Microeconomics
Homework 5: General Equilibirum - Exchange

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1. Consumer A and B preferences, as well as endowments, are given below.

$$\begin{aligned} u_A(x_1, x_2) &= x_1 x_2 \qquad & \omega_A &= (2, 8) \\ u_B(x_1, x_2) &= x_1 x_2 \qquad & \omega_B &= (8, 2) \end{aligned}$$

   (a) Plot the initial endowment as well as the indifference curves going through it in the Edgeworth box.

   (b) Is the initial endowment efficient?

   (c) Plot the set of all efficient allocations.

   (d) Find the competitive equilibrium. Is the equilibrium allocation efficient?

2. Continue working with the previous exercise. Normalize $p_2 = 1$ and $p_1 = p$.

   (a) Find the marshallian demands of consumers A and B and goods 1 and 2 as a function of p. You may have found these demands in part (d) of the previous exercise.

   (b) For different values of p (you can try $p = 0.8, 0.9, 1, 2, 3, 4$) plot the optimal consumption of A: $(x_1^A(p), x_2^A(p))$. This trajectory is called offer curve.

   (c) Do the same for consumer B. You can use $p = 0.25, 0.5, 0.75, 1, 1.1, 1.2$.

   (d) Where do the offer curves intersect each other?

3. Consumer A and B preferences and endowments are given below. α and β represent how much consumers like their own good. In international trade, this is called home bias. Consider $0 < \alpha, \beta < 1$.

$$\begin{aligned} u_A(x_1, x_2) &= x_1^{\alpha} x_2^{1-\alpha} \qquad & \omega_A &= (10, 0) \\ u_B(x_1, x_2) &= x_1^{1-\beta} x_2^{\beta} \qquad & \omega_B &= (0, 10) \end{aligned}$$

   (a) For $\alpha = \beta = \frac{1}{2}$. Find the competitive equilibrium and plot the Edgeworth box.

   (b) Suppose now that $\alpha = \beta = \frac{4}{5}$. Find the competitive equilibrium and plot the Edgeworth box. Compare with (a).

   (c) Suppose now that $\alpha = \frac{4}{5}$, but $\beta = \frac{1}{2}$. Repeat the exercise and compare with your previous answers.

   (d) Finally, suppose $\beta = \frac{1}{2}$ and α is unknown. Find the equilibrium price ratio and explain how it depends on α.

4. Consumer A and B preferences and endowments are given below.

$$u_A(x_1, x_2) = x_1 + x_2 \qquad \omega_A = (4, 8)$$
$$u_B(x_1, x_2) = x_1 x_2 \qquad \omega_B = (16, 2)$$

   (a) Using the fact that in an interior solution $\frac{p_1}{p_2}$ must equal the MRS for both consumers, find the equilibrium price ratio.

   (b) Use the equilibrium prices to find the equilibrium allocation. Hint: focus on consumer B.

5. Consumer A and B preferences and endowments are given below.

$$u_A(x_1, x_2) = x_1 + x_2 \qquad \omega_A = (2, 8)$$
$$u_B(x_1, x_2) = \min\{x_1, x_2\} \qquad \omega_B = (8, 2)$$

   (a) Using the fact that in an interior solution $\frac{p_1}{p_2}$ must equal the MRS (if defined) for both consumers, find the equilibrium price ratio.

   (b) Use the equilibrium prices to find the equilibrium allocation.

   (c) Who is benefiting the most from trade? Can you explain intuitively why?

6. Consider individuals A and B with preferences and endowments described below. During this exercise you can assume all solutions are interior. There are in total 12 units of good 1 and 10 units of good 2. The price vector is $(p_1, p_2) = (1, p)$.

$$u_A(x_1, x_2) = x_1 + \ln(x_2) \quad \text{with} \quad \omega_A = (4, 0)$$
$$u_B(x_1, x_2) = x_1 + \ln(x_2) \quad \text{with} \quad \omega_B = (8, 10)$$

   (a) For each consumer solve the utility maximization problem. Find individual demands as a function of the relative price p.

   (b) Focus on good 2. Find the aggregate demand. Find the equilibrium level of the relative price p. Also find the equilibrium allocations.

   (c) Find the contract curve or Pareto set.

   (d) Plot all your previous answers in an Edgeworth box. This includes: the initial endowment, the equilibrium allocation with the final indifference curves, the budget line, and the contract curve.

7. Consider the following preferences and endowments.

$$u_A(x_1, x_2) = \min\{x_1, x_2\} \qquad \omega_A = (2, 1)$$
$$u_B(x_1, x_2) = \min\{x_1, x_2\} \qquad \omega_B = (1, 2)$$

   (a) Plot the Edgeworth box.

   (b) Show the set of all efficient allocation.

   (c) Argue intuitively that there will not be an unique equilibrium price ratio.

8. Same preferences as the previous exercise, but now the endowments are $\omega_A = (2, 2)$ and $\omega_B = (3, 1)$. Answer intuitively: Which good is more valuable in this economy?

Answers

1. (a) See graph. The dashed indifference curves are the ones associated to the initial endowment.
   (b) It is not.
   (c) It is the 45 degree line.
   (d) $p^* = p_1/p_2 = 1$ and the allocations are $(5,5)$ for A and $(5,5)$ for B.

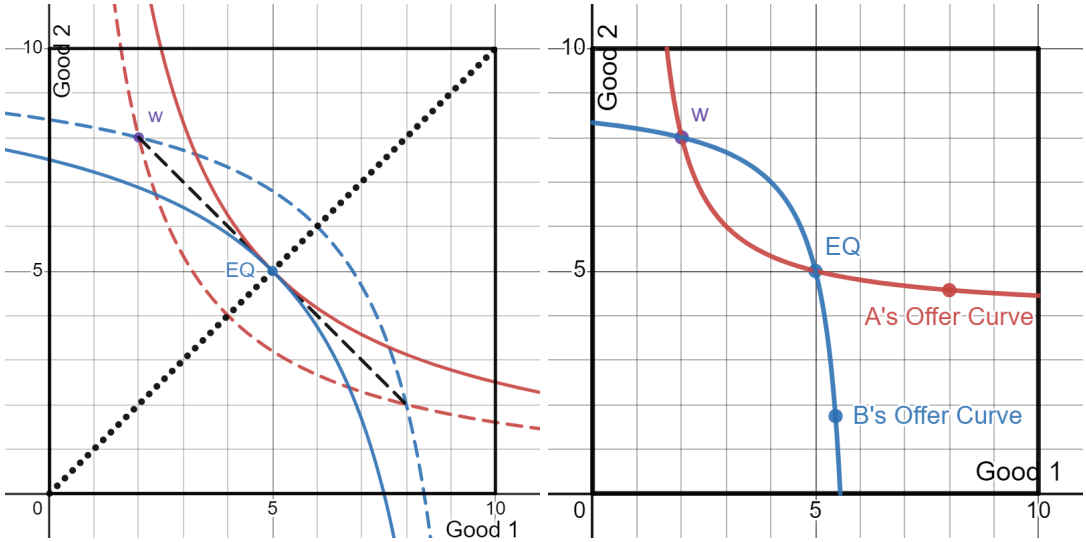

Figure 1: Exercises 1 and 2

2. (a) $x_1^A = \frac{p+4}{p}$ and $x_2^A = p + 4$ for consumer A. $x_1^B = \frac{4p+1}{p}$ and $x_2^B = 4p + 1$ for consumer B.
   (b) See graph.
   (c) See graph.
   (d) They cross in the equilibrium allocation and the initial endowment. This is another way of finding the equilibrium allocation.

3. (a) See picture. $p_1/p_2 = 1$ and the equilibrium allocations are $(5,5)$ for A and $(5,5)$ for B.
   (b) See picture. $p_1/p_2 = 1$ and the equilibrium allocations are $(8,2)$ for A and $(2,8)$ for B. Because now consumers like their own good more, they trade less. Equilibrium price does not change because the home bias is the same for both consumers.
   (c) See picture. $p_1/p_2 = 5/2$. The allocations are $(8,5)$ for A and $(2,5)$ for B. Now the price ratio increased because the home bias is no longer symmetric.
   (d) $\frac{p_1}{p_2} = \frac{1}{2-2\alpha}$. As α increases, consumer A has a higher home bias. Good 1 becomes then relatively scarcer. And the equilibrium price level increases to reflect that.

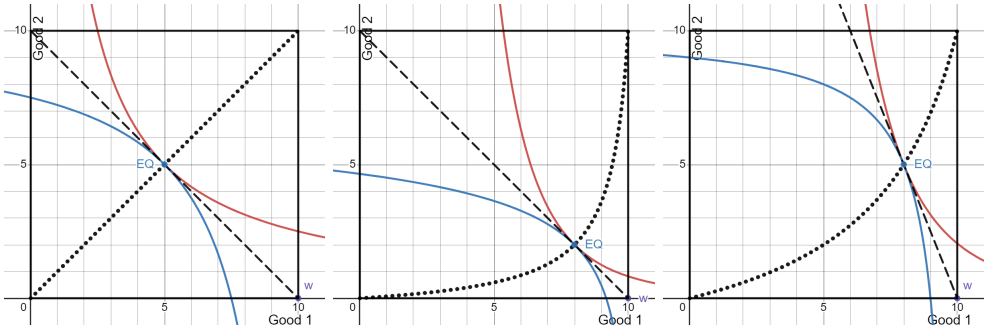

Figure 2: Exercise 3

4. (a) Use the preferences of A. Her MRS is 1 so $p_1/p_2 = 1$.

   (b) Consumer B gets the allocation $(9, 9)$ and consumer A gets $(11, 1)$.

5. (a) $p_1/p_2 = 1$ using the preferences of A.

   (b) $(5, 5)$ for both consumers A and B.

   (c) Consumer B benefits the most. Because A is indifferent between the two goods at the price ratio $p_1/p_2 = 1$, B can trade units with him to increase her utility.

6. (a) $x_1^A = 3$ and $x_2^A = 1/p$ for consumer A.
   $x_1^B = 7 + 10p$ and $x_2^B = 1/p$ for consumer B.

   (b) $p^* = 1/5$. The equilibrium allocations are $(3, 5)$ and $(9, 5)$.

   (c) $x_2^A = x_2^B = 5$.

   (d) See figure.

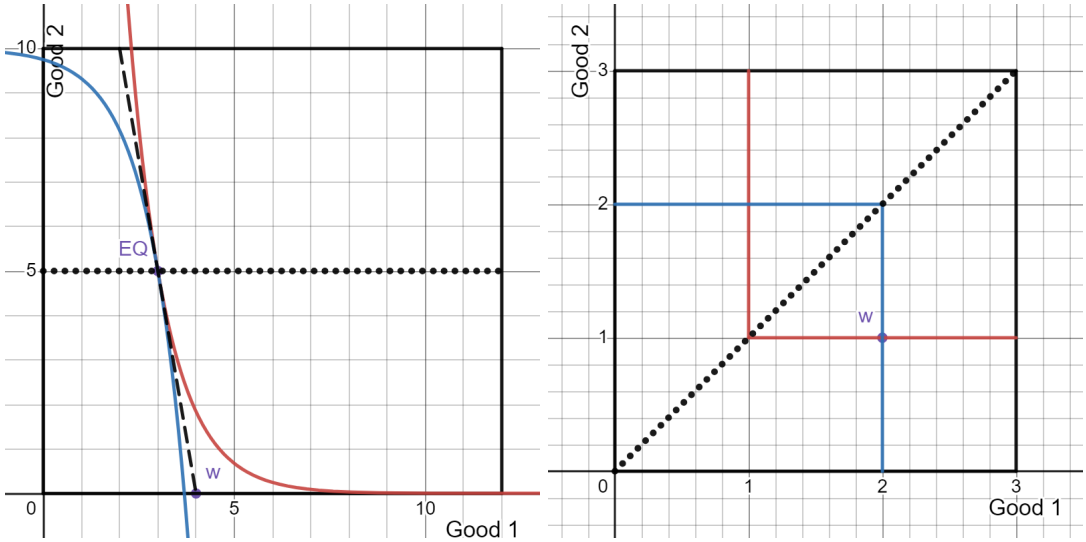

Figure 3: Exercises 6 and 7

7. The efficient allocations are in the 45 degree line. Because the MRS are not defined, there won't be any tangency and we cannot pin down an equilibrium price ratio.

8. Good x_2 is more valuable. Consumer B will demand some of good 2. But consumer A does not want to sell good 2. So there will be excess demand for good 2.