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CS 111 ASSIGNMENT 0

What you need to do in this assignment:

- Put your name and student ID on the top.
- Revise the academic integrity statement.
- In Problem 1, implement the following change: Suppose that the first inner loop prints $3i^2$ letters instead of i^2. (Make this change.) As a result, in the formula for $f(n)$ the first summation will now have coefficient 3. Modify this formula and the rest of the calculation to reflect this change.
- In Problem 2, change the notation, by changing the name of variable a to b.
- Remove these instructions (text in red).

Solution 1: (a) The inner loop of the first **for** loop prints i^2 letters for each $i = 1, 2, ..., n+1$. The inner loop of the second **for** loop prints $2i$ letters for each $i = 1, 2, ..., n^2$. Thus denoting $f(n)$ the number of letters "A" printed we have:

$$f(n) = \sum_{i=1}^{n+1} i^2 + \sum_{i=1}^{n^2} (2i) = \sum_{i=1}^{n+1} i^2 + 2\sum_{i=1}^{n^2} i.$$

(b) Using formulas for the sum of the first k terms of an arithmetic series and the sum of squares of k first integers, we can simplify the above formula as follows:

$$\begin{aligned} f(n) &= \textstyle\sum_{i=1}^{n+1} i^2 + 2\sum_{i=1}^{n^2} i \\ &= \tfrac{1}{6}(n+1)(n+2)(2n+3) + 2\cdot\tfrac{1}{2}n^2(n^2+1) \\ &= \tfrac{1}{6}(2n^3 + 9n^2 + 13n + 6) + (n^4 + n^2) \\ &= n^4 + \tfrac{1}{3}n^3 + \tfrac{5}{2}n^2 + \tfrac{13}{6}n + 1 \end{aligned}$$

(c) We conclude that $f(n) = \Theta(n^4)$, because $f(n)$ is a polynomial of degree 4.

Solution 2: Use induction to prove the formula for the sum of a geometric sequence (where $a \neq 1$):

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$$

Base case: For $n = 0$, LHS $= a^0 = 1$ and RHS $= \frac{a-1}{a-1} = 1$. So it is true for base case.

Inductive Step: Assume the identity holds for some for $n = k$, that is:

$$\sum_{i=0}^{k} a^i = \frac{a^{k+1} - 1}{a - 1}$$

We need to show that it is true for $n = k + 1$:

$$\sum_{i=0}^{k+1} a^i = \frac{a^{k+2} - 1}{a - 1}$$

To do this, starting with the left-hand side, we proceed as follows:

$$\begin{aligned}
\text{LHS} = \sum_{i=0}^{k+1} a^i = \sum_{i=0}^{k} a^i + a^{k+1} & \qquad \text{(separate last term from the sum)} \\
= \frac{a^{k+1} - 1}{a - 1} + a^{k+1} & \qquad \text{(apply inductive assumption)} \\
= \frac{a^{k+1} - 1 + a^{k+1}(a - 1)}{a - 1} & \\
= \frac{a \cdot a^{k+1} - 1}{a - 1} = \frac{a^{k+2} - 1}{a - 1} = \text{RHS} &
\end{aligned}$$

Therefore the claim holds for $n = k + 1$, completing the inductive step.

From the base case and the inductive step, we can conclude that the identity is true for all $n \geq 0$.

Academic integrity declaration. This assignment was done jointly by Marek and his dog Paco.

Paco: I did all the work. I did all the math and typed up the solutions. I verified and have a full understanding of all solutions.

Marek: I was only watching Paco solving these problems, scratching his belly. Afterwards, it took a while, but he managed to explain to me all solutions. I understand them now and verified that they are indeed correct.

We have not used any external resources.