Quiz 1 B

● Graded

Student

Javier Herrera

Total Points

20.5 / 33 pts

Question 1

Problem 1

4 / 10 pts

- ✔ **+ 2 pts** (a) Correct Θ estimate
- **+ 1.5 pts** (a) Partial: Θ estimate is almost correct
- **+ 2 pts** (b) Correct Θ estimate
- **+ 1 pt** (b) Partial: Θ estimate is almost correct
- **+ 2 pts** (c) Correct Θ estimate
- ✔ **+ 1 pt** (c) Partial: Θ estimate is almost correct
- **+ 2 pts** (c) Reasonable justification
- ✔ **+ 1 pt** (c) Partial: Justification, correct idea
- **+ 0.5 pts** (c) Partial: some correct work
- **+ 2 pts** (d) Correct Θ estimate
- **+ 1 pt** (d) Partial: Θ estimate is almost correct, small error
- **+ 0 pts** Incorrect/No solution

Question 2

Problem 2(a)

8.5 / 11 pts

- ✔ **+ 2 pts** (a1) **Correct** Θ-estimate
- **+ 1 pt** (a1) Partial: Almost correct Θ-estimate, a small error
- ✔ **+ 2 pts** (a2) **Correct** Θ-estimate
- **+ 1 pt** (a2) Partial: Almost correct Θ-estimate, a small error
- ✔ **+ 2 pts** (a3) **Correct** Θ-estimate
- **+ 1 pt** (a3) Partial: Almost correct Θ-estimate, a small error
- **+ 1 pt** (a3) **Correct** justification for big-O.
- ✔ **+ 0.5 pts** (a3) Partial: Some correct work for big-O.
- **+ 0.5 pts** (a3) **Correct** justification for big-Ω.
- ✔ **+ 2 pts** (a4) **Correct** Θ-estimate
- **+ 1 pt** (a4) Partial: Almost correct Θ-estimate, a small error
- **+ 1 pt** (a4) **Correct** justification for big-O.
- **+ 0.5 pts** (a4) Partial: Some correct work for big-O.
- **+ 0.5 pts** (a4) **Correct** justification for big-Ω.
- **+ 0 pts** Incorrect/No solution

Question 3

Problem 2(b)

3 / 3 pts

3.1 (no title)

3 / 3 pts

- ✔ **+ 1.5 pts** b) Big-O correct
- **+ 1 pt** b) Partial: Big-O almost correct
- ✔ **+ 1.5 pts** b) Big-Omega correct
- **+ 1 pt** b) Partial: Big-Omega almost correct
- **+ 1 pt** Partial total: some correct work
- **+ 0 pts** incorrect
- **+ 0.01 pts** Need to check the hard copy

Question 4

Problem 3

5 / 9 pts

✔ **+ 1 pt** (a) Base Case

$$125 \geq 91$$

+ 0.5 pts (a) Partial: Base case, small mistake

✔ **+ 1.5 pts** (a) Started, used assumption correctly

+ 2.5 pts (a) Finished the proof correctly

+ 1.5 pts a) Partial: Finishing the proof, a small error

✔ **+ 0.5 pts** (a) Partial: almost finished - small mistakes OR details missing

+ 0.5 pts (a) Partial: the idea is correct

✔ **+ 2 pts** (b) Definition is stated and/or applied correctly; inequality is correct

+ 1 pt (b) Correct c from correct inequality

+ 1 pt (b) Correct n_0 from correct inequality

+ 1 pt (b) Partial: Definition, inequality: simple mistake

+ 0 pts Incorrect

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Date 10/15/25 Time 5:00pm

CS111 Quiz 1 (B)

Rules:

- Time: 45 minutes.
- Closed notes, closed book. No electronic devices, including calculators.
- To receive credit, you must show your work and provide justification when appropriate.
- Only the front pages will be scanned! If you need more room, write on the last blank page. If you need scratch space, use the back side of the previous page.

Problem 1: For each piece of pseudo-code below, give the Θ-estimate for its running time. Include a very brief informal explanation for the third piece of pseudo-code.

Pseudo-code	Θ-estimate	Justification

for $i \leftarrow 3$ **to** n^4 **do**
    for $j \leftarrow 1$ **to** n^2 **do**
        print("X")
        $j \leftarrow j + 2$
	$\Theta(n^6)$	No explanation is required here.

$i \leftarrow 2$
while $i \leq n^3$ **do**
    for $k = 1$ **to** $30n$ **do**
        print("X")
        $i \leftarrow 5 \cdot i$
	$\Theta(n^4)$	No explanation is required here.

$j \leftarrow 3$
while $j < n$ **do**
    for $i \leftarrow 1$ **to** j **do**
        print("X")
    $j \leftarrow 2 \cdot j$
	$\Theta(n \log_2 n)$	The outer loop runs $\Theta(n)$ times while the inner loop runs $\Theta(\log(n))$ sin j is doubling.

$k \leftarrow 1$
for $i \leftarrow 3$ **to** $3n$ **do**
    while $k < 2n^3$ **do**
        print("X")
        $k \leftarrow k + 3$
	$\Theta(n^4)$	No explanation is required here.

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Problem 2: (a) Use the Θ-notation to determine the rate of growth of functions below. Justify your answers for the last two functions only, using asymptotic relations between the basic reference functions. (Reminder: $\log x = \log_2 x$.)

Function	Θ estimate	Justification
$150n + 18n^3 + 30n^8 - 2n^2 + 16n^5$	$\Theta(n^8)$	Don't need to justify
$n^2 \cdot \left(\frac{10}{11}\right)^n + n \cdot (1.25^n) + n^7 \log^2 n$	$\Theta(n \cdot 1.25^n)$	Don't need to justify
$(8)^n \cdot \log^2 n + (5)^n \cdot n^3 + n^{12}$	$\Theta(8^n \cdot \log^2 n)$	Since $8 > 5$, the base will be 8^n since it grows rapidly no matter $\log^2 n$
$5n^4 \log n + n^3 \log^3 n + 5\sqrt{n^9}$	$\Theta(\sqrt{n^9})$	Since $\sqrt{n}$ is also $n^{1/2}$, $n^{1/2} \cdot n^9 = n^{4.5}$, therefore growing the fastest

(b) Justify, using the definition of Θ, that $7n^6 + 5n^4 - 3n^2 + 1 = \Theta(n^6)$.

Assumming $7n^6 + 5n^4 - 3n^2 + 1 = \Theta(n^6)$, we must solve for Ω and O. for $n \geq 1$, $c_1 = 7$ and $5n^4 - 3n^2 + 1 \geq 0$

$5 - 3 + 1 \geq 0$

$3 \geq 0$ ✓

after solving for $n_0 = 1$, we find the statement to be True. Next, we solve $7n^6 + 5n^4 - 3n^2 + 1 = O(n^6)$.

For $n \geq 1$, $c_1 = 12$ and $7n^6 + 5n^4 + 1 \leq 7n^6 + 5n^6 + 1$

$\leq 12n^6 + 1$

Since $7n^6 + 5n^4 - 3n^2 + 1 = \Omega(n^6)$ & $7n^6 + 5n^4 - 3n^2 + 1 = O(n^6)$, for $n = 1$, $13 \leq 13$ ✓

$7n^6 + 5n^4 - 3n^2 + 1 = \Theta(n^6)$ is true.

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Problem 3: (a) Below you are given a sketch of an inductive proof of the following inequality:

$$5^n \geq 2^{n+3} + 9n, \text{ for } n \geq 3.$$

64 + 27 = 91

Complete this proof by filling in the remaining steps.

Base case: For $n = 3$, the left-hand side is 125 and the right-hand side is 91. Since $125 \geq 91$, the base case holds.

Inductive step: Assume that the inequality holds when $n = k$, for some $k \geq 3$: $5^k \geq 2^{k+3} + 9k$.

We need to prove that it holds for the next integer, that is $5^{k+1} \geq 2^{k+4} + 9(k+1)$. Starting with the left-hand side of this inequality, we proceed as follows:

$5^{k+1} = 5 \cdot 5^k$ since $5^k \geq 2^{k+3} + 9k$

$\geq 5 \cdot (2^{k+3} + 9k)$

$\geq 10^{k+3} + 45k$

So the inequality holds for $n = k + 1$. From the base case and the inductive step, it holds for all $n \geq 3$.

(b) Prove, directly from definition and using part (a), that $2^{k+3}+9k = O(5^k)$. (It is sufficient to specify an inequality and two constants from the definition of the big-Oh notation.)

$2^{k+3} + 9k \leq O(5^k)$

$C_1 = 10$

$k_0 = 1$