Simple Implementable Financial Policy Rules*

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Abstract

How important, for welfare, is the counter-cyclical capital buffer (CCyB) relative to other —higher and more permanent— bank capital requirements? While there is a better understanding of the effect of a-cyclical higher capital requirements on banks' resilience and credit supply, much less is known about the marginal effects of introducing a macroprudential counter-cyclical capital requirement. In this paper, we study and rank the welfare gains of introducing several simple and implementable financial policy (CCyB) rules that co-exist with monetary policy. We find that the institutional design of the financial-policy instruments matters. In particular, a zero lower bound on the CCyB interacts with its counter-cyclical nature and provides a rationale for a positive *neutral* level. We build our analysis based on a quantitative macro-banking model with two main frictions; nominal rigidities and financial frictions, which we estimate for Chile.

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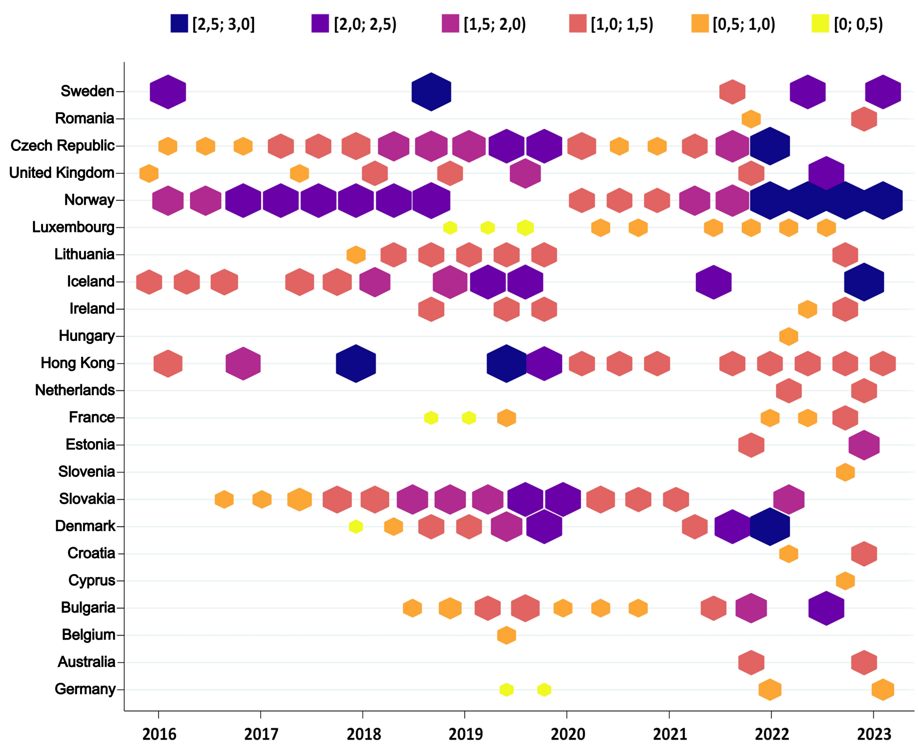
1 Introduction

The 2008 financial crisis put forward the importance of financial intermediation, mainly through banking, in the potential origination and amplification of shocks to the macroeconomy. This observation catalyzed both research on macro-financial linkages and a re-assessment of banking regulation. The latter materialized in the package of reforms we know as Basel III, with one of its main objectives being the incorporation of a system-wide approach to financial risk assessments, and financial policy, thereby explicitly introducing a macroprudential perspective to banks' capital regulation. Basel III introduces two buffers in this direction: the capital conservation buffer (CCoB) and the countercyclical capital buffer (CCyB) (Financial Stability Institute, 2019)[1]. While the CCoB has more automatic guidelines for its replenishment in case of loss-related draw downs, the CCyB can be activated and deactivated according to the the authority's decision. That is, the CCyB is a macroprudential tool. In this paper we examine the implications of different rules guiding this decision in terms of welfare and banks' resilience and emphasize the implications of institutional design on the adequacy of a positive neutral level of CCyB.

Figure 1: Countercyclical capital buffer activation across countries



Note.– This figure reports activation of countercyclical buffer (CCyB) by date and size of requirement. Each hexagon shows the current level of CCyB. No hexagon means deactivated CCyB. Source: Financial Stability Report CBC 2023-S1

Further, the experience from the Covid-19 pandemic suggests that there might be important differences between CCoB and CCyB usability. In particular, banks might be reluctant to exhaust CCoB (Basel Committee on Banking Supervision, 2022), and instead might want to comply with capital requirements deleveraging. In contrast,

[1]Both capital buffers must be met with Common Equity Tier 1 (CET1) capital only. The CCoB is meant to give banks and additional layer of usable capital when idiosyncratic losses are incurred. The CCyB is meant to be raised when system-wide risks, usually associated with high credit growth is perceived to become more important. Both buffers range from 0% to 2.5%.

a system-wide deactivation of the countercyclical capital buffer by instruction of the supervisor, would not attract adverse market reaction or stigma on any particular bank, and might better accomplish its countercyclical objective. Notably, before the Covid-19 pandemic many jurisdictions had activated the CCyB, and deactivated it in early 2020 (see Figure (1)). By the end of 2021, mostly the same economies started activating this buffer again, suggesting that its deactivation was useful during the worst moment of the sanitary crisis.

To comprehensively analyze the macroeconomic implications of different CCyB designs, we build a macro-banking model with two main inefficiencies, as in Carrillo et al. (2021). Monetary policy addresses inefficiencies from staggered pricing by monopolistic input producers, while financial policy addresses inefficiencies from financial frictions in the form of costly state verification. Drawing on the results of Carrillo et al. (2021) we abstract from a one-tool-for-two-objectives policy, and instead start from the Tinbergen rule. Our model includes both a monetary policy rule and a countercyclical capital requirement rule, and features three levels of default by different agents in the economy, including the banking sector, as in Clerc et al. (2014). Hence our model is rich enough to analyze the interaction of monetary and financial policy, yet parsimonious enough to calculate welfare of different policy regimes. In particular, our model is based on a simplified version of Calani et al. (2022), one of the main models used at the Central Bank of Chile. Notably, on the financial side, this model features financial frictions as in Bernanke et al. (1999) and Clerc et al. (2014), long-term debt as in Woodford (2001a), and a bank-related friction in which depositors do not price bank default risk at the margin, as in Mendicino et al. (2018) and Mendicino et al. (2020). Our model is more appropriate for small open economies with both monetary and financial policies, in which bank credit can be short- and long-term.

Using this quantitative model estimated with Chilean data, we explore several simple and implementable financial policy rules in terms of welfare differences (summarized in consumption equivalent terms). Notably, we find that simply following a credit-gap rule may not be optimal. Furthermore, we show that rules which following prices, specifically spread credit rates—commercial or weighted average between commercial and mortgage—generate higher welfare gains. It is because these spread react directly to the risk-socks, whose are the main source of volatility in our model, in line with Christiano et al. (2014).

By design, however, the CCyB ranges from 0 to 2.5 percent of risk weighted assets (RWA), which implies the inclusion of a occasionally binding constrain to model. For instance, if a shock which would be better addressed by deactivating the CCyB, hits the economy, and this instrument is currently not activated, then much of its benefits are not grasped. This mechanism provides a rationale for setting a positive neutral level in case deactivation is suddenly required. We explore this issue quantitatively.

Implementing a novel strategy to avoid this non-linearity we show that a positive level of neutral $CCyB$ raise the welfare gains for all rules. Using a *quadratic filter*—explained in detail in the section 4.2— we show that for all rules a positive level increase the welfare gains in a concave way. Therefore, it implies that there is an optimum level of neutral $CCyB$ which may be different between rules. Moreover, we highlight how critically depend the quantitative results of the optimum level of $CCyB$ in front to the regulatory capital requirements.

Both results, the best rules and the effectiveness of a positive level of neutral $CCyB$ are explained by three distortions. Firstly, since we have a deposits insurance the banks do not internalize the externality of their lending decisions because its increase the probability of default of other banks in line with Malherbe (2020). Secondly, there are nonlinear cost of default in the level of bank capital, so the cost of having a flexible higher capital in terms of the level of credits are compensate by the resilience to the shock and for the capacity to release the level of $CCyB$, as in Mendicino et al. (2018). This two mechanisms explain why the Financial Policy Rules increase the welfare. Finally, due to the nonlinearity in the design of the $CCyB$ and the increase of the marginal utility of the households in crisis, explain why a positive level of neutral $CCyB$ generates welfare gains for all rules. In additions, for that trade-off the welfare benefits of the neutral $CCyB$ are concave.

Related Literature. The literature on the effects of banks' capital requirements on financial and real variables has grown significantly in recent years, in tandem with the number of countries adopting and implementing capital regulation and the availability of micro-data. However, most of the literature has emphasized the aggregate consequences of higher levels of capital requirements. The main trade-off of a higher a-cyclical capital requirements is lower systemic risk —measured as banking sector default probability— and reduced activity in credit, leading to lower economic activity (Van den Heuvel, 2008; Clerc et al., 2014; Mendicino et al., 2018, 2020). Our paper shares this main feature, but focuses on cyclical considerations of capital regulation, i.e., the design of a CCyB rule and its macroeconomic effects. Thus, our paper is more related to Carrillo et al. (2021) and Malherbe (2020). We explore different implementable, simple, policy rules in terms of their welfare implications, exploring the relationship with monetary policy.

The document is structured as follows. In Section 2 we present a detailed description of the theoretical structure of the model. Section 3.2 describes the estimation of the model, the calibration, the choice of priors and presents the results. Section 4 presents the results. Section 5 concludes.

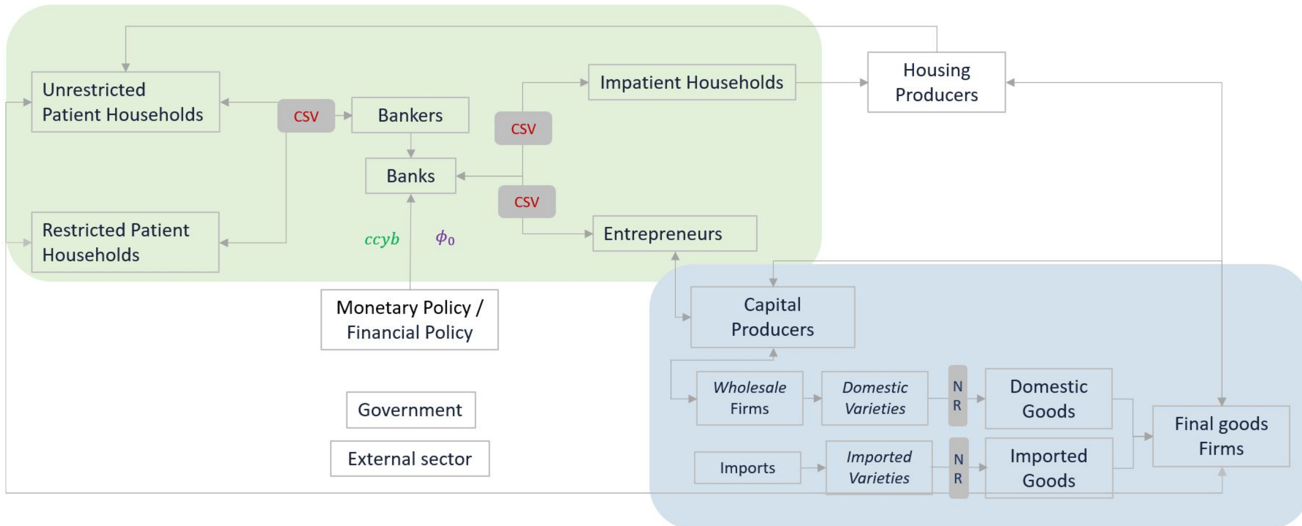
2 A Small Open Economy Model with Nominal and Financial Frictions

Our aim is to study the implications of simple and implementable financial policy rules. Our analysis is based on a rich DSGE model featuring two main inefficiencies: nominal rigidities and financial frictions in the form of costly state verification (CSV). We introduce CSV as in Bernanke et al. (1999) in three layers of the model, following Clerc et al. (2014), to explicitly introduce the notion of default, notably banking-system default probability. We depart from Clerc et al. (2014) by enriching our model to incorporate sticky prices and a role for monetary policy, as much of our analysis builds on welfare implications from taming business-cycle volatility, for which the role of monetary policy is first-order relevant. Thus we can compare different specifications of financial policy rules at the margin, considering its interactions with monetary policy.

Figure (2) shows a sketch of agents and interactions in the model. Households are divided into two groups: patient and impatient, who in equilibrium, save and borrow respectively. Patient households can be "unrestricted", and have access to save in short and long-term assets, or "restricted", and be able to save only in short-term instruments[2]. Impatient households borrow resources from banks to finance housing purchases, subject to CSV

[2]This distinction follows from Andres et al. (2004) and Chen et al. (2012) to introduce market segmentation and preferred habitat

Figure 2: Graphical illustration of agents and frictions of the model



Note.– CSV stands for costly state verification and NR stands for nominal rigidities. Green box emphasizes the financial modules of the model, which are directly affected by Financial Policy. The blue box emphasizes the more standard New-Keynesian modules of the model, more directly affected by nominal rigidities and for which monetary policy is directly relevant.

and can thus default. Households negotiate their wages through unions. Entrepreneurs are the sole owners of productive capital, who finance their capital investment through banking loans, also subject to CSV. Bankers are the owners of bank equity, which in turn finance entrepreneurs and impatient households. From the production side, we introduce capital producers, housing-good producers, and productive firms related to the production of the final good. Wholesale firms produce domestic good varieties, which are combined with imported good varieties produced by importers. Final good producers combine domestic and imported goods. There is a monetary and financial authority besides a government with balanced fiscal budget.

There are two main sources of inefficiencies in this economy, nominal rigidities and financial frictions. Monetary policy and financial policy are motivated by these two. The aim of this paper is to characterize the aggregate and welfare effects of different financial policy rules. Next, we outline the main components of the model emphasizing those important to our results or distinctive in this model, leaving more standard components to be explained in detail in Appendix A

2.1 Households

Preferences depend on consumption of a final good (C_t), housing services from housing stocks (H_{t-1})—both relative to external habits—, and leisure. Households can differ in terms of their discount factor, being patient or impatient. Patient households can further be grouped into Restricted-Patient, and have access only to long-term assets, and Unrestricted-Patient who can access both short- and long-term assets. However, they can save in the long-term asset at a cost which is proportional to the ratio of their holdings of long-term instruments.

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as in Vayanos and Vila (2009)

In equilibrium (restricted and unrestricted) patient households save. Short term assets include one-period deposits in banks, one-period government bonds, and one-period foreign bonds denominated in US dollars. We model long-term debt as instruments that pay geometric-decaying coupons as in Woodford (2001a). Long-term bonds can be issued either by the sovereign or by banks.

Also, in equilibrium, impatient households borrow from banks to finance their purchases of housing goods, subject to a financial friction: costly state verification (CSV). As the project to be financed is the purchase of housing good, it serves as collateral and its price is subject to an idiosyncratic shock which can trigger default. In states when the amount of contracted debt is higher than the value of the house, households default. Indeed, one of the reasons that we choose to model financial frictions through CSV as in Bernanke et al. (1999), is that default is an object that exists in equilibrium, and can vary in time Clerc et al. (2014). These mortgage loans are long-term obligations subject to a small transaction cost in case households need to adjust their debt levels (renegotiation).

2.2 Entrepreneurs

Entrepreneurs are the sole owners of productive capital K_t, which they rent to firms for the production of intermediate goods. They live two periods. In the second period they draw utility from transferring part of their wealth to households as dividends and leaving bequest to the next generation of entrepreneurs (initial net worth). This implies that entrepreneurs will not save their way out of requiring external financing from banks[3].

In the first period, entrepreneurs receive the bequests from the previous generation N_t^e, and maximize expected second period wealth, Ψ_{t+1}^e, by choosing purchases of capital at nominal price Q_t^K, and simultaneously the amount of commercial borrowing L_t^F from commercial banks (F-banks, henceforth).

$$Q_t^K K_t = N_t^e + L_t^F \tag{1}$$

Borrowing is also subject to CSV. After deciding the level of investment K_t in period one, entrepreneurs receive an idiosyncratic shock ω_{t+1}^e to the efficiency units of capital in period two, which affects their ability to pay their debt to banks[4]. This shock is only observable to entrepreneurs. Banks can verify if the reported ω_{t+1}^e is true at a cost μ. If the entrepreneur honors her debt she pays pre-set amount $R_t^F L_t^F$. If she defaults, the bank pays the verification cost and seizes all capital. This lending contract is a standard-debt-contract. It induces truth-telling from the entrepreneur and minimizes the verification cost.

Then, second period entrepreneur's wealth is the proceeds from renting capital R_{t+1}^k and selling depreciated capital at price Q_{t+1}^K, minus debt repayment, only if this difference is positive.

$$\Psi_{t+1}^e = \max\left[\omega_{t+1}^e \left(R_{t+1}^k + (1-\delta_K)\, Q_{t+1}^K\right) K_t - R_t^L L_t^F,\ 0\right] \tag{2}$$

[3]This part of the model follows closely Clerc et al. (2014) who also model a two-period living entrepreneur subject to CSV as in Bernanke et al. (1999)

[4]The ω_{t+1}^e is assumed to be log-normal as in Bernanke et al. (1999). More details in Appendix A

Limited liability defines a threshold $\overline{\omega}^e_{t+1}$ for ω^e_{t+1}, below which the entrepreneur defaults. This conveniently defines a default probability $PD^e_t = F_e(\bar{\omega}^e_t)$ for commercial loans.

In equilibrium the profitability of the project is split between lender and borrower. The share of the gross return that goes to the entrepreneur is $[1 - \Gamma_e(\bar{\omega}^e_{t+1})]$, and the share of gross return that goes to the bank is $\Gamma_e(\bar{\omega}^e_{t+1})$. Banks subtract from this share, the verification costs stemming from non-performing loans, $\left(\bar{\omega}^e_{t+1}\right)$. Then, their net share of return is $\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)$. We can re-write equation (2) using this notation and the accounting identity (1), and write the problem of the entrepreneur in t as

$$\max_{\bar{\omega}^e_{t+1}, K_t} \mathbb{E}_t\left\{\Psi^e_{t+1}\right\} = \mathbb{E}_t\left\{\left[1 - \Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\}, \quad s.t.$$

$$\mathbb{E}_t\left\{\left[1 - \Gamma_F\left(\bar{\omega}^F_{t+1}\right)\right]\left[\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\} \geq \bar{\rho}_t \phi_F L^F_t, \tag{3}$$

where equation (3) is the participation constraint for banks. The first term in brackets in the LHS of (3) will become clearer later, but it comes from the fact that another participation constraint applies also to the owner of bank equity–the banker. The rest of the LHS is the net return for lending to entrepreneurs. The RHS of the same equation is the demanded return $\bar{\rho}_t$ for commercial bank equity $E^F_t = \phi_F L^F_t$, where ϕ_F is the capital requirement for commercial banks.

2.3 Bankers and Banks

Bankers. Just as entrepreneurs, they live two periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital. In the first period, the banker receives a bequest N^b_t from the previous generation and must distribute it between two types of banks: banks specializing in corporate loans (F banks) and in housing loans (H banks). Denote inside equity in each E^F_t and E^H_t, respectively. This allocation, together with realized return ρ^j_{t+1} on each j bank, determines second period total wealth,

$$\Psi^b_{t+1} = \rho^F_{t+1} E^F_t + \xi^{b,roe}_t \rho^H_{t+1}\left(N^b_t - E^F_t\right)$$

where $\xi^{b,roe}_t$ is a relative profit shock. As the banker chooses equity allocation in the first period, her problem is to maximize $\mathbb{E}_t\left\{\Psi^b_{t+1}\right\}$ which results in the following to hold:

$$\mathbb{E}_t\left\{\rho^F_{t+1}\right\} = \mathbb{E}_t\left\{\xi^{b,roe}_t \rho^H_{t+1}\right\} = \bar{\rho}_t$$

where $\bar{\rho}_t$ denotes banks' required expected gross rate of return on equity investment undertaken at time t.

In the second period the banker decide how to distribute his wealth Ψ^b_{t+1} between dividends to households and bequests N^b_{t+1} to the next generation.

Banks. Banks are projects that invest in credit portfolios, financed with internal equity of bankers and households deposits or holdings of (long-term) bank-bonds. In particular, the balance sheet of bank-F is given by

$$L_t^F = E_t^F + D_t^F$$

and balance sheet of banks of class H is given by

$$Q_t^L L_t^H = E_t^H + Q_t^{BB} BB_t$$

Capital requirements are given by $E_t^F \geq \phi_F L_t^F$ and $E_t^H \geq \phi_H Q_t^L L_t^H$, which are binding in equilibrium. We assume a continuum of banks of class $j = \{F, H\}$, with ex-post profits Π_{t+1}^j defined by:

$$\Pi_{t+1}^F = \max\left[\omega_{t+1}^F \tilde{R}_{t+1}^F L_t^F - R_t^D D_t^F, 0\right], \qquad \Pi_{t+1}^H = \max\left[\omega_{t+1}^H \tilde{R}_{t+1}^H Q_t^L L_t^H - R_{t+1}^{BB} Q_{t+1}^{BB} BB_t, 0\right]$$

where $\tilde{R}_{t+1}^j$ is the realized return on a well-diversified portfolio of loans to entrepreneurs or households, R_t^D is the interest rate on deposits, and Q_t^L and Q_t^{BB} are the price of long-term mortgage loans and bank bonds, respectively. Also, let ω_{t+1}^j denote an idiosyncratic portfolio return shock, which is i.i.d across banks of class j with a cdf of $F_j(\omega_{t+1}^j)$ and pdf $f_j(\omega_{t+1}^j)$. Limited liability for bankers defines thresholds $\bar{\omega}_{t+1}^j$:

$$\bar{\omega}_{t+1}^F \equiv \frac{R_t^D D_t^F}{\tilde{R}_{t+1}^F L_t^F}, \qquad \bar{\omega}_{t+1}^H \equiv \frac{R_{t+1}^{BB} Q_{t+1}^{BB} BB_t}{\tilde{R}_{t+1}^H Q_t^L L_t^H}$$

Similar to households and entrepreneurs, let $\Gamma_j(\bar{\omega}_{t+1}^j)$ denote the share of gross returns that goes to the creditor; in this case, depositors or bond holders, implying that $[1 - \Gamma_j(\bar{\omega}_{t+1}^j)]$ is the share that the bankers will keep as profits. We also define $G_j(\bar{\omega}_{t+1}^j)$ as the share of defaulting j banks, and thus $\mu_j G_j(\bar{\omega}_{t+1}^j)$ is the total verification cost of bank j default.

Finally, we are in position to define the realized rate of return of equity invested in a bank of class j:

$$\rho_{t+1}^j = \left[1 - \Gamma_j\left(\bar{\omega}_{t+1}^j\right)\right] \frac{\tilde{R}_{t+1}^j}{\phi_j} \tag{4}$$

2.4 Capital and Housing goods producers

As in Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms. Both, owned by households. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. We depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment.

Capital goods. There is a continuum of competitive capital-producer firms who buy an amount I_t of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital. New units of capital are sold at price Q_t^K. As is usual in the literature, we consider quadratic investment adjustment costs in the accumulation of capital:

$$K_t = (1-\delta_K)\, K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right] \xi_t^i I_t$$

where γ_K controls the adjustment cost, and ξ_t^i is a shock to investment efficiency.

Housing goods. Housing good producers are subject to investment adjustment costs and time-to-build as in Kydland and Prescott (1982) and Uribe and Yue (2006). A continuum of competitive housing firm producers choose housing investment I_t^{AH} in period t, which will increase housing stock N_H periods later: the time it takes to build.[5] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before in interaction with adjustment costs,

$$H_t = (1-\delta_H)\, H_{t-1} + \left[1 - \frac{\gamma_H}{2}\left(\frac{I_{t-N_H}^{AH}}{I_{t-N_H-1}^{AH}} - a\right)^2\right] \xi_{t-N_H}^{ih} I_{t-N_H}^{AH} \tag{5}$$

where ξ_t^{ih} is a shock to housing investment efficiency. Time-to-build implies that firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I_t^H = \sum_{j=0}^{N_H} \varphi_j^H I_{t-j}^{AH}$$

Where φ_j^H (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfies $\sum_{j=0}^{N_H} \varphi_j^H = 1$ and $\varphi_j^H = \rho^{\varphi H} \varphi_{j-1}^H$.[6] The representative housing producer chooses how much to authorize in new projects I_t^{AH} in order to maximize the discounted utility of its profits.

2.5 Final good producing firms

The supply side of the economy is composed by different types of firms, all owned by the households. Monopolistically competitive unions act as wage setters, selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a continuum of monopolistically competitive firms producing different varieties j of a home good Y_{jt}^H, using wholesale good X_t^Z as input; a set of monopolistically competitive firms that import a homogeneous foreign good M_t to transform it into varieties, Y_{jt}^F; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, Y_t^H, one packing the imported varieties into a composite foreign good, Y_t^F, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y_t^C. This final good is purchased by households (C_t^P,C_t^I), capital and housing producers (I_t^K,I_t^H), and the government (G_t).

[5]Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

[6]Notice that $\rho^{\varphi H} > 1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H} < 1$.

Final goods. A representative final-goods firm demands composite home good X_t^H, and composite foreign goods X_t^F, and combines them according to the following technology:

$$Y_t^C = \left[\omega^{1/\eta}\left(X_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(X_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}} \tag{6}$$

where $\omega \in (0,1)$ controls home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The price of the final good is P_t, while P_t^H and P_t^F denote the prices of the home composite and foreign composite goods, respectively.

Home composite goods. A representative home composite goods firm demands all $j \in [0,1]$ varieties of intermediate home goods in amounts X_{jt}^H, and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} \tag{7}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. The firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ choosing input demands X_{jt}^H, subject to (7) and taking prices as given.

Intermediate Home Goods of Variety j. There is measure one of firms, that demand domestic wholesale goods X_t^Z and differentiate into j intermediate home good varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{8}$$

The firm producing variety j satisfies the demand from the home-composite producing firm Y_t^H, and has monopoly power for its variety. Given (8), the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. As every firm buys their input from the same wholesale market, all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{9}$$

Firm j chooses its price P_{jt}^H to maximize profits, taking marginal costs in as given. In setting prices, the firm are subject to Calvo-type nominal rigidities, whereby each period the firm can change its price optimally with probability $1-\theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0,1]$ and $1-\kappa_H$ respectively.

Wholesale Domestic Goods. A representative firm produces a homogeneous wholesale home good, combining capital K_{t-1} and composite labor $\tilde{n}_t$ according to the following technology

$$Y_t^Z = z_t K_{t-1}^{\alpha}\left(A_t \widetilde{n}_t\right)^{1-\alpha} \tag{10}$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t shock. Notably, the firm faces adjustment costs of labor. Profit maximization implies that the price of this wholesale good is equated to marginal cost.

Foreign composite goods. Like with home composite goods, a representative firm demands foreign goods of all $j \in [0,1]$ varieties in amounts X_{jt}^F and combines them into Y_t^F according to the following technology with $\epsilon_F > 0$.

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F \right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj \right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{11}$$

Intermediate foreign goods of variety j. Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad, and convert this good into varieties Y_{jt}^F that are sold domestically. Total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ co-integrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are common across varieties

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{12}$$

Producer of variety j has monopoly power for its variety. Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms. The firm can change its price optimally with probability $1 - \theta_F$, else it indexes its previous price according to a weighted product of past and steady state inflation.

The model then features inefficiencies due to staggered pricing by monopolistic input producers in two markets; the home and foreign intermediate goods markets. This nominal frictions motivate the existence of monetary policy, as in the benchmark NK model.

Wages. Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{13}$$

Differentiated labor $n_t(i)$ is supplied by a continuum of monopolistically competitive unions who set wages subject to the demand of labor-packing firms, and to nominal rigidities à la Calvo. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i, t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1 - \wp_U)\, n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

Commodities. We assume the country receives an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

2.6 Fiscal and Monetary policies

Fiscal Policy. The government consumes an exogenous stream of final goods G_t, pays (through an insurance agency IA_t) for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households T_t^P, issues one-period bonds BS_t^G and long-term bonds BL_t^G, and receives revenue from commodity exports $\chi S_t P_t^{Co\star} Y_t^{Co}$. The government satisfies the following period-by-period constraint where sources of funds (left-hand side) equate uses of funds (right-hand side):

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \tag{14}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G.

Monetary Policy. In turn, following Garcia et al. (2019) monetary policy is follows a Taylor Rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\,\pi_t + \alpha_E \mathbb{E}_t\left\{\pi_{t+4}\right\}}{\pi_t^T}\right)^{\alpha_\pi} \left(\frac{GDP_t/GDP_{t-1}}{a}\right)^{\alpha_y}\right]^{1-\alpha_R} e_t^m \tag{15}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[7]

2.7 Financial Policy

This paper's main contribution to the literature is the examination of how financial policy impacts allocations, prices and ultimately welfare. Financial policy takes the form of counter-cyclical capital (CCyB) requirements. We explore different specifications for such a rule in this paper, among those that are simple and implementable. The CCyB rule depends on its own lag and some endogenous variable X_t as well as its expected value at some future *horizon*. We develop more on the exact functional forms, and explore the parameters governing this policy rule in section 4.

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right) = \left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1} \left(\frac{(1-\alpha_E)X_t + \alpha_E \mathbb{E}(X_{t+horizon})}{X}\right)^{\theta_2} e_t^{req} \tag{16}$$

2.8 Rest of the world

Real exchange rate. Foreigners demand both, the home composite goods and the domestic commodity. The structure of the foreign economy is identical to the domestic economy, but the latter is assumed to be small relative to the foreign economy. This implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. There are no transaction costs or other barriers to trade, so the law of one price holds separately for home composite

[7] We do not need a time-varying target, so we will set it to a constant.

goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$. The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t} \frac{mc_t^F}{\xi_t^m} \tag{17}$$

Interest rate. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position, expressed as a ratio of nominal GDP, as in

$$R_t^\star = R_t^W \exp\left\{ -\frac{\phi^\star}{100} \left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b} \right) \right\} \xi_t^R z_t^R \tag{18}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

2.9 Market clearing and aggregation

This is a large model with many market clearing conditions: final gooods, intermediate goods, factor markets and financial asset markets.

Goods markets. In particular, markets must clear for goods,

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + {}^{\Upsilon_t}/{}_{P_t} \tag{19}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and adjustment costs.

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{20}$$

$$Y_t^F = X_t^F \tag{21}$$

while in the market for home and foreign varieties we have,

$$Y_{jt}^H = X_{jt}^H, \qquad \forall j$$

$$Y_{jt}^F = X_{jt}^F, \qquad \forall j$$

By the same token, in the market for the wholesale domestic good, we have $Y_t^Z = X_t^Z$. Finally, in the market for housing, demand from both households must equal supply from housing producers $H_t = H_t^P + H_t^I$

Factors of production. Labor and capital markets must also clear

Financial Assets. Deposits demand by banks and supply by patient households must equate

$$D_t^F = D_t^{Tot} \tag{22}$$

Similarly, the aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \tag{23}$$

$$BS_t^{Pr} + BS_t^G = 0 \tag{24}$$

where BL_t^{CB} is an exogenous process denoting long-term bond purchases by the Central Bank.

Aggregate demand. GDP is defined as the sum of domestic absorption Y_t^C and trade balance, with nominal trade balance defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{25}$$

Real GDP, in turn, is defined as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP, Y_t^{NoCo}, is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t \left(C_t^P + C_t^I + I_t + I_t^H + G_t \right) + TB_t \tag{26}$$

Note that by combining (26) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods,(19), (21) and (20), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

Taking stock. The purpose of this brief sketch of the model is to inform the reader of the main structure and frictions/inefficiencies present in the model, not a full description of all equlibrium conditions and all details. The interested reader is referred to Appendix A which documents in detail the full model used in this paper, equilibrium conditions in stationary form, and the computation of the steady state.

3 Parameterization strategy and estimation results

The model parameters are calibrated and estimated. The calibrated parameters include those characterizing model dynamics for which we have a data counterpart, those drawn from related studies, and those chosen to match long-run ratios for Chile. In particular, we follow closely the calibration strategy from Garcia et al. (2019) and Clerc et al. (2014), as the models described there form the basis of this paper's framework. We estimate the non-calibrated parameters using Bayesian techniques as discussed below.

3.1 Calibration

Table (1) presents the values of the parameters related to the real sector of the economy that are either chosen from previous studies in the relevant literature or chosen in order to match exogenous steady state moments. The value of the parameters α, α_E, β_U, β_R, χ, ϵ_F, ϵ_H, ϵ_W, ω and π^T are taken from Garcia et al. (2019). We assume that the housing capital depreciation rate, δ_H is equal to the productive capital depreciation rate, δ_K, whose value is taken from Adolfson et al. (2013). The value for β_I is taken from Clerc et al. (2014).

Table 1: Calibration, Real Sector

Parameter	Description	Value	Source
α	Capital share in production function	0.34	Garcia et al. (2019)
α_E	Expected Inflation weight in Taylor Rule	0.50	Garcia et al. (2019)
α^{BSG}	Short-term govt. bonds as percentage of GDP	-0.40	Data: 2009-2019
α^{BLG}	Long-term govt. bonds as percentage of GDP	-4.50	Data: 2009-2019
β_U, β_R	Patient HH Utility Discount Factors	0.99997	Garcia et al. (2019)
β_I	Impatient Utility HH Discount Factor	0.98	Clerc et al. (2014)
δ_K	Capital Annual depreciation rate	0.01	Adolfson et al. (2013)
δ_H	Housing Annual Depreciation rate	0.00529	Same as capital depreciation
ϵ_F	Elasticity of substitution among foreign varieties	11	Garcia et al. (2019)
ϵ_H	Elasticity of substitution among home varieties	11	Garcia et al. (2019)
ϵ_W	Elasticity of substitution among types of workers	11	Garcia et al. (2019)
ϵ_τ	Convergence speed towards SS Gov debt	0.10	Normalization
N_H	Time-to-build periods in housing goods	6	CBC 2018S2 Financial Stability Report
κ	Coupon discount in housing loans	0.975	10 years duration of loan contract
κ_{BL}	Coupon discount in long term government bonds	0.975	10 years bond duration
κ_{BB}	Coupon discount in long term banking bonds	0.975	10 years bond duration
π^T	Annual inflation target of 3%	$1.03^{1/4}$	Garcia et al. (2019)
$\rho_{\varphi h}$	Spending profile for long term housing investment	1	Even investment distribution
σ	Log Utility	1	Garcia et al. (2019)
υ	Strength of households wealth effect	0	No wealth effect
χ	Government share in commodity sector	0.33	Garcia et al. (2019)
ω	Home bias in domestic demand	0.79	Garcia et al. (2019)
$\wp_U$	Fraction of unrestricted patient households	0.70	Chen et al. (2012)
ω_{BL}	Ratio of long term assets to short assets	0.822	Chen et al. (2012)

The parameters that set the steady state value of short term and long term government bonds as a percentage of GDP, α^{BSG} and α^{BLG}, respectively, were calculated from data obtained from Depósito Central de Valores (DCV).[8] The parameters that determine the coupons' geometric decline of the long term housing debt, κ, and government

[8]DCV is an entity that registers ownership of financial instruments take place in several exchange markets.

bonds, κ_{BL}, are set so their duration is 10 years. The duration of the bank bonds, κ_{BB}, is set to 5 years.

The value used for the time that takes a house to be built, N_H is taken from the second semester of 2018 Financial Stability Report (FSR), equal to 6 quarters in order to match the average length of construction projects. We also assume an even investment spending profile for housing capital, consistent with a value of 1 for $\rho_{\varphi h}$. Following Garcia et al. (2019), we set the value of the parameter that determines the strength of the wealth effect, υ, to 0, to avoid undesired dynamics in the labor market.

For the calibration of the parameters related to the financial sector, shown in Table (2), the values of χ_b, χ_e, γ_{bh}, γ_d, μ_e, μ_F, μ_H and μ_I come from Clerc et al. (2014). The values for the parameters related to bank capital requirements, ϕ_F and ϕ_H, are set as the ratio between the average level of TIER I capital of over the risk weighted assets of the banking system from the year 2000 to the year 2020. In particular, we calculate 4.3% excess of TIER I capital in addition to legal 9.75%. For corporate banks we assume 100% weight in corporate loans, while for housing bank we assume 60% weight in housing loans.

Table 2: Calibration, Financial Sector

Parameter	Description	Value	Source
χ_b	Banks dividend policy	0.04	Clerc et al. (2015)
χ_e	Entrepreneurs dividend policy	0.05	Clerc et al. (2015)
γ_{bh}	Household cost bank bonds default	0.10	Clerc et al. (2015)
γ_d	Cost of recovering defaulted bank deposits	0.10	Clerc et al. (2015)
ϕ_F	Bank Capital Requirement (RWA)	0.1683	Data (2000-2022)
ϕ_H	Bank Capital Requirement (RWA)	0.1183	Data (2000-2022)

3.2 Estimation

We provide all model details in Appendix A, summarize equilibrium conditions in Appendix B and compute the non-stochastic steady state in Appendix C and D.The parameters whose values are not calibrated are estimated using Bayesian methods. The data for the estimation, described in Table (3), includes 25 macroeconomic and financial variables from between 2001Q3 and 2019Q3. Data for the real Chilean sector is obtained from the Central Bank of Chile's National Accounts database, while prices and labor statistics are obtained from the National Statistics Institute (INE). Finally, local financial data is obtained from the Financial Markets Committee (CMF), and foreign data is obtained from Bloomberg. Variables regarding the real sector are log-differentiated with respect to the previous quarter. All variables are demeaned. Our estimation strategy also includes i.i.d. measurement errors for all local observables with the exception of the policy rate. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable, as is standard in the literature.

The posterior estimates are obtained using full information maximum likelihood estimation. To facilitate optimization, we scale shocks' standard deviations a similar order of magnitude in the posterior estimation (See Christiano et al. 2011). We choose the type of priors according to the related literature from distributions that have supported distributions consistent with the theoretical values expected for the parameters. In columns three, four and five of Table (4) we show the chosen prior distributions and prior distribution moments of the estimated

Table 3: Observable Data

Non Financial		Financial	
$\Delta \log Y_t^{NoCo}$	Non mining real GDP	R_t^L	Comercial Loans interest Rate
$\Delta \log Y_t^{Co}$	Copper real GDP	R_t^I	Housing Loans Interest Rate
$\Delta \log C_t$	Total Consumption	R_t^D	Nominal Interest Rate on Deposits
$\Delta \log G_t$	Goverment Consumption	R_t^{LG}	10 Year BCP Rate
$\Delta \log I_t^K$	Real Capital Investment	$\Delta \log L_t$	Housing and Corporate Loan
$\Delta \log I_t^H$	Real Housing Investment	ROE_t	Banks ROE
$TB_t/GDPN_t$	Trade Balance-GDP Ratio	R_t^*	LIBOR
$\Delta \log N_t$	Total Employment	Ξ_t^R	EMBI Chile
$\Delta \log WN_t$	Nominal Cost of labor	rer_t	Real Exchange Rate
π_t	Core CPI	R_t	Nominal MPR
$\Delta \log y_t^*$	Real External GDP	π_t^H	Household Price Index
π_t^*	Foreign Price Index		
π_t^M	Imports Deflactor		
π_t^{Co*}	Nominal Copper Price		
π_t^H	Housing Price Index		

Note.– This table summarizes the observable data time-series we feed the model for Bayesian estimation. The symbol $\Delta \log$ implies we take the log of the referred series, take first differences and subtract the mean. For all other variables we subtract the sample mean. Sources: INE, CBC, CMF, and Bloomberg

values of the deep parameters. The sixth and seventh columns of the same table show the posterior mean and the 95% interval of the estimation. On Table (5) we show the estimation priors and results of the parameters related to shock variables. For all autocorrelation coefficient we use a beta distribution while for the standard deviation we use a inverse gamma distribution.

Finally, Table (6) shows the importance of Risk Shock to explain the volatility of the main macro and financial variables. This results are in line with Christiano et al. 2014 in which the variance of the GDP, Investment, Credits and Credit Spread are explain in a same magnitude by a similar configuration of risk shock[9].

4 Results

In this section we dig deeper in the features of the model. First we assess how the CCyB operates and its transmission mechanism. Next we quantify the implications of different financial policy (FP) rules for welfare. In particular, we consider rules that are simple and implementable from the policy-maker perspective. Using the results of this analysis we dig deeper in the implementability in consideration of lower and upper legal bounds considered in Basel III; 0 and 2.5%, respectively.

4.1 Transmission mechanism

For this exercise we explore the effect of activation of the CCyB in the estimated model using the simplest possible financial policy rule; one in which the buffer is activated only as an exogenous shock. Later we will consider more

[9]In Christiano et al. 2014 they separated the effects of risk shocks in a unanticipated component, anticipated component and total effect, Therefore, since we us only an anticipated component, our results must be lived between the anticipated and total effects

Table 4: Estimation

Parameter	Description	Prior			Posterior	
		Dist	Mean	St Dev	Mean	95% Inter
α_π	Inflation weight in Taylor Rule	N	1.70	0.10	1.98	[1.77 2.07]
α_R	Lagged interest rate weight in Taylor Rule	β	0.85	0.025	0.75	[0.73 0.81]
α_W	Weight on past productivity on wage indexation	β	0.25	0.075	0.15	[0.05 0.27]
α_y	Output weight in Taylor Rule	N	0.25	0.075	0.11	[0.01 0.26]
η	Elasticity of subst. home and foreign goods	γ	1.00	0.25	0.96	[0.69 1.24]
$\eta_{\hat{C}}$	Elasticity of subst. consumption and housing goods	γ	0.15	0.03	0.13	[0.07 0.16]
η^*	Foreign demand elasticity of substitution	γ	0.2	0.11	0.19	[0.10 0.27]
γ_H	Housing investment adjustment cost parameter	γ	3.00	0.25	2.44	[2.55 3.40]
γ_K	Capital investment adjustment cost parameter	γ	3.00	0.25	2.83	[2.48 3.41]
γ_n	Labor adjustment cost parameter	γ	3.00	0.25	1.80	[1.45 2.13]
γ_L	Housing debt cost parameter	γ	0.12	0.01	0.12	[0.10 0.13]
κ_F	Weight on past inflation on foreign good indexation	β	0.50	0.075	0.66	[0.55 0.79]
κ_H	Weight on past inflation on home good indexation	β	0.50	0.075	0.76	[0.66 0.86]
κ_W	Weight on past inflation on wages indexation	β	0.85	0.025	0.84	[0.79 0.90]
ϕ^*	Country premium elasticity to NFA position	γ^{-1}	1.00	Inf	0.29	[0.19 0.44]
ϕ_c	Habit formation in good consumption	β	0.85	0.025	0.93	[0.87 0.91]
ϕ_{hh}	Habit formation in housing consumption	β	0.85	0.025	0.76	[0.75 0.86]
θ_F	Calvo param. foreign goods producers	β	0.50	0.075	0.71	[0.68 0.75]
θ_H	Calvo param. domestic goods producers	β	0.50	0.025	0.82	[0.79 0.83]
θ_W	Calvo param. wage setters	β	0.50	0.075	0.65	[0.59 0.71]
φ	Inverse Frisch elasticty	γ	7.50	1.50	7.2	[4.66 9.80]
μ_e	Monitoring cost of corporate loan default	β	0.30	0.05	0.51	[0.42 0.60]
μ_F	Monitoring cost of F bank default	β	0.30	0.05	0.20	[0.12 0.27]
μ_H	Monitoring cost of H bank default	β	0.30	0.05	0.25	[0.16 0.34]
μ_i	Monitoring cost of housing loan default	β	0.30	0.21	0.23	[0.13 0.30]
η_{ζ_L}	Term premium elasticity to relative bond liquidity	γ	0.15	0.03	0.14	[0.08 0.20]

Note.– This table shows the first two moments of the prior distribution of estimated parameters, together with posterior mean and 95% credible intervals, based on maximum likelihood estimation and the Laplace approximation.

realistic and interesting rules. For now, let us consider the following rule

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right)=\left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1}e_t^{CCyB} \tag{27}$$

where $0<\theta_1<1$ is the persistence governing the CCyB, and is set equal to $\theta_1=0.917$ which is equivalent to a rule of mean-life of 8 quarters. Since an activation of the CCyB generates effects with financial origins, we will start with the financial transmission channels and then explain how they affect the real economy.

Financial Transmission Channel. Capital requirements are shocked such that the effective level of extra capital amounts to one percent of RWA in the period following the regulation shock.

On impact, from the Figure (3) we can see that net worth of the banking system stay almost fix and the effects are in the reallocation of the capital between both types of banks. Specifically, it can be seen that the bankers decide to cut credits and also bank H reduce the amount of capital. This could be a little bit counterintuitive but it can be explain for two effects that are happen. First (assets side), and same as bank F, on impact the price of Mortgages q_L decrease dramatically which increase interest rate of the credits R_I and therefore decrease the demand

Table 5: Estimation, exogenous variables AR1 processes

Shock process	**A.R**	**Prior**		**Posterior**		**S.D.**	**Prior**		**Posterior**	
		Mean	**S.D**	**Mean**	**90% HPD**		**Mean**	**S.D**	**Mean**	**90% HPD**
Non stat. productivity	ρ_a	0.5	0.075	0.54	[0.42 0.67]	$100 \times \sigma_a$	0.50	Inf	0.36	[0.27 0.46]
Monetary Policy	ρ_{e^m}	0.25	0.075	0.26	[0.14 0.38]	$1000 \times \sigma_{e^m}$	1.00	Inf	1.63	[1.32 1.95]
Government spending	ρ_g	0.75	0.075	0.74	[0.59 0.88]	$100 \times \sigma_g$	1.00	Inf	1.76	[1.44 2.07]
Copper price	$\rho_{p^{co}}$	0.75	0.075	0.90	[0.85 0.95]	$10 \times \sigma_{p^{co}}$	1.00	Inf	1.07	[0.89 1.26]
Foreign inflation	ρ_{π^*}	0.75	0.075	0.47	[0.39 0.55]	$100 \times \sigma_{\pi^*}$	2.00	Inf	2.15	[1.77 2.53]
Foreign interest rate	ρ_{R^W}	0.75	0.075	0.90	[0.87 0.94]	$1000 \times \sigma_{R^W}$	1.00	Inf	1.12	[0.94 1.32]
Entrepreneurs risk	ρ_{σ^e}	0.75	0.075	0.98	[0.97 0.99]	$100 \times \sigma_{\sigma^e}$	2.00	Inf	2.95	[2.32 3.57]
Corporate bank risk	ρ_{σ^F}	0.75	0.075	0.94	[0.91 0.97]	$10 \times \sigma_{\sigma^F}$	2.00	Inf	1.67	[1.28 2.08]
Housing bank risk	ρ_{σ^H}	0.75	0.075	0.76	[0.61 0.92]	$10 \times \sigma_{\sigma^H}$	1.00	Inf	0.47	[0.07 0.88]
Housing valuation risk	ρ_{σ^I}	0.75	0.075	0.91	[0.81 1.02]	$10 \times \sigma_{\sigma^I}$	2.00	Inf	3.13	[0.21 6.06]
Current consumption prefs.	ρ_{ϱ}	0.75	0.075	0.41	[0.31 0.51]	$10 \times \sigma_{\varrho}$	1.00	Inf	5.06	[2.77 7.33]
Housing consumption prefs	ρ_{ξ^h}	0.75	0.075	0.88	[0.84 0.92]	$10 \times \sigma_{\xi^h}$	1.00	Inf	1.11	[0.70 1.53]
Investment mg. eff.(K)	ρ_{ξ^i}	0.75	0.075	0.63	[0.49 0.77]	$10 \times \sigma_{\xi^I}$	0.50	Inf	1.05	[0.73 1.37]
Investment mg. eff.(H)	$\rho_{\xi^{ih}}$	0.75	0.075	0.24	[0.16 0.32]	$10 \times \sigma_{\xi^{ih}}$	1.00	Inf	8.42	[5.78 11.1]
Import prices	ρ_{ξ^m}	0.75	0.075	0.91	[0.85 0.97]	$100 \times \sigma_{\xi^m}$	1.00	Inf	2.58	[1.98 3.20]
Labor disutility	ρ_{ξ^n}	0.75	0.075	0.66	[0.53 0.80]	$10 \times \sigma_{\xi^n}$	1.00	Inf	6.57	[2.80 10.3]
Country premium	ρ_{ξ^R}	0.75	0.075	0.83	[0.75 0.91]	$1000 \times \sigma_{\xi^R}$	0.50	Inf	0.63	[0.50 0.76]
Banker dividend	$\rho_{\xi^{x_b}}$	0.75	0.075	0.79	[0.69 0.90]	$10 \times \sigma_{\xi^{x_b}}$	0.50	Inf	1.55	[1.11 2.01]
Entrepreneur dividend	$\rho_{\xi^{x_e}}$	0.75	0.075	0.49	[0.34 0.60]	$10 \times \sigma_{\xi^{x_e}}$	1.00	Inf	2.28	[1.81 2.76]
Banker required return	$\rho_{\xi^{roe}}$	0.75	0.075	0.82	[0.74 0.91]	$10 \times \sigma_{\xi^{roe}}$	0.50	Inf	0.4	[0.29 0.51]
Foreign demand	$\rho_{\xi^{y*}}$	0.85	0.075	0.91	[0.80 1.03]	$100 \times \sigma_{\xi^{y*}}$	0.50	Inf	0.21	[0.08 0.34]
Mining productivity	$\rho_{\xi^{yco}}$	0.85	0.075	0.80	[0.64 0.96]	$10 \times \sigma_{\xi^{yco}}$	1.00	Inf	3.20	[2.61 3.80]
Stat. productivity	ρ_z	0.85	0.01	0.84	[0.82 0.86]	$10 \times \sigma_z$	0.50	Inf	1.30	[1.01 1.60]
UIP shock	ρ_{ζ^u}	0.75	0.075	0.96	[0.94 0.98]	$1000 \times \sigma_{z_\tau}$	0.50	Inf	1.74	[1.07 2.41]
Liquidity costs	ρ_{ϵ^L}	0.75	0.05	0.76	[0.66 0.86]	$100 \times \sigma_{\epsilon^L}$	0.20	Inf	0.09	[0.02 0.17]
Bank Balance Sheet	$\rho_{\xi^u_B}$	0.75	0.05	0.76	[0.66 0.86]	$10 \times \sigma_{\xi^u_B}$	0.20	Inf	0.09	[0.02 0.17]
Long-term government bonds supply	ρ_{BL_g}	0.75	0.05	0.76	[0.66 0.86]	$10 \times \sigma_{BL_g}$	0.50	Inf	0.23	[0.04 0.42]

Note.– This table shows the first two moments of the prior distribution of estimated parameters, together with posterior mean and standard deviation, based on maximum likelihood estimation and the Laplace approximation. Note that some standard deviations are scaled by different factors to obtain posterior means that are in the same order of magnitude. All autocorrelations were estimated using the Beta distribution, while standard deviations using the inverse-gamma distribution.

Table 6: Variance Decomposition at Quarterly Frequency

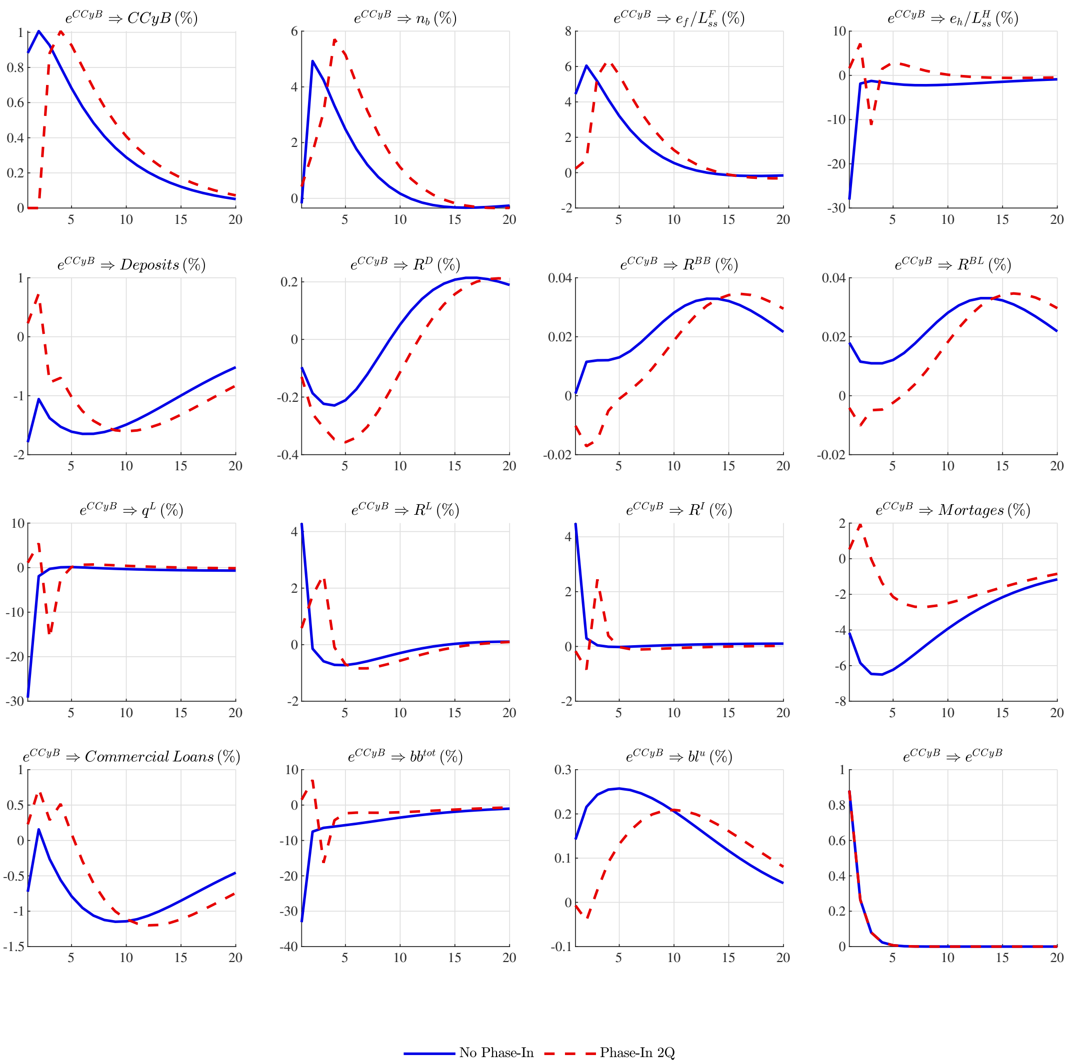
Shock Variable	Risk shocks	M.E.I.	Technology	MP	Demand	Gov. Spend.	Others	Total
GDP without Commodities	23.9	16.1	5.2	1.5	20.1	0.3	32.9	100.0
Total Investment	11.3	40.3	6.3	0.9	5.2	0.0	36.1	100.0
Commercial Loans	71.9	5.3	4.5	0.2	2.5	0.0	15.6	100.0
Commercial Spread	24.8	6.0	8.6	0.9	7.2	0.2	52.3	100.0
Mortgage Spread	11.9	4.0	10.3	0.4	8.7	0.2	64.5	100.0
Mortgage Loans	73.9	1.6	2.6	0.1	1.0	0.0	20.7	100.0

Note.– This table shows the variance decomposition for selected macro and financial variables. The results are generated by model evaluated at the mode of posterior the posterior distribution shows in Tables (4) and (5).

of mortgages. Second (liabilities side), since the long-term-bonds of banks have the government long-term-bonds as substitute, from the optimally condition of unrestricted household and since R^{BL} increase relative to R_{BB}, this households change the demand for long-term-bonds of banks for long-terms-bonds of governments, which increase the effects on the balance-sheet of the Bank H. It should be noted, that this last effect is not so important in the bank F because the deposits d_f do not have a substitute and their interest rate R_D, which follow closely the monetary policy.

However, if the activation of $CCyB$ has a Phase-In period the effects related to the re-allocation of the capital between banks are smoothed a lot (red-dotted lines). Specifically, since the banks know that in two periods the capital requirements will increase, they start to slowly increase the amount of capital and therefore the prices adjust smoothly in the effective activation of $CCyB$. Therefore, although we can observe that there is a re-allocation in the capital between banks, the effects in Commercial Credits, Mortgages and long-term-bonds of the banks H are much more smaller.

Figure 3: Transmission Channel CCyB activation



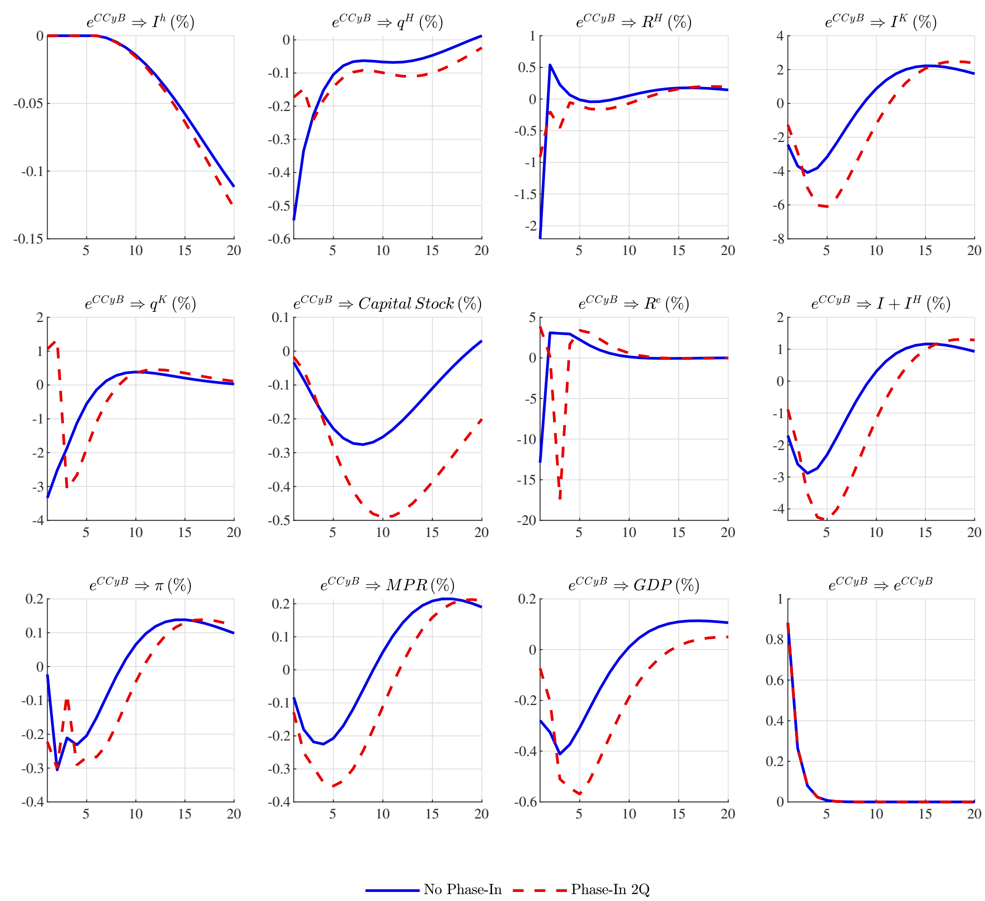
Note.– This figure shows the impulse response functions to an activation of CCyB of 100bp with no phase-in period (blue), which implies CCyB requirement must be met in full the period after its announcement, and the activation with a phase-in period of 2Q. Variables with (%) represent deviations from same variable steady states.

Now, regarding the transmission to the real sector, the channel follows very close to the Bernanke et al. 1999. On impact, since commercial credits and mortgages decrease, investment in capital and new houses (with the time-to-build lag) the price of capital q^K and price of houses decrease, and the return of these assets (R^e and R^H

respectively). These effects decrease the aggregate investment, which in the context of the New-Keynesian model are deflationary, the monetary policy act expansively. All these effects contract the real GDP.

Finally, although a Phase-In period smoothed the effects on Credits, it increases the the effects on the real variables as we can see in the Figure (4). This behavior is explained mostly for the effect in the price of capital q^K. Since the banks start to increase the capital before the effective activation of $CCyB$, and therefore, the commercial credits, it increases the price of capital. Now, once the $CCyB$ is activated, the change in the price of capital is deeper and therefore, the return of the entrepreneurs fall more. This increases the effects on investment and, ultimately, on GDP.

Figure 4: Transmission Channel CCyB activation II



Note.– This figure shows the impulse response functions to an activation of CCyB of 100bp with no phase-in period (blue), which implies CCyB requirement must be met in full the period after its announcement, and the activation with a phase-in period of 2Q. Variables with (%) represent deviations from same variable steady states.

4.2 Simple Implementable Financial Policy Rules

One challenge in discussing the effects of financial policy rules is the lack of consensus on their structure. It is unclear what the neutral CCyB —the CCyB requirement when perceived systemic risk is moderate— level should be, and the literature has not established the minimum arguments on which changes of CCyB would depend. In this subsection, we examine various specifications for a financial policy rule and compare their effects on the long-term welfare of consumers. We focus exclusively on potential rules that may have an empirical counterpart, making them readily implementable. We refer to them as simple and implementable financial policy (SIFPR) rules.

Consumption equivalence. To find the optimal SIR we perform a welfare analysis in the spirit of Carrillo et al. (2021) and denote the welfare of the economy as $\mathbf{W}(\theta)$, using the equation (38) as follows

$$\mathbf{W}(\theta) = \sum_{i \in I,U,R} \wp_i \mathbb{E}_{0,i} \left\{ \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i(\theta) \right)^{1-\sigma} - \Theta_t^i(\theta) A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i(\theta) \right)^{1+\varphi}}{1+\varphi} \right] \right\} \tag{28}$$

We calculate a baseline welfare $\mathbf{W^0} \equiv \mathbf{W}(\theta|\theta = 0)$ that summarizes consumers' welfare in an economy with no financial policy rule. This baseline statistic is useful for comparing gains or losses resulting from any CCyB rule activation under different specifications. Specifically, $\mathbf{W^0}$ is computed as the discounted value of the perpetual stream of constant period-utilities evaluated at the stochastic steady states of endogenous variables, $\hat{C}_{ss}^{i,\mathbf{0}}, \Theta_{ss}^{i,\mathbf{0}}, n_{ss}^{i,\mathbf{0}}$. We solve using a second-order perturbation and the pruning algorithm in Kim et al. (2008).

$$\mathbf{W^0} = \sum_{i \in I,U,R} \wp_i \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^{i,\mathbf{0}} \right)^{1-\sigma} - \Theta_{ss}^{i,\mathbf{0}} A_{ss}^{1-\sigma} \frac{\left(n_{ss}^{i,\mathbf{0}} \right)^{1+\varphi}}{1+\varphi} \right] \tag{29}$$

To more easily represent the gains from a given CCyB rule we compute consumption equivalent units, ce. This represents the permanent change in consumption that equates the welfare of the economy under a CCyB rule, $\mathbf{W}(\theta)$, and the welfare of the economy without a CCyB rule, $\mathbf{W^0}$. In other words, ce is the level of permanent consumption required to offset the welfare gains/losses from implementing a certain rule (i.e., when $\theta \neq 0$).

$$\mathbf{W^0}(ce) = \sum_{i \in I,U,R} \wp_i \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^{i,0}(ce,\theta) \right)^{1-\sigma} - \Theta_{ss}^{i,0}(ce,\theta) A_{ss}^{1-\sigma} \frac{\left(n_{ss}^{i,0}(ce,\theta) \right)^{1+\varphi}}{1+\varphi} \right] = \mathbf{W}(\theta) \tag{30}$$

where we adjust (37), (39), (40) accordingly,

$$\hat{C}_{ss}^{i,0}(ce,\theta) = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(C_{ss}^{i,0}(1-\phi_c)(1-ce) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(H_{ss}^{i,0}(1-\phi_{hh}) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{31}$$

$$\Theta_{ss}^{i,0}(ce,\theta) = \tilde{\chi}_{ss}^{i,0}(ce,\theta) A_{ss}^{\sigma} \left(\hat{C}_{ss}^{i,0}(ce,\theta) \right)^{-\sigma} \tag{32}$$

$$\tilde{\chi}_{ss}^{i,0}(ce,\theta) = A_{ss}^{-\sigma} \left(\hat{C}_{ss}^{i,0}(ce,\theta) \right)^{\sigma} \tag{33}$$

Therefore, when $ce > 0$ there is a welfare gain from implementing the SIR with respect to the baseline scenario. Consumers would need to increase their consumption by ce% in order to be indifferent to living in the no-rule economy. Conversely, if $ce < 0$ then households are worse off because of the implementation of the CCyB rule, as they require a negative consumption wedge in order to be at least as good as in the no-rule economy.

Simple implementable financial policy rules (SIFPR). One of the challenges in assessing the marginal contribution of financial policy rules in models commonly used for monetary policy analysis, is the lack of consensus on how a Financial Policy Rule should look like. Our goal is to study rules which are implementable in the sense that they depend only on observable variables by policymakers, and are simple enough to guide expectation formation.

For monetary policy analysis, there is little disagreement around the most basic policy rule specification. The Taylor (1993) rule is not only a fair description of central banks' actual policy, but it also is a good approximation to the optimal Ramsey policy under fairly general assumptions (Woodford, 2001b). That said, the literature on optimal monetary policy has examined several variations and extensions to the Taylor rule, which is too vast to summarize here.

Counter-cyclical financial policy has become widespread after the global financial crisis, with emerging and advanced economies alike adopting the guidelines of Basel III. Yet we lack a consensus financial policy rule that guides financial policy as the Taylor rule guides monetary policy. Financial policy operates in most countries under "guided discretion" (ESRB, 2014), which combines the prescription of a mechanic rule with expert judgment nurtured by many financial risk indicators. The buffer guide most frequently suggested by the BCBS has been the credit-to-GDP gap rule (see BIS (2010); Drehmann (2013)). The logic behind this indicator is intuitive. Credit booms often precede financial stress. Raising buffers in booms and releasing them in busts helps stabilize the credit cycle and its amplification to real variables. However, there is little evidence that countries that activated the CCyB did so following the credit-to-GDP gap rule (see Herz and Keller (2023); Edge and Liang (2020)). Instead, activation has followed house price booms and the deterioration of banks' credit portfolios. This implies that, in practice, national financial authorities have different assessments of financial policy rules. Our paper aims to inform on the quantitative properties of many options available to them, as long as they are simple (log-linear) and their inputs are observable to the policy maker.

Functional forms. We restrict to simple functional forms for the financial policy rule. In particular, we consider log-linear policy rules which are a function of the CCyB lag and an observable variable,

$$\left(\frac{1+CCyB_t}{1+\overline{CCyB}}\right)=\left(\frac{1+CCyB_{t-1}}{1+\overline{CCyB}}\right)^{\theta_1}\left(\frac{(1-\alpha_E)X_t+\alpha_E\mathbb{E}(X_{t+h})}{X}\right)^{\theta_2} \tag{34}$$

In particular, if the neutral level CCyB is zero, the log-linear equivalent to (34) is

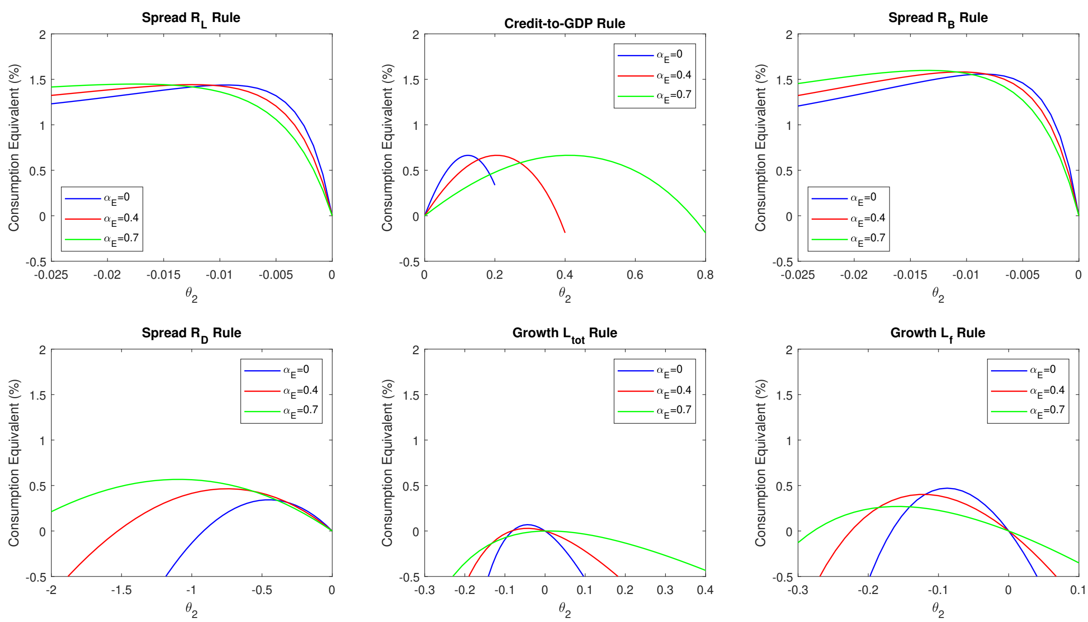
$$CCyB_t=\theta_1 CCyB_{t-1}+\theta_2\log\left(\frac{(1-\alpha_E)X_t+\alpha_E\mathbb{E}X_{t+h}}{\overline{X}}\right) \tag{35}$$

We will consider X_t to represent the rules described in Table (7):

Table 7: CCyB rules

Rule	Model variable	Description
Spread R_L	$R_t^L - R_t$	Observable of the external finance premium as in Carrillo et al. (2021)
Credit-to-GDP	$L_{tot,t}/GDP_t$	Commercial credit as in Drehmann (2013) and BIS (2010)
Spread R_B	$R_{B,t} - R_t$	$R_{B,t}$ is the portfolio-size-weighted average of commercial and mortgage rates
Spread R_D	$R_{D,t} - R_t$	Captures funding premium
Growth L_{tot}	$L_{tot,t}/L_{tot,t-1}$	Aggregate credit growth
Growth L_f	$L_{f,t}/L_{f,t-1}$	Commercial credit growth

Figure 5: Consumption equivalence for different Rules

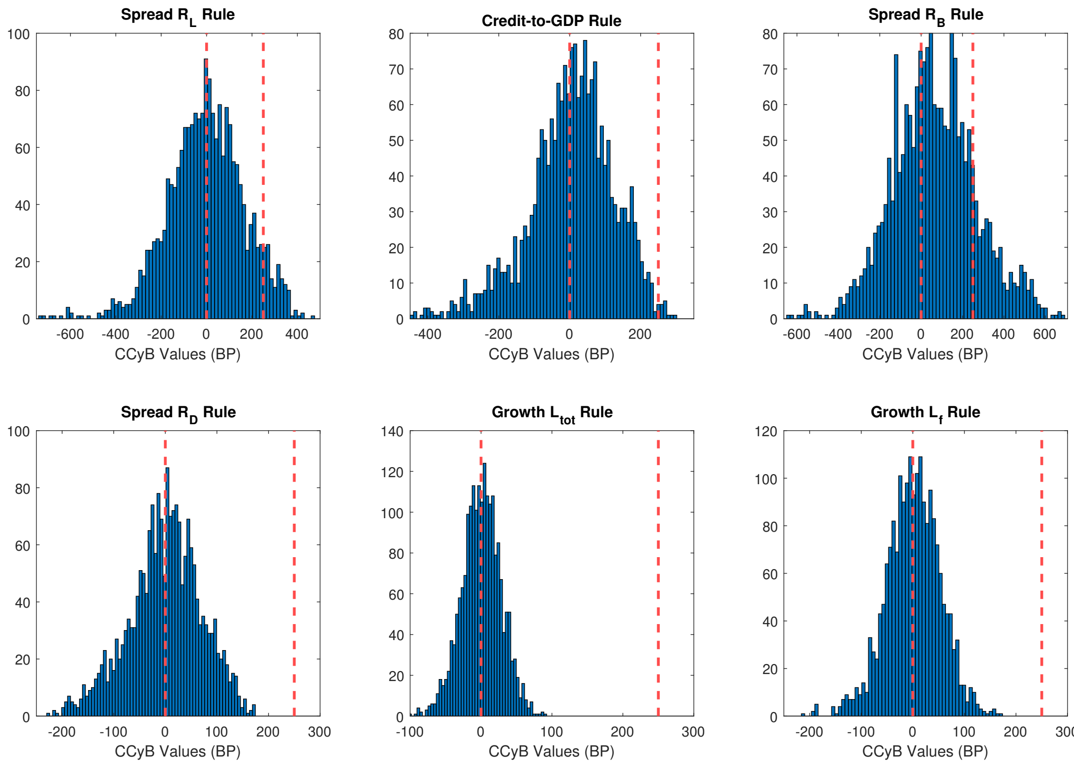


Note.– This figure shows the consumption equivalent for different values of θ_1 (controls inertia in countercylical capital requirements) and θ_2, the weight on the endogenous variable to which the rule reacts. Every sub-figure shows the results for different values of α_E, which indicates the degree to which the rule based on variable X is forward or backward looking, where $\alpha_E = 1$ and $\alpha_E = 0$ correspond to a perfectly forward and backward looking, respectively.

As we observe in the Figure (5) all the rules generates some welfare gains, measured as positive consumption equivalent, but at the same time, they can generate welfare loses for some values of θ_2 and α_E. This observation highlights the importance of choosing a sensible financial policy rule. We can see that the candidates to the best SIFPR are: Spreads R_L and R_B. This rules imply a 1.5% consumption equivalence approximately.

Importantly, however, we solve the model using perturbation around the steady state. This implies that for equation (34) we are not able take in to account that the countercyclical buffer goes from 0% to 2.5%. This implies that our welfare calculations reported in Figure (5) may be positively biased. We report the CCyB simulations for the best performing for each rules in Figure (6), which shows the distribution of CCyB realizations for the simulation considering 2.000 periods.

Figure 6: Distribution of CCyB simulated by optimal rules



Note.– This figure shows the distribution of the CCyB simulations implied best performing rules in terms of consumption equivalence. Each simulation consists of 2 thousand periods. Vertical axis shows frequency, the horizontal axis is expressed in basis points. The red dashed lines show the feasible band for CCyB values.

Although the rules do not generate extreme values for the CCyB, an important amount of values are outside the feasible region, in particular taking negative values. This goes in sharp contrast with implementation of the BIS (2010) principle of raising buffers when credit expands to lower it when the economy enters financial stress. In this business cycle model we see that two things happen instead. First, financial shock realizations can happen at any point without prior credit build-up. Second, it is often the case that the rule requires to lower the CCyB beyond prior accumulation, resulting in negative values. This observation on the implementation for the mechanic credit-to-GDP rule is consistent with many countries first considering to raise the buffer to achieve a "neutral" level, meaning a positive CCyB unrelated to shock realizations. The logic of a neutral level would be to have enough room to lower it without hitting the zero-lower bound implied by the design of the policy.

Implementing Effective Band. As noted earlier, incorporating an effective band for the CCyB rate is essential. Given that the Financial Stability Institute's (2019) design allows for values between 0 and 250 basis points, the results presented in the preceding section might be biased or even unattainable.

However, incorporating such a band presents a non-trivial challenge due to the model's characteristics and solution techniques. With over 300 endogenous variables and the use of second-order perturbation methods, implementing these non-linearities is computationally intensive. While toolkits like *OccBin* (Guerrieri and Iacoviello, 2015) and DynareOBC (Holden, 2016, 2019) facilitate incorporating non-linearities within perturbation frameworks, the model's size and second-order approximation remain obstacles.

To address these challenges and maintain the accuracy of our second-order approximation, we introduce a *Quadratic Filter*. This filter defines an auxiliary variable, $\widehat{CCyB}_t$, designed to closely track the $CCyB_t$ values obtained from the various rules (34) when within the feasible band, and to equal the upper or lower band limits if $CCyB_t$ exceeds them. The *Quadratic Filter* is implemented using the following algorithm:

Quadratic Filter algorithm

1: **for** each rule and each parameters configuration θ **do**
2: Simulate the model and save the $CCyB_t$ simulations.
3: For these simulation, evaluate the values in the *Quadratic Filter* defined as

$$\widehat{CCyB}_t = a + b(CCyB_t - CCyB_{ss}) + c(CCyB_t - CCyB_{ss})^2$$

with $CCyB_{ss}$ deterministic steady state of $CCyB_t$.
4: Find the parameters a, b and c which minimize the error function $\varepsilon(\cdot)$ by solving

$$\min_{a,b,c} \sqrt{\sum_{i=1}^{T} \varepsilon(\widehat{CCyB}_t)}$$

with

$$\varepsilon(\widehat{CCyB}_t) = \begin{cases} (\widehat{CCyB}_t - 250)^2 & \text{if } CCyB_t > 250 \\ (\widehat{CCyB}_t - CCyB_t)^2 & \text{if } 0 \leq CCyB_t \leq 250 \\ (0 - \widehat{CCyB}_t)^2 & \text{if } CCyB_t < 0 \end{cases}$$

5: Change the capital requirement in the model for

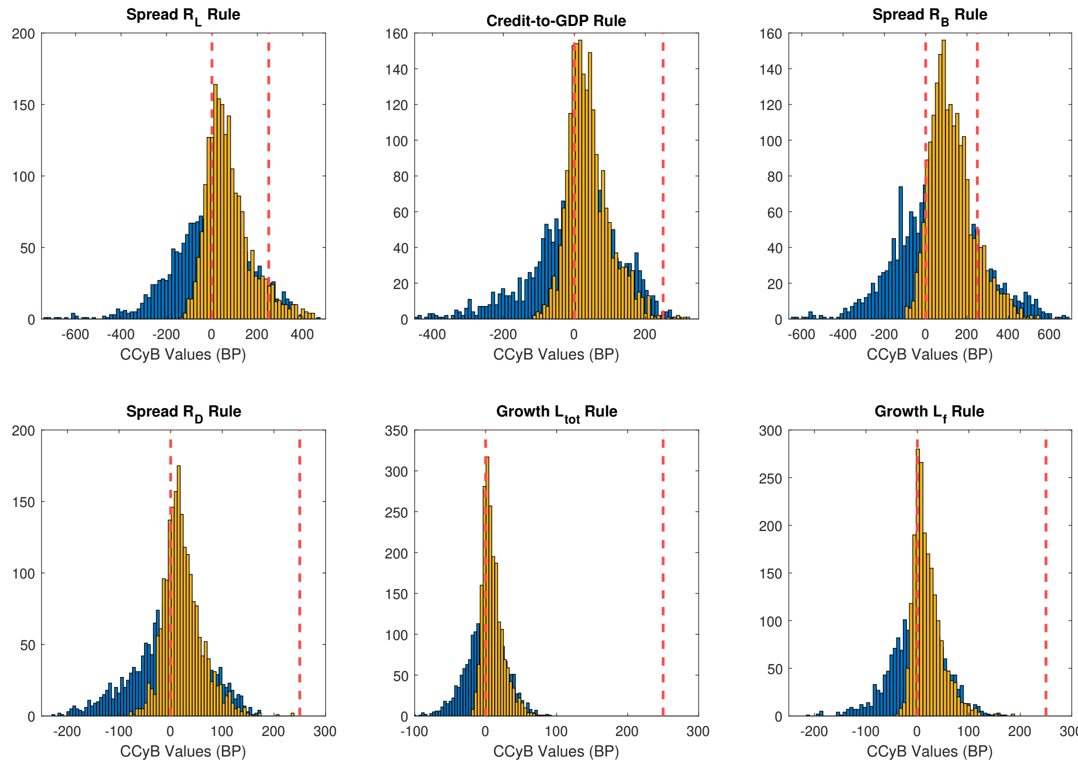
$$\phi_{F,t} = \lambda_f(\phi + \widehat{CCyB}_t)$$
$$\phi_{H,t} = \lambda_h(\phi + \widehat{CCyB}_t)$$

6: Simulate the model.
7: **end for**

It's crucial to note that this algorithm is applied to each rule and parameter setting θ individually. As illustrated in Figure (6), the manner in which each rule violates the feasible band varies. Consequently, it's imperative to determine specific parameters for each rule and configuration. The rationale is that some rules might need an approximation with a better fit for the lower band, the upper band, or both.

Figure (7) effectively illustrates the performance of the *Quadratic Filter* described earlier. This figure mirrors Figure (6), but with the distribution of $CCyB_t$ after applying the *Quadratic Filter* shown in yellow. While not a perfect implementation, as some of the yellow region extends beyond the feasible band (red dotted lines), the filter substantially improves the proportion of the distribution within the feasible band. On average, this proportion increases from 45% to 80% before and after applying the filter, respectively.

Figure 7: Distribution of CCyB simulated by optimal rules with Effective Band



Note.– This figure shows the distribution of the CCyB simulations implied best performing rules in terms of consumption equivalence. Each simulation consists of 2 thousand periods. Vertical axis shows frequency, the horizontal axis is expressed in basis points. The red dashed lines show the feasible band for CCyB values. In blue the same distribution in Figure (6), and in yellow the distribution applying the *Quadratic Filter*.

Table (8) summarizes the outcomes of applying an effective band to CCyB values. The first row shows the same maximum consumption equivalent, for each rule, as in Figure (5). The subsequent rows show the degree in forward-lookingness of the rule and the consumption equivalent for the same parameter θ_2, but after applying the *Quadratic Filter* and the respective degree of forward-lookingness α_E. The main results that we extract from the table are: First, as anticipated, the Consumption Equivalents decrease compared to the scenario without a band, as the restricted CCyB cannot react optimally, thus hindering welfare gains. Second, the rules responsive to prices, particularly commercial spread (R_L) and overall spread (R_B), still produce the higher welfare gains, almost three times higher than the others. Finally, in terms of the forward-lookingness degree of the rules, we observe that the rules inherit the characteristics of their observable. For instance, since the spread R_L and spread R_B rules react quickly in model, there is no reason to have any degree of forward-lookingness, as we see in the table, because the highest consumption equivalent occurs at $\alpha_E = 0$. On the other hand, spread R_L rule, since funding premium of the banks follows closely the monetary policy rate, which is sticky in this framework, we can see that the highest consumption equivalent occurs at $\alpha_E = 0.7$, similar to a standard Taylor Rule.

Table 8: Summary Results for each Rule and Effective Band

	Spread R_L			Credit-to-GDP			Spread R_B		
CE without EB	1.44%			0.66%			1.56%		
α_E	0	0.4	0.7	0	0.4	0.7	0	0.4	0.7
CE with EB	1.23%	1.12%	0.99%	0.49%	0.49%	0.48%	1.28%	1.12%	1.10%

	Spread R_D			Growth L_{tot}			Growth L_f		
CE without EB	0.57%			0.07%			0.47%		
α_E	0	0.4	0.7	0	0.4	0.7	0	0.4	0.7
CE with EB	0.25%	0.34%	0.40%	0.05%	0.02%	0.00%	0.33%	0.25%	0.14%

Note – This table shows the summary of the benefits of the different rules measured as equivalent consumption for the optimal value of θ_2, with and without effective band. For the latter, the results for different levels of foward-looking are also included.

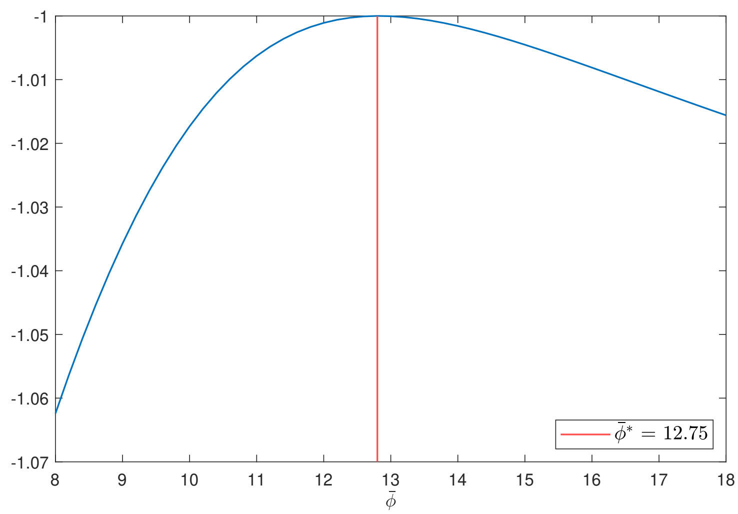
Given that the effective band for the $CCyB$ diminishes consumption equivalents across all rules, the subsequent section explores alternative $CCyB$ neutral levels to assess their potential for generating additional welfare gains, aligning with the preceding analysis.

4.3 Simple Implementable Financial Policy Rules with Neutral $CCyB$

As mentioned above, given that the $CCyB$ can only take values between 0 and 250 basis points, the presence of a neutral level may result in an increase in welfare, acting as a buffer for crises. The rationale for a positive neutral level is that, when a financial crisis occurs, consumption, housing and labor decrease for all households. This implies that the economy's aggregate welfare decreases and the households' marginal utility raises. Therefore, the $CCyB$ rules may want to reduce the $CCyB$ more than what the lower band permits (zero basis points). As a consequence, it could be welfare-optimal to have a positive level of neutral $CCyB$.

Since the regulatory capital requirement is inherently related to the $CCyB$ and $CCyB_N$, before assessing the mechanism described previously, we need to determine the baseline level of capital requirement above which the $CCyB$ and $CCyB_N$ will act. To determine this baseline level, we compute the optimal level of regulatory capital requirement by computing the stochastic steady state value of welfare in an economy without rules, and performing a grid-search to find the $\bar{\phi}$ that maximizes welfare. We find this optimal level to be $\bar{\phi} = 12.75\%$ as shown in Figure (8).

Figure 8: Stochastic Steady State for Welfare without Financial Policy Rule



Note.– This figure shows the stochastic steady state for total welfare defined in (28). Each point in the Figure was calculate as the mean of 2.000 simulations for each value of minimum capital requirement ϕ in percentage.

Having set a baseline level for the regulatory capital requirement, we can now assess the rationale for a positive level of neutral $CCyB$. To do so, we test different levels of neutral countercyclical capital buffer, $CCyB_N$, adding up these extra basis points to the base capital requirement $\bar{\phi}$ and, at the same time, shifting the feasible band in $-CCyB_N$ basis points. This procedure allows us to rule out negative values of $CCyB$ while reaching a higher capital requirement in steady state.

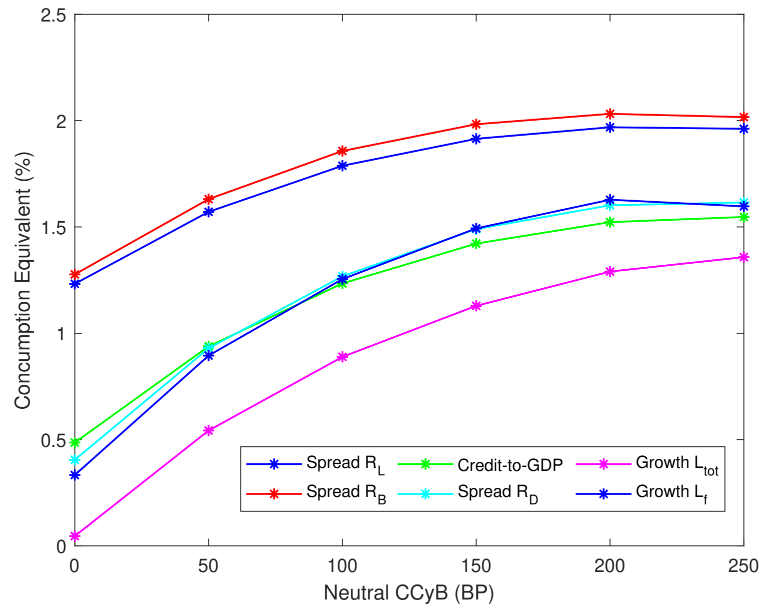
To ensure the Effective Band accurately incorporates the presence of a neutral level, we refine the *Quadratic Filter* by adjusting the error function limits within the algorithm as outlined below.

$$\varepsilon(\widehat{CCyB}_t) = \begin{cases} (\widehat{CCyB}_t - (250 - CCyB_N))^2 & \text{if } CCyB_t > 250 - CCyB_N \\ (\widehat{CCyB}_t - CCyB_t)^2 & \text{if } -CCyB_N \leq CCyB_t \leq 250 - CCyB_N \\ (-CCyB_N - \widehat{CCyB}_t)^2 & \text{if } CCyB_t < -CCyB_N \end{cases} \tag{36}$$

Figure (9) exhibits the welfare gains of a positive level of Neutral $CCyB$ in consumption equivalent units[10]. The figure reveals two main insights. First, for every rule, it is welfare-improving to have a positive level of neutral countercyclical capital buffer $CCyB_N$. These results confirm our intuition. Given the Effective Band, a positive level of $CCyB_N$ improves welfare due to fact that during a crisis, the optimal level of $CCyB$ hits the lower bound of the Effective Band. Secondly, the figure shows that the welfare gains are concave and heterogeneous among rules. We observe that the Growth L_{tot} and Credit-to-GDP rules reach their maximum consumption equivalent at 250bp and, while the other rules at 200bp. This result highlights the trade-off between the extra $CCyB$ available to be released during crises and lower level of credit during normal times, and as a consequence of the latter, a lower level consumption and GDP in steady state.

[10]The $CCyB$ rule parameters are set at their welfare-maximizing values shown in Table (8).

Figure 9: Consumption Equivalent for different levels of Neutral CCyB and Effective Band



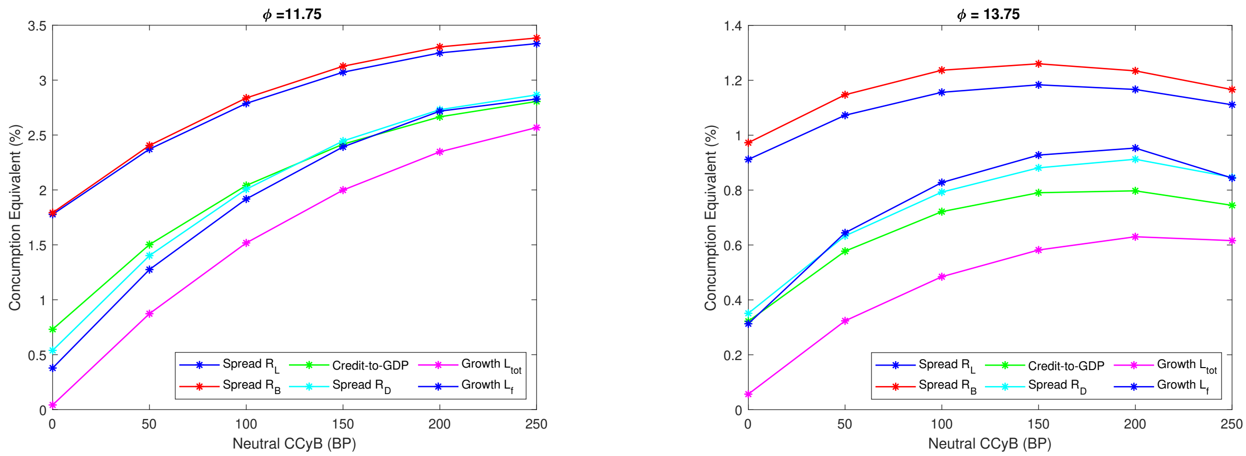
Note.– This figure shows the consumption equivalent for different values of neutral CCyB, for the optimal θ_2 found in Figure 5 for each rule, and $\theta_1 = 0.8409$

As we stated at the beginning of this section, these results are conditional on the capital requirement, which we set at its optimum $\bar{\phi} = 12.75\%$. In the following subsection we discuss the sensibility of our findings to this value.

Neutral $CCyB$ and the regulatory capital requirement. In this section we assess the sensibility of our previous findings to different levels of capital requirement. To do so, we obtain the optimal level of $CCyB_N$ when regulatory capital requirement is set at 11.75% and 13.75%, which corresponds to ±100bp apart from the optimal value. The results are depicted in Figure (10).

Figure (10) depicts the interaction between the neutral level of $CCyN$ and the regulatory capital requirement. We observe that the relationship between the welfare gains and $CCyB_N$ remains concave, finding a maximum beyond 250bp for $\bar{\phi} = 11.75\%$, and between 150 and 200bp in the case of $\bar{\phi} = 13.75\%$. The rationale is that, when the economy is below the optimal, $\bar{\phi} = 12\%$, the $CCyB_N$ acts as extra regulatory capital requirement to reach the optimal 12%, and as a neutral level of $CCyB$, making $CCyB_N$ higher in this case. On the other hand, when the economy is above the optimum, $CCyB_N$ will be lower in order to compensate for the suboptimal level of capital requirement. In addition, it is worth to mention that even when the level of capital requirement is suboptimal, a positive level of $CCyN$ is still justified and optimal.

Figure 10: Consumption Equivalent for different levels of Neutral CCyB and Effective Band



Note – This figure shows the consumption equivalent for different values of neutral $CCyB$, for the optimal θ_2 found previously for each rule, and $\theta_1 = 0.8409$. The figure on the left is for a base capital requirement equal to $\bar{\phi} = 11.75\%$, while the one on the right is for $\bar{\phi} = 13.75\%$.

Further comments. The presence of an Effective Band for the $CCyB$ necessitates a positive Neutral $CCyB$ to enhance welfare. This allows for greater flexibility in lowering the $CCyB$ during crises, even when it's already at its lower bound. The optimal Neutral $CCyB$ isn't independent of the capital requirement level; it's contingent upon the prevailing regulatory capital requirement and its proximity to the optimal level that would exist without a $CCyB$. If the current requirement falls short of its optimum, a higher Neutral $CCyB$ is beneficial, acting as both a capital boost and a crisis buffer. Conversely, if the current requirement is excessive, a lower, and still positive, Neutral $CCyB$ helps offset the overly restrictive capital rules. Finally, a positive Neutral $CCyB$ consistently proves valuable, offering adaptability to effectively manage economic downturns, even when capital requirements aren't perfectly aligned with the theoretical optimum.

5 Conclusion

In this paper we have evaluated the welfare implications of introducing a countercyclical buffer rule which is simple and implementable. We do so by building a macro-banking model with two inefficiencies: nominal rigidities and financial frictions. This gives room for monetary and financial policies to be desirable. We use our model to study the functional form of a SIR for financial policy. Further, we argue that the countercyclical nature of the CCyB and its institutional design (zero lower bound) imply a rationale for a neutral positive level of the buffer.

Using a quantitative model estimated with Chilean data, we explore several simple and implementable financial policy rules in terms of welfare differences (summarized in consumption equivalent terms). We find that i) price-responsive rules generate higher welfare gains, because they better map the effects of financial shocks; ii) the lower and upper limits of 0 and 250bp provides a rationale for a positive level of neutral CCyB, i.e., it makes the consumption equivalence a concave function of the neutral level; and iii) Welfare gains depend critically on the value of the regulatory capital requirement.

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ONLINE APPENDIX

A Full Model Details

A.1 Households

There are two continuums of households, each of measure one, risk-averse and infinitely lived, impatient (I) and patient (P) with discount factors β_I and $\beta_P > \beta_I$, respectively. Both households' preferences depend on consumption of a final good C_t^i relative to external habits $\hat{C}_{t-1}^i$, their stock of housing from last period H_{t-1} relative to external habits $\hat{H}_{t-2}^i$, and labor supplied (hours worked) n_t^i in each period. The consumption of the aggregate good $\hat{C}_t^i \equiv \hat{C}(C_t^i, \tilde{C}_{t-1}^i, H_{t-1}^i, \hat{H}_{t-2}^i)$ for households of type $i = \{P, I\}$ is assumed to be a constant elasticity of substitution (CES) as shown in (37):

$$\hat{C}_t^i = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_C}} \left(C_t^i - \phi_c \tilde{C}_{t-1}^i \right)^{\frac{\eta_C - 1}{\eta_C}} + (o_{\hat{C}})^{\frac{1}{\eta_C}} \left(\xi_t^h \left(H_{t-1}^i - \phi_{hh} \tilde{H}_{t-2}^i \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_C}} \right]^{\frac{\eta_{\hat{C}}}{\eta_C - 1}} \tag{37}$$

where $o_{\hat{C}} \in (0, 1)$ is the weight on housing in the aggregate consumption basket, η_C is the elasticity of substitution between the final good and the housing good, ξ_t^h is an exogenous preference shifter shock and $\phi_c, \phi_{hh} \geq 0$ are parameters guiding the strength of external habits in consumption and housing respectively. Households of type i maximize the following expected utility

$$\max_{\{\hat{C}_t^i, H_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i \right)^{1-\sigma} - \Theta_t^i A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i \right)^{1+\varphi}}{1+\varphi} \right] \tag{38}$$

where $\beta_i \in (0, 1)$ is the respective discount factor, ϱ_t is an exogenous shock to intertemporal preferences, ξ_t^n is a preference shock that affects the (dis)utility from labor,$\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\varphi \geq 0$ is the inverse elasticity of labor supply. As in Galí et al. (2012), we introduce an endogenous preference shifter Θ_t, that satisfies the following conditions

$$\Theta_t^i = \tilde{\chi}_t^i A_t^{\sigma} \left(\hat{C} \left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{-\sigma} \tag{39}$$

and

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i \right)^{1-v} A_t^{-\sigma v} \left(\hat{C} \left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{\sigma v} \tag{40}$$

where the parameter $v \in [0, 1]$ regulates the strength of the wealth effect, and $\tilde{C}_t^i$ and $\tilde{H}_{t-1}^i$ are taken as given by the households. In equilibrium $C_t^i = \tilde{C}_t^i$ and $H_t^i = \tilde{H}_t^i$.

A.1.1 Patient Households

This group is formed by fraction $\wp_P$ of the households. They save in one-period government bonds (BS_t), long-term government bonds (BL_t), short-term bank deposits (D_t), long-term bank-issued bonds (BB_t), and one-period foreign bonds quoted in foreign currency ($B_t^\star$). All non-state-contingent assets. Then, patient households' period budget constraint equates uses and sources of funds,

$$\begin{aligned} BS_t + D_t + S_t B_t^\star + Q_t^{BL} BL_t + Q_t^{BB} BB_t + P_t \hat{C}_t^P + Q_t^H \left(H_t^P - (1-\delta_H) H_{t-1}^P \right) = \\ R_{t-1} BS_{t-1} + Q_t^{BL} R_t^{BL} BL_{t-1} + \tilde{R}_t^D D_{t-1} + \tilde{R}_t^{BB} Q_t^{BB} BB_{t-1} + S_t B_{t-1}^\star R_{t-1}^\star + W_t n_t^P + \Psi_t \end{aligned} \tag{41}$$

where R_t^{BL} and R_t^{BB} are the real gross yield to maturity for long-term government and bank-issued bonds at time t, P_t denotes the price of the consumption good, Q_t^H denotes the nominal price of housing good, δ_H is depreciation rate of housing goods, S_t the nominal exchange rate (units of domestic currency per unit of foreign currency), $R_t^\star$ the foreign one-period bond return, and R_t denotes the short term nominal government bond rate.

Further, $\widetilde{R}_t^D = R_{t-1}^D(1 - \gamma_D PD_t^B)$, $\widetilde{R}_t^{BB} = R_t^{BB}(1 - \gamma_{BB} PD_t^B)$ denote the net return on deposits and net yield on bank-bonds, received by households. Also, R^D_{t-1} is the gross interest rate paid by banks in t, PD^B_t denotes the default probability of banks, and γ_D and γ_{BB}) denote transaction costs that households must pay in order to recover their funds, even under deposit insurance. Finally, W_t denotes the nominal wage and, Ψ_t denotes lump-sum payments that include taxes T_t, dividend income from entrepreneurs C^e_t, bankers C^b_t, rents from ownership of foreign firms REN^*_t and profits from ownership of domestic firms.

Households supply differentiated labor services to a continuum of *unions* which act as wage setters on behalf of the households in monopolistically competitive markets. The unions collect the wage income from all households and distribute it equally among them, providing insurance against wage-income risk. Defining for convenience the multiplier on the budget constraint as $\lambda^U_t A^{-\sigma}_t / P_t$, then, Patient Households solve (38) subject to (37), (39), (40), and (41). From this problem, we obtain the following first-order conditions:

$$[C^P_t]: \quad \lambda^P_t A^{-\sigma}_t = \left(\hat{C}^P_t\right)^{-\sigma} \left(\frac{\left(1-o_{\hat{C}}\right)\hat{C}^P_t}{\left(C^P_t - \phi_c \tilde{C}^P_{t-1}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{42}$$

$$\begin{aligned} [H^P_t]: \quad \varrho_t \frac{\lambda^P_t A^{-\sigma}_t Q^H_t}{P_t} =& \beta_P \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{C}^P_{t+1}\right)^{-\sigma} \xi^h_{t+1} \left(\frac{o_{\hat{C}} \hat{C}^P_{t+1}}{\xi^h_{t+1}\left(H^P_t - \phi_{hh}\tilde{H}^P_{t-1}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \right. \\ & \left. + \left(1-\delta_H\right) \frac{\lambda^P_{t+1} A^{-\sigma}_{t+1} Q^H_{t+1}}{P_{t+1}} \right\} \end{aligned} \tag{43}$$

$$[BS_t]: \quad \varrho_t \lambda_t A^{-\sigma}_t = \beta_P R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda^P_{t+1}}{\pi_{t+1}} A^{-\sigma}_{t+1} \right\} \tag{44}$$

$$[BL_t]: \quad \varrho_t \lambda^P_t A^{-\sigma}_t \left(\frac{Q^{BL}_t}{P_t}\right) = \beta_P \mathbb{E}_t \left\{ \varrho_{t+1}\lambda^P_{t+1} A^{-\sigma}_{t+1} R^{BL}_{t+1} \left(\frac{Q^{BL}_{t+1}}{P_{t+1}}\right) \right\} \tag{45}$$

$$[B^\star_t]: \quad \varrho_t \lambda^P_t A^{-\sigma}_t = \beta_P R^\star_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda^P_{t+1}\pi^s_{t+1}}{\pi_{t+1}} A^{-\sigma}_{t+1} \right\} \tag{46}$$

$$[D_t]: \quad \varrho_t \lambda^P_t A^{-\sigma}_t = \beta_P \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda^P_{t+1}}{\pi_{t+1}} \tilde{R}^D_{t+1} A^{-\sigma}_{t+1} \right\} \tag{47}$$

$$[BB_t]: \quad \varrho_t \lambda^P_t A^{-\sigma}_t = \beta_P \mathbb{E}_t \left\{ \varrho_{t+1}\lambda^P_{t+1} A^{-\sigma}_{t+1} \tilde{R}^{BB}_{t+1} \left(\frac{Q^{BB}_{t+1}}{P_{t+1}}\right) \right\} \tag{48}$$

In equilibrium, we have that $\tilde{C}^P_t = C^P_t$ and $\tilde{H}^P_t = H^P_t$.

A.1.2 Impatient Households

Impatient households work, consume, and purchase housing goods. In addition, they take long-term loans in equilibrium from banks to finance their purchases of housing goods.

Mortgage Default. We follow the Clerc et al. (2014) by assuming that these mortgage loans are non-recourse and limited liability contracts, which enables the possibility of default for households. For the household, the only consequence of default is losing the housing good on which the mortgage is secured, therefore default is optimal when the value of the total outstanding debt is higher than the value of the assets, limited-liability.

$$R^I_t Q^L_t L^H_{t-1} > \omega^I_t Q^H_t \left(1-\delta_H\right) H^I_{t-1} \tag{49}$$

Define ω^I_t as an idiosyncratic shock to the efficiency units of housing of impatient households, which can be interpreted as a reduced-form representation of any shock to the value of houses. The shock ω^I_t is i.i.d. across households and follows a log-normal distribution with pdf $f_I\left(\omega^I_t\right)$ and cdf $F_I\left(\omega^I_t\right)$. After the realization of aggregate and idiosyncratic shocks individual households decide whether to default, and then the resulting net worth is distributed evenly across members of this type, which optimally decide to choose the same debt, consumption, housing and hours worked. Let

$$R^H_t = \frac{Q^H_t\left(1-\delta_H\right)}{Q^H_{t-1}}.$$

Then, in order for the impatient household to pay for its loan, the idiosyncratic shock ω_t^I must exceed the threshold

$$\bar{\omega}_t^I = \frac{R_t^I Q_t^L L_{t-1}^H}{R_t^H Q_{t-1}^H H_{t-1}^I}$$

If $\omega_t^I \geq \bar{\omega}_t^I$ the household pays liabilities due in the period t in the amount $R_t^I Q_t^L L_{t-1}^H$, to obtain positive net worth, $(\omega_t^I - \bar{\omega}_t^I) Q_t^H (1-\delta_H) H_{t-1}^I$. Otherwise, the household debt becomes non-perming, defaults and receives nothing. On the other hand, the bank receives $R_t^I Q_t^L L_{t-1}^H$ from performing loans, but it only recovers $(1-\mu_I)\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I$ from non performing loans. With the definition of the $\bar{\omega}_t^I$ threshold, we can define $PD_t^I = F_I\left(\bar{\omega}_t^I\right)$ as the default rate of impatient households on their housing loans. Note that these defaults are over the value of all loans outstanding, $Q_t^L L_{t-1}^H$.

Budget constraint. The budget constraint for the impatient household equates uses and sources of funds,

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L L_t^H = W_t n_t^I + \int_0^\infty \max\left\{\omega_t^I Q_t^H (1-\delta_H) H_{t-1}^I - R_t^I Q_t^L L_{t-1}^H, 0\right\} dF_I(\omega_t^I) \tag{50}$$

Following Bernanke et al. (1999), the share of the gross return that goes to the bank is denoted as $\Gamma_I\left(\bar{\omega}_t^I\right)$ whereas the share of gross return that goes to the impatient household is $\left(1-\Gamma_I\left(\bar{\omega}_t^I\right)\right)$ where:

$$\Gamma_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I + \bar{\omega}_t^I \int_{\bar{\omega}_t^I}^\infty f_I\left(\omega_t^I\right) d\omega_t^I$$

The first integral on the right denotes the share of the return that is defaulted while the second integral denotes the share of return that is paid in full. This allows us to rewrite the budget condition from (50) as

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L L_t^H = W_t n_t^I + \left[1-\Gamma_I\left(\bar{\omega}_t^I\right)\right] R_t^H Q_{t-1}^H H_{t-1}^I \tag{51}$$

Also, let

$$G_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I$$

denote the part of those returns that comes from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_I G_I\left(\bar{\omega}_t^I\right)$, then the net share of return that goes to the bank is

$$\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right).$$

The terms of the loan must imply the net expected profits of the bank must equal its alternative use of funds, therefore it must satisfy a participation constraint:

$$\begin{aligned} &\mathbb{E}_t\left\{\left[1-\Gamma^H\left(\bar{\omega}_{t+1}^H\right)\right]\left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right] R_{t+1}^H Q_t^H H_t^I\right\} \geq \rho_{t+1}^H \phi_H Q_t^L L_t^H \\ &\lambda_{1,t}^b = \mathbb{E}_t\left[\Lambda_{t+1}^b \rho_{t+1}^H\right] \end{aligned} \tag{52}$$

Where $\lambda_{1,t}^b$ is the shadow value of bank equity (Lagrange multiplier in the resource constrained of the bankers problem), $\bar{x}^b$ is target dividend policy of banks and κ is a parameter for the cost of take a different dividend policy. The equation (52) represent the banks' participation constraint. For details see subsection A.3.

Thus, following the timing described above, the impatient household's optimization problem can be written as maximizing (38) for $i = I$ subject to their budget constraint (51) and the bank participation constraint (52). For this, define for convenience $\frac{\lambda_t^I A_t^{-\sigma}}{P_t}$ and $\frac{\lambda_t^H A_t^{-\sigma}}{P_t}$ as the multipliers for each constraint respectively. Define also $\omega_{t+1}^I \equiv \frac{R_{t+1}^I Q_{t+1}^L L_t^H}{R_{t+1}^H Q_t^H H_t^I}$, a measure of household leverage conditional on chosen optimal control variables. This yields the following FOC's:

$$[C_t^I]: \quad \lambda_t^I A_t^{-\sigma} = \left\{ \left(\hat{C}_t^I\right)^{-\sigma} \right\} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{C}_t^I}{\left(C_t^I - \phi_c \tilde{C}_{t-1}^I\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{53}$$

$$[H_t^I]: \quad \beta_I \varrho_{t+1} \frac{\partial V_{t+1}(H_t)}{\partial H_t} - \lambda_t^I Q_t^H + \lambda_t^H \mathbb{E}_t \left[\Lambda_{t+1}^b \frac{\partial \rho_{t+1}^H}{\partial \tilde{R}_{t+1}^H} \frac{\partial \tilde{R}_{t+1}^H}{\partial H_t} \right] = 0$$

$$\frac{\lambda_t^I A_t^{-\sigma} Q_t^H}{P_t} = \mathbb{E}_t \left\{ \begin{array}{l} \beta_I \varrho_{t+1} \left(\left(\hat{C}_{t+1}^I\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^I}{\xi_{t+1}^h \left(H_t^I - \phi_{hh} \tilde{H}_{t-1}^I\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \right. \\ \left. + \frac{\lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \left[1 - \Gamma_I\left(\bar{\omega}_{t+1}^I\right)\right] R_{t+1}^H Q_t^H \right) \\ + \lambda_t^H \Lambda_{t+1}^b \frac{\left[1 - \Gamma_b(\bar{\omega}_{t+1}^H) - \zeta(\hat{\omega}_{t+1}^H - \Gamma_b(\hat{\omega}_{t+1}^H))\right]}{\phi_t^H} \frac{\left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right] R_{t+1}^H Q_t^H}{Q_t^L L_t^H} \end{array} \right\} \tag{54}$$

$$[L_t^H]: \quad \lambda_t^I Q_t^L + \lambda_t^H \mathbb{E}_t \left[\Lambda_{t+1}^b \frac{\partial \rho_{t+1}^H}{\partial \tilde{R}_{t+1}^H} \frac{\partial \tilde{R}_{t+1}^H}{\partial L_t} \right] = 0$$

$$\lambda_t^I = \lambda_t^H \mathbb{E}_t \left[\Lambda_{t+1}^b \frac{\left[1 - \Gamma_b(\bar{\omega}_{t+1}^H) - \zeta(\hat{\omega}_{t+1}^H - \Gamma_b(\hat{\omega}_{t+1}^H))\right]}{\phi_t^H} \frac{\tilde{R}_{t+1}^H}{L_t^H Q_t^L} \right] \tag{55}$$

$$[\omega_{t+1}^I]: \quad \beta_I \varrho_{t+1} \frac{\partial V_{t+1}(\omega_{t+1}^I)}{\partial \omega_{t+1}^I} + \lambda_t^H \mathbb{E}_t \left[\Lambda_{t+1}^b \frac{\partial \rho_{t+1}^H}{\partial \tilde{R}_{t+1}^H} \frac{\partial \tilde{R}_{t+1}^H}{\partial \omega_{t+1}^I} \right] = 0$$

$$\frac{\lambda_t^H A_t^{-\sigma}}{P_t} \mathbb{E}_t \left[\Lambda_{t+1}^b \frac{\left[1 - \Gamma_b(\bar{\omega}_{t+1}^H) - \zeta(\hat{\omega}_{t+1}^H - \Gamma_b(\hat{\omega}_{t+1}^H))\right]}{\phi_t^H} \frac{\left[\Gamma_I'\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I'\left(\bar{\omega}_{t+1}^I\right)\right]}{L_t^H Q_t^L} \right] = \beta_I \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \Gamma_I'\left(\bar{\omega}_{t+1}^I\right) \right\} \tag{56}$$

Functional forms idiosyncratic shocks ω^I . We draw from Bernanke et al. (1999) and assume that $\ln(\omega_t^I) \sim N(-\frac{1}{2}(\sigma_t^I)^2, (\sigma_t^I)^2)$, therefore its unconditional expectation is $\mathbb{E}\{\omega_t^I\} = 1$, and its average conditional on truncation is

$$\mathbb{E}_t \left\{ \omega_t^I | \omega_t^I \geq \bar{\omega}_t^I \right\} = \frac{1 - \Phi\left(z_t^I - \sigma_t^I\right)}{1 - \Phi\left(z_t^I\right)},$$

where Φ is the c.d.f. of the standard normal and z_t^I is an auxiliary variable defined as $z_t^I \equiv {}^{(\ln(\bar{\omega}_t^I) + 0.5(\sigma_t^I)^2)}/{\sigma_t^I}$. Then, we can obtain the following functional forms:

$$\Gamma_I\left(\bar{\omega}_t^I\right) = \Phi\left(z_t^I - \sigma_t^I\right) + \bar{\omega}_t^I\left(1 - \Phi\left(z_t^I\right)\right)$$

and

$$\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right) = (1 - \mu_I)\,\Phi\left(z_t^I - \sigma_t^I\right) + \bar{\omega}_t^I\left(1 - \Phi\left(z_t^I\right)\right)$$

Finally, we allow for fluctuations in the variance of the idiosyncratic shock σ_t^I, as in Christiano et al. (2014) and Carrillo et al. (2021).

A.2 Entrepreneurs

As in Clerc et al. (2014), we introduce risk-neutral entrepreneurs that follow an overlapping generations structure, where each generation lives across two consecutive periods. The entrepreneurs are the sole owners of productive capital, which is bought from capital producers to be, in turn, rented to the firms that produce different varieties of the home good.

Entrepreneurs born in period t draw utility in $t+1$ from transferring part of final wealth as dividends, C_{t+1}^e, to patient households and from leaving the rest as bequests, N_{t+1}^e, to the next generation of entrepreneurs in the form:

$$\max_{C_{t+1}^e, N_{t+1}^e} \left(C_{t+1}^e\right)^{\xi_{\chi_e} \chi_e} \left(N_{t+1}^e\right)^{1 - \xi_{\chi_e} \chi_e} \text{ subject to}$$

$$C_{t+1}^{e}+N_{t+1}^{e}=\Psi_{t+1}^{e}$$

where Ψ_{t+1}^{e} is entrepreneurial wealth at $t+1$, explained below, and ξ_{χ_e} is a stochastic shock to their preferences. The first order conditions to this problem may be written as:

$$[C_{t+1}^{e}]:\ \xi_{\chi_e}\chi_e(C_{t+1}^{e})^{(\xi_{\chi_e}\chi_e-1)}\left(N_{t+1}^{e}\right)^{1-\xi_{\chi_e}\chi_e}-\lambda_t^{\chi_e}=0$$

$$[N_{t+1}^{e}]:\ (1-\xi_{\chi_e}\chi_e)(C_{t+1}^{e})^{\xi_{\chi_e}\chi_e}\left(N_{t+1}^{e}\right)^{-\xi_{\chi_e}\chi_e}-\lambda_t^{\chi_e}=0$$

$$[\lambda_t^{\chi_e}]:\ C_{t+1}^{e}+N_{t+1}^{e}-\Psi_{t+1}^{e}=0$$

From first order conditions we get the following optimal rules

$$C_{t+1}^{e}=\chi_e\Psi_{t+1}^{e}$$
$$N_{t+1}^{e}=\left(1-\chi_e\right)\Psi_{t+1}^{e}$$

In their first period, entrepreneurs will try to maximize expected second period wealth, Ψ_{t+1}^{e}, by purchasing capital at nominal price Q_t^K, which will be productive (and rented) in the next period. These purchases are financed using the resources left as bequests by the previous generation of entrepreneurs and borrowing an amount L_t^F at nominal rate R_t^L from from F banks. In borrowing from banks, entrepreneurs also face an agency problem of the type faced by impatient households i.e. in $t+1$ entrepreneurs receive an idiosyncratic shock to the efficiency units of capital that will ultimately determine their ability to pay their liabilities to banks. Banks cannot observe these shock, but entrepreneurs can. Depreciated capital is sold in the next period to capital producers at Q_{t+1}^K. Entrepreneurial leverage, as measured by assets over equity, is $lev_t^e={}^{Q_t^K K_t}/_{N_t^e}$.

In this setting, entrepreneurs solve, in their first period,

$$\max_{K_t,L_t^F}\mathbb{E}_t\left(\Psi_{t+1}^{e}\right)\text{ subject to}$$

$$Q_t^K K_t-L_t^F=N_t^e$$
$$\Psi_{t+1}^{e}=\max\left[\omega_{t+1}^{e}\left(R_{t+1}^{k}+\left(1-\delta_K\right)Q_{t+1}^{K}\right)K_t-R_t^L L_t^F,\ 0\right]$$

and a bank participation condition, which will be explained later. The factor ω_{t+1}^{e} represents the idiosyncratic shock to the entrepreneurs efficiency units of capital. This shock takes place after the loan with the bank has taken place but before renting capital to consumption goods producers. It is assumed that this shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one. Let

$$R_{t+1}^{e}=\left[\frac{R_{t+1}^{k}+\left(1-\delta_K\right)Q_{t+1}^{K}}{Q_t^K}\right]\tag{57}$$

be the gross nominal return per efficiency unit of capital obtained in period $t+1$ from capital obtained in period t. Then in order for the entrepreneur to pay for its loan the efficiency shock, ω_{t+1}^{e}, must exceed the threshold

$$\bar{\omega}_{t+1}^{e}=\frac{R_t^L L_t^F}{R_{t+1}^{e}Q_t^K K_t}$$

If $\omega_{t+1}^{e}\geq\bar{\omega}_{t+1}^{e}$ the entrepreneurs pays $R_t^L L_t^F$ to the bank and gets $(\omega_{t+1}^{e}-\bar{\omega}_{t+1}^{e})R_{t+1}^{e}Q_t^K K_t$. Otherwise, the entrepreneurs defaults and receives nothing. While F-banks only recover $(1-\mu_e)\omega_{t+1}^{e}R_{t+1}^{e}Q_t^K K_t$ from non performing loans, and $R_t^L L_t^F$ from performing loans. With the threshold, we can define $PD_t^e=F_e(\bar{\omega}_t^e)$ as the default rate of entrepreneurs on their loans.

The share of the gross return that goes to the bank is denoted as $\Gamma_e(\bar{\omega}_{t+1}^{e})$ whereas the share of gross return that goes to the entrepreneur is $(1-\Gamma_e(\bar{\omega}_{t+1}^{e}))$ where:

$$\Gamma_e\left(\bar{\omega}_{t+1}^{e}\right)=\int_0^{\bar{\omega}_{t+1}^{e}}\omega_{t+1}^{e}f_e\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}+\bar{\omega}_{t+1}^{e}\int_{\bar{\omega}_{t+1}^{e}}^{\infty}f_e\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}$$

also let

$$G_e\left(\bar{\omega}^e_{t+1}\right) = \int_0^{\bar{\omega}^e_{t+1}} \omega^e_{t+1} f_e\left(\omega^e_{t+1}\right) d\omega^e_{t+1}$$

denote the part of those returns that come from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_e G_e\left(\bar{\omega}^e_{t+1}\right)$, then the net share of return that goes to the bank is

$$\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right).$$

Taking this into account then the maximization problem of the entrepreneur can be written as

$$\max_{\bar{\omega}^e_{t+1}, K_t} \mathbb{E}_t\left\{\Psi^e_{t+1}\right\} = \mathbb{E}_t\left\{\left[1 - \Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\}, \text{ subject to} \tag{58}$$

$$\mathbb{E}_t\left\{\left[1 - \Gamma_F\left(\bar{\omega}^F_{t+1}\right)\right]\left[\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t K_t\right\} \geq \rho^F_{t+1} \phi_F L^F_t$$

$$\lambda^b_{1,t} = \mathbb{E}_t\left[\Lambda^b_{t+1} \rho^F_{t+1}\right] \tag{59}$$

that says that banks will participate in the contract only if its net expected profits are at least equal to their alternative use of funds. This yields the following optimality conditions

$$[K_t]: \quad \left[1 - \Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right] R^e_{t+1} Q^K_t + \lambda^e_t \mathbb{E}_t\left[\Lambda^b_{t+1} \frac{\partial \rho^F_{t+1}}{\partial \tilde{R}^F_{t+1}} \frac{\partial \tilde{R}^F_{t+1}}{\partial K_t}\right] = 0$$

$$1 - \Gamma_e(\bar{\omega}^e_{t+1}) = \lambda^e_t \left[\Lambda^b_{t+1} \frac{\left[1 - \Gamma_b(\bar{\omega}^F_{t+1}) - \zeta(\hat{\omega}^F_{t+1} - \Gamma_b(\hat{\omega}^F_{t+1}))\right]}{\phi^F_t} \frac{\left(\mu_e G_e\left(\bar{\omega}^e_{t+1}\right) - \Gamma_e\left(\bar{\omega}^e_{t+1}\right)\right)}{L^F_t}\right] \tag{60}$$

$$[\bar{\omega}^e_{t+1}]: \quad -\Gamma'_e\left(\bar{\omega}^e_{t+1}\right) R^e_{t+1} Q^K_t K_t + \lambda^e_t \mathbb{E}_t\left[\Lambda^b_{t+1} \frac{\partial \rho^F_{t+1}}{\partial \tilde{R}^F_{t+1}} \frac{\partial \tilde{R}^F_{t+1}}{\partial \omega^e_{t+1}}\right] = 0$$

$$\Gamma'_e(\bar{\omega}^e_{t+1}) = \lambda^e_t \left[\Lambda^b_{t+1} \frac{\left[1 - \Gamma_b(\bar{\omega}^F_{t+1}) - \zeta(\hat{\omega}^F_{t+1} - \Gamma_b(\hat{\omega}^F_{t+1}))\right]}{\phi^F_t} \frac{\left(\Gamma'_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G'_e\left(\bar{\omega}^e_{t+1}\right)\right)}{L^F_t}\right] \tag{61}$$

Further, it is assumed that $\ln(\omega^e_t) \sim N(-0.5(\sigma^e_t)^2, (\sigma^e_t)^2)$, leading to analogous properties as with impatient households for $\bar{\omega}^e_t$, Γ_e and G_e.

A.3 Bankers

Bankers are modeled in line with Mendicino et al. (ming). They own and manage a continuum of subsidiaries for two classes of banks, commercial banks (F) and mortgages banks (H). The bankers start each period with a basquest N^b_t and decide dividend payout $x^b_t N^b_t$ and equity allocated to each class of banks E^H_t and E^F_t. Moreover, bankers decide leverage of each class of banks taking into account regulatory constraints and costs of non compliance. Specifically, we define the leverage as

$$\phi^F_t = \frac{E^F_t}{L^F_t} \quad \text{and} \quad \phi^H_t = \frac{E^H_t}{L^H_t Q^H_t} \tag{62}$$

Banks Profits. Since we assume limited liability for the banks, we allowed that bank can make defaults. However, in contrast to the standard literature Clerc et al. (2014) or Mendicino et al. (2018), we define a compliance threshold in order to internalize the cost of being lower than it value, but above the minimal requirement in a similar way as Mendicino et al. (ming). This new threshold reflect the reputational cost of being lower than compliance target, ie, the banks in the region between both threshold still working but have to pay a cost which depends on how far is the bank from the compliance threshold.

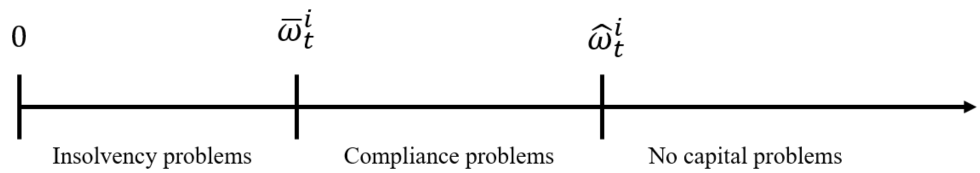
Now, following the same rationality of Bernanke et al. (1999), each of these threshold are determinate for the exogenous capital requirements targets $\bar{\phi}$ and $\hat{\phi}$ for the minimum and compliance capital requirement respectively. So, the thresholds can be find imposing the value for the idiosyncratic portfolio return ω^i_{t+1}, which is i.i.d. across banks and follows a log-normal distribution with pdf $f_b^i\left(\omega_t^i\right)$ and cdf $F_b^i\left(\omega_t^i\right)$, that satisfy

$$\bar{\omega}_{t+1}^i \tilde{R}_{t+1}^i L_t^i - R_t^{L_i}(1-\phi_t^i)L_t^i = \bar{\phi} L_t^i \qquad \Rightarrow \qquad \bar{\omega}_{t+1}^i = \frac{R_t^{L_i}(1-\phi_t^i)+\bar{\phi}}{\tilde{R}_{t+1}^i} \tag{63}$$

$$\text{Analogously} \qquad \hat{\omega}_{t+1}^i = \frac{R_t^{L_i}(1-\phi_t^i)+\hat{\phi}}{\tilde{R}_{t+1}^i} \tag{64}$$

when $\tilde{R}_t^i$ and $R_t^{L_i}$ are the return on loans i and the promised rate on bank debt i respectively determined in equilibrium (competitive banks). Then, with these threshold defined, the profits of each banks depends in which region the shocks ω_t^i lives. The Figure 11 explains the profits

Figure 11: Banks Profits with respect ω_t^i values.



Note.– This figure shows the the different areas and consequences for realizations of idiosyncratic shocks ω_t^i.

using these thresholds we can define a profit function for the banks in function of the value of ω_t^i as following

$$P_t^i(\omega_t^i) = \left\{ \begin{array}{cl} \zeta\left(R_t^{L_i}(1-\phi_t^i)+\hat{\phi}-\omega_t^i\tilde{R}_t\right)L_t^i & \text{for } \ 0 \leq \omega_t^i \leq \bar{\omega}_t^i \\ \left(\omega_t^i\tilde{R}_t - R_t^{L_i}(1-\phi_t^i) - \zeta\left(R_t^{L_i}(1-\phi_t^i)+\hat{\phi}-\omega_t^i\tilde{R}_t\right)\right)L_t^i & \text{for } \ \bar{\omega}_t^i \leq \omega_t^i \leq \hat{\omega}_t^i \\ \left(\omega_t^i\tilde{R}_t - R_t^{L_i}(1-\phi_t^i)\right)L_t^i & \text{for } \ \hat{\omega}_t^i \leq \omega_t^i \end{array} \right. \tag{65}$$

with ζ parameter to calibrated the compliance cost in steady state. The equation (65) makes the assumption that even though when the banks is in the insolvency regions, it still have to pay compliance cost. This try to replicate the reputational cost when a banks close any branch. This phenomenon, with the new compliance threshold, make our approach more realistic regarding the previous works as Clerc et al. (2014) or Mendicino et al. (2018). Then with this profit function, we can define the ex-post gross return for each banks ρ_{t+1}^i as

$$\rho_{t+1}^i \equiv \frac{\mathbb{E}_t(\Pi_{t+1}^i)}{E_t^i} = \frac{\int_0^\infty P_t^i(\omega)dF_{b,t+1}^i(\omega)}{E_t^i} \tag{66}$$

using the definition of bank profit function in (65) with the equation (66) we have

$$\begin{aligned}
\mathbb{E}_t(\Pi_{t+1}^i(\omega_t^i)) &= \int_0^\infty P_t^i(\omega)dF_{b,t+1}^i(\omega) \\
&= \int_0^{\bar{\omega}_{t+1}^i} \zeta\left(R_t^{L_i}(1-\phi_t^i)+\hat{\phi}-\omega\tilde{R}_{t+1}\right)L_t^i\, dF_{b,t+1}^i(\omega) \\
&\quad + \int_{\bar{\omega}_{t+1}^i}^{\hat{\omega}_{t+1}^i} \left(\omega\tilde{R}_{t+1}^i - R_t^{L_i}(1-\phi_t^i) - \zeta\left(R_t^{L_i}(1-\phi_t^i)+\hat{\phi}-\omega\tilde{R}_{t+1}^i\right)\right)L_t^i\, dF_{b,t+1}^i(\omega) \\
&\quad + \int_{\hat{\omega}_{t+1}^i}^{\infty} \left(\omega\tilde{R}_{t+1}^i - R_t^{L_i}(1-\phi_t^i)\right)L_t^i\, dF_{b,t+1}^i(\omega) \\
&= \int_{\bar{\omega}_{t+1}^i}^{\infty} \left(\omega\tilde{R}_{t+1}^i - R_t^{L_i}(1-\phi_t^i)\right)L_t^i\, df_{b,t+1}^i(\omega) - \zeta\int_0^{\hat{\omega}_{t+1}^i}\left(R_t^{L_i}(1-\phi_t^i)+\hat{\phi}-\omega\tilde{R}_{t+1}^i\right)L_t^i\, dF_{b,t+1}^i(\omega)
\end{aligned}$$

Now, following Bernanke et al. (1999) we can define the shares $\Gamma_b(\omega^*)$ and $G_b(\omega^*)$ as follow

$$\Gamma_b\left(\omega^*_{t+1}\right) = \int_0^{\omega^*_{t+1}} \omega^i_{t+1} F_b\left(\omega^i_{t+1}\right) d\omega^i_{t+1} + \omega^*_{t+1} \int_{\omega^*_{t+1}}^{\infty} F_b\left(\omega^i_{t+1}\right) d\omega^i_{t+1}$$

$$G_b\left(\omega^*_{t+1}\right) = \int_0^{\omega^*_{t+1}} \omega^i_{t+1} F_b\left(\omega^i_{t+1}\right) d\omega^i_{t+1}$$

when w^*_{t+1} is one of the thresholds defined in (63) and (64). Then, using the definitions of the shares above and the thresholds, the ex-post profits of banks is

$$\begin{aligned}
\mathbb{E}_t(\Pi^i_{t+1}(\omega^i_t)) &= \int_{\bar{\omega}^i_{t+1}}^{\infty} \left(\omega \tilde{R}^i_{t+1} - R^{L_i}_t(1-\phi^i_t)\right) L^i_t \, df^i_{b,t+1}(\omega) - \zeta \int_0^{\hat{\omega}^i_{t+1}} \left(R^{L_i}_t(1-\phi^i_t) + \hat{\phi} - \omega \tilde{R}^i_{t+1}\right) L^i_t \, dF^i_{b,t+1}(\omega) \\
&= \int_{\bar{\omega}^i_{t+1}}^{\infty} \left(\omega \tilde{R}^i_{t+1} - (\bar{\omega}^i_{t+1}\tilde{R}^i_{t+1} - \bar{\phi})\right) L^i_t \, df^i_{b,t+1}(\omega) - \zeta \int_0^{\hat{\omega}^i_{t+1}} \left(\hat{\omega}\tilde{R}^i_{t+1} - \omega \tilde{R}^i_{t+1}\right) L^i_t \, dF^i_{b,t+1}(\omega) \\
&= L^i_t \left(\int_{\bar{\omega}^i_{t+1}}^{\infty} (\omega - \bar{\omega}^i_{t+1})\tilde{R}^i_{t+1} \, dF^i_{b,t+1}(\omega) + \bar{\phi} \int_{\bar{\omega}^i_{t+1}}^{\infty} dF^i_{b,t+1}(\omega) - \zeta \tilde{R}^i_{t+1} \left[\hat{\omega}^i_{t+1} \int_0^{\hat{\omega}^i_{t+1}} dF^i_{b,t+1} - \int_0^{\hat{\omega}^i_{t+1}} \omega dF^i_{b,t+1} \right] \right) \\
&= L^i_t \left(\tilde{R}^i_{t+1}[1 - \Gamma_b(\bar{\omega}^i_{t+1})] + \bar{\phi}[1 - F_b(\bar{\omega}^i_{t+1})] - \zeta \tilde{R}^i_{t+1}[\hat{\omega}^i_{t+1} F_b(\hat{\omega}^i_{t+1}) - G_b(\hat{\omega}^i_{t+1})] \right)
\end{aligned}$$

when the last equality was obtained applying the definitions of shares and the fundamental theorem of calculus. Then, the ex-post gross return for each banks ρ^i_{t+1} in (66) can be written as

$$\rho^i_{t+1} \equiv \frac{\mathbb{E}_t(\Pi^i_{t+1})}{E^i_t} = \frac{\tilde{R}^i_{t+1}\left[1 - \Gamma_b(\bar{\omega}^i_{t+1}) - \zeta(\hat{\omega}^i_{t+1} - \Gamma_b(\hat{\omega}^i_{t+1}))\right] + \bar{\phi}(1 - F_b(\bar{\omega}^i_{t+1}))}{\phi^i_t} \tag{67}$$

Then, the bankers solves the following maximization problem

$$V^b_t(N^b_t) = \max_{x^b_t, E^H_t, E^F_t, \phi^F_t, \phi^H_t, N^b_{t+1}} \left\{ x^b_t N^b_t + \mathbb{E}_t[\Lambda_{t+1} V^b_{t+1}(N^b_{t+1})] \right\} \quad \text{subject to} \tag{68}$$

$$E^H_t + E^F_t + x^b_t N^b_t = N^b_t - \frac{\kappa}{2}(x^b_t - \bar{x}^b)^2 N^b_t \tag{69}$$

$$N^b_{t+1} = \rho^F_{t+1} E^F_t + \rho^H_{t+1} E^H_t \tag{70}$$

when the equations (69) and (70) corresponds to the resources constraint and the low of motion of banks net-worth respectively. It should be noted that the term $\frac{\kappa}{2}(x^b_t - \bar{x}^b)^2$ is the cost of deviating from the policy dividend target $\bar{x}^b$. So, using the Lagrange representation of the banker's problem defined in (68)-(70) and (67) we obtain

$$\begin{aligned}
\mathcal{L}(\bar{X}, \lambda^b_{1,t}, \lambda^b_{2,t}) =& x^b_t N^b_t + \mathbb{E}_t[\Lambda_{t+1} V^b_{t+1}(N^b_{t+1})] + \lambda^b_{1,t}\left(N^b_t - \frac{\kappa}{2}(x^b_t - \bar{x}^b)^2 N^b_t - E^H_t - E^F_t - x^b_t N^b_t\right) \\
&+ \lambda^b_{2,t}\left(\rho^F_{t+1} E^F_t + \rho^H_{t+1} E^H_t - N^b_{t+1}\right)
\end{aligned}$$

with $\lambda^b_{1,t}$ and $\lambda^b_{2,t}$ the Lagrange multiplier for constraints (69) and (70) respectively. Then applying the first order condition for each control variable in $\bar{X}$ we have

$$
\begin{aligned}
[x_t^b]: \quad & N_t^b - \lambda_{1,t}^b N_t^b(\kappa(x_t^b - \bar{x}^b) + 1) = 0 \\
& \Rightarrow x_t^b = \bar{x}^b + \frac{1}{\kappa}\left(\frac{1}{\lambda_{1,t}^b} - 1\right) && (71) \\
[E_t^F]: \quad & -\lambda_{1,t}^b + \lambda_{2,t}^b \mathbb{E}_t[\rho_{t+1}^F] = 0 \\
& \Rightarrow \lambda_{2,t}^b \mathbb{E}_t[\rho_{t+1}^F] = \lambda_{1,t}^b && (72) \\
[E_t^H]: \quad & \lambda_{2,t}^b \mathbb{E}_t[\rho_{t+1}^H] = \lambda_{1,t}^b && (73) \\
[\phi_t^i]: \quad & \mathbb{E}_t\left[\Lambda_{t+1}\frac{\partial V_{t+1}^b(N_{t+1}^b)}{\partial N_{t+1}^b}\frac{\partial N_{t+1}^b}{\partial \rho_{t+1}^i}\frac{\partial \rho_{t+1}^i}{\partial \phi_t^i}\right] + \lambda_{2,t}^b \mathbb{E}_t\left[\frac{\partial N_{t+1}^b}{\partial \rho_{t+1}^i}\frac{\partial \rho_{t+1}^i}{\partial \phi_t^i}\right] = 0 \\
& \Rightarrow \mathbb{E}_t\left[E_t^i \frac{\partial \rho_{t+1}^i}{\partial \phi_t^i}\right] \cdot \mathbb{E}_t\left[\Lambda_{t+1}\frac{\partial V_{t+1}^b(N_{t+1}^b)}{\partial N_{t+1}^b} + \lambda_{2,t}^b\right] = 0 && (74) \\
[N_{t+1}^b]: \quad & \mathbb{E}_t\left[\Lambda_{t+1}\frac{\partial V_{t+1}^b(N_{t+1}^b)}{\partial N_{t+1}^b}\right] - \lambda_{2,t}^b = 0 && (75)
\end{aligned}
$$

Now, we define an auxiliary function $g(N_t^b) = N_{t+1}^b$ we have

$$
\begin{aligned}
\frac{\partial V_t^b(N_t^b)}{\partial N_t^b} &= x_t^b + \Lambda_{t+1}\frac{\partial V_t^b(g(N_t^b))}{\partial g(N_t^b)}\frac{\partial g(N_t^b)}{\partial N_t^b} + \lambda_{1,t}^b\left(1 - \frac{\kappa}{2}(x_t^b - \bar{x}^2)^2 - x_t^b\right) - \lambda_{2,t}^b \frac{\partial g(N_t^b)}{\partial N_t^b} \\
&= x_t^b + \frac{\partial g(N_t^b)}{\partial N_t^b}\underbrace{\left(\Lambda_t \frac{\partial V_t^b(N_t^b)}{\partial N_t^b} - \lambda_{2,t}^b\right)}_{\text{Envelope Theorem} \Rightarrow 0} + \lambda_{1,t}^b\left(1 - \frac{\kappa}{2}(x_t^b - \bar{x}^2)^2 - x_t^b\right) \\
\Rightarrow \frac{\partial V_{t+1}^b(N_{t+1}^b)}{\partial N_{t+1}^b} &= x_{t+1}^b + \lambda_{1,t+1}^b\left(1 - \frac{\kappa}{2}(x_{t+1}^b - \bar{x}^2)^2 - x_{t+1}^b\right) && (76)
\end{aligned}
$$

Then, combining the equations (71), (72), (73), (75) and (76) we have

$$
\lambda_{1,t}^b = \mathbb{E}_t\left[\underbrace{\Lambda_{t+1}\left(\bar{x}^b + (1-\bar{x}^b)\lambda_{1,t+1}^b + \frac{1}{2\kappa}\frac{(1-\lambda_{1,t+1}^b)^2}{\lambda_{1,t+1}^b}\right)}_{\Lambda_{t+1}^b} \cdot \rho_{t+1}^i\right] \tag{77}
$$

Furthermore, from (75) and using equation (74) imply that

$$
\mathbb{E}_t\left[\frac{\partial \rho_{t+1}^i}{\partial \phi_t^i}\right] = 0 \tag{78}
$$

which imply that the bankers choose their capital ratio to maximize the expected return on equity. So, using (63),

(64) and (67) we have that

$$
\begin{aligned}
\frac{\partial \rho^i_{t+1}}{\partial \phi^i_t} =& \frac{\left(\tilde{R}^i_{t+1}\left[-\frac{\partial \Gamma_b(\bar{\omega}^i_{t+1})}{\partial \bar{\omega}^i_{t+1}}\frac{\partial \bar{\omega}^i_{t+1}}{\partial \phi^i_t} - \zeta\left(\frac{\partial \hat{\omega}^i_{t+1}}{\partial \phi^i_t} - \frac{\partial \Gamma_b(\hat{\omega}^i_{t+1})}{\partial \hat{\omega}^i_{t+1}}\frac{\partial \hat{\omega}^i_{t+1}}{\partial \phi^i_t}\right)\right] - \bar{\phi}\frac{\partial F_b(\bar{\omega}^i_{t+1})}{\partial \bar{\omega}^i_{t+1}}\frac{\partial \bar{\omega}^i_{t+1}}{\partial \phi^i_t}\right)\phi^i_t}{(\phi^i_t)^2} \\
&- \frac{\tilde{R}^i_{t+1}\left[1-\Gamma_b(\bar{\omega}^i_{t+1}) - \zeta(\hat{\omega}^i_{t+1} - \Gamma_b(\hat{\omega}^i_{t+1}))\right] + \bar{\phi}(1-F_b(\bar{\omega}^i_{t+1}))}{(\phi^i_t)^2} \\
=& \frac{\left(\tilde{R}^i_{t+1}\left[(1-F(\bar{\omega}^i_{t+1}))\frac{R^{L_i}_t}{\tilde{R}^i_{t+1}} - \zeta\left(-\frac{R^{L_i}_t}{\tilde{R}^i_{t+1}} + (1-F(\hat{\omega}^i_{t+1}))\frac{R^{L_i}_t}{\tilde{R}^i_{t+1}}\right)\right] + \bar{\phi} f_b(\bar{\omega}^i_{t+1})\frac{R^{L_i}_t}{\tilde{R}^i_{t+1}}\right)\phi^i_t}{(\phi^i_t)^2} \\
&- \frac{\tilde{R}^i_{t+1}\left[1-\Gamma_b(\bar{\omega}^i_{t+1}) - \zeta(\hat{\omega}^i_{t+1} - \Gamma_b(\hat{\omega}^i_{t+1}))\right] + \bar{\phi}(1-F_b(\bar{\omega}^i_{t+1}))}{(\phi^i_t)^2} \\
\Rightarrow \phi^i_t =& \frac{\tilde{R}^i_{t+1}\left[1-\Gamma_b(\bar{\omega}^i_{t+1}) - \zeta(\hat{\omega}^i_{t+1} - \Gamma_b(\hat{\omega}^i_{t+1}))\right] + \bar{\phi}(1-F_b(\bar{\omega}^i_{t+1}))}{1-F(\bar{\omega}^i_{t+1}) + \zeta F(\hat{\omega}^i_{t+1}) + \bar{\phi}\frac{f(\bar{\omega}^i_{t+1})}{\tilde{R}^i_{t+1}}}
\end{aligned}
$$

For completeness, notice that derivations in prior sections imply that following expressions for $\tilde{R}^j_{t+1},\ j = \{F, H\}$:

$$\tilde{R}^F_{t+1} = \left(\Gamma_e\left(\bar{\omega}^e_{t+1}\right) - \mu_e G_e\left(\bar{\omega}^e_{t+1}\right)\right)\frac{R^e_{t+1}Q^K_t K_t}{L^F_t}$$

$$\tilde{R}^H_{t+1} = \left(\Gamma_I\left(\bar{\omega}^I_{t+1}\right) - \mu_I G_I\left(\bar{\omega}^I_{t+1}\right)\right)\frac{R^H_{t+1}Q^H_t H^I_t}{Q^L_t L^H_t}$$

As with households and entrepreneurs, it is assumed that the bank idiosyncratic shock follows a log-normal distribution: $\log(\omega^j_t) \sim N(-\frac{1}{2}(\sigma^j_t)^2, (\sigma^j_t)^2)$, leading to analogous properties for $\bar{\omega}^j_t$, Γ_j and G_j.

Finally, the equations that governs the behavior of bankers are the following

$$x_t^b = \bar{x}^b + \frac{1}{\kappa}\left(\frac{1}{\lambda_{1,t}^b} - 1\right) \tag{79}$$

$$\mathbb{E}_t[\rho_{t+1}^H] = \mathbb{E}_t[\rho_{t+1}^F] \tag{80}$$

$$\lambda_{1,t}^b = \mathbb{E}_t\left[\Lambda_{t+1}\left(\bar{x}^b + (1-\bar{x}^b)\lambda_{1,t+1}^b + \frac{1}{2\kappa}\frac{(1-\lambda_{1,t+1}^b)^2}{\lambda_{1,t+1}^b}\right)\rho_{t+1}^H\right] \tag{81}$$

$$\lambda_{1,t}^b = \mathbb{E}_t\left[\Lambda_{t+1}\left(\bar{x}^b + (1-\bar{x}^b)\lambda_{1,t+1}^b + \frac{1}{2\kappa}\frac{(1-\lambda_{1,t+1}^b)^2}{\lambda_{1,t+1}^b}\right)\rho_{t+1}^F\right] \tag{82}$$

$$\phi_t^F = \frac{\tilde{R}_{t+1}^F\left[1-\Gamma_b(\bar{\omega}_{t+1}^F) - \zeta(\hat{\omega}_{t+1}^F - \Gamma_b(\hat{\omega}_{t+1}^F))\right] + \bar{\phi}(1-F_b(\bar{\omega}_{t+1}^i))}{1 - F(\bar{\omega}_{t+1}^F) + \zeta F(\hat{\omega}_{t+1}^i) + \bar{\phi}\frac{f(\bar{\omega}_{t+1}^F)}{\tilde{R}_{t+1}^F}} \tag{83}$$

$$\phi_t^H = \frac{\tilde{R}_{t+1}^H\left[1-\Gamma_b(\bar{\omega}_{t+1}^H) - \zeta(\hat{\omega}_{t+1}^H - \Gamma_b(\hat{\omega}_{t+1}^H))\right] + \bar{\phi}(1-F_b(\bar{\omega}_{t+1}^H))}{1 - F(\bar{\omega}_{t+1}^H) + \zeta F(\hat{\omega}_{t+1}^H) + \bar{\phi}\frac{f(\bar{\omega}_{t+1}^H)}{\tilde{R}_{t+1}^H}} \tag{84}$$

$$\rho_{t+1}^F = \frac{\tilde{R}_{t+1}^F\left[1-\Gamma_b(\bar{\omega}_{t+1}^F) - \zeta(\hat{\omega}_{t+1}^F - \Gamma_b(\hat{\omega}_{t+1}^F))\right] + \bar{\phi}(1-F_b(\bar{\omega}_{t+1}^i))}{\phi_t^F} \tag{85}$$

$$\rho_{t+1}^H = \frac{\tilde{R}_{t+1}^H\left[1-\Gamma_b(\bar{\omega}_{t+1}^H) - \zeta(\hat{\omega}_{t+1}^H - \Gamma_b(\hat{\omega}_{t+1}^H))\right] + \bar{\phi}(1-F_b(\bar{\omega}_{t+1}^H))}{\phi_t^H} \tag{86}$$

$$\hat{\omega}_{t+1}^F = \frac{R_t^{L_F}(1-\phi_t^F) + \hat{\phi}}{\tilde{R}_{t+1}^F} \tag{87}$$

$$\bar{\omega}_{t+1}^H = \frac{R_t^{L_H}(1-\phi_t^H) + \bar{\phi}}{\tilde{R}_{t+1}^H} \tag{88}$$

$$\bar{\omega}_{t+1}^F = \frac{R_t^{L_F}(1-\phi_t^F) + \bar{\phi}}{\tilde{R}_{t+1}^F} \tag{89}$$

$$\bar{\omega}_{t+1}^H = \frac{R_t^{L_H}(1-\phi_t^H) + \bar{\phi}}{\tilde{R}_{t+1}^H} \tag{90}$$

$$\tilde{R}_{t+1}^F = \left(\Gamma_e\left(\bar{\omega}_{t+1}^e\right) - \mu_e G_e\left(\bar{\omega}_{t+1}^e\right)\right)\frac{R_{t+1}^e Q_t^K K_t}{L_t^F} \tag{91}$$

$$\tilde{R}_{t+1}^H = \left(\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right)\frac{R_{t+1}^H Q_t^H H_t^I}{Q_t^L L_t^H} \tag{92}$$

A.4 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. Monopolistically competitive unions act as wage setters by selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a set of monopolistically competitive firms producing different varieties of a home good, Y_{jt}^H, using wholesale good X_t^Z as input; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties, X_{jt}^F; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, X_t^H, one packing the imported varieties into a composite foreign good, X_t^F, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y_t^C. This final good is purchased by households (C_t^P,C_t^I), capital and housing producers (I_t^K,I_t^H), and the government (G_t).

Similarly to Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms. Both types of firms are owned by patient households and their technology is subject to an adjustment cost. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. However, we depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment.

Finally, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and which follows an exogenous process). The total mass of firms in each sector is normalized to one.

A.4.1 Capital goods

There is a continuum of competitive capital firm producers who buy an amount I_t of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital, i.e. $K_t - (1-\delta_K)K_{t-1}$, where new units of capital are sold at price Q_t^K. As is usual in the literature, we assume that the aggregate stock of new capital considers investment adjustment costs and evolves according to following law of motion:

$$K_t = (1-\delta_K)\,K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right]\xi_t^i I_t$$

Where ξ_t^i is a shock to investment efficiency. Therefore a representative capital producer chooses how much to invest in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i}\left\{Q_{t+i}^K\left[1 - \frac{\gamma_K}{2}\left(\frac{I_{t+i}}{I_{t+i-1}} - a\right)^2\right]\xi_{t+i}^i I_{t+i} - P_{t+i}I_{t+i}\right\}$$

Discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of capital to the level of investment

$$\begin{aligned} P_t &= Q_t^K\left\{\left(1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right) - \gamma_K\left(\frac{I_t}{I_{t-1}} - a\right)\frac{I_t}{I_{t-1}}\right\}\xi_t^i \\ &\quad + E_t\left\{r_{t,t+1}Q_{t+1}^K\gamma_K\left(\frac{I_{t+1}}{I_t} - a\right)\left(\frac{I_{t+1}}{I_t}\right)^2\xi_{t+1}^i\right\} \end{aligned} \tag{93}$$

A.4.2 Housing goods

The structure of housing producers is similar to that of capital good producers with the difference that housing goods also face investment adjustment costs in the form of time to build Kydland and Prescott (1982) and Uribe and Yue (2006). As such, there is a continuum of competitive housing firm producers who authorize housing investment projects I_t^{AH} in period t, which will increase housing stock N_H periods later, the time it takes to build.[11] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before, and includes investment adjustment costs,

$$H_t = (1-\delta_H)\,H_{t-1} + \left[1 - \frac{\gamma_H}{2}\left(\frac{I_{t-N_H}^{AH}}{I_{t-N_H-1}^{AH}} - a\right)^2\right]\xi_{t-N_H}^{ih} I_{t-N_H}^{AH}$$

where ξ_t^{ih} is a shock to housing investment efficiency, and the sector covers all demand for new housing, $H_t - (1-\delta_H)H_{t-1}$, by selling units at price Q_t^H.

The firm's effective expenditure is spread out during the periods that new housing is being built. In particular, the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I_t^H = \sum_{j=0}^{N_H}\varphi_j^H I_{t-j}^{AH}$$

Where φ_j^H (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfy $\sum_{j=0}^{N_H}\varphi_j^H = 1$ and $\varphi_j^H = \rho^{\varphi H}\varphi_{j-1}^H$.[12]

[11] Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

[12] Notice that $\rho^{\varphi H} > 1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H} < 1$.

Therefore a representative housing producer chooses how much to authorize in new projects I_t^{AH} in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^H \left[1 - \frac{\gamma_H}{2} \left(\frac{I_{t-N_H+i}^{AH}}{I_{t-N_H+i-1}^{AH}} - a \right)^2 \right] \xi_{t-N_H+i}^{ih} I_{t-N_H+i}^{AH} - P_{t+i} I_{t+i}^H \right\}$$

Where discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of housing to the level of housing investment

$$\begin{aligned} E_t \sum_{j=0}^{N_H} r_{t,t+j} \varphi_j^H P_{t+j} &= E_t r_{t,t+N_H} Q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right)^2 \right] - \gamma_H \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right) \frac{I_t^{AH}}{I_{t-1}^{AH}} \right\} \xi_t^{ih} \\ &\quad + E_t r_{t,t+N_H+1} Q_{t+N_H+1}^H \left\{ \gamma_H \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} - a \right) \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} \right)^2 \xi_{t+1}^{ih} \right\} \end{aligned} \tag{94}$$

A.4.3 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts X_t^H and X_t^F, respectively, and combines them according to the following technology:

$$Y_t^C = \left[\omega^{1/\eta} \left(X_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(X_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{95}$$

where $\omega \in (0,1)$ is inversely related to the degree of home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by P_t, while the prices of the domestic and foreign inputs are P_t^H and P_t^F, respectively. Subject to the technology constraint (95), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X_t^H, X_t^F} P_t \left[\omega^{1/\eta} \left(X_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(X_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} - P_t^H X_t^H - P_t^F X_t^F$$

The first-order conditions of this problem determine the optimal input demands:

$$X_t^H = \omega \left(\frac{P_t^H}{P_t} \right)^{-\eta} Y_t^C \tag{96}$$

$$X_t^F = (1-\omega) \left(\frac{P_t^F}{P_t} \right)^{-\eta} Y_t^C \tag{97}$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t = \left[\omega \left(P_t^H \right)^{1-\eta} + (1-\omega) \left(P_t^F \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{98}$$

A.4.4 Home composite goods

A representative home composite goods firm demands home goods of all varieties $j \in [0,1]$ in amounts X_{jt}^H and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H \right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj \right]^{\frac{\epsilon_H}{\epsilon_H - 1}} \tag{99}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. Subject to the technology constraint (99), the firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ over the input demands X_{jt}^H taking prices as given:

$$\max_{X_{jt}^H} P_t^H \left[\int_0^1 \left(X_{jt}^H \right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj \right]^{\frac{\epsilon_H}{\epsilon_H - 1}} - \int_0^1 P_{jt}^H X_{jt}^H dj$$

This implies the following first-order conditions for all j:

$$\partial X_{jt}^H : \; P_t^H \left(Y_t^H \right)^{1/\epsilon_H} \left(X_{jt}^H \right)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left(\frac{P_{jt}^H}{P_t^H} \right)^{-\epsilon_H} Y_t^H \tag{100}$$

Substituting (100) into (99) yields the price of home composite goods:

$$P_t^H = \left[\int_0^1 \left(P_{jt}^H \right)^{1-\epsilon_H} dj \right]^{\frac{1}{1-\epsilon_H}} \tag{101}$$

A.4.5 Home goods of variety j

There is a continuum of j's firms, with measure one, that demand a domestic wholesale good X_t^Z and differentiate into home goods varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{102}$$

The firm producing variety j satisfies the demand given by (100) but it has monopoly power for its variety. For varieties, the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. Given that, every firm buys their input from the same wholesale market. It implies that all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{103}$$

Given nominal marginal costs $P_t^H mc_{jt}^H$, firm j chooses its price P_{jt}^H to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability $1 - \theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0, 1]$ and $1 - \kappa_H$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^H$ that maximizes the current market value of the profits generated until it can reoptimize again. [13] As the firms are owned by the households, profits are discounted using the households' stochastic discount factor for nominal payoffs, $r_{t,t+s}$. A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left(P_{jt+s}^H - P_{t+s}^H mc_{jt+s}^H \right) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left(\frac{\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H} \right)^{-\epsilon_H} Y_{t+s}^H$$

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[\left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left(P_{t+s}^H \right)^{\epsilon_H} - mc_{jt+s}^H \left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left(P_{t+s}^H \right)^{1+\epsilon_H} \right] Y_{t+s}^H$$

[13] Therefore, the following relation holds: $P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \ldots \pi_{t+s}^{I,H}$, where $\pi_t^{I,H} = \left(\pi_{t-1}^H \right)^{\kappa_H} \left(\pi_t^T \right)^{1-\kappa_H}$, $\pi_t^H = P_t^H / P_{t-1}^H$, and π_t^T denotes the inflation target in period t.

The first-order conditions determining the optimal price $\tilde{P}_t^H$ can be written as follows:[14]

$$
\begin{aligned}
0 = E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} & \left[(1-\epsilon_H) \left(\tilde{P}_t^H\right)^{-\epsilon_H} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} \right. \\
& \left. + \epsilon_H mc_{t+s}^H \left(\tilde{P}_t^H\right)^{-\epsilon_H - 1} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H} \right] Y_{t+s}^H \\
\Leftrightarrow \quad 0 = E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} & \left[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{\epsilon_H}}{P_t^H} \right. \\
& \left. - mc_{t+s}^H \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{1+\epsilon_H}}{P_t^H} \right] Y_{t+s}^H \\
\Leftrightarrow \quad 0 = E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} & \left[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} \right. \\
& \left. - mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} \right] Y_{t+s}^H
\end{aligned}
$$

where the second step follows from multiplying both sides by $-\tilde{P}_t^H/(P_t^H \epsilon_H)$, and the third by defining $\tilde{p}_t^H = \tilde{P}_t^H / P_t^H$. The first-order condition can be rewritten in recursive form as follows, defining $F_t^{H_1}$ as

$$
\begin{aligned}
F_t^{H_1} &= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s+1}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \right. \\
&\qquad \left. \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{\epsilon_H} Y_{t+s+1}^H \right\} \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} F_{t+1}^{H_1} \right\} \qquad (104)
\end{aligned}
$$

and, analogously, $F_t^{H_2}$ as

$$
\begin{aligned}
F_t^{H_2} &= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} mc_{t+s+1}^H \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} mc_{t+s+1}^H \right. \\
&\qquad \left. \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \right\} \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} F_{t+1}^{H_2} \right\} \qquad (105)
\end{aligned}
$$

[14]Notice that the subscript j has been removed from $\tilde{P}_t^H$; this simplifies notation and underlines that the prices chosen by all firms j that reset prices optimally in a given period are equal as they face the same problem by (103).

such that

$$F_t^{H_1} = F_t^{H_2} = F_t^H \tag{106}$$

Using (101), we have

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H} \right)^{1-\epsilon_H} dj \\ &= (1-\theta_H) \left(\tilde{p}_t^H \right)^{1-\epsilon_H} + \theta_H \left(\frac{P_{t-1}^H \pi_t^{I,H}}{P_t^H} \right)^{1-\epsilon_H} \\ &= (1-\theta_H) \left(\tilde{p}_t^H \right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \end{aligned} \tag{107}$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of aggregate prices in period $t-1$, though with total mass reduced to θ_H.

A.4.6 Wholesale Domestic Goods

There is a representative firm producing a homogeneous wholesale home good, combining capital and labor according to the following technology:

$$Y_t^Z = z_t K_{t-1}^{\alpha} \left(A_t \widetilde{n}_t \right)^{1-\alpha} \tag{108}$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t. Production of the wholesale good composite labor services $\widetilde{n}_t$ and capital K_{t-1}. Additionally, following Lechthaler et al. (2010), the firm faces a quadratic adjustment costs of labor which is a function of parameter γ_n, and of aggregate wholesale domestic goods $\widetilde{Y_t}^Z$, which in equilibrium are equal to Y_t^Z and which the representative firm takes as given. In a first stage, the firm hires composite labor and rents capital to solve the following problem:

$$\begin{aligned} \min_{\widetilde{n}_{t+s}, K_{t+s-1}} \sum_{s=0}^{\infty} r_{t,t+s} &\left\{ W_{t+s} \widetilde{n}_{t+s} + \frac{\gamma_n}{2} \left(\frac{\widetilde{n}_{t+s}}{\widetilde{n}_{t+s-1}} - 1 \right)^2 \widetilde{Y_{t+s}}^Z P_t^Z + R_t K_{t+s-1} \right\} \\ s.t. \quad Y_{t+s}^Z &= X_{t+s}^Z = z_{t+s} K_{t+s-1}^{\alpha} \left(A_{t+s} \widetilde{n}_{t+s} \right)^{1-\alpha} \end{aligned}$$

Then, the optimal capital and labor demands are given by:

$$\widetilde{n}_t = (1-\alpha) \left\{ \frac{mc_t^Z Y_t^Z}{W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) \widetilde{Y_t}^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) \widetilde{Y}_{t+1}^Z P_{t+1}^Z} \right\} \tag{109}$$

$$K_{t-1} = \alpha \left(\frac{mc_t^Z}{R_t^k} \right) Y_t^Z \tag{110}$$

Where mc_t^Z is the lagrangian multiplier on the production function and $r_{t,t+1}$ the households' stochastic discount factor between periods t and $t+1$. The, combining both optimality conditions:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) \widetilde{Y}_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) \widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}$$

Substituting (109) and (110) into (108) we obtain an expression for the real marginal cost in units of the wholesale domestic good:

$$mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{\left(R_t^k\right)^\alpha}{z_t A_t^{1-\alpha}} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right) \widetilde{Y}_t^Z P_t^Z \right.$$
$$\left. - r_{t,t+1}\gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right) \widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}^{1-\alpha}$$

In a second stage, the wholesale firm maximize its profits from the production of Y_t^Z, which is sold as X_t^Z at P_t^Z. The problem is:

$$\max_{Y_t^Z} \left(P_t^Z - mc_t^Z\right) Y_t^Z$$

The first-order condition implies that

$$P_t^Z = mc_t^Z.$$

A.4.7 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts X_{jt}^F and combines them according to the technology

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F\right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{111}$$

with $\epsilon_F > 0$. Let P_{jt}^F denote the price of the foreign good of variety j. Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} Y_t^F \tag{112}$$

for all j, and substituting (112) into (111) yields the price of foreign composite goods:

$$P_t^F = \left[\int_0^1 \left(P_{jt}^F\right)^{1-\epsilon_F} dj\right]^{\frac{1}{1-\epsilon_F}} \tag{113}$$

A.4.8 Foreign goods of variety j

Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad and convert this good into varieties Y_{jt}^F that are sold domestically, and where total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ cointegrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. The firm producing variety j satisfies the demand given by (112) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{114}$$

Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability $1-\theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_F \in [0,1]$ and $1-\kappa_F$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^F$ that maximizes the current market value of the profits generated until it can reoptimize.[15] The solution to this problem is analogous to the case of domestic varieties, implying the

[15] As in the home varieties case, the following relation holds: $P_{jt+s}^F = \tilde{P}_{jt}^F \pi_{t+1}^{I,F} \ldots \pi_{t+s}^{I,F}$, where $\pi_t^{I,F} = (\pi_{t-1}^F)^{\kappa_F} (\pi_t^T)^{1-\kappa_F}$, and, in turn, $\pi_t^F = P_t^F / P_{t-1}^F$.

first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \tag{115}$$

where, defining $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F\right)^{1-\epsilon_F} Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F\right)^{\epsilon_F} F_{t+1}^{F_1} \right\}$$

and

$$F_t^{F_2} = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} F_{t+1}^{F_2} \right\}$$

Using (113), we further have

$$1 = (1 - \theta_F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{116}$$

A.4.9 Wages

Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$. The production function, variety i demand, and aggregate nominal wage are respectively given by:

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}} di \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{117}$$

$$n_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_W} \widetilde{n}_t \tag{118}$$

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon_W} di \right]^{\frac{1}{1-\epsilon_W}}. \tag{119}$$

Regarding the supply of differentiated labor, as in Erceg et al. (2010), there is a continuum of monopolistically competitive unions indexed by $i \in [0,1]$, which act as wage setters for the differentiated labor services supplied by households. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i,t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1-\wp_U) n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

The union supplying variety i satisfies the demand given by (118) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability $1 - \theta_W$. The wages of unions that cannot optimally adjust, are indexed to a weighted average of past and steady state productivity and inflation, with a gross growth rate of

$$\pi_t^{I,W} \equiv a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W}$$

Where $\Gamma_{t,s}^W = \Pi_{i=1}^s \pi_{t+i}^{I,W}$ is the growth of indexed wages s periods ahead of t. A union reoptimizing in period t chooses the wage $\widetilde{W}_t$ (equal for patient and impatient households) that maximizes the households' discounted lifetime utility. This union weights the benefits of wage income by considering the agents' marginal utility of consumption –which will usually differ between patient and impatient households– and weighs each household equally by considering a lagrangian multiplier of $\lambda_t^W = \left(\lambda_t^P + \lambda_t^I\right)/2$, with $\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\lambda_t^R$. We assume, for the sake of simplicity, that $\beta_W = (\beta_P + \beta_I)/2$ with $\beta_P = \wp_U \beta_U + (1-\wp_U)\beta_R$, and $\Theta_t = \left(\Theta_t^P + \Theta_t^I\right)/2$ with $\Theta_t^P = \wp_U \Theta_t^U + (1-\wp_U)\Theta_t^R$.

All things considered, taking the aggregate nominal wage as given, the union i's maximization problem can be expressed as

$$\max_{\widetilde{W}_t(i)} E_t \sum_{s=0}^{\infty} (\beta_U \theta_W)^s \varrho_{t+s} \left(\frac{\lambda^U_{t+s} A^{-\sigma}_{t+s}}{P_{t+s}} \widetilde{W}_t \Gamma^W_{t,s} n_{t+s}(i) - \Theta_{t+s} (A_{t+s})^{1-\sigma} \xi^n_{t+s} \frac{n_{t+s}(i)^{1+\varphi}}{1+\varphi} \right),$$

$$\text{s.t.} \quad n_{t+s}(i) = \left(\frac{\widetilde{W}_t \Gamma^W_{t,s}}{W_{t+s}} \right)^{-\epsilon_W} \widetilde{n}_{t+s},$$

Which, after some derivation, results in the FOCs in a recursive formulation:

$$f^{W1}_t = \tilde{w}_t^{1-\epsilon_W} \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a^{-\sigma}_{t+1} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda^U_{t+1}}{\lambda^U_t} \frac{\pi^W_{t+1}}{\pi_{t+1}} \left(\frac{\pi^{\widetilde{W}}_{t+1}}{\pi^{I,W}_{t+1}} \right)^{\epsilon_W - 1} f^{W1}_{t+1} \right\}$$

$$f^{W2}_t = \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc^W_t \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a^{-\sigma}_{t+1} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda^U_{t+1}}{\lambda^U_t} \frac{\pi^W_{t+1}}{\pi_{t+1}} \left(\frac{\pi^{\widetilde{W}}_{t+1}}{\pi^{I,W}_{t+1}} \right)^{\epsilon_W(1+\varphi)} f^{W2}_{t+1} \right\}$$

Where $f^{W1}_t = f^{W2}_t = f^W_t$ are the LHS and RHS of the FOC respectively, $mc^W_t = -{}^{(U_n/U_C)}\!/\!_{(W_t/A_t P_t)} = {}^{\xi^n_t(\widetilde{n}_t)^{\varphi}}\!/\!_{\lambda^U_t}(\frac{A_t P_t}{W_t})\Theta_t$, is the gap with the efficient allocation when wages are flexible[16], $\pi^W_{t+1} = {}^{W_{t+1}}\!/\!_{W_t}$, $\pi^{\widetilde{W}}_{t+1} = {}^{\widetilde{W}_{t+1}}\!/\!_{\widetilde{W}_t}$ and $\tilde{w}_t = \widetilde{W}_t / W_t$.

Further, let $\Psi^W(t)$ denote the set of labor markets in which wages are not reoptimized in period t. By (119), the aggregate wage index W_t evolves as follows:

$$\begin{aligned}
(W_t)^{1-\epsilon_W} = \int_0^1 W_t(i)^{1-\epsilon_W} di &= (1-\theta_W) \left(\widetilde{W}_t \right)^{1-\epsilon_W} + \int_{\Psi^W(t)} \left[W_{t-1}(i) \pi^{I,W}_t \right]^{1-\epsilon_W} di, \\
&= (1-\theta_W) \left(\widetilde{W}_t \right)^{1-\epsilon_W} + \theta_W \left[W_{t-1} \pi^{I,W}_t \right]^{1-\epsilon_W},
\end{aligned}$$

or, dividing both sides by $(W_t)^{1-\epsilon_W}$:

$$1 = (1-\theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi^{I,W}_t}{\pi^W_t} \right)^{1-\epsilon_W}.$$

The third equality above follows from the fact that the distribution of wages that are not reoptimized in period t corresponds to the distribution of effective wages in period $t-1$, though with total mass reduced to θ_W.

Finally, the clearing condition for the labor market is

$$n_t = \int_0^1 n_t(i)\, di = \widetilde{n}_t \int_0^1 \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_W} di = \widetilde{n}_t \Xi^W_t,$$

Where Ξ^W_t is a wage dispersion term that satisfies

$$\Xi^W_t = (1-\theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_W \left(\frac{\pi^{I,W}_t}{\pi^W_t} \right)^{-\epsilon_W} \Xi^W_{t-1}.$$

A.4.10 Commodities

We assume the country receives an exogenous and stochastic endowment of commodities Y^{Co}_t. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P^{Co\star}_t$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

[16] U_n and U_C are the first derivatives of the utility function with respect to labor and consumption respectively.

A.5 Fiscal and monetary policy

The government consumes an exogenous stream of final goods G_t, pays through an insurance agency IA_t for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households T_t^P, and issues one-period bonds BS_t^G and long-term bonds BL_t^G. Hence, the government satisfies the following period-by-period constraint:

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \tag{120}$$

where

$$T_t = \alpha^T GDPN_t + \epsilon_t \left(BS_{SS}^G - BS_t^G + Q_{SS}^{BL} BL_{SS}^G - Q_t^{BL} BL_t^G \right) \tag{121}$$

and

$$IA_t = \gamma_D PD_t^D R_{t-1}^D D_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} Q_t^{BB} BB_{t-1}^{Pr} \tag{122}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G. In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\,\pi_t + \alpha_E \mathbb{E}_t \{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{GDP_t / GDP_{t-1}}{a} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{123}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[17]

A.6 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. That is, domestic and foreign prices of both goods are identical when expressed in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$ from (114). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t} \frac{mc_t^F}{\xi_t^m} \tag{124}$$

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{125}$$

where $\pi_t^s = S_t / S_{t-1}$. Foreign demand for the home composite good $X_t^{H\star}$ is given by

$$X_t^{H\star} = \left(\frac{P_t^H}{S_t P_t^\star} \right)^{-\eta^\star} Y_t^\star \tag{126}$$

with $\eta^\star > 0$ and where $Y_t^\star$ denotes foreign aggregate demand or GDP. Both $Y_t^\star$ and $\pi_t^\star$ evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position (expressed as a ratio of nominal GDP):

$$R_t^\star = R_t^W \exp\left\{ -\frac{\phi^\star}{100} \left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b} \right) \right\} \xi_t^R z_t^R \tag{127}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

[17] We do not need a time-varying target, so we will set it to a constant.

A.6.1 Aggregation across patient households

Aggregate variables add up the per-capita amounts from unrestricted and restricted patient households, according to their respective mass $\wp_U$ and $1-\wp_U$:

$$C_t^P = \wp_U C_t^U + (1-\wp_U)\, C_t^R$$

$$H_t^P = \wp_U H_t^U + (1-\wp_U)\, H_t^R$$

$$n_t^P = \wp_U n_t^U + (1-\wp_U)\, n_t^R$$

$$n_t^U = n_t^R$$

$$D_t^{Tot} = \wp_U D_t^U$$

$$B_t^{*,Tot} = \wp_U B_t^{\star,U}$$

$$BS_t^{Pr} = \wp_U BS_t^U$$

$$BL_t^{Pr} = \wp_U BL_t^U + (1-\wp_U)\, BL_t^R$$

$$BB_t^{Pr} = \wp_U BB_t^U$$

A.6.2 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + {}^{\Upsilon_t}/_{P_t} \tag{128}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and the cost of adjusting labor.

$$\Upsilon_t = \begin{array}{c} \gamma_D PD_t^B R_{t-1}^D D_{t-1}^{Tot} + \gamma_D PD_t^B Q_t^{BB} R_t^{BB} BB_{t-1}^{Pr} + \mu_e G_e\left(\bar{\omega}_t^e\right) R_t^e Q_{t-1}^K K_{t-1} + \mu_I G_I\left(\bar{\omega}_t^I\right) R_t^H Q_{t-1}^H H_{t-1}^I \\ +\mu_H G_H\left(\bar{\omega}_t^H\right) \tilde{R}_t^H Q_{t-1}^L L_{t-1}^H + \mu_F G_F\left(\bar{\omega}_t^F\right) \tilde{R}_t^F L_{t-1}^F \\ +\frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 Y_t^Z + Q_t^L (L_t^H - \kappa L_{t-1}^H)\left[\frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right] \end{array}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{129}$$

and

$$Y_t^F = X_t^F \tag{130}$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^H = X_{jt}^H \tag{131}$$

and

$$Y_{jt}^F = X_{jt}^F \tag{132}$$

for all j.

In the market for the wholesale domestic good, we have

$$Y_t^Z = X_t^Z \tag{133}$$

Finally, in the market for housing, demand from both households must equal supply from housing producers:

$$H_t = H_t^P + H_t^I \tag{134}$$

A.6.3 Factor market clearing

In the market for labor, the clearing conditions are:

$$n_t^P + n_t^I = n_t = \widetilde{n}_t \Xi_t^W \tag{135}$$

$$n_t^P = n_t^I = \frac{n_t}{2} \tag{136}$$

Combining (110) and (109), the capital-labor ratio satisfies:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) Y_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) Y_{t+1}^Z P_{t+1}^Z \right\} \tag{137}$$

A.6.4 Deposits clearing

Bank F takes deposits, and its demand must equal the supply from unrestricted households:

$$D_t^F = D_t^{Tot} \tag{138}$$

A.6.5 Domestic bonds clearing

The aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \tag{139}$$

$$BS_t^{Pr} + BS_t^G = 0 \tag{140}$$

Where BL_t^{CB} is an exogenous process that represents the long-term government bond purchases done by the Central Bank.

A.6.6 No-arbitrage condition in bond markets

The no-arbitrage condition implies the following relation between short and long-tem interest rates:

$$R_t \left(\frac{1+\zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} \left(\frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \left(\mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \right)^{-1}$$

which can be further rearranged (up to a first order) by using the definition of R_t^{BL}

$$R_t \left(1+\zeta_t^L\right) \quad \approx \quad \mathbb{E}_t \left\{ \left(\frac{Q_{t+1}^{BL}}{Q_t^{BL}} R_{t+1}^{BL} \right) \right\} \tag{141}$$

A.6.7 Inflation and relative prices

The following holds for $j = H, F$:

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

A.6.8 Aggregate supply

Using the productions of different varieties of home goods (102)

$$\int_0^1 Y_{jt}^H dj = X_t^Z$$

Integrating (131) over j and using (100) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 \left(p_{jt}^H\right)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = X_t^Z$$

where Ξ_t^H is a price dispersion term satisfying

$$\begin{aligned}
\Xi_t^H &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} dj \\
&= (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H
\end{aligned}$$

A.6.9 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to $Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t$. The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{142}$$

Integrating (132) over j and using (112) allows us to write imports as

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where Ξ_t^F is a price dispersion term satisfying

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real GDP as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP, Y_t^{NoCo}, is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t\left(C_t^P + C_t^I + I_t + I_t^H + G_t\right) + TB_t \tag{143}$$

Note that by combining (143) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods, (128)-(129), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

A.6.10 Balance of payments

Aggregate nominal profits, dividends, rents and taxes are given by

$$\begin{aligned} \Psi_t =& \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\ & \underbrace{+\int_0^1 Y_{jt}^H \left(P_{jt}^H - P_t^Z\right) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 \left(P_{jt}^F Y_{jt}^F - S_t P_t^{M\star} Y_{jt}^F\right) dj}_{\int_0^1 \Pi_{jt}^F dj} \\ & \underbrace{+Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) - P_t I_t}_{\Pi_t^I} + \underbrace{Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) - P_t I_t^H}_{\Pi_t^{IH}} + \underbrace{\left(P_t^Z - mc_t^Z\right) Y_t^Z}_{\Pi_t^Z} \\ & \underbrace{+\zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U}_{\Pi_t^F} + C_t^e + C_t^b + S_t REN_t^* - T_t \\ =& P_t \left(C_t + G_t\right) + \Upsilon_t + P_t^H X_t^{H\star} - S_t P_t^{M\star} M_t - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) \\ & +Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U \\ =& P_t \left(C_t + G_t\right) + \Upsilon_t + TB_t - S_t P_t^{Co\star} Y_t^{Co} - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) \\ & +Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U \end{aligned}$$

Where the second equality uses the market clearing conditions (128)-(140), and the third equality uses the definition of the trade balance, (142). Substituting out Ψ_t in the households' budget constraint (41) and using the government's budget constraint (120) to substitute out taxes T_t shows that the net foreign asset position evolves according to

$$S_t B_t^\star = S_t B_{t-1}^\star R_{t-1}^\star + TB_t + S_t REN_t^* - (1-\chi) S_t P_t^{Co\star} Y_t^{Co}$$

B Stationary Equilibrium Conditions

In the model described in the previous sections, real variables in uppercase contain a unit root in equilibrium due to the presence of the non-stationary productivity vector A_t. Uppercase nominal variables contain an additional unit root given by the non-stationarity of the price level. In this section we show the stationary version of the model, where we define $a_t = A_t/A_{t-1}$, and all lowercase variables denote the stationary counterpart of the original variables, obtained by dividing them by its co-integration vector(A_t or P_t).

The rational expectations equilibrium of the stationary version of the model is then the set of sequences for the endogenous variables such that for a given set of initial values and exogenous processes the following conditions are satisfied:

B.1 Patient Households

B.1.1 Unrestricted (U)

$$\hat{c}_t^U = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^U}{a_t} - \phi_{hh} \frac{h_{t-2}^U}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

$$\lambda_t^U = \left(\hat{c}_t^U \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^U}{\left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{2}$$

$$\varrho_t \lambda_t^U q_t^H = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^U a_{t+1} \right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^U a_{t+1}}{\xi_{t+1}^h \left(h_t^U - \phi_{hh} \frac{h_{t-1}^U}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1 - \delta_H)\, \lambda_{t+1}^U a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{3}$$

$$\varrho_t \lambda_t^U = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{4}$$

$$\varrho_t \lambda_t^U = \beta_U \mathbb{E}_t \left\{ \frac{\tilde{R}_{t+1}^D}{\pi_{t+1}} \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \right\} \tag{5}$$

$$\varrho_t \lambda_t^U = \beta_U R_t^\star \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} a_{t+1}^{-\sigma} \right\} \tag{6}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L \right) q_t^{BL} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} R_{t+1}^{BL} q_{t+1}^{BL} \right\} \tag{7}$$

$$\varrho_t \lambda_t^U \left(1 + \zeta_t^L \right) q_t^{BB} = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U a_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} q_{t+1}^{BB} \right\} \tag{8}$$

B.1.2 Restricted (R)

$$\hat{c}_t^R = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^R}{a_t} - \phi_{hh} \frac{h_{t-2}^R}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{9}$$

$$\lambda_t^R = \left(\hat{c}_t^R \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^R}{\left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{10}$$

$$\varrho_t \lambda_t^R q_t^H = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{c}_{t+1}^R a_{t+1} \right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^R a_{t+1}}{\xi_{t+1}^h \left(h_t^R - \phi_{hh} \frac{h_{t-1}^R}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + (1 - \delta_H)\, \lambda_{t+1}^R a_{t+1}^{-\sigma} q_{t+1}^H \right\} \tag{11}$$

$$\varrho_t \lambda_t^R q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^R q_{t+1}^{BL} R_{t+1}^{BL} a_{t+1}^{-\sigma} \right\} \tag{12}$$

$$q_t^{BL} bl_t^R + c_t^R + q_t^H h_t^R = q_t^{BL} R_t^{BL} \frac{bl_{t-1}^R}{a_t} + w_t n_t^R + q_t^H (1 - \delta_H) \frac{h_{t-1}^R}{a_t} \tag{13}$$

B.2 Impatient Households

$$\frac{R_t^H}{\pi_t} = \frac{q_t^H (1 - \delta_H)}{q_{t-1}^H} \tag{14}$$

$$\hat{c}_t^I = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(\frac{h_{t-1}^I}{a_t} - \phi_{hh} \frac{h_{t-2}^I}{a_t a_{t-1}} \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{15}$$

$$\lambda_t^I = \left(\hat{c}_t^I \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}_t^I}{\left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{16}$$

$$\hat{q}_t^L = \left(1 - \frac{\kappa l_{t-1}^H}{l_t^H a_t} \right) q_t^L + \left(\frac{\kappa l_{t-1}^H}{l_t^H a_t} \right) \hat{q}_{t-1}^L \tag{17}$$

$$\bar{\omega}_t^I = \frac{\hat{R}_t^I \hat{q}_t^L l_{t-1}^H}{R_t^H q_{t-1}^H h_{t-1}^I} \pi_t \tag{18}$$

$$R_t^I = \frac{1}{q_t^L} + \kappa \tag{19}$$

$$\hat{R}_t^I = \frac{1 + \kappa \hat{q}_{t-1}^L}{\hat{q}_t^L} \tag{20}$$

$$\varrho_t \lambda_t^I q_t^H = \mathbb{E}_t \left\{ \begin{array}{c} \beta_I \varrho_{t+1} \left(\left(\hat{c}_{t+1}^I a_{t+1} \right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{c}_{t+1}^I a_{t+1}}{\xi_{t+1}^h \left(h_t^I - \phi_{hh} \frac{h_{t-1}^I}{a_t} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h + \lambda_{t+1}^I a_{t+1}^{-\sigma} \left[1 - \Gamma_I \left(\bar{\omega}_{t+1}^I \right) \right] \frac{R_{t+1}^H}{\pi_{t+1}} q_t^H \right) \\ + \varrho_t \lambda_t^H \left[1 - \Gamma_H \left(\bar{\omega}_{t+1}^H \right) \right] \left[\Gamma_I \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I \left(\bar{\omega}_{t+1}^I \right) \right] R_{t+1}^H q_t^H \end{array} \right\} \tag{21}$$

$$\beta_I = \mathbb{E}_t \left\{ \frac{\varrho_t \lambda_t^H \pi_{t+1}}{\varrho_{t+1} \lambda_{t+1}^I a_{t+1}^{-\sigma}} \left[1 - \Gamma_H \left(\bar{\omega}_{t+1}^H \right) \right] \frac{\left[\Gamma_I' \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I' \left(\bar{\omega}_{t+1}^I \right) \right]}{\Gamma_I' \left(\bar{\omega}_{t+1}^I \right)} \right\} \tag{22}$$

$$c_t^I + q_t^H h_t^I - q_t^L (l_t^H - \frac{\kappa l_{t-1}^H}{a_t}) \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_t - \bar{a})^2 \right] - \frac{\kappa l_{t-1}^H \hat{q}_{t-1}^L}{a_t} = \frac{w_t n_t}{2} + \left[1 - \Gamma_I \left(\bar{\omega}_t^I \right) \right] \frac{R_t^H q_{t-1}^H h_{t-1}^I}{a_t \pi_t} \tag{23}$$

$$\begin{aligned} \varrho_t q_t^L \left\{ \lambda_t^I \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_t - \bar{a})^2 \right] - \lambda_t^I \nabla \tilde{l}_t \gamma_L (\nabla \tilde{l}_t - \bar{a}) - \lambda_t^H \rho_{t+1}^H \phi_H \right\} = \ldots \\ \ldots\ \beta_I \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^I a_{t+1}^{-\sigma} \left[\kappa q_{t+1}^L \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_{t+1} - \bar{a})^2 \right] - q_{t+1}^L \nabla \tilde{l}_{t+1} \gamma_L (\nabla \tilde{l}_{t+1} - \bar{a})(\nabla \tilde{l}_{t+1} + \kappa) - \kappa \hat{q}_t^L \right] \right\} \end{aligned} \tag{24}$$

$$PD_t^I = F_I \left(\bar{\omega}_t^I \right) \tag{25}$$

B.3 Entrepreneurs

$$q_t^K k_t = n_t^e + l_t^F \tag{26}$$

$$\frac{R_t^e}{\pi_t} = \frac{r_t^k + (1 - \delta_K)\, q_t^K}{q_{t-1}^K} \tag{27}$$

$$\bar{\omega}_t^e = \frac{R_{t-1}^L l_{t-1}^F}{R_t^e q_{t-1}^K k_{t-1}} \tag{28}$$

$$c_t^e = \chi_e \xi_t^{\chi_e} \psi_t^e \tag{29}$$

$$n_t^e = \left(1 - \chi_e \xi_t^{\chi_e} \right) \psi_t^e \tag{30}$$

$$\psi_t^e a_t \pi_t = \left[1 - \Gamma_e \left(\bar{\omega}_t^e \right) \right] R_t^e q_{t-1}^K k_{t-1} \tag{31}$$

$$(1 - \Gamma_{t+1}^e) = \lambda_t^e \left(\frac{\rho_{t+1}^F \phi_t^F}{R_{t+1}^e} - \left(1 - \Gamma_{t+1}^F \right) \left[\Gamma_{t+1}^e - \mu^e G_{t+1}^e \right] \right) \tag{32}$$

$$\Gamma_{t+1}^{e'} = \lambda_t^e \left(1 - \Gamma_{t+1}^F \right) \left[\Gamma_{t+1}^{e'} - \mu^e G_{t+1}^{e'} \right] \tag{33}$$

$$PD_t^e = F_e \left(\bar{\omega}_t^e \right) \tag{34}$$

B.4 Bankers and Banking System

$$\mathbb{E} \left[\rho_{t+1}^F \right] = \xi_t^{b,roe} \mathbb{E} \left[\tilde{\rho}_{t+1}^H \right] \tag{35}$$

$$c_t^b = \xi_t^{\chi_b} \chi_b \psi_t^b \tag{36}$$

$$n_t^b = \left(1 - \xi_t^{\chi_b} \chi_b\right) \psi_t^b \tag{37}$$

$$\psi_t^b a_t \pi_t = \rho_t^F e_{t-1}^F + \tilde{\rho}_t^H e_{t-1}^H \tag{38}$$

$$n_t^b = e_t^F + e_t^H \tag{39}$$

$$PD_t^D = \frac{Q_{t-1}^{BB} BB_{t-1} PD_t^H + d_{t-1}^{Tot} PD_t^F}{Q_{t-1}^{BB} BB_{t-1} + d_{t-1}^{Tot}} \tag{40}$$

B.5 F Banks

$$d_t^F + e_t^F = l_t^F \tag{41}$$

$$\bar{\omega}_t^F = \left(1 - \phi_{F,t-1}\right) \frac{R_{t-1}^D}{\tilde{R}_t^F} \tag{42}$$

$$e_t^F = \phi_{F,t} l_t^F \tag{43}$$

$$\rho_t^F = \left[1 - \Gamma_F\left(\bar{\omega}_t^F\right)\right] \frac{\tilde{R}_t^F}{\phi_{F,t-1}} \tag{44}$$

$$\tilde{R}_t^F = \left[\Gamma_e\left(\bar{\omega}_t^e\right) - \mu_e G_e\left(\bar{\omega}_t^e\right)\right] \frac{R_t^e q_{t-1}^K k_{t-1}}{l_{t-1}^F} \tag{45}$$

$$PD_t^F = F_F\left(\bar{\omega}_t^F\right) \tag{46}$$

B.6 H Banks

$$q_t^{BB} bb_t^{Pr} + e_t^H = \hat{q}_t^L l_t^H \tag{47}$$

$$\bar{\omega}_t^H = \left(1 - \phi_{H,t-1}\right) \frac{R_t^{BB} q_t^{BB}}{\tilde{R}_t^H q_{t-1}^{BB}} \pi_t \tag{48}$$

$$e_t^H = \phi_H \hat{q}_t^L l_t^H \tag{49}$$

$$\rho_t^H = \left[1 - \Gamma_H\left(\bar{\omega}_t^H\right)\right] \frac{\tilde{R}_t^H}{\phi_{H,t-1}} \tag{50}$$

$$\tilde{\rho}_t^H = (1 - \kappa)\, \rho_t^H + \kappa \mathbb{E}\left[\tilde{\rho}_{t+1}^H\right] \tag{51}$$

$$\tilde{R}_t^H = \left[\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right)\right] \frac{R_t^H q_{t-1}^H h_{t-1}^I}{\hat{q}_{t-1}^L l_{t-1}^H} \tag{52}$$

$$PD_t^H = F_H\left(\bar{\omega}_t^H\right) \tag{53}$$

B.7 Capital and Housing Goods

$$k_t = (1 - \delta_K) \frac{k_{t-1}}{a_t} + \left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}} a_t - a\right)^2\right] \xi_t^i i_t \tag{54}$$

$$\begin{aligned} 1 \quad &= \quad q_t^K \left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}} a_t - a\right)^2 - \gamma_K \left(\frac{i_t}{i_{t-1}} a_t - a\right) \frac{i_t}{i_{t-1}} a_t\right] \xi_t^i \\ &+ \quad \beta_P \mathbb{E}_t \left\{\frac{\varrho_{t+1} \lambda_{t+1}^P}{\varrho_t \lambda_t^P} a_{t+1}^{-\sigma} q_{t+1}^K \gamma_K \left(\frac{i_{t+1}}{i_t} a_{t+1} - a\right)\left(\frac{i_{t+1}}{i_t} a_{t+1}\right)^2 \xi_{t+1}^i\right\} \end{aligned} \tag{55}$$

$$h_t = (1 - \delta_H) \frac{h_{t-1}}{a_t} + \left[1 - \frac{\gamma_H}{2}\left(\frac{i_{t-N_H}^{AH}}{i_{t-N_H-1}^{AH}} a_t - a\right)^2\right] \xi_{t-N_H}^{ih} \frac{i_{t-N_H}^{AH}}{\prod_{i=0}^{N_H-1} a_{t-j}} \tag{56}$$

$$
\begin{aligned}
0 \;=\; & E_t \sum_{j=0}^{N_H} \beta_P^j \varrho_{t+j} \lambda_{t+j}^P \varphi_j^H \prod_{i=j+1}^{N_H} \left(a_{t+i}^{\sigma}\right) \\
- \; & E_t \beta_P^{N_H} \varrho_{t+N_H} \lambda_{t+N_H}^P q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right)^2 \right] - \gamma_H \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right) \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t \right\} \xi_t^{ih} \\
- \; & E_t \beta_P^{N_H+1} \varrho_{t+N_H+1} \lambda_{t+N_H+1}^P q_{t+N_H+1}^H a_{t+N_H+1}^{-\sigma} \left\{ \gamma_H \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} - a \right) \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} \right)^2 \xi_{t+1}^{ih} \right\}
\end{aligned} \tag{57}
$$

$$
i_t^H = \sum_{j=0}^{N_H} \varphi_j^H \frac{i_{t-j}^{AH}}{\prod_{i=0}^{j-1} a_{t-j}} \tag{58}
$$

B.8 Final Goods

$$
y_t^C = \left[\omega^{1/\eta} \left(x_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(x_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{59}
$$

$$
x_t^F = (1-\omega) \left(p_t^F \right)^{-\eta} y_t^C \tag{60}
$$

$$
x_t^H = \omega \left(p_t^H \right)^{-\eta} y_t^C \tag{61}
$$

B.9 Home Goods

$$
f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \right)^{1-\epsilon_H} y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\pi_{t+1}^H \right)^{\epsilon_H} f_{t+1}^H \right\} \tag{62}
$$

$$
f_t^H = \left(\tilde{p}_t^H \right)^{-\epsilon_H} mc_t^H y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\pi_{t+1}^H \right)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{63}
$$

$$
1 = (1-\theta_H) \left(\tilde{p}_t^H \right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \tag{64}
$$

$$
\pi_t^{I,H} = \left(\pi_{t-1}^H \right)^{\kappa_H} \left(\pi^T \right)^{1-\kappa_H} \tag{65}
$$

$$
mc_t^H = \frac{p_t^Z}{p_t^H} \tag{66}
$$

B.10 Wholesale Domestic Goods

$$
\begin{aligned}
mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha}{z_t} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\}^{1-\alpha}
\end{aligned} \tag{67}
$$

$$
\begin{aligned}
\frac{k_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) r_t^k} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\} a_t
\end{aligned} \tag{68}
$$

$$
p_t^Z = mc_t^Z \tag{69}
$$

B.11 Foreign Goods

$$
p_t^F mc_t^F = rer_t \xi_t^m \tag{70}
$$

$$
f_t^F = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F \right)^{1-\epsilon_F} y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F \right)^{\epsilon_F} f_{t+1}^F \right\} \tag{71}
$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} f_{t+1}^F \right\} \tag{72}$$

$$1 = (1-\theta_F) \left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{73}$$

$$\pi_t^{I,F} = \left(\pi_{t-1}^F\right)^{\kappa_F} \left(\pi^T\right)^{1-\kappa_F} \tag{74}$$

B.12 Wages

$$\lambda_t^W = \frac{\lambda_t^P + \lambda_t^I}{2} \tag{75}$$

$$\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U) \lambda_t^R \tag{76}$$

$$\Theta_t = \frac{\left(\wp_U \Theta_t^U + (1-\wp_U) \Theta_t^R\right) + \Theta_t^I}{2} \tag{77}$$

$$mc_t^W = \Theta_t \frac{\xi_t^n \left(\tilde{n}_t\right)^{\varphi}}{\lambda_t^U w_t} \tag{78}$$

$$\Theta_t^i = \tilde{\chi}_t^i \left(\hat{c}_t^i\right)^{-\sigma} \quad \forall \quad i = \{U, R, I\} \tag{79}$$

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i\right)^{1-v} \left(\hat{c}_t^i\right)^{\sigma v} \quad \forall \quad i = \{U, R, I\} \tag{80}$$

$$\begin{aligned} f_t^W = & \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \tilde{w}_t^{1-\epsilon_W} \tilde{n}_t \\ & + \left(\frac{\left(\omega_{UP} \beta^{UP} + (1-\omega_{UP}) \beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1} \lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^W \right\} \end{aligned} \tag{81}$$

$$\begin{aligned} f_t^W = & \tilde{w}_t^{-\epsilon_W (1+\varphi)} mc_t^W \tilde{n}_t \\ & + \left(\frac{\left(\omega_{UP} \beta^{UP} + (1-\omega_{UP}) \beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1} \lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W (1+\varphi)} f_{t+1}^W \right\} \end{aligned} \tag{82}$$

$$1 = (1-\theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W} \tag{83}$$

$$\pi_t^{I,W} = a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W} \tag{84}$$

B.13 Fiscal Policy

$$\begin{aligned} \tau_t + R_{t-1} \frac{bs_{t-1}^G}{a_t \pi_t} + q_t^{BL} R_t^{BL} bl_{t-1}^G \frac{1}{a_t} + \chi s_t p_t^{Co\star} y_t^{Co} = & g_t + bs_t^G + q_t^{BL} bl_t^G + \gamma_D \frac{PD_t^D R_{t-1}^D d_{t-1}^F}{a_t \pi_t} \\ & + \gamma_{BH} \frac{PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr}}{a_t} \end{aligned} \tag{85}$$

$$\tau_t = \alpha^T gdpn_t + \epsilon_t \left(bs^G - bs_t^G + q^{BL} bl^G - q_t^{BL} bl_t^G \right) \tag{86}$$

B.14 Monetary Policy and Rest of the World

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E) \pi_t + \alpha_E \mathbb{E}_t \{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{gdp_t}{gdp_{t-1}} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{87}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{88}$$

$$R_t^\star = R_t^W \exp \left\{ \frac{-\phi^\star}{100} \left(\frac{rer_t b_t^\star}{gdpn_t} - \frac{rer b^\star}{gdpn} \right) \right\} \xi_t^R z_t^\tau \tag{89}$$

$$x_t^{H\star} = \left(\frac{p_t^H}{rer_t} \right)^{-\eta^\star} y_t^\star \tag{90}$$

B.15 Aggregation and Market Clearing

$$y_t^C = c_t^P + c_t^I + i_t^K + i_t^H + g_t + \upsilon_t \tag{91}$$

$$c_t^P = \wp_U c_t^U + (1-\wp_U)\, c_t^R \tag{92}$$

$$\begin{aligned}\upsilon_t a_t \pi_t =& \gamma_D PD_t^D R_{t-1}^D d_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr} + \mu_e G_e(\bar{\omega}_t^e) R_t^e q_{t-1}^K k_{t-1} + \mu_I G_I\left(\bar{\omega}_t^I\right) R_t^H q_{t-1}^H h_{t-1}^I \\ &+ \mu_H G_H\left(\bar{\omega}_t^H\right) \tilde{R}_t^H l_{t-1}^H q_{t-1}^L + \mu_F G_F\left(\bar{\omega}_t^F\right) \tilde{R}_t^F l_{t-1}^F + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 y_t^Z p_t^Z\end{aligned} \tag{93}$$

$$y_t^H = x_t^H + x_t^{H\star} \tag{94}$$

$$y_t^F = x_t^F \tag{95}$$

$$h_t = h_t^P + h_t^I \tag{96}$$

$$h_t^P = \wp_U h_t^U + (1-\wp_U)\, h_t^R \tag{97}$$

$$bl_t^{Pr} = \wp_U bl_t^U + (1-\wp_U)\, bl_t^R \tag{98}$$

$$bs_t^{Pr} = \wp_U bs_t^U \tag{99}$$

$$bb_t^{Tot} = \wp_U bb_t^U \tag{100}$$

$$b_t^{*Tot} = \wp_U b_t^{*U} \tag{101}$$

$$bl_t^{Pr} + bl_t^{CB} + bl_t^G = 0 \tag{102}$$

$$bs_t^{Pr} + bs_t^G = 0 \tag{103}$$

$$d_t^F = \wp_U d_t^U \tag{104}$$

$$\zeta_t^L = \left(\frac{q_t^{BL} bl_t^U + q_t^{BB} bb_t^U}{bs_t^U + rer_t b_t^{\star,U} + d_t^U}\right)^{\eta_\zeta} \epsilon_t^{L,S} \tag{105}$$

$$\tilde{R}_t^D = R_{t-1}^D\left(1 - \gamma_D PD_t^D\right) \tag{106}$$

$$\tilde{R}_t^{BB} = R_t^{BB}\left(1 - \gamma_{BH} PD_t^H\right) \tag{107}$$

$$R_t^{BL} = \frac{1}{q_t^{BL}} + \kappa_{BL} \tag{108}$$

$$R_t^{BB} = \frac{1}{q_t^{BB}} + \kappa_{BB} \tag{109}$$

$$R_t^{Nom,BL} = R_t^{BL}\pi_t \tag{110}$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \tag{111}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \tag{112}$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} a_t \pi_t \tag{113}$$

$$\pi_t^{\widetilde{W}} = \frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \pi_t^W \tag{114}$$

$$y_t^H \Xi_t^H = x_t^Z \tag{115}$$

$$y_t^Z = z_t\left(\frac{k_{t-1}}{a_t}\right)^\alpha \widetilde{n}_t^{1-\alpha} \tag{116}$$

$$y_t^Z = x_t^Z \tag{117}$$

$$\Xi_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H\left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \tag{118}$$

$$m_t = y_t^F \Xi_t^F \tag{119}$$

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F\left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F \tag{120}$$

$$n_t = \widetilde{n}_t \Xi_t^W \tag{121}$$

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_W\left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W} \Xi_{t-1}^W \tag{122}$$

$$n_t = n_t^P + n_t^I \tag{123}$$

$$n_t^P = n_t^I \tag{124}$$

$$n_t^P = \wp_U n_t^U + (1-\wp_U)\, n_t^R \tag{125}$$

$$n_t^U = n_t^R \tag{126}$$

$$gdp_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + x_t^{H\star} + y_t^{Co} - m_t \tag{127}$$
$$gdpn_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + tb_t \tag{128}$$
$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \tag{129}$$
$$rer_t b_t^\star = \frac{rer_t}{a_t \pi_t^\star} b_{t-1}^\star R_{t-1}^\star + tb_t + rer_t ren^* - (1-\chi)\, rer_t p_t^{Co\star} y_t^{Co} \tag{130}$$

The exogenous processes are:

$$\begin{gathered}
\log(z_t/z) = \rho_z \log(z_{t-1}/z) + u_t^z \\
\log(a_t/a) = \rho_a \log(a_{t-1}/a) + u_t^a \\
\log(\xi_t^n/\xi^n) = \rho_{\xi^n} \log(\xi_{t-1}^n/\xi^n) + u_t^{\xi^n} \\
\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + u_t^{\xi^h} \\
\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + u_t^{\xi^i} \\
\log(\xi_t^{ih}/\xi^{ih}) = \rho_{\xi^{ih}} \log(\xi_{t-1}^{ih}/\xi^{ih}) + u_t^{\xi^{ih}} \\
\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + u_t^{\xi^R} \\
\log(e_t^m/e^m) = \rho_{e^m} \log(e_{t-1}^m/e^m) + u_t^{e^m} \\
\log(g_t/g) = \rho_g \log(g_{t-1}/g) + u_t^g \\
\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + u_t^{y^{Co}} \\
\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + u_t^{\pi^\star} \\
\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + u_t^{R^W} \\
\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + u_t^{y^\star} \\
\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + u_t^{p^{Co\star}} \\
\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + u_t^{\xi^m} \\
\log(\sigma_t^I/\sigma^I) = \rho_{\sigma^I} \log(\sigma_{t-1}^I/\sigma^I) + u_t^{\sigma^I} \\
\log(\sigma_t^e/\sigma^e) = \rho_{\sigma^e} \log(\sigma_{t-1}^e/\sigma^e) + u_t^{\sigma^e} \\
\log(\sigma_t^F/\sigma^F) = \rho_{\sigma^F} \log(\sigma_{t-1}^F/\sigma^F) + u_t^{\sigma^F} \\
\log(\sigma_t^H/\sigma^H) = \rho_{\sigma^H} \log(\sigma_{t-1}^H/\sigma^H) + u_t^{\sigma^H} \\
\log(\epsilon_t^{L,S}/\epsilon^{L,S}) = \rho_{\epsilon^{L,S}} \log(\epsilon_{t-1}^{L,S}/\epsilon^{L,S}) + u_t^{\epsilon^{L,S}} \\
\log(bl_t^G/bl^G) = \rho_{bl^G} \log(bl_{t-1}^G/bl^G) + u_t^{bl^G} \\
\log(bl_t^{CB}/bl^{CB}) = \rho_{bl^{CB}} \log(bl_{t-1}^{CB}/bl^{CB}) + u_t^{bl^{CB}} \\
\log(\varrho_t/\varrho) = \rho_\varrho \log(\varrho_{t-1}/\varrho) + u_t^\varrho \\
\log(\xi_t^{\chi b}/\xi^{\chi b}) = \rho_\xi^{\chi b} \log(\xi_{t-1}^{\chi b}/\xi^{\chi b}) + u_t^{\xi^{\chi b}} \\
\log(\xi_t^{\chi e}/\xi^{\chi e}) = \rho_\xi^{\chi e} \log(\xi_{t-1}^{\chi e}/\xi^{\chi e}) + u_t^{\xi^{\chi e}} \\
\log(\xi_t^{roe}/\xi^{roe}) = \rho_\xi^{roe} \log(\xi_{t-1}^{roe}/\xi^{roe}) + u_t^{\xi^{roe}} \\
\log(z_t^\tau/z^\tau) = \rho_{z^\tau} \log(z_{t-1}^\tau/z^\tau) + u_t^{z^\tau}
\end{gathered}$$

with $u_t^j \sim \mathcal{N}(0,\, (\sigma^j)^2)$ for all j-exogenous variables defined above

C Steady State Computation

In this section we show how to compute the steady state for a given value of most of the parameters and all exogenous variables in the long run, except for:

$$R^W,\ \pi^\star,\ \sigma^F,\ \sigma^H,\ \sigma^e,\ \sigma^I,\ g,\ y^{Co},\ y^\star,\ o_{\hat{C}},\ ren^*,\ \xi^n.$$

that are determined endogenously by imposing values for the steady state of the following endogenous variables:

$$\pi^s,\ \xi^i = 1,\ \xi^R,\ R^D,\ PD^F = PD^H,\ n,\ R^{nom,BL},\ R^{nom,I},\ R^L,\ p^H,\ r^{h,k} = {}^{q^H h}/_{q^K k},\ s^g = g/gdpn,\ s^{Co} = p^{Co\star} y^{Co} rer/gdpn,$$
$$s^{tb} = tb/gdpn,\ s^{b*} = b^* rer/gdpn,\ \alpha_{BLG} = \frac{bl^G * q^{BL}}{gdpn},\ \alpha_{SG} = \frac{bs^G}{gdpn}$$

Start with (4), (5), (6), (87) (88) and (89):

$$R = \frac{\pi a^\sigma}{\beta_U};\quad \tilde{R}^D = R;\quad R^\star = \frac{R}{\pi^s};\quad \pi = \pi^T;\quad \pi^\star = \frac{\pi}{\pi^s};\quad R^W = \frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^{I,H} = \pi^{I,F} = \pi^H = \pi^F = \pi$$

From (84), (113) and (114) :

$$\pi^{I,W} = \pi^W = \pi^{\widetilde{W}} = a\pi$$

From (64), (73), (83), (62),(63), (71),(72), (81), (82), (118), (120) and (122):

$$\tilde{p}^H = \tilde{p}^F = \widetilde{w} = 1$$
$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}$$
$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}$$
$$mc^W = \frac{\epsilon_W - 1}{\epsilon_W}$$
$$\Xi^H = \Xi^F = \Xi^W = 1$$

From (55) and (57):

$$q^K = 1/\xi^i$$
$$q^H = \frac{a^{N_H \sigma} \varphi_0^H}{\beta_{UP}^{N_H} \xi^{ih}} \left(\frac{1 - \left(\frac{\beta_{UP} \rho^{\varphi H}}{a^\sigma}\right)^{N_H + 1}}{1 - \frac{\beta_P \rho^{\varphi H}}{a^\sigma}} \right)$$

From (14) and (121):

$$R^H = \pi \left(1 - \delta_H\right)$$
$$\widetilde{n} = n$$

From (35), (37), (38), (39) and (51):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (40), (106), R^D and using $PD^F = PD^H$

$$PD^D = \frac{1}{\gamma_D}\left(1 - \frac{\tilde{R}^D}{R^D}\right) = PD^H = PD^F$$

From (12)

$$R^{BL} = \frac{R^{Nom,BL}}{\pi}$$
$$\beta_{RP} = \frac{a^\sigma}{R^{BL}}$$

From (17), (19) and (20)

$$R^I = \frac{R^{Nom,I}}{\pi}$$
$$\hat{R}^I = R^I$$
$$\hat{q}^L = \frac{1}{\hat{R}^I - \kappa_L}$$

$$q^L = \hat{q}^L$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (107)

$$R^{BB} = \frac{\tilde{R}^{BB}}{1 - \gamma_D PD^H}$$

From (109)

$$q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$

$$\Delta l = a$$

Numerical solution for $\bar{\omega}^F$ and σ^F using (42), (44) and (46)

$$\bar{\omega}^F - \left[1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)\right]\left(\frac{1-\phi_F}{\phi_F}\right)\frac{R^D}{\tilde{\rho}^F} = 0$$

$$PD^F - F_F\left(\bar{\omega}^F, \sigma^F\right) = 0$$

Numerical solution for $\bar{\omega}^H$ and σ^H using (48), (50) and (53)

$$\bar{\omega}^H - \left[1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)\right]\left(\frac{1-\phi_H}{\phi_H}\right)\frac{R^{BB}}{\rho^H}\pi = 0$$

$$PD^H - F_H\left(\bar{\omega}^H, \sigma^H\right) = 0$$

Then, from (44) and (50):

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)}$$

$$\tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)}$$

Numerical solution for $\bar{\omega}^e$ and σ^e: Use (33) in (32), then use (44), (45), (26) and (31). Later combine (28) and (45) to obtain

$$\frac{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e'\left(\bar{\omega}^e, \sigma^e\right)}{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right)} - \frac{\left(1-\chi_e\right)\tilde{R}^F}{a\pi} = 0$$

$$R^L - \frac{\tilde{R}^F \bar{\omega}^e}{\Gamma_e\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e\left(\bar{\omega}^e, \sigma^e\right)} = 0$$

From (34):

$$PD^e = F_e\left(\bar{\omega}^e\right)$$

Numerical solution for $\bar{\omega}^I$ and σ^I: use (50) and (24) in (22). Also, use (52) in (18)

$$\frac{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I'\left(\bar{\omega}^I, \sigma^I\right)}{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right)} - \frac{\beta_I \tilde{R}^H}{a^\sigma \pi} = 0$$

$$R^I - \frac{\tilde{R}^H \bar{\omega}^I}{\pi\left[\Gamma_I\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I\left(\bar{\omega}^I, \sigma^I\right)\right]} = 0$$

From (25):

$$PD^I = F_I\left(\bar{\omega}^I\right)$$

From (30), (26), (31) and (45):

$$R^e = \frac{\tilde{R}^F a\pi}{a\pi\left[\Gamma_e\left(\bar{\omega}^e\right) - \mu_e G_e\left(\bar{\omega}^e\right)\right] + \left[1 - \Gamma_e\left(\bar{\omega}^e\right)\right]\left(1-\chi_e\right)\tilde{R}^F}$$

From (27):

$$r^K = q^K\left[\frac{R^e}{\pi} - \left(1 - \delta_K\right)\right]$$

From (66) and (69):

$$p^Z = p^H mc^H$$
$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} mc^Z z}{(r^k)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha}\widetilde{n}\frac{w}{r^k}a$$
$$y^Z = z\left(\frac{k}{a}\right)^{\alpha}\tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k\left[\frac{1-(1-\delta_K)/a}{\xi^i}\right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = \left[1-\Gamma_e\left(\bar{\omega}^e\right)\right]\frac{R^e q^K k}{a\pi}$$
$$n^e = \left(1-\chi_e \xi^{\chi_e}\right)\psi^e$$
$$c^e = \chi_e \xi^{\chi_e}\psi^e$$
$$\lambda^e = \frac{\Gamma^{e'}\left(\bar{\omega}^e\right)}{\left(1-\Gamma^F\left(\bar{\omega}^F\right)\right)\left[\Gamma^{e'}\left(\bar{\omega}^e\right)-\mu^e G^{e'}\left(\bar{\omega}^e\right)\right]}$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F/\wp_U$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k}q^K k}{q^H}$$
$$i^{AH} = \frac{ha^{N_H}}{\xi^{ih}}\left[1-\left(\frac{1-\delta_H}{a}\right)\right]$$
$$i^H = i^{AH}\varphi_0^H\left[\frac{1-\left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1-\frac{\rho^{\varphi H}}{a}}\right]$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega(p^H)^{1-\eta}}{1-\omega}\right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F/\xi^m$$

Numerical solution for l_h iterating over the following equation up until $\Delta^l \approx 0$ (see Appendix C.1)

$$\Delta^l = gdpn - (c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn)$$

From (18):

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

from (49):

$$e^H = \phi_H q^L l^H$$

From (36), (37), (39) and (47):

$$n^b = e^F + e^H$$

$$\psi^b = \frac{n^b}{1-\chi_b \xi^{\chi_b}}$$
$$c^b = \chi_b \xi^{\chi_b} \psi^b$$
$$bb^{Tot} = (1-\phi_H)\frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi}\left(\begin{array}{c} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e(\bar{\omega}^e) R^e q^K k \\ +\mu_I G_I(\bar{\omega}^I) R^H q^H h^I + \mu_H G_H(\bar{\omega}^H)\tilde{R}^H q^L l^H + \mu_F G_F(\bar{\omega}^F)\tilde{R}^F l^F \end{array}\right)$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta}(p^F - rer\xi^m \Xi^F)(1-\omega)\upsilon - \upsilon}{1 - s^{Co} - (1-s^{tb})(p^F)^{-\eta}(p^F - rer\xi^m \Xi^F)(1-\omega)}$$

From their definitions:

$$tb = s^{tb} gdpn$$
$$g = s^g gdpn$$
$$y^{Co} = \frac{s^{Co} gdpn}{p^{Co\star} rer}$$
$$b^{*Tot} = \frac{s^{b*} gdpn}{rer}$$

From (60), (61),(90), (91), (94), (95) , (119) and (128):

$$y^C = gdpn + \upsilon - tb$$
$$x^F = (1-\omega)(p^F)^{-\eta} y^C$$
$$x^H = \omega (p^H)^{-\eta} y^C$$
$$x^{H\star} = y^H - x^H$$
$$y^\star = x^{H\star}\left(\frac{p^H}{rer}\right)^{\eta\star}$$
$$y^F = x^F$$
$$m = y^F \Xi^F$$

From (96):

$$h^P = h - h^I$$

From (23):

$$c^I = \frac{wn}{2} + q^H h^I\left[(1-\Gamma_I)\frac{R^H}{a\pi} - 1\right] + q^L l^H$$

From (21) and (16):

$$o_{\hat{C}} = \left\{(a)^{-\sigma\eta_{\hat{C}}}(\xi^h)^{\eta_{\hat{C}}-1}\left(\frac{ac^I\left(1-\frac{\phi_c}{a}\right)}{h^I\left(1-\frac{\phi_{hh}}{a}\right)}\right)\left(\frac{1}{\beta_I}\left[q^H - (\Gamma_I - \mu_I G_I)\frac{R^H q^H}{\tilde{R}^H}\right] - a^{-\sigma}(1-\Gamma_I)\frac{R^H}{\pi}q^H\right)^{-\eta_{\hat{C}}} + 1\right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c^I\left(1-\frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\frac{\xi^h h^I}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (16):

$$\lambda^I = \left\{(\hat{c}^I)^{-\sigma}\right\}\left(\frac{(1-o_{\hat{C}})\hat{c}^I}{c^I\left(1-\frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (21) and (22)

$$\lambda^H = \frac{\lambda^I}{\rho^H \phi_H}$$

Use ratios $\alpha_{BLG} = \frac{bl^G q^{BL}}{gdpn}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG}\frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG} gdpn$$

Then from (102) and (103), and normalizing $bl^{CB} = 1$

$$bl^{Pr} = -bl^{G}$$
$$bs^{Pr} = -bs^{G}$$

We can solve for bond holdings of the unrestricted households Also, from (99), (100) and (101)

$$bs^{U} = \frac{bs^{Pr}}{\wp^{U}}$$
$$b^{*U} = \frac{b^{*Tot}}{\wp^{U}}$$
$$bb^{U} = \frac{bb^{tot}}{\wp^{U}}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^{u} = \frac{\omega_{BL} * (bs^{u} + rer * b^{*U} + d^{U}) - bb^{U} q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^{R} = \frac{bl^{Pr} - \wp_{U} bl^{U}}{1 - \wp_{U}}$$

From 102

$$bl^{CB} = 1$$

Next, we solve for h^R, c^R, $\hat{c}^R$, λ^R. From (10) and (11) and the restricted household budget constraint (13)

$$h^{R} = \frac{q^{BL} bl^{R} \left(\frac{R^{BL}}{a} - 1 \right) + \frac{wn}{2}}{q^{H} - \frac{q^{H}}{a}(1 - \delta_{H}) + aux_{1}}$$

with aux_1

$$aux_{1} = (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^{h})^{1 - \eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1 - \delta_{H})\, a^{-\sigma} q^{H} \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a} \right)}{o_{\hat{C}} \left(1 - \frac{\phi_{c}}{a} \right)}$$

and

$$c^{R} = h^{R} aux_{1}$$

From (9):

$$\hat{c}^{R} = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{R} \left(1 - \frac{\phi_{c}}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{R}}{a} \left(1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}} - 1}}$$

From (10):

$$\lambda^{R} = \left\{ \left(\hat{c}^{R} \right)^{-\sigma} \right\} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}^{R}}{c^{R} \left(1 - \frac{\phi_{c}}{a} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

Also, from (97) we get

$$h^{U} = \frac{h^{P} - (1 - \wp_{U}) h^{R}}{\wp_{U}}$$

which together with (2) and (3) lets us solve for c^U

$$c^{U} = h^{U} (a)^{\sigma \eta_{\hat{C}} - 1} (\xi^{h})^{1 - \eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1 - \delta_{H})\, a^{-\sigma} q^{H} \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a} \right)}{o_{\hat{C}} \left(1 - \frac{\phi_{c}}{a} \right)}$$

From (1) we solve for $\hat{c}^U$

$$\hat{c}^{U} = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{U} \left(1 - \frac{\phi_{c}}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{U}}{a} \left(1 - \frac{\phi_{hh}}{a} \right) \right)^{\frac{\eta_{\hat{C}} - 1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}} - 1}}$$

and from (2) we obtain λ^U

$$\lambda^{U} = \left(\hat{c}^{U} \right)^{-\sigma} \left(\frac{(1 - o_{\hat{C}})\, \hat{c}^{U}}{c^{U} \left(1 - \frac{\phi_{c}}{a} \right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U)\,\lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U)\,c^R$$
$$c = c^p + c^i$$

From (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

From (79), (80) and (77):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = 1$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = 1$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$

$$\Theta = \frac{\left(\wp_U \Theta^U + (1 - \wp_U)\,\Theta^R\right) + \Theta^I}{2} = 1$$

From (75) and (78):

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}, \qquad \xi^n = \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^{\varphi}}$$

From (127) and (130):

$$gdp = c + i + i^h + g + x^{H\star} + y^{Co} - m$$
$$ren^* = b^\star \left(1 - \frac{R^\star}{a\pi^\star}\right) - \frac{tb}{rer} + (1 - \chi)\,p^{Co\star} y^{Co}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1\right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1\right) - \chi rerp^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP}\theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP}\theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W(1+\varphi)} mc^W \widetilde{n}}{1 - \left(\frac{(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}) + \beta_I}{2}\right)\theta_W a^{1-\sigma}}$$

C.1 Numerical solution for l^H

First, guess l^H. Then, from (18) solve for h^I:

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

From (49) and (47):

$$bb^{Tot} = (1 - \phi_H) \frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi} \begin{pmatrix} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e (\bar{\omega}^e) R^e q^K k \\ +\mu_I G_I (\bar{\omega}^I) R^H q^H h^I + \mu_H G_H (\bar{\omega}^H) \tilde{R}^H q^L l^H + \mu_F G_F (\bar{\omega}^F) \tilde{R}^F l^F \end{pmatrix}$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta} (p^F - rer\xi^m \Xi^F)(1-\omega)\upsilon - \upsilon}{1 - s^{Co} - (1 - s^{tb})(p^F)^{-\eta}(p^F - rer\xi^m \Xi^F)(1-\omega)}$$

From (96):

$$h^P = h - h^I$$

From (23):

From (21) and (16):

$$o_{\hat{C}} = \left\{ (a)^{-\sigma\eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}}-1} \left(\frac{ac^I \left(1 - \frac{\phi_c}{a}\right)}{h^I \left(1 - \frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I} \left[q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1 - \Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Use ratios $\alpha_{BLG} = \frac{bl^G}{gdpnq^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$

$$bs^G = \alpha_{BSG} gdpn$$

Then from (102) and (103), and normalizing $bl^{CB} = 0$

$$bl^{Pr} = -bl^G$$

$$bs^{Pr} = -bs^G$$

Also, from (99) and (100)

$$bs^U = \frac{bs^{Pr}}{\wp^U}, \qquad bb^U = \frac{bb^{tot}}{\wp^U}$$

Use ratio $s^{b*} = b^* rer/gdpn$, and (101)

$$b^{*Tot} = s^{b*} * gdpn/rer, \qquad b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

which using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (10) and (11) and the restricted household budget constraint (13)

$$h^R = \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1 \right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1 - \delta_H) + aux_1}$$

with aux_1

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1 - \delta_H) a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1 - o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

and

$$c^R = h^R aux_1$$

Also, from (97) we get

$$h^U = \frac{h^P - (1-\wp_U)h^R}{\wp_U}$$

which together with (2) and (3) lets us solve for c^U

$$c^U = h^U (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_c}{a}\right)}$$

From (92):

$$c^P = \wp_U c^U + (1-\wp_U)\, c^R$$

Then, the following equation must hold:

$$gdpn = c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn$$

If it does not, update guess of l^H and repeat.

D Steady state for capital requirements comparative statics

For a given value of capital requirements ϕ_f, ϕ_h we use estimated and calibrated parameters: related to real sector α, α^{BSG}, α^{BLG}, β_U, β_R, β_I, δ_K, δ_K, ϵ_F, ϵ_H, ϵ_W, N_H, κ, κ_{BL}, κ_{BB}, σ, χ, ω, ω_U, ω_{BL}, η, η^*, $\eta_{\hat{C}}$, θ_F, θ_H, θ_W, η_{ζ_L}; financial sector : χ_b, χ_e, γ_d, γ_{bh}, μ_e, μ_f, μ_h, μ_i, σ^e, σ^F, σ^H, σ^I, ξ^{χ_e}, ξ^{χ_b}; preference parameters and external sector parameters: $O_{\hat{C}}$, ϕ_c, ϕ_{hh}, $\rho^{\varphi H}$, φ, φ_0^H, a, bl^{cb}, $\epsilon^{L,S}$, g, n, $r^{h,k}$, π^T, p^{Co}, π^*, R^W, ξ^h, ξ^i, ξ^{ih}, ξ^m, ξ^n, ξ^R, y^*, y^{Co}, z, bl^G, bs^G, b^{*Tot} to compute the steady state of the model consistent with capital requirements different from that of the 2001-2019 period

Consider ϕ^F and ϕ^H total capital requirements including regulatory minimum capital, voluntary buffers and the neutral level (if any) for the CCyB requirement.

$$\phi^F = (\phi^F_{Reg} + \phi^F_{Vol} + CCyB)$$
$$\phi^H = 0.6(\phi^H_{Reg} + \phi^H_{Vol} + CCyB)$$

Use (4), (5), (6), (87) (88) and (89):

$$\pi = \pi^T; \quad R = \frac{\pi a^\sigma}{\beta_U}; \quad \tilde{R}^D = R; \quad \pi^s = \frac{\pi}{\pi^\star}; \quad R^\star = \frac{R}{\pi^s}; \quad R^W = \frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^H = \pi^F = \pi^{I,H} = \pi^{I,F} = \pi$$

From (84), (113) and (114):

$$\pi^W = \pi^{\widetilde{W}} = \pi^{I,W} = a\pi$$

From (62),(63),(64), (71),(72),(73), (81), (82), (83), (118), (120) and (122):

$$\tilde{p}^H = \tilde{p}^F = \widetilde{w} = 1$$
$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}$$
$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}$$
$$mc^W = \frac{\epsilon_W - 1}{\epsilon_W}$$
$$\Xi^H = \Xi^F = \Xi^W = 1$$

From (55) and (57):

$$q^K = 1/\xi^i; \quad \nabla l = a$$
$$q^H = \frac{a^{N_H \sigma}\varphi_0^H}{\beta_{UP}^{N_H}\xi^{ih}}\left(\frac{1 - \left(\frac{\beta_{UP}\rho^{\varphi H}}{a^\sigma}\right)^{N_H+1}}{1 - \frac{\beta_P \rho^{\varphi H}}{a^\sigma}}\right)$$

From (14) and (121):

$$R^H = \pi(1 - \delta_H)$$
$$\widetilde{n} = n$$

From (35), (37), (38), (39) and (51):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (12) and (110)

$$R^{BL} = \frac{a^\sigma}{\beta_{RP}}$$
$$R^{Nom,BL} = R^{BL}\pi$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$

Given σ^F and the previous result for $\tilde{R}^D$, use a numerical solution for $\bar{\omega}^F$ and R^D using (42), (44) and (106)

$$\bar{\omega}^F - \left[1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)\right]\left(\frac{1 - \phi_F}{\phi_F}\right)\frac{R^D}{\tilde{\rho}^F} = 0$$
$$PD^F - \frac{1}{\gamma_D}\left(1 - \frac{\tilde{R}^D}{R^D}\right) = 0$$

And, from (44)

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)}$$

Next, given σ^H and previous results for $\tilde{R}^{BB}$, use (48), (50) and (107) to find $\bar{\omega}^H$ and R^{BB} numerically,

$$\bar{\omega}^H - \left[1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)\right]\left(\frac{1-\phi_H}{\phi_H}\right)\frac{R^{BB}}{\rho^H}\pi = 0$$
$$\tilde{R}^{BB} = R^{BB}\left(1 - \gamma_{BH} PD^H\right)$$

Then, from (48), (53) and (109):

$$\tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)}$$
$$PD^H = F_H\left(\bar{\omega}^H, \sigma^H\right)$$
$$q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}$$

Use (33) in (32), then use (44), (45), (26) and (31) to solve for $\bar{\omega}^e$

$$\frac{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e'\left(\bar{\omega}^e, \sigma^e\right)}{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right)} - \frac{\left(1-\chi_e\right)\tilde{R}^F}{a\pi} = 0$$

Then, from (34):

$$PD^e = F_e\left(\bar{\omega}^e\right)$$

Combine (28) and (45) to obtain

$$R^L = \frac{\tilde{R}^F \bar{\omega}^e}{\Gamma_e\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e\left(\bar{\omega}^e, \sigma^e\right)}$$

Go back to (33) in (32) to obtain

$$\lambda^e = \frac{\Gamma^{e'}\left(\bar{\omega}^e\right)}{\left(1 - \Gamma^F\left(\bar{\omega}^F\right)\right)\left[\Gamma^{e'}\left(\bar{\omega}^e\right) - \mu^e G^{e'}\left(\bar{\omega}^e\right)\right]}$$

$$R^e = \left\{\frac{\left[1 - \Gamma_e\left(\bar{\omega}^e\right)\right]}{\lambda^e} + \left[1 - \Gamma_F(\bar{\omega}^F)\right]\left[\Gamma_e\left(\bar{\omega}^e\right) - \mu_e G_e\left(\bar{\omega}^e\right)\right]\right\}^{-1}\rho^F \phi_F$$

From (27):

$$r^K = q^K\left[\frac{R^e}{\pi} - \left(1 - \delta_K\right)\right]$$

Numerical solution for $\bar{\omega}^I$ using (50) and (22)

$$\frac{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I'\left(\bar{\omega}^I, \sigma^I\right)}{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right)} - \frac{\beta_I \tilde{R}^H}{a^\sigma \pi} = 0$$

From (25):

$$PD^I = F_I\left(\bar{\omega}^I\right)$$

From (18) and (52)

$$\hat{R}^I = \frac{\tilde{R}^H \bar{\omega}^I}{\pi\left[\Gamma_I\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I\left(\bar{\omega}^I, \sigma^I\right)\right]}$$

and from (17), (19) and (20)

$$\hat{q}^L = \frac{1}{\hat{R}^I - \kappa_L}$$
$$q^L = \hat{q}^L$$
$$\hat{R}^I = R^I$$

From (20)

$$R^{Nom,I} = R^I \pi$$

Using the normalization $p^H = 1$, and from (66) and (69):

$$p^Z = p^H mc^H$$

$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha} mc^Z z}{(r^k)^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha} \tilde{n} \frac{w}{r^k} a$$
$$y^Z = z \left(\frac{k}{a} \right)^{\alpha} \tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k \left[\frac{1-(1-\delta_K)/a}{\xi^i} \right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = \left[1 - \Gamma_e\left(\bar{\omega}^e\right)\right] \frac{R^e q^K k}{a\pi}$$
$$n^e = \left(1 - \chi_e \xi^{\chi_e}\right) \psi^e$$
$$c^e = \chi_e \xi^{\chi_e} \psi^e$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F / \wp_U$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega (p^H)^{1-\eta}}{1-\omega} \right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F / \xi^m$$

Next, we can find l^H, h^I, c^I solving the three equation system by (18), (23) and (21)

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$
$$c^I = \frac{wn}{2} + q^H h^I \left[(1-\Gamma_I) \frac{R^H}{a\pi} - 1 \right] + q^L l^H$$
$$\Delta^l = o_{\hat{C}} - \left\{ (a)^{-\sigma\eta_{\hat{C}}} \left(\xi^h\right)^{\eta_{\hat{C}}-1} \left(\frac{ac^I \left(1 - \frac{\phi_c}{a}\right)}{h^I \left(1 - \frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I} \left[q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1-\Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c^I \left(1 - \frac{\phi_c}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\frac{\xi^h h^I}{a} \left(1 - \frac{\phi_{hh}}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (16) and (24), respectively:

$$\lambda^I = \left\{ \left(\hat{c}^I\right)^{-\sigma} \right\} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^I}{c^I \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \quad ; \quad \lambda^H = \frac{\lambda^I}{\rho^H \phi^H}$$

Also, from (49):

$$e^H = \phi_H q^L l^H$$

From (39), (37), (36), and (47):

$$n^b = e^F + e^H$$
$$\psi^b = \frac{n^b}{1 - \chi_b \xi^{\chi_b}}$$
$$c^b = \chi_b \xi^{\chi_b} \psi^b$$
$$bb^{Tot} = \frac{q^L l^H - e^H}{q^{BB}}$$

From (40)

$$PD^D = \frac{q^{BB} bb^{Tot} PD^H + d^F PD^F}{q^{BB} bb^{Tot} + d^F}$$

From (90), (94), (61), (60) (95) and (119)

$$x^{H\star} = \frac{y^\star}{\left(\frac{p^H}{rer}\right)^{\eta\star}}$$
$$x^H = y^H - x^{H\star}$$
$$y^C = \frac{x^H}{\omega (p^H)^{-\eta}}$$
$$x^F = (1-\omega)(p^F)^{-\eta} y^C$$
$$y^F = x^F$$
$$m = y^F \Xi^F$$

From (129)

$$tb = p^H x^{H\star} + p^{Co\star} y^{Co} rer - m\xi^m rer$$

From (93):

$$\upsilon = \frac{1}{a\pi} \left(\begin{array}{c} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e \left(\bar{\omega}^e\right) R^e q^K k \\ +\mu_I G_I \left(\bar{\omega}^I\right) R^H q^H h^I + \mu_H G_H \left(\bar{\omega}^H\right) \tilde{R}^H q^L l^H + \mu_F G_F \left(\bar{\omega}^F\right) \tilde{R}^F l^F \end{array} \right)$$

Combine (91) and (128)

$$gdpn = y^C - \upsilon + tb$$

From their definitions:

$$s^g = \frac{g}{gdpn}$$
$$s^{Co} = \frac{y^{Co} p^{Co\star} rer}{gdpn}$$
$$s^{tb} = \frac{tb}{gdpn}$$

Supply of soverign debt instruments is inelastic, thus use ratios $\alpha_{BLG} = \frac{bl^G}{gdpn q^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$

$$bs^G = \alpha_{BSG} gdpn$$

From (102) and (103)

$$bl^{Pr} = -bl^G$$
$$bs^{Pr} = -bs^G$$
$$bs^U = \frac{bs^{Pr}}{\wp^U}$$
$$bb^U = \frac{bb^{tot}}{\wp^U}$$

Also, from (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

Next, we implement a numerical search for s^{b*} and $r^{h,k}$ (see Appendix D.1) using (78) and (128)

$$\xi^n = \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^\varphi}$$
$$gdpn = c^P + c^I + i^K + i^H + g + tb$$

Then from its definition, we have

$$b^{*,Tot} = \frac{s^{b*} gdpn}{rer}$$

From (130)

$$ren^{*} = b^{*Tot}\left(1 - \frac{R^{\star}}{a\pi^{\star}}\right) - \frac{tb}{rer} + (1-\chi)\, p^{Co\star} y^{Co}$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$\begin{aligned}
h &= \frac{r^{h,k} q^K k}{q^H} \\
i^{AH} &= \frac{h a^{N_H}}{\xi^{ih}}\left[1 - \left(\frac{1-\delta_H}{a}\right)\right] \\
i^{H} &= i^{AH}\varphi_0^H\left[\frac{1 - \left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1 - \frac{\rho^{\varphi H}}{a}}\right]
\end{aligned}$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1-\wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

$$\begin{aligned}
h^R &= \frac{q^{BL} bl^R\left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1} \\
c^R &= h^R aux_1
\end{aligned}$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c^R\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\xi^h\frac{h^R}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{\left(\hat{c}^R\right)^{-\sigma}\right\}\left(\frac{(1-o_{\hat{C}})\,\hat{c}^R}{c^R\left(1 - \frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U)h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U(a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c^U \left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a} \left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + \left(1 - \wp_U\right) \lambda^R$$

From (92):

$$c^P = \wp_U c^U + \left(1 - \wp_U\right) c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = \tilde{\chi}^U \left(\hat{c}^U\right)^{-\sigma}$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = \tilde{\chi}^I \left(\hat{c}^I\right)^{-\sigma}$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$
$$\Theta = \frac{\left(\omega_{UP} \Theta^U + \left(1 - \omega_U\right) \Theta^R\right) + \Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1\right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1\right) - \chi rerp^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{\left(\tilde{p}^H\right)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP} \theta_H a^{1-\sigma}}, \quad f^F = \frac{\left(\tilde{p}^F\right)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP} \theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W (1+\varphi)} mc^W \widetilde{n}}{1 - \left(\frac{\left(\omega_{UP} \beta^{UP} + (1 - \omega_{UP}) \beta^{RP}\right) + \beta_I}{2} \right) \theta_W a^{1-\sigma}}$$

D.1 Numerical solution for $(s^{b*},\ r^{h,k})$

Iterate on $(s^{b*},\ r^{h,k})$ until $\Delta \approx 0$

$$\Delta = \begin{bmatrix} \xi^n - \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{bmatrix}$$

For each guess of $(s^{b*},\ r^{h,k})$ we have

$$b^{*,Tot} = \frac{s^{b*} gdpn}{rer}$$

From $r^{h,k} = {}^{q^H h}/{}_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$
$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}} \left[1 - \left(\frac{1-\delta_H}{a}\right)\right]$$
$$i^H = i^{AH} \varphi_0^H \left[\frac{1 - \left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1 - \frac{\rho^{\varphi H}}{a}}\right]$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$
$$h^R = \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1}$$
$$c^R = h^R aux_1$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^R\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^R}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{\left(\hat{c}^R\right)^{-\sigma}\right\} \left(\frac{(1-o_{\hat{C}})\,\hat{c}^R}{c^R\left(1 - \frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U) h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^U\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U)\, \lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U)\, c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = \tilde{\chi}^U \left(\hat{c}^U\right)^{-\sigma}$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = \tilde{\chi}^I \left(\hat{c}^I\right)^{-\sigma}$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$
$$\Theta = \frac{\left(\omega_{UP} \Theta^U + (1 - \omega_U)\, \Theta^R\right) + \Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

Check if $\Delta = 0$

$$\Delta = \left[\begin{array}{l} \xi^n - \frac{mc^W \lambda^W w}{\Theta \tilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{array} \right]$$