid S_3) = 1$
$\pi(S_4 \mid KQ) = 0$	$\pi(S_4 \mid \text{Say white}) = 1$	$\pi(\text{Say Black} \mid S_4) = -10000$
$\pi(S_5 \mid KQ) = 0$	$\pi(S_5 \mid \text{Say white}) = 1$	$\pi(\text{Say Black} \mid S_5) = -10000$
$\pi(S_6 \mid KQ) = 0$	$\pi(S_6 \mid \text{Say white}) = 1$	$\pi(\text{say black} \mid S_6) = -10000$

1 = Yes
0 = No

JFRKQ

If Roberto talked the game is over and they won or are dead

Does never occur

$b_2^F(S_0 \mid B0) = 0.25$	$b_2^F(S_0 \mid B1) = NA$	$b_2^F(S_0 \mid W0) = 0$	$b_2^F(S_0 \mid W1) = NA$
$(S_1 \mid B0) = 0$	$(S_1 \mid B1) = NA$	$(S_1 \mid W0) = 0.5$	$(S_1 \mid W1) = NA$
$(S_2 \mid B0) = 0.25$	$(S_2 \mid B1) = NA$	$(S_2 \mid W0) = 0$	$(S_2 \mid W1) = NA$
$(S_3 \mid B0) = NA$	$(S_3 \mid B1) = NA$	$(S_3 \mid W0) = NA$	$(S_3 \mid W1) = NA$
$(S_4 \mid B0) = 0.25$	$(S_4 \mid B1) = NA$	$(S_4 \mid W0) = 0$	$(S_4 \mid W1) = NA$
$(S_5 \mid B0) = 0$	$(S_5 \mid B1) = NA$	$(S_5 \mid W0) = 0.5$	$(S_5 \mid W1) = NA$
$(S_6 \mid B0) = 0.25$	$(S_6 \mid B1) = NA$	$(S_6 \mid W0) = 0$	$(S_6 \mid W1) = NA$

from This point I will ignore impossible cases (Ex. RT) S_1 and S_5 both have black root

-

b_2^F	B	W
S0	0.25	0
S_1	0	0.5
S_2	0.25	0
S_4	0.25	0
S_5	0	0.5
S_6	0.25	0

- $\pi(B)$ Keep Quiet
  $\pi(W)$ say black

- If we again read it then we see he indeed observed that Roberto kept quiet and Jolly's root is black, thus he is 66% certain he has a black which is not safe enough and decides to again shut up

- It informed Filipe that State S_3 (so B, W, W) did not occur, because if it was the case the Roberto would have talked. By eliminating this state we can make it a dont care condition which simplifies our statespace and modifies directly our beliefs towards more certainty, freeing some information that allows Filipe to 100% know he has the black colour if Filipe has white because thanks to the RKQ he knows both can not have white.

$\pi_S(KQ) = 0_S$ → all states

-

$\pi^F(S_0)$ = say black	$\pi(\text{Black} \mid S_0) = 1$	$\pi(\text{white} \mid S_0)$ -10000
$\pi^F(S_1)$ = say black	$\pi(\text{Black} \mid S_1) = 1$	$\pi(\text{white} \mid S_1)$ -10000
$\pi^F(S_2)$ = say white	$\pi(\text{Black} \mid S_2) = -10000$	$\pi(\text{white} \mid S_2)$ 1
$\pi^F(S_4)$ = say black	$\pi(\text{Black} \mid S_4) = 1$	$\pi(\text{white} \mid S_4)$ -10000
$\pi^F(S_5)$ = Say black	$\pi(\text{Black} \mid S_5) = 1$	$\pi(\text{white} \mid S_5)$ -10000
$\pi^F(S_6)$ = say white	$\pi(\text{Black} \mid S_6) = -10000$	$\pi(\text{white} \mid S_6)$ 1

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Note. Jolly cannot have S_1 nor S_5 nor S_3 nor FT (All tables with 0s / Xs)

- $b_3^J(S_0 \mid FKQ) = 0.25$
  $(S_2 \mid FKQ) = 0.25$
  $(S_4 \mid FKQ) = 0.25$
  $(S_6 \mid FKQ) = 0.25$
  All these states have Jolly's feet as black

	FKQ
S_0	0.25
S_2	0.25
S_4	0.25
S_6	0.25

- π^J = Say Black. Jolly should always say black. The game (In this formulation) cannot reach Jolly's turn without it being black.

- Jolly thus chose to say his foot was black, saved the three musketeers (or wait!) and kept the foot. He did this by observing that if Roberto didnt talk (which means they (Filipe and Jolly) both didn't have white) and ~~the~~ then Filipe also didnt talk (which in turn means that Jolly didnt have a white, remember, if Jolly would have had white then Filipe could be sure his was black and would talk) by this Jolly can know with certainty that his would be black. By setting the whites to only 2 foot Geraldo shot himself in the foot and ensured the musketeers could always win given they were bright (or took some IMAS class). The game is set up so that by state space reduction once it gets to Jolly it is essentially solved due to elimination of (uncertain) conflicting states, especially here.

$\pi^J(S_0) =$, $\pi^J(S_2) =$, $\pi^J(S_4) =$, $\pi^J(S_6) =$ } say black

$\pi^J(Black \mid S_0) = 1$	$\pi^J(white \mid S_0) = 0$	$\pi^J(KQ \mid S_0) = 0$
$\pi^J(Black \mid S_2) = 1$	$\pi^J(white \mid S_2) = 0$	$\pi^J(KQ \mid S_2) = 0$
$\pi^J(Black \mid S_4) = 1$	$\pi^J(white \mid S_4) = 0$	$\pi^J(KQ \mid S_4) = 0$
$\pi^J(Black \mid S_6) = 1$	$\pi^J(white \mid S_6) = 0$	$\pi^J(KQ \mid S_6) = 0$

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- There is never a 4th step, no matter the initial state. If we begin with S_3 then at $t=1$ Roberto solves it successfully. If we begin at S_1 or S_5 Filipe solves it succesfully at $t=2$, if we have all the others then Jolly solves it at $t=3$. The reason why is in these 6 pages. to make it go for more we would need 3 black 3 white foots