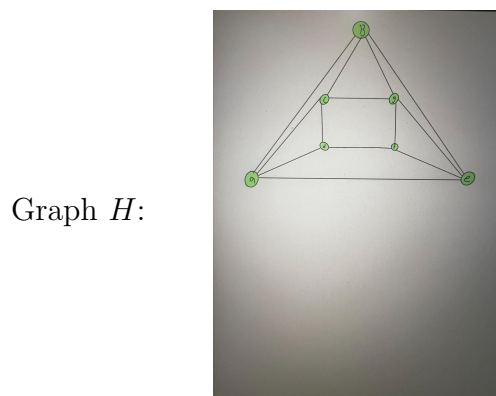
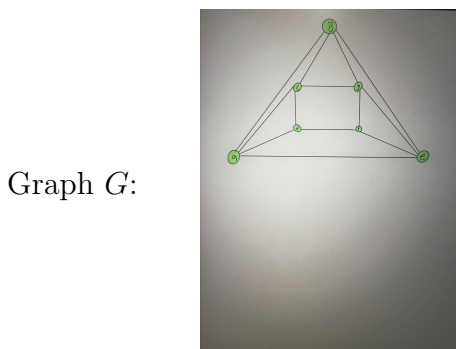
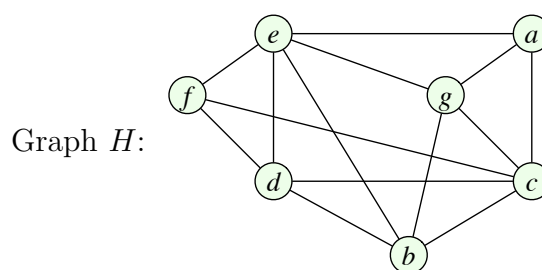
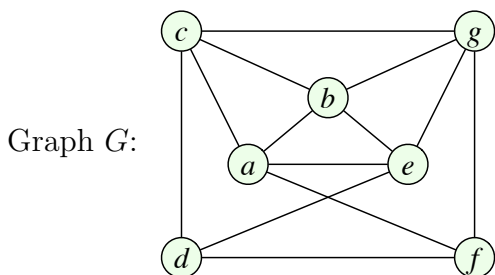


CS/MATH111 ASSIGNMENT 5

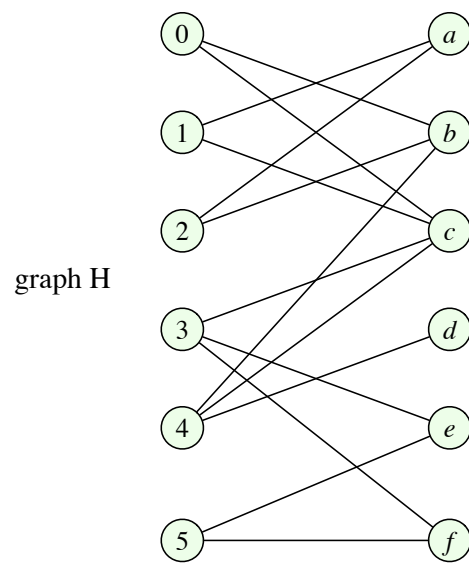
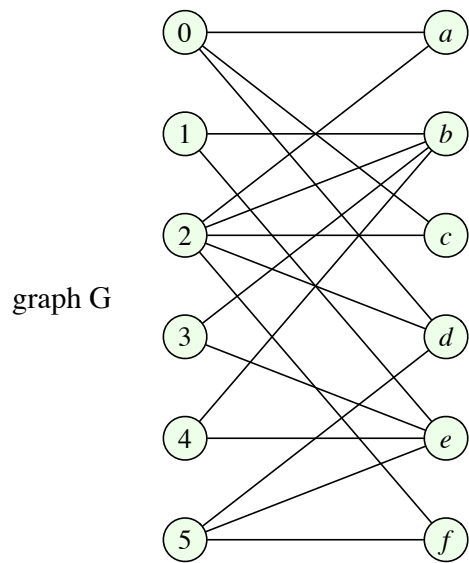
Problem 1. Determine whether the two graphs below are planar or not. To show planarity, give a planar embedding. To show that a graph is not planar, use Kuratowski's theorem.



- Graph G is planar. The planar embedding is shown above.

- Graph H is not planar. It contains a subgraph that is a subdivision of K_5 , which is a non-planar graph. Therefore, by Kuratowski's theorem, H is not planar.

Problem 2. You are given two bipartite graphs G and H below. For each graph determine whether it has a perfect matching. Justify your answer, either by listing the edges that are in the matching or using Hall's Theorem to show that the graph does not have a perfect matching.



- Graph G does not have a perfect matching. By Hall's Theorem, there is no perfect matching in H because the number of neighbors of X is less than $|X|$ for the set $X = \{a, b, c, d, e, f\}$.

- Graph H has a perfect matching. The perfect matching is $\{(c, 0), (a, 1), (b, 2), (e, 3), (d, 4), (f, 5)\}$.

Problem 3. (a) For each degree sequence below, determine whether there is a graph with 6 vertices where vertices have these degrees. If a graph exists, (i) draw it, (ii) find the chromatic number and justify, (iii) determine whether the graph has an Euler tour and justify, (iv) determine whether the graph has a Hamiltonian cycle and justify. If no such graph exists, justify.

(a1) 5, 5, 4, 4, 3, 1.

(a2) 5, 5, 4, 3, 3, 1.

(a3) 5, 5, 5, 4, 4, 3.

(b) For each degree sequence below, determine whether there is a planar graph with 6 vertices where vertices have these degrees. If a planar graph exists, (i) draw it, (ii) find the chromatic number and justify, (iii) determine whether the graph has an Euler tour and justify, (iv) determine whether the graph has a Hamiltonian cycle and justify. If no such planar graph exists, justify.

(b1) 5, 5, 3, 3, 2, 2.

(b2) 5, 5, 4, 4, 4, 4.

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a short paragraph where you briefly explain *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

- For all the problems above, I referenced all the examples shown in the planar and bipartite graph lecture notes and slides. I also used 2 youtube videos to better understand the concepts of planar graphs and bipartite graphs. I did this homework individually.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment. Remember that only L^AT_EX papers are accepted.