

Preoperative Atelectasis

Part 4: Outcomes

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Setup

Packages used

```
if (!require("pacman", quietly = TRUE)) {  
  install.packages("pacman")  
}
```

```

pacman::p_load(
  tidyverse, # Used for basic data handling and visualization.
  table1, #Used to add labes to variables.
  RColorBrewer, #Color palettes for data visualization.
  gridExtra, #Used to arrange multiple ggplots in a grid.
  grid, #Used to arrange multiple ggplots in a grid.
  mgcv, #Used to model non-linear relationships with a general additive model.
  ggmosaic, #Used to create mosaic plots.
  car, #Used assess distribution of continuous variables (stacked Q-Q plots).
  simpleboot, boot, # Used to calculate mean atelectasis coverage and
    # 95%CI through bootstrapping.
  gt, #Used to present tables in html format.
  report #Used to cite packages used in this session.
)

```

Session and package dependencies

R version 4.3.2 (2023-10-31 ucrt)
 Platform: x86_64-w64-mingw32/x64 (64-bit)
 Running under: Windows 11 x64 (build 22621)

Matrix products: default

locale:
 [1] LC_COLLATE=Spanish_Mexico.utf8 LC_CTYPE=Spanish_Mexico.utf8
 [3] LC_MONETARY=Spanish_Mexico.utf8 LC_NUMERIC=C
 [5] LC_TIME=Spanish_Mexico.utf8

time zone: Europe/Berlin
 tzcode source: internal

attached base packages:
 [1] grid stats graphics grDevices datasets utils methods
 [8] base

other attached packages:
 [1] report_0.5.7 gt_0.10.0 boot_1.3-28.1 simpleboot_1.1-7
 [5] car_3.1-2 carData_3.0-5 ggmosaic_0.3.3 mgcv_1.9-0
 [9] nlme_3.1-164 gridExtra_2.3 RColorBrewer_1.1-3 table1_1.4.3
 [13] lubridate_1.9.3 forcats_1.0.0 stringr_1.5.1 dplyr_1.1.4

```
[17] purrr_1.0.2      readr_2.1.4      tidyr_1.3.0      tibble_3.2.1
[21] ggplot2_3.4.4    tidyverse_2.0.0  pacman_0.5.1
```

Set seed (for reproducibility of bootstrapping) as the current year 2023:

```
seed <- 2023
```

Outcome variable

Corroborate that atelectasis(Yes/No) matches atelectasis percent equal or different to 0%:

	atelectasis_percent								
atelectasis	0	2.5	5	7.5	10	12.5	15	17.5	27.5
Yes	0	11	14	33	6	1	4	7	1
No	159	0	0	0	0	0	0	0	0

Yes, these do match.

Prevalence of atelectasis

	Yes	No
frequencies	77.0	159.0
percent	32.6	67.4

Prevalence of atelectasis with 95% confidence interval

```
1-sample proportions test without continuity correction

data:  frequencies, null probability 0.5
X-squared = 28.492, df = 1, p-value = 9.411e-08
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.2696526 0.3884549
sample estimates:
      p
0.3262712
```

The prevalence of atelectasis was **32.6 (95%CI: 26.97, 38.85)**.

Atelectasis - obesity class

Mean expected frequency:

```
mean_expected_freq
1          39.33333
```

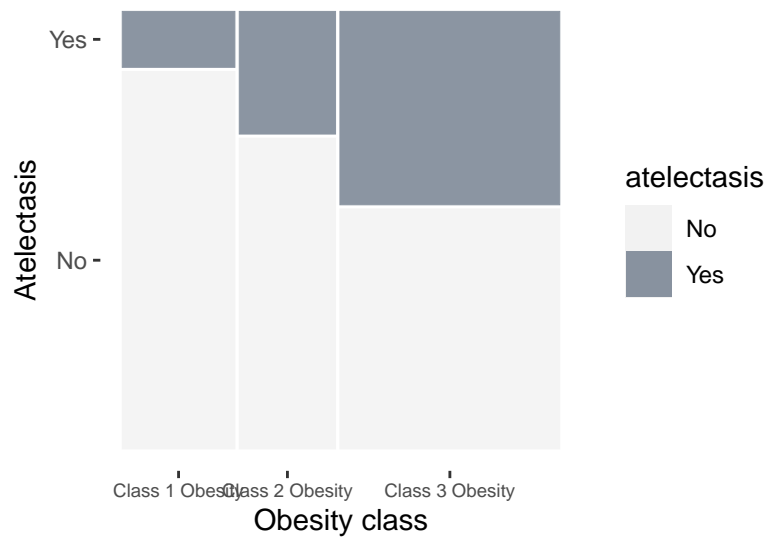
Frequencies:

```
          atelectasis
type_obesity  Yes No
Class 1 Obesity   8 54
Class 2 Obesity  15 38
Class 3 Obesity  54 67
```

Percentage:

```
          atelectasis
type_obesity  Yes   No
Class 1 Obesity 12.90 87.10
Class 2 Obesity 28.30 71.70
Class 3 Obesity 44.63 55.37
```

Mosaic Plot



Pearson's Chi-squared test

data: frequencies

X-squared = 19.352, df = 2, p-value = 6.279e-05

Atelectasis location by obesity class

Mean expected frequency:

```
mean_expected_freq
1          12.83333
```

Mean expected frequency is greater than 5.0, so chi-squared without continuity correction is adequate.

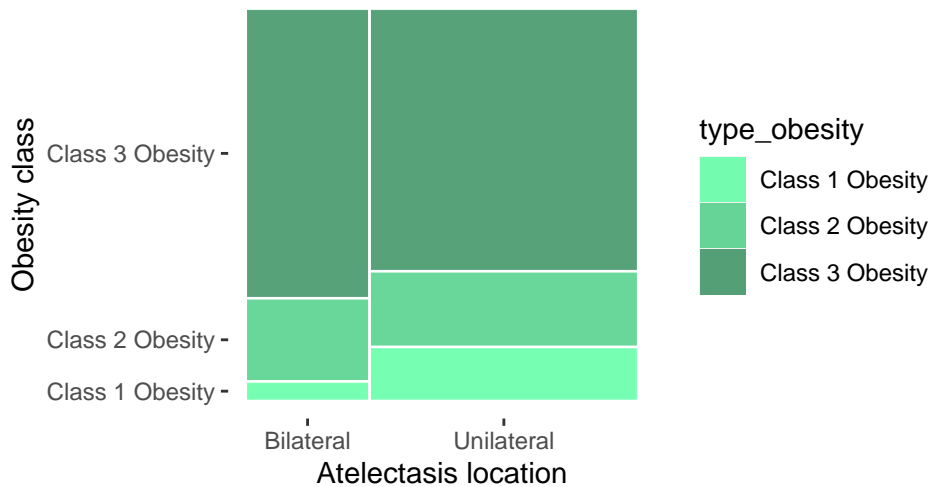
Frequencies:

type_obesity	atelectasis_location	
	Bilateral	Unilateral
Class 1 Obesity	1	7
Class 2 Obesity	5	10
Class 3 Obesity	18	36

Percentage:

type_obesity	atelectasis_location	
	Bilateral	Unilateral
Class 1 Obesity	12.50	87.50
Class 2 Obesity	33.33	66.67
Class 3 Obesity	33.33	66.67

Mosaic Plot



Pearson's Chi-squared test

```
data: frequencies
X-squared = 1.4503, df = 2, p-value = 0.4843
```

Prevalence of atelectasis with 95% confidence intervals calculated with sourced script ***Prevalence_atelectasis.R***

The prevalence of atelectasis was greater in higher obesity classes: class 1, n=8 (12.9%, 95%CI:6.13 - 24.4); class 2, n=15 (28.3%, 95%CI:17.2 - 42.56); and class 3, n=54 (44.63%, 95%CI:35.68 - 53.92) (p<0.001).

Of those who had atelectasis, the most frequent presentation was unilateral n=53 (68.83%), compared to bilateral n=24 (31.17%). When examining this by obesity class, the observed distribution was not significantly different for those with class 1, 2, and 3 obesity categories (n=7 (87.5%), n=10 (66.67%), and n=36 (66.67%), respectively) (p=0.484).

Atelectasis Percent

Mean atelectasis percentage

The following would be the mean atelectasis percentage coverage if a normal distribution were assumed, which is what has been done in some prior studies:

```

      mean      sd
1 2.658898 4.687145

```

And by obesity class:

```

# A tibble: 3 x 3
  type_obesity    mean    sd
  <fct>         <dbl> <dbl>
1 Class 1 Obesity 0.927  2.91
2 Class 2 Obesity 1.56   3.15
3 Class 3 Obesity 4.03   5.52

```

As is evident from these numbers, assuming normality causes standard deviation to capture negative values, which is impossible in reality for this variable.

Thus, bootstrapping the mean and 95%CI is expected to lead to more appropriate estimates.

Mean by bootstrapping for the total sample:

```
[1] 2.664441
```

Bootstrap 95% confidence intervals:

```

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
Based on 10000 bootstrap replicates

```

```

CALL :
boot.ci(boot.out = boot_atel)

```

```

Intervals :
Level      Normal              Basic
95%    ( 2.058,  3.248 )    ( 2.044,  3.220 )

```

```

Level      Percentile          BCa
95%    ( 2.097,  3.273 )    ( 2.076,  3.263 )
Calculations and Intervals on Original Scale

```

The bias-corrected and accelerated (BCa) bootstrap interval is known to lead to more stable intervals with better coverage. Will report this. However, it is a good thing that here 95%CI through different methods do not lead to widely different results.

Now, I will calculate this for different BMI categories:

Class 1

Mean:

[1] 0.9268065

95% CI:

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS

Based on 10000 bootstrap replicates

CALL :

boot.ci(boot.out = boot_class1)

Intervals :

Level	Normal	Basic
95%	(0.1968, 1.6593)	(0.1210, 1.5726)

Level	Percentile	BCa
95%	(0.2823, 1.7339)	(0.3226, 1.8145)

Calculations and Intervals on Original Scale

Class 2

Mean:

[1] 1.552382

95% CI:

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS

Based on 10000 bootstrap replicates

CALL :

boot.ci(boot.out = boot_class2)

Intervals :

Level	Normal	Basic
95%	(0.723, 2.399)	(0.660, 2.311)

Level	Percentile	BCa
95%	(0.802, 2.453)	(0.755, 2.453)

Calculations and Intervals on Original Scale

Class 3

Mean:

```
[1] 4.036455
```

95% CI:

BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS

Based on 10000 bootstrap replicates

CALL :

```
boot.ci(boot.out = boot_class3)
```

Intervals :

Level	Normal	Basic
95%	(3.043, 5.000)	(2.996, 4.979)

Level	Percentile	BCa
95%	(3.079, 5.062)	(3.058, 5.021)

Calculations and Intervals on Original Scale

The mean atelectasis percentage coverage in the sample was 2.66% (95%CI:2.08-3.26) and according to obesity categories: class 1 (0.93%, 95%CI:0.32-1.81), class 2 (1.55%, 95%CI:0.75-2.45), and class 3 (4.04%, 95%CI:3.06-5.02).

What could happen if I took random subsamples between n=20 and n=30 similar to what has been done in other studies and calculate mean BMI and mean atelectasis percentage assuming normal distributions?

Random sample

	mean_BMI	mean_atelectasis
1	42.024	2.375

	mean_BMI	mean_atelectasis
1	40.9836	2.1

	mean_BMI	mean_atelectasis
1	40.712	1.75

Atelectasis percentage by obesity class

Now, I will continue assessing atelectasis percentage if assumed to be categorical ordinal:

Mean expected frequency:

```
mean_expected_freq
1      8.740741
```

Mean expected frequency is greater than 5.0, so chi-squared without continuity correction is adequate.

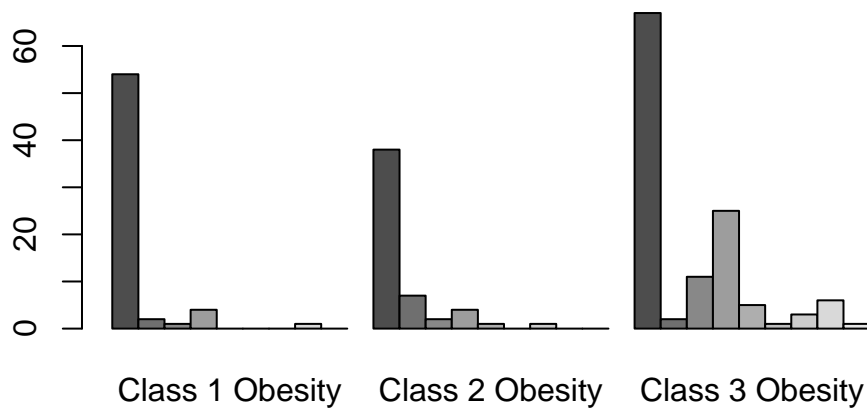
Frequencies:

	type_obesity		
atelectasis_percent	Class 1 Obesity	Class 2 Obesity	Class 3 Obesity
0	54	38	67
2.5	2	7	2
5	1	2	11
7.5	4	4	25
10	0	1	5
12.5	0	0	1
15	0	1	3
17.5	1	0	6
27.5	0	0	1

Percentage by obesity class

	type_obesity		
atelectasis_percent	Class 1 Obesity	Class 2 Obesity	Class 3 Obesity
0	87.10	71.70	55.37
2.5	3.23	13.21	1.65
5	1.61	3.77	9.09
7.5	6.45	7.55	20.66
10	0.00	1.89	4.13
12.5	0.00	0.00	0.83
15	0.00	1.89	2.48
17.5	1.61	0.00	4.96
27.5	0.00	0.00	0.83

Barplot of absolute frequencies:

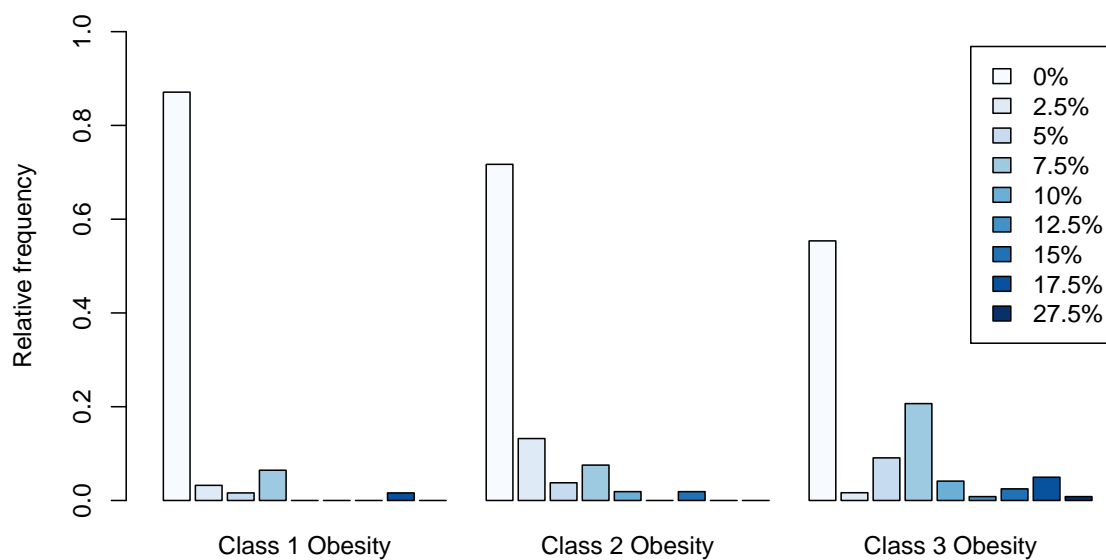


Pearson's Chi-squared test

data: frequencies

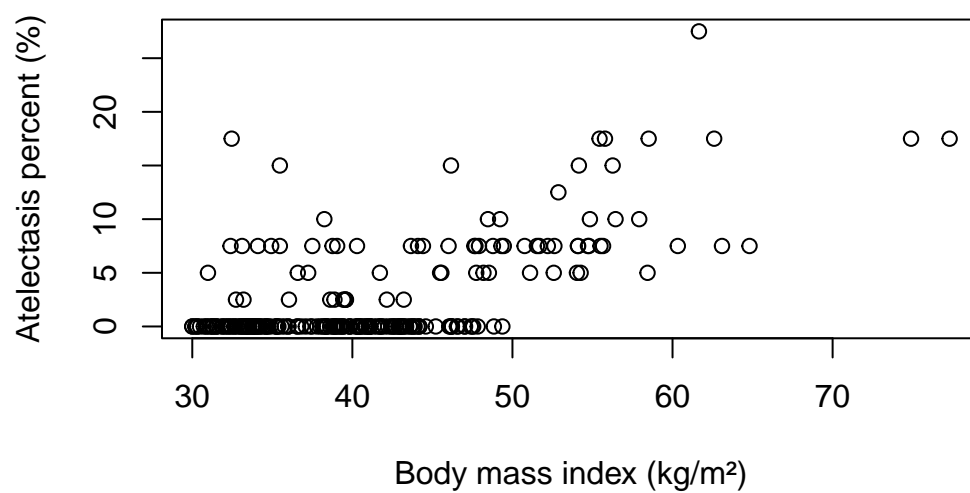
X-squared = 39.434, df = 16, p-value = 0.0009412

Barplot of atelectasis percentage by obesity class category



Smooth term?

Scatterplot



Atelectasis percent seems to increase as BMI increases. However, relationship is not linear.

Models evaluated with the accompanying sourced script ***nonlinear_BMI_Atelectasis.R***

All models are significantly better than linear. Thus, using a smooth term for BMI to predict atelectasis percent is better than modelling a linear relationship.

Best AIC:

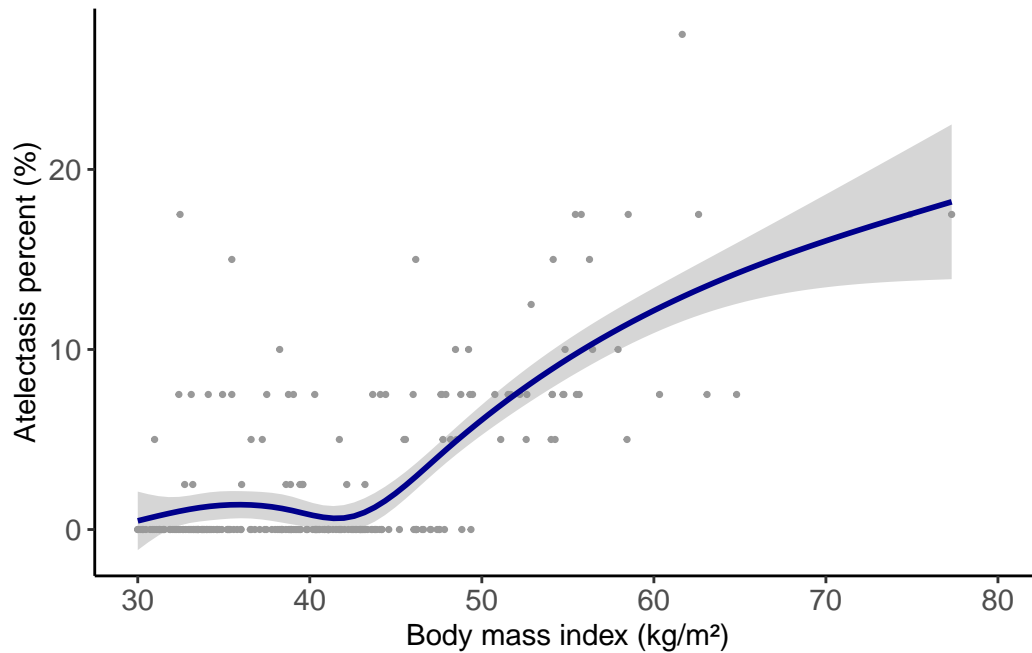
model	AIC
k=2	1259.7
k=4	1237.6
k=6	1232.3
k=8	1233.6

Regarding AIC, greatest improvement in AIC is k=6. Will model with k=5 and k=7 to compare

Best AIC:

model	AIC
k=5	1235.7
k=6	1232.3
k=7	1233.2

k=6 offers the lowest AIC. Will keep k=6 to model.



Positive non-monotonic relationship since atelectasis increases as BMI increases only after ~BMI equal to 42.

Will assess Spearman's correlation again only to have a rough idea (will not report this in the paper since the relationship is not monotonic):

Spearman's rank correlation rho

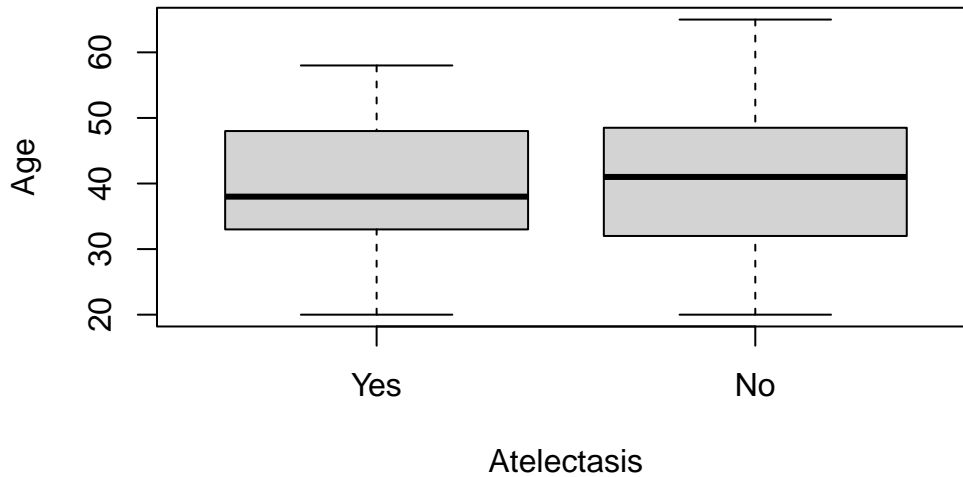
```
data:  spo2_VP0 and atelectasis_percent
S = 3883525, p-value < 2.2e-16
alternative hypothesis: true rho is not equal to 0
sample estimates:
      rho
-0.7727568
```

Atelectasis percent exhibited a negative non-linear non-monotonic relationship with SpO2 (**Figure 1A**, $\rho = -0.773$, $p < 0.001$).

Note that this p-value refers to the smooth term vs linear as assessed in GAM models.

Interestingly, this figure is almost a mirror image of the priorly created plot for SpO2 ~ BMI.

Atelectasis - age



Assess distribution of age by atelectasis (yes/no):

Distribution near-normal, will assess mean and variance for further testing:

Atelectasis	n	age_mean	sd	variance
Yes	77	39.65	9.30	86.5728
No	159	40.55	10.14	102.8816

Variances near-similar, but group sizes differ. Welch's t-test more suitable:

Welch Two Sample t-test

```
data: age by atelectasis
```

```
t = -0.67931, df = 162.72, p-value = 0.4979
```

```
alternative hypothesis: true difference in means between group Yes and group No is not equal
```

```
95 percent confidence interval:
```

```
-3.532230  1.724013
```

```
sample estimates:
```

```
mean in group Yes  mean in group No
      39.64935      40.55346
```

Age was similarly distributed among patients without atelectasis (40.5, sd:10.1) and those with atelectasis (39.6, sd:9.3) ($p=0.498$).

Atelectasis - sex

Mean expected frequency:

```

      mean_expected_freq
1                59

```

Mean expected frequency is greater than 5.0, so chi-squared without continuity correction is adequate.

Frequencies:

```

      atelectasis
sex    Yes  No
  Man    10  12
  Woman   67 147

```

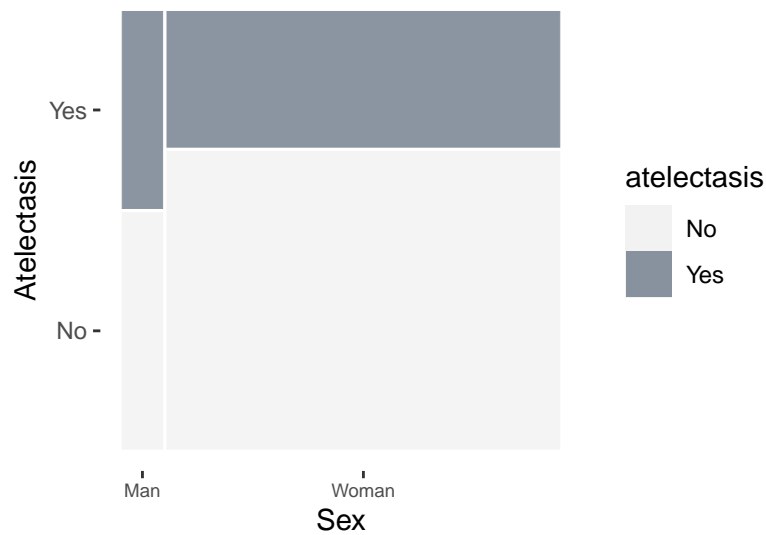
Percentage:

```

      atelectasis
sex    Yes  No
  Man  45.45 54.55
  Woman 31.31 68.69

```

Mosaic Plot



Pearson's Chi-squared test

```
data: frequencies
X-squared = 1.8161, df = 1, p-value = 0.1778
```

There were no significant differences in atelectasis occurrence between men (31.3%) and women (45.5%) ($p=0.178$).

Atelectasis - OSA

Mean expected frequency:

```
mean_expected_freq
1                59
```

Mean expected frequency is greater than 5.0, so chi-squared without continuity correction is adequate.

Frequencies:

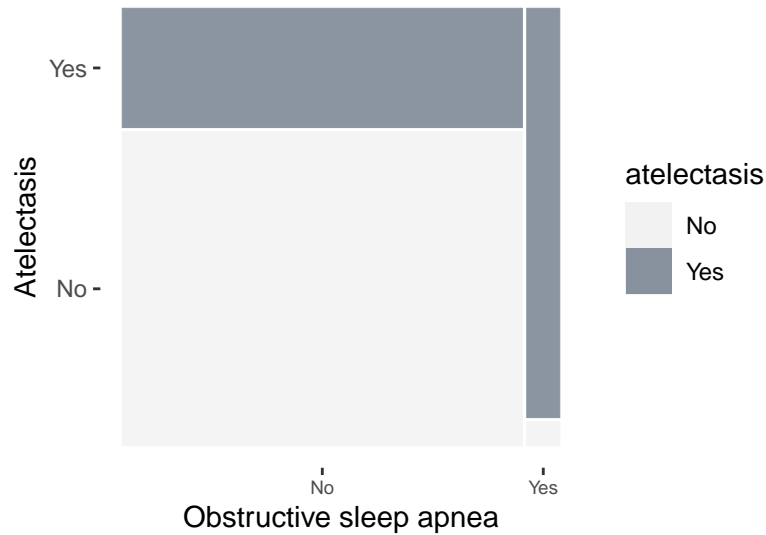
```

      atelectasis
sleep_apnea Yes  No
No         60 158
Yes        17   1
```

Percentage:

		atelectasis	
sleep_apnea		Yes	No
No	27.52	72.48	
Yes	94.44	5.56	

Mosaic Plot



Pearson's Chi-squared test

data: frequencies
X-squared = 33.875, df = 1, p-value = 5.876e-09

Patients with a diagnosis of obstructive sleep apnea had atelectasis more frequently (94.4%) than those without the diagnosis (27.5%) ($p < 0.001$).

Atelectasis location by OSA

Mean expected frequency:

	mean_expected_freq
1	19.25

Mean expected frequency is greater than 5.0, so chi-squared without continuity correction is adequate.

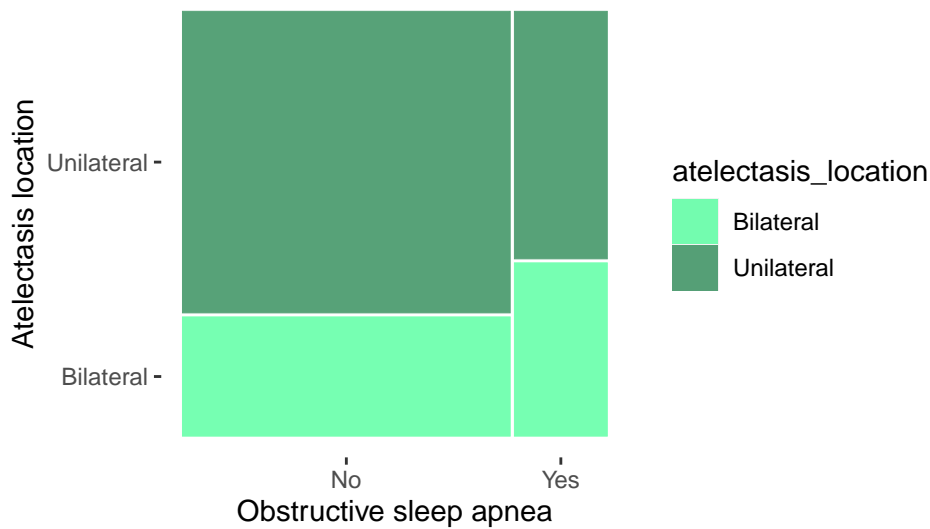
Frequencies:

atelectasis_location		
sleep_apnea	Bilateral	Unilateral
No	17	43
Yes	7	10

Percentage:

atelectasis_location		
sleep_apnea	Bilateral	Unilateral
No	28.33	71.67
Yes	41.18	58.82

Mosaic Plot



Pearson's Chi-squared test

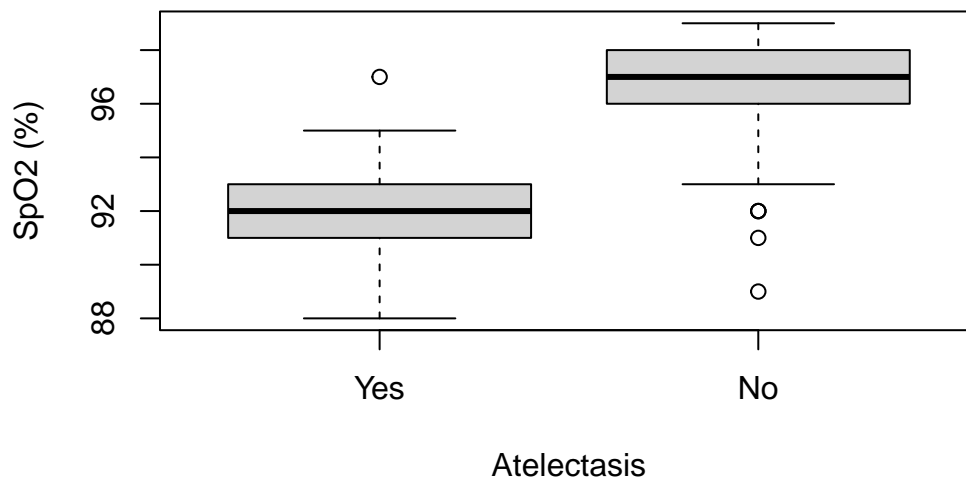
data: frequencies

X-squared = 1.0185, df = 1, p-value = 0.3129

The location of atelectasis was not different among patients with and without OSA (p=0.313).

Atelectasis - SpO2

Atelectasis	n	spo2_median	Q1	Q3	min	max
Yes	77	92	91	93	88	97
No	159	97	96	98	89	99



Distribution not normal and influential outliers. Will assess non-parametrically.

Wilcoxon rank sum test with continuity correction

data: spo2_VP0 by atelectasis

W = 465.5, p-value < 2.2e-16

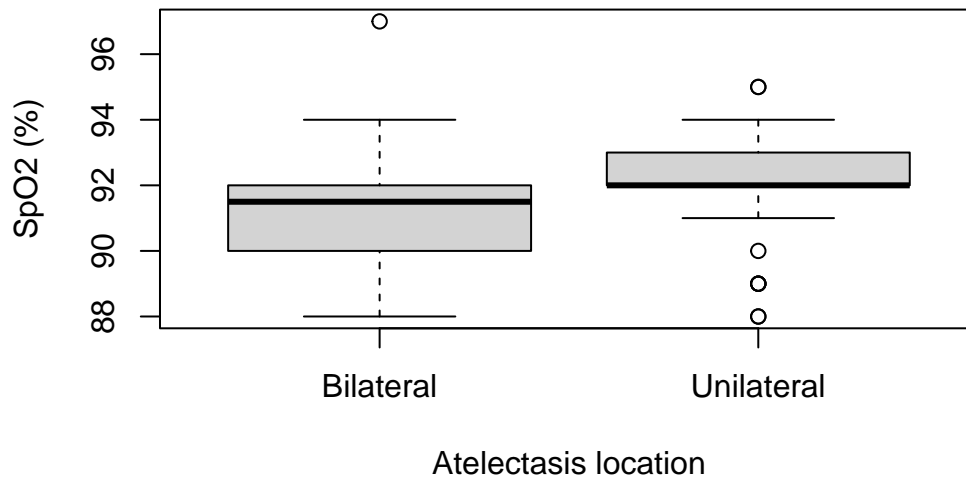
alternative hypothesis: true location shift is not equal to 0

The median SpO2 was significantly lower in patients with atelectasis (92, IQR: 91-93) compared to those without (97, IQR: 96-98) ($p < 0.001$).

Atelectasis location - SpO2

Atelectasis location	n	spo2_median	Q1	Q3	min	max
----------------------	---	-------------	----	----	-----	-----

Bilateral	24	91.5	90	92	88	97
Unilateral	53	92.0	92	93	88	95



Distribution not normal and likely influential outliers. Will assess non-parametrically.

Wilcoxon rank sum test with continuity correction

data: spo2_VP0 by atelectasis_location

W = 393, p-value = 0.006227

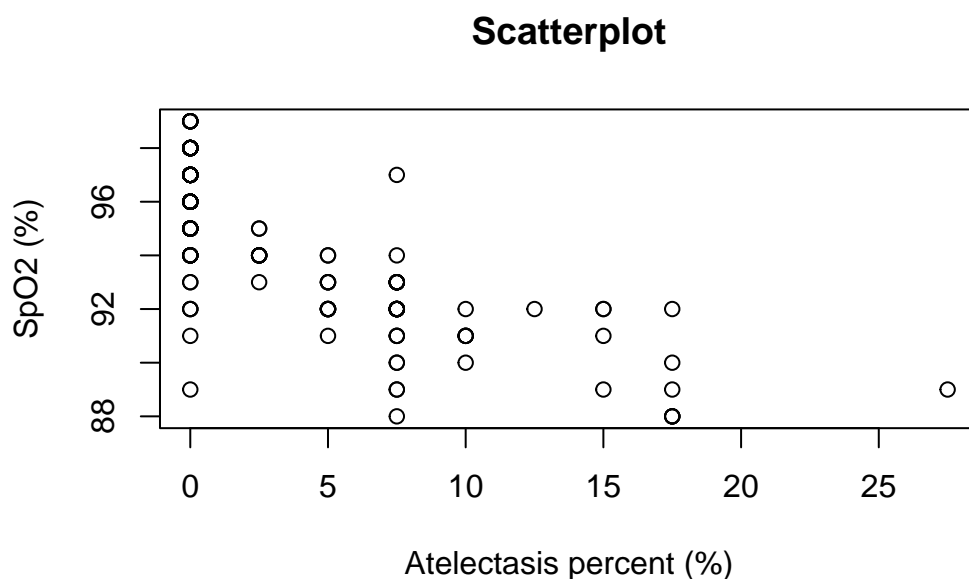
alternative hypothesis: true location shift is not equal to 0

The median SpO2 was significantly lower in patients with bilateral atelectasis (92, IQR: 92-93) compared to those with unilateral atelectasis (91.5, IQR: 90-92) (p=0.006).

Atelectasis percent - SpO2

Smooth term?

Scatterplot



Decreasing SpO2 as atelectasis percent increases.

Would a smooth term be more useful to model SpO2?

Models evaluated with the accompanying sourced script ***nonlinear_Atelectasis_SpO2.R***

All models are significantly better than linear. Thus, using a smooth term for atelectasis percent is better than modelling a linear relationship.

Best AIC:

model	AIC
k=2	892.1
k=4	888.6
k=6	885.8
k=8	885.8

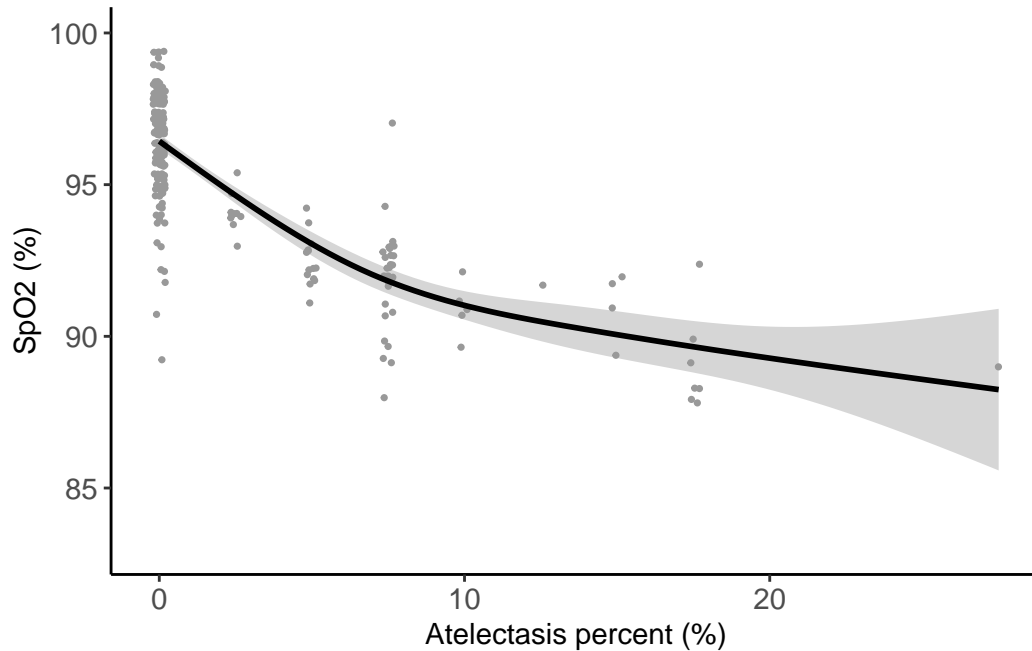
Regarding AIC, no model offers greater improvement in AIC than k=6. Will try a model with k=5.

Best AIC:

model	AIC
k=4	888.6

k=5	885.1
k=6	885.8

There is a drop in AIC for k=5, which also offers the best k-index. Nonetheless, one problem with this is that the extra knot is explaining a clump around 12.%, for which there was only one single observation. Thus, it is likely that this clump and additional knot is only explaining noise in the data, and would thus not be a good representation of the trend in the variable. Thus, will keep k=4 to model as this model offers the best visual representation of the trend in all categories.



Negative monotonic relationship since SpO2 decreases as BMI increases. Will assess Spearman's correlation coefficient to report in paper:

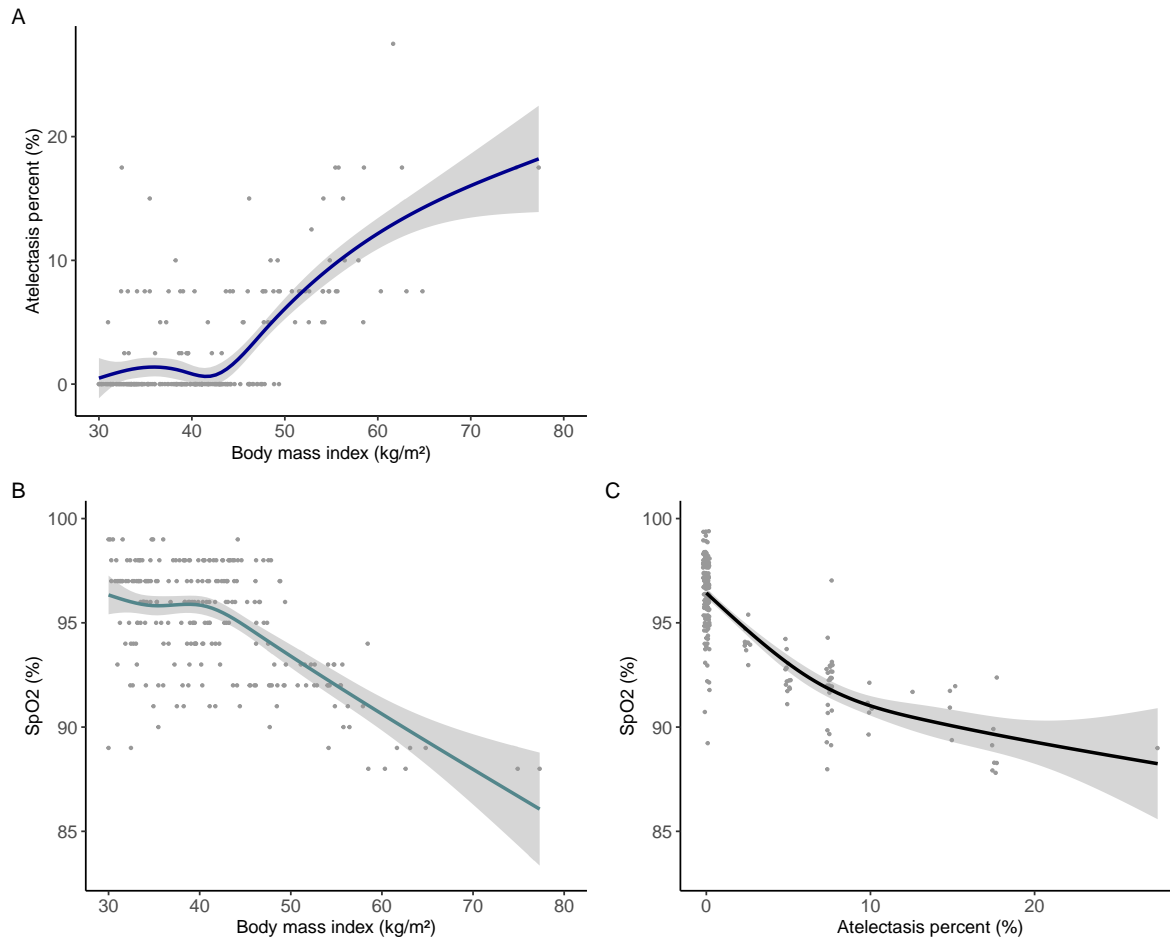
Spearman's rank correlation rho

```
data: spo2_VP0 and atelectasis_percent
S = 3883525, p-value < 2.2e-16
alternative hypothesis: true rho is not equal to 0
sample estimates:
rho
-0.7727568
```

Atelectasis percent exhibited a negative non-linear monotonic relationship with SpO2 (**Figure 1C**, rho= -0.773, p<0.001).

Figure 1

Created with the accompanying sourced script *Figure1.R*



Ordinal variable

Since there is only one participant in the 30% category, will collapse with the 20 category for further analyses:

0%	2.5%	5%	7.5%	10%	12.5%	15%	17.5%
159	11	14	33	6	1	4	8

Distribution not normal, group sizes are different and there are outliers in both directions, depending where you are located. Thus, will proceed with non-parametric assessment:

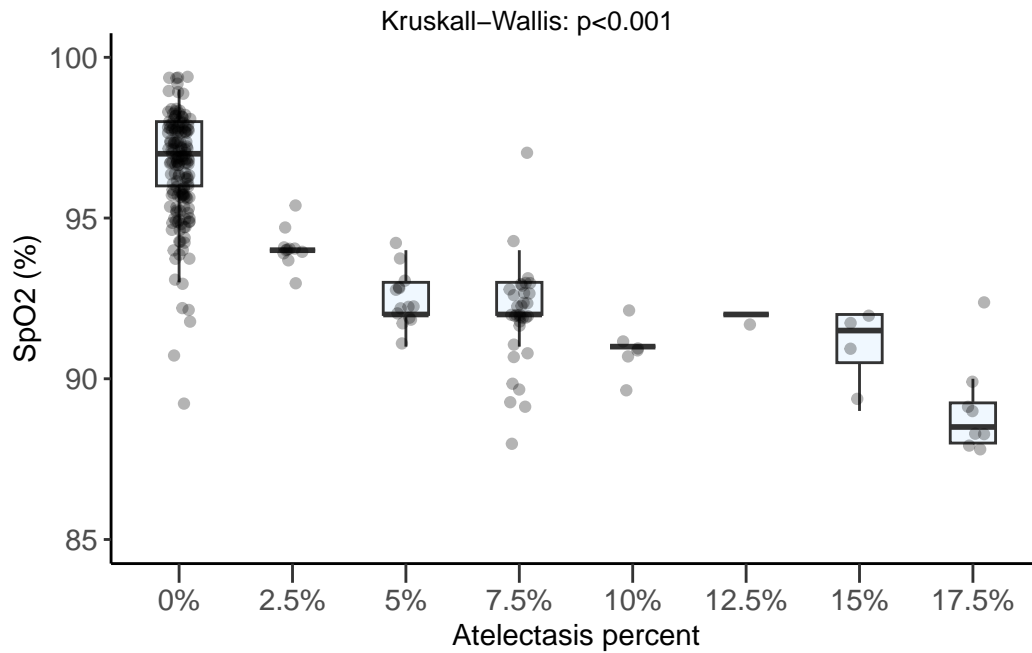
atelectasis_percent	n	spo2_median	Q1	Q3	min	max
0%	159	97.0	96.0	98.00	89	99
2.5%	11	94.0	94.0	94.00	93	95
5%	14	92.0	92.0	93.00	91	94
7.5%	33	92.0	92.0	93.00	88	97
10%	6	91.0	91.0	91.00	90	92
12.5%	1	92.0	92.0	92.00	92	92
15%	4	91.5	90.5	92.00	89	92
17.5%	8	88.5	88.0	89.25	88	92

Kruskal-Wallis rank sum test

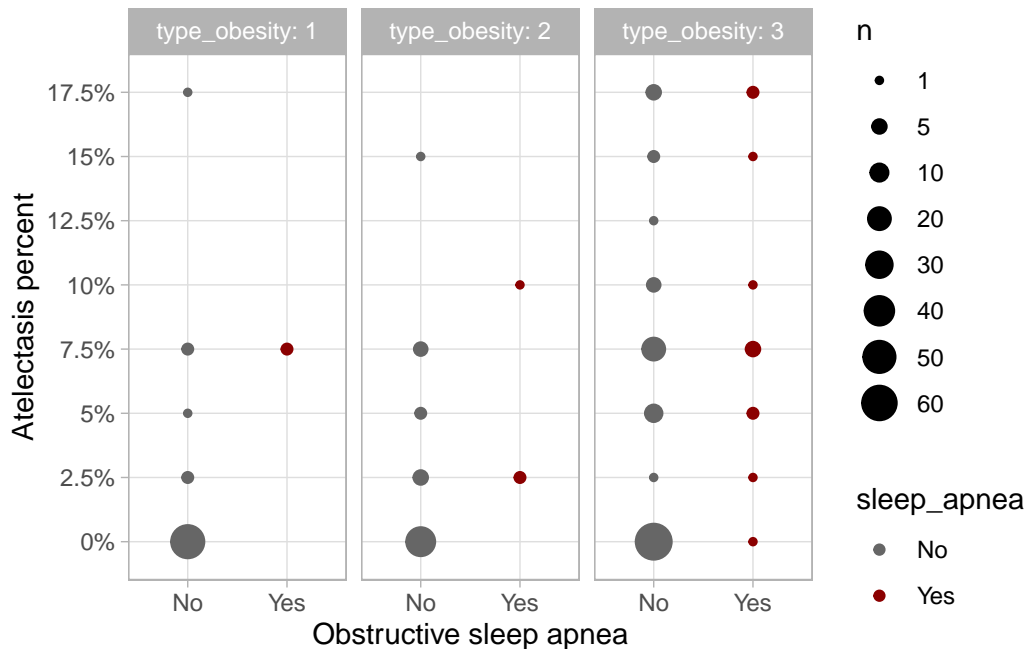
data: spo2_VP0 by atelectasis_percent

Kruskal-Wallis chi-squared = 141.19, df = 7, p-value < 2.2e-16

There was a decreasing trend in median SpO2 with higher atelectasis percentage extension ($p < 0.001$).



Relationship between OSA, obesity type and atelectasis percent:



Sleep apnea was more common with higher BMI categories and also with higher atelectasis percentage. Atelectasis percent increases at higher obesity classes.

Package References

- Auguie B (2017). *gridExtra: Miscellaneous Functions for “Grid” Graphics*. R package version 2.3, <https://CRAN.R-project.org/package=gridExtra>.
- Canty A, Ripley BD (2022). *boot: Bootstrap R (S-Plus) Functions*. R package version 1.3-28.1. Davison AC, Hinkley DV (1997). *Bootstrap Methods and Their Applications*. Cambridge University Press, Cambridge. ISBN 0-521-57391-2, <http://statwww.epfl.ch/davison/BMA/>.
- Fox J, Weisberg S (2019). *An R Companion to Applied Regression*, Third edition. Sage, Thousand Oaks CA. <https://socialsciences.mcmaster.ca/jfox/Books/Companion/>.
- Fox J, Weisberg S, Price B (2022). *carData: Companion to Applied Regression Data Sets*. R package version 3.0-5, <https://CRAN.R-project.org/package=carData>.
- Grolemund G, Wickham H (2011). “Dates and Times Made Easy with lubridate.” *Journal of Statistical Software*, 40(3), 1-25. <https://www.jstatsoft.org/v40/i03/>.
- Iannone R, Cheng J, Schloerke B, Hughes E, Lauer A, Seo J (2023). *gt: Easily Create Presentation-Ready Display Tables*. R package version 0.10.0, <https://CRAN.R-project.org/package=gt>.

- Jeppson H, Hofmann H, Cook D (2021). *ggmosaic: Mosaic Plots in the ‘ggplot2’ Framework*. R package version 0.3.3, <https://CRAN.R-project.org/package=ggmosaic>.
- Makowski D, Lüdtke D, Patil I, Thériault R, Ben-Shachar M, Wiernik B (2023). “Automated Results Reporting as a Practical Tool to Improve Reproducibility and Methodological Best Practices Adoption.” *CRAN*. <https://easystats.github.io/report/>.
- Müller K, Wickham H (2023). *tibble: Simple Data Frames*. R package version 3.2.1, <https://CRAN.R-project.org/package=tibble>.
- Neuwirth E (2022). *RColorBrewer: ColorBrewer Palettes*. R package version 1.1-3, <https://CRAN.R-project.org/package=RColorBrewer>.
- Peng RD (2019). *simpleboot: Simple Bootstrap Routines*. R package version 1.1-7, <https://CRAN.R-project.org/package=simpleboot>.
- Pinheiro J, Bates D, R Core Team (2023). *nlme: Linear and Nonlinear Mixed Effects Models*. R package version 3.1-164, <https://CRAN.R-project.org/package=nlme>. Pinheiro JC, Bates DM (2000). *Mixed-Effects Models in S and S-PLUS*. Springer, New York. doi:10.1007/b98882 <https://doi.org/10.1007/b98882>.
- R Core Team (2023). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. <https://www.R-project.org/>.
- Rich B (2023). *table1: Tables of Descriptive Statistics in HTML*. R package version 1.4.3, <https://CRAN.R-project.org/package=table1>.
- Rinker TW, Kurkiewicz D (2018). *pacman: Package Management for R*. version 0.5.0, <http://github.com/trinker/pacman>.
- Wickham H (2016). *ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York. ISBN 978-3-319-24277-4, <https://ggplot2.tidyverse.org>.
- Wickham H (2023). *forcats: Tools for Working with Categorical Variables (Factors)*. R package version 1.0.0, <https://CRAN.R-project.org/package=forcats>.
- Wickham H (2023). *stringr: Simple, Consistent Wrappers for Common String Operations*. R package version 1.5.1, <https://CRAN.R-project.org/package=stringr>.
- Wickham H, Averick M, Bryan J, Chang W, McGowan LD, François R, Golemund G, Hayes A, Henry L, Hester J, Kuhn M, Pedersen TL, Miller E, Bache SM, Müller K, Ooms J, Robinson D, Seidel DP, Spinu V, Takahashi K, Vaughan D, Wilke C, Woo K, Yutani H (2019). “Welcome to the tidyverse.” *Journal of Open Source Software*, 4(43), 1686. doi:10.21105/joss.01686 <https://doi.org/10.21105/joss.01686>.
- Wickham H, François R, Henry L, Müller K, Vaughan D (2023). *dplyr: A Grammar of Data Manipulation*. R package version 1.1.4, <https://CRAN.R-project.org/package=dplyr>.
- Wickham H, Henry L (2023). *purrr: Functional Programming Tools*. R package version 1.0.2, <https://CRAN.R-project.org/package=purrr>.
- Wickham H, Hester J, Bryan J (2023). *readr: Read Rectangular Text Data*. R package version 2.1.4, <https://CRAN.R-project.org/package=readr>.
- Wickham H, Vaughan D, Girlich M (2023). *tidyr: Tidy Messy Data*. R package version 1.3.0, <https://CRAN.R-project.org/package=tidyr>.
- Wood SN (2011). “Fast stable restricted maximum likelihood and marginal likelihood estimation of semiparametric generalized linear models.” *Journal of the Royal Statistical*

Society (B), 73(1), 3-36. Wood S, N., Pya, S, and Raftery B (2016). "Smoothing parameter and model selection for general smooth models (with discussion)." *Journal of the American Statistical Association*, 111, 1548-1575. Wood SN (2004). "Stable and efficient multiple smoothing parameter estimation for generalized additive models." *Journal of the American Statistical Association*, 99(467), 673-686. Wood S (2017). *Generalized Additive Models: An Introduction with R*, 2 edition. Chapman and Hall/CRC. Wood SN (2003). "Thin-plate regression splines." *Journal of the Royal Statistical Society (B)*, 65(1), 95-114.