CS111 Winter'25 ASSIGNMENT 1

Problem 1: Give an asymptotic estimate for the number $D(n)$ of "D"s printed by Algorithm PRINT_DS below. Your solution *must* consist of the following steps:

(a) First express $D(n)$ using the summation notation $\sum$.

(b) Next, give a closed-form expression[1] for $D(n)$.

(c) Finally, give the asymptotic value of $D(n)$ using the Θ-notation.

Show your work and include justification for each step.

Algorithm PRINT_DS $(n : \textbf{integer})$
    for $i \leftarrow 1$ **to** $4n$ **do**
        for $j \leftarrow 3$ **to** $(2i+1)^2$ **do** print("D")
    for $i \leftarrow 5$ **to** $n-1$ **do**
        for $j \leftarrow 1$ **to** $4i$ **do** print("D")

Note: If you need any summation formulas for this problem, you are allowed to look them up. You do not need to prove them, you can just state in the assignment when you use them.

(a) In order to express $D(n)$ using summation notation, we first have to analyze how each part of the for loop runs. We notice in the outer for loop, its iteration is from $i = 1$ to $i = 4n$, which is the range of the summation. Secondly, the inner loop suggests the range for $j = 3$ to $(2i+1)^2$

Problem 2: (a) Use properties of quadratic functions to prove that $5x^2 \geq (x+1)^2$ for all real $x \geq 1$.

(b) Use mathematical induction and the inequality from part (a) to prove that $3 \cdot 5^n \geq 4^{n+1} + n \cdot 3^n + n^2$ for all integers $n \geq 3$.

(c) Let $g(n) = 4^{n+1} + n \cdot 3^n + n^2$ and $h(n) = 5^n$. Using the inequality from part (b), prove that $g(n) = O(h(n))$. You need to give a rigorous proof derived directly from the definition of the O-notation, without using any theorems from class. (First, give a complete statement of the definition. Next, show how $g(n) = O(h(n))$ follows from this definition.)

Problem 3: Give asymptotic estimates, using the Θ-notation, for the following functions:

(a) $7n^5 + 5n^3 - 2n^2 + 3$

(b) $n^3 \log^2 n + n^{2.5} \log^5 n + 5n^2 \log_5 n$

(c) $3n^4 + n^3 + 2^{\log n} + n^2 \cdot (0.5)^n$

(d) $2n^5 + n^3 \log^4 n + n \cdot (1.5)^n$

(e) $n^7 + n^3 \cdot 7^n + n^5 \cdot 4^n$

Justify your answer in part (a) using the definition of Θ. Justify your answer in parts (b) - (e) using asymptotic relations between the basic reference functions: n^b, $\log n$, and c^n.

Academic integrity declaration. The homework papers must include at the end an academic integrity declaration. This should be a brief paragraph where you state *in your own words* (1) whether you did the homework individually or in collaboration with a partner student (if so, provide the name), and (2) whether you used any external help or resources.

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[1] A closed-form expression is a formula that can be evaluated in some fixed number of arithmetic operations, independent of n. For example, $3n^5 + n - 1$ and $n2^n + 5n^2$ are closed-form expressions, but $\sum_{i=1}^{n} i^2$ is not, as it involves $n-1$ additions.

Submission. To submit the homework, you need to upload the pdf file to Gradescope. If you submit with a partner, you need to put two names on the assignment and submit it as a group assignment.

Reminders. Remember that only LATEX papers are accepted.