PARAMETRIZACION DENAVIT-HARTENBERG

CINEMATICA DE ROBOTS

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>> robot1

Código:

```
syms theta1
syms theta2
syms theta3
T1=[cos(theta1),-sin(theta1),0,0;0,0,1,0;-sin(theta1),-
cos(theta1),0,0;0,0,0,1]
syms L1
syms L2
T2=[cos(theta2),-
sin(theta2),0,L1;sin(theta2),cos(theta2),0,0;0,cos(theta2),1,0;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,cos(theta3),1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$\boxed{T_1^0} \quad \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_2^1} \quad \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & \cos(\theta_2) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^2} \quad \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & \cos(\theta_3) & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^0} \quad \begin{pmatrix} \cos(\theta_3)\,\sigma_2 - \sin(\theta_3)\,\sigma_3 & \sigma_1 & 0 & L_2\,\sigma_2 + L_1\cos(\theta_1) \\ \cos(\theta_2)\,\sin(\theta_3) & \cos(\theta_3) + \cos(\theta_2)\,\cos(\theta_3) & 1 & 0 \\ \sigma_1 & \sin(\theta_3)\,\sigma_3 - \cos(\theta_3)\,\sigma_2 & 0 & -L_2\,\sigma_3 - L_1\sin(\theta_1) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = -\cos(\theta_3)\,\sigma_3 - \sin(\theta_3)\,\sigma_2$$

$$\sigma_2 = \cos(\theta_1)\,\cos(\theta_2) - \sin(\theta_1)\,\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\,\sin(\theta_2) + \cos(\theta_2)\,\sin(\theta_1)$$

>> Robot2

Código:

```
syms theta1
syms theta2
syms theta3
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-sin(theta2),0,L1;0,0,1,0;-sin(theta2),-
cos(theta2),0,0;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$\boxed{T_1^0} \quad \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_2^1} \quad \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^2} \quad \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^0} \quad \begin{pmatrix} \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) & -\sin(\theta_1) & L_1\cos(\theta_1) + L_2\cos(\theta_1)\cos(\theta_2) \\ \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1) & L_1\sin(\theta_1) + L_2\cos(\theta_2)\sin(\theta_1) \\ -\cos(\theta_2)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_2) & \sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3) & 0 & -L_2\sin(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

>> robot3

Código:

```
syms theta1
syms theta2
syms theta3
syms d2
T1=[cos(theta1),-sin(theta1),0,0;0,0,1,0;-sin(theta1),-
cos(theta1),0,0;0,0,0,1]
syms L1
```

```
T2=[cos(theta2),-sin(theta2),0,L1;0,0,-1,-d2;-sin(theta2),-
cos(theta2),0,0;0,0,0,1]
syms L2
T3=[cos(theta3),-sin(theta3),0,L2;0,0,1,0;-sin(theta3),-
cos(theta3),0,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$\boxed{T_1^0}$
$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_2^1}$
$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ 0 & 0 & -1 & -d_2 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_3^2}$
$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_3) & -\cos(\theta_3) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_3^0}$
$$\begin{pmatrix} \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3) & -\cos(\theta_3)\sin(\theta_1) - \cos(\theta_1)\cos(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\sin(\theta_2) & L_1\cos(\theta_1) + d_2\sin(\theta_1) + L_2\cos(\theta_1)\cos(\theta_2) \\ -\cos(\theta_3)\sin(\theta_2) & \sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2) & -L_2\sin(\theta_2) \\ -\cos(\theta_1)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) & \cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_3) & \sin(\theta_1)\sin(\theta_2) & d_2\cos(\theta_1) - L_1\sin(\theta_1) - L_2\cos(\theta_2)\sin(\theta_1) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

>> robot4

Código:

```
syms theta1
syms theta2
syms theta3
syms d1
syms d2
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-
sin(theta2),0,3/4L1;sin(theta2),cos(theta2),0,0;0,0,1,d1;0,0,0,1]
syms L2
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$\boxed{T_1^0}$

$$\begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_2^1}$

$$\begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_3^2}$

$$\begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{T_3^0}$

$$\begin{pmatrix} \sigma_1 & -\cos(\theta_3)\,\sigma_3 - \sin(\theta_3)\,\sigma_2 & 0 & L_2\,\sigma_2 + L_1\cos(\theta_1) \\ \cos(\theta_3)\,\sigma_3 + \sin(\theta_3)\,\sigma_2 & \sigma_1 & 0 & L_2\,\sigma_3 + L_1\sin(\theta_1) \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\theta_3)\,\sigma_2 - \sin(\theta_3)\,\sigma_3$$

$$\sigma_2 = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)$$

>> robot5

Código:

```
syms theta1
syms theta2
syms theta3
syms d1
syms d2
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,d1;0,0,0,1]
syms L1
T2=[cos(theta2),-sin(theta2),0,0;0,0,-
1,d2;cos(theta2),sin(theta2),0,0;0,0,0,1]
T3=[cos(theta3),-
sin(theta3),0,L1;sin(theta3),cos(theta3),0,0;0,0,1,0;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$\boxed{T_1^0} \quad \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_2^1} \quad \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & -1 & d_2 \\ \cos(\theta_2) & \sin(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^2} \quad \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_1 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^0} \quad \begin{pmatrix} \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - \cos(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_1)\cos(\theta_2)\sin(\theta_3) - \cos(\theta_1)\cos(\theta_3)\sin(\theta_2) & \sin(\theta_1) & L_1\cos(\theta_1)\cos(\theta_2) - d_2\sin(\theta_1) \\ \cos(\theta_2)\cos(\theta_3)\sin(\theta_1) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) & -\cos(\theta_2)\sin(\theta_1)\sin(\theta_3) - \cos(\theta_3)\sin(\theta_1)\sin(\theta_2) & -\cos(\theta_1) & d_2\cos(\theta_1) + L_1\cos(\theta_2)\sin(\theta_1) \\ \cos(\theta_2)\cos(\theta_3) + \sin(\theta_2)\sin(\theta_3) & \cos(\theta_3)\sin(\theta_2) - \cos(\theta_2)\sin(\theta_3) & 0 & d_1 + L_1\cos(\theta_2) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

>> robot6

Código:

```
syms theta1
syms theta2
syms theta3
syms d1
T1=[cos(theta1),-
sin(theta1),0,0;sin(theta1),cos(theta1),0,0;0,0,1,0;0,0,0,1]
syms L1
T2=[cos(theta2),-
sin(theta2),0,L1;sin(theta2),cos(theta2),0,0;0,0,1,0;0,0,0,1]
T3=[cos(theta3),-
sin(theta3),0,L2;sin(theta3),cos(theta3),0,0;0,0,1,d1;0,0,0,1]
syms ans
ans =T1*T2*T3
```

$$\boxed{T_1^0} \quad \begin{pmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_2^1} \quad \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & L_1 \\ \sin(\theta_2) & \cos(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^2} \quad \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & L_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\boxed{T_3^0} \quad \begin{pmatrix} \sigma_1 & -\cos(\theta_3)\,\sigma_3 - \sin(\theta_3)\,\sigma_2 & 0 & L_2\,\sigma_2 + L_1\cos(\theta_1) \\ \cos(\theta_3)\,\sigma_3 + \sin(\theta_3)\,\sigma_2 & \sigma_1 & 0 & L_2\,\sigma_3 + L_1\sin(\theta_1) \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\theta_3)\,\sigma_2 - \sin(\theta_3)\,\sigma_3$$

$$\sigma_2 = \cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)$$

$$\sigma_3 = \cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)$$