Homework I, Econ 714

Be sure to include all the material that shows your work and to present your results in a way that is easy to understand. Remember: ignorance and inability to communicate are observationally equivalent. Go as far as you can in each exercise.

1. Github

Set up a Github repo for this class. You should upload all the codes and results of this homework to the repo. Write the documentation required for a third party to look at your code and results.

2. Integration

Compute:

$$\int_0^T e^{-\rho t} u(1 - e^{-\lambda t}) dt$$

for $T = 100$, $\rho = 0.04$, $\lambda = 0.02$, and $u(\cdot) = -e^{-c}$ using quadrature (midpoint, Trapezoid, and Simpson rule) and a Monte Carlo. Compare both the performance of methods.

3. Optimization: basic problem

Use the Newton-Raphson, BFGS, steepest descent, and conjugate descent method to solve:

$$\min_{x,y} 100(y - x^2)^2 + (1 - x)^2$$

Compare the performance of each method.

4. Computing Pareto efficient allocations

Consider an endowment economy with m different goods and n agents. Each agent $j = 1, ..., n$ has an endowment $e_j^i > 0$ for every $i = 1, ..., m$ and a utility function of the form:

$$u^i(x) = \sum_{j=1}^{m} \alpha_j \frac{x_j^{1+\omega_j^i}}{1 + \omega_j^i}$$

where $\alpha_j > 0 > \omega_j^i$ are agent-specific parameters.

Given some social weights, $\lambda_j > 0$, solve for the social planner's problem for $m = n = 3$ using your favorite optimization method. Try different values of α_j, ω_j^i and $\lambda_j > 0$. For example, you can compute first the case where all the agents have the same parameters and social weights and later a case where there is a fair degree of heterogeneity.

How does the method perform? How does heterogeneity in the agent-specific parameters affect the results? Can you handle the case $m = n = 10$?

5. Computing Equilibrium allocations

Using the same model as in the previous exercise, can you find the equilibrium prices p^i? Hint: Solve for the first-order conditions of each agent, aggregate the excess demands, and solve the resulting system of nonlinear equations.