A theoretical sea ice drift linear model: Technical report

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1 Introduction

MET Norway provides sea ice drift products derived from data obtained through several spaceborne platforms. The algorithms employed to construct such products are based on the technique known as *motion tracking* which consists in following the displacements of brightness temperature patterns observed from consecutive images of passive microwave satellite sensors [Bischof, 2000]. Similarly to other motion tracking applications in geosciences (e.g. sea surface currents, winds), sea ice motion vectors are computed by cross-correlation of pairs of satellite images [Lavergne et al., 2010]. Due to surface melting and a denser atmosphere, the latter being a direct consequence of its increased water vapour content, sea ice drift vectors cannot be retrieved reliably during summer from the satellite instruments and channels currently in use, which results in noisy data [Lavergne, 2015]. Our contribution consists in an independent estimate of sea ice drift based, not on direct observations or a numerical model, but on a theoretical model. The hope is that MET Norway can make a more reliable summer drift estimate by combining their remote sensing estimate with our model estimate.

The following pages describe the functionality of a series of Python codes specifically designed for solving a theoretical linear model of sea ice drift and some details about the theory on which it is based.

2 A linear model

The theoretical model derives from the two-dimensional equation of motion of sea ice when applied to one simple drift regime known as *free drift*, where internal ice stresses are neglected. The motion of sea ice can be treated as a two-dimensional phenomenon on the sea surface, after integrating the three-dimensional momentum equation through the thickness of the ice.

$$h\rho\left(\frac{\partial\vec{u}}{\partial t}+\vec{u}\cdot\nabla\vec{u}+f\vec{k}\times\vec{u}\right)=\nabla\cdot\vec{\sigma}+\vec{\tau}_a+\vec{\tau}_w-h\rho g\nabla\vec{\eta}-h\nabla p_a \tag{1}$$

where h is the sea ice thickness, ρ is the sea ice density, $\vec{u}$ is the sea ice velocity, $\nabla\cdot\vec{\sigma}$ is the internal sea ice stress, $\vec{\tau}_a$ is the stress of the wind, $\vec{\tau}_w$ is the stress of the water, g is the gravitational acceleration, η is the sea surface elevation and p_a is the atmospheric pressure. (For a throughout derivation see Leppäranta, 2011, pp. 143-150.)
Under certain assumptions it results in an algebraic equation

$$\vec{\tau}_a+\vec{\tau}_w+h\rho f\vec{k}\times(\vec{U}_{wg}-\vec{u})=0 \tag{2}$$

with a linear solution of the form [Thorndike and Colony, 1982]

$$\vec{u}=A\vec{U}_a+\vec{U}_{wg}+\vec{e} \tag{3}$$

A is a complex coefficient

$$A=|A|e^{-i\theta} \tag{4}$$

where $|A|$ is the wind scaling factor, also known as the Nansen number [Leppäranta, 2011], and θ is the deviation angle measured from the wind-driven ice drift direction to the wind direction.

The usefulness of this linear model for sea ice drift resides in its capability to predict time evolving sea ice velocities, $\vec{u}$, from time-evolving surface winds, $\vec{U}_a$. In order to achieve that, sea ice drift and surface wind observations have to be used firstly to solve the linear equation for the model parameters, A and the geostrophic sea currents $\vec{U}_{wg}$, in an inverse sense. The inverse theory [Wunsch, 1996; Menke, 2012] has been applied for that purpose, with the following solution to solve for the model parameters

$$m^{est}=[G^HG]^{-1}G^Hd^{obs} \tag{5}$$

where m^{est} represents a 2×1 matrix that contains the model parameters A and $\vec{U}_{wg}$, G is a matrix containing the wind data, G^H is the Hermitian (complex transposed conjugate) of G and d^{obs} represents the sea ice drift observations.

The inverse theory also allows for a random noise term or *residual*,

$$e=(I-G[G^HG]^{-1}G^H)d^{obs} \tag{6}$$

where I is the identity matrix, that will provide information about the validity of the model by checking where it actually takes on a Gaussian structure and is also free of autocorrelations, both requirements for true random noise.

The theory behind this linear solution is best applicable in summer conditions, when there are free paths between floes and open-water areas do not freeze [Leppäranta, 2011].

3 Solving the inverse problem for the model parameters A and $\vec{U}_{wg}$

The script *params_from_inv.py* contains a main function *compute_params* which requires 5 arguments (starting and ending years to compute, season, wind and ice drift velocity files directories) and solves for the model parameters by calling 4 auxiliary functions, each performing a specific task:

- *load_files*: looks for the correct wind and sea ice drift velocity files in the appropriate directories for a particular season and year range, and returns 2 lists, containing respectively the wind and sea ice drift files, and also their respective sizes.

- *load_data*: extracts the data from each of the file lists created by the previous function and stores them by their x and y components in four 2D matrices (2 per variable).

- *inversion*: inverts for the model parameters A and $\vec{U}_{wg}$ and computes the corresponding RMS of the residuals for every grid point on both sea ice drift and wind time series according to the inverse problem solution (equation 5), and the residuals solution (equation 6), after transforming the variables to complex ones. The regressions are performed for a specific season, whether summer or winter. A minimum number of days with sea ice cover (20 in winter, 10 in summer) is required per every grid point for the regression to be performed, in order to avoid outliers and provide robustness to our results.

- *average*: The results produced by the previous function are then averaged and reshaped to fit a pre-defined polar stereographic grid (as per function *getarea_def*).

The function *write_params* is then invoked, after importing the script *write_params_Athc.py* that contains it and the aforementioned *getarea_def* function. This two functions combine to store the results produced for the model parameters and the residuals in a netCDF file named *inversion_parameters_{X}-{Y}.nc*, where X represents the initial year of the period and Y the final year.

4 Solving the forward problem for sea ice drift

The model parameters estimates, m^{est}, contained in that netCDF file can then be used to solve for the linear model in a forward sense, by combining them with wind velocities data from observations, G, and make predictions of sea ice drift, d^{est}.

$$d^{est} = Gm^{est} \tag{7}$$

This is performed by function *compute_drift* (from script *icedrift_from_winds.py*) which requires two arguments: a date and a directory path that it will lead to a wind velocities file for the period between the specified date and two days later. The wind data will then be combined with the model parameters data from file *inversion_parameters_{X}-{Y}.nc* as per equation 7 to compute the estimated sea ice drift. The values obtained will be stored in another netCDF file named *ice_drift_{a}_polstere-625_{b}_{X}1200-{Y}1200.nc*, where a represents the hemisphere, b the method, X the initial year of the period and Y the final year, after callings function *write_icedrift* and *getarea_def* from script *WriteLRSID.py*

All files and functions mentioned above are available at `https://github.com/javimozo/LinearModelSeaIceDrift`

References

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