From Capital Gains to Wealth Taxation Distributional Consequences

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Abstract

In this paper I study the distributional consequences of shifting from a capital-gains tax system to a wealth tax system. I argue that there are efficiency gains from taxing more an unproductive stock of wealth than a productive flow. I pay special attention to the winners and losers of this policy in terms of the wealth distribution of the economy over the life cycle, the wealth distribution of each agent, and the Life Cycle profiles (the richs and the poors). I find that, in line with previous literature, there are gains from wealth taxation instead of return to investment. People with high returns get disproportionate benefits from this policy, hence efficient goverment redistribution may mitigate the resulting inequality, and make everyone better off.

1 Introduction

Optimal taxation theory has been a rich source for discussion in economics. It dates back to the classical Ramsey problem, that essentially recommends to tax more those goods and services whose elasticity of substitution is low, i.e complementary goods. For instance, other papers like [Mirrlees, 1976] brought many pieces of theory together building an unified corp of knowledge. The question aroused in macroeconomics when economists realized the government levy tax on household but not once for all. In a dynamic framework, the question is no longer how can design incentives in order to distort market as least as possible, now the question shifts to the time dimension: how can the government levy taxes over many years, without distorting the market, and giving the right incentives to household not once, but during many years. On a dynamic framework households evolve, some die, some get rich, even some stay alive infinitely many years. Household earnings are constantly changing, hence each period there are new incentives to pay taxes (i.e substitute labor for leisure if τ_w is high). Moreover, in line with what here concerns us, the welfare that households derive from paying taxes changes. Taxes finance public goods that make families happy. Among the most important public goods financed by taxes are the social insurance provided by the government, like unemployment checks and healthcare assistance (see [De Nardi et al., 2016a] for the importance of medical expenses driving savings in old ages). However, optimal taxation may affect welfare and boost economic growth indirectly, just by shifting the tax burden from productive agents/activities to unproductive ones.

In this paper I look at the distributional consequences from changing the tax policy in the mentioned direction, shifting the burden from productive, to unproductive agents. In spite of the previous discussion on optimal dynamic taxation, and for reasons I explain in the next section, I do not pay attention to efficiency, just compare the facts and welfare of two tax schemes. Further discussion on this will be on following sections, for now disregard it.

I build a simplified version of [Guvenen et al., 2019] that allows us to test the same facts, using a clean version of their economy, with some crucial departures. The best way to show the intuition of the paper is by looking at a simple example and two budget constraints. Consider the government must pick one of the following tax schemes: Taxing the stock of wealth with a low rate each year (τ_a) or taxing the return of invested wealth at high rate (τ_r). The constraint when r is taxed is

$$c_t + \frac{a_{t+1}}{(1 + r(1 - \tau_k))} \leq y_t + a_t$$

and when stock of capital is taxed,

$$c_t + \frac{a_{t+1}}{(1+r)} \leq y_t + a_t(1-\tau_a)$$

There are two identical agents except for the fact that one of them is a more successful investor, hence she has more chances of getting a high return (r). This difference can be explained by entrepreneurial ability like in the original paper (further reference is [Quadrini, 2000]), whatsoever we take it as exogenous. In this setting, the first tax system charges the burden on the productive guy, because she enjoys higher returns. In fact, the productive person is the one who contributes more to the economy, saving more and boosting economic growth. On the other hand, the second tax system shifts the burden to the unproductive person. Since the stock of wealth is taxed now, the productive one puts more money into investment accounts, while the unproductive does not, for that reason the government raises more revenues (relatively) from the unproductive than from the productive.
I use a partial equilibrium model, agents behave optimally given their information of the world at each point in time, hence more savings from the productive do not boost economic growth (no firms using capital). However, this do happens if we generalize to general equilibrium, like authors do in the original paper. In fact, savings boost wages, they do not act just as precautionary, like in our framework.
All in all, just to conclude the introduction, I will show how shifting the tax burden from the productive agent, to the unproductive, or which is the same, taxing the stock of wealth instead of the flow of new generated wealth, boosts the accumulation of wealth. The productive agent will gain, and the unproductive will lose. Using lump-Transfers form the government we can make everyone better off.

2 Formulation of the Model

2.1 Some Considerations

Before diving into the model, I want to make clear three facts with respect to the description: **1)** Taxes are left out from the general definition. **2)** Distinctions between agents are out of the definition too. **3)** Unless it is strictly necessary, I write the model without the standard notation for stochastic models, for instance I use c to denote consumption, instead of $c_t(r_t^t)$ where r is the stochastic variable.

2.2 The Model

Discrete-Time Life Cycle Economy in which agents live $t = 0, ..., T$ years. Each person has its own saving-consumption decision, which is affected by the states of nature. **Markets are Incomplete**: there is one saving vehicle, (a) and two possible states which are the rate of return (r). (r) follows a discrete-time Markov process with transition probabilities $\mathbf{\Gamma}^{(i)}$, where $i = Agents$. Each agent has its own transition matrix, as a matter of fact, this is the main source of heterogeneity. The productive agent is more likely to obtain the high level of return. Agents maximize expected discounted utility

$$\max_{\{c,a'\}_{t=0}^{T}} \mathbb{E}_0 \Big\{ \sum_{t=0}^{T} \beta^t u(c) \Big\}$$

Subject to the flow constraint, borrowing limit, and transversality condition

$$a' = (1+r)(a+y-c) \tag{1}$$
$$a' \geq -\underline{\text{a}} \tag{2}$$
$$a'_{T+1} = 0 \tag{3}$$

Variables are straightforward to define, c is consumption, a present wealth, a' savings and r the stochastic rate of return. Lastly, y is a life cycle profile of earnings that generates roughly a 90% increases from $t = 0$ to the peak, and gives the agent 50% of their last wage as pension from the retirement age until the final period. See Figure 1.

To solve the model I rely on Dynamic Programming techniques seen in class. At each date, agents solve their problem taking expectations of a Markov Process. The recursive formulation that allows to solve the

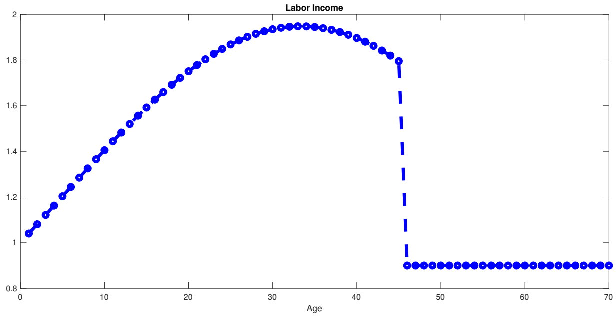

Figure 1: Life Cycle Income Profile

model by backward iterations, in terms of just one choice variable (a) is

$$V(a,r) = \max_{a' \in [\underline{a},(y+a)(1+r)]} \Big\{ u\Big(a + y - \frac{a'}{1+r}\Big) + \beta \mathbb{E}_t \big[V(a',r')\big|r\big] \Big\}$$

The first order conditions of this problem immediately yields a standard Euler Equations that may not be binding each period. Depending on the operativeness of the borrowing limit $\underline{a}$, the Euler Equation holds with equality or not. On the appendix there is a full derivation of the Euler Equation.

2.3 The Case for Taxation

Once the theoretical background for the model has been laid out, I discuss how taxes are implemented. Recall the objective is to study distributional consequences as a result of shifting from capital gains (τ_r) to wealth taxation (τ_a). The underlying assumption is that there is heterogeneity in the rate of return, and also the one who gets high return is the productive person. Actually, it is a genuine assumption, and micro evidence strongly backs it. Here I show how the Euler Equation looks with each taxation scheme. Two aspects, the first one is that I write the equation in terms of the state (a and a'), second, for now I do not impose any functional form to $u()$ just a generic continuous, concave and twice differentiable function. The Euler Equation is $u'(c) = \beta(1+r)\mathbb{E}_t[u'(c')]$. The subscript in t makes explicit that the expectation is conditional on todays's state of the world. r is observed, hence it is not denoted by anything new. There is uncertainty about tomorrow's return (r'). The case for capital gains taxation (τ_r) is:

$$u'\Big(y + a - \frac{a'}{1+r(1-\tau_r)}\Big) = \beta(1+r(1-\tau_r))\mathbb{E}_t\Big[u'\Big(y' + a' - \frac{a''}{1+r'(1-\tau_r)}\Big)\Big]$$

Whereas, wealth taxation (τ_a) requires

$$u'\Big(y + a(1-\tau_a) - \frac{a'}{1+r}\Big) = \beta(1+r)\mathbb{E}_t\Big[u'\Big(y' + a'(1-\tau_a) - \frac{a''}{1+r'}\Big)\Big]$$

Undoubtedly, in absence of transfers (will be included) taxes reduce welfare. To see how tax incidence works, consider the very simple static case in which we just draw a two-period restriction for each scheme:

$$\tau_r \;:\; a' = (1 + r(1-\tau_r))(a + y - c)$$

$$\tau_a \;:\; a' = (1+r)(a(1-\tau_a) + y - c)$$

The partial effect of taxes on savings (wealth tomorrow) is

$$\tau_r \;:\; \frac{\partial a'}{\partial \tau_r} = -r(a + y - c)$$

$$\tau_a \;:\; \frac{\partial a'}{\partial \tau_a} = -a(1+r)$$

In the first case, the tax affects the amount saved, income+assets - consumption. We know high interest rate drives high savings, therefore this tax has much more incidence on the guy with high return than on the guy with low return. Alternatively, the second equation directly relates assets holdings with the incidence of the tax. Of course, wealthy households (which will be the one with high interest rate) will pay more taxes. However, with this system the guy with low interest rate pays more than with the capital gains tax. In both cases the richer pays more. However, the second scheme distributes the tax incidence over the two households, making the unproductive agent bear a larger burden.

3 Calibration and Parametrization

The baseline model, on which I have made the calibration, is the one that charges the return of capital. Many countries levy taxes on capital gains, hence I have decided to take that scenario as the reference. The parametrization of the model has two steps. First, I take as many parameters as possible from the literature, like the curvature of the utility function or the discount factor. The remaining parameters have been chosen in order to match two properties of the wealth distribution:

- Significant proportion of people with 0 wealth.
- Right-Skewed wealth distribution.

A vast amount of literature, theoretical and empirical confirm that wealth is more concentrated and dispersed than income. Our model displays a perfect distribution of income across agent, both in the cross section and in the life cycle, however wealth changes a lot depending on each agent. For a detailed description of those facts for the US see [Kuhn et al., 2016]. The purpose of this section is to match a significant amount of poor people over the Life-Cycle distribution, and certain degree of skewness. For that purpose, I have chosen the transition probabilities of each agent, and the rate of return that they face.
The vector of returns of each agent is

$$\mathbf{r}^{(1)} = \begin{pmatrix} 0.15 & 0.05 \end{pmatrix} \qquad\qquad \mathbf{r}^{(2)} = \begin{pmatrix} 0.1 & 0.01 \end{pmatrix} \tag{4}$$

Whose transition probabilities are

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{pmatrix} \qquad\qquad \mathbf{\Gamma}^{(2)} = \begin{pmatrix} 0.3 & 0.7 \\ 0.2 & 0.8 \end{pmatrix} \tag{5}$$

Those numbers define how agents shift from one state to another. For instance, the productive guy obtains a 15% of return investing, while the unproductive only 10%. Moreover, the productive guy good luck is much more persistent than the good luck for the unproductive one. With an 80% of probability the productive one remains at the good state of the world, while with 50% switch to the good state if her previous state was the bad. For the unproductive agent there is a completely different story. This person has a very persistent bad luck, and very low chances of remain at the good state. For instance, if the unproductive guy is at the good state, with 70% of probability she will shift to the low state, and 30% she remains at the good state. On the other hand, bad luck is as persistent for her as good luck is for the productive guy. If the unproductive find herself at the low state, with 80% probability she will remain there for the next period. This transition probabilities drive the accumulation of assets over the Life Cycle. Since markets are incomplete, agents save to smooth consumption taking into account the uncertainty. Savings serves for a precautionary purpose, the only available insurance against bad states of the world is owned wealth.
Another key instrument influencing the shape of the Life-Cycle wealth distribution is the use of taxes. For the baseline model I consider a lump-sum transfer that is flat across agents (all receive the same) but is contingent on the state of the world. Government transfers follows the scheme $\mathbf{G}^{(i)} = \begin{pmatrix} 1 & 1.1 \end{pmatrix}$ regardless of the revenue. For obvious reasons, people who have bad luck benefits slightly more from transfers.
The choice of the utility function is a standard CES with curvature parameter $\gamma = 2$ $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The following parameters have been chosen from the literature or class:

Table 1: Baseline Calibration

$\beta = 0.98$	$T = 70$
$\gamma = 2$	Retirement Age $= 45$
$\tau_r = 25$	Retirement Benefit $=$ 50% last wage
$\underline{a} = 0$	

4 Baseline Analysis

This section looks at the results of the baseline economy with a 25% tax rate on capital gains. The analysis has three sections: **1)** Aggregate statistics over the life cycle. Concretely, I show the joint (two agents) distribution of assets in the economy, and relevant aggregate profiles of assets by age. **2)** Dis-aggregated statistics: how each agent behaves individually in term of distribution of assets on the life cycle, and assets for profiles by age. **3)** Lastly I draw some moments from the policy function (consumption).

The resulting Life-Cycle wealth distribution is depicted by the left side of Figure 2. The distribution should be interpreted as follows: consider wealth equals to 0, then the (joint) probability of that event happening on the life cycle is around 15%. Or, what is the same, 15% of the time, some of those agents have 0 wealth. Furthermore, we can extrapolate this conclusion to a more general cross sectional setting, and argue that around 15% of the population have zero net wealth. The mapping to this case is not 1-to-1, but there is room for such interpretation if we are interested in generalize the results.

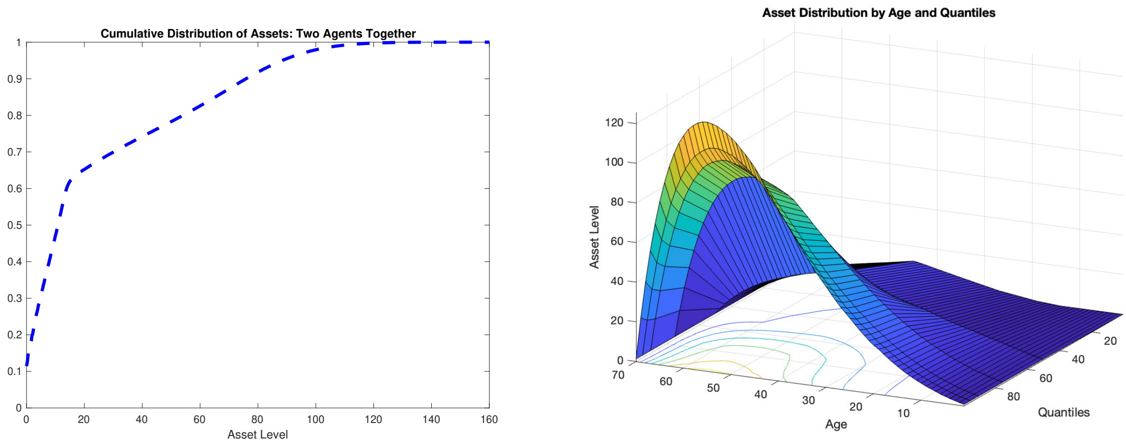

Figure 2: **Left:** Cumulative distribution of assets on the economy, over the Life cycle. Computed jointly for both agents. **Right:** Level of assets by age and quantiles. Countor lines for the level of assets on ground.

The cross sectional distribution in this case is easy to spot: wealth of one agent plus the wealth of the other. It is a 4-dimensional object that does not bring additional insights, therefore it is omitted. The proportion of poor people is close to 13%, being the unproductive agent the one who contributes the most to this proportion. The kink at $a = 20$ reflects there is one agent able to reach higher level of assets, hence from 20 onward only the productive agent contributes to the cdf. The right figure shows level of assets reached by age and quantiles. Regardless of the quantile, the peak is found close to the retirement age. This fact is, of course, in line with the data, that signals correlation of income and wealth (some papers find correlation but not that strong as may be thought, [Kuhn et al., 2016]), we see that the assets distribution over age follows the same pattern as income does. One (another) departure of this model from the data is that I force agents to slowly di-save until they die. This model feature contrasts with facts found by [De Nardi et al., 2009] and surveyed in [De Nardi et al., 2016b], that find people still save at the end of life. Here there is no bequest motives or uncertainty on death. As a result, people accumulate assets for self-insure, but they have certainty about the life span, therefore they drastically di-save at the end of life.

Figure 3 shows the cdf over the life cycle of each agent. In contrast with the previous figure, this one allows to look at the heterogeneity in the economy. The unproductive agent (red) is not able to attain the level of assets that the productive accumulates. This figure makes explicit two matters 1) the unproductive agent has a disproportionate share over the poor people, which is not surprising, and 2) the kink at 20 on the overall distribution is caused by this differences in wealth across agents.

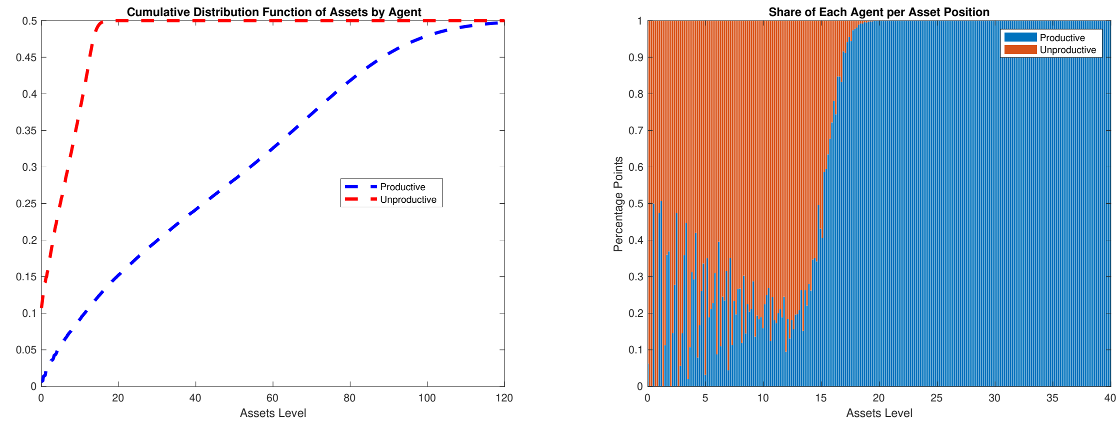

Figure 3: **Left:** Individual cummulative distribution function. **Right:** Agents' share for each asset level.

Right figure depicts the participation of each agent on that asset position. For instance, consider the empirical cumulative distribution functions (plotted in Figure 2) $F_i(a)$, we use the marginal distribution for each i

$$f_i(a) = \int_{\underline{a}}^{\bar{a}} F_i(a)da$$

And compute the participation of each agent for all asset grid points as

$$share_i(a) = \frac{f_i(a)}{\sum_{j \in I} f_j(a)}$$

The productive guy dominates the share until $a = 10$, point in which her participation starts to decreases, until it completely vanishes from 20 onward.

In order to finish the analysis of the baseline tax rate, Figure 4 plots the two first moments of consumption for each agent, over the life cycle. The productive guy (left figure) has non-stationary moments. There is an upward trend in mean consumption and volatility. This trend arises as a result of the very high persistence in good luck. The productive person has very high chances of getting high returns. Furthermore, she accumulates a lot of wealth, hence the average consumption increases every period. In contrast, the unproductive guy has fluctuating moments. In addition, it is worth to take a closer look to the scale of the figures. The unproductive guy barely has an average consumption larger than 1.5 units in any period. On the other hand, the productive guy easily double and triplicate the average consumption of the unproductive guy. The heterogeneity in tax returns drives very large inequalities, not just in assets holding as we have seen, but in consumption too.

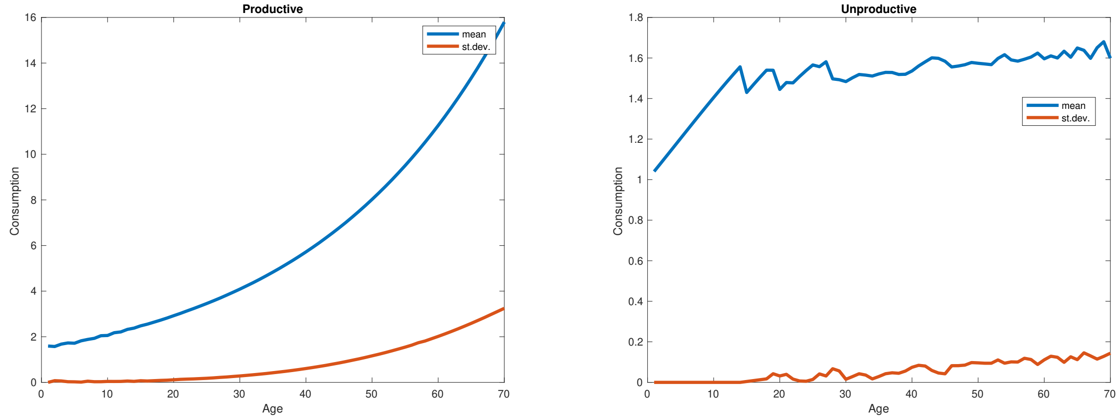

Figure 4: **Left:** mean and standard deviation of consumption productive person. **Right:** "" Unproductive person.

5 Shifting to Wealth Taxation

This section employs the model to quantitatively disentangle consequences of moving from the baseline tax on capital gains, to a tax on the stock of wealth. The idea is to shift taxation burden from the productive agent, to the unproductive. As it is being along the paper, special attention is dedicated to the movements in distributions. The only change falls on the tax scheme, the rest of parameters remain unchanged.
Following the original paper, the stock of assets is taxed at a rate of 1.5%. Figure 5 (right) shows how the

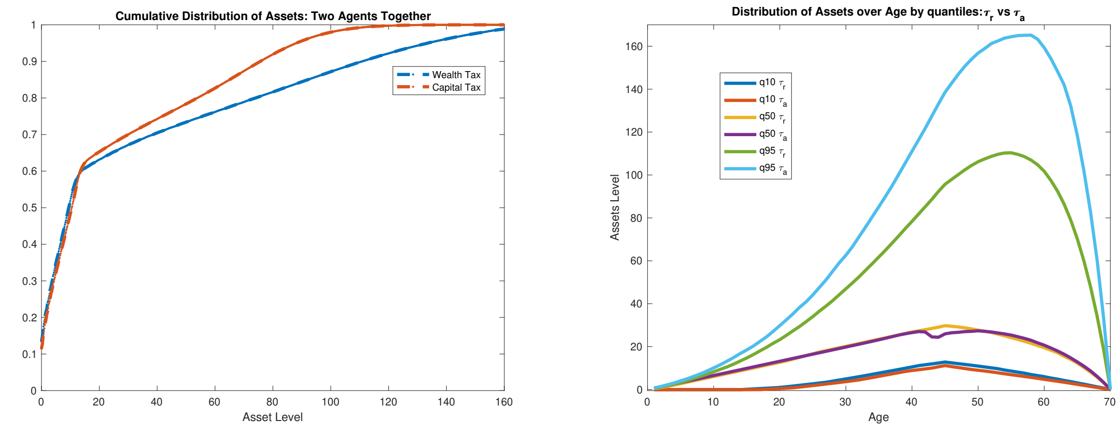

Figure 5: **Left:** Aggregate Cumulative Distribution function for the baseline tax orange) vs the wealth tax (blue). **Right:** Distribution of assets over the life cycle for various quantiles and tax schemes.

new tax menu affects households conditional on their income profile. The lowest quantile in the figure (10th) is better of under the baseline system (τ_r). The median household is practically unaffected by the measure. Depending on the point at which you look some line may be above/below the other but essentially the same. However, those households that belong to the right tail (95q) significantly improve their asset position on a Life-Cycle perspective. The cumulative distribution of assets (right figure) provides information that closely relate with the quantile analysis. Capital gains taxation (τ_r) is the orange line, while the blue is the cdf of the new system (τ_a). The updated cdf shows that from 20 onward, there is a larger probability mass toward the right. Now, under wealth taxation, (productive)people accumulate more assets.
Conclusions drawn by the previous paragraph: poor people is worse off, rich people is better off, are made explicit by Figure 6. The dashed green line (unproductive with τ_a) shifts upward, hence with the new system a larger fraction of the population has zero assets, and the mass of probability to the right (more assets)

is lower. Now the unproductive agent is poorer. She is less productive, and now additionally bears larger taxes. The story for the productive guy is the opposite. Her investments pays off because there is no capital gains taxation, and even though her wealth is taxed, she invest a lot obtaining larger gains. Finally, the productive person is richer. For practically any asset choice, her mass of probability to the right is larger, thus the probability of reaching higher levels of wealth has enlarged.

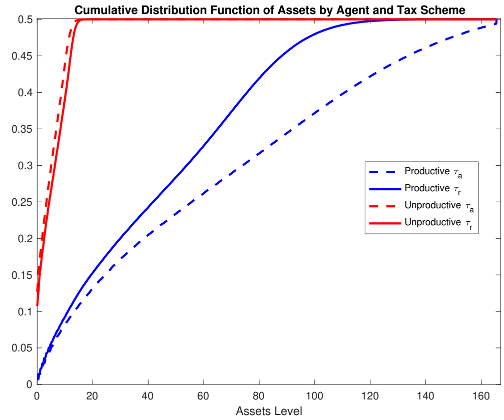

Figure 6: Cumulative Distribution function of assets by agent. Red is for the unproductive, blue for the productive. Solid line for capital gains tax (τ_r), dashed line wealth tax (τ_a).

In order to close the analysis and present the same set of facts for both tax systems I show the first two moments of consumption. The productive consumes much more on average with the new tax system, while the unproductive barely changes. Notice that the average consumption of the unproductive agent increases until the age of 20, that is because in the beginning she is affected by the borrowing limit. Average consumption obviously increases because income increases, however from 20 onward average consumption remains roughly constant and volatile.

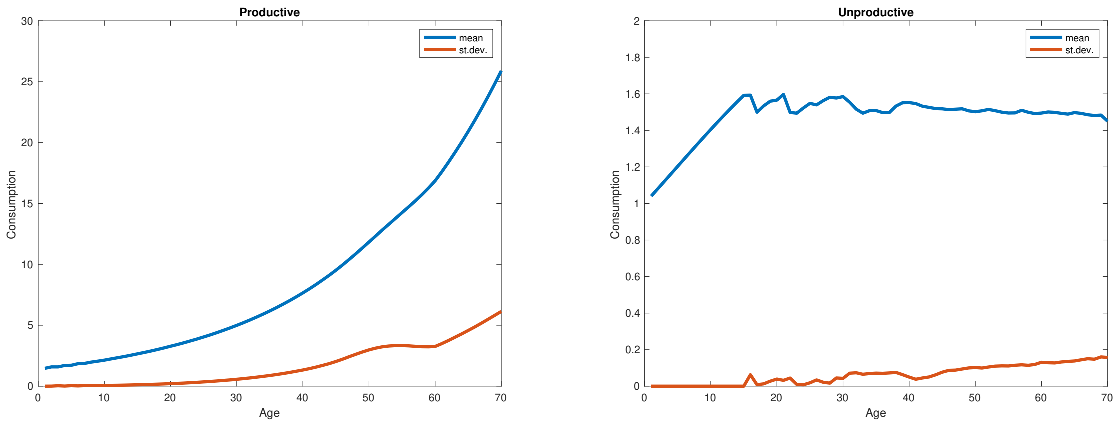

Figure 7: Mean and Standard deviation of Consumption over the Life Cycle. **Left** Productive, **Right** Unproductive.

6 Government's Role

In this section I analyze the role played by the government. This ending section closes the "circle" of the paper. Remember that in the introduction I started the discussion talking about optimal taxation, how the government dynamically imposes duties, and, overall, how households benefit from government redistribution.
The two tax systems that have been tested have completely neglected the government's role. Each household receives a lump-sum transfers that depends on the state of the world, however both receive the same amount. Usually there are two approaches to government modelling

- Time-varying taxes that adjust to keep a balanced budget and,
- Constant taxes with fluctuating public budgets according to the state of the economy.

In our Life-Cycle economy income grows year to year until t = 40, so flat lump-sum transfers produce non-constant government budgets with certainty. However, the balance may be positive or negative depending on the amount transferred. Imposing an optimal tax each year, and adjust the tax rate to raise similar revenues year-to-year than the other system is hard task for this paper. Hence, to study the role of government I compare distributional outcomes of different transfers schedules. The benchmark economy and the tax reform run with a transfer equal to 1 for the good state, and 1.1 for the bad state. To accomplish the redistributive role of the government I pay special attention to **1)** the share of poor people.
Consider that the unproductive guy receives a transfer G which is x-times larger than the one for the productive guy. Moreover, consider the government only helps when the agent is at the bad state. Basically, the unproductive guy now is backed up by the government when she is in bad luck, but doesn't receive anything otherwise. As Figure 8 shows, a small increase in the lump sum, from 1 to 5, the mass of probability

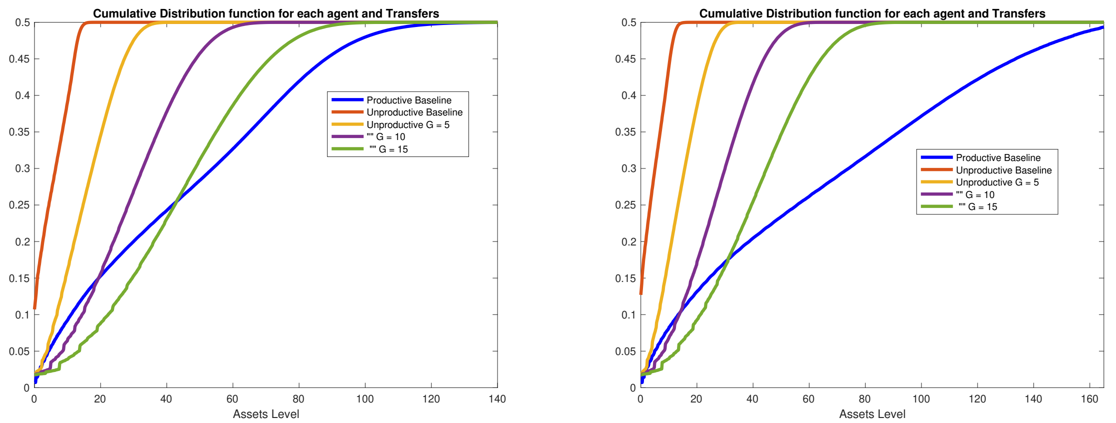

Figure 8: **Left:** Tax on capital income, **Left:** Tax on wealth. Individual Cumulative Distribution function. Blue Line represents High productivity guy *Ceteris Paribus*, the other are for the low productivity with different transfer schemes.

over zero vanishes. That is, with a five times more generous (which is within the budget) transfer it is possible to eradicate the proportion of poor households. As expected, the distribution function of the productive guy is unchanged under both scenarios.

7 Discussion and Conclusions

Up to here I have shown how moving from capital gains taxation to wealth taxation generates large gains for those investor who get high returns. For instance, high productivity agents become wealthier when τ_r is inoperative. The setting I am using has a particularity that posses some difficulty mapping the model to the reality. Here, the productive agent, who may be considered as the rich, gets richer under the new taxing schedule. This may not be true in reality . For example, considered a young entrepreneur with high potential, and an old businessmen who was very productive early, but now he just holds wealth. In that case, the model predicts that the person with high potential and low assets (consider financial constraints) is better than the person who just accumulate wealth. For this model, the scale of numbers matters. Also, it is interesting to consider that heterogeneity in r_t is common however productive people may not be aware of it. For nay case, shifting the burden of taxation from capital gains to assets' stocks induces the formation of new capital. There are very interesting general equilibrium considerations here that are left out of the analysis. For instance, the huge wealth accumulation of the productive person may drive er interest rate down, therefore she can be a productive person at some point in the life cycle, and an unproductive one at other point.

8 GitHub Repository

Here you will find all matlab codes to replicate figures and results.

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9 Appendix

Considering the standard CES utility function,

The general Bellman equation is:

$$V(a,r) = \max_{a' \in [a,\bar{a}]} = \Big\{ \frac{\big(a+y-\frac{a'}{1+r}\big)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}[V(a',r')] \Big\}$$

Whose FOCs are

$$\frac{\partial V(a,r)}{\partial a'} = \frac{-1}{1+r}\Big(a+y-\frac{a'}{1+r}\Big)^{-\gamma} + \beta \mathbb{E}\Big[\frac{\partial V(a',r')}{\partial a'}\Big| r\Big]$$

Writting the value function one period ahead, we obtain the following first derivative for the last term

$$\frac{\partial V(a',r')}{\partial a'} = \Big(a'+y'-\frac{a''}{1+r'}\Big)^{1-\gamma}$$

Note that the expected value of the derivative is the probability of going from one state to another, conditional on the actual state, evolving according to a Markov matrix.

$$\frac{\partial V(a,r)}{\partial a'} = -\frac{\big(a+y-\frac{a'}{1+r}\big)^{1-\gamma}}{1+r} + \beta\Big[\mathbb{P}(r'=\bar{r}|r)\Big(a'+y'-\frac{a''}{1+\bar{r}}\Big)^{1-\gamma} + \mathbb{P}(r'=r|r)\Big(a'+y'-\frac{a''}{1+r}\Big)^{1-\gamma}\Big]$$

The first order condition immediately yields the Euler Equation

$$(a+y-\frac{a'}{1+r})^{1-\gamma} = \beta(1+r)\mathbb{E}\big[(a'+y'-\frac{a''}{1+r'})^{1-\gamma}\big| r\big]$$

10 Computation

$$\mathbf{\Gamma}^{(1)} = \begin{pmatrix} \pi_1^{(1)} & \pi_2^{(1)} \\ \pi_3^{(1)} & \pi_4^{(1)} \end{pmatrix} \qquad \mathbf{\Gamma}^{(2)} = \begin{pmatrix} \pi_1^{(2)} & \pi_2^{(2)} \\ \pi_3^{(2)} & \pi_4^{(2)} \end{pmatrix} \tag{6}$$

$$\mathbf{\Gamma} = \underset{4\times 4}{\begin{bmatrix} \mathbf{\Gamma}^{(1)} & \mathbf{0} \\ \mathbf{0} & \mathbf{\Gamma}^{(2)} \end{bmatrix}}$$