Problem 1:

(a) General solution: $D_n = \alpha_1 \cdot (-1)^n + \alpha_2 \cdot 2^n + \alpha_3 \cdot 3^n$

(b) Characteristic equation is $x^2 = 16$. Its roots are 4 and -4.

General form of the solution: $U_n = \alpha_1 \cdot 4^n + \alpha_2 \cdot (-4)^n$

(c) Characteristic equation is $x^2 = 5x + 6$. Its roots are -1 and 6.

General solution form: $R_n = \alpha_1(-1)^n + \alpha_2 6^n$.
Initial condition equations:

$$\begin{cases} \alpha_1 + \alpha_2 = 1 \\ -\alpha_1 + 6\alpha_2 = 13 \end{cases}$$

So $\alpha_1 = -1$ and $\alpha_2 = 2$. This gives us the final answer: $R_n = -(-1)^n + 2 \cdot 6^n$.

Problem 2:

Initial conditions for T_n: $T_0 = 1$, $T_1 = 3$
Recurrence equation for T_n: $T_n = 2T_{n-1} + T_{n-2}$
Justification: `A`, `B` can be appended to valid strings of length $n-1$ to make valid strings of length n. `AC` can be appended to valid strings of length $n-2$ to make valid strings of length n.

Problem 3: (i) Characteristic equation and its solution:

$$x^2 - 2x - 3 = 0 \Rightarrow x_1 = 3,\ x_2 = -1$$

(ii) General solution of the homogeneous equation:

$$f'_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot (-1)^n$$

(iii) Particular solution of the nonhomogeneous equation:

- Particular solution: $f''_n = \beta n 3^n$
- Plug in:

$$\begin{aligned} &\beta n 3^n = 2\beta(n-1)3^{n-1} + 3\beta(n-2)3^{n-2} + 2 \cdot 3^n \\ &\Rightarrow 9\beta n = 6\beta(n-1) + 3\beta(n-2) + 18 \\ &\Rightarrow \beta = \tfrac{3}{2} \end{aligned}$$

- Thus, $f''_n = \frac{3}{2}n3^n$

(iv) General solution of the nonhomogeneous equation:

$$f_n = f'_n + f''_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot (-1)^n + \tfrac{3}{2}n3^n$$

Problem 4: (i) Characteristic equation and its solution:

$$x^2 - 2x - 3 = 0 \Rightarrow x_1 = 3,\, x_2 = -1$$

(ii) General solution of the homogeneous equation:

$$f'_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot (-1)^n$$

(iii) Particular solution of the nonhomogeneous equation:

- Particular solution: $f''_n = \beta$
- Plug in:

$$\beta = 2\beta + 3\beta + 3 \quad \Rightarrow \quad \beta = -\tfrac{3}{4}$$

- So $f''_n = -\frac{3}{4}$.

(iv) General solution of the nonhomogeneous equation:

$$f_n = f'_n + f''_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot (-1)^n - \tfrac{3}{4}$$

initial conditions equations and their solution

$$\begin{cases} \alpha_1 + \alpha_2 - \frac{3}{4} = 1 \\ \alpha_1 \cdot 3 + \alpha_2 \cdot (-1) - \frac{3}{4} = -2 \end{cases} \Rightarrow \begin{cases} \alpha_1 = \frac{1}{8} \\ \alpha_2 = \frac{13}{8} \end{cases}$$

final answer:

$$f_n = \tfrac{1}{8} \cdot 3^n + \tfrac{13}{8} \cdot (-1)^n - \tfrac{3}{4}$$
