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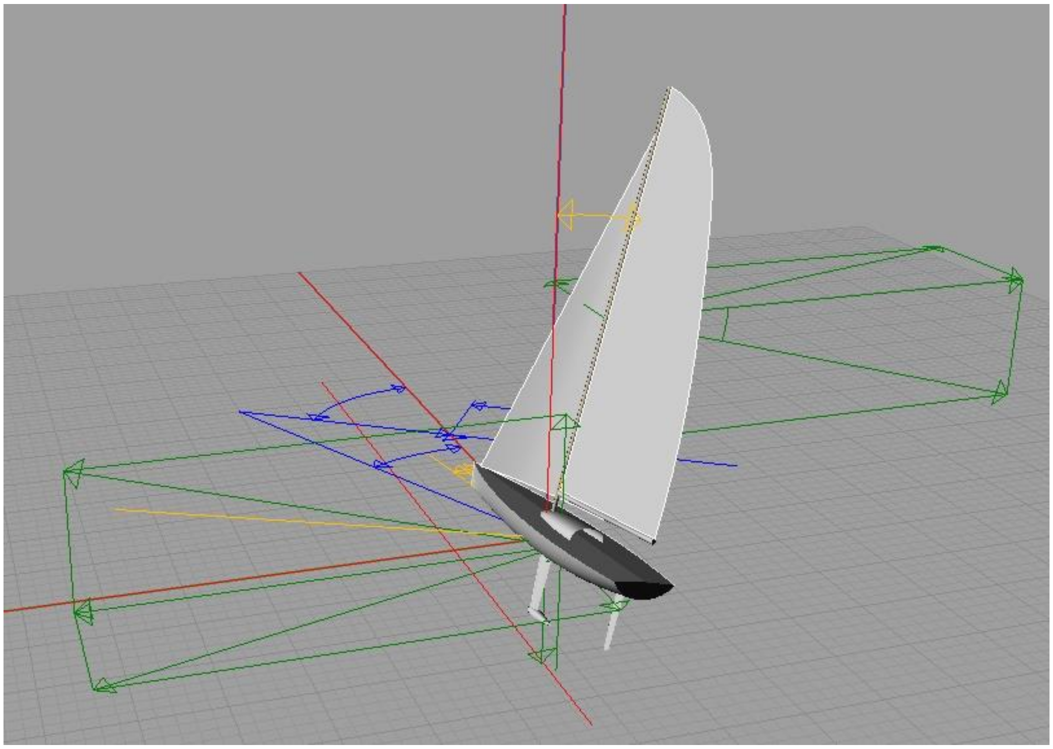
Yacht Velocity Prediction Program



Predicting the speed of a sailing boat under any given condition requires solving the following system of equations,

$$R_{tot}(V, \phi) = F_{drive}(V, \phi)$$
$$M_{right}(\phi) = M_{heel}(V, \phi)$$

where

- R_{tot}, the advance resistance
- F_{drive}, the force due to the wind
- M_{right}, the righting moment
- M_{heel}, the heeling moment
- V, velocity of the boat
- ϕ, heeling angle

Then,

$$R_{tot}(V,\phi) = R_{tot}(V^*,\phi^*) + (V - V^*)\frac{\partial R_{tot}}{\partial V}\bigg|_{V^*,\phi^*} + (\phi - \phi^*)\frac{\partial R_{tot}}{\partial \phi}\bigg|_{V^*,\phi^*} =$$

$$= \underbrace{V\frac{\partial R_{tot}}{\partial V}\bigg|_{V^*,\phi^*}}_{B_1} + \underbrace{\phi\frac{\partial R_{tot}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{C_1} + \underbrace{R_{tot}(V^*,\phi^*) - V^*\frac{\partial R_{tot}}{\partial V}\bigg|_{V^*,\phi^*} - \phi^*\frac{\partial R_{tot}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{D_1} = B_1V + C_1\phi + D_1$$

$$F_{drive}(V,\phi) = F_{drive}(V^*,\phi^*) + (V - V^*)\frac{\partial F_{drive}}{\partial V}\bigg|_{V^*,\phi^*} + (\phi - \phi^*)\frac{\partial F_{drive}}{\partial \phi}\bigg|_{V^*,\phi^*} =$$

$$= \underbrace{V\frac{\partial F_{drive}}{\partial V}\bigg|_{V^*,\phi^*}}_{B_2} + \underbrace{\phi\frac{\partial F_{drive}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{C_2} + \underbrace{F_{drive}(V^*,\phi^*) - V^*\frac{\partial F_{drive}}{\partial V}\bigg|_{V^*,\phi^*} - \phi^*\frac{\partial F_{drive}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{D_2} = B_2V + C_2\phi + D_2$$

$$M_{right}(V,\phi) = M_{right}(V^*,\phi^*) + (V - V^*)\frac{\partial M_{right}}{\partial V}\bigg|_{V^*,\phi^*} + (\phi - \phi^*)\frac{\partial M_{right}}{\partial \phi}\bigg|_{V^*,\phi^*} =$$

$$= \underbrace{V\frac{\partial M_{right}}{\partial V}\bigg|_{V^*,\phi^*}}_{B_3=0} + \underbrace{\phi\frac{\partial M_{right}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{C_3} + \underbrace{M_{right}(V^*,\phi^*) - V^*\frac{\partial M_{right}}{\partial V}\bigg|_{V^*,\phi^*} - \phi^*\frac{\partial M_{right}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{D_3} = C_3\phi + D_3$$

$$M_{heel}(V,\phi) = M_{heel}(V^*,\phi^*) + (V - V^*)\frac{\partial M_{heel}}{\partial V}\bigg|_{V^*,\phi^*} + (\phi - \phi^*)\frac{\partial M_{heel}}{\partial \phi}\bigg|_{V^*,\phi^*} =$$

$$= \underbrace{V\frac{\partial M_{heel}}{\partial V}\bigg|_{V^*,\phi^*}}_{B_4} + \underbrace{\phi\frac{\partial M_{heel}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{C_4} + \underbrace{M_{heel}(V^*,\phi^*) - V^*\frac{\partial M_{heel}}{\partial V}\bigg|_{V^*,\phi^*} - \phi^*\frac{\partial M_{heel}}{\partial \phi}\bigg|_{V^*,\phi^*}}_{D_4} = B_4V + C_4\phi + D_4$$

and balancing forces and moments

$$R_{tot}(V,\phi) = F_{drive}(V,\phi) => B_1V + C_1\phi + D_1 = B_2V + C_2\phi + D_2 => (B_1 - B_2)V + (C_1 - C_2)\phi = D_2 - D_1$$

$$M_{right}(\phi) = M_{heel}(V,\phi) => C_3\phi + D_3 = B_4V + C_4\phi + D_4 => (-B_4)V + (C_3 - C_4)\phi = D_4 - D_3$$

it is possible to build the following iterative scheme

$$\begin{pmatrix} B_1 - B_2 & C_1 - C_2 \\ -B_4 & C_3 - C_4 \end{pmatrix}\begin{pmatrix} V \\ \phi \end{pmatrix} = \begin{pmatrix} D_2 - D_1 \\ D_4 - D_3 \end{pmatrix}$$

where the spectral radius of the main matrix is less or equal 1, so it is expected linear convergence. The initial values are

$$V^* = 1.25 \cdot \sqrt{Lwl}$$

$$\phi^* = 0$$

The calculation of the partial derivatives of each one of the coefficients relays on the splines of order 1 obtained from the corresponding values obtained experimentally and publicly available.

References,

- Principles of Yacht Design, Larsson & Eliasson
- Applied Naval Architecture, Zubaly
- Numerical Methods using MATLAB, Mathews & Fink
- Approximation of the Hydrodynamic Forces on a Sailing Yacht base on the Delft Systematic Yacht Hull Series, Keuning and Sonnenberg, The International HISWA Symposium
- The MATLAB software gvpp.

–	Effective apparent wind angle		alfa_eff	
–	True wind angle	α_{TW}	alfa_tw	[deg]
–	Average freeboard	$\overline{fb}$	AVGFREB	[m]
–	Design waterplane area	Aw	AW	[m^2]
–	Total lateral area of yacht		ALT	[m^2]
–	Rudder planform area		APR	[m^2]
–	Nominal sail area		AN	
–	Mainsail area		AM	
–	Jib area		AJ	
–	Spinnaker area		AS	
–	Foretriangle area		AF	
–	Crew arm	b	b	[m]
–	beam of waterline	Bwl	BWL	[m]
–	Design of maximum beam	B	B	[m]
–	Bottom height above deck		BAD	[m]
–	Midship section coefficient	Cm	CMS	[-]
–	Longitudinal prismatic coefficient		CPL	[m]
–	Mean chord length		CHMEK	[m]
–	Root chord length		CHRTK	[m]
–	Tip chord length		CHTPK	[m]
–	Mean chord length		CHMER	[m]
–	Root chord length		CHRTR	[m]
–	Tip chord length		CHTPR	[m]
–	Mean thickness ratio of keel section		DELTTK	[-]
–	Mean thickness ratio of rudder section		DELTTR	[m]
–	Volume of displacement of canoe	∇_c	DIVCAN	[m^3]
–	Displaced volume of keel		DVK	[m^3]
–	Rudder displacement volume		DVR	[m^3]
–	Mainsail base		E	[m]
–	Mast's height above deck		EHM	[m]
–	Mast's average diameter		EMDC	[m]
–	Flattening factor	F	F	[-]
–	Flattening factor		F	[-]
–	Drive force		Fdrive	
–	Side force		Fside	
–	Gravitational acceleration	g	g	[m/s^2]
–	Hull form factor		HULLFF	[m]
–	Foretriangle height		I	[m]
–	Foretriangle base		J	[m]
–	Center of gravity above moulded base or keel		KG	[m]
–	Transverse metacenter above moulded base or keel		KM	[m]
–	Keel's form factor		KEELFF	[-]
–	Perpendicular of largest jib		LPG	[m]
–	Waterline length	Lwl	LWL	[m]
–	Heeling moment		Mheel	
–	Movable crew mass		MMVBLCRW	[kg]
–	Correction for mainsail roach		MROACH	[-]
–	Full length battens in main (0:no, 1:yes)		MFLB	[-]

– Heeling angle	ϕ	Phi	[deg]
– Density of water	ρ_w	rho_w	[kg/m^3]
– Water kinematic viscosity	ν_w	ni_w	[m^2/s]
– Air density	ρ_a	rho_a	[kg/m^3]
– Total resistance		Rtot	
– Hull viscous resistance	R_{vh}	Rvh	
– Hull residuary resistance	R_{rh}	Rrh	
– Keel viscous resistance	R_{vk}	Rvk	
– Rudder viscous resistance	R_{vr}	Rvr	
– Keel residuary resistance	R_{rk}	Rrk	
– Change in hull viscous resistance due to heel	R_{vhH}	RvhH	
– Change in hull residuary resistance due to heel	R_{rhH}	RrhH	
– Change in keel residuary resistance due to heel	R_{rkH}	RrkH	
– Keel induced resistance	R_i	Ri	
– Change in hull residuary resistance due to trim	R_{rhT}	RrhT	
– Rudders form factor		RUDDFF	[-]
– Sails used in the calculation		SAILSET	[-]
– Wetted area of the hull at zero speed	S_c	SC	[m^2]
– Spinnaker length		SL	[m]
– Keel's wetted surface		SK	[m^2]
– Rudder wetted surface		SR	[m^2]
– Total draft	T	T	[m]
– draft of canoe body	T_c	TCAN	[m]
– Taper ratio of keel (CHRTK/CHTPK)		TAK	[-]
– Effective span of the keel		Te	
– Mainsail height		P	[m]
– The forward velocity of the yacht	V	V	[m/s]
– Apparent wind velocity opposite to direction of motion		V1	
Apparent wind velocity at right angles to mast and direction of motion		V2	
– Effective apparent wind velocity		Veff	
– True wind speed	V_{TW}	V_tw	[m/s]
– Boat Speed		V	[m/s]
– Boat Velocity Made Good		VMG	[m/s]
– LCB from forward perpendicular	LCB_{fpp}	XFB	[m]
– LCF from forward perpendicular	LCF_{fpp}	XFF	[m]
– Keel's vertical center of buoyancy (above keel)	$Zcbk$	ZCBK	[m]

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$$F_{drive}(V^*, \phi^*)$$

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: $V_1 = V^* + V_{tw} \cdot \cos\alpha_{tw}$

: $V_2 \approx V_{tw} \cdot \sin\alpha_{tw} \cdot \cos\phi^*$

: $V_{eff} = \sqrt{V_1^2 + V_2^2}$

: $\alpha_{eff} = \operatorname{atan}\left(\frac{V_2}{V_1}\right)$

: $AM = \frac{1}{2} \cdot P \cdot E \cdot MROACH$

: $AJ = \frac{1}{2} \cdot LPG \cdot \sqrt{I^2 + J^2}$

: $AS = 1.15 \cdot SL \cdot J$

: $AF = \frac{1}{2} \cdot I \cdot J$

: $AN = AF + AM$

: $ZCEM = 0.39 \cdot P + BAD$

: $ZCEJ = 0.39 \cdot I$

: $ZCES = 0.59 \cdot I$

: $k\_M, k\_S, k\_J \in (0,1)$

: $C_L = F \cdot \frac{k_M \cdot Cl_M \cdot A_M + k_S \cdot Cl_S \cdot A_S + k_J \cdot Cl_J \cdot A_J}{AN}$

: $Cdp = \frac{k\_M \cdot Cdp\_M \cdot A\_M + k\_S \cdot Cdp\_S \cdot A\_S + k\_J \cdot Cdp\_J \cdot A\_J}{AN}$

: $ZCE = \frac{k\_M \cdot ZCEM \cdot AM + k\_S \cdot ZCES \cdot AS + k\_J \cdot ZCEJ \cdot AJ}{k\_M \cdot AM + k\_S \cdot AS + k\_J \cdot AJ}$

: $AR = \begin{cases} 1.1 \cdot \frac{(EHM + AVGFREB)^2}{AN} & \text{if } \alpha_{eff} < 45^o \\ 1.1 \cdot \frac{EHM^2}{AN} & \text{if } \alpha_{eff} \geq 45^o \end{cases}$

: $CdI = C_L^2 \cdot \left(\frac{1}{\pi \cdot AR} + 0.005\right)$

: $\mathrm{Cd0} = 1.13 \cdot \frac{(\mathrm{B} \cdot \mathrm{AVGFREB} + \mathrm{EHM} \cdot \mathrm{EMDC})}{\mathrm{AN}}$

: $\mathrm{Cd} = \mathrm{Cdp} + \mathrm{Cd0} + \mathrm{CdI}$

: $\mathrm{L} = \frac{1}{2} \cdot \rho_a \cdot V_{\mathrm{eff}}^2 \cdot \mathrm{AN} \cdot C_L$

: $\mathrm{D} = \frac{1}{2} \cdot \rho_a \cdot V_{\mathrm{eff}}^2 \cdot \mathrm{AN} \cdot C_D$

: $D_2 = \mathrm{Fdrive} = \mathrm{L} \cdot \sin(\alpha_{\mathrm{eff}}) - \mathrm{D} \cdot \cos(\alpha_{\mathrm{eff}})$

α_{eff}	Cl_M	Cl_J	Cl_S	Cdp_M	Cdp_J	Cdp_S
0	0	0	0	0	0	0
27	1.725*	1.5	0	0.02	0.02	0
50	1.5	0.5	1.5	0.15	0.25	0.25
80	0.95	0.3	1	0.8	0.15	0.9
100	0.85	0	0.85	1.0	0	1.2
180	0	0	0	0.9	0	0.66

<table>
<tr><th>α_{eff}</th><th>Cl_M</th><th>Cl_J</th><th>Cl_S</th></tr>
<tr><td>0</td><td rowspan="2">$0.0638888\alpha_{\mathrm{eff}}$</td><td rowspan="2">$0.0555555\alpha_{\mathrm{eff}}$</td><td rowspan="2">0</td></tr>
<tr><td rowspan="2">27</td></tr>
<tr><td rowspan="2">$-9.782 \cdot 10^{-3}\alpha_{\mathrm{eff}} + 1.9891304$</td><td rowspan="2">$-0.043478 \cdot \alpha_{\mathrm{eff}} + 2.673913$</td><td rowspan="2">$0.0652173 \cdot \alpha_{\mathrm{eff}} - 1.760869$</td></tr>
<tr><td rowspan="2">50</td></tr>
<tr><td rowspan="2">$-0.018333 \cdot \alpha_{\mathrm{eff}} + 2.4166666$</td><td rowspan="2">$-6.666 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 0.8333333$</td><td rowspan="2">$-0.016666 \cdot \alpha_{\mathrm{eff}} + 2.3333333$</td></tr>
<tr><td rowspan="2">80</td></tr>
<tr><td rowspan="2">$-5 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 1.35$</td><td rowspan="2">$-0.015 \cdot \alpha_{\mathrm{eff}} + 1.5$</td><td rowspan="2">$-7.5 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 1.6$</td></tr>
<tr><td rowspan="2">100</td></tr>
<tr><td rowspan="2">$-0.010625 \cdot \alpha_{\mathrm{eff}} + 1.9125$</td><td rowspan="2">0</td><td rowspan="2">$-0.010625 \cdot \alpha_{\mathrm{eff}} + 1.9125$</td></tr>
<tr><td>180</td></tr>
</table>

<table>
<tr><th>α_{eff}</th><th>Cdp_M</th><th>Cdp_J</th><th>Cdp_S</th></tr>
<tr><td>0</td><td rowspan="2">$7.4074 \cdot 10^{-4} \cdot \alpha_{\mathrm{eff}}$</td><td rowspan="2">$7.4074 \cdot 10^{-4} \cdot \alpha_{\mathrm{eff}}$</td><td rowspan="2">0</td></tr>
<tr><td rowspan="2">27</td></tr>
<tr><td rowspan="2">$5.6521 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} - 0.132608$</td><td rowspan="2">$0.01 \cdot \alpha_{\mathrm{eff}} - 0.25$</td><td rowspan="2">$0.0108695 \cdot \alpha_{\mathrm{eff}} - 0.293478$</td></tr>
<tr><td rowspan="2">50</td></tr>
<tr><td rowspan="2">$0.0216666 \cdot \alpha_{\mathrm{eff}} - 0.933333$</td><td rowspan="2">$-3.333 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 0.4166666$</td><td rowspan="2">$0.0216666 \cdot \alpha_{\mathrm{eff}} - 0.833333$</td></tr>
<tr><td rowspan="2">80</td></tr>
<tr><td rowspan="2">$0.01 \cdot \alpha_{\mathrm{eff}}$</td><td rowspan="2">$-7.5 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 0.75$</td><td rowspan="2">$0.015 \cdot \alpha_{\mathrm{eff}} - 0.3$</td></tr>
<tr><td rowspan="2">100</td></tr>
<tr><td rowspan="2">$-1.25 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 1.125$</td><td rowspan="2">0</td><td rowspan="2">$-6.75 \cdot 10^{-3} \cdot \alpha_{\mathrm{eff}} + 1.875$</td></tr>
<tr><td>180</td></tr>
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$$M_{heel}(V^*, \phi^*)$$

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: $\mathrm{Fheel} = \mathrm{L} \cdot \cos(\alpha_{\mathrm{eff}}) + \mathrm{D} \cdot \sin(\alpha_{\mathrm{eff}})$

: $\mathrm{Mheel} = \mathrm{Fheel} \cdot (\mathrm{ZCE} + \mathrm{T} - \mathrm{ZCBK})$

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$$\mathrm{M_{right}}(\mathrm{V}^*, \phi^*)$$

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: $\mathrm{M1} = (\mathrm{KM} - \mathrm{KG}) \cdot \sin(\phi^*) \cdot \mathrm{g} \cdot \rho_{\mathrm{w}} \cdot (\mathrm{DIVCAN} + \mathrm{DVK})$

: $\mathrm{M2} = \mathrm{MMVBLCRW} \cdot \mathrm{g} \cdot \mathrm{b} \cdot \cos(\phi^*)$

: $\mathrm{Mright} = \mathrm{M_1} + \mathrm{M_2}$

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$$\mathrm{R_t}(\mathrm{V}^*, \phi^*)$$

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: $\mathrm{Rt} = \mathrm{Ri} + \mathrm{Rrh} + \mathrm{RrhH} + \mathrm{Rrk} + \mathrm{RrkH} + \mathrm{Rvh} + \mathrm{RvhH} + \mathrm{Rvk} + \mathrm{Rvr}$

$$\mathrm{R_i}$$

: $\mathrm{Ri} = \dfrac{\mathrm{Fheel}^2}{\pi \cdot \mathrm{Te}^2 \cdot \dfrac{1}{2} \cdot \rho_{\mathrm{w}} \cdot \mathrm{V}^{*2}}$

: $\mathrm{T_e} = \mathrm{T} \cdot \left(\mathrm{A_1} \cdot \dfrac{\mathrm{TCAN}}{\mathrm{T}} + \mathrm{A_2} \cdot \left(\dfrac{\mathrm{TCAN}}{\mathrm{T}}\right)^2 + \mathrm{A_3} \cdot \dfrac{\mathrm{BWL}}{\mathrm{TCAN}} + \mathrm{A_4} \cdot \dfrac{\mathrm{CHTPK}}{\mathrm{CHRTK}}\right) \cdot (\mathrm{B_0} + \mathrm{B_1} \cdot \mathrm{Fn})$

: $\mathrm{Fn} = \dfrac{\mathrm{V}^*}{\sqrt{\mathrm{g} \cdot \mathrm{Lwl}}}$

ϕ^*	A_1	A_2	A_3	A_4	B_0	B_1
0	3.7455	-3.6246	0.0589	-0.0296	1.2306	-0.7256
10	4.4892	-4.8454	0.0294	-0.0176	1.4231	-1.2971
20	3.9592	-3.9804	0.0283	-0.0075	1.5450	-1.5622
30	3.4891	-2.9577	0.0250	-0.0272	1.4744	-1.3499

ϕ^*	A_1	A_2	A_3	A_4
0 10	$0.07437 \cdot \phi^* + 3.7455$	$-0.12208 \cdot \phi^* - 3.6246$	$-2.95 \cdot 10^{-3} \cdot \phi^* + 0.0589$	$1.2 \cdot 10^{-3} \cdot \phi^* - 0.0296$
10 20	$-0.053 \cdot \phi^* + 5.0192$	$0.0865 \cdot \phi^* - 5.7104$	$-1.1 \cdot 10^{-4} \cdot \phi^* + 0.0305$	$1.01 \cdot 10^{-3} \cdot \phi^* - 0.0277$
20 30	$-0.04701 \cdot \phi^* + 4.8994$	$0.10227 \cdot \phi^* - 6.0258$	$-3.3 \cdot 10^{-4} \cdot \phi^* + 0.0349$	$-1.97 \cdot 10^{-3} \cdot \phi^* + 0.0319$

ϕ^*	B_0	B_1
0 10	$0.01925 \cdot \phi^* + 1.2306$	$-0.05715 \cdot \phi^* - 0.7256$
10 20	$0.01219 \cdot \phi^* + 1.3012$	$-0.02651 \cdot \phi^* - 1.032$
20 30	$-7.06 \cdot 10^{-3} \cdot \phi^* + 1.6862$	$0.02123 \cdot \phi^* - 1.9868$

Rrh

Fn	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
0	0	0	0	0	0	0	0	0	0
0.10	-0.0014	0.0403	0.0470	-0.0227	-0.0119	0.0061	-0.0086	-0.0307	-0.0553
0.15	0.0004	-0.1808	0.1793	-0.0004	0.0097	0.0118	-0.0055	0.1721	-0.1728
0.20	0.0014	-0.1071	0.0637	0.0090	0.0153	0.0011	0.0012	0.1021	-0.0648
0.25	0.0027	0.0463	-0.1263	0.0150	0.0274	-0.0299	0.0110	-0.0595	0.1220
0.30	0.0056	-0.8005	0.4891	0.0269	0.0519	-0.0313	0.0292	0.7314	-0.3619
0.35	0.0032	-0.1011	-0.0813	-0.0382	0.0320	-0.1481	0.0837	0.0223	0.1587
0.40	-0.0064	2.3095	-1.5152	0.0751	-0.0858	-0.5349	0.1715	-2.4550	1.1865
0.45	-0.0171	3.4017	-1.9862	0.3242	-0.1450	-0.8043	0.2952	-3.5284	1.3575
0.50	-0.0201	7.1576	-6.3304	0.5829	0.1630	-0.3966	0.5023	-7.1579	5.2534
0.55	0.0495	1.5618	-6.0661	0.8641	1.1702	1.7610	0.9176	-2.1191	5.4281
0.60	0.0808	-5.3233	-1.1513	0.9663	1.6084	2.7459	0.8491	4.7129	1.1089

: $$\mathrm{Rrh} = \nabla_c g \rho_w \left[a_0 + \frac{\nabla^{\frac{1}{3}}}{\mathrm{Lwl}} \left(a_1 \frac{\mathrm{LCB}}{\mathrm{Lwl}} + a_2 \mathrm{Cp} + a_3 \frac{\nabla_c^{2/3}}{A_w} + a_4 \frac{\mathrm{Bwl}}{\mathrm{Lwl}} + a_5 \frac{\nabla_c^{2/3}}{S_c} + a_6 \frac{\mathrm{LCB}}{\mathrm{LCF}} + a_7 \left(\frac{\mathrm{LCB}}{\mathrm{Lwl}} \right)^2 + a_8 \mathrm{Cp}^2 \right) \right]$$

Fn	a_0	a_1	a_2
0 – 0.10	$-0.014 \cdot Fn$	$0.403 \cdot Fn$	$0.47 \cdot Fn$
0.10 – 0.15	$0.036 \cdot Fn - 0.005$	$-4.422 \cdot Fn + 0.4825$	$2.646 \cdot Fn - 0.2176$
0.15 – 0.20	$0.02 \cdot Fn - 0.0026$	$1.474 \cdot Fn - 0.4019$	$-2.312 \cdot Fn + 0.5261$
0.20 – 0.25	$0.026 \cdot Fn - 0.0038$	$3.068 \cdot Fn - 0.7207$	$-3.8 \cdot Fn + 0.8237$
0.25 – 0.30	$0.058 \cdot Fn - 0.0118$	$-16.936 \cdot Fn + 4.2803$	$12.308 \cdot Fn - 3.2033$
0.30 – 0.35	$-0.048 \cdot Fn + 0.02$	$13.988 \cdot Fn - 4.9969$	$-11.408 \cdot Fn + 3.9115$
0.35 – 0.40	$-0.192 \cdot Fn + 0.0704$	$48.212 \cdot Fn - 16.9753$	$-28.678 \cdot Fn + 9.956$
0.40 – 0.45	$-0.214 \cdot Fn + 0.0792$	$21.844 \cdot Fn - 6.4281$	$-9.42 \cdot Fn + 2.2528$
0.45 – 0.50	$-0.06 \cdot Fn + 0.0099$	$75.118 \cdot Fn - 30.4014$	$-86.884 \cdot Fn + 37.1116$
0.50 – 0.55	$1.392 \cdot Fn - 0.7161$	$-111.916 \cdot Fn + 63.1156$	$5.286 \cdot Fn - 8.9734$
0.55 – 0.60	$0.626 \cdot Fn - 0.2948$	$-137.702 \cdot Fn + 77.2979$	$98.296 \cdot Fn - 60.1289$

Fn	a_3	a_4	a_5
0 – 0.10	$-0.227 \cdot Fn$	$-0.119 \cdot Fn$	$0.061 \cdot Fn$
0.10 – 0.15	$0.446 \cdot Fn - 0.0673$	$0.432 \cdot Fn - 0.0551$	$0.114 \cdot Fn - 0.0053$
0.15 – 0.20	$0.188 \cdot Fn - 0.0286$	$0.112 \cdot Fn - 0.0071$	$-0.214 \cdot Fn + 0.0439$
0.20 – 0.25	$0.12 \cdot Fn - 0.015$	$0.242 \cdot Fn - 0.0331$	$-0.62 \cdot Fn + 0.1251$
0.25 – 0.30	$0.238 \cdot Fn - 0.0445$	$0.49 \cdot Fn - 0.0951$	$-0.028 \cdot Fn - 0.0229$
0.30 – 0.35	$-1.302 \cdot Fn + 0.4175$	$-0.398 \cdot Fn + 0.1713$	$-2.336 \cdot Fn + 0.6695$
0.35 – 0.40	$2.266 \cdot Fn - 0.8313$	$-2.356 \cdot Fn + 0.8566$	$-7.736 \cdot Fn + 2.5595$
0.40 – 0.45	$4.982 \cdot Fn - 1.9177$	$-1.184 \cdot Fn + 0.3878$	$-5.388 \cdot Fn + 1.6203$
0.45 – 0.50	$5.174 \cdot Fn - 2.0041$	$6.16 \cdot Fn - 2.917$	$8.154 \cdot Fn - 4.4736$
0.50 – 0.55	$5.624 \cdot Fn - 2.2291$	$20.144 \cdot Fn - 9.909$	$43.152 \cdot Fn - 21.9726$
0.55 – 0.60	$2.044 \cdot Fn - 0.2601$	$8.764 \cdot Fn - 3.65$	$19.698 \cdot Fn - 9.0729$

Fn	a_6	a_7	a_8
0			
	$-0.086 \cdot Fn$	$-0.307 \cdot Fn$	$-0.553 \cdot Fn$
0.10			
	$0.062 \cdot Fn - 0.0148$	$4.056 \cdot Fn - 0.4363$	$-2.35 \cdot Fn + 0.1797$
0.15			
	$0.134 \cdot Fn - 0.0256$	$-1.4 \cdot Fn + 0.3821$	$2.16 \cdot Fn - 0.4968$
0.20			
	$0.196 \cdot Fn - 0.038$	$-3.232 \cdot Fn + 0.7485$	$3.736 \cdot Fn - 0.812$
0.25			
	$0.364 \cdot Fn - 0.08$	$15.818 \cdot Fn - 4.014$	$-9.678 \cdot Fn + 2.5415$
0.30			
	$1.09 \cdot Fn - 0.2978$	$-14.182 \cdot Fn + 4.986$	$10.412 \cdot Fn - 3.4855$
0.35			
	$1.756 \cdot Fn - 0.5309$	$-49.546 + 17.3634$	$20.556 \cdot Fn - 7.0359$
0.40			
	$2.474 \cdot Fn - 0.8181$	$-21.468 \cdot Fn + 6.1322$	$3.42 \cdot Fn - 0.1815$
0.45			
	$4.142 \cdot Fn - 1.5687$	$-72.59 + 29.1371$	$77.918 \cdot Fn - 33.7056$
0.50			
	$8.306 \cdot Fn - 3.6507$	$100.776 \cdot Fn - 57.5459$	$3.494 \cdot Fn + 3.5064$
0.55			
	$-1.37 \cdot Fn + 1.6711$	$136.64 \cdot Fn - 77.2711$	$-86.384 \cdot Fn + 52.9393$
0.60			

RrhH

: $$RrhH = \nabla_c g \rho_w \cdot \left[u_0 + u_1 \frac{Lwl}{Bwl} + u_2 \frac{Bwl}{T_c} + u_3 \left(\frac{Bwl}{T_c}\right)^2 + u_4 \frac{XFB}{Lwl} + b_5 \left(\frac{XFB}{Lwl}\right)^2\right] \cdot 6.0 \cdot \phi^{*1.7}$$

Fn	u_0	u_1	u_2	u_3	u_4	u_5
0	0	0	0	0	0	0
0.20	0	0	0	0	0	0
0.25	$0.0268 \cdot 10^{-3}$	$-0.0014 \cdot 10^{-3}$	$-0.0057 \cdot 10^{-3}$	$0.0016 \cdot 10^{-3}$	$-0.0070 \cdot 10^{-3}$	$-0.0017 \cdot 10^{-3}$
0.30	$0.6628 \cdot 10^{-3}$	$-0.0632 \cdot 10^{-3}$	$-0.0699 \cdot 10^{-3}$	$0.0069 \cdot 10^{-3}$	$0.0459 \cdot 10^{-3}$	$-0.0004 \cdot 10^{-3}$
0.35	$1.6433 \cdot 10^{-3}$	$-0.2144 \cdot 10^{-3}$	$-0.1640 \cdot 10^{-3}$	$0.0199 \cdot 10^{-3}$	$-0.0540 \cdot 10^{-3}$	$-0.0268 \cdot 10^{-3}$
0.40	$-0.8659 \cdot 10^{-3}$	$-0.0354 \cdot 10^{-3}$	$0.2226 \cdot 10^{-3}$	$0.0188 \cdot 10^{-3}$	$-0.5800 \cdot 10^{-3}$	$-0.1133 \cdot 10^{-3}$
0.45	$-3.2715 \cdot 10^{-3}$	$0.1372 \cdot 10^{-3}$	$0.5547 \cdot 10^{-3}$	$0.0268 \cdot 10^{-3}$	$-1.0064 \cdot 10^{-3}$	$-0.2026 \cdot 10^{-3}$
0.50	$-0.1976 \cdot 10^{-3}$	$-0.1480 \cdot 10^{-3}$	$-0.6593 \cdot 10^{-3}$	$0.1862 \cdot 10^{-3}$	$-0.7489 \cdot 10^{-3}$	$-0.1648 \cdot 10^{-3}$
0.55	$1.5873 \cdot 10^{-3}$	$-0.3749 \cdot 10^{-3}$	$-0.7105 \cdot 10^{-3}$	$0.2146 \cdot 10^{-3}$	$-0.4818 \cdot 10^{-3}$	$-0.1174 \cdot 10^{-3}$

Fn	u_0	u_1	u_2
0 – 0.20	0	0	0
0.20 – 0.25	$0.000536 \cdot \mathrm{Fn} - 0.0001072$	$-0.000028 \cdot \mathrm{Fn} + 0.0000056$	$-0.000114 \cdot \mathrm{Fn} + 0.0000228$
0.25 – 0.30	$0.0007896 \cdot \mathrm{Fn} - 0.0001706$	$-0.001236 \cdot \mathrm{Fn} + 0.0003076$	$-0.001284 \cdot \mathrm{Fn} + 0.0003153$
0.30 – 0.35	$0.0315404 \cdot \mathrm{Fn} - 0.00939584$	$-0.003024 \cdot \mathrm{Fn} + 0.000844$	$-0.001882 \cdot \mathrm{Fn} + 0.0004947$
0.35 – 0.40	$-0.050184 \cdot \mathrm{Fn} + 0.0192077$	$0.00358 \cdot \mathrm{Fn} - 0.0014674$	$0.007732 \cdot \mathrm{Fn} - 0.0028702$
0.40 – 0.45	$-0.048112 \cdot \mathrm{Fn} + 0.0183789$	$0.003452 \cdot \mathrm{Fn} - 0.0014162$	$0.006642 \cdot \mathrm{Fn} - 0.0024342$
0.45 – 0.50	$0.061478 \cdot \mathrm{Fn} - 0.0309366$	$-0.005704 \cdot \mathrm{Fn} + 0.002704$	$-0.02428 \cdot \mathrm{Fn} + 0.0114807$
0.50 – 0.55	$0.035698 \cdot \mathrm{Fn} - 0.0180466$	$-0.004538 \cdot \mathrm{Fn} + 0.002121$	$-0.001024 \cdot \mathrm{Fn} - 0.0001473$

Fn	u_3	u_4	u_5
0 – 0.20	0	0	0
0.20 – 0.25	$0.000032 - 0.0000064$	$-0.00014 \cdot \mathrm{Fn} + 0.000028$	$-0.000034 \cdot \mathrm{Fn} + 0.0000068$
0.25 – 0.30	$0.000106 \cdot \mathrm{Fn} - 0.0000249$	$0.001058 \cdot \mathrm{Fn} - 0.0002715$	$0.000026 \cdot \mathrm{Fn} - 0.0000082$
0.30 – 0.35	$0.00026 \cdot \mathrm{Fn} - 0.0000711$	$-0.001998 \cdot \mathrm{Fn} + 0.0006453$	$-0.000528 \cdot \mathrm{Fn} + 0.000158$
0.35 – 0.40	$-0.000022 \cdot \mathrm{Fn} + 0.0000276$	$-0.01052 \cdot \mathrm{Fn} + 0.003628$	$-0.00173 \cdot \mathrm{Fn} + 0.0005787$
0.40 – 0.45	$0.00016 \cdot \mathrm{Fn} - 0.0000452$	$-0.008528 \cdot \mathrm{Fn} + 0.0028312$	$0.006318 \cdot \mathrm{Fn} - 0.0026405$
0.45 – 0.50	$0.003188 \cdot \mathrm{Fn} - 0.0014078$	$0.00515 \cdot \mathrm{Fn} - 0.0033239$	$-0.007348 \cdot \mathrm{Fn} + 0.0035092$
0.50 – 0.55	$0.000568 \cdot \mathrm{Fn} - 0.0000978$	$0.005342 \cdot \mathrm{Fn} - 0.0034199$	$0.000948 \cdot \mathrm{Fn} - 0.0006388$

Rrk

: $\mathrm{Rrk} = \nabla_k \cdot \rho_w \cdot g \cdot \left[A_0 + A_1 \cdot \frac{T}{\mathrm{Bwl}} + A_2 \cdot \frac{(T_C + \mathrm{Zcbk})^3}{\nabla_k} + A_3 \cdot \frac{\nabla_c}{\nabla_k}\right]$

Fn	A_0	A_1	A_2	A_3
0	0	0	0	0
0.15	0	0	0	0
0.20	-0.00104	0.00172	0.00117	-0.00008
0.25	-0.00550	0.00597	0.00390	-0.00009
0.30	-0.01110	0.01421	0.00069	0.00021
0.35	-0.00713	0.02632	-0.00232	0.00039
0.40	-0.03581	0.08649	0.00999	0.00017
0.45	-0.00470	0.11592	-0.00064	0.00035
0.50	0.00553	0.07371	0.05991	-0.00114
0.55	0.04822	0.00660	0.07048	-0.00035
0.60	0.01021	0.14173	0.06409	-0.00192

Fn	A_0	A_1	A_2	A_3
0 0.15	0	0	0	0
0.15 0.20	$-0.0208 \cdot Fn + 0.00312$	$0.0344 \cdot Fn - 0.00516$	$0.0234Fn - 0.00351$	$-0.0016Fn + + 0.00024$
0.20 0.25	$-0.0892 \cdot Fn + 0.0168$	$0.085 \cdot Fn - 0.01528$	$0.0546 \cdot Fn - 0.00975$	$-0.0002 \cdot Fn - - 0.00004$
0.25 0.30	$-0.112 \cdot Fn + 0.0225$	$0.1648 \cdot Fn - 0.03523$	$-0.0642 \cdot Fn + + 0.01995$	$0.006 \cdot Fn - 0.00159$
0.30 0.35	$0.0794 \cdot Fn - 0.03492$	$0.2422 \cdot Fn - 0.05845$	$-0.0602 \cdot Fn + + 0.01875$	$0.0036 \cdot Fn - 0.00087$
0.35 0.40	$-0.5736 \cdot Fn + 0.19363$	$1.2034 \cdot Fn - 0.39487$	$0.2462 \cdot Fn - 0.08849$	$-0.0044 \cdot Fn + + 0.00193$
0.40 0.45	$0.6222 \cdot Fn - 0.28469$	$0.5886 \cdot Fn - 0.14895$	$-0.2126 \cdot Fn + + 0.09503$	$0.0036 \cdot Fn - 0.00127$
0.45 0.50	$0.2046 \cdot Fn - 0.09677$	$-0.8442 \cdot Fn + + 0.49581$	$1.211 \cdot Fn - 0.54559$	$-0.0298 \cdot Fn + + 0.01376$
0.50 0.55	$0.8538 \cdot Fn - 0.42137$	$-1.3422 \cdot Fn + + 0.74481$	$0.2114 \cdot Fn - 0.04579$	$0.0158 \cdot Fn - 0.00904$
0.55 0.60	$-0.7602 \cdot Fn + 0.46633$	$2.7026 \cdot Fn - 1.47983$	$-0.1278 \cdot Fn + + 0.14077$	$-0.0314 \cdot Fn + + 0.01692$

RrkH

: $$RrkH = \nabla_k \cdot \rho_w \cdot g \cdot \left[H_1 \cdot \frac{T_c}{T} + H_2 \cdot \frac{Bwl}{T_c} + H_3 \cdot \frac{T_c}{T} \cdot \frac{Bwl}{T_c} + H_4 \cdot \frac{Lwl}{\nabla_c^{1/3}} \right] \cdot Fn^2 \cdot \phi^*$$

H_1	H_2	H_3	H_4
-3.5837	-0.0518	0.5958	0.2055

Rvh

: $Rn = \frac{V \cdot 0.7 \cdot Lwl}{\nu}$

: $C_f = \frac{0.075}{(\log(Rn) - 2)^2}$

: $Sc = \left(1.97 + 0.171 \cdot \frac{Bwl}{Tc}\right) \cdot \left(\frac{0.65}{Cm}\right)^{1/3} \cdot (\nabla_c \cdot Lwl)^{1/2}$

: $Rfh = \frac{1}{2} \cdot \rho_w \cdot V^2 \cdot Sc \cdot C_f$

: $Rvh = (1 + k) \cdot Rfh$

RvhH

: $Rn = \frac{V \cdot 0.7 \cdot Lwl}{\nu}$

: $C_f = \frac{0.075}{(\log(Rn) - 2)^2}$

: $Sc_\phi = S_c \cdot \left[1 + \frac{1}{100} \cdot \left(s_0 + s_1 \cdot \frac{Bwl}{T_c} + s_2 \cdot \left(\frac{Bwl}{Tc}\right)^2 + s_3 \cdot Cms\right)\right]$

: $RfhH = \frac{1}{2} \cdot \rho_w \cdot V^2 \cdot C_f \cdot Sc_\phi$

: $RvhH = (1 + k) \cdot Rfh$

ϕ	s_0	s_1	s_2	s_3
0	0	0	0	0
5	-4.112	0.054	-0.027	6.329
10	-4.522	-0.132	-0.077	8.738
15	-3.291	-0.389	-0.118	8.949
20	1.850	-1.200	-0.109	5.364
25	6.510	-2.305	-0.066	3.443
30	12.334	-3.911	0.024	1.767
35	14.648	-5.182	0.102	3.497

ϕ	s_0	s_1	s_2	s_3
0–5	$-0.8224 \cdot \phi$	$0.0108 \cdot \phi$	$-0.0054 \cdot \phi$	$1.2658 \cdot \phi$
5–10	$-0.082 \cdot \phi - 3.702$	$-0.0372 \cdot \phi + 0.24$	$-0.01 \cdot \phi + 0.023$	$0.4818 \cdot \phi + 3.92$
10–15	$0.2462 \cdot \phi - 6.984$	$-0.0514 \cdot \phi - 0.382$	$-0.0082 \cdot \phi + 0.005$	$0.0422 \cdot \phi + 8.316$
15–20	$1.0282 \cdot \phi - 18.714$	$-0.1622 \cdot \phi + 2.044$	$0.0018 \cdot \phi - 0.145$	$-0.717 \cdot \phi + 19.704$
20–25	$0.932 \cdot \phi - 16.79$	$-0.221 \cdot \phi + 3.22$	$0.0086 \cdot \phi - 0.281$	$-0.3842 \cdot \phi + 13.048$
25–30	$1.1648 \cdot \phi - 22.61$	$-0.3212 \cdot \phi + 5.725$	$0.018 \cdot \phi - 0.516$	$-0.3352 \cdot \phi + 11.823$
30–35	$0.4628 \cdot \phi - 1.55$	$-0.2542 \cdot \phi + 3.715$	$0.0156 \cdot \phi - 0.444$	$0.346 \cdot \phi - 8.613$

Rvk

: $\mathrm{Rn} = \dfrac{\mathrm{CHMEK} \cdot V}{\nu}$

: $\mathrm{Cf} = \dfrac{0.075}{(\log(\mathrm{Rn}) - 2)^2}$

: $\mathrm{Rfk} = \dfrac{1}{2} \cdot \rho_w \cdot V^2 \cdot S_k \cdot \mathrm{Cf}$

: $(1 + k_k) = \left(1 + 2 \cdot \left(\dfrac{t}{c}\right)_k + 60 \cdot \left(\dfrac{t}{c}\right)_k^4\right)$

: $\mathrm{Rvk} = \mathrm{Rfk} \cdot (1 + k_k)$

Rvr

: $\mathrm{Rn} = \dfrac{\mathrm{CHMER} \cdot \mathrm{V}}{\nu}$

: $\mathrm{Cf} = \dfrac{0.075}{(\log(\mathrm{Rn}) - 2)^2}$

: $\mathrm{Rfr} = \dfrac{1}{2} \cdot \rho_w \cdot \mathrm{V}^2 \cdot \mathrm{S}_r \cdot \mathrm{Cf}$

: $(1 + k_r) = \left(1 + 2 \cdot \left(\dfrac{t}{c}\right)_r + 60 \cdot \left(\dfrac{t}{c}\right)_r^4\right)$

: $\mathrm{Rvr} = \mathrm{Rfr} \cdot (1 + k_r)$

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$$C_2 = \left.\frac{\partial F_{\mathrm{drive}}}{\partial \phi}\right|_{V^*,\phi^*}$$

****

: $C_2 = \dfrac{\partial F_{\mathrm{drive}}}{\partial \phi} = \dfrac{\partial}{\partial \phi}(L \cdot \sin(\alpha_{\mathrm{eff}}) - D \cdot \cos(\alpha_{\mathrm{eff}})) = \dfrac{\partial}{\partial \phi}(L \cdot \sin(\alpha_{\mathrm{eff}})) - \dfrac{\partial}{\partial \phi}(D \cdot \cos(\alpha_{\mathrm{eff}}))$

: $\dfrac{\partial}{\partial \phi}(L \cdot \sin(\alpha_{\mathrm{eff}})) = (\sin \alpha_{\mathrm{eff}})\dfrac{\partial L}{\partial \phi} + L \cdot \dfrac{\partial}{\partial \phi}(\sin(\alpha_{\mathrm{eff}}))$

: $\dfrac{\partial}{\partial \phi}(D \cdot \cos(\alpha_{\mathrm{eff}})) = (\cos \alpha_{\mathrm{eff}})\dfrac{\partial D}{\partial \phi} + D \cdot \dfrac{\partial}{\partial \phi}(\cos(\alpha_{\mathrm{eff}}))$

: $\dfrac{\partial L}{\partial \phi} = \dfrac{\partial}{\partial \phi}\left(\dfrac{1}{2} \cdot \rho_a \cdot V_{\mathrm{eff}}^2 \cdot AN \cdot C_L\right) = \dfrac{1}{2} \cdot \rho_a \cdot AN \cdot \dfrac{\partial}{\partial \phi}\left(V_{\mathrm{eff}}^2 \cdot C_L\right)$

: $\dfrac{\partial}{\partial \phi}\left(V_{\mathrm{eff}}^2 \cdot C_L\right) = C_L \cdot \dfrac{\partial}{\partial \phi}\left(V_{\mathrm{eff}}^2\right) + V_{\mathrm{eff}}^2 \cdot \dfrac{\partial C_L}{\partial \phi} = 2 \cdot V_{\mathrm{eff}} \cdot C_L \cdot \dfrac{\partial V_{\mathrm{eff}}}{\partial \phi} + V_{\mathrm{eff}}^2 \cdot \dfrac{\partial C_L}{\partial \phi}$

: $\dfrac{\partial V_{\mathrm{eff}}}{\partial \phi} = \dfrac{\partial}{\partial \phi}\left(\sqrt{V_1^2 + V_2^2}\right) = \dfrac{1}{2}(V_1^2 + V_2^2)^{-1/2} \cdot \dfrac{\partial}{\partial \phi}(V_1^2 + V_2^2) = \dfrac{1}{2}(V_1^2 + V_2^2) \cdot \dfrac{\partial}{\partial \phi}(V_2^2) = V_2 \cdot (V_1^2 + V_2^2) \cdot \dfrac{\partial V_2}{\partial \phi}$

: $\frac{\partial V_2}{\partial \phi} = \frac{\partial}{\partial \phi}(V_{tw} \cdot \sin \alpha_{tw} \cdot \cos \phi^*) = V_{tw} \cdot \sin \alpha_{tw} \cdot \frac{\partial}{\partial \phi}(\cos \phi^*) = -V_{tw} \cdot \sin \alpha_{tw} \cdot \sin \phi^*$

: $\frac{\partial C_L}{\partial \phi} = \frac{\partial}{\partial \phi}\left(k_M \cdot Cl_M \cdot AM + k_S \cdot Cl_S \cdot AS + k_J \cdot Cl_J \cdot AJ\right) = k_M \cdot AM \cdot \frac{\partial Cl_M}{\partial \phi} + k_S \cdot AS \cdot \frac{\partial Cl_S}{\partial \phi} + k_J \cdot AJ \cdot \frac{\partial Cl_J}{\partial \phi}$

: $AM = \frac{1}{2} \cdot P \cdot E \cdot MROACH$

: $AJ = \frac{1}{2} \cdot LPG \cdot \sqrt{I^2 + J^2}$

: $AS = 1.15 \cdot SL \cdot J$

: $AF = \frac{1}{2} \cdot I \cdot J$

: $AN = AF + AM$

: k_M, k_S, k_J ∈ (0,1)

: $\frac{\partial Cl_M}{\partial \phi} = \frac{\partial}{\partial \phi}(m_o \cdot \alpha_{eff} + m_1) = m_o \frac{\partial \alpha_{eff}}{\partial \phi}$

: $\frac{\partial Cl_J}{\partial \phi} = \frac{\partial}{\partial \phi}(j_o \cdot \alpha_{eff} + j_1) = j_o \frac{\partial \alpha_{eff}}{\partial \phi}$

: $\frac{\partial Cl_S}{\partial \phi} = \frac{\partial}{\partial \phi}(s_o \cdot \alpha_{eff} + s_1) = s_o \frac{\partial \alpha_{eff}}{\partial \phi}$

: $\frac{\partial \alpha_{eff}}{\partial \phi} = \frac{\partial}{\partial \phi}\left(\tan^{-1}\left(\frac{V_2}{V_1}\right)\right) = \frac{1}{1 + \left(\frac{V_2}{V_1}\right)^2} \cdot \frac{\partial}{\partial \phi}\left(\frac{V_2}{V_1}\right) = \frac{V_1^2}{V_1^2 + V_2^2} \cdot \frac{1}{V_1} \cdot \frac{\partial V_2}{\partial \phi} = \frac{V_1}{V_{eff}^2} \cdot \frac{\partial V_2}{\partial \phi}$

: $\frac{\partial V_2}{\partial \phi} = V_{tw} \cdot \sin \alpha_{tw} \cdot \frac{\partial \cos \phi^*}{\partial \phi} = -V_{tw} \cdot \sin \alpha_{tw} \cdot \sin \phi^*$

: $\frac{\partial}{\partial \phi}(\sin(\alpha_{eff})) = \cos(\alpha_{eff}) \cdot \frac{\partial \alpha_{eff}}{\partial \phi}$

: $\frac{\partial}{\partial \phi}(\cos(\alpha_{eff})) = -\sin(\alpha_{eff}) \cdot \frac{\partial \alpha_{eff}}{\partial \phi}$

: $\frac{\partial D}{\partial \phi} = \frac{\partial}{\partial \phi}\left(\frac{1}{2} \cdot \rho_a \cdot V_{eff}^2 \cdot AN \cdot C_D\right) = \frac{1}{2} \cdot \rho_a \cdot AN \cdot \frac{\partial}{\partial \phi}\left(V_{eff}^2 \cdot C_D\right) = \frac{1}{2} \cdot \rho_a \cdot AN \cdot \left(V_{eff}^2 \cdot \frac{\partial C_D}{\partial \phi} + 2 \cdot V_{eff} \cdot \frac{\partial V_{eff}}{\partial \phi}\right)$

: $\frac{\partial C_D}{\partial \phi} = \frac{\partial}{\partial \phi}(Cdp + Cd0 + CdI) = \frac{\partial Cdp}{\partial \phi} + \frac{\partial Cd0}{\partial \phi} + \frac{\partial CdI}{\partial \phi}$

: $\frac{\partial CdI}{\partial \phi} = \left(\frac{1}{\pi \cdot AR} + 0.005\right)\frac{\partial}{\partial \phi} C_L^2 = 2 \cdot C_L \cdot \left(\frac{1}{\pi \cdot AR} + 0.005\right)\frac{\partial C_L}{\partial \phi}$

$$:\ \frac{\partial Cd0}{\partial \phi} = \left(1.13 \cdot \frac{(B \cdot AVGFREB + EHM \cdot EMDC)}{AN}\right) = 0$$

$$:\ \frac{\partial Cdp}{\partial \phi} = \frac{1}{AN}\left[k_M A_M \frac{\partial Cdp_M}{\partial \phi} + k_S A_S \frac{\partial Cdp_S}{\partial \phi} + k_J A_J \frac{\partial Cdp_J}{\partial \phi}\right]$$

$$:\ \frac{\partial Cdp_M}{\partial \phi} = \frac{\partial}{\partial \phi}(m_2 \alpha_{eff} + m_3) = m_2 \cdot \frac{\partial \alpha_{eff}}{\partial \phi}$$

$$:\ \frac{\partial Cdp_S}{\partial \phi} = \frac{\partial}{\partial \phi}(s_2 \alpha_{eff} + s_3) = s_2 \cdot \frac{\partial \alpha_{eff}}{\partial \phi}$$

$$:\ \frac{\partial Cdp_J}{\partial \phi} = \frac{\partial}{\partial \phi}(j_2 \alpha_{eff} + j_3) = j_2 \cdot \frac{\partial \alpha_{eff}}{\partial \phi}$$

<table>
<tr><th>α_{eff}</th><th>m_o</th><th>j_o</th><th>s_o</th><th>m_2</th><th>j_2</th><th>s_2</th></tr>
<tr><td>0</td><td rowspan="2">0.0638888</td><td rowspan="2">0.0555555</td><td rowspan="2">0</td><td rowspan="2">$7.4074 \cdot 10^{-4}$</td><td rowspan="2">$7.4074 \cdot 10^{-4}$</td><td rowspan="2">0</td></tr>
<tr><td rowspan="2">27</td></tr>
<tr><td rowspan="2">$-9.782 \cdot 10^{-3}$</td><td rowspan="2">-0.043478</td><td rowspan="2">0.0652173</td><td rowspan="2">$5.6521 \cdot 10^{-3}$</td><td rowspan="2">$0.01 \cdot \alpha_{eff}$</td><td rowspan="2">0.0108695</td></tr>
<tr><td rowspan="2">50</td></tr>
<tr><td rowspan="2">-0.018333</td><td rowspan="2">$-6.666 \cdot 10^{-3}$</td><td rowspan="2">-0.016666</td><td rowspan="2">0.0216666</td><td rowspan="2">$-3.333 \cdot 10^{-3}$</td><td rowspan="2">0.0216666</td></tr>
<tr><td rowspan="2">80</td></tr>
<tr><td rowspan="2">$-5 \cdot 10^{-3}$</td><td rowspan="2">-0.015</td><td rowspan="2">$-7.5 \cdot 10^{-3}$</td><td rowspan="2">0.01</td><td rowspan="2">$-7.5 \cdot 10^{-3}$</td><td rowspan="2">0.015</td></tr>
<tr><td rowspan="2">100</td></tr>
<tr><td rowspan="2">-0.010625</td><td rowspan="2">0</td><td rowspan="2">-0.010625</td><td rowspan="2">$1.25 \cdot 10^{-3}$</td><td rowspan="2">0</td><td rowspan="2">$-6.75 \cdot 10^{-3}$</td></tr>
<tr><td>180</td></tr>
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$$C_4 = \left.\frac{\partial M_{heel}}{\partial \phi}\right|_{V^*,\phi^*}$$

****

$$:\ \frac{\partial Mheel}{\partial \phi} = (ZCE + T - ZCBK) \cdot \frac{\partial Fheel}{\partial \phi}$$

$$:\ \frac{\partial Fheel}{\partial \phi} = \frac{\partial}{\partial \phi}(L \cdot \cos(\alpha_{eff})) + \frac{\partial}{\partial \phi}(D \cdot \sin(\alpha_{eff}))$$

$$:\ \frac{\partial}{\partial \phi}(L \cdot \cos(\alpha_{eff})) = \cos(\alpha_{eff}) \cdot \frac{\partial L}{\partial \phi} + L \cdot \frac{\partial}{\partial \phi}(\cos(\alpha_{eff}))$$

$$:\ \frac{\partial}{\partial \phi}(D \cdot \sin(\alpha_{eff})) = \sin(\alpha_{eff}) \cdot \frac{\partial D}{\partial \phi} + D \cdot \frac{\partial}{\partial \phi}(\sin(\alpha_{eff}))$$

****

$$C_3 = \left.\frac{\partial M_{right}}{\partial \phi}\right|_{V^*,\phi^*}$$

****

: $\frac{\partial M_{right}}{\partial \phi} = \frac{\partial M_1}{\partial \phi} + \frac{\partial M_2}{\partial \phi}$

: $\frac{\partial M_1}{\partial \phi} = (KM - KG) \cdot g \cdot \rho_w \cdot (DIVCAN + DVK) \cdot \cos(\phi^*)$

: $\frac{\partial M_2}{\partial \phi} = -MMVBLCRW \cdot g \cdot b \cdot \sin(\phi^*)$

****

$$C_1 = \left.\frac{\partial R_t}{\partial \phi}\right|_{V^*,\phi^*}$$

****

: $\frac{\partial R_{tot}}{\partial \phi} = \frac{\partial R_i}{\partial \phi} + \frac{\partial R_{rh}}{\partial \phi} + \frac{\partial R_{rhH}}{\partial \phi} + \frac{\partial R_{rk}}{\partial \phi} + \frac{\partial R_{rkH}}{\partial \phi} + \frac{\partial R_{vh}}{\partial \phi} + \frac{\partial R_{vhH}}{\partial \phi} + \frac{\partial R_{vk}}{\partial \phi} + \frac{\partial R_{vr}}{\partial \phi}$

$$\frac{\partial Ri}{\partial \phi}$$

: $\frac{\partial Ri}{\partial \phi} = \frac{\partial}{\partial \phi}\left(Fheel^2 \cdot \frac{1}{\pi \cdot Te^2 \cdot \frac{1}{2} \cdot \rho_w \cdot V^{*2}}\right) = \frac{2}{\pi \cdot Te^2 \cdot \frac{1}{2} \cdot \rho_w \cdot V^{*2}} \frac{\partial Fheel}{\partial \phi} - \frac{2 \cdot Fheel^2}{\pi \cdot \frac{1}{2} \cdot \rho_w \cdot V^{*2}} \cdot Te^{-3} \cdot \frac{\partial Te}{\partial \phi}$

: $\frac{\partial Te}{\partial \phi} = \frac{\partial}{\partial \phi}\left[T \cdot \left(A_1 \cdot \frac{TCAN}{T} + A_2 \cdot \left(\frac{TCAN}{T}\right)^2 + A_3 \cdot \frac{BWL}{TCAN} + A_4 \cdot \frac{CHTPK}{CHRTK}\right) \cdot (B_0 + B_1 \cdot Fn)\right] =$

$$= (B_0 + B_1 \cdot Fn) \cdot T \cdot \frac{\partial}{\partial \phi}\left(A_1 \cdot \frac{TCAN}{T} + A_2 \cdot \left(\frac{TCAN}{T}\right)^2 + A_3 \cdot \frac{BWL}{TCAN} + A_4 \cdot \frac{CHTPK}{CHRTK}\right) + T \cdot \\ \cdot \left(A_1 \cdot \frac{TCAN}{T} + A_2 \cdot \left(\frac{TCAN}{T}\right)^2 + A_3 \cdot \frac{BWL}{TCAN} + A_4 \cdot \frac{CHTPK}{CHRTK}\right) \cdot \frac{\partial}{\partial \phi}[(B_0 + B_1 \cdot Fn)] =$$

$$= (B_0 + B_1 \cdot Fn) \cdot T \cdot \left(\frac{TCAN}{T} \cdot \frac{\partial A_1}{\partial \phi} + \left(\frac{TCAN}{T}\right)^2 \cdot \frac{\partial A_2}{\partial \phi} + \frac{BWL}{TCAN} \cdot \frac{\partial A_3}{\partial \phi} + \frac{CHTPK}{CHRTK} \cdot \frac{\partial A_4}{\partial \phi}\right) + T \cdot \\ \cdot \left(A_1 \cdot \frac{TCAN}{T} + A_2 \cdot \left(\frac{TCAN}{T}\right)^2 + A_3 \cdot \frac{BWL}{TCAN} + A_4 \cdot \frac{CHTPK}{CHRTK}\right) \cdot \left(\frac{\partial B_0}{\partial \phi} + Fn \cdot \frac{\partial B_1}{\partial \phi}\right)$$

: $\mathrm{Fn} = \dfrac{V^*}{\sqrt{g \cdot \mathrm{Lwl}}}$

<table>
<tr><th>ϕ^*</th><th>$\frac{\partial A_1}{\partial \phi}$</th><th>$\frac{\partial A_2}{\partial \phi}$</th><th>$\frac{\partial A_3}{\partial \phi}$</th><th>$\frac{\partial A_4}{\partial \phi}$</th><th>$\frac{\partial B_0}{\partial \phi}$</th><th>$\frac{\partial B_1}{\partial \phi}$</th></tr>
<tr><td>0</td><td rowspan="2">0.07437</td><td rowspan="2">-0.12208</td><td rowspan="2">$-2.95 \cdot 10^{-3}$</td><td rowspan="2">$1.2 \cdot 10^{-3}$</td><td rowspan="2">0.01925</td><td rowspan="2">-0.05715</td></tr>
<tr><td rowspan="2">10</td></tr>
<tr><td rowspan="2">-0.053</td><td rowspan="2">0.0865</td><td rowspan="2">$-1.1 \cdot 10^{-4}$</td><td rowspan="2">$1.01 \cdot 10^{-3}$</td><td rowspan="2">0.01219</td><td rowspan="2">-0.02651</td></tr>
<tr><td rowspan="2">20</td></tr>
<tr><td rowspan="2">-0.04701</td><td rowspan="2">0.10227</td><td rowspan="2">$-3.3 \cdot 10^{-4}$</td><td rowspan="2">$-1.97 \cdot 10^{-3}$</td><td rowspan="2">$-7.06 \cdot 10^{-3}$</td><td rowspan="2">0.02123</td></tr>
<tr><td>30</td></tr>
</table>

$$\frac{\partial \mathrm{Rrh}}{\partial \phi}$$

: $\dfrac{\partial \mathrm{Rrh}}{\partial \phi} = 0$

$$\frac{\partial \mathrm{RrhH}}{\partial \phi}$$

: $$\frac{\partial \mathrm{RrhH}}{\partial \phi} = \nabla_c g \rho_w \cdot \left[u_0 + u_1 \frac{\mathrm{Lwl}}{\mathrm{Bwl}} + u_2 \frac{\mathrm{Bwl}}{T_c} + u_3 \left(\frac{\mathrm{Bwl}}{T_c}\right)^2 + u_4 \frac{\mathrm{XFB}}{\mathrm{Lwl}} + u_5 \left(\frac{\mathrm{XFB}}{\mathrm{Lwl}}\right)^2\right] \cdot 6.0 \cdot \frac{\partial \phi^{*1.7}}{\partial \phi} =$$
$$= \nabla_c g \rho_w \cdot \left[u_0 + u_1 \frac{\mathrm{Lwl}}{\mathrm{Bwl}} + u_2 \frac{\mathrm{Bwl}}{T_c} + u_3 \left(\frac{\mathrm{Bwl}}{T_c}\right)^2 + u_4 \frac{\mathrm{XFB}}{\mathrm{Lwl}} + u_5 \left(\frac{\mathrm{XFB}}{\mathrm{Lwl}}\right)^2\right] \cdot 1.7 \cdot 6.0 \cdot \phi^{*0.7}$$

Fn	u_0	u_1	u_2
0 – 0.20	0	0	0
0.20 – 0.25	$0.000536 \cdot \mathrm{Fn} - 0.0001072$	$-0.000028 \cdot \mathrm{Fn} + 0.0000056$	$-0.000114 \cdot \mathrm{Fn} + 0.0000228$
0.25 – 0.30	$0.0007896 \cdot \mathrm{Fn} - 0.0001706$	$-0.001236 \cdot \mathrm{Fn} + 0.0003076$	$-0.001284 \cdot \mathrm{Fn} + 0.0003153$
0.30 – 0.35	$0.0315404 \cdot \mathrm{Fn} - 0.00939584$	$-0.003024 \cdot \mathrm{Fn} + 0.000844$	$-0.001882 \cdot \mathrm{Fn} + 0.0004947$
0.35 – 0.40	$-0.050184 \cdot \mathrm{Fn} + 0.0192077$	$0.00358 \cdot \mathrm{Fn} - 0.0014674$	$0.007732 \cdot \mathrm{Fn} - 0.0028702$
0.40 – 0.45	$-0.048112 \cdot \mathrm{Fn} + 0.0183789$	$0.003452 \cdot \mathrm{Fn} - 0.0014162$	$0.006642 \cdot \mathrm{Fn} - 0.0024342$
0.45 – 0.50	$0.061478 \cdot \mathrm{Fn} - 0.0309366$	$-0.005704 \cdot \mathrm{Fn} + 0.002704$	$-0.02428 \cdot \mathrm{Fn} + 0.0114807$
0.50 – 0.55	$0.035698 \cdot \mathrm{Fn} - 0.0180466$	$-0.004538 \cdot \mathrm{Fn} + 0.002121$	$-0.001024 \cdot \mathrm{Fn} - 0.0001473$

Fn	u_3	u_4	u_5
0 – 0.20	0	0	0
0.20 – 0.25	$0.000032 - 0.0000064$	$-0.00014 \cdot \mathrm{Fn} + 0.000028$	$-0.000034 \cdot \mathrm{Fn} + 0.0000068$
0.25 – 0.30	$0.000106 \cdot \mathrm{Fn} - 0.0000249$	$0.001058 \cdot \mathrm{Fn} - 0.0002715$	$0.000026 \cdot \mathrm{Fn} - 0.0000082$
0.30 – 0.35	$0.00026 \cdot \mathrm{Fn} - 0.0000711$	$-0.001998 \cdot \mathrm{Fn} + 0.0006453$	$-0.000528 \cdot \mathrm{Fn} + 0.000158$
0.35 – 0.40	$-0.000022 \cdot \mathrm{Fn} + 0.0000276$	$-0.01052 \cdot \mathrm{Fn} + 0.003628$	$-0.00173 \cdot \mathrm{Fn} + 0.0005787$
0.40 – 0.45	$0.00016 \cdot \mathrm{Fn} - 0.0000452$	$-0.008528 \cdot \mathrm{Fn} + 0.0028312$	$0.006318 \cdot \mathrm{Fn} - 0.0026405$
0.45 – 0.50	$0.003188 \cdot \mathrm{Fn} - 0.0014078$	$0.00515 \cdot \mathrm{Fn} - 0.0033239$	$-0.007348 \cdot \mathrm{Fn} + 0.0035092$
0.50 – 0.55	$0.000568 \cdot \mathrm{Fn} - 0.0000978$	$0.005342 \cdot \mathrm{Fn} - 0.0034199$	$0.000948 \cdot \mathrm{Fn} - 0.0006388$

$$\frac{\partial \mathrm{Rrk}}{\partial \phi}$$

: $$\frac{\partial \mathrm{Rrk}}{\partial \phi} = \nabla_k \cdot \rho_w \cdot g \cdot \frac{\partial}{\partial \phi}\left[A_0 + A_1 \cdot \frac{T}{\mathrm{Bwl}} + A_2 \cdot \frac{(T_C + \mathrm{Zcbk})^3}{\nabla_k} + A_3 \cdot \frac{\nabla_c}{\nabla_k}\right] = 0$$

$$\frac{\partial \mathrm{RrkH}}{\partial \phi}$$

: $$\frac{\partial \mathrm{RrkH}}{\partial \phi} = \nabla_k \cdot \rho_w \cdot g \cdot \left[H_1 \cdot \frac{T_c}{T} + H_2 \cdot \frac{\mathrm{Bwl}}{T_c} + H_3 \cdot \frac{T_c}{T} \cdot \frac{\mathrm{Bwl}}{T_c} + H_4 \cdot \frac{\mathrm{Lwl}}{\nabla_c^{1/3}}\right] \cdot \mathrm{Fn}^2 \cdot \frac{\partial \phi}{\partial \phi} =$$

$$= \nabla_k \cdot \rho_w \cdot g \cdot \left[H_1 \cdot \frac{T_c}{T} + H_2 \cdot \frac{\mathrm{Bwl}}{T_c} + H_3 \cdot \frac{T_c}{T} \cdot \frac{\mathrm{Bwl}}{T_c} + H_4 \cdot \frac{\mathrm{Lwl}}{\nabla_c^{1/3}}\right] \cdot \mathrm{Fn}^2$$

H_1	H_2	H_3	H_4
-3.5837	-0.0518	0.5958	0.2055

$$\frac{\partial \mathrm{Rvh}}{\partial \phi}$$

: $$\frac{\partial \mathrm{Rvh}}{\partial \phi} = 0$$

$$\frac{\partial \mathrm{Rvk}}{\partial \phi}$$

: $$\frac{\partial \mathrm{Rvk}}{\partial \phi} = 0$$

: $\dfrac{\partial Rvr}{\partial \phi}$

: $\dfrac{\partial Rvr}{\partial \phi} = 0$

$\dfrac{\partial RvhH}{\partial \phi}$

: $\dfrac{\partial RvhH}{\partial \phi} = (1 + k) \cdot \dfrac{\partial RfhH}{\partial \phi}$

: $\dfrac{\partial RfhH}{\partial \phi} = \dfrac{1}{2} \cdot \rho_w \cdot V^2 \cdot C_f \cdot \dfrac{\partial Sc_\phi}{\partial \phi}$

: $\dfrac{\partial Sc_\phi}{\partial \phi} = S_c \cdot \left[1 + \dfrac{1}{100} \cdot \left(\dfrac{\partial s_0}{\partial \phi} + \dfrac{Bwl}{T_c} \cdot \dfrac{\partial s_1}{\partial \phi} + \left(\dfrac{Bwl}{Tc}\right)^2 \cdot \dfrac{\partial s_2}{\partial \phi} + Cm \cdot \dfrac{\partial s_3}{\partial \phi}\right)\right]$

: $Rn = \dfrac{V \cdot 0.7 \cdot Lwl}{\nu}$

: $C_f = \dfrac{0.075}{(\log(Rn) - 2)^2}$

<table>
<tr><th>ϕ</th><th>$\frac{\partial s_0}{\partial \phi}$</th><th>$\frac{\partial s_1}{\partial \phi}$</th><th>$\frac{\partial s_2}{\partial \phi}$</th><th>$\frac{\partial s_3}{\partial \phi}$</th></tr>
<tr><td>0</td><td rowspan="2">-0.8224</td><td rowspan="2">0.0108</td><td rowspan="2">-0.0054</td><td rowspan="2">1.2658</td></tr>
<tr><td rowspan="2">5</td></tr>
<tr><td rowspan="2">-0.082</td><td rowspan="2">-0.0372</td><td rowspan="2">-0.01</td><td rowspan="2">0.4818</td></tr>
<tr><td rowspan="2">10</td></tr>
<tr><td rowspan="2">0.2462</td><td rowspan="2">-0.0514</td><td rowspan="2">-0.0082</td><td rowspan="2">0.0422</td></tr>
<tr><td rowspan="2">15</td></tr>
<tr><td rowspan="2">1.0282</td><td rowspan="2">-0.1622</td><td rowspan="2">0.0018</td><td rowspan="2">-0.717</td></tr>
<tr><td rowspan="2">20</td></tr>
<tr><td rowspan="2">0.932</td><td rowspan="2">-0.221</td><td rowspan="2">0.0086</td><td rowspan="2">-0.3842</td></tr>
<tr><td rowspan="2">25</td></tr>
<tr><td rowspan="2">1.1648</td><td rowspan="2">-0.3212</td><td rowspan="2">0.018</td><td rowspan="2">-0.3352</td></tr>
<tr><td rowspan="2">30</td></tr>
<tr><td rowspan="2">0.4628</td><td rowspan="2">-0.2542</td><td rowspan="2">0.0156</td><td rowspan="2">0.346</td></tr>
<tr><td>35</td></tr>
</table>

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$$B_2 = \frac{\partial F_{drive}}{\partial V}\bigg|_{V^*,\phi^*}$$

****

: $B_2 = \frac{\partial F_{drive}}{\partial V} = \frac{\partial}{\partial V}(L \cdot \sin(\alpha_{eff}) - D \cdot \cos(\alpha_{eff})) = \frac{\partial}{\partial V}(L \cdot \sin(\alpha_{eff})) - \frac{\partial}{\partial V}(D \cdot \cos(\alpha_{eff}))$

: $\frac{\partial}{\partial V}(L \cdot \sin(\alpha_{eff})) = L \cdot \frac{\partial}{\partial V}(\sin(\alpha_{eff})) + \sin(\alpha_{eff})\frac{\partial L}{\partial V}$

: $\frac{\partial L}{\partial V} = \frac{1}{2} \cdot \rho_a \cdot AN \cdot \frac{\partial}{\partial V}\left(V_{eff}^2 \cdot C_L\right)$

: $\frac{\partial}{\partial V}\left(V_{eff}^2 \cdot C_L\right) = 2 \cdot V_{eff} \cdot C_L \cdot \frac{\partial V_{eff}}{\partial V} + V_{eff}^2 \cdot \frac{\partial C_L}{\partial V}$

: $\frac{\partial V_{eff}}{\partial V} = \frac{\partial}{\partial V}\left(\sqrt{V_1^2 + V_2^2}\right) = \frac{1}{2} \cdot (V_1^2 + V_2^2)^{-1/2} \cdot \frac{\partial}{\partial V}(V_1^2 + V_2^2)$

: $\frac{\partial}{\partial V}(V_1^2 + V_2^2) = \frac{\partial}{\partial V}(V_1^2) + \frac{\partial}{\partial V}(V_2^2)$

: $\frac{\partial}{\partial V}(V_1^2) = 2 \cdot V_1 \cdot \frac{\partial V_1}{\partial V} = 2 \cdot V_1 \cdot \frac{\partial}{\partial V}(V + V_{tw} \cdot \cos\alpha_{tw}) = 2 \cdot V_1 = 2 \cdot (V^* + V_{tw} \cdot \cos\alpha_{tw})$

: $\frac{\partial}{\partial V}(V_2^2) = 2 \cdot V_2 \cdot \frac{\partial V_2}{\partial V} = 2 \cdot V_2 \cdot \frac{\partial}{\partial V}(V_{tw} \cdot \sin\alpha_{tw} \cdot \cos\phi^*) = 0$

: $\frac{\partial C_L}{\partial V} = \frac{F}{AN} \cdot \left(k_M \cdot AM \cdot \frac{\partial Cl_M}{\partial V} + k_S \cdot AS \cdot \frac{\partial Cl_s}{\partial V} + k_J \cdot AJ \cdot \frac{\partial Cl_j}{\partial V}\right)$

: $\frac{\partial Cl_M}{\partial V} = \frac{\partial}{\partial V}(m_0 \cdot \alpha_{eff} + m_1) = m_0 \cdot \frac{\partial \alpha_{eff}}{\partial V}$

: $\frac{\partial Cl_J}{\partial V} = \frac{\partial}{\partial V}(j_o \cdot \alpha_{eff} + j_1) = j_o \frac{\partial \alpha_{eff}}{\partial V}$

: $\frac{\partial Cl_S}{\partial V} = \frac{\partial}{\partial V}(s_o \cdot \alpha_{eff} + s_1) = s_o \frac{\partial \alpha_{eff}}{\partial V}$

: $\frac{\partial \alpha_{eff}}{\partial V} = \frac{\partial}{\partial V}\left(\tan^{-1}\left(\frac{V_2}{V_1}\right)\right) = \frac{1}{1 + \left(\frac{V_2}{V_1}\right)^2} \cdot \frac{\partial}{\partial V}\left(\frac{V_2}{V_1}\right) = \frac{V_1^2 \cdot V_2}{V_1^2 + V_2^2} \cdot \frac{\partial}{\partial V}\left(\frac{1}{V_1}\right) = -1 \cdot \frac{1}{V_1^2} \cdot \frac{V_1^2 \cdot V_2}{V_1^2 + V_2^2} \cdot \frac{\partial V_1}{\partial V} =$

$$= -1 \cdot \frac{V_2}{V_1^2 + V_2^2}$$

: $\frac{\partial}{\partial V}(\sin(\alpha_{eff})) = \cos(\alpha_{eff}) \cdot \frac{\partial \alpha_{eff}}{\partial V}$

$$:\ \frac{\partial}{\partial V}(D\cdot\cos(\alpha_{eff})) = D\cdot\frac{\partial}{\partial V}(\cos(\alpha_{eff})) + \cos(\alpha_{eff})\cdot\frac{\partial D}{\partial V}$$

$$:\ \frac{\partial}{\partial V}(\cos(\alpha_{eff})) = -\sin(\alpha_{eff})\cdot\frac{\partial \alpha_{eff}}{\partial V}$$

$$:\ \frac{\partial D}{\partial V} = \frac{\partial}{\partial V}\left(\frac{1}{2}\cdot\rho_a\cdot V_{eff}^2\cdot AN\cdot C_D\right) = \frac{1}{2}\cdot\rho_a\cdot AN\cdot\frac{\partial}{\partial V}\left(V_{eff}^2\cdot C_D\right)$$

$$:\ \frac{\partial}{\partial V}\left(V_{eff}^2\cdot C_D\right) = C_D\cdot\frac{\partial}{\partial V}\left(V_{eff}^2\right) + V_{eff}^2\cdot\frac{\partial C_D}{\partial V} = 2\cdot V_{eff}\cdot C_D\cdot\frac{\partial V_{eff}}{\partial V} + V_{eff}^2\cdot\frac{\partial C_D}{\partial V}$$

$$:\ \frac{\partial C_D}{\partial V} = \frac{\partial}{\partial V}(Cdp + Cd0 + CdI) = \frac{\partial Cdp}{\partial V} + \frac{\partial Cd0}{\partial V} + \frac{\partial CdI}{\partial V}$$

$$:\ \frac{\partial CdI}{\partial V} = \left(\frac{1}{\pi\cdot AR} + 0.005\right)\frac{\partial}{\partial V}\left(C_L^2\right) = 2\cdot C_L\cdot\left(\frac{1}{\pi\cdot AR} + 0.005\right)\cdot\frac{\partial C_L}{\partial V}$$

$$:\ \frac{\partial Cd0}{\partial V} = 0$$

$$:\ \frac{\partial Cdp}{\partial V} = \frac{1}{AN}\left[k_M A_M\frac{\partial Cdp_M}{\partial V} + k_S A_S\frac{\partial Cdp_S}{\partial V} + k_J A_J\frac{\partial Cdp_J}{\partial V}\right]$$

$$:\ \frac{\partial Cdp_M}{\partial V} = \frac{\partial}{\partial V}(m_2\alpha_{eff} + m_3) = m_2\cdot\frac{\partial \alpha_{eff}}{\partial V}$$

$$:\ \frac{\partial Cdp_S}{\partial V} = \frac{\partial}{\partial V}(s_2\alpha_{eff} + s_3) = s_2\cdot\frac{\partial \alpha_{eff}}{\partial V}$$

$$:\ \frac{\partial Cdp_J}{\partial V} = \frac{\partial}{\partial V}(j_2\alpha_{eff} + j_3) = j_2\cdot\frac{\partial \alpha_{eff}}{\partial V}$$

$$:\ V_1 = V^* + V_{tw}\cdot\cos\alpha_{tw}$$

$$:\ V_2 \approx V_{tw}\cdot\sin\alpha_{tw}\cdot\cos\phi^*$$

α_{eff}	m_o	j_o	s_o	m_2	j_2	s_2
0 – 27	0.0638888	0.0555555	0	$7.4074\cdot10^{-4}$	$7.4074\cdot10^{-4}$	0
27 – 50	$-9.782\cdot10^{-3}$	-0.043478	0.0652173	$5.6521\cdot10^{-3}$	$0.01\cdot\alpha_{eff}$	0.0108695
50 – 80	-0.018333	$-6.666\cdot10^{-3}$	-0.016666	0.0216666	$-3.333\cdot10^{-3}$	0.0216666
80 – 100	$-5\cdot10^{-3}$	-0.015	$-7.5\cdot10^{-3}$	0.01	$-7.5\cdot10^{-3}$	0.015
100 – 180	-0.010625	0	-0.010625	$1.25\cdot10^{-3}$	0	$-6.75\cdot10^{-3}$

****

$$B_4 = \frac{\partial M_{heel}}{\partial V}\bigg|_{V^*,\phi^*}$$

****

: $B_4 = \frac{\partial M_{heel}}{\partial V} = (ZCE + T - ZCBK)\frac{\partial Fheel}{\partial V}$

: $\frac{\partial Fheel}{\partial V} = \frac{\partial}{\partial V}(L \cdot \cos(\alpha_{eff}) + D \cdot \sin(\alpha_{eff})) = \frac{\partial}{\partial V}(L \cdot \cos(\alpha_{eff})) + \frac{\partial}{\partial V}(D \cdot \sin(\alpha_{eff}))$

: $\frac{\partial}{\partial V}(L \cdot \cos(\alpha_{eff})) = L \cdot \frac{\partial}{\partial V}(\cos(\alpha_{eff})) + \cos(\alpha_{eff}) \cdot \frac{\partial L}{\partial V}$

: $\frac{\partial}{\partial V}(D \cdot \sin(\alpha_{eff})) = D \cdot \frac{\partial}{\partial V}(\sin(\alpha_{eff})) + \sin(\alpha_{eff}) \cdot \frac{\partial D}{\partial V}$

****

$$B_3 = \frac{\partial M_{right}}{\partial V}\bigg|_{V^*,\phi^*}$$

****

: $\frac{\partial M_{right}}{\partial V} = 0$

****

$$B_1 = \frac{\partial R_t}{\partial V}\bigg|_{V^*,\phi^*}$$

****

$$\frac{\partial R_{tot}}{\partial V}$$

: $\frac{\partial R_{tot}}{\partial V} = \frac{\partial R_i}{\partial V} + \frac{\partial R_{rh}}{\partial V} + \frac{\partial R_{rhH}}{\partial V} + \frac{\partial R_{rk}}{\partial V} + \frac{\partial R_{rkH}}{\partial V} + \frac{\partial R_{vh}}{\partial V} + \frac{\partial R_{vhH}}{\partial V} + \frac{\partial R_{vk}}{\partial V} + \frac{\partial R_{vr}}{\partial V}$

$$\frac{\partial R_i}{\partial V}$$

: $\frac{\partial Ri}{\partial V} = \frac{\partial}{\partial V}\left(\frac{Fheel^2}{\pi \cdot Te^2 \cdot \frac{1}{2} \cdot \rho_w \cdot V^{*2}}\right) = \frac{Fheel^2}{\pi \cdot \frac{1}{2} \cdot \rho_w} \cdot \frac{\partial}{\partial V}\left(\frac{1}{Te^2 \cdot V^{*2}}\right) + \frac{1}{\pi \cdot Te^2 \cdot \frac{1}{2} \cdot \rho_w \cdot V^{*2}} \cdot \frac{\partial}{\partial V}(Fheel^2)$

: $\frac{\partial}{\partial V}(Fheel^2) = 2 \cdot Fheel \cdot \frac{\partial Fheel}{\partial V}$

: $\frac{\partial}{\partial V}\left(\frac{1}{Te^2 \cdot V^{*2}}\right) = \frac{1}{Te^2} \cdot \frac{\partial}{\partial V}\left(\frac{1}{V^2}\right) + \frac{1}{V^{*2}} \cdot \frac{\partial}{\partial V}\left(\frac{1}{Te^2}\right)$

: $\frac{\partial}{\partial V}\left(\frac{1}{V^2}\right) = -2 \cdot \frac{1}{V^3}$

: $\frac{\partial}{\partial V}\left(\frac{1}{Te^2}\right) = -2 \cdot \frac{1}{Te^3} \cdot \frac{\partial Te}{\partial V}$

: $\frac{\partial Te}{\partial V} = T \cdot \left(A_1 \cdot \frac{TCAN}{T} + A_2 \cdot \left(\frac{TCAN}{T}\right)^2 + A_3 \cdot \frac{BWL}{TCAN} + A_4 \cdot \frac{CHTPK}{CHRTK}\right) \cdot \frac{\partial}{\partial V}(B_0 + B_1 \cdot Fn)$

: $\frac{\partial}{\partial V}(B_0 + B_1 \cdot Fn) = B_1 \cdot \frac{\partial Fn}{\partial V}$

: $\frac{\partial Fn}{\partial V} = \frac{\partial}{\partial V}\left(\frac{V}{\sqrt{g \cdot Lwl}}\right) = \frac{1}{\sqrt{g \cdot Lwl}}$

<table>
<tr><th>ϕ^*</th><th>A_1</th><th>A_2</th><th>A_3</th><th>A_4</th></tr>
<tr><td>0</td><td rowspan="2">$0.07437 \cdot \phi^* + 3.7455$</td><td rowspan="2">$-0.12208 \cdot \phi^* - 3.6246$</td><td rowspan="2">$-2.95 \cdot 10^{-3} \cdot \phi^* + 0.0589$</td><td rowspan="2">$1.2 \cdot 10^{-3} \cdot \phi^* - 0.0296$</td></tr>
<tr><td rowspan="2">10</td></tr>
<tr><td rowspan="2">$-0.053 \cdot \phi^* + 5.0192$</td><td rowspan="2">$0.0865 \cdot \phi^* - 5.7104$</td><td rowspan="2">$-1.1 \cdot 10^{-4} \cdot \phi^* + 0.0305$</td><td rowspan="2">$1.01 \cdot 10^{-3} \cdot \phi^* - 0.0277$</td></tr>
<tr><td rowspan="2">20</td></tr>
<tr><td rowspan="2">$-0.04701 \cdot \phi^* + 4.8994$</td><td rowspan="2">$0.10227 \cdot \phi^* - 6.0258$</td><td rowspan="2">$-3.3 \cdot 10^{-4} \cdot \phi^* + 0.0349$</td><td rowspan="2">$-1.97 \cdot 10^{-3} \cdot \phi^* + 0.0319$</td></tr>
<tr><td>30</td></tr>
</table>

<table>
<tr><th>ϕ^*</th><th>B_1</th></tr>
<tr><td>0</td><td rowspan="2">$-0.05715 \cdot \phi^* - 0.7256$</td></tr>
<tr><td rowspan="2">10</td></tr>
<tr><td rowspan="2">$-0.02651 \cdot \phi^* - 1.032$</td></tr>
<tr><td rowspan="2">20</td></tr>
<tr><td rowspan="2">$0.02123 \cdot \phi^* - 1.9868$</td></tr>
<tr><td>30</td></tr>
</table>

$$\frac{\partial \mathrm{Rrh}}{\partial \mathrm{V}}$$

$$:\ \frac{\partial \mathrm{Rrh}}{\partial \mathrm{V}} =$$

$$= \nabla_c g \rho_w \cdot \frac{\partial}{\partial \mathrm{V}}\left[a_0 + \frac{\nabla^{\frac{1}{3}}}{\mathrm{Lwl}}\left(a_1 \frac{\mathrm{LCB}}{\mathrm{Lwl}} + a_2 \mathrm{Cp} + a_3 \frac{\nabla_c^{\frac{2}{3}}}{A_w} + a_4 \frac{\mathrm{Bwl}}{\mathrm{Lwl}} + a_5 \frac{\nabla_c^{\frac{2}{3}}}{S_c} + a_6 \frac{\mathrm{LCB}}{\mathrm{LCF}} + a_7 \left(\frac{\mathrm{LCB}}{\mathrm{LCF}}\right)^2 + a_8 \mathrm{Cp}^2\right)\right] =$$

$$= \nabla_c g \rho_w \cdot$$

$$\cdot \left[\frac{\partial a_0}{\partial \mathrm{V}} + \frac{\nabla^{\frac{1}{3}}}{\mathrm{Lwl}} \cdot\right.$$

$$\left.\cdot \left(\frac{\mathrm{LCB}}{\mathrm{Lwl}} \cdot \frac{\partial a_1}{\partial \mathrm{V}} + \mathrm{Cp} \cdot \frac{\partial a_2}{\partial \mathrm{V}} + \frac{\nabla_c^{2/3}}{A_w} \cdot \frac{\partial a_3}{\partial \mathrm{V}} + \frac{\mathrm{Bwl}}{\mathrm{Lwl}} \cdot \frac{\partial a_4}{\partial \mathrm{V}} + \frac{\nabla_c^{2/3}}{S_c} \cdot \frac{\partial a_5}{\partial \mathrm{V}} + \frac{\mathrm{LCB}}{\mathrm{LCF}} \cdot \frac{\partial a_6}{\partial \mathrm{V}} + \left(\frac{\mathrm{LCB}}{\mathrm{LCF}}\right)^2 \cdot \frac{\partial a_7}{\partial \mathrm{V}} + \mathrm{Cp}^2 \cdot \frac{\partial a_8}{\partial \mathrm{V}}\right)\right]$$

Fn	$\frac{\partial a_0}{\partial V}$	$\frac{\partial a_1}{\partial V}$	$\frac{\partial a_2}{\partial V}$
0			
	$-0.014 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$0.403 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$0.47 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.10			
	$0.036 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-4.422 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$2.646 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.15			
	$0.02 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$1.474 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-2.312 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.20			
	$0.026 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$3.068 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-3.8 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.25			
	$0.058 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-16.936 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$12.308 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.30			
	$-0.048 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$13.988 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-11.408 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.35			
	$-0.192 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$48.212 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-28.678 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.40			
	$-0.214 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$21.844 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-9.42 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.45			
	$-0.06 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$75.118 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-86.884 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.50			
	$1.392 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-111.916 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$5.286 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.55			
	$0.626 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$-137.702 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$	$98.296 \cdot (g \cdot \mathrm{Lwl})^{-1/2}$
0.60			

Fn	$\frac{\partial a_3}{\partial V}$	$\frac{\partial a_4}{\partial V}$	$\frac{\partial a_5}{\partial V}$
0			
	$-0.227 \cdot (g \cdot Lwl)^{-1/2}$	$-0.119 \cdot (g \cdot Lwl)^{-1/2}$	$0.061 \cdot (g \cdot Lwl)^{-1/2}$
0.10			
	$0.446 \cdot (g \cdot Lwl)^{-1/2}$	$0.432 \cdot (g \cdot Lwl)^{-1/2}$	$0.114 \cdot (g \cdot Lwl)^{-1/2}$
0.15			
	$0.188 \cdot (g \cdot Lwl)^{-1/2}$	$0.112 \cdot (g \cdot Lwl)^{-1/2}$	$-0.214 \cdot (g \cdot Lwl)^{-1/2}$
0.20			
	$0.12 \cdot (g \cdot Lwl)^{-1/2}$	$0.242 \cdot (g \cdot Lwl)^{-1/2}$	$-0.62 \cdot (g \cdot Lwl)^{-1/2}$
0.25			
	$0.238 \cdot (g \cdot Lwl)^{-1/2}$	$0.491 \cdot (g \cdot Lwl)^{-1/2}$	$-0.028 \cdot (g \cdot Lwl)^{-1/2}$
0.30			
	$-1.302 \cdot (g \cdot Lwl)^{-1/2}$	$-0.398 \cdot (g \cdot Lwl)^{-1/2}$	$-2.336 \cdot (g \cdot Lwl)^{-1/2}$
0.35			
	$2.266 \cdot (g \cdot Lwl)^{-1/2}$	$-2.356 \cdot (g \cdot Lwl)^{-1/2}$	$-7.736 \cdot (g \cdot Lwl)^{-1/2}$
0.40			
	$4.982 \cdot (g \cdot Lwl)^{-1/2}$	$-1.184 \cdot (g \cdot Lwl)^{-1/2}$	$-5.388 \cdot (g \cdot Lwl)^{-1/2}$
0.45			
	$5.174 \cdot (g \cdot Lwl)^{-1/2}$	$6.16 \cdot (g \cdot Lwl)^{-1/2}$	$8.154 \cdot (g \cdot Lwl)^{-1/2}$
0.50			
	$5.624 \cdot (g \cdot Lwl)^{-1/2}$	$20.144 \cdot (g \cdot Lwl)^{-1/2}$	$43.152 \cdot (g \cdot Lwl)^{-1/2}$
0.55			
	$2.044 \cdot (g \cdot Lwl)^{-1/2}$	$8.764 \cdot (g \cdot Lwl)^{-1/2}$	$19.698 \cdot (g \cdot Lwl)^{-1/2}$
0.60			

Fn	$\frac{\partial a_6}{\partial V}$	$\frac{\partial a_7}{\partial V}$	$\frac{\partial a_8}{\partial V}$
0			
	$-0.086 \cdot (g \cdot Lwl)^{-1/2}$	$-0.307 \cdot (g \cdot Lwl)^{-1/2}$	$-0.553 \cdot (g \cdot Lwl)^{-1/2}$
0.10			
	$0.062 \cdot (g \cdot Lwl)^{-1/2}$	$4.056 \cdot (g \cdot Lwl)^{-1/2}$	$-2.35 \cdot (g \cdot Lwl)^{-1/2}$
0.15			
	$0.134 \cdot (g \cdot Lwl)^{-1/2}$	$-1.4 \cdot (g \cdot Lwl)^{-1/2}$	$2.16 \cdot (g \cdot Lwl)^{-1/2}$
0.20			
	$0.196 \cdot (g \cdot Lwl)^{-1/2}$	$-3.232 \cdot (g \cdot Lwl)^{-1/2}$	$3.736 \cdot (g \cdot Lwl)^{-1/2}$
0.25			
	$0.364 \cdot (g \cdot Lwl)^{-1/2}$	$15.818 \cdot (g \cdot Lwl)^{-1/2}$	$-9.678 \cdot (g \cdot Lwl)^{-1/2}$
0.30			
	$1.09 \cdot (g \cdot Lwl)^{-1/2}$	$-14.182 \cdot (g \cdot Lwl)^{-1/2}$	$10.412 \cdot (g \cdot Lwl)^{-1/2}$
0.35			
	$1.756 \cdot (g \cdot Lwl)^{-1/2}$	$-49.546 \cdot (g \cdot Lwl)^{-1/2}$	$20.556 \cdot (g \cdot Lwl)^{-1/2}$
0.40			
	$2.474 \cdot (g \cdot Lwl)^{-1/2}$	$-21.468 \cdot (g \cdot Lwl)^{-1/2}$	$3.42 \cdot (g \cdot Lwl)^{-1/2}$
0.45			
	$4.142 \cdot (g \cdot Lwl)^{-1/2}$	$-72.59 \cdot (g \cdot Lwl)^{-1/2}$	$77.918 \cdot (g \cdot Lwl)^{-1/2}$
0.50			
	$8.306 \cdot (g \cdot Lwl)^{-1/2}$	$100.776 \cdot (g \cdot Lwl)^{-1/2}$	$3.494 \cdot (g \cdot Lwl)^{-1/2}$
0.55	$a_6 = -1.37 \cdot Fn + 1.6711$		

0.60		$136.64 \cdot (g \cdot Lwl)^{-1/2}$	$-86.384 \cdot (g \cdot Lwl)^{-1/2}$

$$\frac{\partial RrhH}{\partial V}$$

$$: \ \frac{\partial RrhH}{\partial V} = \nabla_c g \rho_w \cdot 6.0 \cdot \phi^{*1.7} \cdot \frac{\partial}{\partial V}\left[u_0 + u_1 \frac{Lwl}{Bwl} + u_2 \frac{Bwl}{T_c} + u_3 \left(\frac{Bwl}{T_c}\right)^2 + u_4 \frac{XFB}{Lwl} + b_5 \left(\frac{XFB}{Lwl}\right)^2\right] =$$

$$= \nabla_c g \rho_w \cdot 6.0 \cdot \phi^{*1.7} \cdot \left[\frac{\partial u_0}{\partial V} + \frac{Lwl}{Bwl}\frac{\partial u_1}{\partial V} + \frac{Bwl}{T_c}\frac{\partial u_2}{\partial V} + \left(\frac{Bwl}{T_c}\right)^2 \frac{\partial u_3}{\partial V} + \frac{XFB}{Lwl}\frac{\partial u_4}{\partial V} + \left(\frac{XFB}{Lwl}\right)^2 \frac{\partial u_5}{\partial V}\right]$$

Fn	$\frac{\partial u_0}{\partial V}$	$\frac{\partial u_1}{\partial V}$	$\frac{\partial u_2}{\partial V}$
0	0	0	0
0.20	$0.000536 \cdot (g \cdot Lwl)^{-1/2}$	$-0.000028 \cdot (g \cdot Lwl)^{-1/2}$	$-0.000114 \cdot (g \cdot Lwl)^{-1/2}$
0.25	$0.0007896 \cdot (g \cdot Lwl)^{-1/2}$	$-0.001236 \cdot (g \cdot Lwl)^{-1/2}$	$-0.001284 \cdot (g \cdot Lwl)^{-1/2}$
0.30	$0.0315404 \cdot (g \cdot Lwl)^{-1/2}$	$-0.003024 \cdot (g \cdot Lwl)^{-1/2}$	$-0.001882 \cdot (g \cdot Lwl)^{-1/2}$
0.35	$-0.050184 \cdot (g \cdot Lwl)^{-1/2}$	$0.00358 \cdot (g \cdot Lwl)^{-1/2}$	$0.007732 \cdot (g \cdot Lwl)^{-1/2}$
0.40	$-0.048112 \cdot (g \cdot Lwl)^{-1/2}$	$0.003452 \cdot (g \cdot Lwl)^{-1/2}$	$0.006642 \cdot (g \cdot Lwl)^{-1/2}$
0.45	$0.061478 \cdot (g \cdot Lwl)^{-1/2}$	$-0.005704 \cdot (g \cdot Lwl)^{-1/2}$	$-0.02428 \cdot (g \cdot Lwl)^{-1/2}$
0.50	$0.035698 \cdot (g \cdot Lwl)^{-1/2}$	$-0.004538 \cdot (g \cdot Lwl)^{-1/2}$	$-0.001024 \cdot (g \cdot Lwl)^{-1/2}$
0.55			

Fn	$\dfrac{\partial u_3}{\partial V}$	$\dfrac{\partial u_4}{\partial V}$	$\dfrac{\partial u_5}{\partial V}$
0	$u_3 = 0$	$u_4 = 0$	$u_5 = 0$
0.20			
	$0.000032 \cdot (g \cdot Lwl)^{-1/2}$	$-0.00014 \cdot (g \cdot Lwl)^{-1/2}$	$-0.000034 \cdot (g \cdot Lwl)^{-1/2}$
0.25			
	$0.000106 \cdot (g \cdot Lwl)^{-1/2}$	$0.001058 \cdot (g \cdot Lwl)^{-1/2}$	$0.000026 \cdot (g \cdot Lwl)^{-1/2}$
0.30			
	$0.00026 \cdot (g \cdot Lwl)^{-1/2}$	$-0.001998 \cdot (g \cdot Lwl)^{-1/2}$	$-0.000528 \cdot (g \cdot Lwl)^{-1/2}$
0.35			
	$-0.000022 \cdot (g \cdot Lwl)^{-1/2}$	$-0.01052 \cdot (g \cdot Lwl)^{-1/2}$	$-0.00173 \cdot (g \cdot Lwl)^{-1/2}$
0.40			
	$0.00016 \cdot (g \cdot Lwl)^{-1/2}$	$-0.008528 \cdot (g \cdot Lwl)^{-1/2}$	$0.006318 \cdot (g \cdot Lwl)^{-1/2}$
0.45			
	$0.003188 \cdot (g \cdot Lwl)^{-1/2}$	$0.00515 \cdot (g \cdot Lwl)^{-1/2}$	$-0.007348 \cdot (g \cdot Lwl)^{-1/2}$
0.50			
	$0.000568 \cdot (g \cdot Lwl)^{-1/2}$	$0.005342 \cdot (g \cdot Lwl)^{-1/2}$	$0.000948 \cdot (g \cdot Lwl)^{-1/2}$
0.55			

$$\frac{\partial Rrk}{\partial V}$$

$$: \frac{\partial Rrk}{\partial V} = \nabla_k \cdot \rho_w \cdot g \cdot \frac{\partial}{\partial V}\left[A_0 + A_1 \cdot \frac{T}{Bwl} + A_2 \cdot \frac{(T_C + Zcbk)^3}{\nabla_k} + A_3 \cdot \frac{\nabla_c}{\nabla_k}\right] =$$

$$= \nabla_k \cdot \rho_w \cdot g \cdot \left[\frac{\partial A_0}{\partial V} + \frac{T}{Bwl} \cdot \frac{\partial A_1}{\partial V} + \frac{(T_C + Zcbk)^3}{\nabla_k}\frac{\partial A_2}{\partial V} + \frac{\nabla_c}{\nabla_k}\frac{\partial A_3}{\partial V}\right]$$

Fn	$\dfrac{\partial A_0}{\partial V}$	$\dfrac{\partial A_1}{\partial V}$	$\dfrac{\partial A_2}{\partial V}$	$\dfrac{\partial A_3}{\partial V}$
0	0	0	0	0
0.15				
	$-0.0208 \cdot (g \cdot Lwl)^{-1/2}$	$0.0344 \cdot (g \cdot Lwl)^{-1/2}$	$0.0234 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0016 \cdot \cdot (g \cdot Lwl)^{-1/2}$
0.20				
	$-0.0892 \cdot (g \cdot Lwl)^{-1/2}$	$0.085 \cdot (g \cdot Lwl)^{-1/2}$	$0.0546 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0002 \cdot \cdot (g \cdot Lwl)^{-1/2}$
0.25				
	$-0.112 \cdot (g \cdot Lwl)^{-1/2}$	$0.1648 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0642 \cdot \cdot (g \cdot Lwl)^{-1/2}$	$0.006 \cdot (g \cdot Lwl)^{-1/2}$
0.30				
	$0.0794 \cdot (g \cdot Lwl)^{-1/2}$	$0.2422 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0602 \cdot \cdot (g \cdot Lwl)^{-1/2}$	$0.0036 \cdot (g \cdot Lwl)^{-1/2}$
0.35				
	$-0.5736 \cdot (g \cdot Lwl)^{-1/2}$	$1.2034 \cdot (g \cdot Lwl)^{-1/2}$	$0.2462 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0044 \cdot \cdot (g \cdot Lwl)^{-1/2}$
0.40				
	$0.6222 \cdot (g \cdot Lwl)^{-1/2}$	$0.5886 \cdot (g \cdot Lwl)^{-1/2}$	$-0.2126 \cdot \cdot (g \cdot Lwl)^{-1/2}$	$0.0036 \cdot (g \cdot Lwl)^{-1/2}$
0.45				

0.50	$0.2046 \cdot (g \cdot Lwl)^{-1/2}$	$-0.8442 \cdot (g \cdot Lwl)^{-1/2}$	$1.211 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0298 \cdot (g \cdot Lwl)^{-1/2}$
0.55	$0.8538 \cdot (g \cdot Lwl)^{-1/2}$	$-1.3422 \cdot (g \cdot Lwl)^{-1/2}$	$0.2114 \cdot (g \cdot Lwl)^{-1/2}$	$0.0158 \cdot (g \cdot Lwl)^{-1/2}$
0.60	$-0.7602 \cdot (g \cdot Lwl)^{-1/2}$	$2.7026 \cdot (g \cdot Lwl)^{-1/2}$	$-0.1278 \cdot (g \cdot Lwl)^{-1/2}$	$-0.0314 \cdot (g \cdot Lwl)^{-1/2}$

$$\frac{\partial RrkH}{\partial V}$$

: $$\frac{\partial RrkH}{\partial V} = \nabla_k \cdot \rho_w \cdot g \cdot \left[H_1 \cdot \frac{T_c}{T} + H_2 \cdot \frac{Bwl}{T_c} + H_3 \cdot \frac{T_c}{T} \cdot \frac{Bwl}{T_c} + H_4 \cdot \frac{Lwl}{\nabla_c^{1/3}} \right] \cdot \phi^* \cdot \frac{\partial}{\partial V}(Fn^2)$$

: $$\frac{\partial}{\partial V}(Fn^2) = 2 \cdot Fn \cdot \frac{\partial Fn}{\partial V} = 2 \cdot Fn \cdot \frac{1}{\sqrt{g \cdot Lwl}} = 2 \cdot \frac{V}{g \cdot Lwl}$$

H_1	H_2	H_3	H_4
-3.5837	-0.0518	0.5958	0.2055

$$\frac{\partial Rvh}{\partial V}$$

: $$\frac{\partial Rvh}{\partial V} = (1 + k) \cdot \frac{\partial Rfh}{\partial V}$$

: $$\frac{\partial Rfh}{\partial V} = \frac{\partial}{\partial V}\left(\frac{1}{2} \cdot \rho_w \cdot V^2 \cdot Sc \cdot C_f\right) = \frac{1}{2} \cdot \rho_w \cdot Sc \cdot \frac{\partial}{\partial V}(V^2 \cdot C_f)$$

: $$\frac{\partial}{\partial V}(V^2 \cdot C_f) = C_f \cdot \frac{\partial}{\partial V}(V^2) + V^2 \cdot \frac{\partial C_f}{\partial V} = 2 \cdot V \cdot C_f + V^2 \cdot \frac{\partial C_f}{\partial V}$$

: $$\frac{\partial C_f}{\partial V} = \frac{\partial}{\partial V}\left(\frac{0.075}{(\log(Rn) - 2)^2}\right) = 0.075 \cdot \frac{\partial}{\partial V}\left(\frac{1}{(\log(Rn) - 2)^2}\right) = \frac{-2 \cdot 0.075}{(\log(Rn) - 2)^3} \cdot \frac{\partial}{\partial V}(\log(Rn))$$

: $$\frac{\partial}{\partial V}(\log(Rn)) = \frac{\log e}{Rn} \cdot \frac{\partial Rn}{\partial V}$$

: $$\frac{\partial Rn}{\partial V} = \frac{\partial}{\partial V}\left(\frac{V \cdot 0.7 \cdot Lwl}{\nu_w}\right) = \frac{0.7 \cdot Lwl}{\nu_w}$$

: $$Sc = \left(1.97 + 0.171 \cdot \frac{Bwl}{Tc}\right) \cdot \left(\frac{0.65}{Cm}\right)^{1/3} \cdot (\nabla_c \cdot Lwl)^{1/2}$$

$$\frac{\partial RvhH}{\partial V}$$

: $\frac{\partial RvhH}{\partial V} = (1 + k) \cdot \frac{\partial RfhH}{\partial V}$

: $\frac{\partial RfhH}{\partial V} = \frac{1}{2} \cdot \rho_w \cdot \left(Sc_\phi - S_c\right) \cdot \frac{\partial}{\partial V}(V^2 \cdot C_f)$

: $\frac{\partial}{\partial V}(V^2 \cdot C_f) = C_f \cdot \frac{\partial}{\partial V}(V^2) + V^2 \cdot \frac{\partial C_f}{\partial V} = 2 \cdot V \cdot C_f + V^2 \cdot \frac{\partial C_f}{\partial V}$

: $\frac{\partial C_f}{\partial V} = \frac{\partial}{\partial V}\left(\frac{0.075}{(\log(Rn) - 2)^2}\right) = 0.075 \cdot \frac{\partial}{\partial V}\left(\frac{1}{(\log(Rn) - 2)^2}\right) = \frac{-2 \cdot 0.075}{(\log(Rn) - 2)^3} \cdot \frac{\partial}{\partial V}(\log(Rn))$

: $\frac{\partial}{\partial V}(\log(Rn)) = \frac{\log e}{Rn} \cdot \frac{\partial Rn}{\partial V}$

: $\frac{\partial Rn}{\partial V} = \frac{\partial}{\partial V}\left(\frac{V \cdot 0.7 \cdot Lwl}{\nu_w}\right) = \frac{0.7 \cdot Lwl}{\nu_w}$

: $Sc_\phi = S_c \cdot \left[1 + \frac{1}{100} \cdot \left(s_0 + s_1 \cdot \frac{Bwl}{T_c} + s_2 \cdot \left(\frac{Bwl}{Tc}\right)^2 + s_3 \cdot Cm\right)\right]$

ϕ	s_0	s_1	s_2	s_3
0 – 5	$-0.8224 \cdot \phi^*$	$0.0108 \cdot \phi^*$	$-0.0054 \cdot \phi^*$	$1.2658 \cdot \phi^*$
5 – 10	$-0.082 \cdot \phi^* - 3.702$	$-0.0372 \cdot \phi^* + 0.24$	$-0.01 \cdot \phi^* + 0.023$	$0.4818 \cdot \phi^* + 3.92$
10 – 15	$0.2462 \cdot \phi^* - 6.984$	$-0.0514 \cdot \phi^* - 0.382$	$-0.0082 \cdot \phi^* + 0.005$	$0.0422 \cdot \phi^* + 8.316$
15 – 20	$1.0282 \cdot \phi^* - 18.714$	$-0.1622 \cdot \phi^* + 2.044$	$0.0018 \cdot \phi^* - 0.145$	$-0.717 \cdot \phi^* + 19.704$
20 – 25	$0.932 \cdot \phi^* - 16.79$	$-0.221 \cdot \phi^* + 3.22$	$0.0086 \cdot \phi^* - 0.281$	$-0.3842 \cdot \phi^* + 13.048$
25 – 30	$1.1648 \cdot \phi^* - 22.61$	$-0.3212 \cdot \phi^* + 5.725$	$0.018 \cdot \phi^* - 0.516$	$-0.3352 \cdot \phi^* + 11.823$
30 – 35	$0.4628 \cdot \phi^* - 1.55$	$-0.2542 \cdot \phi^* + 3.715$	$0.0156 \cdot \phi^* - 0.444$	$0.346 \cdot \phi^* - 8.613$

$$\frac{\partial Rvk}{\partial V}$$

: $\dfrac{\partial Rvk}{\partial V} = (1 + k_k) \cdot \dfrac{\partial Rfk}{\partial V}$

: $\dfrac{\partial Rfk}{\partial V} = \dfrac{1}{2} \cdot \rho_w \cdot S_k \cdot \dfrac{\partial}{\partial V}(V^2 \cdot C_f)$

: $\dfrac{\partial}{\partial V}(V^2 \cdot C_f) = C_f \cdot \dfrac{\partial}{\partial V}(V^2) + V^2 \cdot \dfrac{\partial C_f}{\partial V} = 2 \cdot V \cdot C_f + V^2 \cdot \dfrac{\partial C_f}{\partial V}$

: $\dfrac{\partial C_f}{\partial V} = \dfrac{\partial}{\partial V}\left(\dfrac{0.075}{(\log(Rn) - 2)^2}\right) = 0.075 \cdot \dfrac{\partial}{\partial V}\left(\dfrac{1}{(\log(Rn) - 2)^2}\right) = \dfrac{-2 \cdot 0.075}{(\log(Rn) - 2)^3} \cdot \dfrac{\partial}{\partial V}(\log(Rn))$

: $\dfrac{\partial}{\partial V}(\log(Rn)) = \dfrac{\log e}{Rn} \cdot \dfrac{\partial Rn}{\partial V}$

: $\dfrac{\partial Rn}{\partial V} = \dfrac{\partial}{\partial V}\left(\dfrac{CHMEK \cdot V}{\nu_w}\right) = \dfrac{CHMEK}{\nu_w}$

: $(1 + k_k) = \left(1 + 2 \cdot \left(\dfrac{t}{c}\right)_k + 60 \cdot \left(\dfrac{t}{c}\right)_k^4\right)$

$$\frac{\partial Rvr}{\partial V}$$

: $\dfrac{\partial Rvr}{\partial V} = (1 + k_r) \cdot \dfrac{\partial Rfr}{\partial V}$

: $\dfrac{\partial Rfr}{\partial V} = \dfrac{1}{2} \cdot \rho_w \cdot S_r \cdot \dfrac{\partial}{\partial V}(V^2 \cdot C_f)$

: $\dfrac{\partial}{\partial V}(V^2 \cdot C_f) = C_f \cdot \dfrac{\partial}{\partial V}(V^2) + V^2 \cdot \dfrac{\partial C_f}{\partial V} = 2 \cdot V \cdot C_f + V^2 \cdot \dfrac{\partial C_f}{\partial V}$

: $\dfrac{\partial C_f}{\partial V} = \dfrac{\partial}{\partial V}\left(\dfrac{0.075}{(\log(Rn) - 2)^2}\right) = 0.075 \cdot \dfrac{\partial}{\partial V}\left(\dfrac{1}{(\log(Rn) - 2)^2}\right) = \dfrac{-2 \cdot 0.075}{(\log(Rn) - 2)^3} \cdot \dfrac{\partial}{\partial V}(\log(Rn))$

: $\dfrac{\partial}{\partial V}(\log(Rn)) = \dfrac{\log e}{Rn} \cdot \dfrac{\partial Rn}{\partial V}$

: $\dfrac{\partial Rn}{\partial V} = \dfrac{\partial}{\partial V}\left(\dfrac{CHMER \cdot V}{\nu_w}\right) = \dfrac{CHMER}{\nu_w}$

$$: \ (1+k_k)=\left(1+2\cdot\left(\frac{t}{c}\right)_r+60\cdot\left(\frac{t}{c}\right)_r^4\right)$$

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%%%%%% PHYSICAL PARAMETERS %%%%%%%%%
rho_w   1025.9
ni_w    1.18838E-6
rho_a   1.125
g       9.80665
%%%%%% WIND %%%%%%%%%
% the speeds and incidence angles may be provided as a list
% of values or bya a starting value, a step and an end value
V_tw 7.7167        % [m/s] true wind speeds
alfa_tw 30 5 180   % [deg] true wind angle
%%%%% HULL %%%%%%
DIVCAN  1.549      %[m^3] Displaced volume of canoe body
LWL     6.096      % [m]  Design waterline?s length
BWL     1.737      % [m]  Design waterlins?s beam
B       2.591      % [m]  Design maximum beam
AVGFREB 0.853      % [m]  Average freeboard
XFB     3.483      % [m]  Longitudinal center of buoyancy LCB from fpp
XFF     3.483      % [m]  Longitudinal center of flotation LCF from fpp
CPL     0.550      % [-]  Longitudinal prismatic coefficient
HULLFF  0.0        % [-]  Hull form factor
AW      6.503      %[m^2] Waterplane area
CMS     0.710      % [-]  Midship section coefficient
T       1.372      % [m]  Total draft
TCAN    0.305      % [m]  Draft of canoe body
ALT     5.528      %[m^2] Total lateral area of yacht
KG      0.305      % [m]  Center of gravity above moulded base or keel
KM      2.511      % [m]  Transverse metacentre above moulded base or keel
%%%%%% KEEL %%%%%%%
DVK     0.046      %[m^3] Displaced volume of keel
APK     1.007      %[m^2] Keel?s planform area
ASK     0.850      % [-]  Keel?s aspect ratio
SK      2.014      %[m^2] Keel?s wetted surface
ZCBK    0.653      % [m]  Keel?s vertical center of buoyancy (below free surface)
CHMEK   0.925      % [m]  Mean chord length
CHRTK   1.197      % [m]  Root chord length
CHTPK   0.653      % [m]  Tip chord length
KEELFF  1          % [-]  Keel's form factor
DELTTK  0          % [-]  Mean thickness ratio of keel section
TAK     0.545      % [-]  Taper ratio of keel (CHRTK/CHTPK)
%%%%%% RUDDER %%%%%%
DVR     0          %[m^3] Rudder?s displaced volume
APR     0.480      %[m^2] Rudder?s planform area
SR      0.960      %[m^2] Rudder?s wetted surface
CHMER   0.490      % [m]  Mean chord length
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CHRTR   0.544       % [m]  Root chord length
CHTPR   0.435       % [m]  Tip chord length
DELTTR  0           % [m]  Mean thikness ratio of rudder section
RUDDFF  1           % [m]  Rudder?s form factor
%%%%%%% SAILS %%%%%%%%
%sailset - sails used in THIS calculation
% 3 - main & jib; 5 - main & spi; 7 - main, jib, & spinnaker;
SAILSET 5
P       8.900       % [m]  Mainsail heigth
E       4.084       % [m]  Mainsail base
MROACH  1.3         % [-]  Correction for mainsail roach [-]
MFLB    1           % [0/1] Full main battens in main
BAD     0.610       % [m]  Boom heigth above deck
I       8.626       % [m]  Foretriangle heigth
J       1.890       % [m]  Foretriangle base
LPG     4.115       % [m]  Perpendicular of longest jib
SL      8.077       % [m]  Spinnaker length
EHM     9.754       % [m]  Mast?s heigth above deck
EMDC    0.254       % [m]  Mast?s avarage diameter
F       1           % [-]   flattening factor
%%%%%%% CREW %%%%%%%%%%
MMVBLCRW 228        % [kg] Movable Crew Mass
b       0           % [m]   crew arm
```