A set of mixed-element transition patterns for adaptive 3D meshing

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Abstract

In this technical report, we introduce a set of mixed-element patterns that are meant to be used with the Octree. In difference with previous works, we programmed the whole set of patterns, considering all possible combinations when an Octant and its neighbors differ, at most, in one level of refinement. By doing this, we expect to have more control over the quantity of nodes in the final mesh, because other techniques that implement a subset of transition patterns, must refine the Octant whenever the pattern is not found in their list.

1 Introduction

The Octree technique [8, 7] is one of the most used techniques to concentrate nodes in a given region of a mesh. The basic idea is to split an Octant into several new Octants in a recursive way. The decision for splitting or not an Octant is made regarding some geometric constraint. The original idea was to split each Octant that intersected a portion of input domain's boundary. Once the splitting process (which can be seen as a refinement process) is finished, the mesh is topologically inconsistent. This is due to the fact that not all Octants are refined to the same level.

There are two different approaches for achieving correct transitions between coarse and fine regions: using only hexahedra and using mixed–elements. In the first case, whenever an Octant must be refined, it is split into 27 new Octants (a grid of $3 \times 3 \times 3$ new cubes). On the other hand, when using mixed–elements, each Octant is split into 8 new Octants when refinement is needed, which gives more control over the quantity of nodes in the final mesh.

Several techniques exists to perform the transitions using only hexahedra [5, 6, 10, 9, 3, 2]. On the other hand, some works using mixed–elements are the ones of Conti et al. [1], and more recently, the work of Nicolas and Fouquet in [4]. It is important to mention that all these techniques implement a set of patterns, for managing 1-irregular cases; meaning that an Octant differs at most, in one level of refinement with their neighbors. To be more specific, they all implement a subset of all possible 1-irregular patterns (no matter what type of elements they employ). Therefore, whenever a given configuration is not found in the set of patterns, they refine the Octant, which will increase the number of nodes in the mesh. Moreover, this unnecessary refinement may be extended throughout large regions until a pattern is found.

For this reason, in this technical report we show our whole set of transition patterns that consider all possible 1-irregular configurations. The goal to perform this important task, is to avoid the insertion of unnecessary nodes in the final mesh, due to non existence of a transition pattern for a given configuration.

2 Element and pattern conventions

The four basic element types considered in this work are shown in Figure 1. This convention will be used by all transition pattern to define their sub–elements. For instance if we define a prism (wedge) with the following indexes: 11 20 19 0 8 4, it means that one triangular face t, is defined by indexes: 11, 20 and 19. Moreover, nodes 0, 8 and 4 will have a positive distance w.r.t. to t, when its normal is computed counterclockwise.

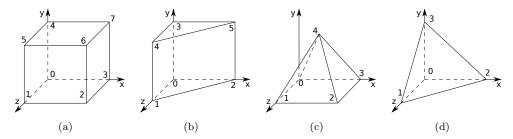


Figure 1: Basic elements: (a) hexahedron, (b) prism (wedge), (c) pyramid and (d) tetrahedron.

These patterns are meant to be applied when the mesh was generated by an Octree technique. Before managing transitions, all Octants are cubes, and they must be 1-irregular. Therefore, each edge may count with at most 1 node inserted in the middle of it, and same occurs with each face. All possible indexes in an Octant are shown in Figure 2.

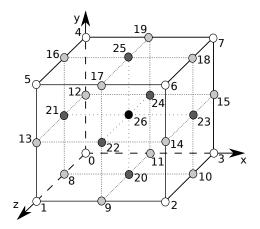


Figure 2: All possible nodes inserted in an hexahedron.

The only configurations allowed in the faces of a pattern, are the ones presented in figure 3. Following these "face patterns" we programmed a code capable of generating all possible combinations. Then we erased invalid configurations. Finally we reduced them to a total of 325 different patterns by using rotational operations. In other words, this set of 325 patterns, through rotational operations, considers all possible configurations of 1–irregular Octants.

As mentioned before, the goal of these patterns is to reduce the number of overall nodes in the final mesh. Unfortunately, for some configurations it was impossible to discretize the space with basic elements (shown in Figure 1) and, at the same time, comply with the constraints imposed by the patterns over the faces (shown in Figure 3). From a total of 325 patterns, 260 achieve correct subdivision, without inserting Octant's central node. They are introduced in section 3. The other 65 patterns are introduced in section 4, at page 134. Note that all values of the Jacobian Ratio quality measure, have the value 1. This is due to no displacement is performed over the nodes. Therefore, they are all perfect elements in terms of this quality measure.

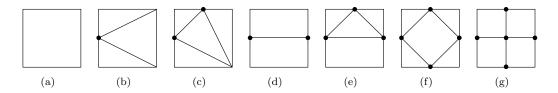


Figure 3: All possible cases for nodes inserted in edges or face central position.

The Aspect Ratio (AR) measure, on the other hand, give us more information on the actual quality of the patterns. Each pattern shows the average AR for its new sub–elements.

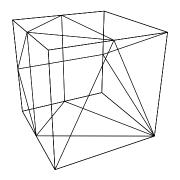
3 Patterns without cube's central node

Pattern 1

Node index:

8 11 12 15 16 17 19 24

List of Elements:

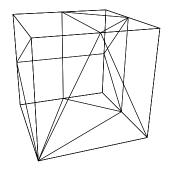


Aspect Ratio: 0.643957

Node index:

10 11 12 15 17 18 19 24

List of Elements:



Aspect Ratio: 0.643957

Jacobian Ratio: 1

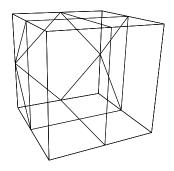
Pattern 3

Node index:

8 9 10 11 12 15 16 17 18 19 20 25

List of Elements:

Pyramid 3 11 20 10 15 Pyramid 0 8 20 11 12 Pyramid 7 18 25 19 15 Pyramid 4 19 25 16 12 Pyramid 12 11 15 19 25 Tetrahedra 11 25 20 15 Tetrahedra 11 20 25 12 Pyramid 10 20 25 18 15 Pyramid 8 16 25 20 12 Hexaedra 1 9 20 8 5 17 25 16 Hexaedra 2 10 20 9 6 18 25 17

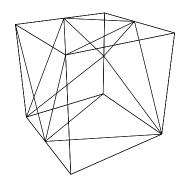


Aspect Ratio: 0.668527

Node index:

8 15 18 19

List of Elements:



Aspect Ratio: 0.530245

Jacobian Ratio: 1

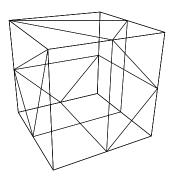
Pattern 5

Node index:

8 9 10 11 12 14 15 17 18 20 23

List of Elements:

Pyramid 1 5 6 2 18

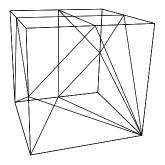


Aspect Ratio: 0.634707

Node index:

11 13 15 16 17 18 19 25

List of Elements:



Aspect Ratio: 0.622529

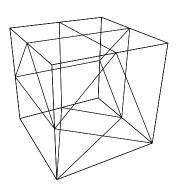
Jacobian Ratio: 1

Pattern 7

Node index:

 $10\ 11\ 12\ 13\ 15\ 16\ 17\ 18\ 19\ 24\ 25$

List of Elements:



Aspect Ratio: 0.603082

Node index:

8 9 10 11 13 15 19 20

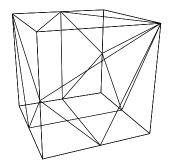
List of Elements:

Pattern 9

Node index:

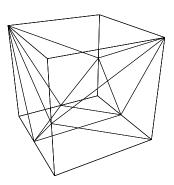
 $8\ 10\ 11\ 13\ 15$

List of Elements:



Aspect Ratio: 0.621381

Jacobian Ratio: 1



Aspect Ratio: 0.599592

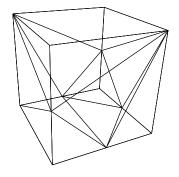
Node index: 9 10 11 13 15

List of Elements:

Pattern 11

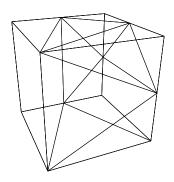
Node index: 11 14 15 18 19

List of Elements:



Aspect Ratio: 0.60588

Jacobian Ratio: 1



Aspect Ratio: 0.633551

Node index: 11 12 13 16 19

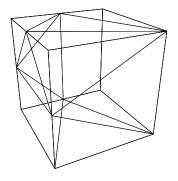
List of Elements:

Pyramid 0 12 13 1 11 Prism 2 3 11 6 7 19 Tetrahedra 1 2 11 13 Tetrahedra 4 12 19 16 Tetrahedra 11 19 12 13 Tetrahedra 12 16 13 19 Pyramid 2 6 19 11 13 Tetrahedra 6 16 19 13 Tetrahedra 5 16 6 13



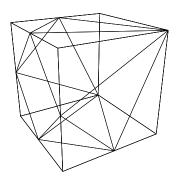
Node index: 8 9 12 13 16 19

List of Elements:



Aspect Ratio: 0.633551

Jacobian Ratio: 1

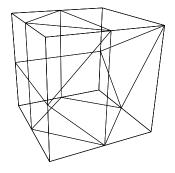


Aspect Ratio: 0.56748

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 15\ 16\ 17\ 19\ 21\ 24$

List of Elements:



Aspect Ratio: 0.721171

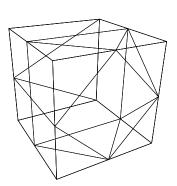
Jacobian Ratio: 1

Pattern 15

Node index:

8 9 10 12 13 14 16 17 18 21

List of Elements:



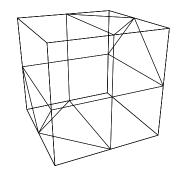
Aspect Ratio: 0.697042

Node index:

 $8\ 9\ 11\ 12\ 13\ 14\ 15\ 17\ 18\ 19\ 22\ 24$

List of Elements:

Hexaedra 3 15 24 11 2 14 22 9 Pyramid 6 14 22 17 18 Pyramid 17 22 24 19 18 Pyramid 7 19 24 15 18 Pyramid 14 15 24 22 18 Pyramid 1 13 22 9 8 Pyramid 9 22 24 11 8 Pyramid 0 11 24 12 8 Pyramid 13 12 24 22 8 Hexaedra 5 17 22 13 4 19 24 12



Aspect Ratio: 0.626599

Jacobian Ratio: 1

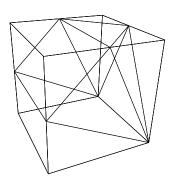
Pattern 17

Node index:

12 13 17 18 19

List of Elements:

Tetrahedra 6 17 18 2 Tetrahedra 7 18 19 3 Pyramid 2 3 19 17 18 Prism 3 19 12 2 17 13 Prism 0 3 12 1 2 13 Prism 4 12 19 5 13 17



Aspect Ratio: 0.577624

Node index: 8 9 10 11 13 16 18 20

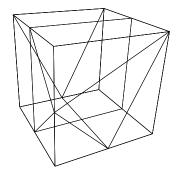
List of Elements:



Node index:

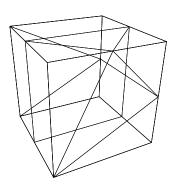
8 10 14 15 16 17 18 23

List of Elements:



Aspect Ratio: 0.547036

Jacobian Ratio: 1



Aspect Ratio: 0.552012

Node index:

8 9 10 11 12 15 18 19 24

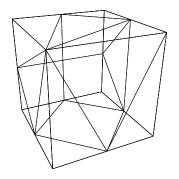
List of Elements:



Node index:

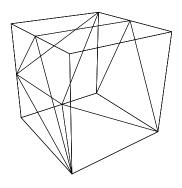
11 12 16 18

List of Elements:



Aspect Ratio: 0.709654

Jacobian Ratio: 1



Aspect Ratio: 0.544065

Node index: 12 15 16 17

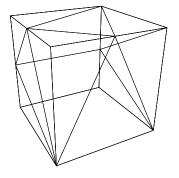
List of Elements:



Node index:

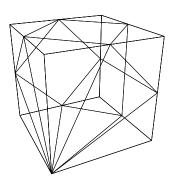
10 11 12 14 15 16 17 18 19 23

List of Elements:



Aspect Ratio: 0.544065

Jacobian Ratio: 1

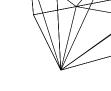


Aspect Ratio: 0.743026

Node index:

10 11 12 14 15 17 18 19 23

List of Elements:



Aspect Ratio: 0.709654

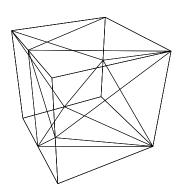
Jacobian Ratio: 1

Pattern 25

Node index:

8 11 13 15 16

List of Elements:



Aspect Ratio: 0.590326

Node index:

 $8\ 9\ 10\ 11\ 13\ 15\ 18\ 19\ 20$

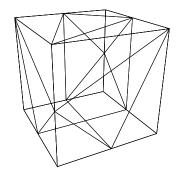
List of Elements:

Pattern 27

Node index:

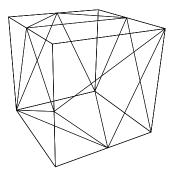
9 13 16 18 19

List of Elements:



Aspect Ratio: 0.589138

Jacobian Ratio: 1



Aspect Ratio: 0.547171

Node index:

 $8\ 9\ 11\ 12\ 15\ 16\ 17\ 19\ 24$

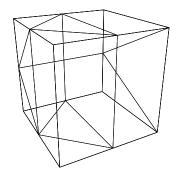
List of Elements:

Pattern 29

Node index:

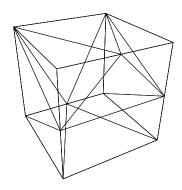
11 13 14 17

List of Elements:



Aspect Ratio: 0.618749

Jacobian Ratio: 1



Aspect Ratio: 0.548601

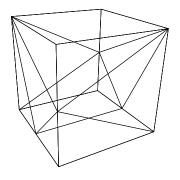
Node index: 8 10 13 15

List of Elements:

Pattern 31

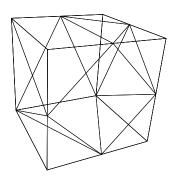
Node index: 9 13 14 17 18 19

List of Elements:



Aspect Ratio: 0.604172

Jacobian Ratio: 1



Aspect Ratio: 0.630342

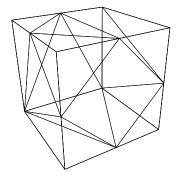
Node index: 9 13 14 16 17 19

List of Elements:

Pattern 33

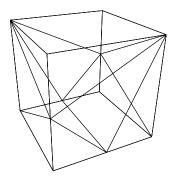
Node index: 9 11 13 15

List of Elements:



Aspect Ratio: 0.630342

Jacobian Ratio: 1



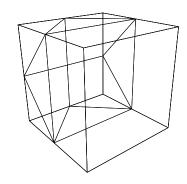
Aspect Ratio: 0.604172

Node index:

8 10 11 12 15 16 18 19 24

List of Elements:

Pyramid 0 11 24 12 8 Pyramid 3 15 24 11 10 Pyramid 4 12 24 19 16 Pyramid 24 15 7 19 18 Tetrahedra 16 19 18 24 Tetrahedra 10 18 15 24 Tetrahedra 8 10 11 24 Tetrahedra 8 12 16 24 Pyramid 8 16 18 10 24 Hexaedra 8 10 18 16 1 2 6 5



Aspect Ratio: 0.712592

Jacobian Ratio: 1

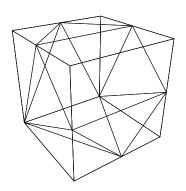
Pattern 35

Node index:

9 13 14 16 18 19

List of Elements:

Tetrahedra 0 19 4 16 Tetrahedra 3 7 19 18 Pyramid 0 16 18 3 19 Tetrahedra 0 1 9 13 Tetrahedra 2 3 9 14 Pyramid 0 3 14 13 9 Prism 3 14 18 0 13 16 Prism 5 13 16 6 14 18



Aspect Ratio: 0.543306

Node index:

8 10 11 12 15 17 19 24

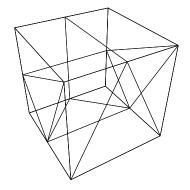
List of Elements:



Node index:

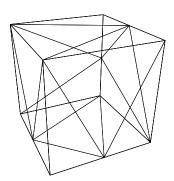
8 9 15 18

List of Elements:



Aspect Ratio: 0.583703

Jacobian Ratio: 1



Aspect Ratio: 0.57022

Node index: 9 13 15 18

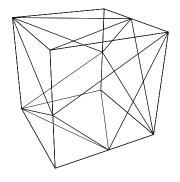
List of Elements:

Pattern 39

Node index:

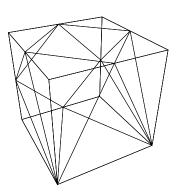
11 12 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.574248

Jacobian Ratio: 1

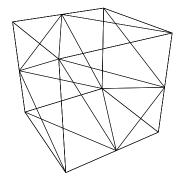


Aspect Ratio: 0.657945

Node index:

9 12 14 19

List of Elements:



Aspect Ratio: 0.555405

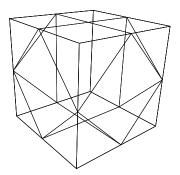
Jacobian Ratio: 1

Pattern 41

Node index:

 $8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 16 \ 17 \ 18 \ 19 \ 20 \ 25$

List of Elements:



Aspect Ratio: 0.704974

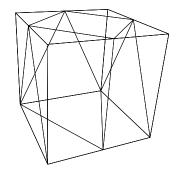
Node index: 9 16 17 18 19

List of Elements:



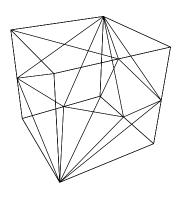
Node index: 10 11 12 14 17

List of Elements:



Aspect Ratio: 0.633527

Jacobian Ratio: 1



Aspect Ratio: 0.593547

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 19\ 20$

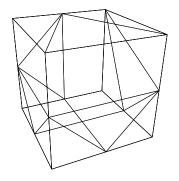
List of Elements:



Node index:

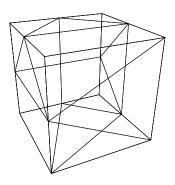
10 11 12 13 15 16 18 19 24

List of Elements:



Aspect Ratio: 0.649862

Jacobian Ratio: 1



Aspect Ratio: 0.649862

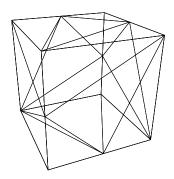
Node index: 9 13 15 18 19

List of Elements:

Pattern 47

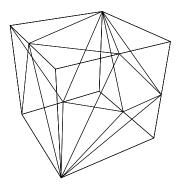
Node index: 10 11 14 16 17

List of Elements:



Aspect Ratio: 0.54986

Jacobian Ratio: 1



Aspect Ratio: 0.543907

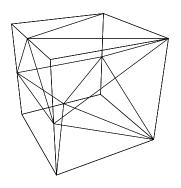
Node index: 11 12 13 15 16

List of Elements:

Pattern 49

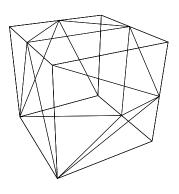
Node index: 10 14 16 18 19

List of Elements:



Aspect Ratio: 0.632368

Jacobian Ratio: 1



Aspect Ratio: 0.632368

Node index:

8 9 12 13 14 18 19

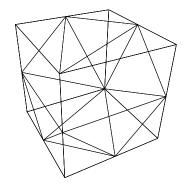
List of Elements:



Node index:

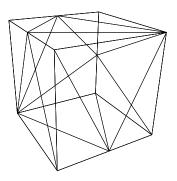
9 15 16 19

List of Elements:



Aspect Ratio: 0.599451

Jacobian Ratio: 1

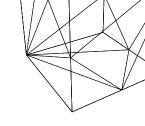


Aspect Ratio: 0.56003

Node index:

9 10 13 15 16 19

List of Elements:



Aspect Ratio: 0.586226

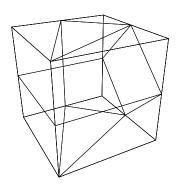
Jacobian Ratio: 1

Pattern 53

Node index:

10 11 12 13 14 15 18 19 24

List of Elements:



Aspect Ratio: 0.623327

Node index: 8 14 15 17 18

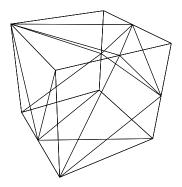
List of Elements:

Pattern 55

Node index:

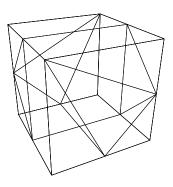
 $8\ 9\ 10\ 11\ 12\ 14\ 16\ 18\ 20$

List of Elements:



Aspect Ratio: 0.580319

Jacobian Ratio: 1



Aspect Ratio: 0.585572

Node index: 8 9 12 18

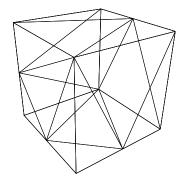
List of Elements:



Node index:

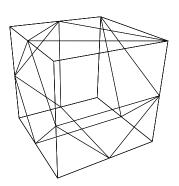
 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 16\ 19\ 20$

List of Elements:



Aspect Ratio: 0.566426

Jacobian Ratio: 1

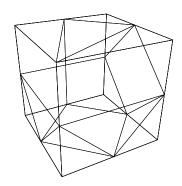


Aspect Ratio: 0.673675

Node index:

8 9 10 11 12 13 14 15 18 19 24

List of Elements:



Aspect Ratio: 0.730002

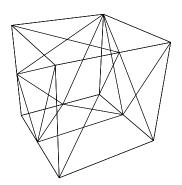
Jacobian Ratio: 1

Pattern 59

Node index:

8 10 11 12 17

List of Elements:



Aspect Ratio: 0.56836

Node index: 9 10 13 14 16 19

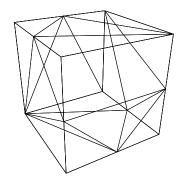
List of Elements:



Node index:

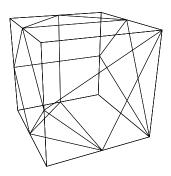
8 9 11 12 13 15 16 18 19 24

List of Elements:



Aspect Ratio: 0.52625

Jacobian Ratio: 1



Aspect Ratio: 0.673675

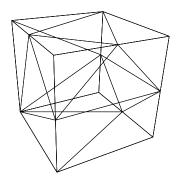
Node index: 10 13 14 15 16 17

List of Elements:

Pattern 63

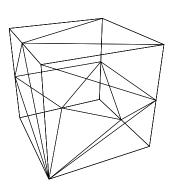
Node index: 10 11 12 14 15 16

List of Elements:



Aspect Ratio: 0.53559

Jacobian Ratio: 1

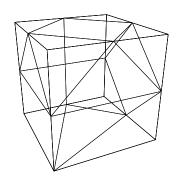


Aspect Ratio: 0.589858

Node index:

10 11 12 13 14 15 16 17 18 19 24

List of Elements:



Aspect Ratio: 0.726483

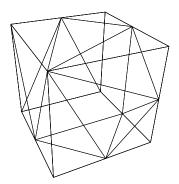
Jacobian Ratio: 1

Pattern 65

Node index:

8 9 10 14 18 19

List of Elements:



Aspect Ratio: 0.589858

Node index: 10 12 14 16 17 19

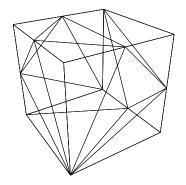
List of Elements:



Node index:

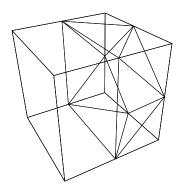
9 10 11 14 15 17 18 19

List of Elements:



Aspect Ratio: 0.52625

Jacobian Ratio: 1



Aspect Ratio: 0.640494

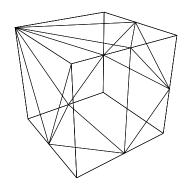
Node index: 8 9 11 14 15 17 18

List of Elements:

Pattern 69

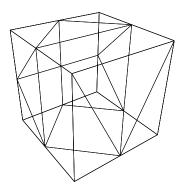
Node index: 8 9 10 11 12 15 16 18 19 24

List of Elements:



Aspect Ratio: 0.608635

Jacobian Ratio: 1



Aspect Ratio: 0.735034

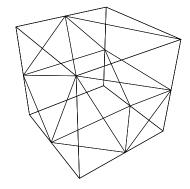
Node index: 8 9 10 12 14 15 19

List of Elements:

Pattern 71

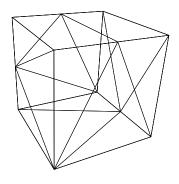
Node index: 10 12 17 19

List of Elements:



Aspect Ratio: 0.568155

Jacobian Ratio: 1



Aspect Ratio: 0.574119

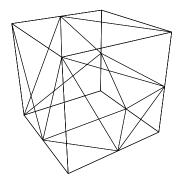
Node index: 8 9 10 14 15 19

List of Elements:

Pattern 73

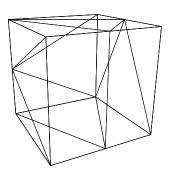
Node index: 9 12 17 18

List of Elements:



Aspect Ratio: 0.612352

Jacobian Ratio: 1



Aspect Ratio: 0.574119

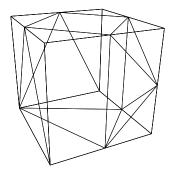
Node index: 9 10 14 16 17 18 19

List of Elements:

Pattern 75

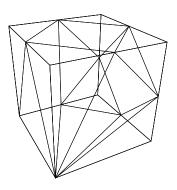
Node index: 10 11 14 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.529231

Jacobian Ratio: 1



Aspect Ratio: 0.648848

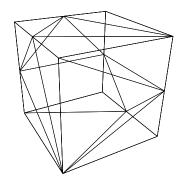
Node index: 10 12 14 15 16 19

List of Elements:

Pattern 77

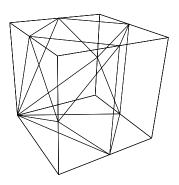
Node index: 9 10 13 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.599315

Jacobian Ratio: 1



Aspect Ratio: 0.58597

Node index:

8 9 10 11 12 14 15 17 19 20

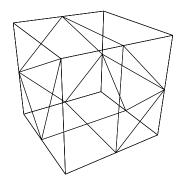
List of Elements:

Pattern 79

Node index:

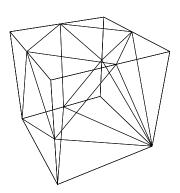
11 13 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.671754

Jacobian Ratio: 1



Aspect Ratio: 0.578921

Node index:

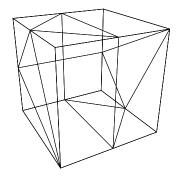
9 10 11 12 15 16 17 19 24

List of Elements:



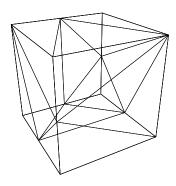
Node index: 8 10 11 13 15 19

List of Elements:



Aspect Ratio: 0.581173

Jacobian Ratio: 1

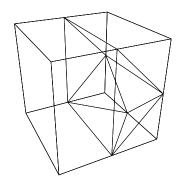


Aspect Ratio: 0.558171

Node index: 9 10 11 14 15 17 19

List of Elements:

Tetrahedra 3 11 10 15 Tetrahedra 2 10 9 14 Pyramid 11 15 14 9 10 Prism 17 6 14 19 7 15 Prism 17 14 9 19 15 11 Hexaedra 0 1 9 11 4 5 17 19



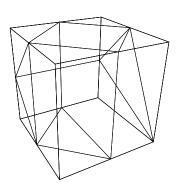
Aspect Ratio: 0.599844

Jacobian Ratio: 1

Pattern 83

Node index: 8 9 11 12 15 16 17 18 19 24

List of Elements:



Aspect Ratio: 0.6686

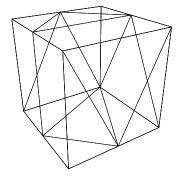
Node index: 8 9 16 18 19

List of Elements:

Pattern 85

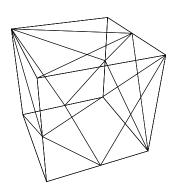
Node index: 9 11 13 15 18

List of Elements:



Aspect Ratio: 0.537716

Jacobian Ratio: 1

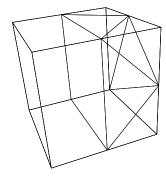


Aspect Ratio: 0.546867

Node index: 9 11 14 17 18 19

List of Elements:

Pyramid 19 11 3 7 18 Pyramid 9 2 3 11 14 Tetrahedra 18 3 14 11 Tetrahedra 11 9 14 18 Pyramid 11 19 17 9 18 Tetrahedra 9 17 14 18 Tetrahedra 17 18 6 14 Hexaedra 0 1 9 11 4 5 17 19



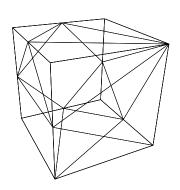
Aspect Ratio: 0.541496

Jacobian Ratio: 1

Pattern 87

Node index: 10 11 12 13 15 16 19

List of Elements:



Aspect Ratio: 0.599939

Node index: 10 11 16 17 19

List of Elements:

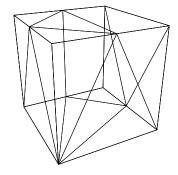
Pyramid 19 11 3 7 10 Pyramid 4 0 11 19 16 Pyramid 19 7 6 17 10 Tetrahedra 0 1 11 16 Pyramid 11 16 17 10 19 Pyramid 10 17 16 11 1 Tetrahedra 5 16 17 1 Tetrahedra 10 2 6 17 Tetrahedra 1 2 10 17

Pattern 89

Node index: 10 12 15 16 17 18 19

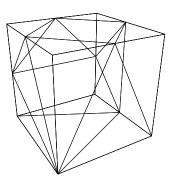
10 12 10 10 11 10 10

List of Elements:



Aspect Ratio: 0.537716

Jacobian Ratio: 1



Aspect Ratio: 0.618184

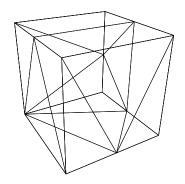
Node index: 9 10 15 16 18

List of Elements:

Pattern 91

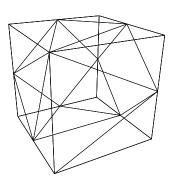
Node index: 8 10 11 12 14 15 18 19

List of Elements:



Aspect Ratio: 0.569168

Jacobian Ratio: 1



Aspect Ratio: 0.648848

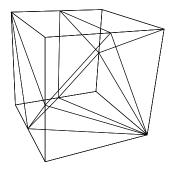
Node index: 8 11 13 15 17 19

List of Elements:

Pattern 93

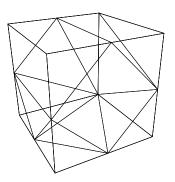
Node index: 8 9 12 13 14 17 19

List of Elements:



Aspect Ratio: 0.516302

Jacobian Ratio: 1



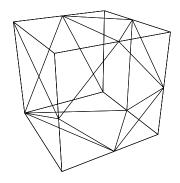
Aspect Ratio: 0.531708

Node index:

9 10 13 14 17 18 19

List of Elements:

Pyramid 9 13 17 14 10



Aspect Ratio: 0.665023

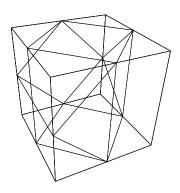
Jacobian Ratio: 1

Pattern 95

Node index:

8 9 10 11 12 13 15 16 17 18 19 21

List of Elements:



Aspect Ratio: 0.798948

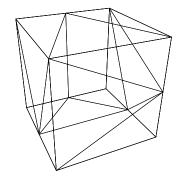
Node index: 8 10 11 14 19

List of Elements:

Pattern 97

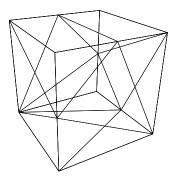
Node index: 10 13 15 17 19

List of Elements:



Aspect Ratio: 0.611976

Jacobian Ratio: 1



Aspect Ratio: 0.550399

Node index:

9 10 12 13 15 16

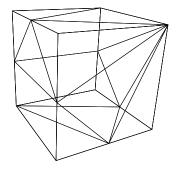
List of Elements:



Node index:

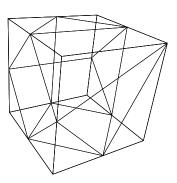
8 9 10 11 12 13 15 16 18 19 24

List of Elements:



Aspect Ratio: 0.563038

Jacobian Ratio: 1



Aspect Ratio: 0.747646

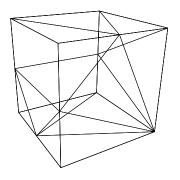
Node index: 8 12 13 15 17

List of Elements:

Pattern 101

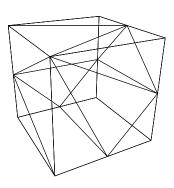
Node index: 9 11 12 14 15 18

List of Elements:



Aspect Ratio: 0.569168

Jacobian Ratio: 1



Aspect Ratio: 0.607479

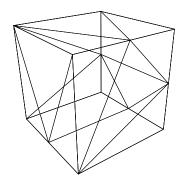
Node index: 8 10 14 15 17

List of Elements:

Pattern 103

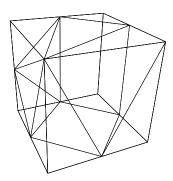
Node index: 8 9 10 11 12 18 19

List of Elements:



Aspect Ratio: 0.611976

Jacobian Ratio: 1



Aspect Ratio: 0.577

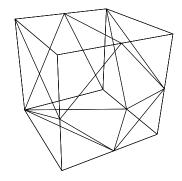
Node index: 9 10 13 14 17 19

List of Elements:

Pattern 105

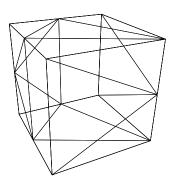
Node index: 8 11 12 14 16 19

List of Elements:



Aspect Ratio: 0.597526

Jacobian Ratio: 1



Aspect Ratio: 0.511314

Node index:

 $8\ 15\ 17$

List of Elements:

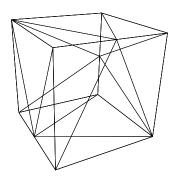


Node index:

 $9\ 11\ 12\ 13\ 14\ 17$

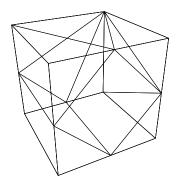
List of Elements:

Prism 1 13 9 0 12 11 Pyramid 11 9 2 3 14 Tetrahedra 3 11 14 7 Pyramid 11 12 17 14 7 Prism 17 13 12 14 9 11 Tetrahedra 7 6 17 14 Tetrahedra 7 4 12 17 Pyramid 12 4 5 13 17



Aspect Ratio: 0.630845

Jacobian Ratio: 1



Aspect Ratio: 0.567645

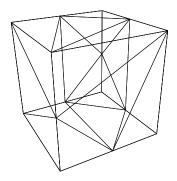
Node index: 9 10 11 13 15 18 19

List of Elements:

Pattern 109

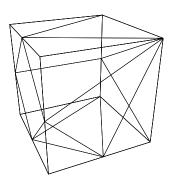
Node index: 8 9 12 13 15 16

List of Elements:



Aspect Ratio: 0.504729

Jacobian Ratio: 1



Aspect Ratio: 0.597526

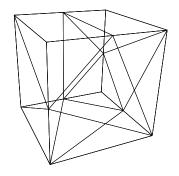
Node index: 10 11 13 15 17 19

List of Elements:

Pattern 111

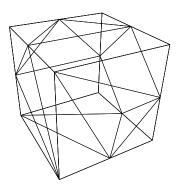
Node index: 9 10 12 14 15 16 18 19

List of Elements:



Aspect Ratio: 0.515055

Jacobian Ratio: 1

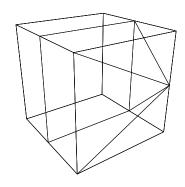


Aspect Ratio: 0.554274

Node index: 8 10 14 16 18

List of Elements:

Pyramid 5 16 18 6 14 Pyramid 16 5 1 8 14 Pyramid 8 1 2 10 14 Pyramid 18 16 8 10 14 Hexaedra 0 8 10 3 4 16 18 7



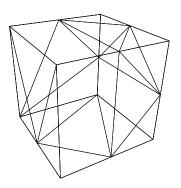
Aspect Ratio: 0.538436

Jacobian Ratio: 1

Pattern 113

Node index: 8 9 14 15 17 18 19

List of Elements:



Aspect Ratio: 0.530555

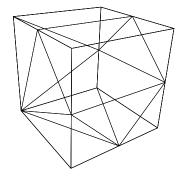
Node index: 9 13 14 15 16 18

List of Elements:

Pattern 115

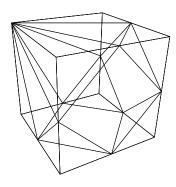
Node index: 8 9 10 11 13 14 15 17 18

List of Elements:



Aspect Ratio: 0.607479

Jacobian Ratio: 1

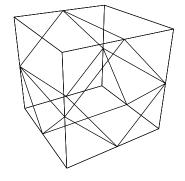


Aspect Ratio: 0.664971

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 17\ 19\ 20$

List of Elements:



Aspect Ratio: 0.759181

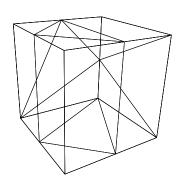
Jacobian Ratio: 1

Pattern 117

Node index:

8 9 15 16 17 19

List of Elements:

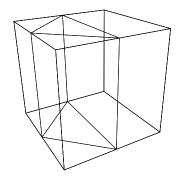


Aspect Ratio: 0.469957

Node index: 8 9 11 16 17 19

List of Elements:

Prism 11 0 8 19 4 16 Prism 11 8 9 19 16 17 Prism 8 1 9 16 5 17 Hexaedra 9 2 3 11 17 6 7 19



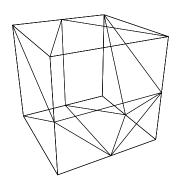
Aspect Ratio: 0.551777

Jacobian Ratio: 1

Pattern 119

Node index: 9 10 11 13 14 19

List of Elements:



Aspect Ratio: 0.530196

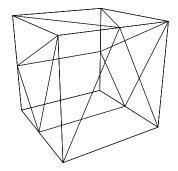
Node index: 8 10 12 15 17

List of Elements:

Pattern 121

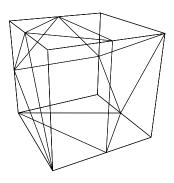
Node index: 9 10 12 15 16 17 19

List of Elements:



Aspect Ratio: 0.474382

Jacobian Ratio: 1



Aspect Ratio: 0.553396

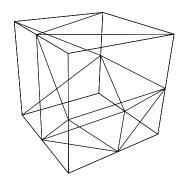
Node index: 8 9 10 13 14 15 16

List of Elements:

Pattern 123

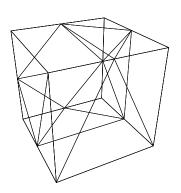
Node index: 8 10 11 12 15 17 18 19

List of Elements:



Aspect Ratio: 0.548909

Jacobian Ratio: 1



Aspect Ratio: 0.577605

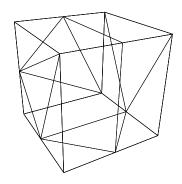
Node index: 8 9 10 12 17 19

List of Elements:

Pattern 125

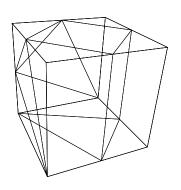
Node index: 9 10 12 16 17 18 19

List of Elements:



Aspect Ratio: 0.55525

Jacobian Ratio: 1



Aspect Ratio: 0.532186

Node index:

8 9 12 14 15 16 17 19

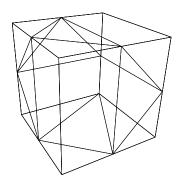
List of Elements:

Pattern 127

Node index:

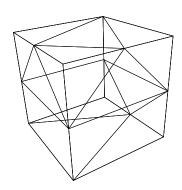
 $10\ 12\ 13\ 14\ 15\ 16\ 17$

List of Elements:



Aspect Ratio: 0.557624

Jacobian Ratio: 1



Aspect Ratio: 0.547649

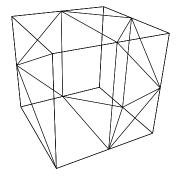
Node index: 9 10 11 12 14 17 18 19

List of Elements:

Pattern 129

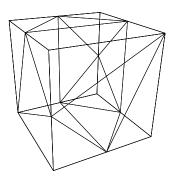
Node index: 9 10 11 13 15 16 18 19

List of Elements:



Aspect Ratio: 0.587915

Jacobian Ratio: 1



Aspect Ratio: 0.499085

Node index:

 $8\ 10\ 11\ 12\ 13\ 15\ 18\ 19$

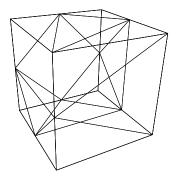
List of Elements:

Pattern 131

Node index:

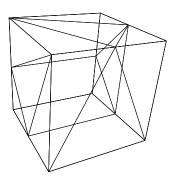
8 10 12 15 17 18

List of Elements:



Aspect Ratio: 0.577605

Jacobian Ratio: 1



Aspect Ratio: 0.469957

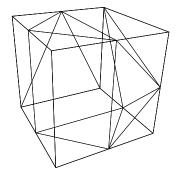
Node index: 8 9 10 14 16 17 19

List of Elements:

Pattern 133

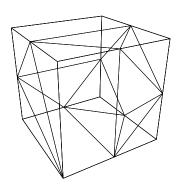
Node index: 9 10 11 12 14 15 16 17 18

List of Elements:



Aspect Ratio: 0.523819

Jacobian Ratio: 1



Aspect Ratio: 0.637702

Node index: 8 11 12 14 17 18

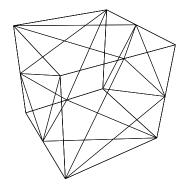
List of Elements:

Pattern 135

Node index:

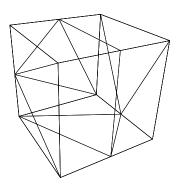
$9\ 10\ 12\ 17\ 19$

List of Elements:



Aspect Ratio: 0.506184

Jacobian Ratio: 1



Aspect Ratio: 0.517394

Node index:

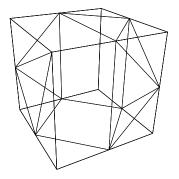
8 9 10 11 12 14 17 18 19

List of Elements:

Pattern 137

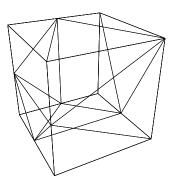
Node index: 8 10 11 12 13 19

List of Elements:



Aspect Ratio: 0.581977

Jacobian Ratio: 1



Aspect Ratio: 0.628867

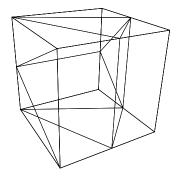
Node index: 9 10 12 15 17 18

List of Elements:

Pattern 139

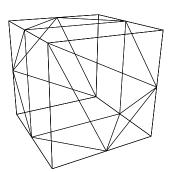
Node index: 8 9 10 12 14 16 18 19

List of Elements:



Aspect Ratio: 0.516024

Jacobian Ratio: 1



Aspect Ratio: 0.547424

Node index:

8 10 12 13 15 16 18 19

List of Elements:

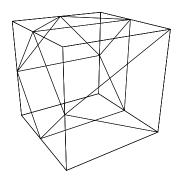


Node index:

8 9 10 15 16 17 18

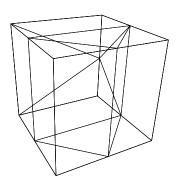
List of Elements:

Pyramid 0 8 10 3 15 Pyramid 0 4 16 8 15 Pyramid 4 7 18 16 15 Pyramid 10 8 16 18 15 Prism 8 1 9 16 5 17 Prism 8 9 10 16 17 18 Prism 9 2 10 17 6 18



Aspect Ratio: 0.586947

Jacobian Ratio: 1



Aspect Ratio: 0.557041

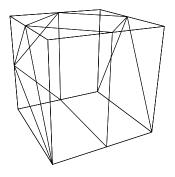
Node index: 9 11 12 16 17 18 19

List of Elements:

Pattern 143

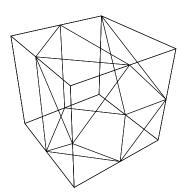
Node index: 8 9 10 11 13 14 16 17 19

List of Elements:



Aspect Ratio: 0.558543

Jacobian Ratio: 1



Aspect Ratio: 0.609416

Node index:

8 9 12 13 14 15 17 18 19

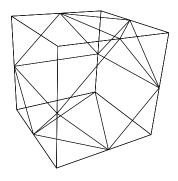
List of Elements:

Pattern 145

Node index:

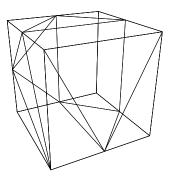
9 10 11 12 16 18 19

List of Elements:



Aspect Ratio: 0.581977

Jacobian Ratio: 1



Aspect Ratio: 0.56148

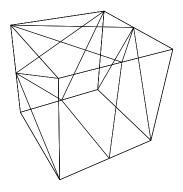
Node index: 9 11 12 17 18

List of Elements:

Pattern 147

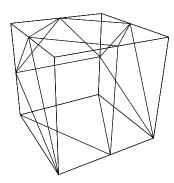
Node index: 9 12 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.517394

Jacobian Ratio: 1



Aspect Ratio: 0.55937

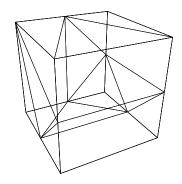
Node index: 8 10 11 13 14 15 19

List of Elements:

Pattern 149

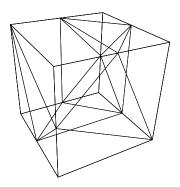
Node index: 8 10 11 13 15 17 18 19

List of Elements:



Aspect Ratio: 0.543173

Jacobian Ratio: 1



Aspect Ratio: 0.523066

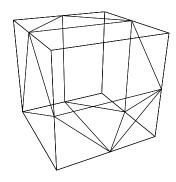
Node index: 9 10 11 13 14 16 18 19

List of Elements:

Pattern 151

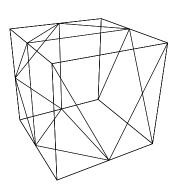
Node index: 8 9 11 12 16 18 19

List of Elements:



Aspect Ratio: 0.557624

Jacobian Ratio: 1



Aspect Ratio: 0.548909

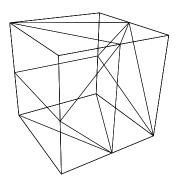
Node index: 9 12 13 15 17 18

List of Elements:

Pattern 153

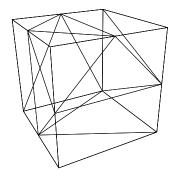
Node index: 8 13 14 16 17 19

List of Elements:



Aspect Ratio: 0.55525

Jacobian Ratio: 1



Aspect Ratio: 0.628867

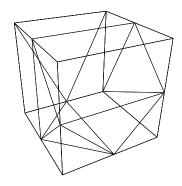
Node index: 8 9 11 13 14 16 18

List of Elements:

Pattern 155

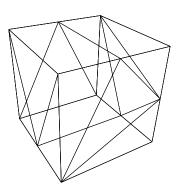
Node index: 8 10 14 17 19

List of Elements:



Aspect Ratio: 0.543173

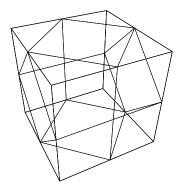
Jacobian Ratio: 1



Aspect Ratio: 0.517394

Node index:

List of Elements:



Aspect Ratio: 0.744161

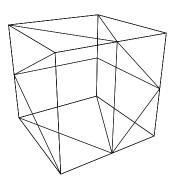
Jacobian Ratio: 1

Pattern 157

Node index:

9 12 14 15 17

List of Elements:



Aspect Ratio: 0.542742

Node index: 8 10 13 15 16 18

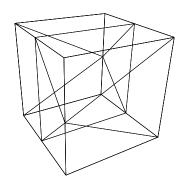
List of Elements:

Pyramid 0 8 10 3 15 Pyramid 0 4 16 8 15 Pyramid 4 7 18 16 15 Pyramid 8 16 18 10 15 Pyramid 16 8 10 18 13 Pyramid 1 2 10 8 13 Pyramid 2 6 18 10 13 Pyramid 6 5 16 18 13

Pattern 159

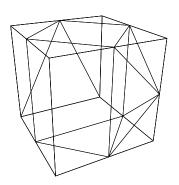
Node index: 8 9 10 14 16 17 18 19

List of Elements:



Aspect Ratio: 0.548045

Jacobian Ratio: 1



Aspect Ratio: 0.586947

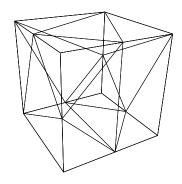
Node index: 9 10 11 13 15 16 17

List of Elements:

Pattern 161

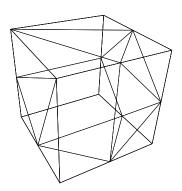
Node index: 8 9 10 12 14 15 17 18

List of Elements:



Aspect Ratio: 0.516779

Jacobian Ratio: 1



Aspect Ratio: 0.57575

Node index:

 $8\ 9\ 10\ 12\ 13\ 14\ 15\ 17\ 18\ 19$

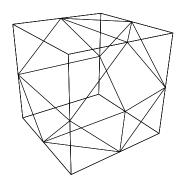
List of Elements:



Node index:

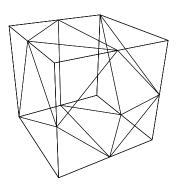
9 10 11 13 14 16 17 19

List of Elements:



Aspect Ratio: 0.670569

Jacobian Ratio: 1

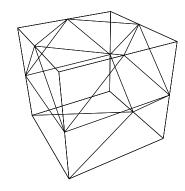


Aspect Ratio: 0.555983

Node index:

10 12 13 14 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.632675

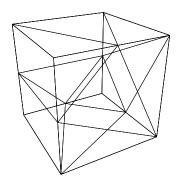
Jacobian Ratio: 1

Pattern 165

Node index:

10 11 12 13 15 17

List of Elements:



Aspect Ratio: 0.515826

Node index: 9 11 14 16 17 19

List of Elements:

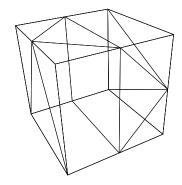
Pyramid 4 0 11 19 16 Pyramid 2 3 11 9 14 Pyramid 3 7 19 11 14 Pyramid 17 19 7 6 14 Pyramid 9 11 19 17 14 Pyramid 19 11 9 17 16 Pyramid 0 1 9 11 16 Pyramid 1 5 17 9 16



Node index:

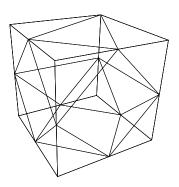
 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17$

List of Elements:



Aspect Ratio: 0.548045

Jacobian Ratio: 1



Aspect Ratio: 0.651127

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 17\ 19$

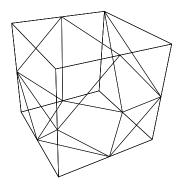
List of Elements:



Node index:

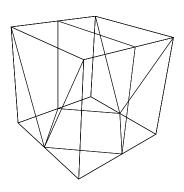
 $8\ 9\ 10\ 11\ 17\ 19$

List of Elements:



Aspect Ratio: 0.612786

Jacobian Ratio: 1



Aspect Ratio: 0.512638

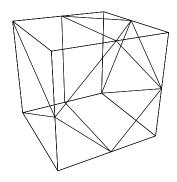
Node index: 9 11 13 14 17 18 19

List of Elements:

Pattern 171

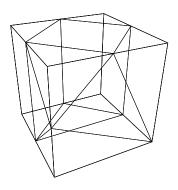
Node index: 8 10 11 13 16 17 18 19

List of Elements:



Aspect Ratio: 0.558543

Jacobian Ratio: 1



Aspect Ratio: 0.56125

Node index:

 $8\ 9\ 10\ 11\ 13\ 14\ 15\ 18\ 19$

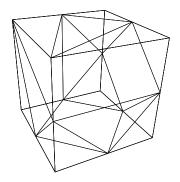
List of Elements:

Pattern 173

Node index:

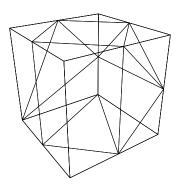
8 9 14 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.59977

Jacobian Ratio: 1



Aspect Ratio: 0.552356

Node index:

 $9\ 12\ 13\ 14\ 15\ 16\ 17\ 18$

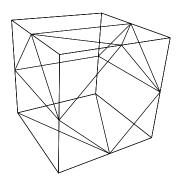
List of Elements:

Pattern 175

Node index:

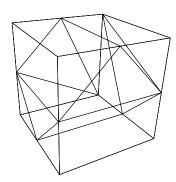
8 10 12 13 14 17 19

List of Elements:



Aspect Ratio: 0.583721

Jacobian Ratio: 1



Aspect Ratio: 0.53192

Node index: 8 10 12 16 17 18

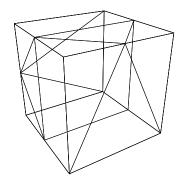
List of Elements:

Pyramid 0 8 10 3 12 Pyramid 7 3 10 18 12 Pyramid 4 7 18 16 12 Pyramid 8 16 18 10 12 Pyramid 8 10 18 16 17 Pyramid 1 2 10 8 17 Pyramid 1 8 16 5 17 Pyramid 10 2 6 18 17

Pattern 177

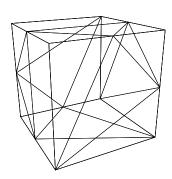
Node index: 8 11 12 14 16 17 18

List of Elements:



Aspect Ratio: 0.548045

Jacobian Ratio: 1

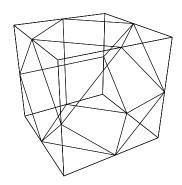


Aspect Ratio: 0.513302

Node index:

 $8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 24$

List of Elements:



Aspect Ratio: 0.827007

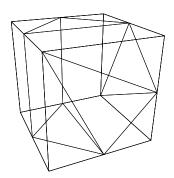
Jacobian Ratio: 1

Pattern 179

Node index:

8 9 11 14 16 18 19

List of Elements:



Aspect Ratio: 0.494248

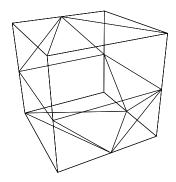
Node index: 9 10 12 13 14 15 19

List of Elements:

Pattern 181

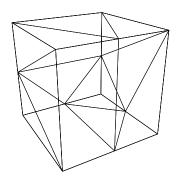
Node index: 9 10 11 12 15 17

List of Elements:



Aspect Ratio: 0.547649

Jacobian Ratio: 1



Aspect Ratio: 0.543705

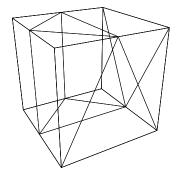
Node index: 8 10 11 16 17 19

List of Elements:

Pattern 183

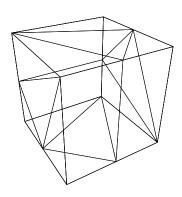
Node index: 8 9 12 15 17 18

List of Elements:



Aspect Ratio: 0.516227

Jacobian Ratio: 1



Aspect Ratio: 0.495869

Node index:

 $8\ 9\ 11\ 13\ 14\ 15\ 16\ 18\ 19$

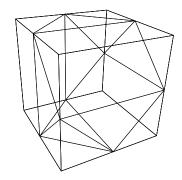
List of Elements:

Pattern 185

Node index:

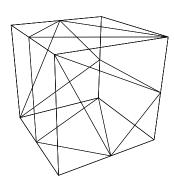
8 9 14 15 16 19

List of Elements:



Aspect Ratio: 0.609483

Jacobian Ratio: 1

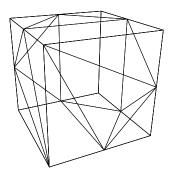


Aspect Ratio: 0.495097

Node index:

9 10 11 12 14 16 18 19

List of Elements:



Aspect Ratio: 0.558291

Jacobian Ratio: 1

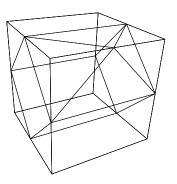
Pattern 187

Node index:

 $8\ 10\ 12\ 13\ 14\ 15\ 16\ 17\ 18$

List of Elements:

Prism 18 7 15 16 4 12 Prism 15 3 10 12 0 8 Prism 10 2 14 8 1 13 Hexaedra 15 10 14 18 12 8 13 16 Pyramid 16 13 14 18 17 Tetrahedra 16 17 5 13 Tetrahedra 17 18 6 14



Aspect Ratio: 0.58558

Node index:

10 11 13 14 16 18 19

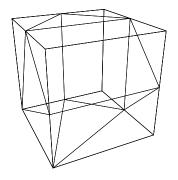
List of Elements:

Pattern 189

Node index:

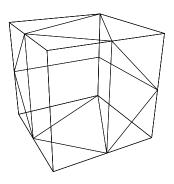
8 9 12 14 15 16 17

List of Elements:



Aspect Ratio: 0.53192

Jacobian Ratio: 1



Aspect Ratio: 0.557666

Node index:

8 10 11 12 13 15 16 17 19

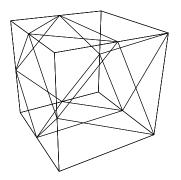
List of Elements:

Pattern 191

Node index:

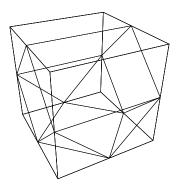
8 10 12 13 14 15 16 17 18 19

List of Elements:



Aspect Ratio: 0.59977

Jacobian Ratio: 1



Aspect Ratio: 0.673637

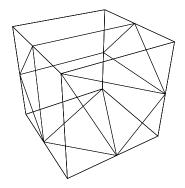
Node index: 8 9 12 14 15 16 18

List of Elements:

Pattern 193

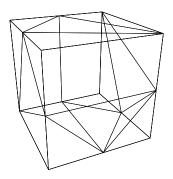
Node index: 9 10 11 13 14 16 19

List of Elements:



Aspect Ratio: 0.494248

Jacobian Ratio: 1



Aspect Ratio: 0.507545

Node index:

8 10 12 13 14 15 16 19

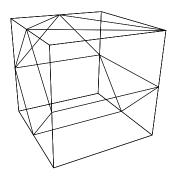
List of Elements:

Pattern 195

Node index:

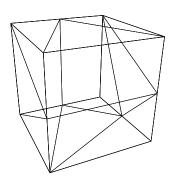
10 11 13 14 19

List of Elements:



Aspect Ratio: 0.56125

Jacobian Ratio: 1



Aspect Ratio: 0.517394

Node index: 9 11 13 14 15 19

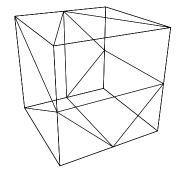
List of Elements:

Pattern 197

Node index:

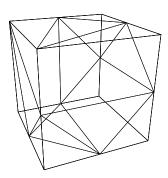
 $8\ 9\ 11\ 13\ 14\ 15\ 18\ 19$

List of Elements:



Aspect Ratio: 0.516227

Jacobian Ratio: 1



Aspect Ratio: 0.558291

Node index:

 $9\ 10\ 12\ 13\ 14\ 15\ 17\ 18$

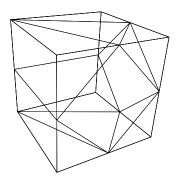
List of Elements:

Pattern 199

Node index:

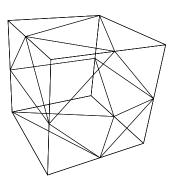
9 10 12 13 14 15 16 17

List of Elements:



Aspect Ratio: 0.587786

Jacobian Ratio: 1



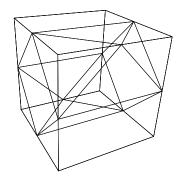
Aspect Ratio: 0.555983

Node index:

8 10 11 12 13 14 15 16 17 18

List of Elements:

Tetrahedra 11 0 8 12 Tetrahedra 10 3 11 15 Pyramid 8 12 15 10 11 Prism 18 7 15 16 4 12 Hexaedra 15 10 14 18 12 8 13 16 Tetrahedra 16 17 5 13 Tetrahedra 17 18 6 14 Pyramid 18 16 13 14 17 Prism 10 2 14 8 1 13



Aspect Ratio: 0.610113

Jacobian Ratio: 1

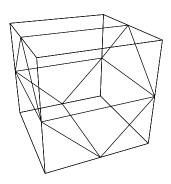
Pattern 201

Node index:

9 11 12 13 14 15 16 18

List of Elements:

Prism 12 0 11 13 1 9 Prism 11 3 15 9 2 14 Prism 14 9 13 15 11 12 Prism 18 7 15 16 4 12 Prism 14 18 15 13 16 12 Prism 16 5 13 18 6 14

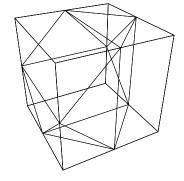


Aspect Ratio: 0.569036

Node index:

8 9 10 12 13 15 17 18 19

List of Elements:



Aspect Ratio: 0.609483

Jacobian Ratio: 1

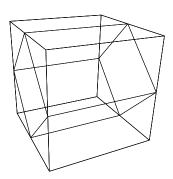
Pattern 203

Node index:

 $8\ 10\ 12\ 13\ 14\ 15\ 16\ 18$

List of Elements:

Prism 18 7 15 16 4 12 Prism 14 6 18 13 5 16 Prism 10 2 14 8 1 13 Prism 15 3 10 12 0 8 Hexaedra 10 14 18 15 8 13 16 12



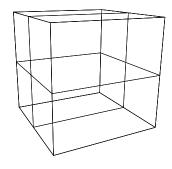
Aspect Ratio: 0.541421

Node index:

8 10 12 13 14 15 16 18 21 23

List of Elements:

 $\begin{array}{c} {\rm Hexaedra}\ 23\ 10\ 2\ 14\ 21\ 8\ 1\ 13 \\ {\rm Hexaedra}\ 15\ 3\ 10\ 23\ 12\ 0\ 8\ 21 \\ {\rm Hexaedra}\ 18\ 23\ 14\ 6\ 16\ 21\ 13\ 5 \\ {\rm Hexaedra}\ 7\ 15\ 23\ 18\ 4\ 12\ 21\ 16 \end{array}$



Aspect Ratio: 0.5

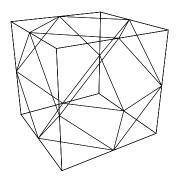
Jacobian Ratio: 1

Pattern 205

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19$

List of Elements:



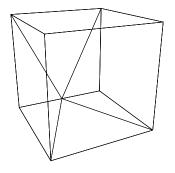
Aspect Ratio: 0.762955

Node index:

11

List of Elements:

Pyramid 4 5 1 0 11 Pyramid 5 6 2 1 11 Pyramid 6 7 3 2 11 Pyramid 4 7 6 5 11



Aspect Ratio: 0.880226

Jacobian Ratio: 1

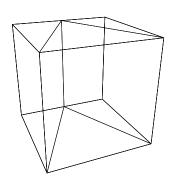
Pattern 207

Node index:

11 19

List of Elements:

Prism 0 1 11 4 5 19 Prism 1 2 11 5 6 19 Prism 2 3 11 6 7 19



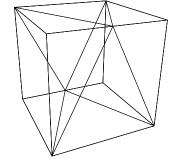
Aspect Ratio: 0.804738

Node index:

 $11 \ 17$

List of Elements:

Tetrahedra 17 2 1 11 Tetrahedra 4 11 7 17 Tetrahedra 5 17 1 11 Tetrahedra 4 17 5 11 Tetrahedra 17 6 2 11 Tetrahedra 7 6 17 11 Pyramid 4 5 1 0 11 Pyramid 6 7 3 2 11



Aspect Ratio: 0.675428

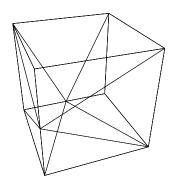
Jacobian Ratio: 1

Pattern 209

Node index:

11 13

List of Elements:



Aspect Ratio: 0.615762

Node index:

11 14

List of Elements:

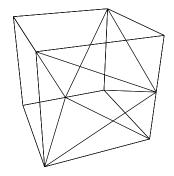
Tetrahedra 11 1 14 5 Tetrahedra 11 1 2 14 Tetrahedra 11 5 14 6 Tetrahedra 11 2 3 14 Tetrahedra 11 14 3 7 Tetrahedra 11 6 14 7 Pyramid 4 5 1 0 11 Pyramid 5 4 7 6 11



Node index:

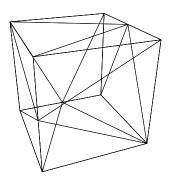
11 13 18

List of Elements:



Aspect Ratio: 0.615762

Jacobian Ratio: 1



Aspect Ratio: 0.527607

Node index: 11 13 19

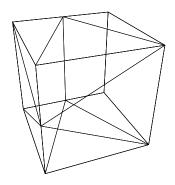
List of Elements:

Tetrahedra 13 1 0 11 Tetrahedra 19 5 4 13 Tetrahedra 13 2 1 11 Tetrahedra 5 6 13 19 Pyramid 11 19 4 0 13 Pyramid 6 19 11 2 13 Prism 2 3 11 6 7 19



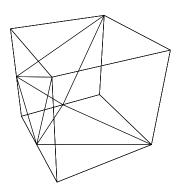
Node index: 8 11 12

List of Elements:



Aspect Ratio: 0.579529

Jacobian Ratio: 1



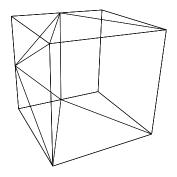
Aspect Ratio: 0.725086

Node index:

11 12 19

List of Elements:

Tetrahedra 0 12 1 11 Tetrahedra 5 12 4 19 Pyramid 5 19 11 1 12 Prism 2 3 11 6 7 19 Prism 1 2 11 5 6 19



Aspect Ratio: 0.658984

Jacobian Ratio: 1

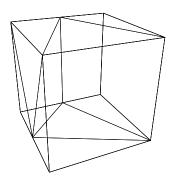
Pattern 215

Node index:

8 11 19

List of Elements:

Tetrahedra 19 5 4 8 Tetrahedra 6 19 8 5 Pyramid 0 11 19 4 8 Pyramid 5 6 2 1 8 Pyramid 2 6 19 11 8 Prism 2 3 11 6 7 19



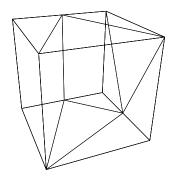
Aspect Ratio: 0.724006

Node index:

10 11 19

List of Elements:

Tetrahedra 19 7 6 10 Tetrahedra 5 6 10 19 Pyramid 7 19 11 3 10 Pyramid 5 6 2 1 10 Pyramid 11 19 5 1 10 Prism 0 1 11 4 5 19



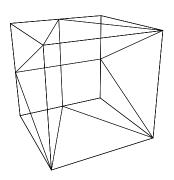
Aspect Ratio: 0.724006

Jacobian Ratio: 1

Pattern 217

Node index: 11 12 15 19 24

List of Elements:

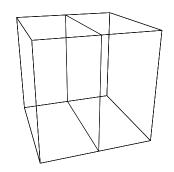


Aspect Ratio: 0.583794

Node index:

9 11 17 19

List of Elements:



Aspect Ratio: 0.5

Jacobian Ratio: 1

Pattern 219

Node index:

11 13 14 19

List of Elements:

Tetrahedra 0 1 11 13

Tetrahedra 4 13 19 5

Tetrahedra 19 $14\ 7\ 6$

Tetrahedra 11 2 3 14

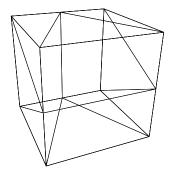
Tetrahedra 11 13 14 19

Pyramid 1 13 14 2 11

Pyramid 5 6 14 13 19

Pyramid 3 7 19 11 14

Pyramid 11 19 4 0 13



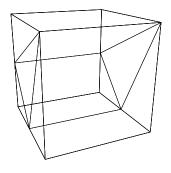
Aspect Ratio: 0.496472

Node index:

8 10 12 15

List of Elements:

Prism 5 8 12 6 10 15 Prism 2 6 10 1 5 8 Prism 6 7 15 5 4 12 Prism 8 0 12 10 3 15



Aspect Ratio: 0.646258

Jacobian Ratio: 1

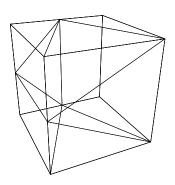
Pattern 221

Node index:

11 12 13 19

List of Elements:

Tetrahedra 11 1 2 13 Tetrahedra 11 12 13 19 Tetrahedra 19 13 6 5 Pyramid 12 13 1 0 11 Pyramid 4 5 13 12 19 Pyramid 11 2 6 19 13 Prism 3 11 2 7 19 6



Aspect Ratio: 0.607299

Node index: 9 11 12 15 19 24

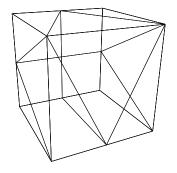
List of Elements:

Tetrahedra 24 5 6 19 Tetrahedra 24 5 9 6 Pyramid 19 24 15 7 6 Pyramid 4 12 24 19 5 Pyramid 12 1 9 24 5 Pyramid 24 9 2 15 6 Prism 0 12 1 11 24 9 Prism 11 24 9 3 15 2

Pattern 223

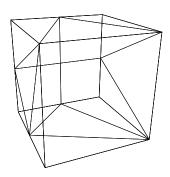
Node index: 8 11 12 15 19 24

List of Elements:



Aspect Ratio: 0.545265

Jacobian Ratio: 1



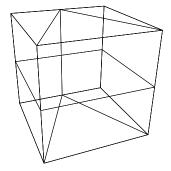
Aspect Ratio: 0.571559

Node index:

11 12 13 14 15 19 24

List of Elements:

Prism 0 1 11 12 13 24 Prism 2 3 11 14 15 24 Prism 14 15 24 6 7 19 Prism 12 13 24 4 5 19 Prism 1 2 11 13 14 24 Prism 13 14 24 5 6 19



Aspect Ratio: 0.5

Jacobian Ratio: 1

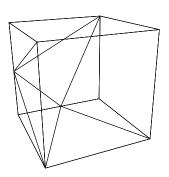
Pattern 225

Node index:

 $11 \ 12$

List of Elements:

Tetrahedra 1 11 0 12 Tetrahedra 1 12 5 11 Tetrahedra 7 12 11 6 Tetrahedra 11 12 5 6 Pyramid 7 6 5 4 12 Pyramid 6 7 3 2 11 Pyramid 5 6 2 1 11



Aspect Ratio: 0.665591

Node index:

 $8\ 9\ 10\ 11\ 12\ 15\ 19\ 20\ 24$

List of Elements:

Tetrahedra 5 24 6 20

Pyramid 12 24 19 4 5

Pyramid 24 15 7 19 6

Pyramid 9 20 8 1 5

Pyramid 9 2 10 20 6 Pyramid 12 8 20 24 5

Pyramid 24 20 10 15 6

Prism 11 20 24 0 8 12

Prism 15 3 10 24 11 20

Pattern 227

Node index:

 $8\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 18\ 19\ 21\ 23\ 24$

List of Elements:

Prism 15 23 24 3 10 11

 $Prism\ 7\ 18\ 19\ 15\ 23\ 24$

 $Prism\ 0\ 8\ 11\ 12\ 21\ 24$

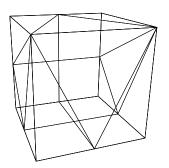
Prism 12 21 24 4 16 19

Prism 11 8 10 24 21 23

Prism 24 21 23 19 16 18

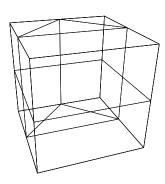
 ${\bf Hexaedra~21~13~14~23~16~5~6~18}$

 ${\bf Hexaedra~8~1~2~10~21~13~14~23}$



Aspect Ratio: 0.594068

Jacobian Ratio: 1

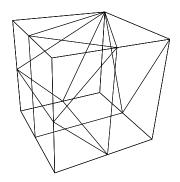


Aspect Ratio: 0.730327

Node index:

8 9 10 11 12 13 16 17 20 21

List of Elements:



Aspect Ratio: 0.627381

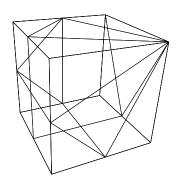
Jacobian Ratio: 1

Pattern 229

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 16\ 19\ 20\ 21$

List of Elements:



Aspect Ratio: 0.627381

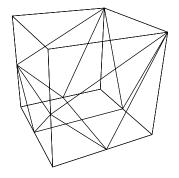
Node index: 8 9 10 11 12 13 20

List of Elements:

Pattern 231

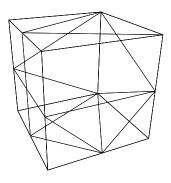
Node index: 8 9 12 13 14 16 21

List of Elements:



Aspect Ratio: 0.62464

Jacobian Ratio: 1



Aspect Ratio: 0.616803

Node index: 8 11 12 13 15 16 21

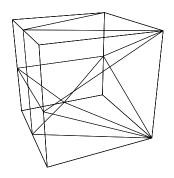
List of Elements:

Pattern 233

Node index: 8 9 10 13

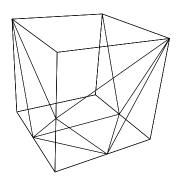
List of Elements:

Tetrahedra 10 13 9 6 Tetrahedra 10 8 9 13 Tetrahedra 9 8 1 13 Tetrahedra 10 9 2 6 Prism 7 3 10 4 0 8 Pyramid 8 10 7 4 13 Tetrahedra 7 10 6 13 Pyramid 7 6 5 4 13



Aspect Ratio: 0.616803

Jacobian Ratio: 1



Aspect Ratio: 0.696609

Node index: 8 10 11 12

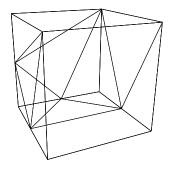
List of Elements:

Tetrahedra 11 12 10 7 Tetrahedra 11 8 10 12 Tetrahedra 0 8 11 12 Tetrahedra 3 11 10 7 Prism 5 1 8 6 2 10 Pyramid 5 6 10 8 12 Tetrahedra 7 10 6 12 Pyramid 5 4 7 6 12

Pattern 235

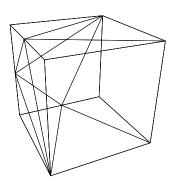
Node index: 11 12 16

List of Elements:



Aspect Ratio: 0.696609

Jacobian Ratio: 1

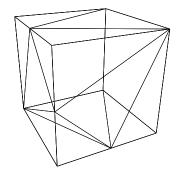


Aspect Ratio: 0.595003

Node index:

 $9\ 13\ 16$

List of Elements:



Aspect Ratio: 0.595003

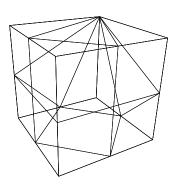
Jacobian Ratio: 1

Pattern 237

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 16\ 17\ 21\ 22$

List of Elements:

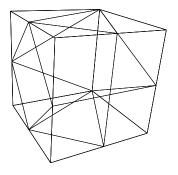


Aspect Ratio: 0.677251

Node index:

 $8\ 9\ 12\ 13\ 14\ 16\ 17\ 22$

List of Elements:



Aspect Ratio: 0.691731

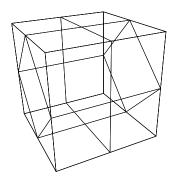
Jacobian Ratio: 1

Pattern 239

Node index:

List of Elements:

Prism 19 24 25 4 12 16 Prism 7 15 18 19 24 25 Prism 22 17 25 13 5 16 Prism 14 6 18 22 17 25 Prism 8 0 12 20 11 24 Prism 20 11 24 10 3 15 Prism 1 8 13 9 20 22 Prism 9 20 22 2 10 14 Hexaedra 8 12 16 13 20 24 25 22 Hexaedra 20 24 25 22 10 15 18 14



Aspect Ratio: 0.856963

Node index:

8 11 12 15 18 19 24

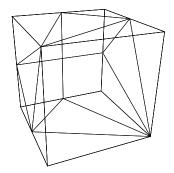
List of Elements:

Pattern 241

Node index:

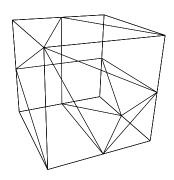
 $9\ 10\ 11\ 12\ 14\ 15\ 19\ 24$

List of Elements:



Aspect Ratio: 0.56309

Jacobian Ratio: 1



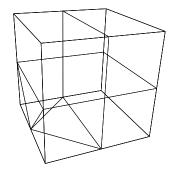
Aspect Ratio: 0.611825

Node index:

 $8\ 9\ 11\ 12\ 13\ 14\ 15\ 17\ 19\ 22\ 24$

List of Elements:

Pyramid 11 24 12 0 8 Pyramid 22 24 11 9 8 Pyramid 13 22 9 1 8 Pyramid 24 22 13 12 8 Hexaedra 12 13 22 24 4 5 17 19 Hexaedra 2 3 11 9 14 15 24 22 Hexaedra 14 15 24 22 6 7 19 17



Aspect Ratio: 0.590428

Jacobian Ratio: 1

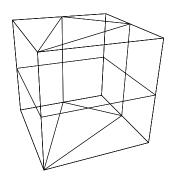
Pattern 243

Node index:

10 11 12 13 14 15 18 19 23 24

List of Elements:

Prism 12 13 24 4 5 19 Prism 0 1 11 12 13 24 Prism 7 18 19 15 23 24 Prism 15 23 24 3 10 11 Prism 13 23 24 5 18 19 Prism 1 10 11 13 23 24 Prism 13 14 23 5 6 18 Prism 1 2 10 13 14 23



Aspect Ratio: 0.58541

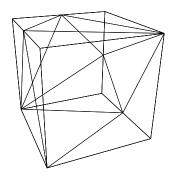
Node index: 10 15 16 19

List of Elements:

Pattern 245

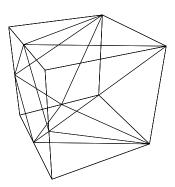
Node index: 8 11 12 13 16

List of Elements:



Aspect Ratio: 0.615839

Jacobian Ratio: 1



Aspect Ratio: 0.569491

Node index:

 $8\ 9\ 10\ 11\ 14\ 15\ 17\ 18\ 19\ 20\ 23$

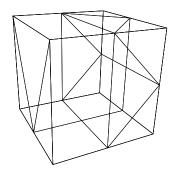
List of Elements:



Node index:

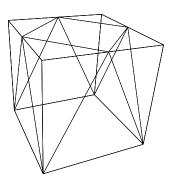
 $16\ 17\ 18\ 19$

List of Elements:



Aspect Ratio: 0.695267

Jacobian Ratio: 1



Aspect Ratio: 0.551876

Node index: 8 9 10 13 14 17 18 22

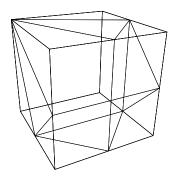
List of Elements:

Pattern 249

Node index: 9 10 14 17 18

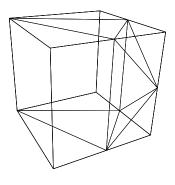
List of Elements:

Prism 4 7 18 0 3 10 Prism 4 18 17 0 10 9 Prism 0 1 9 4 5 17 Pyramid 9 10 18 17 14 Tetrahedra 6 14 17 18 Tetrahedra 2 9 14 10



Aspect Ratio: 0.632542

Jacobian Ratio: 1



Aspect Ratio: 0.662831

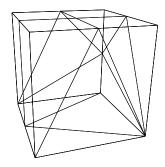
Node index: 8 15 16 17 18 19 25

List of Elements:

Pattern 251

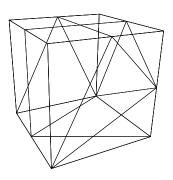
Node index: 8 14 16 17 18 19 25

List of Elements:



Aspect Ratio: 0.570002

Jacobian Ratio: 1



Aspect Ratio: 0.570002

Node index:

 $10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18\ 19\ 23\ 24$

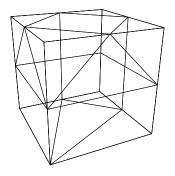
List of Elements:



Node index:

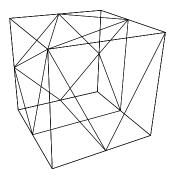
8 9 10 11 12 15 18 19 20

List of Elements:



Aspect Ratio: 0.708832

Jacobian Ratio: 1



Aspect Ratio: 0.687518

Node index: 10 13 14 17

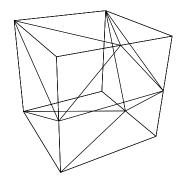
List of Elements:

Pyramid 0 3 7 4 10 Tetrahedra 0 1 10 13 Tetrahedra 0 4 13 10 Pyramid 4 7 14 13 10 Pyramid 13 14 2 1 10 Pyramid 13 14 7 4 17 Tetrahedra 17 13 5 4 Tetrahedra 6 14 17 7

Pattern 255

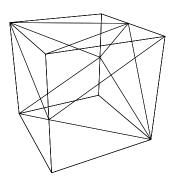
Node index: 13 15 18

List of Elements:



Aspect Ratio: 0.653093

Jacobian Ratio: 1



Aspect Ratio: 0.563806

Node index: 14 16 19

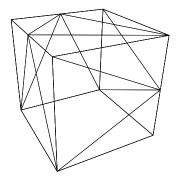
List of Elements:

Pattern 257

Node index: 10 12 15 19

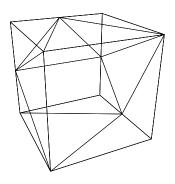
List of Elements:

Pyramid 5 6 2 1 10 Tetrahedra 10 0 1 12 Tetrahedra 12 5 1 10 Pyramid 12 15 6 5 10 Pyramid 0 3 15 12 10 Pyramid 5 6 15 12 19 Tetrahedra 4 12 19 5 Tetrahedra 19 15 7 6



Aspect Ratio: 0.640968

Jacobian Ratio: 1



Aspect Ratio: 0.653093

Node index:

12 14 18

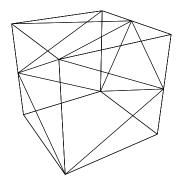
List of Elements:

Pattern 259

Node index:

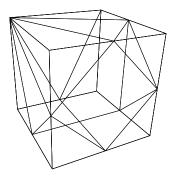
 $8\ 9\ 10\ 11\ 13\ 14\ 17\ 18\ 20$

List of Elements:



Aspect Ratio: 0.563806

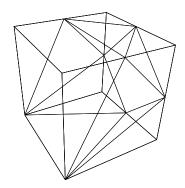
Jacobian Ratio: 1



Aspect Ratio: 0.687518

Node index: 10 14 15 17 18 19

List of Elements:



Aspect Ratio: 0.597636

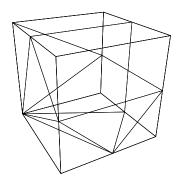
4 Patterns with cube's central node

Pattern 261

Node index:

 $9\ 10\ 13\ 14\ 15\ 16\ 18\ 23\ 26$

List of Elements:

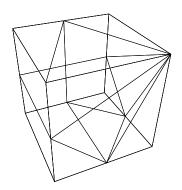


Aspect Ratio: 0.652412

Node index:

 $9\ 10\ 11\ 12\ 13\ 15\ 19\ 24\ 26$

List of Elements:

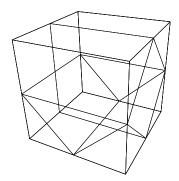


Aspect Ratio: 0.652412

Node index:

8 9 10 11 12 13 14 15 17 19 20 24 26

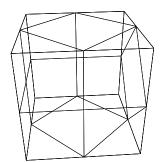
List of Elements:



Aspect Ratio: 0.647321

Node index:

List of Elements:

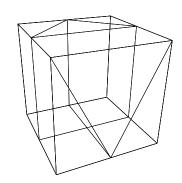


Aspect Ratio: 0.694288

Node index:

 $11\ 12\ 13\ 14\ 15\ 17\ 19\ 24\ 26$

List of Elements:

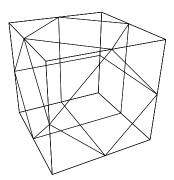


Aspect Ratio: 0.622961

Node index:

8 9 10 11 12 13 14 15 16 17 18 19 20 23 24 26

List of Elements:

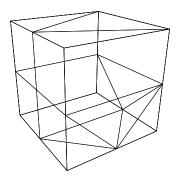


Aspect Ratio: 0.723503

Node index:

 $8\ 9\ 10\ 12\ 13\ 14\ 15\ 16\ 21\ 26$

List of Elements:

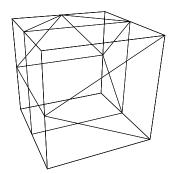


Aspect Ratio: 0.675695

Node index:

 $8\ 10\ 12\ 13\ 15\ 16\ 18\ 19\ 21\ 26$

List of Elements:

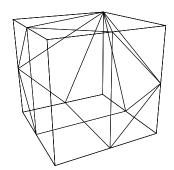


Aspect Ratio: 0.675695

Node index:

 $8\ 9\ 10\ 11\ 12\ 14\ 16\ 17\ 20\ 26$

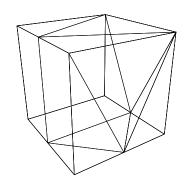
List of Elements:



Aspect Ratio: 0.652009

Node index: 8 9 10 16 26

List of Elements:

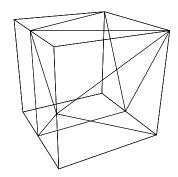


Aspect Ratio: 0.609553

Node index:

8 10 13 16 26

List of Elements:

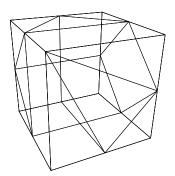


Aspect Ratio: 0.600631

Node index:

 $8\ 9\ 10\ 11\ 12\ 14\ 15\ 16\ 18\ 19\ 20\ 24\ 26$

List of Elements:

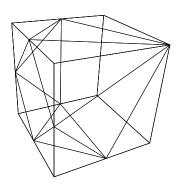


Aspect Ratio: 0.669459

Node index:

 $8\ 10\ 11\ 12\ 15\ 18\ 19\ 26$

List of Elements:

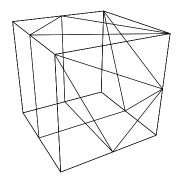


Aspect Ratio: 0.721494

Node index:

 $8\ 9\ 10\ 11\ 14\ 16\ 19\ 20\ 26$

List of Elements:

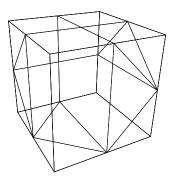


Aspect Ratio: 0.624707

Node index:

 $8 \ 9 \ 11 \ 12 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 24 \ 25 \ 26$

List of Elements:

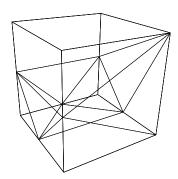


Aspect Ratio: 0.67577

Node index:

8 10 11 12 13 15 26

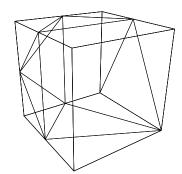
List of Elements:



Aspect Ratio: 0.678774

Node index: 8 11 12 16 18 19 26

List of Elements:



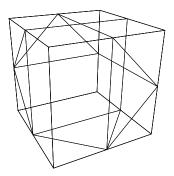
Aspect Ratio: 0.583794

Node index:

 $8\ 9\ 10\ 12\ 14\ 15\ 16\ 17\ 18\ 19\ 23\ 25\ 26$

List of Elements:

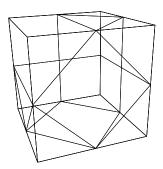
Pyramid 1 5 17 9 26



Aspect Ratio: 0.67577

Node index:

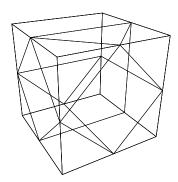
List of Elements:



Aspect Ratio: 0.697085

Node index:

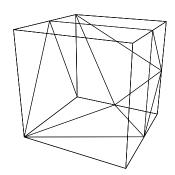
List of Elements:



Aspect Ratio: 0.679936

Node index: 8 11 12 17 19 26

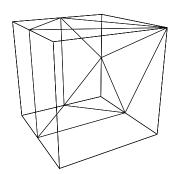
List of Elements:



Aspect Ratio: 0.629632

Node index: 8 10 11 15 16 19 26

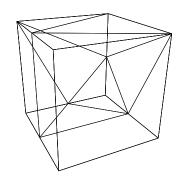
List of Elements:



Aspect Ratio: 0.639766

Node index: 8 10 11 15 16 26

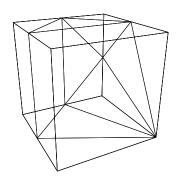
List of Elements:



Aspect Ratio: 0.629632

Node index: 8 11 15 16 18 19 26

List of Elements:

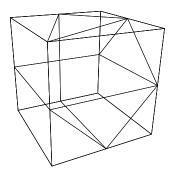


Aspect Ratio: 0.639766

Node index:

 $9\ 11\ 12\ 13\ 14\ 15\ 18\ 19\ 24\ 26$

List of Elements:

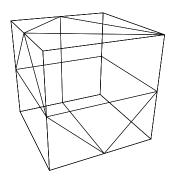


Aspect Ratio: 0.626916

Node index:

 $9\ 11\ 12\ 13\ 14\ 15\ 16\ 19\ 24\ 26$

List of Elements:

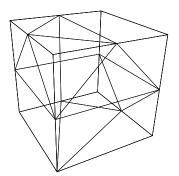


Aspect Ratio: 0.626916

Node index:

 $10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 19\ 24\ 26$

List of Elements:

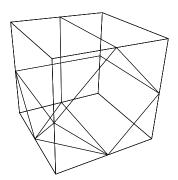


Aspect Ratio: 0.672256

Node index:

 $8\ 9\ 11\ 12\ 13\ 14\ 15\ 17\ 19\ 24\ 26$

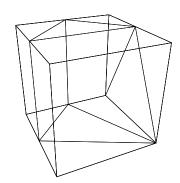
List of Elements:



Aspect Ratio: 0.658686

Node index: 8 11 16 18 19 26

List of Elements:

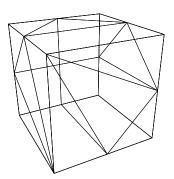


Aspect Ratio: 0.567768

Node index:

 $9\ 11\ 12\ 14\ 15\ 16\ 18\ 19\ 24\ 26$

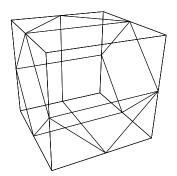
List of Elements:



Aspect Ratio: 0.633986

Node index:

List of Elements:

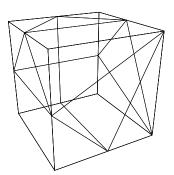


Aspect Ratio: 0.667607

Node index:

 $9\ 11\ 12\ 13\ 15\ 16\ 18\ 19\ 24\ 26$

List of Elements:

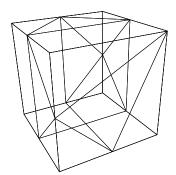


Aspect Ratio: 0.633986

Node index:

 $8\ 9\ 10\ 11\ 13\ 15\ 16\ 18\ 19\ 20\ 26$

List of Elements:



Aspect Ratio: 0.672256

Node index: 9 10 13 16 18 26

List of Elements:

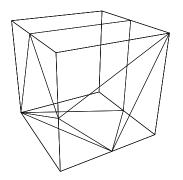
Pattern 295

Node index:

8 10 13 18 26

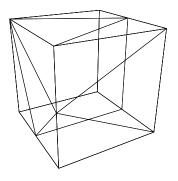
List of Elements:

Pattern 296



Aspect Ratio: 0.585455

Jacobian Ratio: 1



Aspect Ratio: 0.579676

Node index:

8 9 10 11 16 17 18 19 20 26

List of Elements:

Tetrahedra 7 18 19 26 Tetrahedra 5 16 17 26 Tetrahedra 6 17 18 26 Pyramid 16 19 18 17 26

Pyramid 1 5 17 9 26 Pyramid 9 17 6 2 26

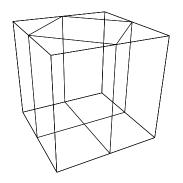
Pattern 297

Node index:

10 11 16 18 26

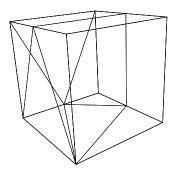
List of Elements:

Pattern 298



Aspect Ratio: 0.60295

Jacobian Ratio: 1

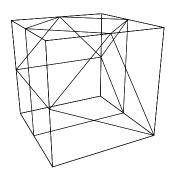


Aspect Ratio: 0.579676

Node index:

 $8\ 10\ 12\ 13\ 15\ 16\ 17\ 18\ 19\ 21\ 26$

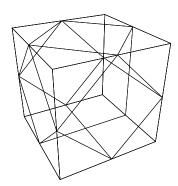
List of Elements:



Aspect Ratio: 0.662868

Node index:

List of Elements:

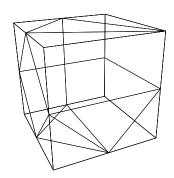


Aspect Ratio: 0.716624

Node index:

 $8\ 9\ 11\ 12\ 13\ 14\ 15\ 16\ 19\ 24\ 26$

List of Elements:

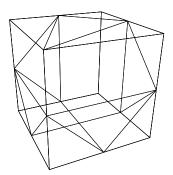


Aspect Ratio: 0.662868

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 14\ 18\ 19\ 20\ 26$

List of Elements:

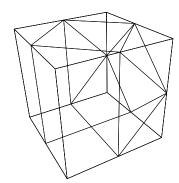


Aspect Ratio: 0.654466

Node index:

 $8\ 9\ 10\ 11\ 14\ 15\ 16\ 17\ 18\ 19\ 20\ 26$

List of Elements:

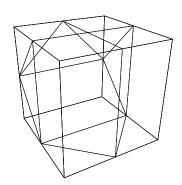


Aspect Ratio: 0.691751

Node index:

 $8\ 9\ 10\ 12\ 15\ 16\ 17\ 18\ 19\ 25\ 26$

List of Elements:

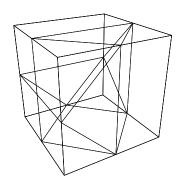


Aspect Ratio: 0.634681

Node index:

 $8\ 9\ 10\ 11\ 12\ 13\ 15\ 16\ 17\ 18\ 21\ 26$

List of Elements:

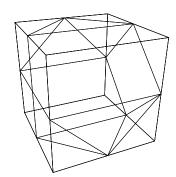


Aspect Ratio: 0.670531

Node index:

 $8\ 9\ 10\ 12\ 13\ 14\ 15\ 16\ 18\ 19\ 21\ 26$

List of Elements:

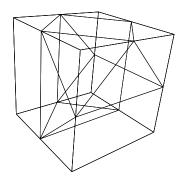


Aspect Ratio: 0.699882

Node index:

 $8\ 9\ 10\ 12\ 13\ 15\ 16\ 17\ 18\ 19\ 21\ 26$

List of Elements:

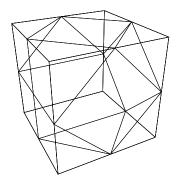


Aspect Ratio: 0.670531

Node index:

 $8 \ 9 \ 10 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 21 \ 26$

List of Elements:

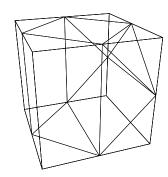


Aspect Ratio: 0.719305

Node index:

 $8\ 9\ 11\ 14\ 15\ 16\ 17\ 18\ 19\ 26$

List of Elements:



Aspect Ratio: 0.665092

Node index:

 $8\ 9\ 10\ 12\ 14\ 15\ 16\ 17\ 18\ 19\ 26$

List of Elements:

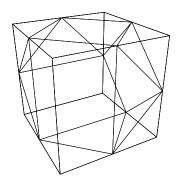
Pattern 310

Node index:

 $8\ 10\ 12\ 13\ 14\ 16\ 18\ 26$

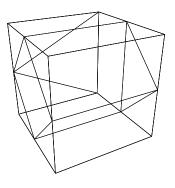
List of Elements:

Pattern 311



Aspect Ratio: 0.742599

Jacobian Ratio: 1

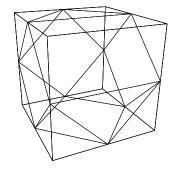


Aspect Ratio: 0.552754

Node index:

8 9 10 11 12 13 14 15 16 18 19 26

List of Elements:



Aspect Ratio: 0.79392

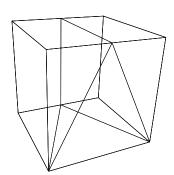
Jacobian Ratio: 1

Pattern 312

Node index:

 $11\ 17\ 19\ 26$

List of Elements:



Aspect Ratio: 0.565479

Jacobian Ratio: 1

Pattern 313

Node index:

8 10 12 15 18 26

List of Elements:

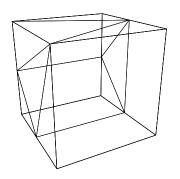
Pattern 314

Node index:

9 11 12 14 15 19 24 26

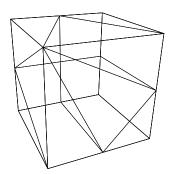
List of Elements:

Pattern 315



Aspect Ratio: 0.567768

Jacobian Ratio: 1



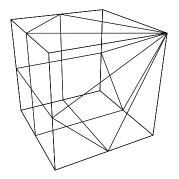
Aspect Ratio: 0.636729

Node index:

8 9 10 11 12 13 15 16 19 20 21 24 26

List of Elements:

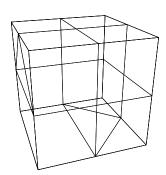
Pyramid 19 24 15 7 26 Pyramid 20 9 2 10 26



Aspect Ratio: 0.709421

Node index:

List of Elements:



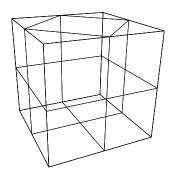
Aspect Ratio: 0.746954

Node index:

8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 26

List of Elements:

Tetrahedra 7 18 19 26

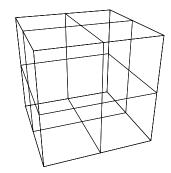


Aspect Ratio: 0.772564

Node index:

List of Elements:

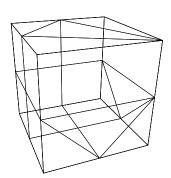
Hexaedra 0 8 20 11 12 21 26 24 Hexaedra 8 1 9 20 21 13 22 26 Hexaedra 12 21 26 24 4 16 25 19 Hexaedra 21 13 22 26 16 5 17 25 Hexaedra 11 20 10 3 24 26 23 15 Hexaedra 24 26 23 15 19 25 18 7 Hexaedra 20 9 2 10 26 22 14 23 Hexaedra 26 22 14 23 25 17 6 18



Aspect Ratio:

Node index:

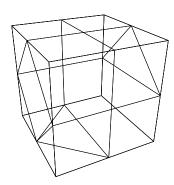
List of Elements:



Aspect Ratio: 0.711262

Node index:

List of Elements:

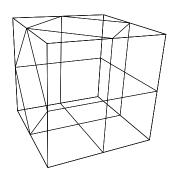


Aspect Ratio: 0.707773

Node index:

8 9 10 11 12 13 14 15 16 17 18 19 20 22 23 24 26

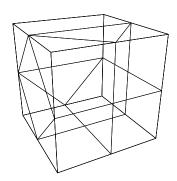
List of Elements:



Aspect Ratio: 0.732011

Node index:

List of Elements:

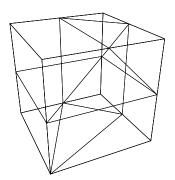


Aspect Ratio: 0.705803

Node index:

 $10\ 11\ 12\ 13\ 14\ 15\ 17\ 18\ 19\ 23\ 24\ 26$

List of Elements:

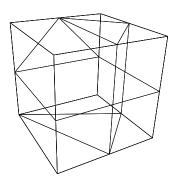


Aspect Ratio: 0.670669

Node index:

 $9\ 10\ 12\ 13\ 14\ 15\ 17\ 18\ 19\ 22\ 23\ 26$

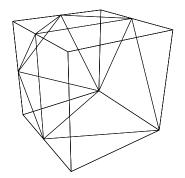
List of Elements:



Aspect Ratio: 0.670669

Node index: 8 12 16 18 19 26

List of Elements:



Aspect Ratio: 0.681061

Jacobian Ratio: 1

5 Conclusion

In this technical report, we have introduced 325 patterns that allow to manage transitions between coarse and fine regions in a mesh generated with the Octree. In 260 cases, the task of splitting an Octant into mixed–elements following the convention adopted over square faces, was accomplished without adding any new node. In the other 65 cases, it was necessary to add the central node of the original cube to achieve the task.

With respect to quality, these patterns present acceptable values in a stationary state. This is shown in Table 1. However, if the nodes start to be displaced, it is likely these quality values will get worse rapidly. More studies over the quality of these patterns should be performed.

	tetrahedron	pyramid	prism	hexahedron	overall
worst	0.240653	0.353553	0.353553	0.5	0.240653
best	0.999221	1	1	1	1
average	0.612236	0.632454	0.610255	0.643872	0.619641

Table 1: AR quality values for each type of element in all transition patterns.

Acknowledgement

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- [3] Y. Ito, A. Shih, and B. Soni. Octree-based reasonable-quality hexahedral mesh generation using a new set of refinement templates. *International Journal for Numerical Methods in Engineering*, 77(13):1809–1833, March 2009.
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