Problem A: Use the Θ-notation to determine the rate of growth of the following functions:

Function	Θ estimate
$5n + 3n^2 + 3$	$\Theta(n^2)$
$17n + 3n^2 \log n + 1$	$\Theta(n^2 \log n)$
$7n^9 + (1.5)^n$	$\Theta((1.5)^n)$
$n^3 4^n + 5^n + 16\sqrt{n}$	$\Theta(5^n)$
$\sqrt{n} + 11 \log n$	$\Theta(\sqrt{n})$

Problem B: For each piece of pseudo-code below, give its asymptotic running time as a function of n. Express this running time using the $\Theta()$ notation. (You don't need to give any justification.)

Pseudo-code	Running time

for $i \leftarrow 1$ **to** $2n$ **do**
      for $j \leftarrow 1$ **to** i **do**
          $x \leftarrow 2x + 7$
	$\Theta(n^2)$

$j \leftarrow 1$
 while $j < n$ **do**
      $x \leftarrow 2x + 7$
      $j \leftarrow j + 2$
	$\Theta(n)$

for $i \leftarrow 1$ **to** n **do**
      $j \leftarrow 1$
      while $j < n$
          $x \leftarrow 2x + 7$
          $j \leftarrow 3j$
	$\Theta(n \log n)$

for $i \leftarrow n/2$ **to** n **do**
      $x \leftarrow 2x + 7$
 for $j \leftarrow 1$ **to** $3n$ **do**
      $x \leftarrow 2x + 7$
	$\Theta(n)$

Note 1: "$\leftarrow$" denotes the assignment statement. The scope of and nesting loops is indicated by the indentation.

Problem C: Use the Θ-notation to determine the rate of growth of the following functions:

Function	big-Θ estimate
$5n + 3n^4 + 3$	$\Theta(n^4)$
$n \log^2 n + n^{1.5} + \sqrt{n}$	$\Theta(n^{1.5})$
$17\sqrt{n} + n3^n \log n + 4^n$	$\Theta(4^n)$
$\sqrt{n} + 11 \log^5 n$	$\Theta(\sqrt{n})$
$1 + 1/\log n$	$\Theta(1)$

Problem D: For each piece of pseudo-code below, give its asymptotic running time as a function of n. Express this running time using the $\Theta()$ notation. Include a brief justification (at most 15 words).

Pseudo-code	Running time	Justification

for $i \leftarrow 1$ **to** n **do**
    $z \leftarrow z + 5$
    $k \leftarrow 1$
    while $k < n$ **do**
        $z \leftarrow z^2$
        $k \leftarrow 2k$
	$\Theta(n \log n)$	k doubles each time, so internal loop makes $\Theta(\log n)$ iterations. External loop makes n iterations.

for $i \leftarrow 1$ **to** $2n + 3$ **do**
    $z \leftarrow z + 5$
for $i \leftarrow 1$ **to** $7n$ **do**
    $z \leftarrow z^2$
	$\Theta(n)$	Two disjoint loops, each making $\Theta(n)$ iterations.

$j \leftarrow 1$
while $j < n$ **do**
    $z \leftarrow z + 5$
    for $i \leftarrow 1$ **to** j **do**
        $z \leftarrow z^2$
    $j \leftarrow 2j$
	$\Theta(n)$	Internal loop takes time j, with j's forming a geometric sequence $1, 2, 4, 8, \dots$ So total is $\Theta(n)$.

for $i \leftarrow 1$ **to** n **do**
    $z \leftarrow z + 2$
    for $j \leftarrow 1$ **to** i **do**
        $z \leftarrow z^2$
	$\Theta(n^2)$	Internal loop takes time i. Adding over all i (arithmetic sequence), we get $\Theta(n^2)$.

Note: "$\leftarrow$" denotes the assignment statement. The scope and nesting of loops is indicated by the indentation.

Problem E:

Pseudo-code	Running time	Justification

for $i \leftarrow 1$ **to** $3n^2$ **do**
 $\quad x \leftarrow x^2$
 for $j \leftarrow 1$ **to** $n+3$ **do**
 $\quad z \leftarrow x + z$
	$\Theta(n^2)$	Two independent loops with running times $\Theta(n^2)$ and $\Theta(n)$.

for $i \leftarrow 1$ **to** n **do** $\quad j \leftarrow 1$ $\quad$ **while** $j < n$ **do** $\quad\quad j \leftarrow 4j$ $\quad\quad x \leftarrow j \cdot x$	$\Theta(n \log n)$	The external loop makes n iterations. For each iteration of the external loop, the internal loop makes $\Theta(\log n)$ iterations.
for $i \leftarrow 1$ **to** n^2 **do** $\quad k \leftarrow 1$ $\quad$ **while** $k < n$ $\quad\quad x \leftarrow x^2$ $\quad\quad k \leftarrow k + 3$	$\Theta(n^3)$	The external loop makes $\Theta(n^2)$ iterations. For each iteration of the external loop, the internal loop makes $\Theta(n)$ iterations.
for $i \leftarrow n/2$ **to** n **do** $\quad x \leftarrow 2x - 1$ $\quad$ **for** $j \leftarrow 1$ **to** $2i$ **do** $\quad\quad x \leftarrow 2j \cdot x$	$\Theta(n^2)$	For any given i, the internal loop makes $2i$ iterations. As i ranges from $n/2$ to n, these numbers will add up to $\Theta(n^2)$ (the sum of an arithmetic sequence).
$k \leftarrow 1$ **for** $i \leftarrow 1$ **to** n **do** $\quad$ **while** $k < 9i$ **do** $\quad\quad k \leftarrow k + 1$ $\quad\quad x \leftarrow x^2$	$\Theta(n)$	The external loop makes n iterations. For each i, the while loop will make only 9 iterations.