Intelligent Multi Agent Systems

TECHNISCHE UNIVERSITÄT DARMSTADT

Summer Semester 2016, Homework 2 (66 points)

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Due date: Tuesday 7$^{\text{th}}$ June, 2016, 23:59 CEST (that's 02:59 AM on the following day in the French Southern Territories!)

How to hand in: The submission and well commented code have to be handed in as a zip-file via moodle. Working in groups of up to three people is highly encouraged and every group needs to submit only once with the names of all collaborators. We will not accept any late hand-ins. The department rules regarding plagiarism apply.

Problem 2.1 Theoretical Questions

Keep your answers short! As a rule of thumb: not more than two sentences per point! Explain your answers.

a) **Extensive Form Games** **[6 Points]**

1. What is the difference between extensive form games and normal form games?
2. Can every extensive form game be converted into a normal form game?
3. What are the pros and cons of expressing an extensive form game as a normal form game?
4. Can every normal form game be converted into an extensive form game?
5. Explain what a threat is with respect to extensive form games.
6. Why does every perfect information game in extensive form have a pure strategy Nash equilibrium? Elaborate on the answer given in the slides.

b) **Sub-Game Perfect Equilibrium** **[5 Points]**

1. Draw the tree diagram of a simple extensive game, with two players and two options each, and use it to explain the concept of a sub-game.
2. What is a sub-game perfect equilibrium?
3. Why is every sub-game perfect equilibrium a Nash equilibrium?
4. Can there be multiple sub-game perfect equilibria for the same extensive game? Give an example game with two players and two actions each.

c) **Bayesian Games** **[5 Points]**

1. What is the main problem that Bayesian games deal with?
2. Describe how Bayesian games can be converted into normal form games?
3. Explain the three meaningful notions of expected utility introduced in the lecture. Explain in your own words how they are connected.

Problem 2.2 Pen & Paper Exercises

a) **All About Histories** **[10 Points]**

It is noon in the black water valley, the sun is burning on a lonely cactus and vaporizes the last drop of water in the river bed. Big kahuna fiery Filipe's dark eyes gaze at the massive silhouette of Roberto the Kid. Roberto points his glossy single-shot carbine at the chief and demands Filipe's loyal horse Jolly Jumper. Filipe hops off Jolly's back, draws his last arrow from his quiver and aims it at Roberto. Being sick of playing *Hoppe, Hoppe, Reiter* for decades, Jolly jumps on his rear hooves and reveals his ninja star-like right horse shoe, ready to kill.

The only thing ridin me gonna be mares from now on, stupid monkeys!, Jolly Jumper nickers.

Damn ol' horse, Filipe thinks, *on damn strike again. When will they finally finish the damn rail tracks?!*

Two cold-blooded killers and fiery Filipe, one shot each, classic IAS stand-off. Roberto shoots first. Because his carbine is a little bit decalibrated, he hits his targets with a chance of only 40%. Next comes the horse. Jolly practiced a bit so he will hit with a probability of 50 %. Finally it's Filipe's turn. As an expert marksman, 60% of his shots are lethal. Since you always meet twice in the Wild West, everybody tries to minimize the danger for the next encounter, where the danger is proportional to the hitting probabilities.

- Write down the tree diagram of the extensive perfect information game with chance moves.
- Compute all sub-game equilibria. (Exploit reoccurring sub-trees.)
- What are the sub-game perfect equilibria of the game?

All similarities to any people, living or dead, or any events, in Darmstadt or elsewhere, are purely coincidental.

b) **Bayesian Game** **[10 Points]**

Consider the Bayesian game in Figure 1. The particular game is chosen by an act of God (or whomever)

Game A

0 2	2 0
2 0	0 2

$p = 0.3$

Game B

2 2	3 0
0 3	1 1

$p = 0.1$

Game C

2 2	0 0
0 0	1 1

$p = 0.2$

Game D

1 2	0 0
0 0	2 1

$p = 0.4$

Figure 1: Bayesian game.

according to the given probabilities without informing the players.

1. Write down the name for each game.
2. Assume that Player 1 knows the row and Player 2 knows the column of the game they play.
   i. What notion of expected utility is suitable for this Bayesian game.
   ii. Compute all pure strategy Nash equilibria. (Show all your steps.)
3. Assume now that both players have no information at all about the game they play. Hence, the strategies are chosen independent of the type.
   i. What notion of expected utility is suitable for this Bayesian game.
   ii. Compute all pure strategy Nash equilibria. (Show all your steps.)

c) **The Big Cake Fight** **[10 Points]**
Vera and Mona are sitting in front of a sweet smelling, mouth-watering, freshly baked marble cake. They are facing the problem of dividing the cake between them such that everybody is d'accord with it. We will assume that the cake has a standardized length of 1 cake unit and that the two kids may only use one straight cut to divide the cake. Vera proposes a cut $x \in [0,1]$ to Mona and Mona might either accept or refuse. If Mona refuses, the two will fight and the winner takes it all, however, both of them will carry away some costs c_V and c_M in terms of bruises, ripped off hair and being grounded. From earlier battles, we know that Vera wins with a probability p_V. (Thus, Mona wins with a probability $p_M = 1 - p_V$.) The utility for both is the stake of the cake, that is, the size of the slice. The costs of battling are also measured in cake units.

1. Assume the costs of the cake battle for both parties are common knowledge.
   i. Compute the expected utilities of having a fight for both parties.
   ii. What is rationally the best cut Vera should propose?
   iii. Does it make sense for Mona to fight for cake?
2. Assume now that Mona's costs c_M are uniformly drawn from the interval $[0,1]$ and known solely to Mona. Vera's costs c_V are still fixed and common knowledge.
   i. From Mona's point of view, what offers will she still accept and what offers will she reject?
   ii. What is the probability that an offer from Vera will be rejected by Mona?
   iii. Compute Vera's expected utility of offering to cut at x.
   iv. With respect to the expected utility, what is rationally the best cut x^* Vera should offer?
   v. How should Mona react to offered cuts?
3. What is your favorite cake? **[0 points]**
   Bake it, bring it to the lecture and share it with the TA staff. **[1 bonus point]**

Hint: take the variables as variables and let them be variables.

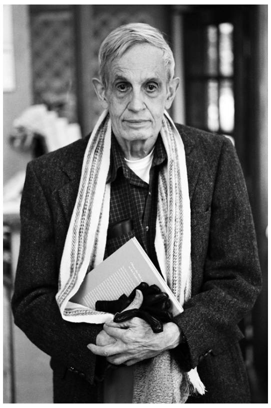
Problem 2.3 Matlab Exercises

a) **Tic-Tac-Toe** **[20 Points]**
Download the Matlab templates from moodle. Implement the functions `getActions`, `getPlayer`, `isTerminalState` and `backwardInduction`. Do not change the signatures of these function.

1. If the computer begins the game, the first move is chosen according to the best action returned by `backwardInduction` function. Is this necessary? Use the function `backwardInduction` to justify your answer.
2. Given the insights of the previous question, copy the code of `play` to a new function `playFaster` and let the computer chose the first move based on a faster heuristic that also maximizes the fun of playing tic-tac-toe against a computer.

3. Could we also apply this heuristic for the second move if the human player begins the game? Write a Matlab script `testSecondMove` that uses `backwardInduction` to prove your answer or to show a counter example.
4. What additional insights of the game could we use to improve the run-time of the function `backwardInduction`? Name at least two. You do not need to implement them.

John Forbes Nash, Jr. (June 13, 1928 – May 23, 2015) was an American mathematician with notable contributions in game theory, differential geometry, and partial differential equations. Nash's work has provided insight into the factors that govern chance and events inside complex systems in daily life.



His theories are used in economics, computing, evolutionary biology, artificial intelligence, accounting, computer science (minimax algorithm which is based on Nash Equilibrium), games of skill, politics and military theory. Serving as a Senior Research Mathematician at Princeton University during the latter part of his life, he shared the 1994 Nobel Memorial Prize in Economic Sciences with game theorists Reinhard Selten and John Harsanyi. In 2015, he was awarded the Abel Prize for his work on nonlinear partial differential equations.

In 1959, Nash began showing clear signs of mental illness, and spent several years at psychiatric hospitals being treated for paranoid schizophrenia. After 1970, his condition slowly improved, allowing him to return to academic work by the mid-1980s. His struggles with his illness and his recovery became the basis for Sylvia Nasar's biography, A Beautiful Mind, as well as a film of the same name starring Russell Crowe.

On May 23, 2015, Nash and his wife, Alicia Nash, while riding in a taxi, were killed in a motor vehicle accident in New Jersey.

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