Javier Herrera Jr

Testing

CS111 Homework 1

due Wednesday, January 20th

Problem 1. (a) The inner loop of the first **for** loop prints i^2 letters for each $i = 1, 2, ..., 2n+1$. The inner loop of the second **for** loop prints $2i$ letters for each $i = 1, 2, ..., n^2$. Thus denoting $f(n)$ the number of letters "A" printed we have:

$$f(n) = \sum_{i=1}^{2n+1} i^2 + \sum_{i=1}^{n^2} 2i.$$

(b) Using formulas for the sum of the first k terms of an arithmetic series and the sum of squares of k first integers, we can simplify the above formula as follows:

$$\begin{aligned} f(n) &= \sum_{i=1}^{2n+1} i^2 + \sum_{i=1}^{n^2} 2i. \\ &= \frac{1}{6}(2n+1)(2n+2)(4n+3) + 2 \cdot \frac{1}{2} n^2(n^2+1) \\ &= \frac{1}{6}(16n^3 + 36n^2 + 26n + 6) + (n^4 + n^2) \\ &= n^4 + \frac{8}{3}n^3 + 7n^2 + \frac{13}{3}n + 1 \end{aligned}$$

(c) $f(n) = \Omega(n^4)$, since $n^4 + 8/3\, n^3 + 7n^2 + 13/3\, n + 1 \geq n^4$ for $n \geq 0$
$f(n) = O(n^4)$, since $n^4 + 8/3\, n^3 + 7n^2 + 13/3\, n + 1 \leq 7(n^4 + n^4 + n^4 + n^4 + n^4) = 35n^4$ for $n \geq 1$
We conclude that $f(n) = \Theta(n^4)$.

Problem 2. Use induction to prove the formula for the sum of a geometric sequence:

$$\sum_{i=0}^{n} a_i = \frac{a^{n+1} - 1}{a - 1}$$

- Base case: $n = 0$: LHS $= a^0 = 1$ and RHS $= \frac{a-1}{a-1} = 1$. So it is true for base case.
- Inductive Step: Assume the identity holds for some for $n = k$ that is:

$$\sum_{i=0}^{k} a_i = \frac{a^{k+1} - 1}{a - 1}$$

Prove that it is true for $n = k + 1$:

$$\sum_{i=0}^{k+1} a_i = \frac{a^{k+2} - 1}{a - 1}$$

We have:

$$\begin{aligned}
\text{LHS} &= \sum_{i=0}^{k+1} a_i = \sum_{i=0}^{k} a_i + a^{k+1} \quad \text{(separate last term from the sum)} \\
&= \frac{a^{k+1}-1}{a-1} + a^{k+1} \quad \text{(apply inductive assumption)} \\
&= \frac{a^{k+1}-1+a^{k+1}(a-1)}{a-1} \\
&= \frac{a \cdot a^{k+1}-1}{a-1} = \frac{a^{k+2}-1}{a-1} = \text{RHS}
\end{aligned}$$

- Conclusion: The claim holds for $n = k+1$. From the base case and the inductive step, it holds for $n \geq 0$

Problem 3. Give asymptotic values for this function using Θ-notation: $f(n) = \dfrac{n^2 3^n}{4} + n^4 2^n$

- $\frac{1}{4}n^2 3^n + n^4 2^n \geq \frac{1}{4}n^2 3^n$ for $n \geq 0$, so $f(n) = \Omega(n^2 3^n)$
- We also have:

$$\begin{aligned}
f(n) &= \frac{1}{4}n^2 3^n + n^4 2^n \\
&= O(n^2 3^n) + n^2 \cdot n^2 \cdot 2^n \\
&= O(n^2 3^n) + n^2 \cdot O(1.5^n) \cdot 2^n \quad (\text{because } n^2 = O(1.5^n)) \\
&= O(n^2 3^n) + O(n^2 3^n) = O(n^2 3^n)
\end{aligned}$$

We have shown that $f(n) = \Omega(n^2 3^n)$ and $f(n) = O(n^2 3^n)$. Therefore $f(n) = \Theta(n^2 3^n)$

Academic integrity declaration. I did the assignment with my cat Tora.