CS 111 Practice Quiz 1

Problem 1:

(a) Use the Θ-notation to determine the rate of growth of the following functions:

Function	Θ estimate
$n^4 + n^3 - 10n + 3$	
$7n^3 + 3 \cdot 5^n + 12 \log(n^4) + 2 \log^7 n$	
$\frac{10 \log^5 n}{n^2 \sqrt{n}} + 5n^2 \log n + 12n \log^7 n$	
$15 \cdot 2^n + 5 \cdot 3^n$	
$n3^n + n^3 5^n + 3n^5 \log n^8 + n^3 14^{-n}$	

(b) Justify, using the definition of Θ, that $6n^5 + 2n^3 - 1 = \Theta(n^5)$.

Problem 2:

For each piece of pseudo-code below, give:

(a) The expression for the exact number of "A"s printed by that pseudo-code as a function of n (you don't have to simplify the expressions, leaving them in summation notation is fine).

(b) Its asymptotic running time as a function of n. Express this running time using the Θ notation.

Include a brief explanation for both, (a) and (b).

Pseudo-code	Exact number	Running time	Explanation

for $i \leftarrow 1$ **to** $2n$ **do**
  for $j \leftarrow 1$ **to** n^2 **do**
  $k \leftarrow x + 7$
  Print A			

for $i \leftarrow 0$ **to** n^2 **do**
  for $j \leftarrow 1$ **to** 5 **do**
  $x \leftarrow x^4$
  Print A
for $k \leftarrow 1$ **to** $2n$ **do**
  $z \leftarrow x + z$
  Print A			

for $i \leftarrow 0$ **to** n **do**
  for $j \leftarrow 1$ **to** i **do**
  $x \leftarrow x - z$
  Print A			

for $i \leftarrow 1$ **to** 10 **do**
  $x \leftarrow 3x$
  Print A			

for $i \leftarrow 1$ **to** $2n$ **do**
  $j \leftarrow 1$
  Print A
  while $j < n$ **do**
  $x \leftarrow 7x$
  $j \leftarrow 3j$
  Print A			

Problem 3: Using mathematical induction, prove that the following identity holds for all integers $n \geq 1$:

$$\sum_{i=1}^{n} i \cdot i\,! = (n+1)\,! - 1.$$

Academic integrity declaration. Please provide a statement confirming that you completed this assignment all by yourself. (For example, *"Hereby I affirm that I completed this test on my own, without any unauthorized help."*) and sign it. Submissions without the signed statement will not be graded.