Introduction to Game Theory

Javier Tasso

Introduction

Prisoners' Dilemma

Pigs' Game

Battle of the Sexes

Rock-Paper-Scissors

Types of Games

According to their timing, we classify games:

1. Simultaneous move games.
    - Players choose their actions at the same time, or without knowing the other player's choice.
    - Examples: Prisoners' Dilemma, Battle of the Sexes, Rock-Paper-Scissors.
2. Sequential move games.
    - Players move one after another, observing previous actions before making their own.
    - Examples: Ultimatum game, Centipede game, Entry deterrance game.

Prisoners’ Dilemma

Premise

Melvin Dresher

- First introduced by Merrill Flood and Melvin Dresher (picture). Premise:
- Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a deal. If he testifies against his partner, he will go free while the partner will get three years in prison on the main charge. There's a catch: If both prisoners testify against each other, both will be sentenced to two years in jail. The prisoners are given a little time to think this over, but in no case may either learn what the other has decided until he has irrevocably made his decision. Each is informed that the other prisoner is being offered the very same deal. Each prisoner is concerned only with his own welfare—with minimizing his own prison sentence.

Payoff Matrix

		P2 C	P2 D
P1	C	(-1,-1)	(-3,0)
P1	D	(0,-3)	(-2,-2)

- Each prisoner can Cooperate (C) (remain silent) or Defect (D) (betray the other prisoner).
- Represent the outcomes and utilities in a payoff matrix.
  - P1 is the row player. We list his utility first.
  - P2 is the column player. We list his utility second.

Strategy

Strategy. An algorithm for playing a game, telling a player what to do in every possible situation. Determines what action to take in every stage of the game.

		P2	
		C	D
P1	C	(-1,-1)	(-3,0)
	D	(0,-3)	(-2,-2)

- P1 has $s_1 = C$, $s_1 = D$, and more.
- P2 has $s_2 = C$, $s_2 = D$, and more.
- Because this game has only one stage, strategies coincide with actions. But this doesn't have to be the case.

Best Response

Best Response. The $BR_1(s_2)$ is the strategy (or strategies) that produces the most favorable outcome when the other player chooses s_2.

		P2	
		C	D
P1	C	(-1,-1)	(-3,0)
	D	(0,-3)	(-2,-2)

- Similarly define $BR_2(s_1)$.
- Example.
- Doesn't have to be unique.
- Doesn't have to be the same.

Strictly Dominated Strategy

Strictly Dominated Strategy. A strategy is strictly dominated if there's another strategy that always gives a better outcome. No matter what the other player does.

		P2	
		C	D
P1	C	(-1,-1)	(-3,0)
	D	(0,-3)	(-2,-2)

- For P1 $s_1 = C$ is strictly dominated by $s_1 = D$.
- Similarly for P2.

Strictly Dominant Strategy

Strictly Dominant Strategy. A strategy is strictly dominant if it strictly dominates every other possible strategy.

		P2	
		C	D
P1	C	(-1,-1)	(-3,0)
	D	(0,-3)	(-2,-2)

- For P1 $s_1 = D$ is strictly dominant.
- Similarly for P2.
- If a player has a strictly dominant strategy, we can be pretty sure that's what he end up playing.

Dominant Strategy Equilibrium

Dominant Strategy Equilibrium. If both players have a dominant strategy, each player plays its dominant strategy. The outcome of the game is known as a dominant strategy equilibrium.

- Assumes players are rational.
- Gives a clear prediction of the outcome of a game.
- In the Prisoners' Dilemma the dominant strategy equilibrium is $s_1 = D$ and $s_2 = D$.

Prediction

- When all players have a strictly dominant strategy, we can be sure they will play that strategy.
- The outcome of the game is unique.
- We only require that players are rational to make this prediction.
- The prediction of a game isn't always efficient.

Pigs’ Game

Premise

- Inspired by B. A. Baldwin and G. B. Meese study of social behavior of pigs. Premise:
- A dominant and a submissive pig share a pen. On one side of the pen is a large button, which if pushed releases food into a dish at the other side of the pen. Each pig has the option of pushing the button (P) or not (D). If neither pushes, the pigs go hungry. If the submissive pig pushes the button and the dominant one does not, the released food is eaten entirely by the dominant pig because it gets to the food first. (Here the submissive pig is even worse off than if neither played P, because it expended the effort to push the button but got no food.) If the dominant pig pushes the button, then the submissive pig can enjoy some of the food before the dominant one reaches the dish.

Payoff Matrix

		Submissive P	
		P	D
Dominant P	P	(4,2)	(2,3)
	D	(6,-1)	(0,0)

- Submissive P has a dominant strategy.
- But Dominant P has not.
- Find BR.
- Rationality is not enough to reach an unique prediction of what will happen in this game.

Iterative Removal of Strictly Dominated Strats: Idea

		Sub. P	
		P	D
Dom. P	P	(4,2)	(2,3)
	D	(6,-1)	(0,0)

		Sub. P	
		P	D
Dom. P	P	(4,2)	(2,3)
	D	(6,-1)	(0,0)

- Assume the Dominant Pig knows the Submissive Pig is rational.
- He anticipates S. Pig will play D.
- And we arrive to an unique prediction for the outcome.
- Stronger assumption:
  - Players are rational.
  - They know their opponent is rational.

Common Knowledge of Rationality

Common Knowledge of Rationality. P1 and P2 are rational. They both know each other is rational. They both know that they both know each other is rational. And so on...

- Stronger than rationality.
- Not always needed. Sometimes I do not need to assume this much to make a prediction.
- Allows for Iterative Dominance type of reasoning.

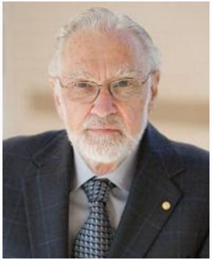
Iterated Removal of Strictly Dominated Strategies

		P2		
		L	C	R
P1	T	(3,3)	(0,5)	(0,4)
	B	(0,0)	(3,1)	(1,2)

- Example. Find BR.
- Apply iterated dominance.
- Builds on two ideas:
  - Rational players will not play strictly dominated strategies.
  - It's common knowledge that players are rational.
- How does common knowledge shows up in our example?

Battle of the Sexes

Premise



Duncan Luce

- Coordination game introduced by Duncan Luce (picture) and Howard Raiffa. Premise:
- Imagine that a man and a woman hope to meet this evening, but have a choice between two events to attend: a prize fight and a ballet. The man would prefer to go to prize fight. The woman would prefer the ballet. Both would prefer to go to the same event rather than different ones. If they cannot communicate, where should they go?

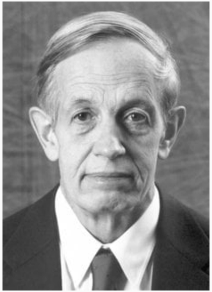
Payoff Matrix

		W	
		F	B
M	F	(3,2)	(0,0)
	B	(0,0)	(2,3)

- Each member can go see the fight (F) or ballet (B).
- No dominated strategies.
- Find BR.
- Can we make predictions in these kind of games?

Nash Equilibrium

Nash Equilibrium. A pair of strategies is a NE if they are mutual best responses.



John Forbes Nash Jr.

- No player can (individually) do better by choosing another strategy.
- A good prediction for games.
- Not necessarily unique. Ex: Battle of the Sexes.
- Not necessarily efficient. Ex: Prisoners' Dilemma.

Prediction

- If there’s a Dominant Strategy Equilibrium, that’s our prediction of the game.
    - Issue: This doesn’t always exists.
- Nash Equilibrium always exists.
    - Most widely used solution concept.
- Consistency and stability of NE.
- NE may involve randomization.
    - See rock-paper-scissors example next.

Rock-Paper-Scissors

Payoff Matrix

		P2		
		R	P	S
P1	R	(0,0)	(-1,1)	(1,-1)
	P	(1,-1)	(0,0)	(-1,1)
	S	(-1,1)	(1,-1)	(0,0)

- Find the BR.

Pure vs Mixed Strategies

- So far we've discussed pure strategies.
- Mixed strategy: randomly choose between pure strategies.
- Every (finite) game has (at least) a Nash Equilibrium.
- Sometimes this equilibrium involves mixed strategies.
- In Rock-Paper-Scissor the NE has both players choosing randomly between the three alternatives.