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Modeling and Market
Making in Foreign Exchange
IEOR E4722
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Lecture 4 (Options Markets) Assignment

Due start of class, Wednesday June 21, 2017

Question 1 (4 marks)

The current time is Wednesday at 1pm and you see the overnight implied volatility (for 10am expiration on Thursday) trading at 9%. The FX markets are open for trading every hour between now and tomorrow at 10am.

The Federal Reserve Chairwoman is speaking about the economy from 2-3pm, and that event adds an extra 0.5 trading days worth of variance on top of the usual variance for that time period.

What should the overnight implied volatility be at 3pm, all else being equal?

From the slides 2, 3, and 4 of the Lecture 4 – Option Markets ppt it's possible to identify the following parameters,

Expiration time (from t_o) measured in working hours	T_o
Expiration time (from t) measured in working hours	T
Estimated effect expressed in hours of the event over Implied Volatility	T_{event}
Expiration time (from t_o) in functional trading hours	T_T
Nondimensional expiration time in calendar days	T_{cal}
Variance of the calendar time interval	Var_{cal}
Variance of the trading time interval	Var_T
Implied volatility at time t_o	σ_o
Volatility of the trading time interval	σ_T
Implied volatility at time t	σ

Then, walking back from the page 4 to the page 2 it's possible to identify the following equations,

$$\sigma = \sqrt{\frac{Var_T}{T_{cal}}}$$

$$Var_T = \sigma_T^2 \cdot T$$

$$\sigma_T = \sqrt{\frac{Var_{cal}}{T_T}}$$

$$Var_{cal} = \sigma_o^2 \cdot T_{cal}$$

$$T_T = T_o + T_{event}$$

Then,

$$\sigma = \sqrt{\frac{Var_T}{T_{cal}}} = \sqrt{\frac{\sigma_T^2 \cdot T}{T_{cal}}} => \sigma^2 = \sigma_T^2 \frac{T}{T_{cal}} = \frac{Var_{cal}}{T_T} \frac{T}{T_{cal}} = \sigma_o^2 \frac{T_{cal} \cdot T}{T_T \cdot T_{cal}} => \sigma^2 = \sigma_o^2 \frac{T}{T_o + T_e}$$

In this case,

- $\sigma_o = 9\%$
- T, from 3pm to 10am of the following day, $T = 19\ h$
- T_o, from 1pm to 10am of the following day, $T_o = 21\ h$
- T_e, estimated as 0.5 trading day, $T_2 = 12\ h$

Then,

$$\sigma = \sigma_o \sqrt{\frac{T}{T_o + T_e}} = 9\% \cdot \sqrt{\frac{19}{21 + 12}} = 6.83\%$$

However, there is something I don't quite understand about this result.

Let's say that the news that will impact on the implied volatility will be released exactly at T. If we knew the volatility immediately before T, let's call it σ_{T-} at T^{-}, then we would be able to calculate σ, as

$$\sigma^2 = \sigma_{T-}^2 \cdot \frac{T}{T+T_e}$$

And σ_o and σ_{T-} must be related by a discount factor

$$\sigma_{T-}^2 = \sigma_o^2 \cdot K$$

Then we can rewrite

$$\sigma^2 = \sigma_o^2 \cdot K \cdot \frac{T}{T+T_e} = \sigma_o^2 \cdot \frac{T}{T_o+T_e} => K = \frac{T+T_e}{T} \cdot \frac{T}{T_o+T_e} = \frac{T+T_e}{T_o+T_e}$$

And then

$$\sigma_{T-}^2 = \sigma_o^2 \cdot \frac{T+T_e}{T_o+T_e} = \sigma_o^2 \cdot \left(1 - \frac{\Delta t}{T_o+T_e}\right)$$

Where Δt is the interval between T_o and T. And

$$\sigma^2 = \sigma_{T-}^2 \cdot \frac{T}{T+T_e} = \sigma_o^2 \cdot \frac{T+T_e}{T_o+T_e} \cdot \frac{T}{T+T_e} = \sigma_o^2 \cdot \frac{T}{T_o+T_e}$$

As expected.

However, σ_{T-}^2. is the implied volatility at T <u>before</u> the news were released. It seems assumed that the event will have an impact in the implied volatility even before the event itself occur.

The alternative would lead to calculate σ_{T-}^2 as

$$\sigma_{T-}^2 = \sigma_o^2 \cdot \frac{T}{T_o}$$

then

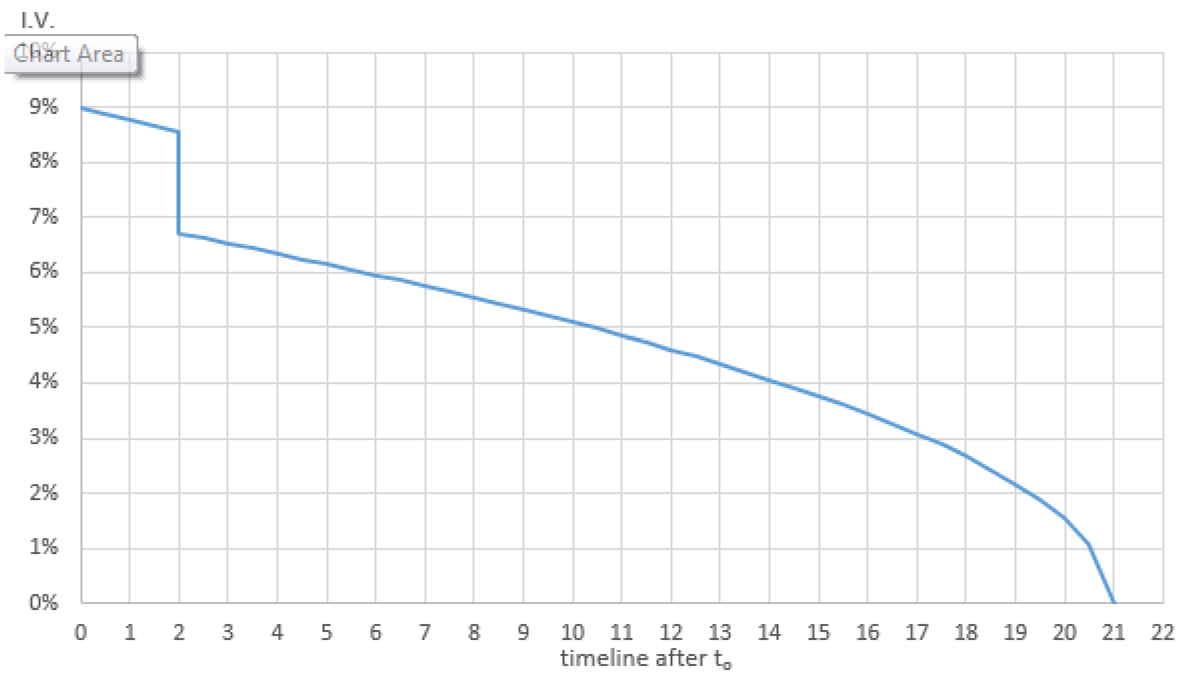
$$\sigma^2 = \sigma_{T-}^2 \cdot \frac{T}{T+T_e} = \sigma_o^2 \cdot \frac{T}{T_o} \cdot \frac{T}{T+T_e} = \sigma_o^2 \cdot \frac{T^2}{T_o \cdot (T+T_e)}$$

and then,

$$\sigma = \sigma_o \sqrt{\frac{T^2}{T_o \cdot (T + T_e)}} = 9\% \cdot 19 \cdot \sqrt{\frac{1}{21 \cdot (19 + 12)}} = 6.70\%$$

Also, by defining Δt as any given interval of time after T_o, it's possible to see the evolution of the implied volatility smoothly decaying with one step at the moment of the event.

$$\sigma_{\Delta t}^2 = \sigma_o^2 \cdot \frac{T_o - \Delta t}{T_o} = \sigma_o^2 \cdot \left(1 - \frac{\Delta t}{T_o}\right)$$



Question 2 (3 marks)

In stochastic volatility models, why is there a smile? Describe the genesis of the smile in terms of vega gamma.

Similarly, describe why stochastic volatility models generate a skew, in terms of vega dspot.

Because the fact that options might be priced assuming constant volatility doesn't mean that their value doesn't change when volatility changes. In fact, because an option can always be not executed if it's out of the money, having high volatility increases the expected return, and so increases the value of the option. Thus, if an option is initially underpriced -e.g. because it was assumed constant volatility in the range of strikes- can always be traded in the secondary market. But then, how to identify an underpriced option in the primary market?

An analogy, the best way to predict how far a Foucault pendulum will arrive is by evaluating the first and second partial derivative respect to the floor. In particular, if the pendulum has max acceleration and zero velocity, we know that it has way to go.

Same thing with options, with underlying's asset price in one axis and volatility in another, flat vega with either long vega gamma $(\partial^2 V/\partial\sigma^2)$, or long vega dspot $(\partial^2 V/\partial\sigma\partial S)$ guarantee profit. Since flat vega can be achieved vega hedging with ATM vanilla options, the market adapts pricing higher these options with higher volatility, and lower those ones with lower volatility (the ATM vanilla), creating the volatility smile.

The slope in each one of the points of the smile depend of vega gamma and vega dspot. In a sense, an increment of volatility in the curve with a correspondent standardized increment in strike, provides a metric of the variation of volatility respect to strike prices, which is precisely vega dspot.

Question 3 (2 marks)

Why do most FX shops use a "sticky delta" volatility market model when defining delta for hedging purposes, even though that might not give the most accurate estimate of how implied volatilities, and hence portfolio prices, change when spot moves?

"Sticky delta" refers to volatility by delta that stays fix as spot moves. In FX markets vol-by-delta is a quoting convention, so traders tend to evaluate portfolio risk in terms of vol-by-delta an "sticky delta" approach makes it easier.

Question 4 (4 marks)

Consider an ATM EURGBP option with 0.5y to expiration. Assume the EURGBP ATM volatility is 3.5%, the EURUSD ATM volatility is 8.5%, and the GBPUSD ATM volatility is 7.5%. What is the implied correlation between EURUSD and GBPUSD spots?

EURUSD spot is 1.25 and GBPUSD spot is 1.56; assume zero interest rates.

Use the Black-Scholes vega formula to calculate the vegas of all three options and determine the notionals of EURUSD and GBPUSD options needed to hedge the vegas of 1 EUR notional of the EURGBP option, assuming correlation stays constant.

The implied correlation,

$$\sigma_{EG}^2 = \sigma_{EU}^2 + \sigma_{GU}^2 - 2\rho\sigma_{EU}\sigma_{GU} => \rho = \frac{\sigma_{EU}^2 + \sigma_{GU}^2 - \sigma_{EG}^2}{2\sigma_{EU}\sigma_{GU}} = \frac{0.085^2 + 0.075^2 - 2 \cdot 0.035^2}{2 \cdot 0.085 \cdot 0.075} = 0.912$$

The strikes,

$$K_{EU} = S_{EU} \cdot e^{\left(R-Q-\frac{\sigma^2}{2}\right)T} = 1.25 \cdot e^{\left(-\frac{0.085^2}{2}\right)\cdot 0.5} = 1.252$$

$$K_{GU} = S_{GU} \cdot e^{\left(R-Q-\frac{\sigma^2}{2}\right)T} = 1.56 \cdot e^{\left(-\frac{0.075^2}{2}\right)\cdot 0.5} = 1.562$$

$$K_{EG} = S_{EG} \cdot e^{\left(R-Q-\frac{\sigma^2}{2}\right)T} = \frac{1EUR}{1.25\,USD} \cdot \frac{1.56\,USD}{1GBP} \cdot e^{\left(-\frac{0.035^2}{2}\right)\cdot 0.5} = 1.248$$

The coefficient d_1

$$d_1 = \left[\ln\frac{S}{K} + \left(R - Q + \frac{\sigma^2}{2}\right)T\right] \cdot \frac{1}{\sigma\sqrt{T}} = \left[\left(R - Q - \frac{\sigma^2}{2}\right) + \left(R - Q + \frac{\sigma^2}{2}\right)\right] \cdot \frac{T}{\sigma\sqrt{T}} = 0$$

The Vegas,

$$\nu_{EU} = \left(\frac{\partial V}{\partial \sigma}\right)_{EU} = S_{EU} \cdot e^{-QT} \cdot \sqrt{T} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-d_1^2}{2}} = 1.25 \cdot \sqrt{\frac{0.5}{2\pi}} = 0.353$$

$$\nu_{GU} = \left(\frac{\partial V}{\partial \sigma}\right)_{GU} = S_{GU} \cdot e^{-QT} \cdot \sqrt{T} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-d_1^2}{2}} = 1.56 \cdot \sqrt{\frac{0.5}{2\pi}} = 0.440$$

$$\nu_{EG} = \left(\frac{\partial V}{\partial \sigma}\right)_{EG} = S_{EG} \cdot e^{-QT} \cdot \sqrt{T} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{\frac{-d_1^2}{2}} = \frac{1.56}{1.25} \cdot \sqrt{\frac{0.5}{2\pi}} = 0.352$$

Then we can build the portfolio Π,

$$\Pi = V_{EG} + N_{EU}V_{EU} + N_{GU}V_{GU}$$

and choose N_{EU} and N_{GU} to make the portfolio vega neutral.

However, it is interesting to notice that if we had to quantify how much money this portfolio worth at any given time, the form in which it has been written considers the EURUSD and GBPUSD options expressed in dollars, but the EURGBP option is expressed in pounds. Then,

$$V_{EG}^{\$} = \frac{1.56\ USD}{1\ GBP} \cdot V_{EG}^{£} = S_{GU} \cdot V_{EG}$$

and,

$$\Pi^{\$} = S_{GU}V_{EG} + N_{EU}V_{EU} + N_{GU}V_{GU}$$

Now, to vega-hedge the portfolio

$$\left.\begin{aligned} \frac{\partial \Pi}{\partial \sigma_{EU}} = S_{GU}\frac{\partial V_{EG}}{\partial \sigma_{EU}} + N_{EU}\frac{\partial V_{EU}}{\partial \sigma_{EU}} + N_{GU}\frac{\partial V_{GU}}{\partial \sigma_{EU}} = 0 => S_{GU} \cdot \frac{\partial V_{EG}}{\partial \sigma_{EG}} \cdot \frac{\partial \sigma_{EG}}{\partial \sigma_{EU}} + N_{EU} \cdot \nu_{EU} = 0 \\ \frac{\partial \Pi}{\partial \sigma_{GU}} = S_{GU}\frac{\partial V_{EG}}{\partial \sigma_{GU}} + N_{EU}\frac{\partial V_{EU}}{\partial \sigma_{GU}} + N_{GU}\frac{\partial V_{GU}}{\partial \sigma_{GU}} = 0 => S_{GU} \cdot \frac{\partial V_{EG}}{\partial \sigma_{EG}} \cdot \frac{\partial \sigma_{EG}}{\partial \sigma_{GU}} + N_{GU} \cdot \nu_{GU} = 0 \end{aligned}\right\} =>$$

$$=> \begin{cases} S_{GU} \cdot \nu_{EG} \cdot \dfrac{\sigma_{EU} - \rho\sigma_{GU}}{\sigma_{EG}} + N_{EU} \cdot \nu_{EU} = 0 => N_{EU} = -S_{GU} \cdot \dfrac{\nu_{EG}}{\nu_{EU}} \cdot \dfrac{\sigma_{EU} - \rho\sigma_{GU}}{\sigma_{EG}} => \\ S_{GU} \cdot \nu_{EG} \cdot \dfrac{\sigma_{GU} - \rho\sigma_{EU}}{\sigma_{EG}} + N_{GU} \cdot \nu_{GU} = 0 => N_{GU} = -S_{GU} \cdot \dfrac{\nu_{EG}}{\nu_{GU}} \cdot \dfrac{\sigma_{GU} - \rho\sigma_{EU}}{\sigma_{EG}} => \end{cases}$$

$$=> N_{EU} = -1.56 \cdot \frac{0.352}{0.353} \cdot \frac{0.085 - 0.912 \cdot 0.075}{0.035} = -0.738$$

$$=> N_{GU} = -1.56 \cdot \frac{0.352}{0.440} \cdot \frac{0.075 - 0.912 \cdot 0.085}{0.035} = 0.090$$

Question 5 (10 marks)

In this question you will look at implied correlations and see how much moves in implied correlation contribute to moves in cross volatility, versus moves in the underlying USD-pair volatilities.

Consider the AUDJPY market, where the underlying USD pairs are AUDUSD and USDJPY.

For a given expiration tenor, one can calculate the market-implied correlation between moves in AUDUSD spot and USDJPY spot through the implied volatilities for the three pairs.

First step: write code to calculate these correlations in a window from 1Jan2007 to 31May2013. I have posted a spreadsheet with the ATM implied volatilities for AUDUSD, USDJPY, and AUDJPY for various expiration tenors on the class forum.

You should write a function that takes in the names of the three pairs (as strings like 'AUDJPY', 'AUDUSD', and 'USDJPY'), a string tenor (like '3m'), a flag to define whether the cross spot is the product or the ratio of the two USD spots (which affects the sign of the correlation), and the start and end dates of the historical window.

It should start by loading the data for the ATM implied volatility for the three tenors from the spreadsheet into pandas DataFrames and then calculate a pandas DataFrame of implied correlations.

The next step: use the correlation from date i, along with the implied volatilities for the USD pairs on date i+1, to predict the cross volatility on date i+1. Do this with the pandas DataFrames you have already created.

Finally, construct two DataFrames: one holding day-to-day changes in the cross ATM volatility, and one holding differences between the predicted cross volatility (assuming the implied correlation from the day before) and the true cross volatility.

The function should print out statistics on both those series.

Run this for the following list of tenors: 1w, 1m, 6m, and 1y. Comment on any differences across tenors, and whether this seems like a good hedging strategy for hedging AUDJPY volatility. Make sure to refer to statistics of the two series, both standard deviations as well as maximum and minimum deviations.

The output of the program,

```
 ------- --------------------------- ------------------------------
|       |     StdDev of intraday     | StdDv of intraday "perceived" |
|       |    volatility variation    |  volatility variation of a    |
| Tenor | (volatility of volatility) | Vega Neutral Hedged AUDJPY    |
|       |  of AUDJPY currency pair   |      options portfolio        |
|-------|----------------------------|-------------------------------|
|  1w   |        1.79492632882       |          1.18567162937        |
|  1m   |        1.09618446789       |          0.742171918568       |
|  6m   |        0.574872056273      |          0.383107979546       |
|  1y   |        0.431274160587      |          0.315865839923       |
 ------- --------------------------- ------------------------------
```

The vega hedging strategy over the AUDJPY options is only able to reduce the standard deviation of the variation of the implied volatility a 33% in the 1w, 1m, and 6m tenors; and 25% in the 1y tenor.

The Python code is copied below.

```
import scipy as sp
import pandas as pd
def main():
  T = pd.read_csv('fx_vol_data.csv')
  label=['1w','1m','6m','1y']
  print(' ------- --------------------------- ------------------------------ ')
  print('|       |     StdDev of intraday     | StdDv of intraday "perceived" |')
  print('|       |    volatility variation    |  volatility variation of a    |')
  print('| Tenor | (volatility of volatility) |  Vega Neutral Hedged AUDJPY   |')
  print('|       |  of AUDJPY currency pair   |      options portfolio        |')
  print('|-------|----------------------------|-------------------------------|')
  for i in range(4):
    delta_vol=sp.diff(T.iloc[:,9+i],n=1,axis=0)
    crr = sp.array((T.iloc[:,1+i]**2+T.iloc[:,5+i]**2-T.iloc[:,9+i]**2)/(2*T.iloc[:,1+i]*T.iloc[:,5+i]))
    vv=pd.Series(sp.concatenate((crr[0:1],crr[0:-1]),axis=0))
    err_est_cor=sp.sqrt(T.iloc[:,1+i]**2+T.iloc[:,5+i]**2-2*vv*T.iloc[:,1+i]*T.iloc[:,5+i])-T.iloc[:,9+i]
    print('|  '+label[i]+'\t|\t\t'+str(delta_vol.std()) + '\t\t |\t\t\t' + str(err_est_cor.std())+'\t\t |')
  print(' ------- --------------------------- ------------------------------ ')

if __name__ == '__main__':
  main()
```