

**Problem A:** Use the  $\Theta$ -notation to determine the rate of growth of the following functions:

Function	$\Theta$ estimate
$5n + 3n^2 + 3$	
$17n + 3n^2 \log n + 1$	
$7n^9 + (1.5)^n$	
$n^3 4^n + 5^n + 16\sqrt{n}$	
$\sqrt{n} + 11 \log n$	

**Problem B:** For each piece of pseudo-code below, give its asymptotic running time as a function of  $n$ . Express this running time using the  $\Theta()$  notation. (You don't need to give any justification.)

Pseudo-code	Running time
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>2n</math> <b>do</b>     <b>for</b> <math>j \leftarrow 1</math> <b>to</b> <math>i</math> <b>do</b>         <math>x \leftarrow 2x + 7</math>     </pre>	
<pre> <math>j \leftarrow 1</math> <b>while</b> <math>j &lt; n</math> <b>do</b>     <math>x \leftarrow 2x + 7</math>     <math>j \leftarrow j + 2</math>     </pre>	
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n</math> <b>do</b>     <math>j \leftarrow 1</math>     <b>while</b> <math>j &lt; n</math>         <math>x \leftarrow 2x + 7</math>         <math>j \leftarrow 3j</math>     </pre>	
<pre> <b>for</b> <math>i \leftarrow n/2</math> <b>to</b> <math>n</math> <b>do</b>     <math>x \leftarrow 2x + 7</math> <b>for</b> <math>j \leftarrow 1</math> <b>to</b> <math>3n</math> <b>do</b>     <math>x \leftarrow 2x + 7</math>     </pre>	

Note 1: “ $\leftarrow$ ” denotes the assignment statement. The scope of and nesting loops is indicated by the indentation.

**Problem C:** Use the  $\Theta$ -notation to determine the rate of growth of the following functions:

Function	big- $\Theta$ estimate
$5n + 3n^4 + 3$	
$n \log^2 n + n^{1.5} + \sqrt{n}$	
$17\sqrt{n} + n3^n \log n + 4^n$	
$\sqrt{n} + 11 \log^5 n$	
$1 + 1/\log n$	

**Problem D:** For each piece of pseudo-code below, give its asymptotic running time as a function of  $n$ . Express this running time using the  $\Theta()$  notation. Include a brief justification (at most 15 words).

Pseudo-code	Running time	Justification
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n</math> <b>do</b>     <math>z \leftarrow z + 5</math>     <math>k \leftarrow 1</math>     <b>while</b> <math>k &lt; n</math> <b>do</b>         <math>z \leftarrow z^2</math>         <math>k \leftarrow 2k</math> </pre>		
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>2n + 3</math> <b>do</b>     <math>z \leftarrow z + 5</math> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>7n</math> <b>do</b>     <math>z \leftarrow z^2</math> </pre>		
<pre> <math>j \leftarrow 1</math> <b>while</b> <math>j &lt; n</math> <b>do</b>     <math>z \leftarrow z + 5</math>     <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>j</math> <b>do</b>         <math>z \leftarrow z^2</math>     <math>j \leftarrow 2j</math> </pre>		
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n</math> <b>do</b>     <math>z \leftarrow z + 2</math>     <b>for</b> <math>j \leftarrow 1</math> <b>to</b> <math>i</math> <b>do</b>         <math>z \leftarrow z^2</math> </pre>		

**Note:** “ $\leftarrow$ ” denotes the assignment statement. The scope and nesting of loops is indicated by the indentation.

**Problem E:** For each piece of pseudo-code below, give its asymptotic running time as a function of  $n$ . Express this running time using the  $\Theta()$  notation. Include a brief justification (at most 25 words).

Pseudo-code	Running time	Justification
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>3n^2</math> <b>do</b>     <math>x \leftarrow x^2</math> <b>for</b> <math>j \leftarrow 1</math> <b>to</b> <math>n + 3</math> <b>do</b>     <math>z \leftarrow x + z</math> </pre>		
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n</math> <b>do</b>     <math>j \leftarrow 1</math>     <b>while</b> <math>j &lt; n</math> <b>do</b>         <math>j \leftarrow 4j</math>         <math>x \leftarrow j \cdot x</math> </pre>		
<pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n^2</math> <b>do</b>     <math>k \leftarrow 1</math>     <b>while</b> <math>k &lt; n</math>         <math>x \leftarrow x^2</math>         <math>k \leftarrow k + 3</math> </pre>		
<pre> <b>for</b> <math>i \leftarrow n/2</math> <b>to</b> <math>n</math> <b>do</b>     <math>x \leftarrow 2x - 1</math>     <b>for</b> <math>j \leftarrow 1</math> <b>to</b> <math>2i</math> <b>do</b>         <math>x \leftarrow 2j \cdot x</math> </pre>		
$k \leftarrow 1$ <pre> <b>for</b> <math>i \leftarrow 1</math> <b>to</b> <math>n</math> <b>do</b>     <b>while</b> <math>k &lt; 9i</math> <b>do</b>         <math>k \leftarrow k + 1</math>         <math>x \leftarrow x^2</math> </pre>		

Note 1: “ $\leftarrow$ ” denotes the assignment statement. The scope and nesting of loops is indicated by the indentation.