Continuous Distributions

Javier Tasso

University of Pennsylvania

General Random Variables

Continuous Random Variables

- Defined over an interval (or an union of intervals) of the real line.
- No particular value has positive probability.
- We talk about density. We do not talk about probability mass.

Let X be a continuous RV. A probability density function is a function $f(x)$ such that for any two numbers a and b (with $a \leq b$),

$$P(a \leq X \leq B) = \int_a^b f(x)dx$$

- $f(x)$ is sometimes called the density.
- $f(x) \geq 0$.
- $\int_{-\infty}^{\infty} f(x)dx = 1$

Cumulative Distribution Function

Let X be a continuous RV. The cumulative distribution function (cdf) is:

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

Moments

$$\mu = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)dx$$
$$\sigma^2 = V(X) = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)dx$$

- All their properties hold for continuous RVs.
- You may use the shortcut formula also in this case.

Some Continuous Distributions

Uniform Distribution

$X \sim \mathrm{U}(a, b)$

$$f(x) = \frac{1}{b-a} \quad \text{and} \quad F(x) = \frac{x-a}{b-a}$$

- $E(X) = \frac{a+b}{2}$ and $V(X) = \frac{(b-a)^2}{12}$.
- Particular case when $a = 0$ and $b = 1$.

Normal Distribution

$X \sim \mathrm{N}(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{2\pi} \cdot \sigma^2} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

- $E(X) = \mu$ and $V(X) = \sigma^2$.
- Linear combination of independent Normals is also Normal.
- Standardization.
- Empirical rule (68, 95, 99).