# Trading strategy using Hidden Markov Models

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## Abstract

In 1906, Andrei Andréyevich Markov analysed the poem "*Eugenio Oneguin"*. But he was no poet. His analysis consisted in dismounting the text, reducing it to a single string of letters. Once he had arranged the text that way, he converted it into a binary sequence, distinguishing each letter in two groups: vowels and consonants.

He observed that 43% of all letters in the poem were vowels. Therefore, one can assume that if a concrete letter was chosen, the next one would always have about 43% chance of being a vocal. That assumption is wrong.

What Markov discovered was that the probabilities of a letter being a vowel or a consonant were dependent on the letter before. Therefore, if a vocal was chosen, the next letter had a probability of just 14% of being a vocal, much lower than the expected 43%.

In this example, Markov designed a Markov Chain. These models have states and transition probabilities. Markov's first model had two states (vowel and consonant).

With this experiment, Markov demonstrated that for some events, the probability of the next event can depend on the current event instead of occurring independently. This was groundbreaking because many probability results (such as the law of large numbers) were thought to apply only when events were independent.

Sequences of dependent random events can be analyzed using transition probabilities.

## Introduction to the idea behind the model

In the stock market, a stock price over time can be understood as sequences of dependent random events. Therefore, in theory, an analysis using Markov Chains could allow us to construct accurate estimations. Of course, the reality is always more complex and such accurate estimations are not possible.

However, though they don't allow us to see directly into the future, Markov Models can still be very useful for stock price prediction, but several challenges must be addressed.

Firstly, one cannot look at the stock price and confidently say what is the current state. On the poem, states were as objective and simple as vowel or consonant. Now states can be as many as one wants and subjective. In the model, the number of states chosen is 3. The current state is estimated using two metrics: the volatility of the stock price at the moment and the returns obtained in the last day.

Secondly, the Markov Model constructed will have different transition probability values each training window (training windows will be explained in the next paragraph). In order to avoid the model becoming obsolete.

Finally, the transition probabilities (arranged in a transition probability matrix) must be estimated. In the model, the walk-forward method was chosen. A machine learning algorithm will use the information of the past to train a model. The model will be trained using stock price values available before year X. Later, it will be used to invest during the year X. The next year (X+1), all information available before and during year X will be used for training. The training window for this estimation would be from the first year (2010) to year X. Therefore, no look-ahead is introduced.

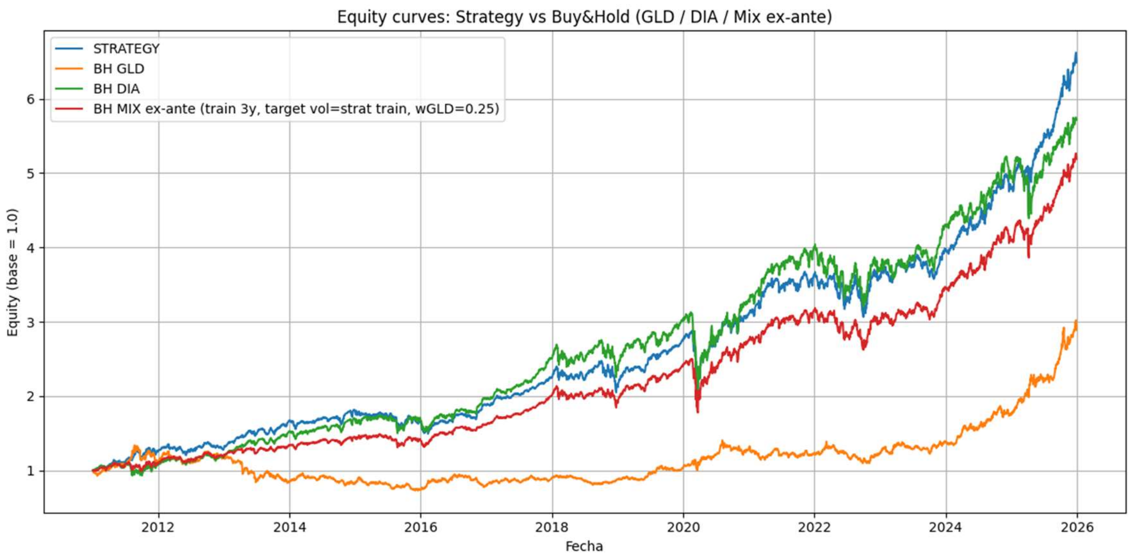
## Model structure

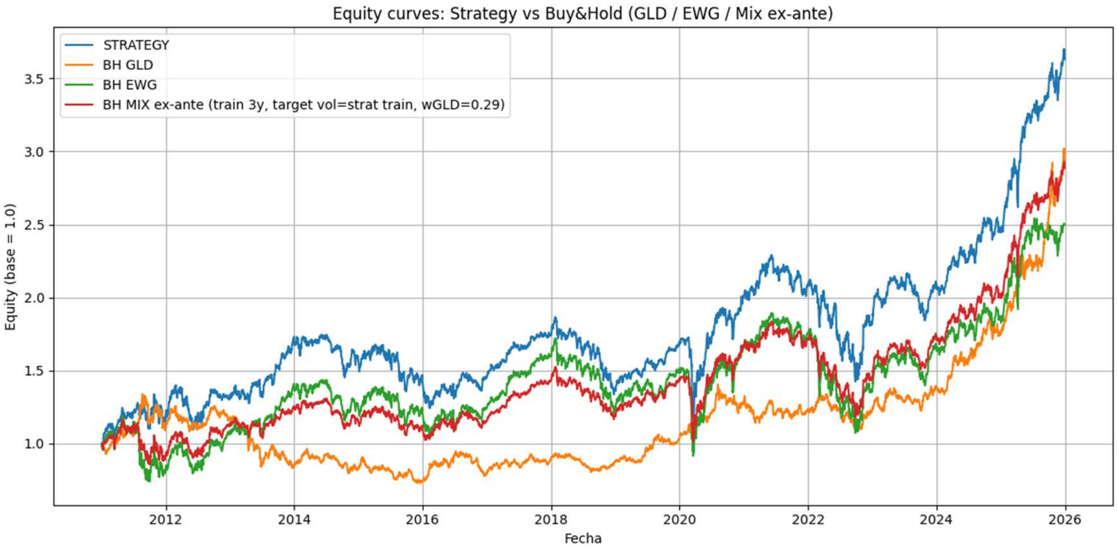
The framework relies on a Hidden Markov Model (HMM), where the hidden market regime is characterized by a first-order Markov process. As impossible to directly identify the regime, estimation of the underlying regime is instead carried out using the daily returns and absolute returns acting as a volatility proxy. The parameters of the model are instead calculated via the Baum-Welch algorithm. Additionally, a walk-forward technique is adopted as a procedure to eliminate look-ahead bias. The model delivers a probability distribution at each step instead of the regime identifier.

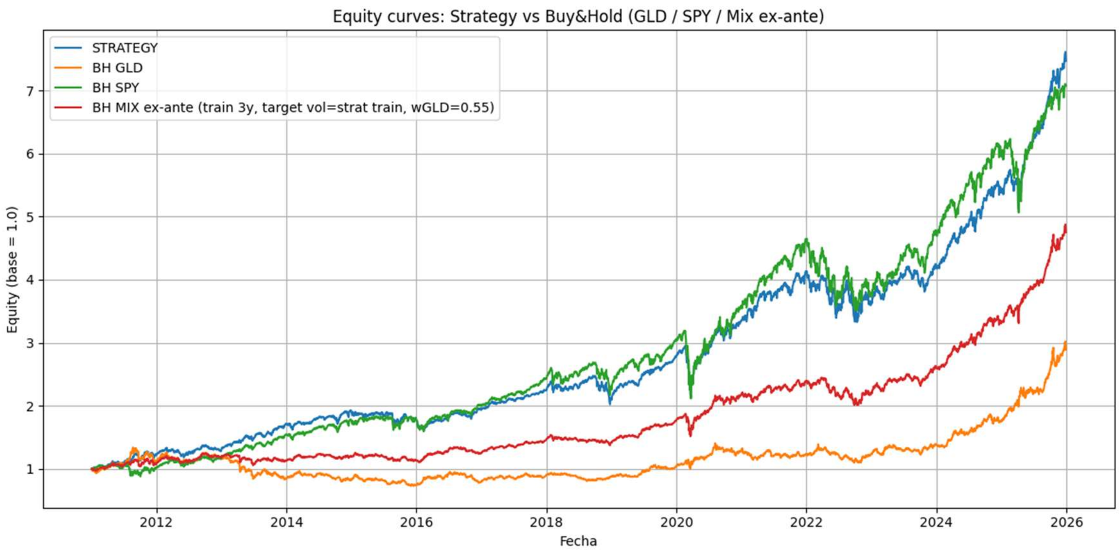
These regime likelihoods are then combined with the state-dependent measures of return on, or the variability of, the asset to calculate the weighted average expected Sharp Ratio. This is then smoothed, normalized by a sigmoid mapping to provide a bounded stable portfolio exposure. The allocation is then filtered based on a trend filter that uses the Exponential Moving Average, as well as the Volatility Targeting Filter, to ensure that the risk remains constant. Rebalanced constraints as well as delay parameters are incorporated to account for turnover costs.

## Results

For research (and flexing) purposes, the results presented initially assume **zero transaction costs**. The model is tested using **gold (GLD)** as the studied asset and is employed to determine the fraction of capital invested in gold on a daily basis. The remaining capital is allocated to a secondary asset, represented by an ETF. The strategy is evaluated across **12 different ETFs** spanning American, Japanese, and European markets. The resulting performance metrics are presented below.




Equity curves: Strategy vs Buy&Hold (GLD / EWG / Mix ex-ante)
STRATEGY
BH GLD
BH EWG
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.29)
Equity (base = 1.0)
3.5
3.0
2.5
2.0
1.5
1.0
2012
2014
2016
2018
2020
2022
2024
2026
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / SPY / Mix ex-ante)
STRATEGY
BH GLD
BH SPY
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.55)
Equity (base = 1.0)
7
6
5
4
3
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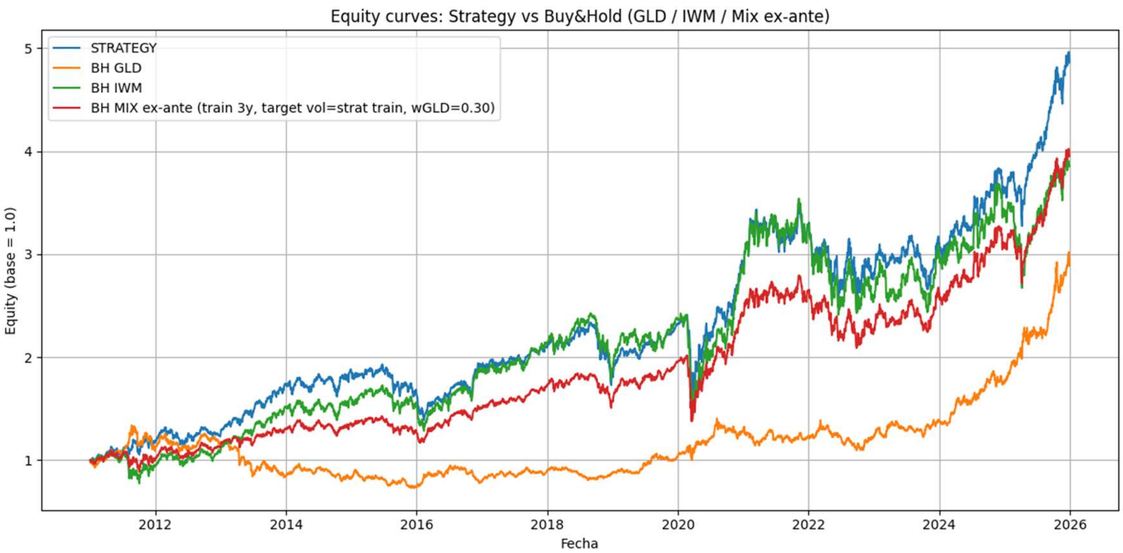
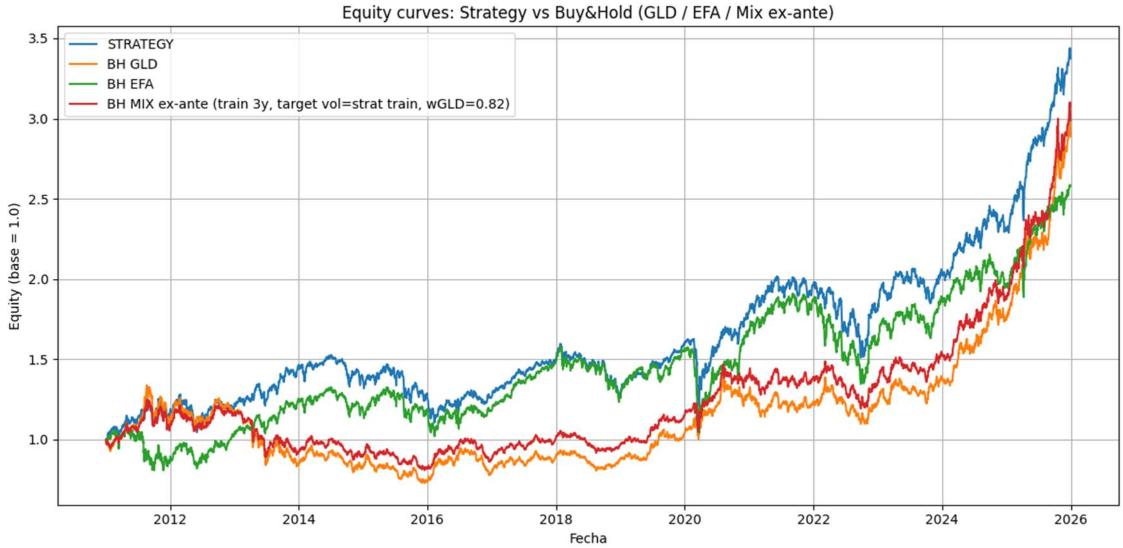
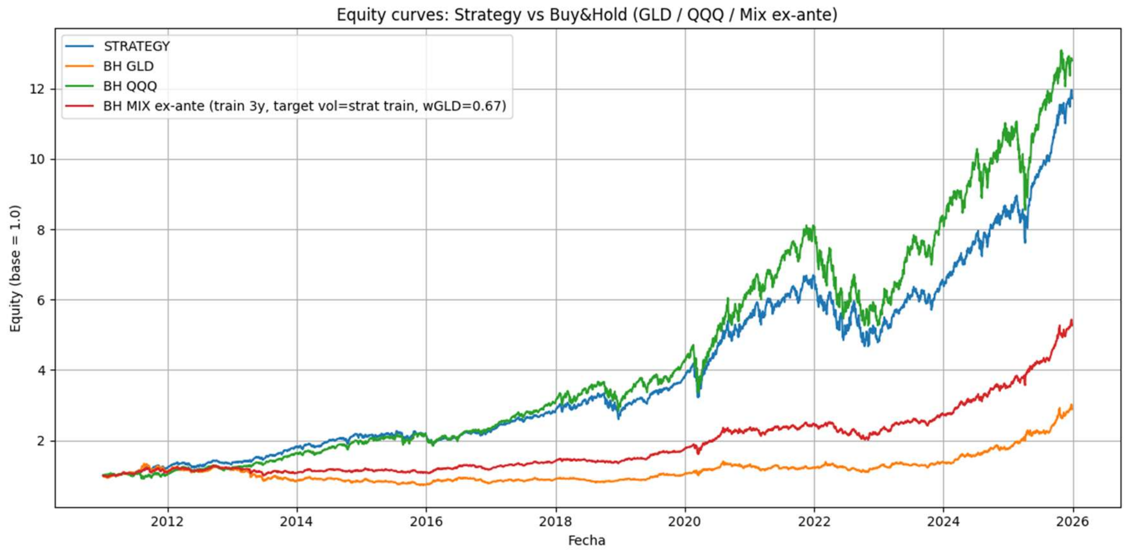
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / IWM / Mix ex-ante)
STRATEGY
BH GLD
BH IWM
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.30)
Equity (base = 1.0)
5
4
3
2
1
2012
2014
2016
2018
2020
2022
2024
2026
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EFA / Mix ex-ante)
STRATEGY
BH GLD
BH EFA
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.82)
Equity (base = 1.0)
Fecha
3.5
3.0
2.5
2.0
1.5
1.0
2012
2014
2016
2018
2020
2022
2024
2026


Equity curves: Strategy vs Buy&Hold (GLD / QQQ / Mix ex-ante)
STRATEGY
BH GLD
BH QQQ
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.67)
Equity (base = 1.0)
Fecha
12
10
8
6
4
2
2012
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2018
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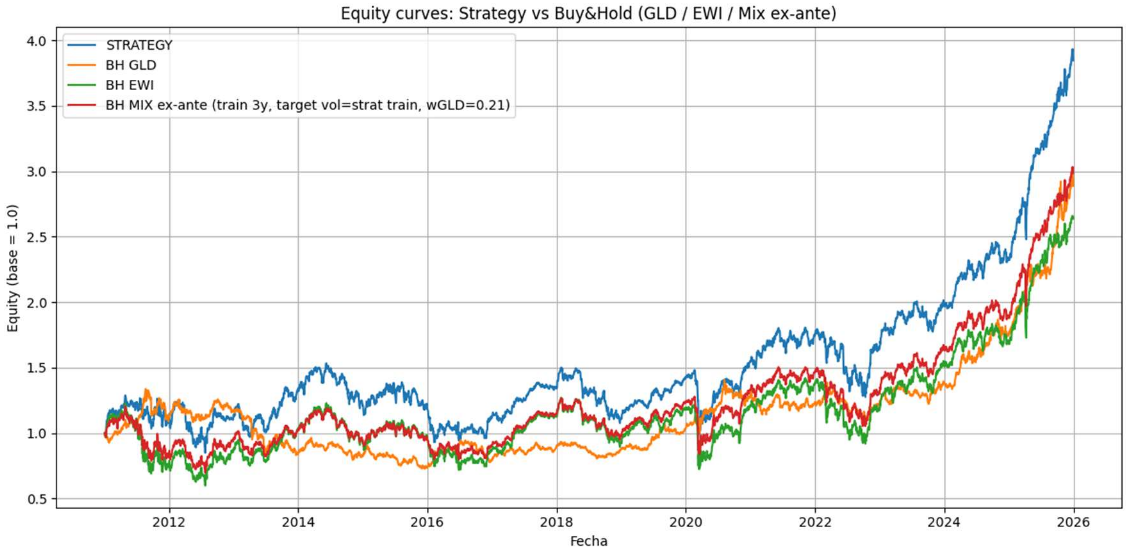
2022
2024
2026


Equity curves: Strategy vs Buy&Hold (GLD / EWI / Mix ex-ante)
STRATEGY
BH GLD
BH EWI
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.21)
Equity (base = 1.0)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
2012
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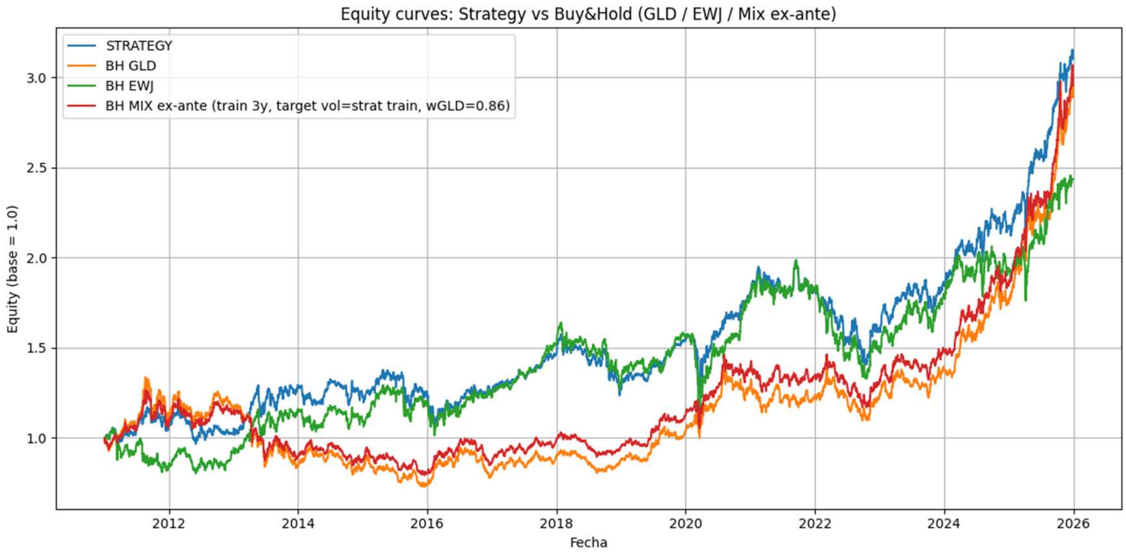
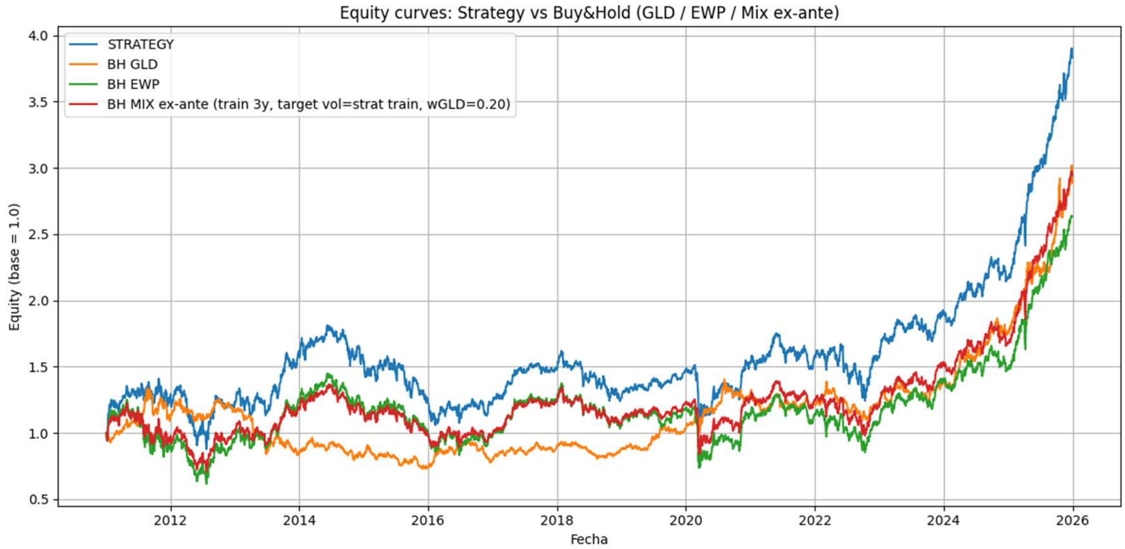
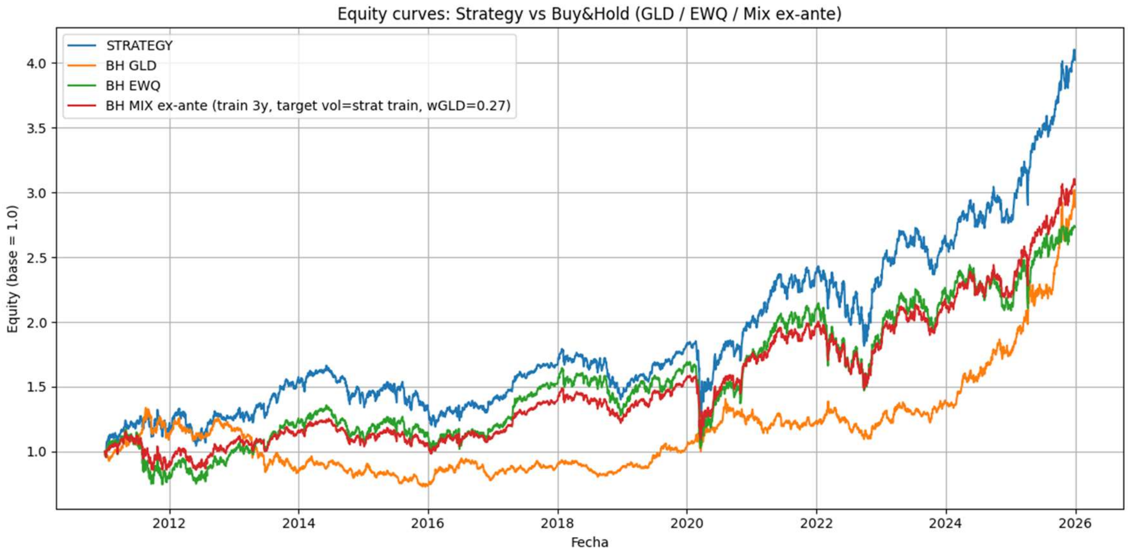
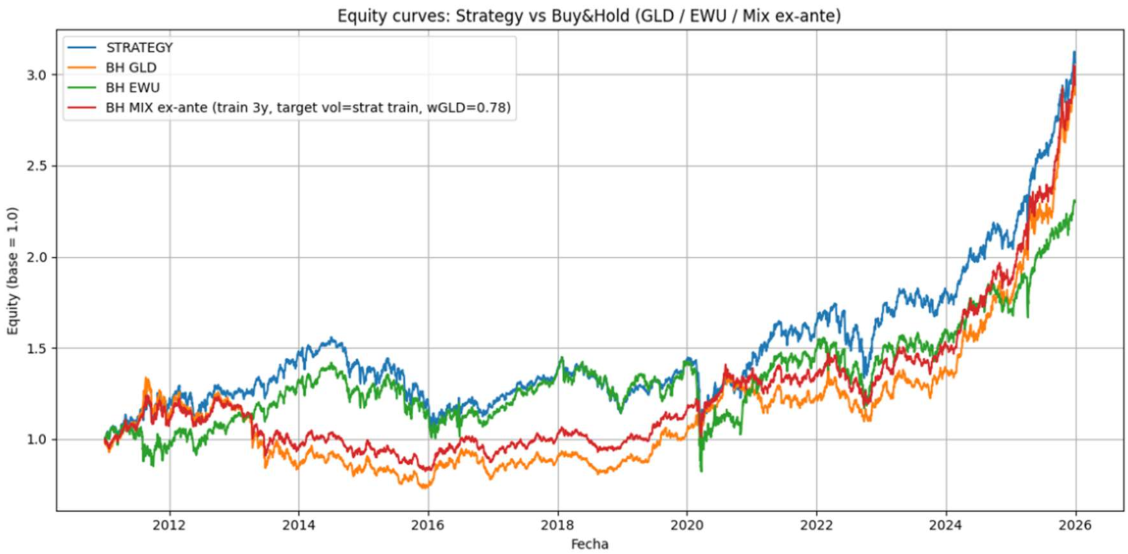
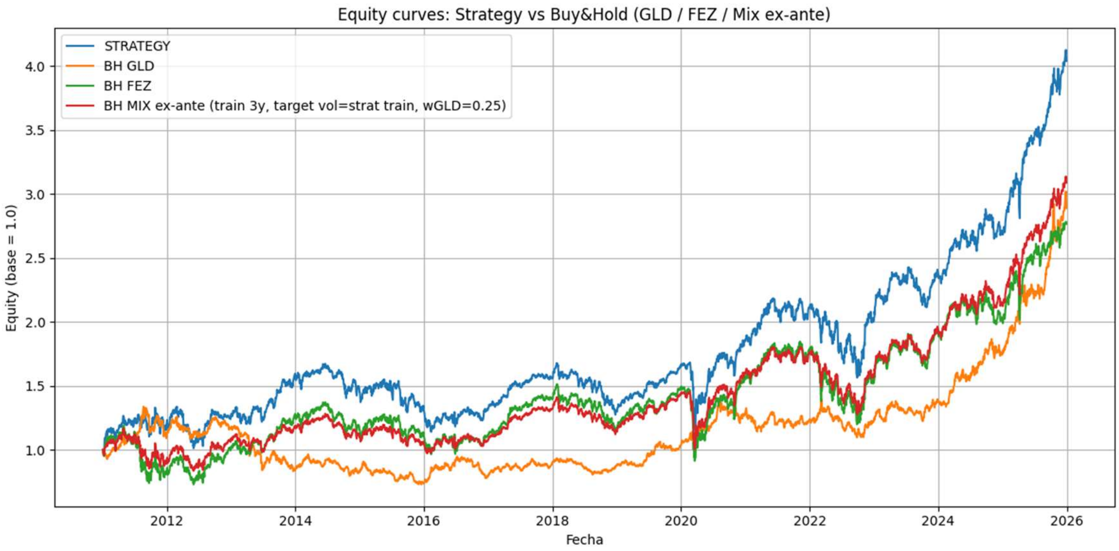
2020
2022
2024
2026
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EWJ / Mix ex-ante)
STRATEGY
BH GLD
BH EWJ
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.86)
Equity (base = 1.0)
1.0
1.5
2.0
2.5
3.0
2012
2014
2016
2018
2020
2022
2024
2026
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EWP / Mix ex-ante)
STRATEGY
BH GLD
BH EWP
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.20)
Equity (base = 1.0)
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EWQ / Mix ex-ante)
STRATEGY
BH GLD
BH EWQ
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.27)
Equity (base = 1.0)
Fecha

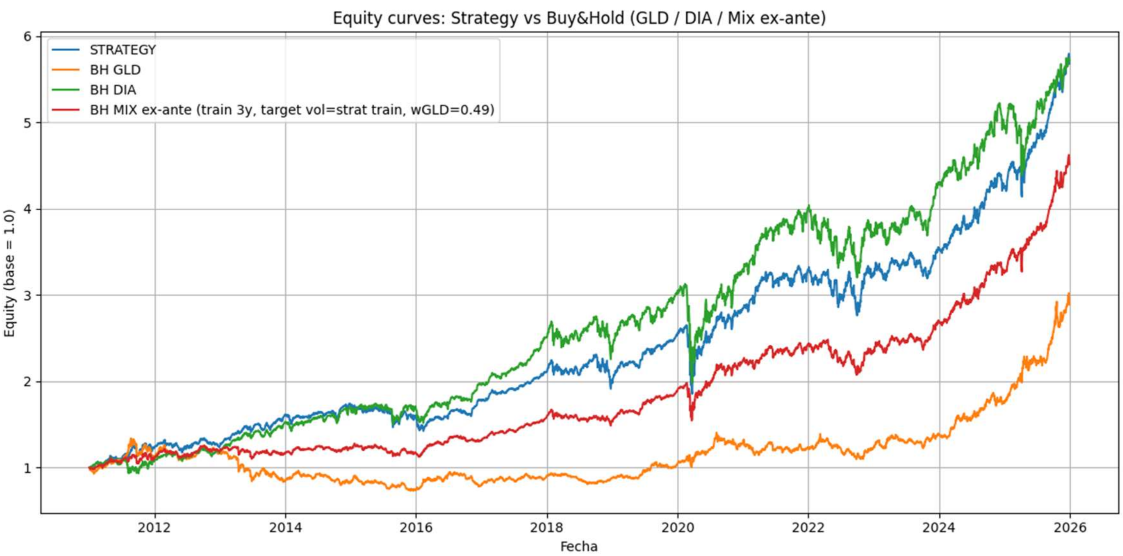

Equity curves: Strategy vs Buy&Hold (GLD / EWU / Mix ex-ante)
STRATEGY
BH GLD
BH EWU
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.78)
Equity (base = 1.0)
Fecha

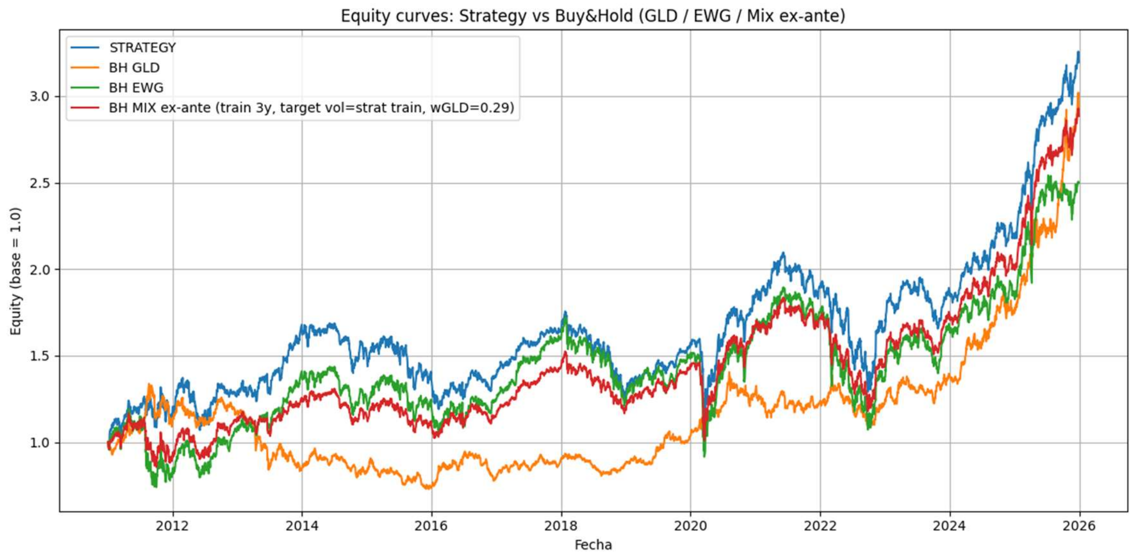


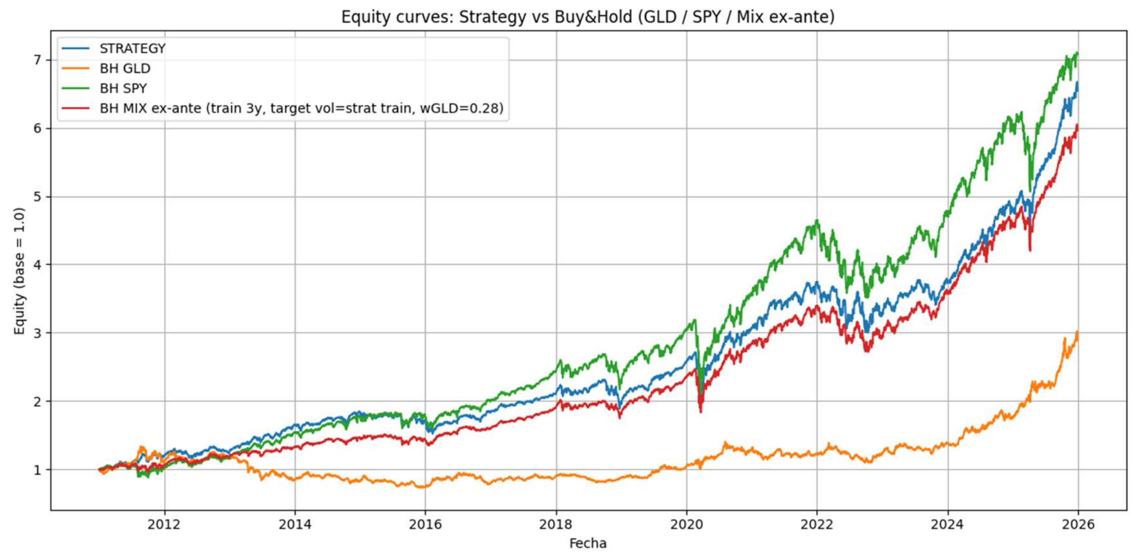

As can be seen, the model works like a charm. It manages to combinate two assets and overperforms each of them individually, as well as a benchmark constructed using both assets (without using leverage, of course). The exception is Nasdaq (QQQ). A pre-model also managed it with Nasdaq, but since it did way too many transactions, the pre-model was adapted to avoid some microtransactions. Therefore, some performance was lost, but in exchange the model became more realistic to use.

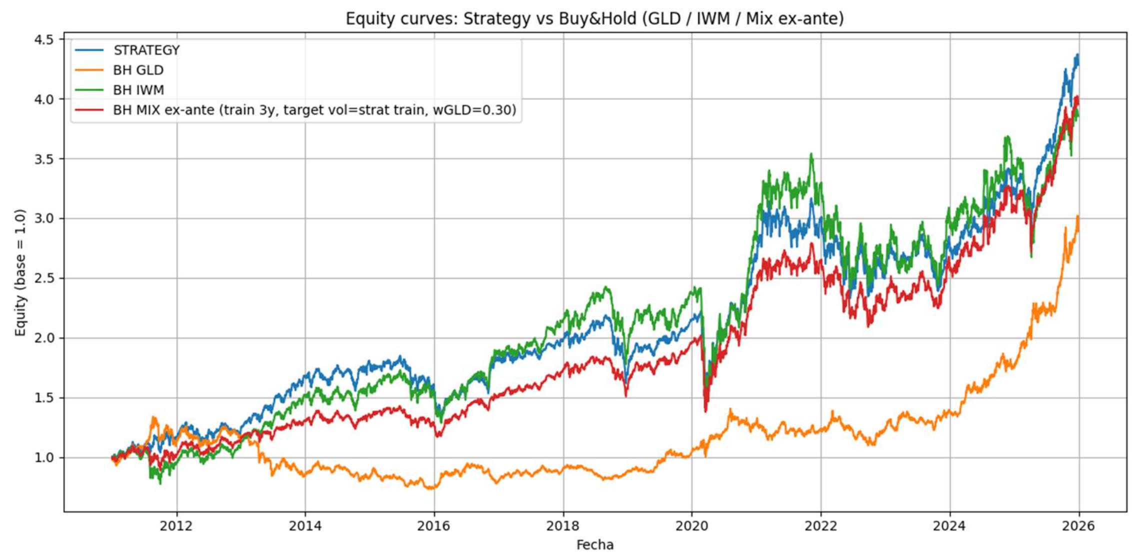
As will be seen later, the model has an edge over the market. However, it is not realistic since transaction costs are not considered.

The next step is obvious: considering transaction costs. Therefore, to simplify the model, a constant fee was considered for each transaction. In reality, costs are more complex, but for our purposes, this will do. Results were the next:




Equity curves: Strategy vs Buy&Hold (GLD / EWG / Mix ex-ante)
STRATEGY
BH GLD
BH EWG
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.29)
Equity (base = 1.0)
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / SPY / Mix ex-ante)
STRATEGY
BH GLD
BH SPY
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.28)
Equity (base = 1.0)
Fecha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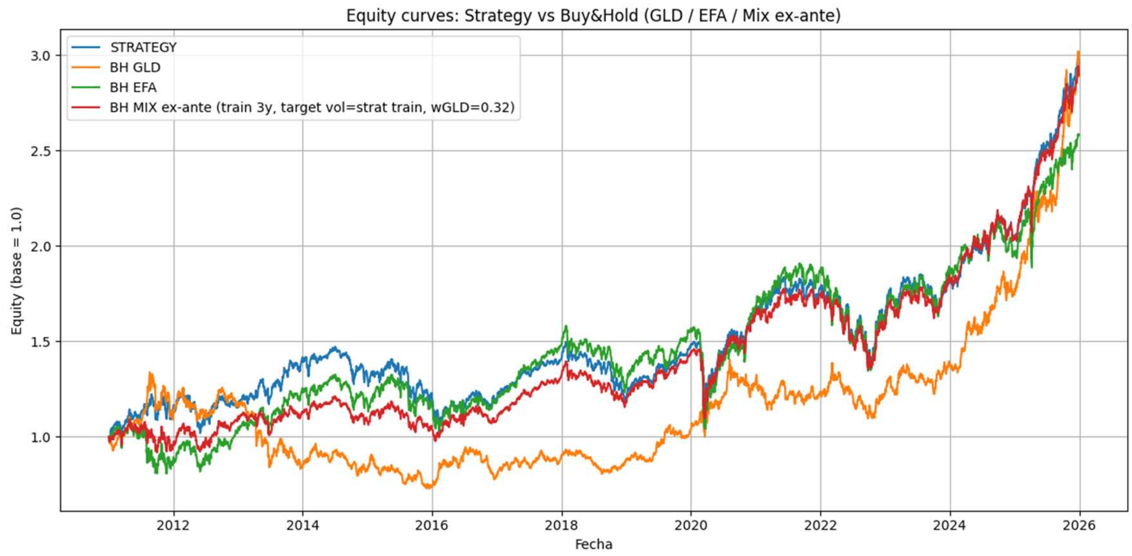
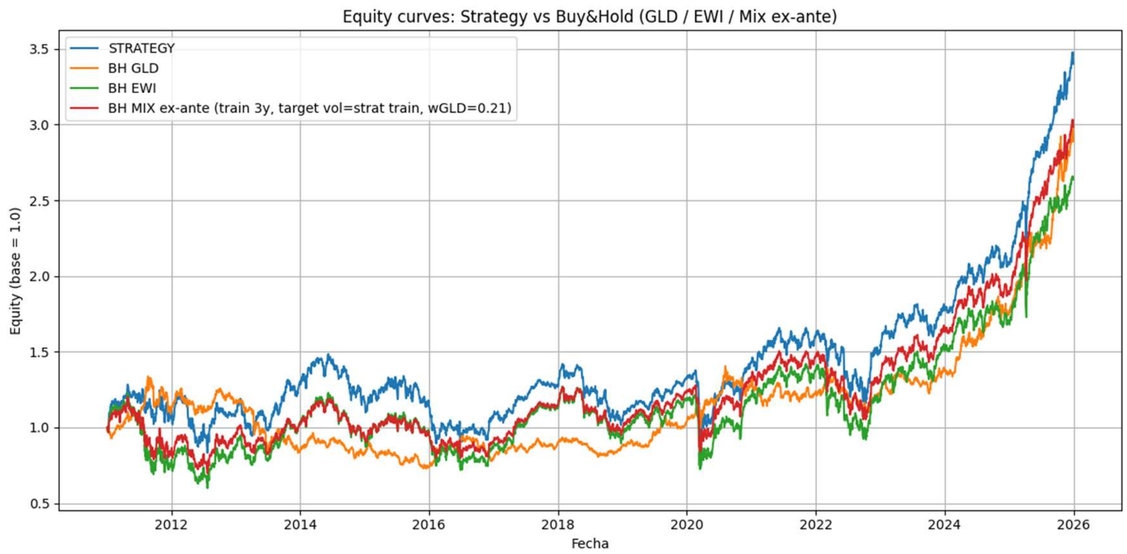
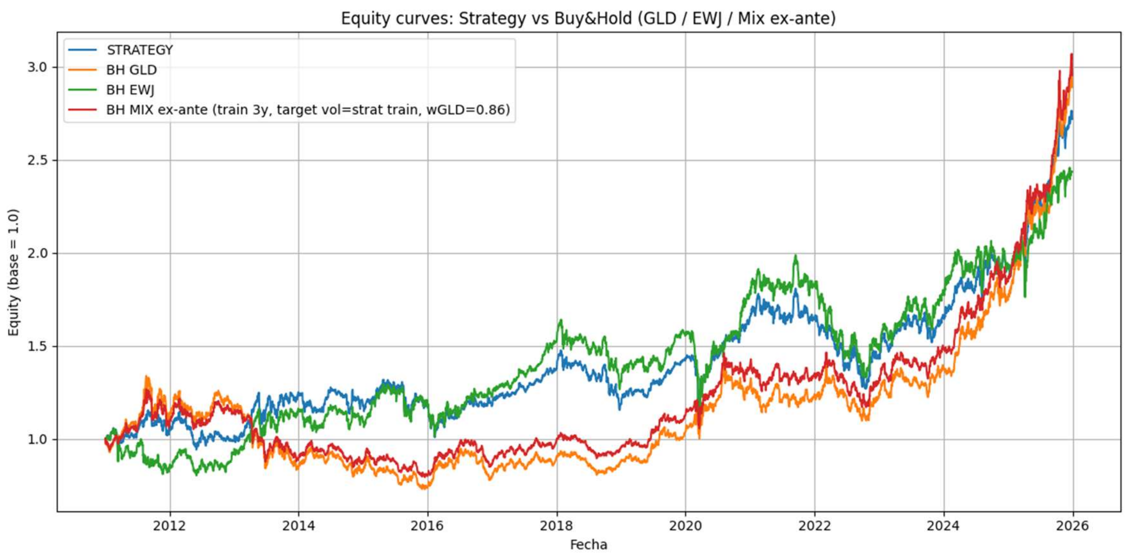
STRATEGY
BH GLD
BH IWM
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.30)
Equity (base = 1.0)
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EFA / Mix ex-ante)
STRATEGY
BH GLD
BH EFA
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.32)
Equity (base = 1.0)
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EWI / Mix ex-ante)
STRATEGY
BH GLD
BH EWI
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.21)
Equity (base = 1.0)
Fecha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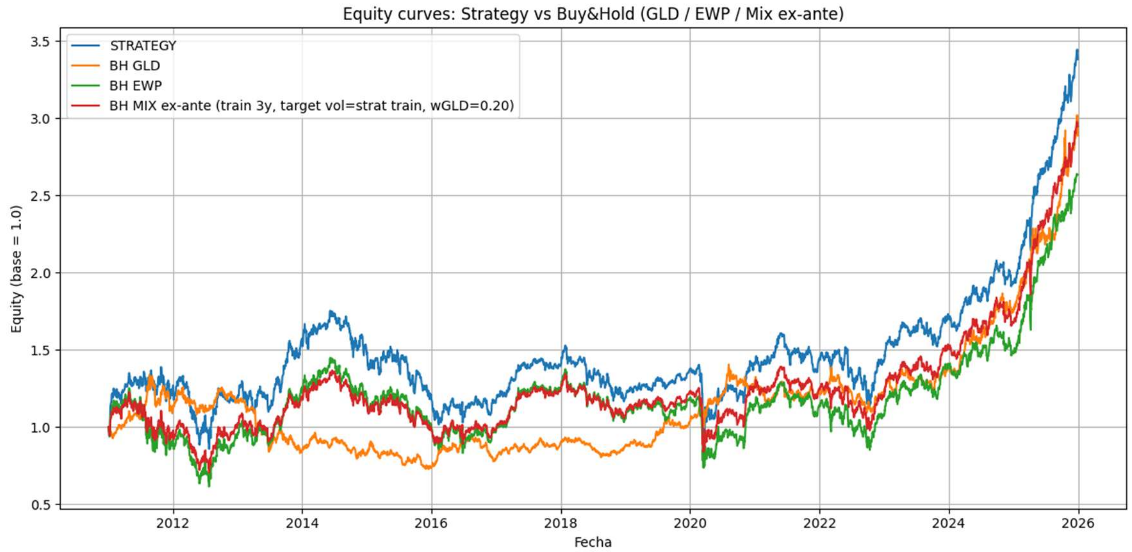
Equity (base = 1.0)
Fecha


Equity curves: Strategy vs Buy&Hold (GLD / EWP / Mix ex-ante)
STRATEGY
BH GLD
BH EWP
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.20)
Equity (base = 1.0)
Fecha
0.5
1.0
1.5
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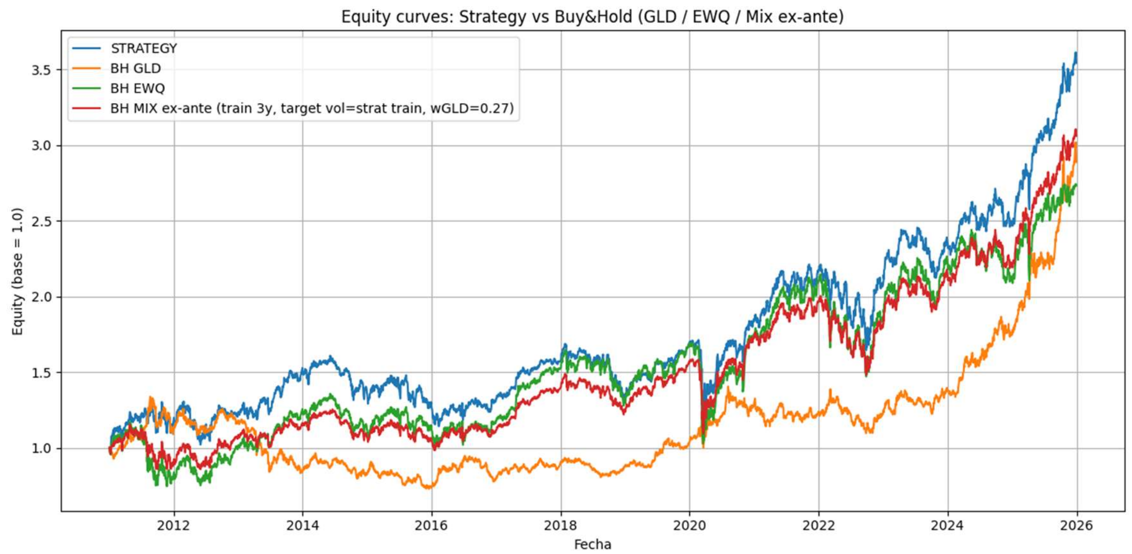
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STRATEGY
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BH EWQ
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.27)
Equity (base = 1.0)
Fecha
1.0
1.5
2.0
2.5
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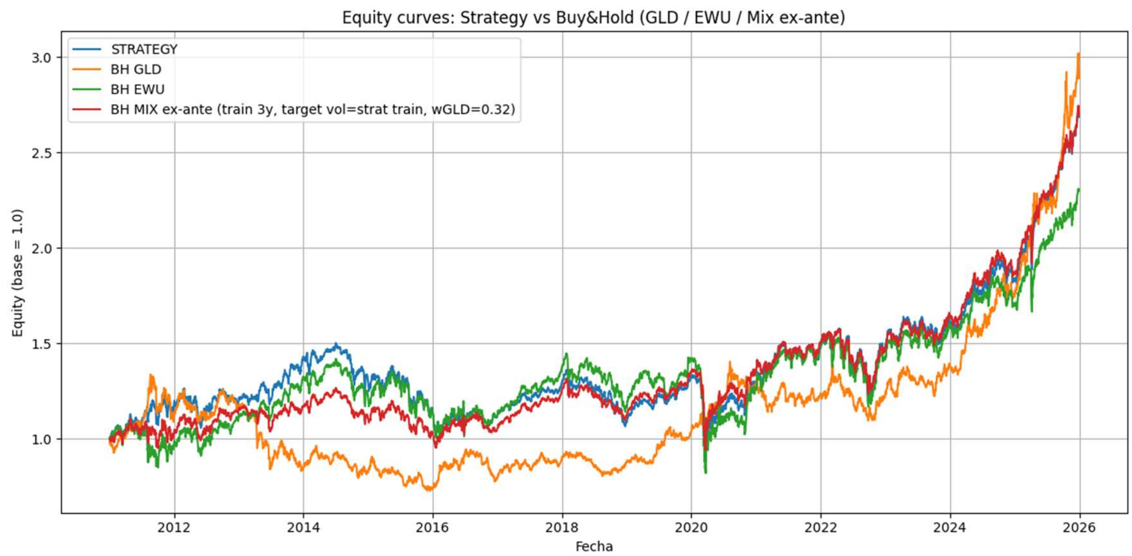
Equity curves: Strategy vs Buy&Hold (GLD / EWU / Mix ex-ante)
STRATEGY
BH GLD
BH EWU
BH MIX ex-ante (train 3y, target vol=strat train, wGLD=0.32)
Equity (base = 1.0)
Fecha
1.0
1.5
2.0
2.5
3.0
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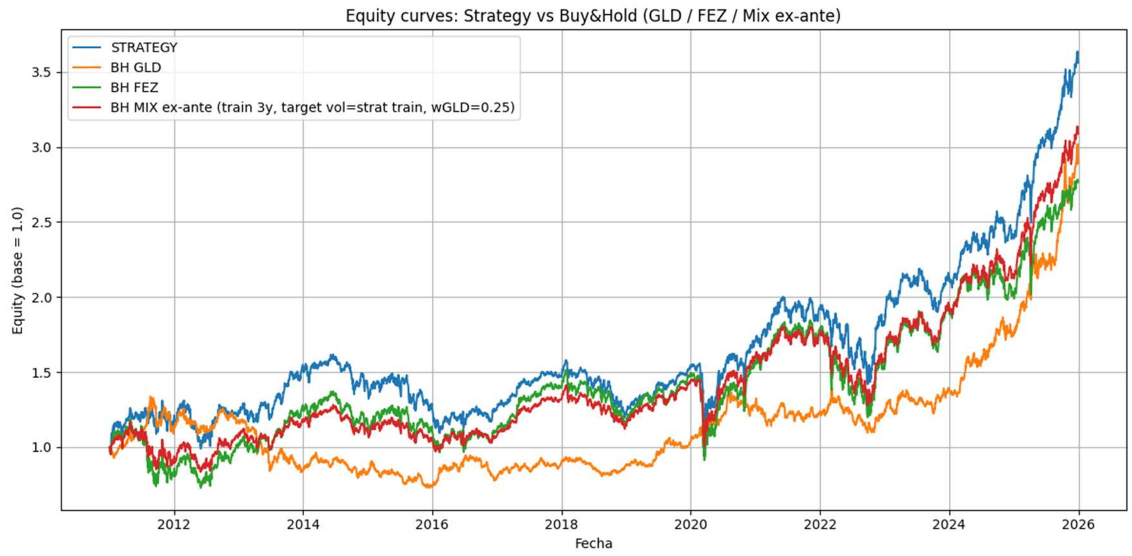


As can be seen, the edge, if still existing, is much smaller than before.

## Results analysis and comparison

Returns were analysed using multi-factor linear regression. For this, it is assumed that the strategy return can be written as a linear combination of the benchmark returns plus a constant term. That constant term is called **alpha**, and it represents the part of the return that cannot be explained by simple exposure to the benchmarks.

Therefore, alpha represents the edge over the market.

$$returns = \alpha + \beta_1 * returns_{GLD} + \beta_2 * returns_{ETF} + \varepsilon$$

$\varepsilon$ represents the residual return, the part not explained by the regression and associated with randomness.

These were the results on the linear regression. T-stat measures how large an estimated coefficient is relative to the uncertainty in its estimation. A large absolute t-stat means the estimate is precise and unlikely to be driven by noise. The p-value translates the t-statistic into a probability. It measures how likely it would be to observe an alpha at least as extreme as the one estimated if the true alpha were actually zero. A low p-value means that such a result would be very unlikely to arise by chance, providing evidence that the strategy generates genuine excess returns.

| Pair (GLD &) | Annualized Alpha | Annualized Alpha (with costs) | t-stat | t-stat (with costs) | p-value | p-value (with costs) |
|---|---|---|---|---|---|---|
| **SPY** | 1.48% | 0.59% | 1.23 | 0.50 | 0.218 | 0.620 |
| **DIA** | 1.55% | 0.66% | 1.32 | 0.56 | 0.186 | 0.573 |
| **QQQ** | 1.07% | 0.21% | 0.85 | 0.17 | 0.397 | 0.868 |
| **IWM** | 1.56% | 0.71% | 1.21 | 0.55 | 0.227 | 0.580 |
| **EFA** | 1.39% | 0.50% | 1.22 | 0.44 | 0.223 | 0.658 |
| **EWJ** | 1.08% | 0.19% | 0.89 | 0.16 | 0.375 | 0.874 |
| **EWG** | 1.87% | 1.01% | 1.42 | 0.77 | 0.154 | 0.440 |
| **EWQ** | 2.10% | 1.25% | 1.66 | 0.99 | 0.097 | 0.323 |

| | | | | | | |
|---|---|---|---|---|---|---|
| **EWI** | 1.78% | 0.96% | 1.35 | 0.73 | 0.176 | 0.465 |
| **EWP** | 1.81% | 0.97% | 1.33 | 0.71 | 0.185 | 0.475 |
| **FEZ** | 2.04% | 1.19% | 1.59 | 0.93 | 0.111 | 0.351 |
| **EWU** | 1.23% | 0.35% | 1.09 | 0.31 | 0.274 | 0.755 |

On all 12 pairs, the annualized alpha is positive before and after transaction costs. This is significant. It demonstrates that the system adds value on a constant linear hedge of GLD and the secondary ETF independent of market region. This suggests that the system has a structural basis and does not necessarily involve favorable data.

In terms of economics, the raw alphas are significant, ranging between 1.0% and 2.1% per year on an annualized basis, and the largest alphas are evident in European ETFs (EWQ, FEZ, and EWG). Taking into account transaction costs, the alphas are reduced by between 40% and 60%, as expected in a scheme involving dynamic allocation, although they are still significant in certain cases (approximately 1%).

From a statistical point of view, however, the evidence is less robust. The absolute values of the t-statistics are mostly between 1.1 and 1.6, indicating p-values well beyond the significance levels. Only EWQ and FEZ come close to the margin of significance before costs (with p-values of about 10%), but none of the test results are significant at the 5% significance level. In contrast, costs reduce t-statistics below 1 for all pairs, while p-values increase significantly.

The existence of these costs diminishes alpha by a considerable margin as well as reduces statistical significance. It is thus clear that the interaction does capture the true underlying effects, yet the cost related to turnover does embed a considerable portion of the gross alpha. Notably, this does not imply anything negative about the underlying signal; instead, it only points out that it is very important to pay heed to implementation efficiencies.

The strategy has consistent positive alpha across markets, although although economically significant before costs, the results are not strongly statistically significant when proper transaction costs are taken into account, which indicates that the issue here is with the efficiency of implementation rather than the signal strength.