

Problem A: Use the Θ -notation to determine the rate of growth of the following functions:

Function	Θ estimate
$5n + 3n^2 + 3$	
$17n + 3n^2 \log n + 1$	
$7n^9 + (1.5)^n$	
$n^3 4^n + 5^n + 16\sqrt{n}$	
$\sqrt{n} + 11 \log n$	

Problem B: For each piece of pseudo-code below, give its asymptotic running time as a function of n . Express this running time using the $\Theta()$ notation. (You don't need to give any justification.)

Pseudo-code	Running time
for $i \leftarrow 1$ to $2n$ do for $j \leftarrow 1$ to i do $x \leftarrow 2x + 7$	
$j \leftarrow 1$ while $j < n$ do $x \leftarrow 2x + 7$ $j \leftarrow j + 2$	
for $i \leftarrow 1$ to n do $j \leftarrow 1$ while $j < n$ $x \leftarrow 2x + 7$ $j \leftarrow 3j$	
for $i \leftarrow n/2$ to n do $x \leftarrow 2x + 7$ for $j \leftarrow 1$ to $3n$ do $x \leftarrow 2x + 7$	

Note 1: “ \leftarrow ” denotes the assignment statement. The scope of and nesting loops is indicated by the indentation.

Problem C: Use the Θ -notation to determine the rate of growth of the following functions:

Function	big- Θ estimate
$5n + 3n^4 + 3$	
$n \log^2 n + n^{1.5} + \sqrt{n}$	
$17\sqrt{n} + n3^n \log n + 4^n$	
$\sqrt{n} + 11 \log^5 n$	
$1 + 1/\log n$	

Problem D: For each piece of pseudo-code below, give its asymptotic running time as a function of n . Express this running time using the $\Theta()$ notation. Include a brief justification (at most 15 words).

Pseudo-code	Running time	Justification
for $i \leftarrow 1$ to n do $z \leftarrow z + 5$ $k \leftarrow 1$ while $k < n$ do $z \leftarrow z^2$ $k \leftarrow 2k$		
for $i \leftarrow 1$ to $2n + 3$ do $z \leftarrow z + 5$ for $i \leftarrow 1$ to $7n$ do $z \leftarrow z^2$		
$j \leftarrow 1$ while $j < n$ do $z \leftarrow z + 5$ for $i \leftarrow 1$ to j do $z \leftarrow z^2$ $j \leftarrow 2j$		
for $i \leftarrow 1$ to n do $z \leftarrow z + 2$ for $j \leftarrow 1$ to i do $z \leftarrow z^2$		

Note: “ \leftarrow ” denotes the assignment statement. The scope and nesting of loops is indicated by the indentation.

Problem E: For each piece of pseudo-code below, give its asymptotic running time as a function of n . Express this running time using the $\Theta()$ notation. Include a brief justification (at most 25 words).

Pseudo-code	Running time	Justification
for $i \leftarrow 1$ to $3n^2$ do $x \leftarrow x^2$ for $j \leftarrow 1$ to $n + 3$ do $z \leftarrow x + z$		
for $i \leftarrow 1$ to n do $j \leftarrow 1$ while $j < n$ do $j \leftarrow 4j$ $x \leftarrow j \cdot x$		
for $i \leftarrow 1$ to n^2 do $k \leftarrow 1$ while $k < n$ $x \leftarrow x^2$ $k \leftarrow k + 3$		
for $i \leftarrow n/2$ to n do $x \leftarrow 2x - 1$ for $j \leftarrow 1$ to $2i$ do $x \leftarrow 2j \cdot x$		
$k \leftarrow 1$ for $i \leftarrow 1$ to n do while $k < 9i$ do $k \leftarrow k + 1$ $x \leftarrow x^2$		

Note 1: “ \leftarrow ” denotes the assignment statement. The scope and nesting of loops is indicated by the indentation.