CS111 Quiz 0 (B) Solutions

Problem 1: (a) Determine the numerical values of the expressions below. To receive credit, you must show your work for each subproblem. Simplify your answer to a single number.

$$3^{\log_9 16} = 3^{\log_{3^2} 4^2} = 3^{\log_3 4} = 4$$

$$15 \log_3 \frac{1}{81} = 15 \cdot \log_3 (3)^{-4} = -4 \cdot 15 \cdot \log_3 3 = -60$$

$$\binom{20}{18} = \frac{20!}{18!2!} = \frac{20 \cdot 19}{2} = 10 \cdot 19 = 190$$

$$\sum_{i=1}^{40} 3i = 3 \cdot \left(\sum_{i=1}^{40} i \right) = 3 \cdot \frac{40 \cdot 41}{2} = 2460$$

$$\sum_{i=0}^{\infty} \left(\frac{1}{5} \right)^i = \frac{1}{1 - \frac{1}{5}} = \frac{5}{4}$$

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Problem 2:

(a)

$$\binom{150}{2} = \frac{150!}{2!148!}$$

or

$$\sum_{i=1}^{149} i = \frac{149 \cdot 150}{2}$$

(b)

$$\text{Number of lineups} = 150!$$

(c)

$$2^{150}$$

(d)

$$\binom{9}{5} = \binom{9}{4} = \frac{9!}{5!4!}$$

(e)

$$\frac{150!}{(150-3)!} = \frac{150!}{147!} = 150 \cdot 149 \cdot 148$$

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Problem 3:
(a) **False**.
The candidate roots are $\pm 1, \pm 3$.
Let $x = -1$: $(-1)^3 + 3(-1)^2 + 5(-1) + 3 = -1 + 3 - 5 + 3 = 0$.
This gives a counterexample.

(b) **True**.
Consider the equation $8x^2 - 6x + 1 = 0$:

$$\begin{aligned} 8x^2 - 6x + 1 &= 0 \\ (4x - 1)(2x - 1) &= 0 \\ x = \frac{1}{4}, x &= \frac{1}{2} \end{aligned}$$

When $x > \frac{1}{2}$ or $x < \frac{1}{4}$, $8x^2 - 6x + 1 > 0$. This contains all integers $x \in \mathbb{Z}$.

(c) **False**.
Let $x = 1$, then $x^2 - 4x = -3$ while $\forall y \in \mathbb{R} : y^2 \geq 0$.

(d) **False**.
Let $x = 0$, then $2^{x^2} - 2 = -1$ while $\forall y \in \mathbb{R} : y^2 \geq 0$.

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