

MACHINE LEARNING FOR DATA STREAMS

DECISION TREES

Tips for theory:

- Paper “Online Classification of Nonstationary Data Streams (2002)” : Continuous streams of data pose new problems to classification methods of data mining, like CART [4], ID3 [28], C4.5 [29], IFN [23], and many others. The common approach of these methods is to store and process the entire set of training examples. The growing amounts of training data increase the processing requirements of data mining systems up to a point, where they either run out of memory, or their computation time becomes prohibitively long. Furthermore, even if all the available examples can be handled by the system, the patterns discovered by an algorithm in the data from the past, may be hardly valid and useful for the new data obtained hours or even minutes later due to unexpected changes in the data-generating process (e.g., a political event which affects the stock prices). -> We could use it to start the comparatives of decision trees mentioning it. Also mentioned in the survey (1)
- Survey (1) -> See decision trees section to write the decision trees’ comparative
- https://en.wikipedia.org/wiki/Incremental_decision_tree -> Very Fast Decision Trees learner reduces training time for large incremental data sets by subsampling the incoming data stream.
- Paper “Incremental Decision Tree based on order statistics (2013)” : The error rate of this kind of algorithm (incremental algorithm) is more important in the early learning stage than a batch algorithm as C4.5. But having learnt several hundreds of thousand examples, this error rate becomes lower than C4.5 since C4.5 is not able to deal with millions of examples and thus has to use only a part of the available information.
- Paper “Incremental Decision Tree based on order statistics (2013)” : Constructing an online decision tree is based on three main choices.
 - In the first place, it is impossible in the data stream context, potentially of infinite size, to keep all the examples. The use of **data summaries** of limited size is necessary to be able to **control the tree memory consumption**. The fact that decisions are local to the leaf justifies storing summaries in each leaf.
 - Secondly, cut points are chosen by the evaluation in every leaf of a criterion (generally the Gini or the entropy criterion). This choice being a definitive action has to be robust and made with a certain confidence.
 - Finally before a split occurs, the available information in leaves is not used. Using a local model in each leaf allows exploiting this information to improve the global prediction of the tree The quality of a decision tree depends on: (i) the summaries in the leaves, (ii) the split criterion, (iii)

the local model. -> They dedicate sections to summaries in the leaves, split criterion and local model.

- The paper “An adapted incremental graded multi-label classification model for recommendation systems (2018):” mentions different strategies for handling missing values and avoiding over-fitting in decision trees. It also explains incremental decision trees proposals (contains old proposals)

Note: A number between parenthesis corresponds to a certain survey:

1. Mining High-Speed Data Streams (2000)_VFDT_NOT READ YET: **No aparece en el survey (6)**

○ (1) Pages 3,4

- This method essentially subsamples the data in order to achieve scalability in the construction of the decision tree. The idea is to show that the **entire decision tree constructed would be the same as the one built on sub-sampled data with high probability**.
- The idea is to determine a random sample of sufficient size so that the tree constructed on the sample is the same as that constructed on the entire data set. The **Hoeffding bound** is used to show that the decision tree on the sub-sampled tree would make the same split as on the full stream with high probability. This approach can be used with a variety of criteria such as the gini-index, or information gain.
- The number of examples required to produce the same split as the original data (with high probability) is determined. The Hoeffding bound is used to determine the number of relevant examples, so that this probabilistic guarantee may be achieved. If all splits in the decision tree are the same, then the same decision tree will be created.
- The Hoeffding tree can also be **applied to data streams**, by building the tree incrementally, as more examples stream in, from the higher levels to the lower levels. At any given node, one needs to wait until **enough tuples are available in order to make decisions about lower levels**. The memory requirements are modest, because only the counts of the different discrete values of the attributes (over different classes) need to be maintained in order to make Split decisions.
- The **VFDT algorithm** is also based on the Hoeffding tree algorithm, though it makes a number of modifications. Specifically, it is **more aggressive about making choices in the tie breaking** of different attributes for splits. It also allows the deactivation of less promising leaf nodes. It is generally more memory efficient, because of these optimizations.
- **The original VFDT method is not designed for cases where the stream is evolving over time.**

- <http://www.otnira.com/2013/03/28/hoeffding-tree-for-streaming-classification/>
 - Hoeffding bound gives certain level of confidence on the best attribute to split the tree, hence we can build the model based on certain number of instances that we have seen.
 - Hoeffding-tree, which is a new decision-tree learning method for streaming that solves these following challenges:
 - Uncertainty in learning time. Learning in Hoeffding tree is **constant time per example** (instance) and this means Hoeffding tree is suitable for mining data streaming.
 - The resulting trees are nearly identical with trees built by conventional batch learner, given enough example to train the and build the Hoeffding trees
 - To achieve the streaming classification characteristics, the authors introduce Hoeffding bound to decide **how many examples of instances needed to achieve certain level of confidence** (i.e. the chosen instance attribute using the bound is the close to the attribute chosen when infinite examples are presented into the classifier).
 - What makes Hoeffding bound attractive is its ability to give the same results **regardless the probability distribution generating the observations**. However, the number of observations needed to reach certain values of δ and ϵ are different across probability distributions.
 - With probability $1-\delta$, one attribute is superior compared to others when observed difference of information gain is greater than ϵ .
 - The authors implemented the Hoeffding tree algorithm into Very Fast Decision Tree learner (VFDT) which includes some enhancements for practical use, such as node-limiting strategy, introduction of as tie breaking parameter, grace period of bound calculation, poor attributes removal, fast initialization by using conventional RAM-based learner and ability to rescan previously-seen examples when data rate is slow.
- (2) Page 21
- (3) Page 12
 - VFDT [57] is considered as a reference article for learning on data streams with millions/billions of examples. This article is widely referenced and compared to new approaches on the same problems since 2000. In VFDT, the tree is built incrementally and no examples are kept. The error rate is higher in the early stage of the learning than an algorithm as C4.5. However after processing millions of examples, the error rate become lower than C4.5 which is not able to deal with millions of examples but has to use just a subset of them.

- Moreover Domingos and Hulten proved that the “Hoeffding Tree” is similar to the tree that would have been learned with an off-line algorithm. In order to suit better the use case of stream mining VFDT can be tuned. The two main parameters are: (i) the maximum amount of memory to use, (ii) the minimal number of examples seen before calculating the criterion.
- HT cautiously works toward the asymptotic batch tree, ignoring, and thus not benefiting from potential improvements on the current state of the tree, until it is sufficiently confident that they will not need to be subsequently revised.
- Paper “Moderated VFDT in Stream Mining Using Adaptive Tie Threshold and Incremental Pruning (2011)”: The original version of VFDT requires a user-defined tie threshold by which a split will be forced to break to control the tree size. It is an open problem that the tree size grows tremendously with noise as continuous data stream in and the classifier's accuracy drops.
- Paper “Moderated VFDT in Stream Mining Using Adaptive Tie Threshold and Incremental Pruning (2011)”: Although VFDT is able to progressively construct a decision tree from the unbounded data stream, VFDT suffers from tree size explosion and the deterioration of prediction accuracy when the data streams are impaired by noise.

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: Among them, the Very Fast Decision Tree (VFDT) algorithm (Domingos and Hulten, 2000) is one of the most well-known for stream classification, being capable of constructing a decision tree in an online fashion by taking advantage of a statistical property called Hoeffding Bound (HB). By doing so, the VFDT obtains a predictive performance similar to conventional decision tree induction algorithms applied to static datasets. Although VFDT is somewhat memory-friendly, learning from data streams can lead to unnecessary tree growth, increasing memory usage and even compromising its application on memory-scarce scenarios.

Explains the VFDT proposal in an acceptable way.

RESUMEN DEL PAPER “Online Classification of Nonstationary Data Streams (2002)”: A recent paper by Domingos and Hulten [6] deals directly with the problem of mining high-speed streams of data. Their data mining system, called VFDT, builds decision trees from symbolic attributes by using sub-sampling of the entire data stream generated by a stationary process. A similar assumption of stationary concepts is used by the incremental method of Fan et al. [7]. The sample size is determined in VFDT from distribution-free Hoeffding bounds.

RESUMEN DEL PAPER “Incremental Decision Tree based on order statistics (2013)”: Among the methods in incremental learning, models based on decision trees inspired by the algorithm “Very Fast Decision Tree” (VFDT) [8] are widely used. The tree construction is incremental and leaves are transformed into nodes as examples

arrive. The new examples go down into the tree and are inserted variable by variable in a summary. A criterion (Gini or Entropy) uses this summary to find the cut points to transform a leaf into a node.

RESUMEN DEL PAPER “Moderated VFDT in Stream Mining Using Adaptive Tie Threshold and Incremental Pruning (2011)”: Its underlying principle is a dynamic decision tree building process that uses a Hoeffding bound (HB) to determine the conversion of a tree leaf to a tree node by accumulating sufficient statistics from the new samples.

Moreover, this paper explains VFDT in a section and the effects of tie breaking in Hoeffding trees in another one.

2. Mining Time-Changing Data Streams (2001)_CVFDT_NOT READ YET: No aparece en los surveys (5), (6) y (7)

○ (1) Pages 4,5

- The original VFDT method is not designed for cases where the stream is evolving over time. The work in [47] extends this method to the case of concept-drifting data streams. This method is referred to as **CVFDT**. CVFDT incorporates two main ideas in order to address the additional challenges of drift:
 - A sliding window of training items is used to limit the impact of historical behavior.
 - Alternate subtrees at each internal node i are constructed.
- Because of the sliding window approach, the main issue here is the update of the attribute frequency statistics at the nodes, as the sliding window moves forward. For the incoming items, their statistics are added to the attribute frequencies in the current window, and the statistics of the expiring items at the other end of the window are decreased. Therefore, **when these statistics are updated, some nodes may no longer meet the Hoeffding bound, and somehow need to be replaced.**

○ (2) Page 21

- Hoeffding tree algorithm has high accuracy and works well with large datasets. However, it is not able to handle concept drifts in data streams as no node can be changed once created. CVFDT [Hulten et al. 2001] is an extension of the Hoeffding tree to address concept drifts in data streams. CVFDT maintains sufficient statistics at every tree node to monitor the validity of its previous decisions. When data come, it continually updates the statistics stored in tree nodes. Using the sliding window, CVFDT removes the effect of outdated data by decreasing the corresponding statistics at the tree nodes. It periodically scans the tree nodes to detect concept drifts. If concept drift appears, CVFDT concurrently grows alternative branches with the new best attribute and removes the old branches with alternative branches if it becomes less accurate.

○ Pages 12, 13

- CVFDT [49] is an extension of VFDT to deal with concept drift. CVFDT grows an alternative subtree whenever an old one seems to be out-of-date, and replaces the old subtree when the new one becomes more accurate. This allows one to make adjustments when concept drift occurs. To limit the memory usage only the promising sub-trees are kept. On the dataset “rotating hyperplane” the experiments show that the tree contains four times less nodes but the time spent to build it is five times longer comparatively to VFDT.

RESUMEN PAPER “Learning Decision Trees from Data Streams with Concept Drift (2016)”: CVFDT is able to adapt to concept-drift in streams, first growing alternate subtrees at each node of the decision tree, and then replacing the current subtree with the alternate whenever the latter becomes more accurate. This is achieved by maintaining sufficient statistics on a time-window moving over the data stream.

RESUMEN DEL PAPER “Extremely Fast Decision Tree (2018)”: Hulten et al [18] follow up on the Hoeffding Tree work with a procedure for drift adaptation (Concept-adapting Very Fast Decision Tree, CVFDT). CVFDT has a moving window that diminishes statistics recorded at a node due to an example that has fallen out of a window at a given time step. The example statistics at each internal node change as the window moves, and existing splits are replaced if the split attribute is no longer the winning attribute and one of a set of alternate subtrees grown by splitting on winning attributes registers greater accuracy.

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: Other VFDT modification, the Concept-adapting Very Fast Decision Tree (CVFDT) algorithm (Hulten et al., 2001), keeps secondary trees in memory, constantly assessed to check if they outperform the original tree, allowing adaptation to concept drifts. Also, CVFDT uses a sliding window to discard old instances. In the absence of concept drifts, the additional memory costs to store secondary trees makes CVFDT less efficient than VFDT-0.05, as shown in (Optimized very fast decision tree with balanced classification accuracy and compact tree size). In concept drift scenarios, CVFDT predictive performance is much lower than those of ensemble-based solutions (Krawczyk et al., 2017).

RESUMEN DEL PAPER “Online Classification of Nonstationary Data Streams (2002)” A new version of the VFDT system, called CVFDT[16], learns decision trees from continuously changing data streams by repeatedly applying the VFDT algorithm to a sliding window of fixed size. CVFDT is aimed at detecting only one type of concept drift at the node level of the tree: namely, the importance of the current input attribute vs. other attributes. The algorithm grows an alternative subtree for each attribute having a relatively high information gain and replaces the old subtree when a new one becomes more accurate.

3. Online Classification of Nonstationary Data Streams (2002)_OLIN_NOT READ YET: **No aparece en los surveys (1), (2), (3), (4), (5), (6) y (7)**

- (8) Page 65

- Last [26] has proposed an online classification system which can adapt to concept drift. The system re-builds the classification model with the most recent examples. By using the error-rate as a guide to concept drift, the frequency of model building and the window size is adjusted over time.
 - The system uses info-fuzzy techniques for building a tree-like classification model. It uses information theory to calculate the window size. The main idea behind the system is to change the sliding window of the model reconstruction according to the classification error rate. If the model is stable, the window size increases. Thus the frequency of model building decreases.
 - The info-fuzzy technique for building a tree-like classification model is referred to as the **Info-Fuzzy Network (IFN)**. The tree is different than conventional decision trees in that each level of the tree represents only one attribute except the root node layer. The nodes represent different values of the attribute.
 - The process of inducing the class label is similar to the one of conventional decision trees.
 - The process of constructing this tree has been termed as Information Network (IN). The IN technique uses a similar procedure of building conventional decision trees. OLIN system repeatedly uses the IN algorithm for building a new classification model.
 - The system uses the information theory to calculate the window size (refers to number of examples).
 - This measure is derived from the mutual conditional information in the IN algorithm by applying the likelihood ratio test to assess the statistical significance of the mutual information. Subsequently, we change the window size of the model reconstruction according to the classification error rate. The error rate is calculated by measuring the difference between the error rate during the training at one hand and the error rate during the model validation at the other hand. A significance increase in the error rate indicates a high probability of a concept drift. The window size changes according to the value of this increase.
- This paper describes and evaluates OLIN, an online classification system, which dynamically adjusts the size of the training window and the number of new examples between model re-constructions to the current rate of concept drift. By using a fixed amount of computer resources, OLIN produces models, which have nearly the same accuracy as the ones that would be produced by periodically re-constructing the model from all accumulated instances.
 - This paper proposes an online classification system, which uses an info-fuzzy network [23], or IFN, as a base classifier. As shown in [19] [23], the IFN method is able to produce much more compact models than other decision-tree methods, like CART and C4.5, while preserving nearly the

same level of predictive accuracy. Moreover, it can also be used as an efficient **feature selection method**.

- OLIN saves computer resources by increasing the update cycle when the concept appears to be stable and it shrinks the size of the training window, whenever a concept drift is detected.
- The cumulative accuracy of the models produced by OLIN tends to be higher than the accuracy obtained with a fixed-size sliding window, though it may be slightly lower than the accuracy of an incremental system that does not “forget” any past examples.
- The empirical results
- presented in this paper show that when applied to nonstationary data, OLIN tends to be more accurate than the static windowing methods.
- However, OLIN is usually less accurate than the extremely inefficient “no-forgetting” approach due to apparent presence of long-term concepts, which are eventually forgotten by OLIN, while being retained by the no-forgetting learner.

- Comparison with VFDT:

- It uses a less conservative measure than Hoeffding bound used in VFDT.

4. Efficient Decision Tree Construction on Streaming Data (2003)_NOT READ

YET: No aparece en los surveys (2), (3), (4), (5), (6) y (7)

- (1) Page 4
 - One of the major challenges of the VFDT family of methods is that it is naturally suited to categorical data. This is a natural derivative of the fact that decision tree splits implicitly assume categorical data. Furthermore, discretization of numerical to categorical data is often done offline in order to ensure good distribution of records into the discretized intervals. Of course, it is always possible to test all possible split points, while dealing with numerical data, but the number of possible split points may be very large, when the data is numerical. The bounds used for ensuring that the split is the same for the sampled and the original data may also not apply in this case. The technique in [48] uses **numerical interval pruning in order to reduce the number of possible split points**, and thereby make the approach more effective.
 - Furthermore, the work uses the properties of the gain function on the entropy in order to achieve the **same bound as the VFDT method** with the use of a smaller number of samples.

RESUMEN DEL PAPER “Extremely Fast Decision Tree (2018)”: There is a sizable literature that adapts HT in sometimes substantial ways [12, 19, 23] that do not, to the best of our knowledge, lead to the same fundamental change in learning premise as does HATT. Substitutes the Hoeffding Test with the “Normal” test.

5. Accurate Decision Trees for Mining High-speed Data Streams (2003)_VFDTc_NOT READ YET: No aparece en los surveys (1), (2), (4), (5), (6) y (7)

○ (1) Page 13

- VFDTc [58] is an extension of VFDT able to deal with numerical attributes and not only categorical. In each leaf and for each attribute counters for numerical values are kept. These counters are stored using a binary tree so that it is efficient to find the best split value for each attribute to transform this leaf into a node. The authors of VFDTc observe that it needs 100 to 1,000 examples to transform a leaf into a node. In order to improve the performance of the tree, they propose to add a local model in the leaves. The naive Bayes classifier is known to have good performance with few data and therefore this classifier is used in the leaves of VFDTc. Local models improve the performance of the tree without using extra amount of memory since all the needed statistics are already available.

RESUMEN DEL PAPER “Learning Decision Trees from Data Streams with Concept Drift (2016)”: Another extension of VFDT is VFDTc, which is able to deal with numerical attributes, i.e. not only categorical ones [12]. The algorithm in each leaf stores counters for numerical values. Additionally, to improve performance authors proposed to add a local model in the leaves (i.e. a naïve Bayes).

RESUMEN DEL PAPER “Extremely Fast Decision Tree (2018)”: Adds support for Naive Bayes at leaves.

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: Later, to increase VFDT predictive performance, instead of using a traditional most common (MC) prediction at the leaves, a Naive Bayes (NB) or Adaptive Naive Bayes (ANB) algorithm can be employed (Gama et al., 2003).

RESUMEN DEL PAPER “Incremental Decision Tree based on order statistics (2013)”: The tree prediction can be improved by the addition of a local model in each leaf as in VFDTc [12].

Exhaustive binary trees (EBT): Gama et al. use this method for their VFDTc tree [12]. A binary search tree, for each numerical variable, is built incrementally. This tree also keeps in each node the counts of values smaller and bigger than the cut point. This structure allows an immediate access to the counts on both sides of a cut point. The tree memory consumption depends on the number of different values arriving in the leaf. -> Summary for numerical attributes

6. Tie-breaking in Hoeffding trees (2005)_NOT READ YET

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: The Genuine Tie Detection (Holmes et al., 2005) has a mechanism to automatically choose the tie-breaking parameter during training.

Despite the VFDT simplification by removing one hyperparameter, there was a decrease in the predictive performance for most of the datasets used in the experiments.

7. Handling numeric attributes in hoeffding trees (2008)_CHAPTER

- Not in the proposals.
- Referenced in the paper “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: The first version of the VFDT only handled nominal features. Afterwards, many estimators for continuous features were proposed. Pfahringer et al. (2008) reviewed these estimators and observed that the **Gaussian estimator** is the least sensitive to hyperparameter value and induced the most accurate models,
- becoming the default estimator in recent works.

8. Adaptive Learning from Evolving Data Streams (2009)_HWT_HAT_NOT READ YET.

RESUMEN Y COMPARACIÓN DEL PAPER “Learning Decision Trees from Data Streams with Concept Drift (2016)”: Since development of the Hoeffding bounds, a number of modifications have been proposed. Bifet and Gavalda proposed the Hoeffding Window Tree (HWT) and the Hoeffding Adaptive Tree (HAT). HWT differs from CVFDT since it creates subtrees without waiting for a fixed number of instances (faster reaction for drift) and updates a subtree as soon as there is a benefit from building the new one., instead of using fixed-size sliding window to detect changes, HAT employs an adaptive window at each internal node.

RESUMEN DEL PAPER “Extremely Fast Decision Tree (2018)”: This method builds a tree that grows alternate subtrees if a subtree is observed to have poorer prequential accuracy on more recent examples, and substitutes an alternate when it has better accuracy than the original subtree. HAT uses an error estimator, such as ADWIN [5] at each node to determine whether the prediction error due to a recent sequence of examples is significantly greater than the prediction error from a longer historical sequence so it can respond to drift.

9. An incremental decision tree learning methodology regarding attributes in medical data mining (2009): No aparece en los surveys (1), (2), (3), (4), (5), (6), (7), (8) y (9)

10. Fast Perceptron Decision Tree Learning from Evolving Data Streams (2010)_NOT READ YET: No aparece en los surveys (2), (4), (5), (6) y (7)

- (1) Page 5

- Bifet et al [19] proposed a method, that shares similarities with the work in [40]. However, the work in [19] replaces the naive Bayes with perceptron classifiers. The idea is to gain greater efficiency, while maintaining competitive accuracy.

RESUMEN DEL PAPER “Cost-Sensitive Perceptron Decision Trees for Imbalanced Drifting Data Streams (2017)”: We propose to build our learning algorithm for imbalanced and drifting data streams on top of the Fast Perceptron Decision Tree [1], as it provides both high accuracy and update speed, making it highly suitable for the task at hand. Its main advantage lies in using a linear perceptron at each leaf. This allows to speed-up the decision making process, as well as improve the overall accuracy. This hybrid solution combines the advantages of trees and neural models, allowing for efficient processing of data streams.

Original implementation of Fast Perceptron Decision Tree used Hoeffding inequality to determine the amount of instances needed for conducting a split [1]. However, recent study discussed flaws in the Hoeffding bound (paper “Decision Trees for Mining Data Streams Based on the McDiarmid's Bound (2013)”).

11. Moderated VFDT in Stream Mining Using Adaptive Tie Threshold and Incremental Pruning (2011): **No aparece en los surveys (2), (3), (4), (5), (6), (7), (8) y (9)**

- In this paper, they propose a Moderated VFDT (M-VFDT), which uses an **adaptive tie threshold for node splitting control by incremental computing**. The tree building process is as fast as that of the original VFDT. The accuracy of M-VFDT improves significantly even under the presence of noise in the data stream. To solve the explosion of tree size, which is still an inherent problem in VFDT, they propose two lightweight pre-pruning mechanisms for stream mining (post-pruning is not appropriate here because of the streaming operation).
- Their contribution is a new model that can efficiently achieve a compact decision tree and good accuracy as an optimal balance in data stream mining.
- They devise a new version of VFDT called Moderated VFDT (M-VFDT) that can provide sustainable prediction accuracy and regulate the growth of decision tree size to a reasonable extent, even in the presence of noise. This is achieved by **revising the decision tree building process** – in particular, the conditional check of whether a leaf should be split as a new tree node is modified. The new checking condition is made adaptive to the distribution of the incoming data samples, which in turn influences the value of the HB that is a key factor in the decision tree construction.
- Improved accuracy is achieved by an adaptive tie threshold rather than a userdefined tie threshold, but the tree size is still as big as in VFDT. To solve this problem, incremental pruning methods are proposed to

complement the adaptive tie threshold mechanism for controlling the tree size as well as maintaining the accuracy.

- They propose an alternative design of a tie threshold parameter (instead of [6]) that is adaptive and is calculated directly from the **mean of the HB**, which is found to be proportionally related to the input stream samples. They were inspired to modify the node splitting function, based on the mean of HB, instead of modifying the HB formulation. Holding on to the mean of HB is equivalent to avoiding the fluctuation of HB values, thereby reducing the noise effects.
- They assign an adaptive tie threshold, equal to the dynamic mean of HB as the splitting tie threshold, which controls the node splitting during the tree building process.
- To rectify the tree size problem, they propose two pruning approaches, the strict pruning mechanism and the loose pruning mechanism, each of which reflects a strong and weak pruning strength, respectively.
- Comparison with VFDT and its variants:
 - M-VFDT with a pruning mechanism shows a better performance than the original VFDT at all times.
 - Although VFDT and its variants have been extensively studied, many models assume a perfect data stream and have sub-optimal performance under imperfect data streams.
 - The tree initializing process is the same as the original VFDT.
 - In general, we observe that M-VFDT with strict pruning keeps the tree size smallest, but the accuracy is worse than that of others. The loose pruning method for M-VFDT yields reasonable accuracy that is on par with VFDT, but its tree size is more compact than that of VFDT, although it is still larger than the tree by strict pruning.

12. Optimized very fast decision tree with balanced classification accuracy and compact tree size (2011a)_NOT READ YET

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: In a similar work, Yang and Fong (2011a) proposed Optimised-VFDT (OVFDT), whose goal was also to increase accuracy avoiding tree size explosion, substituting the tie-breaking parameter by statistics about the HB. OVFDT was compared with three algorithms: VFDT (with multiple tie-breaking values); Genuine Tie Detection; and Hoeffding Option Tree (HOT) (New Options for Hoeffding Trees, Ensemble). It must be observed that none of the compared algorithms try to reduce tree size. When compared with VFDT with tie-breaking = 0.05 (VFDT-0.05), OVFDT obtained a small accuracy improvement (3%) at the cost of creating trees 2.4 times larger.

13. OVFDT with Functional Tree Leaf - Majority Class, Naive Bayes and Adaptive Hybrid Integrations (2011b)_NOT READ YET

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: Yang and Fong (2011b, 2013) extended the OVFDt adding statistical constraints related to leaf accuracy. When compared with VFDT-0.05, despite the small improvement in predictive performance, they always produced larger trees.

14. Incremental optimization mechanism for constructing a decision tree in data stream mining (2013) _NOT READ YET:

RESUMEN DEL PAPER “Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018)”: Yang and Fong (2011b, 2013) extended the OVFDt adding statistical constraints related to leaf accuracy. When compared with VFDT-0.05, despite the small improvement in predictive performance, they always produced larger trees.

15. Decision Trees for Mining Data Streams Based on the McDiarmid's Bound (2013)_NOT READ YET:

RESUMEN DEL PAPER “Learning Decision Trees from Data Streams with Concept Drift (2016)”: There are also other bounds – like the McDiarmid. In [21] authors proved that the Hoeffding’s inequality is not suitable for solving the underlying problem.

RESUMEN DEL PAPER “Extremely Fast Decision Tree (2018)”: There is a sizable literature that adapts HT in sometimes substantial ways [12, 19, 23] that do not, to the best of our knowledge, lead to the same fundamental change in learning premise as does HATT. Substitutes the Hoeffding Test with McDiarmid’s test.

16. Incremental Decision Tree based on order statistics (2013): No aparece en los surveys (1), (2), (3), (4), (5), (6), (7), (8) y (9)

- In this paper, they propose a new decision tree method based on order statistics. The construction of an online tree usually needs summaries in the leaves. Their solution uses **bounded error quantiles summaries**. A robust and performing discretization or grouping method uses these summaries to provide, at the same time, a criterion to find the best split and better density estimations. This estimation is then used to **build a naïve Bayes classifier in the leaves** to improve the prediction in the early learning stage.
- For summaries we choose methods based on **order statistics** and addressing the precision / memory tradeoff. For the criterion, the choice turns to the **MODL** [5] method which finds Bayes optimal cut points with order statistics (a Bayes optimal discretization method for continuous attributes). The MODL approach also provides robust density estimation that can be used by a local model. In our case the naïve Bayes classifier is chosen.
- The MODL approach, designed for discretization and value groupings, also returns the quality of a cut or a grouping. This indication I, named

- 'level' (l pertenece a $[0; 1]$), corresponds to a compression ratio. It indicates the information contained in a numerical or categorical variable when considering a target variable.
- The summaries used in the proposed approach have at the same time a fixed memory consumption and strong guarantees on the error over the counts.
- Summaries in the leaves -> Quantiles summaries for numerical attributes and Count min Sketch for categorical attributes
- Quantiles provides order statistics on the data.
- The MODL criterion based on order statistics is chosen to find cuts and groups respectively for a numerical and a categorical variable.
- The tree, proposed in this article, is built online in the same manner as VFDT. It uses the Hoeffding bound but the Entropy Gain is replaced by the MODL criterion.
- They choose the **naïve Bayes** classifier as the local model in the leaves
- for our tree. Naïve Bayes has good performances when it is built with few data and its prediction has low algorithmic complexity. This classifier requires an estimation of the class conditional density. This density is estimated for all intervals or groups of values calculated by the MODL method applied respectively to GK or CMS summaries contained in the leaves.
- The concept drift was not studied in this paper.
- Comparison with VFDT and different types of Hoeffding trees:
 - The MODL criterion has an additional advantage because it returns a value of criterion $l > 0$ if and only if the discretization model / grouping model is better than the model which returns the majority class. This property allows an automatic "pre-pruning" of the tree while in VFDT this pruning must be separately implemented. What is more this criterion estimates not only binary cut points but can also estimate many cuts for each attribute, which allows to build trees having nodes with more than two sons.
 - The MODL split criterion applied on the GK summaries, for numerical variables, and on the counts per class, for categorical variables, is globally better than the Entropy Gain criterion calculated on Gaussian summaries, for numerical variables, and the counts per class, for the categorical variables.
 - The contribution of the naïve Bayes classifier in leaves is debatable with Gaussian summaries because sometimes the accuracy of the global classifier is either significantly improved or significantly degraded. With their two levels summaries, the naïve Bayes classifier improves the accuracy of the entire tree especially in the beginning of training. There is no degradation thanks to the robustness of the MODL approach. It creates intervals only if they contain information. If the variable is not informative no discretization model is proposed. The estimation based on these intervals is then provided to the naïve Bayes classifier.

- The local classifier always improves the prediction in the early learning stage.

17. Hellinger Distance Trees for Imbalanced Streams (2014)_NOT READ YET:

RESUMEN DEL PAPER “Cost-Sensitive Perceptron Decision Trees for Imbalanced Drifting Data Streams (2017)” : Combination of Hoeffding decision tree with Hellinger distance splitting criterion.

While decision trees are popular both in static imbalanced or balanced streaming data mining areas [13], for online skewed data there exists only a modification of Hoeffding Tree using Hellinger distance for conducting splits [5]. This metric, although skew insensitive, may still fail for difficult imbalanced datasets with complex class structures. On the other hand, it imposes minimal additional computational cost on the classifier - a highly desirable property in data stream mining. In non-stationary scenarios using data preprocessing is challenging and may lead to a prohibitively increased computational complexity. Therefore, algorithm-level solutions are worth pursuing and we will concentrate on them in this paper.

Another decision tree algorithm for imbalanced data streams.

18. Learning Decision Trees from Data Streams with Concept Drift (2016)_CEVOT: No aparece en los surveys (1), (2), (3), (4), (5), (6) y (7)

- The proposed algorithm, named **Concept-adapting Evolutionary Algorithm For Decision Tree** does not require any knowledge of the environment such as numbers and rates of drifts. The novelty of the approach is **combining tree learner and evolutionary algorithm**, where the decision tree is learned incrementally and all information is stored in an internal structure of the trees' population. The proposed algorithm is experimentally compared with state-of-the-art stream methods on several real live and synthetic datasets.
- There is lack of adaptive approaches for evolutionary tree learning.
- Proposed algorithm is extension of our batch (offline) algorithm EVO-Tree
- CEVOT learns from the sliding windows without making any assumption about the nature or type of a drift nor on presence or lack of new concept classes. The novelty of the approach is **combining tree learner and evolutionary algorithm, where the decision tree is learned incrementally and all information (knowledge) is stored in the internal structure of the population of trees.**
- This method has other advantages:
 - A natural variable-length encoding structure is used, because the optimal size of a tree for a given data set is not known a priori;
 - The initial algorithm allows for random generation of unbalanced trees of different sizes;

- The fitness function allows for simultaneous optimization of both, the accuracy and the tree size;
 - All crossover and mutation operators are designed in such a way that as a result only correct individuals, i.e. decision trees, are created.
- CEVOT inherits from EVO-Tree an evolutionary computation to process population of trees. The difference is its ability to handle data streams and gather knowledge.
- CEVOT uses **fixed size sliding window** that takes a chunk of data of size w and retrains the model with the last w examples. Because nonstationary environment is considered, CEVOT algorithm will evolve with data.
- Algorithm starts by random generation of an initial population. Nonetheless, this action takes place only when the first data chunk is received. For each subsequent chunks, CEVOT starts with population which remained from the previous run – this is a key feature which maintains “memory”. The idea is that the algorithm adapts itself to data. With every incoming chunk, population of trees shall converge to a current state (i.e. concept) and improve their accuracy.
- **The main limitation of the blind approaches is slow reaction to the concept drift in data. CEVOT forgets old concepts at a constant speed, independently of whether changes are happening or not (individuals selection process). To discard old information we implemented a special destructive mutation mechanism. It works as follows. Randomly selected internal nodes are converted to leaves and the sub-trees rooted at those nodes are pruned.**
- Comparison with Hoeffding Adaptive Tree (HAT), Hoeffding Tree (HT), Naïve Bayes (NB), Accuracy Updated Ensemble (AUE) (based on HT), Accuracy Weighted Ensemble (AWE) (based on HT):
 - CEVOT creates the smallest model for prediction (a single tree classifier). This result is achieved through the optimal tree structure encoding and minimizing size of the tree in evolutionary algorithm.

The fastest classifiers were NB, HT and HAT, respectively. Unfortunately, these algorithms also have the lowest prediction performance. This fact can be explained by the simplicity of their models.

 - -Ensemble models received not much worse times than single classifier models (NB, HT, HAT) but AUE predicts much better than AWE. The details are given in Table 4. CEVOT turned out to be the slowest classifier. Evolutionary computation are very time-consuming processes. The positive aspect is the fact that most of real live datasets processing time was about five second

19. Cost-Sensitive Perceptron Decision Trees for Imbalanced Drifting Data Streams (2017): **No aparece en los surveys (1), (2), (3), (4), (5), (6) y (7)**

- They propose an efficient and fast **cost-sensitive decision tree learning scheme for handling online class imbalance**. In each leaf of the tree they

train a perceptron with output adaptation to compensate for skewed class distributions, while McDiarmid's bound is used for controlling the splitting attribute selection. The cost matrix automatically adapts itself to the current imbalance ratio in the stream, allowing for a smooth compensation of evolving class relationships. Furthermore, they analyze characteristics of minority class instances and incorporate this information during the model update process. It allows their classifier to focus on most difficult instances, while a sliding window keeps track of changes in class structures.

- In this paper they propose a novel decision tree learning approach for handling imbalanced and drifting data streams. As a base for our model, they use **fast perceptron trees** and improve them to become skew-insensitive by using a **moving threshold** solution. It aims at re-balancing the supports for each class during the decision making step, thus alleviating the skew bias with almost no additional computational cost. This is achieved by weighting support functions for each class according to a specified cost function. In our solution the cost matrix evolves over time and adapts to the current state of the stream. This allows them to propose an adaptive cost-sensitive solution that is able to learn from both binary and multi-class imbalanced data streams. We augment it with drift detection and use McDiarmid's bound for controlling the splitting attribute selection. Additionally, they show how to analyze the structure of minority classes in an online manner by using a sliding window. This allows us to estimate the **difficulty of incoming minority class instances**, giving an additional insight into the current state of the stream. They propose an efficient method of **incorporating this background information into the update process of the proposed decision tree in order to better capture the minority class characteristics**.
- They propose to build their learning algorithm for imbalanced and drifting data streams on top of the Fast Perceptron Decision Tree (paper Fast perceptron decision tree learning from evolving data streams).
- They use an online perceptron approach with sigmoid activation function (as suggested by Bifet et al. [1]) with squared error optimization.
- Bifet et al. [1] proposed to use sigmoid activation function instead of a traditional threshold and they follow this approach.
- As they deal with online learning, a stochastic gradient descent is being used with weights updated after each instance [1]. A single perceptron is trained per each class, making it suitable for both binary and multi-class problems. To obtain a final prediction regarding the class of new instance they select the highest value of support functions returned by each perceptron.
- They propose to take an advantage of **using perceptrons in leafs of the decision tree and enhance them with cost-sensitive approach**. This will be achieved by modifying the output of each perceptron, instead of changing the structure of the training data or the training algorithm.
- This solution is highly compatible with data stream mining requirements, as it does not impose significant additional computational needs, do not

rely on data preprocessing and can be easily included in the proposed decision tree learning scheme, taking the advantage of McDiarmid's inequality. Additionally, it is easily applicable for both binary and multi-class data streams, making it a versatile approach.

- They propose a simple, yet effective approach of monitoring the current imbalance ratio among classes and setting the cost according to local pairwise imbalance ratios. This will allow for an easy modeling of multi-minority and multimajority cases.
- As the stream evolved over time one cannot keep all of previous information regarding class relationships. Therefore, we propose to use a **fixed time threshold**, as well as **time stamps with each recorded label** and use them to remove outdated cases from imbalance ratio counting. This allows for dynamically adapting our cost matrix to changes and drifts in the data stream.
- In order to efficiently learn from imbalanced and drifting data streams, we require tools that will be able to monitor the imbalance ratio and the appearance of concept drifts. We propose **to combine their Cost-Sensitive Perceptron Decision Tree with Drift Detection Method for Online Class Imbalance (DDM-OCI)**.
- DDM-OCI was proposed for binary online imbalance, but can be easily extended to multi-class cases. Here, we monitor the averaged recall over all of minority classes.
- Imbalance ratio among classes is not the sole source of learning difficulty. The underlying class structures, overlapping and noisy instances have significant impact on the decision boundaries being estimated. Therefore, one may assume that minority class instances may pose a different level of difficulty to the learning procedure. In this work, they propose to analyze the difficulty of incoming minority class objects, while taking into account the evolving structure of classes.
- They propose six levels of difficulty λ that can be assigned to each new minority instance based on how contaminated is its neighborhood. This is measured by parameter ρ that states how many of k neighbors belong to the same minority class. They propose to label each new minority class instance based on this analysis.
- As they deal with an online scenario, they cannot nor want to keep the entire stream in the memory. Therefore, they propose to analyze the types of minority instances using a small **sliding window** that will keep only the most recent instances, allowing for a fast neighborhood search within it. Additionally, they will incorporate the information from the drift detector.
- Another issue lies in how to utilize this information regarding minority class structure during the online learning process. Each new minority instance will be presented to the cost-sensitive perceptron tree λ times during online learning, where λ is the difficulty level associated with this instance. This will shift their classifier towards concentrating on difficult instances, which in turn should lead to a better predictive performance.

- Comparison with Fast Perceptron Decision Tree (PDT) and Hellinger Hoeffding Decision Tree (HHT):
 - Original implementation of Fast Perceptron Decision Tree used Hoeffding inequality to determine the amount of instances needed for conducting a Split [1]. However, recent study discussed flaws in the Hoeffding bound [8]. In this work, they propose to modify the underlying base of the original Fast Perceptron Decision Tree and use a McDiarmid's inequality for controlling the splitting criteria. It is a generalization of the Hoeffding's inequality, being applicable to both numerical and non-numerical data, as well as better describing the Split measures.
 - While decision trees are popular both in static imbalanced or balanced streaming data mining areas [13], for online skewed data there exists only a modification of Hoeffding Tree using Hellinger distance for conducting splits [5].
 - Their algorithm is directly applicable for both binary and multi-class data streams, while for multi-class cases they modify HHT to conduct binary decompositions in each split in an identical fashion as its static version. Furthermore, they evaluate the performance of the proposed cost-sensitive algorithm without taking into account the types of minority instances (CSPT) and with this extension included (CSPT+).
 - Binary Imbalanced Data Streams: Standard PDT cannot tackle skewed distributions and becomes easily biased towards the majority class. HHT performs much better. yet CSPT and CSPT+ outperform it on 10 out of 12 data streams. This can be explained by Hellinger split criterion not being enough to counter severe class imbalance and difficult minority class structures.
 - Binary Imbalanced Data Streams: In most cases CSPT+ returns the superior performance, showing that the proposed online analysis of minority instances difficulty can be beneficial to the learning process (most of datasets). They may conclude that difficult minority instances are bound to happen in online learning scenarios, especially when the minority class structure is constantly evolving. Therefore, it is worthwhile to incorporate such information during online classifier updating.
 - Binary Imbalanced Data Streams: When taking into account both time and memory resources being used, one can see that perceptron-based solutions are faster than HHT. CSPT displays almost identical resource usage as the native PDT, proving that the proposed cost-sensitive modification and adaptive cost matrix does not impose any significant additional costs. CSPT+ displays slightly higher computational requirements, which was to be expected as for each new minority instance it analyzes its type and needs to store instances in a sliding window. However, this search is conducted only for minority instances, leading only to a slight

increase in overall resource consumption which is far from being prohibitive.

- Multi-class Imbalanced Data Streams: PDT fails to deliver satisfactory performance. However, we can see much bigger discrepancies between HHT and CSPT/CSPT+. As Hellinger distance is a binary metric, to adapt it for multiclass problems one must use a binary decomposition at each node and then average the metric results when conducting splits. Our experiments show that this fails for multi-class imbalanced data streams. CSPT+ always returns the superior performance.
- When analyzing the resource usage, we can see that perceptron-based solutions increased their costs. This is due to higher number of perceptrons being trained at each leaf. Additionally, CSPT+ needs to store more instances in the sliding window and conduct more instance difficulty analyses, as minority instances may arrive from multiple classes. However, the displayed complexity does not seem prohibitive and shows that CSPT+ can be used in real-life scenarios with multi-class imbalanced data streams.

20. Strict Very Fast Decision Tree: a memory conservative algorithm for data stream mining (2018): **No aparece en los surveys (1), (2), (3), (4), (5), (6) y (7)**

- This study proposes the Strict VFDT (SVFDT), a novel algorithm based on the VFDT. The SVFDT algorithm minimises unnecessary tree growth, substantially reducing memory usage and keeping competitive predictive performance.
- In order to deal with memory cost restrictions, keeping the predictive performance, we propose a new base learner, called Strict Very Fast Decision Tree (SVFDT).
- Two SVFDT versions were proposed, one designed to consume less memory and training time (SVFDT-I) and another (SVFDT-II) with a higher predictive performance. In both versions, the following assumptions hold:
 - A leaf node should split only if there is a minimum uncertainty of class assumption associated with the instances, according to previous and current statistics;
 - All leaf nodes should observe a similar number of instances to be turned into split nodes;
 - The feature used for splitting should have a minimum relevance according to previous statistics.
- SVFDT modifies VFDT by strongly controlling tree growth without degrading predictive performance.
- A leaf can satisfy the VFDT split conditions (according to the HB or tiebreak value) and still remain a leaf if SVFDT considers this split unnecessary.
- Comparison with VFDT, OVFD, OVFD with functional tree leaf - majority class, naive bayes and adaptive hybrid integrations and Tie-breaking in Hoeffding Trees:

- Since it creates much more shallow trees than VFDT, SVFDT can achieve a shorter processing time.
- In the last years, (Holmes et al., 2005; Yang and Fong, 2011b, 2013) proposed a series of modifications to increase the predictive performance of the VFDT algorithm. However, this came with a substantial increase in the memory cost.
- SVFDT is faster than the VFDT in some cases.
- SVFDT uses significantly less memory in comparison to VFDT, reaching similar predictive performance
- With respect related work: All of these previous modifications to VFDT provided better predictive performance, at the cost of an increase in memory and processing time. Their proposal aims at reducing these drawbacks while keeping a competitive predictive performance. In this way, they evaluate our algorithm using VFDT as the baseline.
- The accuracy values for both versions of the SVFDT are very close to those of the VFDT.
- Regarding the size of the induced trees, it is possible to see a significant discrepancy. In none of the tests performed, the size of SVFDT trees was larger than those of the VFDT trees.
- According to the statistical tests, there were no statistically significant differences between VFDT predictive performance and the predictive performance of the two proposed algorithms. However, for memory used, there is a statistically significant difference only between SVFDT-I and VFDT.
- When considering training time, there was statistical difference between SVFDT-I and VFDT, but not for SVFDT-II and VFDT.
- SVFDT-II presented higher predictive accuracy than the SVFDT-I, together with significantly reducing tree size.
- SVFDTs can be an efficient alternative to the VFDT in data stream mining applications.
- Datasets with concept drift and noise were analysed. Concept drifts are present in the sea, spam and usenet datasets. SVFDTs and VFDT predictive performance in the presence of concept drifts were similar, except in the dataset sea, when SVFDTs' predictive performance was better, using less than half of the memory used by VFDT.

21. Extremely Fast Decision Tree (2018)_HAT_: **No aparece en los surveys (1), (2), (3), (4), (5), (6) y (7)**

- Novel incremental decision tree learning algorithm, Hoeffding Anytime Tree, that is statistically **more efficient than current state-of-the-art, Hoeffding Tree**.
- Hoeffding Anytime Tree produces the asymptotic batch tree in the limit, is naturally resilient to concept drift, and can be used as a higher accuracy replacement for Hoeffding Tree in most scenarios, at a small additional computational cost.

- In practice, if no split attribute exists at a node, rather than splitting only when the top candidate split attribute outperforms the second-best candidate, HATT will split when the information gain due to the top candidate split is non-zero with the required level of confidence. At later stages, HATT will split when the difference in information gain between the current top attribute and the current split attribute is non-zero, assuming this is better than having no split.
- On nominal with data with d attributes, v values per attribute, and c classes, HATT requires $O(dvc)$ memory to store node statistics at each node, as does HT.
- There are two primary operations associated with learning for HT: (i) incorporating a training example by incrementing leaf statistics and (ii) evaluating potential splits at the leaf reached by an example. The same operations are associated with HATT, but we also increment internal node statistics and evaluate potential splits at internal nodes on the path to the relevant leaf.

Comparison with VFDT, CVFDT and HAT:

- Our implementation of the Hoeffding Anytime Tree algorithm, the Extremely Fast Decision Tree (EFDT), achieves higher prequential accuracy than the Hoeffding Tree implementation Very Fast Decision Tree (VFDT) on many standard benchmark tasks.
- HT constructs a tree incrementally, delaying the selection of a split at a node until it is confident it has identified the best split, and never revisiting that decision. In contrast, HATT seeks to select and deploy a split as soon as it is confident the split is useful, and then revisits that decision, replacing the split if it subsequently becomes evident that a better split is available.
- The HT strategy is more efficient computationally, but HATT is more efficient statistically, learning more rapidly from a stationary distribution and eventually learning the asymptotic batch tree if the distribution from which the data are drawn is stationary. Further, false acceptances are inevitable, and since HT never revisits decisions, increasingly greater divergence from the asymptotic batch learner results as the tree size increases.
- They observe VFDT taking longer and longer to learn progressively more difficult concepts obtained by increasing the number of classes. EFDT learns all of the concepts very quickly. and keeps adjusting for potential overfitting as fresh examples are observed.
- In scenarios where information distribution among attributes is skewed, with some attributes containing more information than others, such a policy (building structure that improves on the current state but making subsequent corrections when further alternatives are found to be even better) can be highly effective because of the limited cost of rebuilding the tree when replacing a

higher-level attribute with a highly informative one. However, where information is more uniformly distributed among attributes, Hoeffding Tree will struggle to split and might have to resort to using a tie-breaking threshold that depends on the number of random variables, while HATT will pick an attribute to begin with and switch when necessary, leading to faster learning.

- The idea common to both CVFDT and HATT is that of split reevaluation. However, the circumstances, objectives, and methods are entirely different. CVFDT is explicitly designed for a drifting scenario; HATT for a stationary one. CVFDT's goal is to reduce prequential error for the current window in the expectation that this is the best way to respond to drift; HATT's goal is reduce prequential error overall for a stationary stream so that it asymptotically approaches that of a batch learner. CVFDT builds and substitutes alternate subtrees; HATT does not. CVFDT deliberately employs a range of forgetting mechanisms; HATT only forgets as a side effect of replacing splits—when a subtree is discarded, so too are all the historical distributions recorded therein. CVFDT always compares the top attributes, while HATT compares with either the current split attribute or the null split. However, CVFDT is not incompatible with the core idea of Hoeffding Anytime Tree; it would be interesting to examine whether the idea of comparing with the null split or the current split attribute when applied to CVFDT will boost its performance on concept drifting streams.
- Hoeffding Adaptive Tree (HAT) builds a tree that grows alternate subtrees if a subtree is observed to have poorer prequential accuracy on more recent examples, and substitutes an alternate when it has better accuracy than the original subtree. HAT uses an error estimator, such as ADWIN [5] at each node to determine whether the prediction error due to a recent sequence of examples is significantly greater than the prediction error from a longer historical sequence so it can respond to drift. HATT, on the other hand, does not rely on prediction results or error, and does not aim to deliberately replace splits in response to drift. HATT has some inbuilt tolerance to concept drift, though it is not specifically designed as a learner for drift. It is easy to conceive of ensemble, forgetting, decay, or subtree replacement approaches built upon HATT to deal with concept drift, along the lines of approaches that have been proposed for HT.
- Thus, they expect HATT to have an advantage over HT in situations where HT considerably delays splits at each level—such as when the difference in information gain between the top attributes at a node is low enough to require a large number of examples in order to overcome the Hoeffding bound, though the information gains themselves happen to be significant. This would

lead to a potentially useful split in HT being delayed, and poor performance in the interim.

- Conversely, when the differences in information gain between top attributes as well as the information gains themselves are low, it is possible that HATT chooses a split that would require a large number of examples to readjust. However, since we expect this to keep up with VFDT on the whole, the main source of underperformance for EFDT is likely to be an overfitted model making low-level adjustments.
- Hoeffding AnyTime Tree makes a simple change to the current de facto standard for incremental tree learning. The current state-of-the-art Hoeffding Tree aims to only split at a node when it has identified the best possible split and then to never revisit that decision. In contrast HATT aims to split as soon as a useful split is identified, and then to replace that split as soon as a better alternative is identified. Their results demonstrate that this strategy is highly effectively on benchmark datasets.
- HT cautiously works toward the asymptotic batch tree, ignoring, and thus not benefiting from potential improvements on the current state of the tree, until it is sufficiently confident that they will not need to be subsequently revised.

22. An adapted incremental graded multi-label classification model for recommendation systems (2018): No aparece en los surveys (1), (2), (3), (4), (5), (6), (7), (8) y (9)

- This paper presents their proposed incremental GMLC method (Graded multi-label classification, the task of assigning to each data a set of relevant labels with corresponding membership grades) that can be applied to build a recommender system.
- In this paper, we focus only on decision trees because they can be intuitively interpreted, they are easy to implement, and they output predictions rapidly.
- The problem addressed in this paper is to learn a recommender system based on GMLC. The main idea of our proposed recommender system is to build two incremental GMLC models. One model HU named **User-incremental- GMLC** is built considering users as instances, and user characteristics and item ratings as descriptive attributes. The other model HI named **Item-incremental-GMLC** is built considering items as instances, and item characteristics and user ratings as descriptive attributes.
- Their proposed framework is based on $(\text{users} + \text{items}) \times (\text{possible ratings} - 1)$ incremental binary classifiers.
- Received ratings first update statistics for decision trees. After collecting enough statistics, a decision tree may grow by splitting leaf nodes if it is worthy according to the minimum description length measure. Decision trees keep growing and fitting better the dataset while the Hamming loss is decreasing. When a concept drift occurs, the Hamming loss keep

increasing until the decision tree complexity or the prediction error reaches a threshold value, then the concept drift is confirmed and the decision tree is rebuilt. As a result the Hamming loss keep decreasing until another concept drift occurs.

- Since each decision tree is built for a target attribute, it receives only instances having a value for that attribute. Hence decision trees don't learn necessarily from the same instances, and therefore they are not grown or rebuilt at the same time.
- Our framework based on decision trees ensures giving a recommendation in a constant time less than one second in the worst case.