CONSUMER THEORY

Intermediate Microeconomics — Javier Tasso

Cobb Douglas

$$u(x_1, x_2) = x_1 x_2$$

Utility Maximization:

$$v(p_1, p_2, m) = \frac{m^2}{4p_1p_2}$$
$$x_1(p_1, p_2, m) = \frac{m}{2p_1}$$
$$x_2(p_1, p_2, m) = \frac{m}{2p_2}$$

Expenditure Minimization:

$$e(p_1, p_2, u) = 2\sqrt{p_1p_2u}$$
$$h_1(p_1, p_2, u) = \frac{\sqrt{p_1p_2u}}{p_1}$$
$$h_2(p_1, p_2, u) = \frac{\sqrt{p_1p_2u}}{p_2}$$

Leontief

$$u(x_1, x_2) = \min\{x_1, x_2\}$$

Utility Maximization:

$$v(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$
$$x_1(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$
$$x_2(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

Expenditure Minimization:

$$e(p_1, p_2, u) = (p_1 + p_2)u$$
$$h_1(p_1, p_2, u) = u$$
$$h_2(p_1, p_2, u) = u$$

Perfect Substitutes

$$u(x_1, x_2) = x_1 + x_2$$

Utility Maximization[a]:

$$v(p_1, p_2, m) = \frac{m}{\min\{p_1, p_2\}}$$
$$x_1(p_1, p_2, m) = \begin{cases} \frac{m}{p_1} & \text{if} \quad p_1 < p_2 \\ 0 & \text{if} \quad p_1 > p_2 \end{cases}$$
$$x_2(p_1, p_2, m) = \begin{cases} 0 & \text{if} \quad p_1 < p_2 \\ \frac{m}{p_2} & \text{if} \quad p_1 > p_2 \end{cases}$$

Expenditure Minimization[b]:

$$e(p_1, p_2, u) = \min\{p_1, p_2\}u$$
$$h_1(p_1, p_2, u) = \begin{cases} u & \text{if} \quad p_1 < p_2 \\ 0 & \text{if} \quad p_1 > p_2 \end{cases}$$
$$h_2(p_1, p_2, u) = \begin{cases} 0 & \text{if} \quad p_1 < p_2 \\ u & \text{if} \quad p_1 > p_2 \end{cases}$$

[a] If $p_1 = p_2$ any bundle on the budget constraint is a solution.
[b] If $p_1 = p_2$ any bundle on the target indifference curve is a solution.

Quasi-Linear

$$u(x_1, x_2) = \ln(x_1) + x_2$$

Utility Maximization[a]:

$$v(p_1, p_2, m) = \begin{cases} \ln\left(\frac{p_2}{p_1}\right) + \frac{m-p_2}{p_2} & \text{if} \quad m > p_2 \\ \ln\left(\frac{m}{p_1}\right) & \text{if} \quad m \leq p_2 \end{cases}$$
$$x_1(p_1, p_2, m) = \begin{cases} \frac{p_2}{p_1} & \text{if} \quad m > p_2 \\ \frac{m}{p_1} & \text{if} \quad m \leq p_2 \end{cases}$$
$$x_2(p_1, p_2, m) = \begin{cases} \frac{m-p_2}{p_2} & \text{if} \quad m > p_2 \\ 0 & \text{if} \quad m \leq p_2 \end{cases}$$

Expenditure Minimization[b]:

$$e(p_1, p_2, u) = \begin{cases} p_2[1 + u - \ln\left(\frac{p_2}{p_1}\right)] & \text{if} \quad u > \ln\left(\frac{p_2}{p_1}\right) \\ p_1e^u & \text{if} \quad u \leq \ln\left(\frac{p_2}{p_1}\right) \end{cases}$$
$$h_1(p_1, p_2, u) = \begin{cases} \frac{p_2}{p_1} & \text{if} \quad u > \ln\left(\frac{p_2}{p_1}\right) \\ e^u & \text{if} \quad u \leq \ln\left(\frac{p_2}{p_1}\right) \end{cases}$$
$$h_2(p_1, p_2, u) = \begin{cases} u - \ln\left(\frac{p_2}{p_1}\right) & \text{if} \quad u > \ln\left(\frac{p_2}{p_1}\right) \\ 0 & \text{if} \quad u \leq \ln\left(\frac{p_2}{p_1}\right) \end{cases}$$

[a] Alternatively: $x_1(p_1, p_2, m) = \min\left\{\frac{p_2}{p_1}, \frac{m}{p_1}\right\}$ and $x_2(p_1, p_2, m) = \max\left\{\frac{m-p_2}{p_2}, 0\right\}$.
[b] Alternatively: $h_1(p_1, p_2, u) = \min\left\{\frac{p_2}{p_1}, e^u\right\}$ and $h_2(p_1, p_2, u) = \max\left\{u - \ln\left(\frac{p_2}{p_1}\right), 0\right\}$.