Problem 1: *Characteristic equation.* The characteristic equation is $x^2 + x - 12 = 0$. Its roots are $r_1 = -4$ and $r_2 = 3$, each with multiplicity one.

General form. The general solution form is $P_n = \alpha_1 \cdot (-4)^n + \alpha_2 \cdot 3^n$.

Initial condition equations. Substituting $n = 0$ and $n = 1$ into the general form, gives us equations

$$\alpha_1 + \alpha_2 = 3$$
$$-4\alpha_1 + 3\alpha_2 = 2$$

Multiplying the first equation by -3 and adding to the second equation gives $-7\alpha_1 = -7$, so $\alpha_1 = 1$. From the first equation, $\alpha_2 = 2$.

Final solution. We plug in the values of α_1 and α_2 into the general form, obtaining the final solution:

$$P_n = (-4)^n + 2 \cdot 3^n$$

Problem 2: *General solution for the homogeneous equation.* From the factorization of the characteristic polynomial, the roots are: $r_1 = -1$ with multiplicity 2 and $r_2 = 3$ with multiplicity 1. So this general form is $Q'_n = \alpha_1(-1)^n + \alpha_2 n(-1)^n + \alpha_3 3^n$.

Particular solution. Since 3 is a root of the characteristic equation, we need to look for a particular solution of the form $Q''_n = \beta n 3^n$. We plug this into the original equation:

$$\beta n 3^n = \beta(n-1)3^{n-1} + 5\beta(n-2)3^{n-2} + 3\beta(n-3)3^{n-3} + 3^n$$

This must hold for each natural number n, so we can plug in any n and solve for β. We plug in $n = 3$:

$$81\beta = 18\beta + 15\beta + 0 + 27$$

So $\beta = \frac{9}{16}$. Thus the general form is

$$Q_n = \alpha_1(-1)^n + \alpha_2 n(-1)^n + \alpha_3 3^n + \tfrac{9}{16} n 3^n$$

Problem 3: We will call a string "valid" if it satisfies the condition in the problem. We partition the set of valid strings of length n into several types.

Type 1*:* Strings ending with `C`. Adding `C` at the end of a string does not affect its validity. So the number of valid strings of length n of Type 1 is R_{n-1}.

Type 2*:* Strings ending with `B`. By the same argument as for Type 1, we have R_{n-1} strings of Type 2.

Type 3*:* Strings ending with `A`. Any valid string ending with `A` cannot have `A` or `B` as the second last letter. Thus all valid strings of Type 3 end with `CA`, so they consist of a valid string of length $n-2$ followed by `CA`. Therefore there are R_{n-2} such strings.

This gives us $R_n = 2R_{n-1} + R_{n-2}$. The empty string is valid and the three strings of length 1 are valid, so $R_0 = 1$ and $R_1 = 3$. Thus the overall recurrence equation is

$$
\begin{aligned}
R_n &= 2R_{n-1} + R_{n-2} \\
R_0 &= 1 \\
R_1 &= 3
\end{aligned}
$$